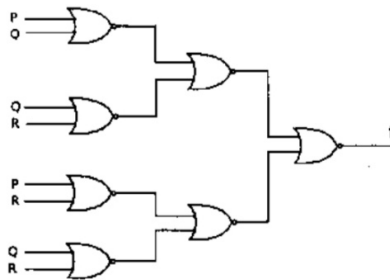


**GATE CS-2010**

**Q.31**

What is the Boolean expression for the output  $f$  of the combinational logic circuit of NOR gates given below?



**Options:**

- (A)  $\overline{Q + R}$
- (B)  $\overline{P + Q}$
- (C)  $\overline{P + R}$
- (D)  $\overline{P + Q + R}$

**Solution:**

Let each NOR gate output be analyzed step-by-step:

$$X_1 = \overline{P + Q}$$

$$X_2 = \overline{Q + R}$$

$$X_3 = \overline{P + R}$$

$$X_4 = \overline{Q + R}$$

$$Y_1 = \overline{X_1 + X_2} = \overline{\overline{P + Q} + \overline{Q + R}}$$

$$Y_2 = \overline{X_3 + X_4} = \overline{\overline{P + R} + \overline{Q + R}}$$

$$f = \overline{Y_1 + Y_2}$$

Using De Morgan's laws and simplification:

$$Y_1 = \overline{\overline{P + Q} + \overline{Q + R}} = (P + Q)(Q + R)$$

$$Y_2 = \overline{\overline{P + R} + \overline{Q + R}} = (P + R)(Q + R)$$

$$f = \overline{(P + Q)(Q + R) + (P + R)(Q + R)}$$

$$f = \overline{(Q + R)[(P + Q) + (P + R)]}$$

$$\text{Since: } (P + Q) + (P + R) = P + Q + R$$

$$\text{So: } f = \overline{(Q + R)(P + Q + R)}$$

$$\text{Since } (Q + R)(P + Q + R) = Q + R \text{ (absorption law):}$$

$$f = \overline{Q + R}$$

**Answer:** Option (A)  $\overline{Q + R}$