analytic psf

November 12, 2021

[3]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import jv
import pandas as pd
```

1 Diffraction limited PSF

$$I(\theta) = \frac{(2J_1(x))^2}{x^2}$$

$$x = \frac{2\pi d_a \sin\theta}{2\lambda}$$

λ: wavelength in [mm]
d_a: aperture diameter θ: angle in radians

[4]:
```
def psf(theta, diameter_mm, lambda_mm):
    x = np.pi * diameter_mm * np.sin(theta)/lambda_mm
    I = (2 * jv(1, x)/x)**2
    return I

def get_fwhm(lambda_mm, dimeters):
    fwhms = np.rad2deg(1.025 * lambda_mm/diameters) * 60.
    return fwhms
```

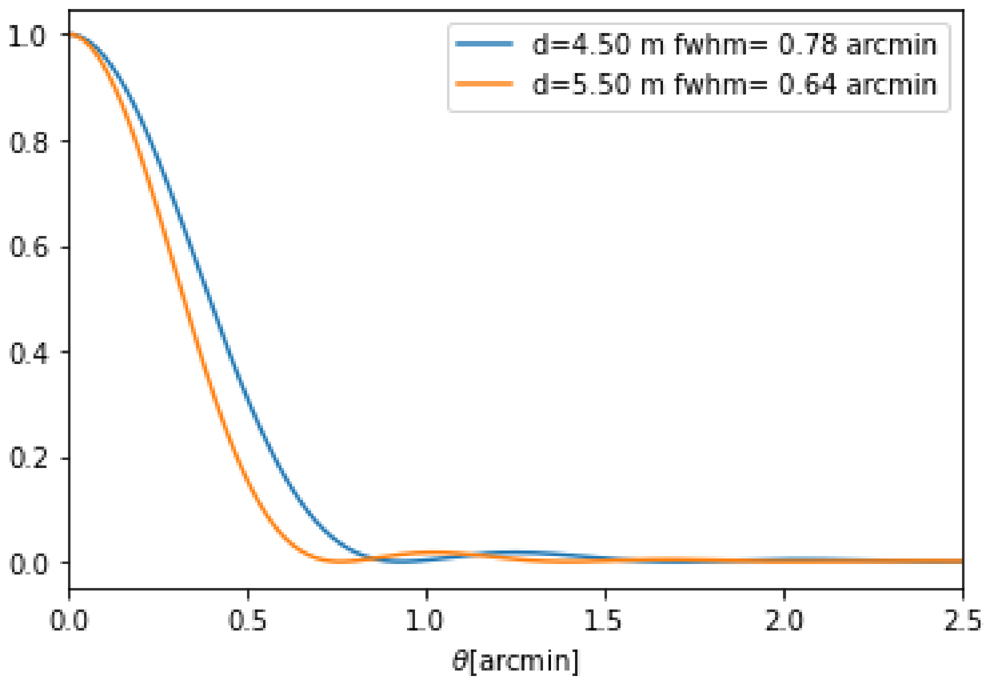
[5]:
```
diameters = 1e3 * np.array([4.5, 5.5])
THETA_MAX_ARCMIN = 10
LAMBDA_MM = 1.0
EE_THRESHOLDS = np.array([0.8, 0.9, 0.95, 0.97, 0.98])
theta_arcmin = np.linspace(0, THETA_MAX_ARCMIN, 1000); theta_arcmin[0]=1e-5
theta_rad = np.deg2rad(theta_arcmin/60.)
```

[6]:
```
fwhms = get_fwhm(LAMBDA_MM, diameters)
```

[7]:
```
for j, diameter in enumerate(diameters):
    plt.plot(theta_arcmin,
             psf(theta_rad,
                 diameter_mm=diameter,
```

```
                 lambda_mm=LAMBDA_MM),
             label='d=%1.2f m fwhm= %1.2f arcmin' % (diameter/1e3, fwhms[j]))
plt.legend();
plt.xlabel('$\\theta $[arcmin]');
plt.xlim([0, 2.5])
```

[7]: (0.0, 2.5)



2 FWHM

The full width at half max for a diffraction limited beam is given by:

$$\theta_{FWHM} = 1.025\frac{\lambda}{D}$$

Which gives

[8]:
```
for j in range(len(diameters)):
    fwhms = np.rad2deg(1.025 * LAMBDA_MM/diameters) * 60.
    print("FWHM for D=%1.2f m at %1.2f mm:  %1.2f arcmin" % (diameters[j]/1e3,␣
 ↪LAMBDA_MM, fwhms[j]))
```

```
FWHM for D=4.50 m at 1.00 mm:  0.78 arcmin
FWHM for D=5.50 m at 1.00 mm:  0.64 arcmin
```

3 Encircled energies

The encircled energy within a disc of radius θ is given by the integral of the psf

$$L(\theta) \propto \int_0^{\theta} \int_0^{2\pi} I(\theta)\theta d\theta d\psi$$

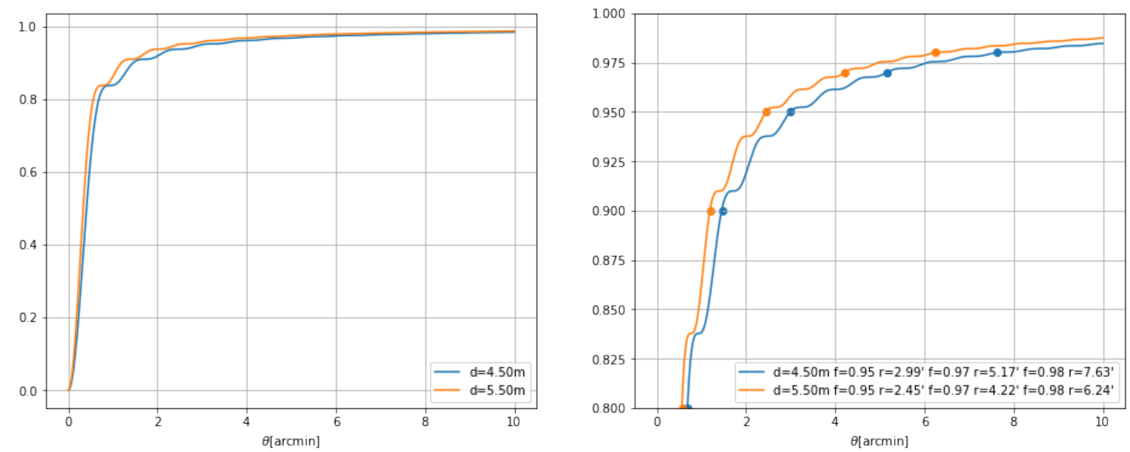
This integral has a closed form for a circular aperture

$$L(\theta) = 1 - J_0^2(ka\sin(\theta)) - J_1^2(ka\sin\theta)$$

[9]:
```
def get_ee(theta, diameter_mm, lambda_mm ):
    x = np.pi * diameter_mm * np.sin(theta)/lambda_mm
    L = 1 - jv(0, x)**2 - jv(1, x)**2
    return L

def get_x_given_y0s(x, y, y0s):
    xs_to_return = np.zeros(len(y0s))
    for j, y0 in enumerate(y0s):
        sel = y > y0
        xs_to_return[j] = x[sel][0]
    return xs_to_return
```

[10]:
```
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=[16, 6])
fig.suptitle('')
theta_thresholds = []
for j, diameter in enumerate(diameters):
    ee = get_ee(theta_rad,diameter_mm=diameter, lambda_mm=LAMBDA_MM)
    threshold_locations = get_x_given_y0s(theta_arcmin, ee, EE_THRESHOLDS)
    theta_thresholds.append(threshold_locations)
    string = ["f=%1.2f r=%1.2f'" % (EE_THRESHOLDS[j], threshold_locations[j])␣
 ↪for j in range(2, 5)]
    string = " ".join(string)
    ax1.plot(theta_arcmin,
             ee,
             label='d=%1.2fm' % (diameter/1e3))
    ax2.plot(theta_arcmin,
             ee,
             label='d=%1.2fm ' % (diameter/1e3) + string )
    ax2.scatter(threshold_locations, EE_THRESHOLDS)
ax1.grid()
ax2.grid()
ax2.set_ylim([0.8, 1.0])
ax1.legend();
ax2.legend();
ax1.set_xlabel('$\\theta $[arcmin]');
ax2.set_xlabel('$\\theta $[arcmin]');
```



[11]:
```
columns = ["d=%1.2f m" %(diameter/1000) for diameter in diameters]
rows = ["th:%1.2f" %(threshold) for threshold in EE_THRESHOLDS]
```

[12]:
```
df = pd.DataFrame(np.array(theta_thresholds).T,
                  columns=columns,
                  index=rows)
```

[13]:
```
print("Radius [arcmin] at a given encircled energy"); df
```

Radius [arcmin] at a given encircled energy

[13]:

	d=4.50 m	d=5.50 m
th:0.80	0.690691	0.570571
th:0.90	1.461461	1.191191
th:0.95	2.992993	2.452452
th:0.97	5.165165	4.224224
th:0.98	7.627628	6.236236

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