



# How Strong Is Your Tinder Game? Two-Sided Search in Swipe-Based Dating Platforms

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## Abstract

In today's love market, swipe-based dating platforms (SBDPs) such as Tinder or Bumble have a well-established presence, but novel platform features can add significant complexities to the user's search problem in ways that have been largely under-studied in existing literature. This paper formulates a model of two-sided search within SBDPs, where agents with heterogeneous preferences seek multiple romantic partners whilst facing intertemporal action constraints. Using numerical methods, I approximate stationary equilibria and perform comparative statics on various exogenous parameters that help explain stylised empirical facts. Finally, agent-based simulations are used to assess the structure of stationary equilibria as well as its attainability under myopic best-response dynamics.

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# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Related Work . . . . .	2
<b>2</b>	<b>Theoretical Model</b>	<b>4</b>
2.1	Setup . . . . .	4
2.2	The Dating Market . . . . .	6
2.3	The Search Problem . . . . .	7
<b>3</b>	<b>Equilibrium &amp; Comparative Statics</b>	<b>10</b>
3.1	Stationary Equilibrium and Computation . . . . .	10
3.2	Best Response Analysis . . . . .	11
3.3	Market Configuration Analysis . . . . .	12
<b>4</b>	<b>Agent-Based Simulations</b>	<b>14</b>
4.1	Steady State Convergence . . . . .	14
4.2	Myopic Best-Response Dynamics . . . . .	16
<b>5</b>	<b>Conclusion</b>	<b>16</b>
	<b>References</b>	<b>18</b>
	<b>Appendix</b>	<b>20</b>
<b>A</b>	<b>Mathematical Appendix</b>	<b>20</b>
A.1	Proof for Proposition 1 and Corollary 1 . . . . .	20
A.2	Proof for Proposition 2 . . . . .	21
<b>B</b>	<b>Exogenous Model Specifications</b>	<b>22</b>

# 1 Introduction

It is widely acknowledged that the search for love is a complex social phenomenon, but in today's world, swipe-based dating platforms (SBDPs) seem to only make it trickier. These platforms, exemplified by Tinder, Bumble, and Hinge, provide a gamified way of browsing through potential romantic partners by swiping through a stack of suggested candidates to indicate likes or dislikes for these, one profile at a time. In the search and matching literature, settings like these fall under the category of decentralised two-sided matching markets with search frictions (Kanoria and Saban, 2021) and, despite broad differences with traditional dating sites that perform centralised static matching, SBDPs have come to dominate the modern love market, with Tinder alone boasting 75 million monthly active users and 9.6 million paid subscribers as of 2021 (Business of Apps, 2022).

From a theoretical standpoint, search within SBDPs involves several complexities induced by platform-specific features, such as swiping caps, asynchronicity, and directed search algorithms. These impose non-trivial constraints on the way utility-maximising agents strategise their search, but they have been sparsely studied in the economics literature due to the relative novelty of these platforms. Overall, the prevalent role of SBDPs in shaping modern romantic interactions and their largely understudied nature motivates many different questions. Nevertheless, exploring these requires a fundamental understanding of how users make decisions in these platforms: to put it simply, *when should a utility-maximising user swipe right?*

This dissertation will explore the above within an SBDP setting, where agents with heterogeneous preferences on both sides of the market search sequentially for multiple romantic partners. Crucially, I focus on explaining (what I refer to as) the 'Fast-Swiping Men' puzzle: that is, the empirical observation that men in SBDPs respond with significantly higher swipe rates and face considerably worse matching outcomes than women. This phenomenon has been both a subject of empirical research (Tyson et al., 2016) and a contentious discussion point within mainstream media (Vice News, 2016; The Washington Post, 2016), and yet a significant gap persists within the literature for an exploration of this through a theoretical lens<sup>1</sup>. Such analysis would add significant value since the potential causes of this phenomenon (user patience, differential preferences, and strategic dominance) are all systematically endogenous with one another, thus demanding a rigorous model that can isolate these individual effects and trace their propagation across the SBDP market. Fundamentally, I show how sex imbalances within the platform (which arise due to several exogenous factors) can explain the above disparities and, expanding on this, I outline a possible intervention under which differential swiping caps between sexes can alleviate inefficiencies stemming from asymmetric selectiveness in the market.

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<sup>1</sup>Among the surveyed literature, perhaps the only partial examination of this phenomenon is provided by Kanoria and Saban (2021)

This work presents two main contributions to existing literature on the topic. Firstly, it constitutes one of a handful of attempts to model the market configurations arising within SBDPs, which is unsurprising due to the novelty of these platforms, but important given their current social relevance. Furthermore, it distinguishes itself from other similar works by directly considering the ‘Fast-Swiping Men’ puzzle as well as the impact of swiping caps both as a constraint in the agent’s search problem and a potential market correction mechanism. Finally, this work provides an interesting case study for the use of computational methods within game theory (a field that has traditionally emphasised pure mathematical analysis), showing how the two approaches can be jointly applied to complicated questions, with computational methods providing quick explorations that can serve as an intuitive stepping stone towards formalising mathematical arguments.

The remainder of the paper is structured as follows. In Section 2, I outline the theoretical framework for the model developed in this paper, and derive necessary conditions for both the platform steady state and agent best-responses. In Section 3, I present a refined definition for the stationary equilibrium of the model and perform computational comparative statics on several parameters, with the aim of explaining the ‘Fast-Swiping Men’ phenomenon and a possible market intervention. In Section 4, I utilise agent-based simulations to analyse the convergence and dynamics of my model, while Section 5 presents concluding remarks and outlines potential avenues for future research.

## 1.1 Related Work

The present work draws inspiration from two key branches of economics literature: that of search and matching theory, which studies the decision-making process of agents who seek, for example, a job, a business partner, or a spouse, and that of mean-field game theory, which models complex dynamic games involving a large number of players. I discuss each of these in turn, and then contrast this work with the handful of papers that have focused on specifically analysing SBDP markets.

Despite the abundance of papers within the search and matching literature, which has been amply surveyed by Chade et al. (2017), I focus on works that consider the three defining features of SBDP markets: decentralised matching, non-transferable utility, and sequential search with frictions. A seminal paper at this intersection is that of Burdett and Coles (1997), which studies the marriage market for ex-ante heterogeneous agents under uniform random search, extending the work of Becker (1973) by showing that positive assortative matching can arise in a setting with search frictions. Several extensions follow from this, considering idiosyncratic preferences (Burdett and Wright, 1998), noisy attractiveness observations (Chade, 2006), and even convergence onto the set of stable matchings (Adachi, 2003). The framework outlined in this dissertation is perhaps most similar to that of Burdett and Wright (1998), with three major differences between

the two. Firstly, the model developed in this paper extends the above by allowing for multiple partners within an agent’s lifetime, a feature which was probably not significant within the labour market context considered by Burdett and Wright (1998), but which is nevertheless quintessential of SBDPs given their role in enabling casual relationships. Furthermore, I extend the work of Burdett and Wright by allowing for sex-specific mass differences in the platform, as well as exogenous agent arrival flows; a point that was of noted interest for the authors themselves, and which is fundamental when considering the effects of sex imbalances within the platform. Finally, the model in this dissertation adopts a discrete time framework, departing from Burdett and Wright (1998) and most of the recent matching literature. Although continuous-time models provide sharper analysis and more flexible empirical specifications (Burdett and Coles, 1999), this modelling choice lends itself naturally to agent-based simulations, which are used to explore equilibria convergence and dynamics in a richer manner.

On the other hand, mean-field game theory focuses on dynamic games with a large number of agents, for which curses of dimensionality arise due to intractable state spaces. Mean-field models tackle this issue by conditioning gameplay on the invariant state distribution amongst players, rather than tracking and considering the individual state of each opponent (Light and Weintraub, 2022). This simplifying assumption is cemented by a *consistency check*, such that equilibria arise when rational play conditional on an aggregate state distribution maintains this as a fixed point. This approach, perhaps first considered by Jovanovic and Rosenthal (1988) and Hopenhayn (1992), has been successfully applied to settings such as network routing (Calderone and Sastry, 2017), dynamic auctions with learning (Iyer et al., 2014) and, perhaps most relevantly, online matching platforms (Kanoria and Saban, 2021; Immorlica et al., 2021). In this paper, I rely on mean-field assumptions to abstract away from observability considerations: within SBDPs, the market history is unknown to players, therefore concepts such as Perfect Bayesian Equilibrium would require players to maintain and update beliefs over uncountable history spaces, and even beliefs over the beliefs of other players (a complication known as nested beliefs (Brandenburger and Dekel, 1993)). This yields two central problems: first, that agent best-responses and equilibria become virtually impossible to compute, and, by extension, that the model assumes an unreasonable level of rationality on behalf of agents (Iyer et al., 2014). Thus, by conditioning interactions on the stationary platform state only, the model in this paper characterises equilibria that are both insightful and representative of real-life behaviour and dynamics.

Among the few papers specifically considering SBDPs, Kanoria and Saban (2021) propose a dynamic two-sided model with vertically-differentiated agents, and show that platforms with unbalanced markets can improve welfare by forcing the short side to ‘propose’ in all interactions. Furthermore, Immorlica et al. (2021) focus on the problem of designing a directed search algorithm for SBDPs by endogenising type-contingent meet-

ing rates for agents. Finally, Arnosti et al. (2021) study the welfare losses that can arise in congested matching markets, and propose limiting the visibility of agents as a mechanism for overcoming this issue. All of these papers present theoretical models with similar features, and these have largely influenced my work in several ways. Despite this, the above papers all model settings with one-to-one matchings only, thus differing fundamentally with my work in terms of the nature of endogeneity for agent departures. On one hand, this modelling choice makes their work and insights applicable to a wider variety of online matching platforms (such as AirBnb, TaskRabbit, etc.), but it also fails to capture an essential aspect specific to SBDPs that could have significant behavioural implications. Other than this (and some minor technical differences), the main point of distinction between my work and theirs is mostly one of perspective: whilst the above papers focus mostly on platform mechanism design (by considering features such as information constraints or directed search algorithms), I instead seek to explain, from first-principles, how empirically-observed phenomena arises in SBDPs.

## 2 Theoretical Model

### 2.1 Setup

In this section, I establish the theoretical framework for the model developed throughout this paper. Fix a non-atomic continuum of male and female agents and consider the dynamic two-sided market formed by the SBDP, which agents can join to search for potential romantic partners. For ease of exposition, I assume that this market is heteronormative such that male agents search exclusively for female agents and vice-versa. Time is discrete and indexed by  $t = 0, 1, 2, \dots$  over an infinite horizon. Each period, masses  $\lambda_m, \lambda_w > 0$  of new men and women enter the platform, where agents are then paired and presented a candidate partner from the opposite side of the market.

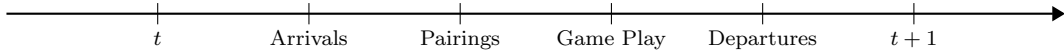
We model agents with heterogeneous preferences (capturing the notion that ‘beauty lies in the eye of the beholder’) and thus, after being paired, each agent observes an *idiosyncratic attractiveness value*  $\theta \in \Theta := [0, 1]$  for their candidate. These values are i.i.d according to a pair of absolutely continuous CDF’s,  $F_m$  and  $F_w$ , with corresponding PDF’s  $f_m, f_w$ . Female agents draw male candidate values from  $F_m$  and vice versa but, importantly, the value man  $i$  draws for woman  $j$  does not necessarily equal the value that  $j$  draws for  $i$  and, for simplicity, these are modelled as independent from one another.

After observing their candidate’s attractiveness, agents then choose to swipe left (dislike) or right (like) on them, yielding an action space  $\mathcal{A} = \{\text{Swipe Left}, \text{Swipe Right}\}$ . If either agent swipes left, then they both receive a payoff of zero; however, if both agents swipe right, they are said to have *matched* and both receive a matching payoff  $u(\theta)$ , where  $u(\cdot)$  is a continuous, strictly increasing function that satisfies  $u(0) = 0$ . This last property

stems from the fact that, in most SBDPs, users are allowed to unmatched with each other, and therefore matching with even the least attractive individual on the other side of the market is weakly preferred to not matching.

After payoffs have been received, agents are paired with different candidates and the stage interaction is repeated. Given the continuum of agents, I assume that interactions take place *anonymously* in the style of Jovanovic and Rosenthal (1988). Furthermore, to the agents' knowledge, pairings are determined in an unknown manner (since SBDPs are generally secretive regarding the algorithms they use), effectively making their problem one of uniform random search.

Figure 1: Sequence of events within each time period



Since swiping right in the above stage game is weakly dominant for all agents, one would not expect the repeated interaction to be massively revealing of real-life preferences, where initiating a romantic encounter is often perceived by humans as a costly endeavour (Dawkins and Davis, 2017). This becomes problematic since the main selling point of SBDPs is a reduction in searching costs for individuals seeking romantic encounters, and this is only accomplished if matches have a high likelihood of resulting in real-life romantic attraction. Because of this, SBDPs like Tinder place a cap on the total number of right swipes for each user, thus enabling this as a form of costly signalling. I refer to the total number of right-swipes a user has left as its *budget*,  $b$ , which evolves dynamically according to the law of motion:

$$b_{t+1} = b_t - \mathbb{1}\{a_t = \text{Swipe Right}\}$$

The budget sets for men and women are thus defined by  $\mathcal{B}_s = \{b \in \mathbb{Z} : 1 \leq b \leq B_s\}$ , for each sex  $s = m, w$ , with budget caps  $B_m$  and  $B_w$  determined exogenously. Importantly, agents depart from the platform in one of two ways: they can leave *endogenously*, if they expend their swiping budget, or *exogenously* with probability  $(1 - \delta)$  in each time period. This admits to the interpretation of a geometrically distributed lifetime, such that agents discount future payments by a factor of  $\delta \in (0, 1)$ .

One final remark for this setup is that, in a continuum market with anonymous interactions, the mean-field assumption established in subsection 2.3 effectively restricts focus onto the set of (pure) symmetric stationary strategies. This is argued in more detail in the following sections but, for now, denote these strategies by functions  $\mu : \Theta \times \mathcal{B}_m \rightarrow \mathcal{A}$  for men and  $\omega : \Theta \times \mathcal{B}_w \rightarrow \mathcal{A}$  for women.

## 2.2 The Dating Market

With the above framework in place, I now outline the platform state variables that make up the SBDP market. Let  $N_{mt}(b), N_{wt}(b)$  denote the mass of male and female agents (respectively) with a budget of  $b \in \mathcal{B}$  in a given time period  $t$ . Since gender imbalances can leave some agents in the long side of the market unpaired, a pairings process must also be determined. Given the automated nature of SBDPs, I assume an efficient matching technology and model pairings as a Bernoulli process parametrised by market tightness; thus, the probability of being paired with a candidate is defined for both sides as:

$$\tau_{mt} := \min \left\{ \frac{\sum_{b \in \mathcal{B}_w} N_{wt}(b)}{\sum_{b \in \mathcal{B}_m} N_{mt}(b)}, 1 \right\}, \quad \tau_{wt} := \left( \frac{\sum_{b \in \mathcal{B}_m} N_{mt}(b)}{\sum_{b \in \mathcal{B}_w} N_{wt}(b)} \right) \tau_{mt}$$

From the above, the platform state in time period  $t$  can be defined as  $\Psi_t = (N_{mt}, N_{wt})$ . For most of this paper, I focus on characterising user behaviour and its resulting implications in a stationary setting (which is denoted by omitted time subscripts), although some discussion of coupled strategy and market dynamics is provided in Section 4. As a necessary requirement, the market steady state  $\Psi_t = \Psi_{t+1} = \dots = \Psi$  must satisfy the balanced flow conditions<sup>2</sup> for our continuum model; these are presented below for the female agents, but they apply analogously to the male side of the market. Firstly, the entry flow of agents into the platform must equal the departure flow:

$$\lambda_w = \underbrace{(1 - \delta) \sum_{b \in \mathcal{B}_w} N_{wt}(b)}_{\text{Exogenous Outflow}} + \underbrace{N_w(1) \delta \tau_w \int_{\Theta} \omega(\theta, 1) dF_m(\theta)}_{\text{Endogenous Outflow}} \quad (2.1)$$

Secondly, for both sides, the flow of agents into any particular budget level must equal the outflow of agents from that same level. Thus, for all  $b \in \mathcal{B}_w$ :

$$\underbrace{N_w(b+1) \delta \tau_w \int_{\Theta} \omega(\theta, b+1) dF_m(\theta)}_{\text{Inflow into } b} = \underbrace{N_w(b) \left[ (1 - \delta) + \delta \tau_w \int_{\Theta} \omega(\theta, b) dF_m(\theta) \right]}_{\text{Outflow from } b} \quad (2.2)$$

Finally, the entry flow of agents into the platform must equal the outflow from the top budget level, hence:

$$\lambda_w = \underbrace{N_w(B_w) \left[ (1 - \delta) + \tau_w \delta \int_{\Theta} \omega(\theta, B_w) dF_m(\theta) \right]}_{\text{Outflow from } B_w} \quad (2.3)$$

Importantly, the above conditions take the strategy profile  $(\mu, \omega)$  as exogenously fixed.

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<sup>2</sup>Formally, these conditions rely on the exact law of large numbers, which was rigorously developed in discrete-time settings by Duffie et al. (2018), but a technical discussion of this lies outside the scope of this paper.



Over the next section, these are endogenously derived by characterising the agents' best-responses in an SBDP setting, which themselves depend on the market steady state  $\Psi$ .

### 2.3 The Search Problem

With the model framework outlined above, I now present the decision problem faced by female agents in the market given some platform steady state  $\Psi$ , with analogous results and implications for the male side. Consider now a woman  $i$  who is paired with a man  $j$  in the platform. Since  $j$ 's swiping behaviour will depend on his own budget, which is unknown to woman  $i$ , then (under strict rationality) she would have to compute her expected payoff conditional on her beliefs for the market history.

This behaviour is both unreasonable and intractable in an SBDP setting, as explained in subsection 1.1; instead, given a stationary platform and a continuum of anonymous agents, it is reasonable to conjecture that the average swiping rate for men,  $\bar{\mu}$ , is also stationary, and that any individual agent's actions have a negligible effect on the platform state dynamics<sup>3</sup>. As such, the expected ex-interim payoff for woman  $i$  is the following:

$$U(\theta, a) = \left( \mathbb{1}\{a = \text{Swipe Right}\} \right) \bar{\mu} u(\theta)$$

The above imposes a mean-field assumption, such that woman  $i$  accounts for  $j$ 's behaviour only through  $\bar{\mu}$  conditional on the platform state  $\Psi$ . This modelling choice, which has been employed by Immorlica et al. (2021) and Iyer et al. (2014) among others, simplifies the full dynamic game by collapsing it onto a pair of Markov Decision Processes (MDPs), one for each side of the market, such that strategy  $\omega$  is a best-response for women iff it is an optimal policy for the corresponding MDP. Additionally, this also condenses  $i$ 's state of payoff-dependent variables to include only the value  $\theta$  that she observes for  $j$  and her own budget  $b$ . Therefore, since all agents in the same side solve the same MDP, which depends only on their individual state, this effectively justifies the restricted focus on symmetric stationary strategies, as established in subsection 2.1.

Let the unrealized jump times of the pairing process for woman  $i$  be indexed by  $k$ . Given that, at the time of pairing, this women has a budget of  $b$  right swipes left, she then solves the constrained MDP presented below, captured by the value function  $V_w(\theta, b)$ :

$$\begin{aligned} V_w(\theta, b) = \max_{\{a_k\}_{k=0}^{\infty}} \quad & \mathbb{E}_{\theta} \left[ \sum_{k=0}^{\infty} \delta^k U(\theta_k, a_k) \mid \theta_0 = \theta, b_0 = b \right] \\ \text{s.t.} \quad & b_{k+1} = b_k - \mathbb{1}\{a_k = \text{Swipe Right}\} \\ & b_k \in \mathcal{B}_w \cup \{0\}, \\ & a_k \in \mathcal{A} \end{aligned}$$

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<sup>3</sup>I also assume that  $\bar{\mu} > 0$  to prune out degenerate equilibria where one side never swipes right.

Importantly, the first two constraints, along with the exogenous departure process, make this problem non-trivial: by limiting woman  $i$ 's right-swiping budget, the platform imposes an opportunity cost for swiping right on  $j$  and foregoing potential future matches with more attractive men, whilst the exogenous departure process removes the possibility of simply waiting around to swipe right on the top- $B_w$  most attractive men in the platform. By standard dynamic programming arguments, this problem can be captured by two Bellman equations; one for when  $j$  is paired and another for when she isn't:

$$V_w^P(\theta, b) = \max \left\{ \bar{\mu} u(\theta) + \delta \tau_w \mathbb{E}_\theta \left[ V_w^P(\theta', b-1) \right] + \delta(1-\tau) V_w^{NP}(b-1), \right. \\ \left. \delta \tau_w \mathbb{E}_\theta \left[ V_w^P(\theta', b) \right] + \delta(1-\tau_w) V_w^{NP}(b) \right\} \quad (2.4)$$

$$V_w^{NP}(b) = \delta \tau_w \mathbb{E}_\theta \left[ V_w^P(\theta', b) \right] + \delta(1-\tau_w) V_w^{NP}(b) \quad (2.5)$$

With some straightforward algebra, the above two equations can be merged into the full Bellman equation below. Note that, by imposing the swiping budget constraint from the above MDP, it must be the case that  $V_w(\theta, 0) = 0$  for all  $\theta \in \Theta$ , since agents who expend their budget must leave the platform and can't accumulate any additional payoffs:

$$V_w(\theta, b) = \max \left\{ \bar{\mu} u(\theta) + \alpha \mathbb{E}_\theta \left[ V_w(\theta', b-1) \right], \alpha \mathbb{E}_\theta \left[ V_w(\theta', b) \right] \right\} \quad (2.6)$$

Here,  $\alpha$  is the effective discount rate accounting for the exogenous possibilities of both departures and pairings, defined as:

$$\alpha := \frac{\tau_w \delta}{1 - \delta(1 - \tau_w)}.$$

Upon inspection, it is clear that the value function is of a piecewise nature over  $\Theta$ . This is formally stated below (with the corresponding derivation included in Appendix A):

**Proposition 1.** *Fix some  $b \in \mathcal{B}_w$ . Then there exists some unique reservation value  $\tilde{\omega}_b \in \Theta$  such that  $V_w(\theta, b)$  admits the following piecewise form over  $\Theta$ :*

$$V_w(\theta, b) = \begin{cases} \bar{\mu} u(\theta) + \alpha \mathbb{E}_\theta \left[ V_w(\theta', b-1) \right], & \theta \geq \tilde{\omega}_b \\ \alpha \mathbb{E}_\theta \left[ V_w(\theta', b) \right], & \theta \leq \tilde{\omega}_b \end{cases}$$

In the result above, all reservation values  $\{\tilde{\omega}\}_{b \in \mathcal{B}_w}$  are such that woman  $i$  is indifferent between swiping left or right. Therefore, an optimal policy for woman  $i$ 's MDP involves swiping right for partners who exceed the reservation value for her current budget:

**Corollary 1.** *The following threshold policy  $\tilde{\omega}$ , parametrised by  $\{\tilde{\omega}\}_{b \in \mathcal{B}_w}$ , attains  $V_w(\theta, b)$ :*

$$\tilde{\omega}(\theta, b) = \begin{cases} \text{Swipe Right,} & \theta \geq \tilde{\omega}_b \\ \text{Swipe Left,} & \theta < \tilde{\omega}_b \end{cases}$$

Using both of these results, I now derive the following explicit characterisation for reservation values  $\{\tilde{\omega}_b\}_{b \in \mathcal{B}_w}$  (with a corresponding proof included in Appendix A).

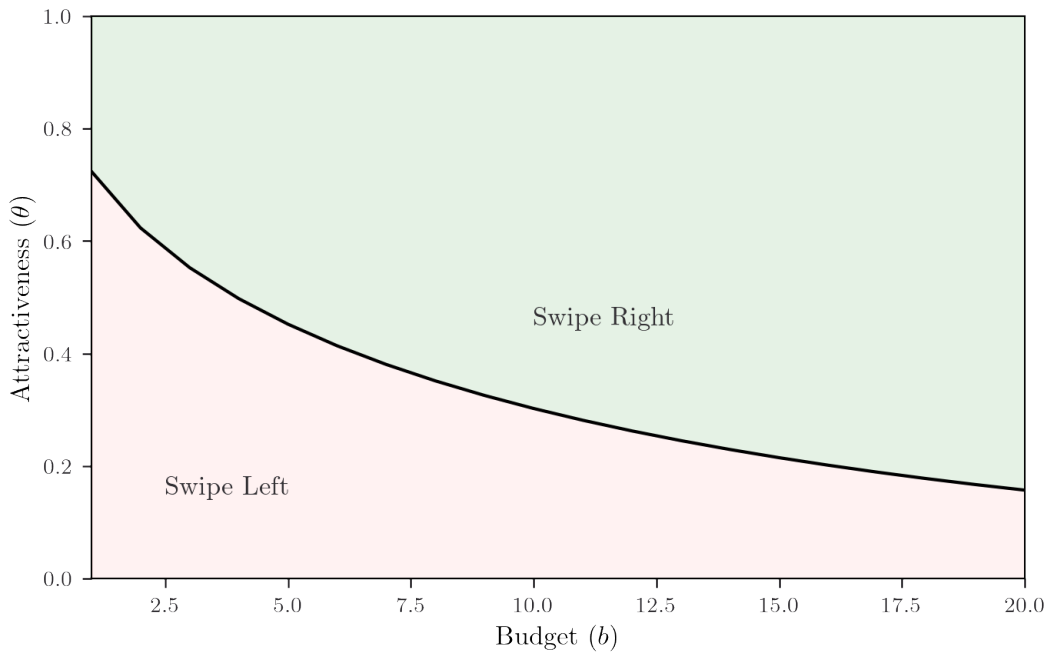
**Proposition 2.** *The set of reservation values for women,  $\{\tilde{\omega}_b\}_{b \in \mathcal{B}_w}$ , uniquely satisfies the recurrence relation and initial condition below, over the budget set  $\mathcal{B}_w$ :*

$$u(\tilde{\omega}_b) = \alpha u(\tilde{\omega}_b) F_m(\tilde{\omega}_b) + \alpha u(\tilde{\omega}_{b-1}) [1 - F_m(\tilde{\omega}_{b-1})] + \int_{\tilde{\omega}_b}^{\tilde{\omega}_{b-1}} \alpha u(\theta') dF_m(\theta') \quad (2.7)$$

$$u(\tilde{\omega}_1) = \alpha u(\tilde{\omega}_1) F(\tilde{\omega}_1) + \alpha \int_{\tilde{\omega}_1}^1 u(\theta') dF(\theta') \quad (2.8)$$

This explicit characterisation allows for an improved computation of  $\{\tilde{\omega}_b\}_{b \in \mathcal{B}_w}$  over traditional numerical methods such as value iteration (Bellman and Dreyfus, 2015), which has a runtime complexity of  $\mathcal{O}(|\mathcal{B}_w \times \Theta|^2 |\mathcal{A}|)$  and can become expensive for large budget sets or fine discretisations of  $\Theta$ . Using the recurrence relation in Proposition 2, the optimal policy was computed for an arbitrary set of exogenous parameters, with results in Figure 2 showing a clear cut-off rule for swiping right. These cut-off values are decreasing in the agent's budget, which captures the notion that an agent's current swipe is more valuable than all preceding ones given the increasing opportunity cost.

Figure 2: The Optimal Swiping Rule



### 3 Equilibrium & Comparative Statics

#### 3.1 Stationary Equilibrium and Computation

Using the framework and results above, I now present a refined definition for stationary market equilibria:

**Definition 1.** *A Stationary Equilibrium (SE) is a triplet  $(\mu^*, \omega^*, \Psi^*)$  such that:*

1.  $\mu^*(\theta, b)$  attains  $V_m(\theta, b)$ , for all pairs  $\theta, b \in \Theta \times \mathcal{B}_m$ , given  $\omega^*, \Psi^*$ .
2.  $\omega^*(\theta, b)$  attains  $V_w(\theta, b)$ , for all pairs  $\theta, b \in \Theta \times \mathcal{B}_w$ , given  $\mu^*, \Psi^*$ .
3.  $\Psi^*$  satisfies Equations 2.1, 2.2, and 2.3 given the strategy profile  $(\mu^*, \omega^*)$ .

Intuitively, the above definition establishes two requirements that must be satisfied by an equilibrium market configuration. Firstly, it must be the case that  $\mu^*$  and  $\omega^*$  are mutual best responses given the platform state  $\Psi^*$  for which, as previously outlined, a necessary and sufficient condition would have them each solve the sex-specific MDP. These two conditions alone demand *partially rational expectations*, as per Burdett and Coles (1997), since they require agents to play optimally for some fixed steady state  $\Psi^*$ , imposing rationality on all game aspects other than the platform state dynamics. Furthermore, in line with other works in mean-field game theory, a consistency check is imposed by the third condition, such that the platform state to which agents are best-responding with  $(\mu^*, \omega^*)$  is sustained as a fixed point under that same strategy profile.

Although formal proofs for the existence and uniqueness of SE are outside the scope of this paper, I rely on numerical procedures<sup>4</sup> to approximate equilibria under various exogenous settings. This approach has been frequently employed by related works (see Iyer et al., 2014; Gummadi et al., 2011) as it can help uncover insights provided by mean-field models. To compute model equilibria, I frame the recurrence relation presented in Proposition 2, as well as Equations 2.1, 2.2, and 2.3, as a system of  $2(|\mathcal{B}_m| + |\mathcal{B}_w| + 1)$  non-linear equations, and solve this using a modified version of the hybrid Powell method, as implemented by the MINPACK 1 routine (Moré et al., 1980). In what follows, I present SE results for a number of comparative statics experiments. For all of these, the exogenous settings used to calibrate the model are provided in Appendix B, and convergence of numerical procedures was assured by computing the squared loss of the above system, which was exactly equal to zero in all cases (although the runtime required for convergence varied slightly across experiments).

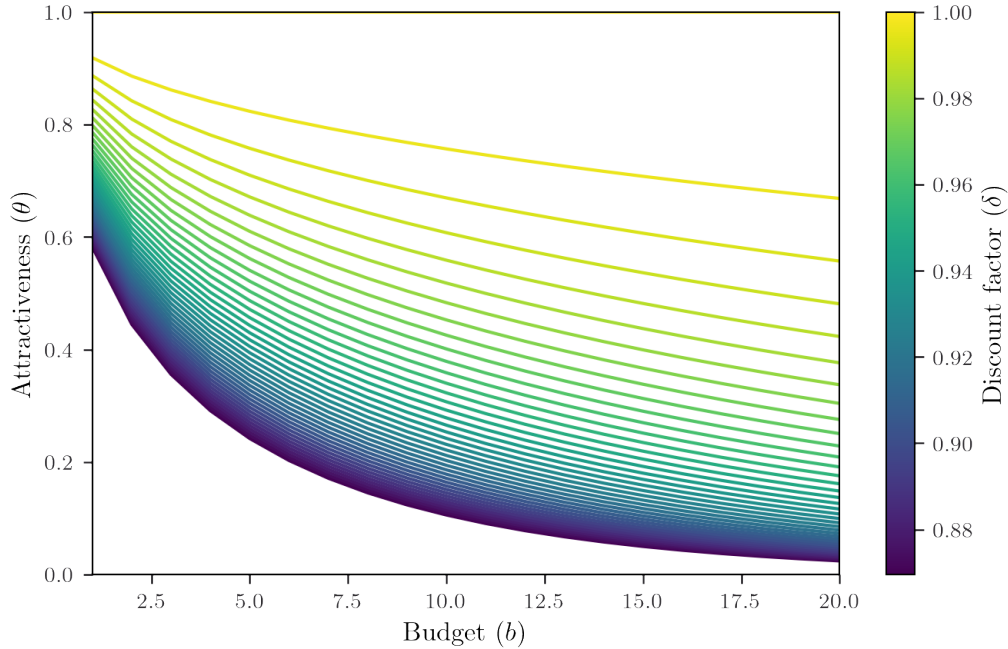
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<sup>4</sup>The code required to reproduce all presented analysis is accessible under the GitHub repository [patohdzs/project-swipe](https://github.com/patohdzs/project-swipe), with most dependencies covered by the SciPy Stack packages.

### 3.2 Best Response Analysis

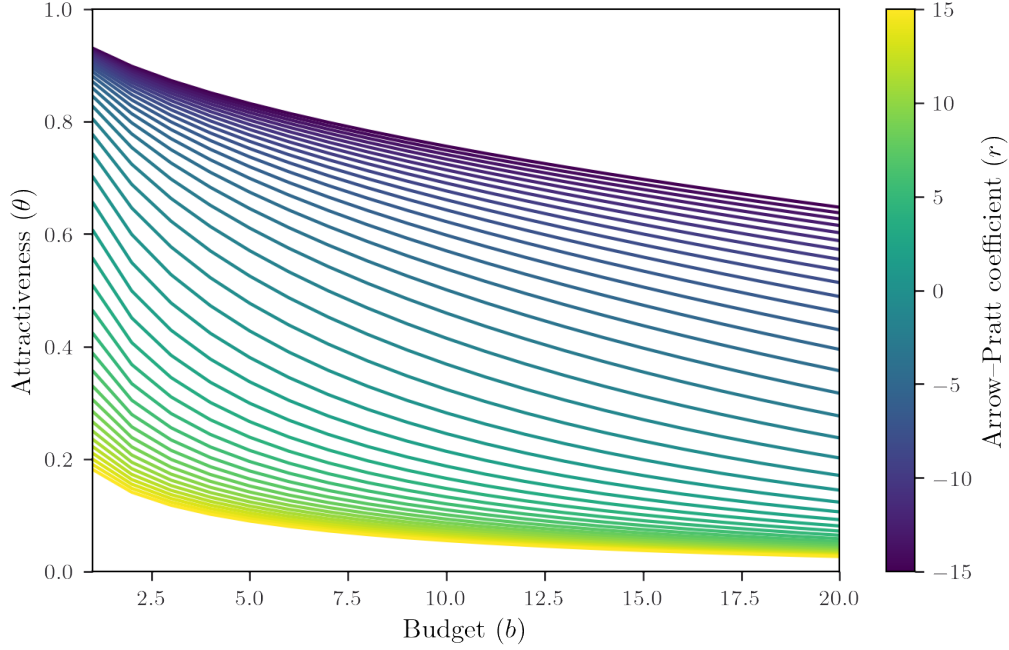
Using the computational procedures outlined above, a number of insights can be uncovered related to how exogenous parameters affect an agent's optimal swiping behaviour. The first parameter I analyse is the discount factor, which represents the probability of remaining inside the platform for an additional time period, but is often interpreted as the representative agent's patience level. To determine the effects of changes in the discount factor, I computed the best-response policy over a range of different values for  $\delta$  (using an arbitrary set of exogenous parameters), with results shown in Figure 3. Evidently, as the agent becomes less patient, they 'lower their standards' for potential matches in the platform, shifting their swiping curve downwards.

Figure 3: Comparative Statics on the Discount Factor



Another interesting parameter to examine is the absolute risk aversion of agents, which I choose to interpret as their 'desperateness' for matching. In the platform, risk-averse agents prefer a greater likelihood of matching (even if this yields relatively lower payoffs), whilst risk-loving agents prefer to save their swipes for high-yield candidates. To perform comparative statics on this parameter, I fix a CARA utility function for agents, with parameter  $r$  corresponding to the Arrow-Pratt coefficient for absolute risk aversion. I then compute the optimal swiping rule for various different values of  $r$ , with results for this shown on Figure 4. From here, it is evident that as absolute risk aversion rises, agents become 'more desperate' for matches, inducing them to lower their standards for right-swiping on a candidate, and thus shifting their swiping curve downwards.

Figure 4: Comparative Statics on Absolute Risk Aversion

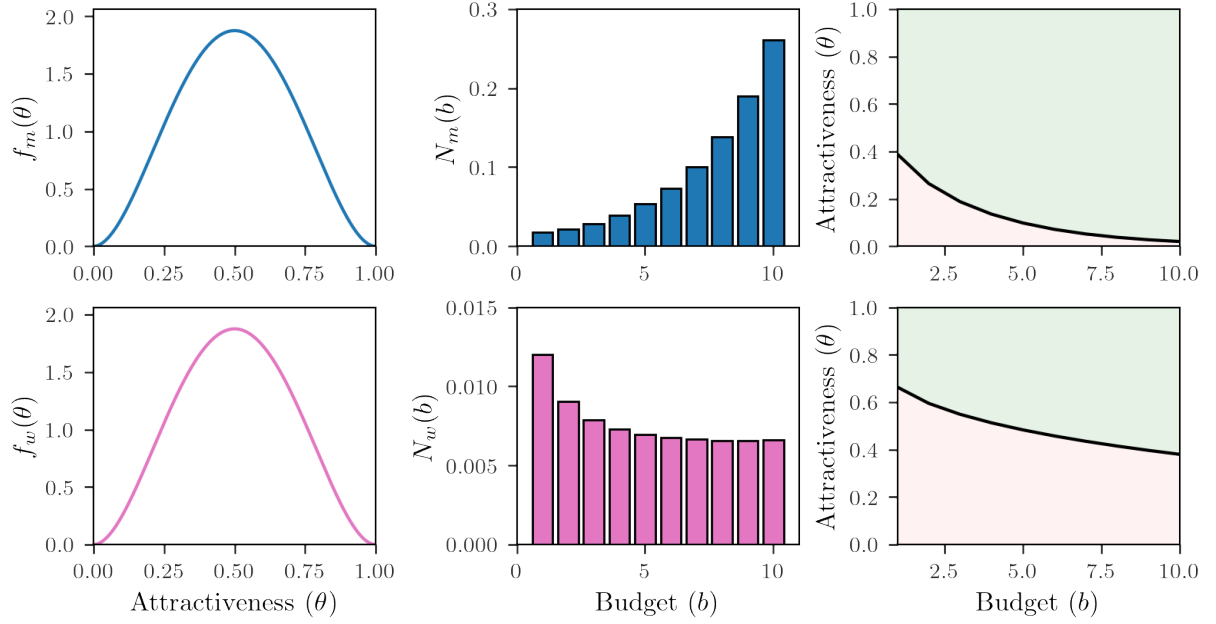


### 3.3 Market Configuration Analysis

Finally, I perform comparative statics at the platform level to explore how different exogenous factors affect market configurations. This is especially important as it considers not only the effects on best-responses for one sex, but also how these propagate across the market, ultimately affecting the other side's swiping behaviour as well. More specifically, I focus on the aforementioned 'Fast-Swiping Men' puzzle, investigating the discrepancies in swiping rates and matching outcomes between men and women. There are a number of potential explanations for this behaviour, most of which concern sex-specific differential preferences which are compatible with my model. Indeed, it is reported that women spend around 20% more time than men on a single Tinder session (The New York Times, 2018), potentially indicating a higher level of patience which would induce 'higher standards' and a lower swiping rate, as explained above. Departing from these, I consider below a scenario that could explain the 'Fast-Swiping Men' puzzle purely as a result of tightness-induced search frictions, for which I analyse the market SE computed under a 6:1 ratio between exogenous arrival rates  $\lambda_m$  and  $\lambda_w$ . This ratio was calibrated such that the steady state mass of men in the platform is around ten times greater than that of women, in line with demographic estimates for the UK (Business of Apps, 2022), and results for such SE are shown in Figure 5.

Under the above scenario, men overcrowd the market and struggle to get paired with female candidates, as evidenced by the top-center plot in Figure 5 which shows that male agents are highly concentrated in the top budget levels. Due to the effect of market tightness on the effective discount rate, male agents become more impatient than

Figure 5: Market Configuration Under Differential Agent Inflows

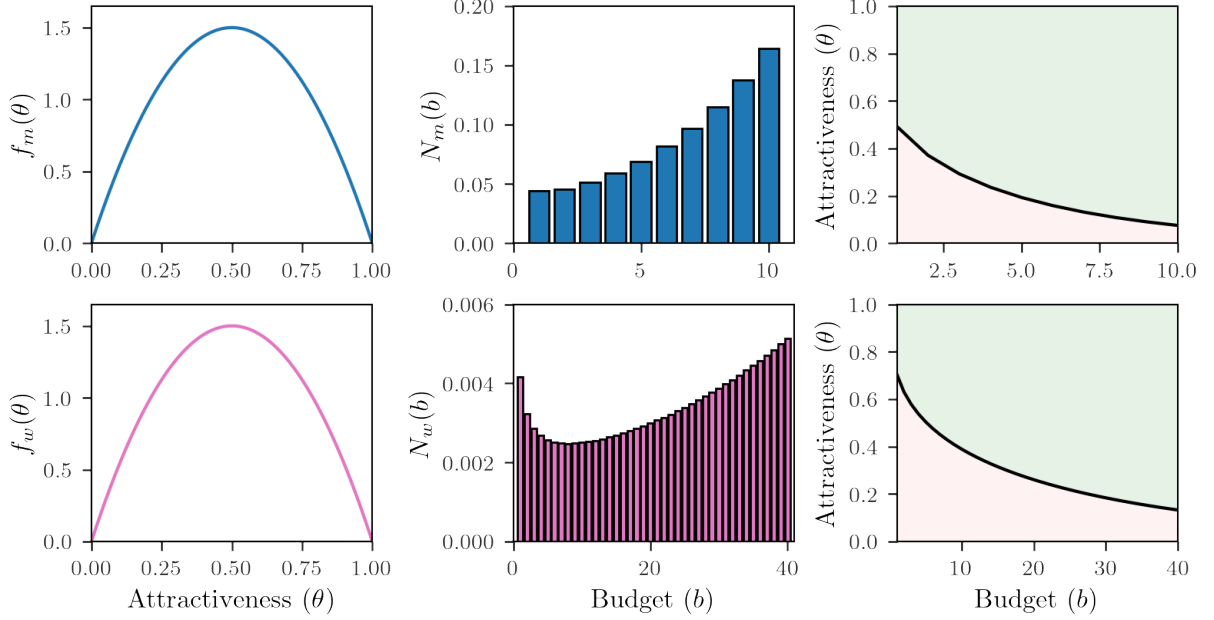


women, and this shock is absorbed by their optimal swiping policy, which sits considerably lower than the female swiping curve, effectively showing how a tight market lowers male patience and by extension, their standards, leading them to swipe right on most women. Ultimately, this explains the ‘Fast-Swiping Men’ puzzle given that, under this particular SE, men swipe right with probability  $\bar{\mu} = 0.988$ , compared to  $\bar{\omega} = 0.491$  for women, thus replicating the observed phenomenon.

One potential intervention to correct this phenomenon would involve relaxing the swiping cap for the shorter side of the market. To analyse such measure, the SE was computed under the same conditions as above but with a 4:1 swiping cap ratio between women and men; the results for this, shown in Figure 6, indicate the presence of two separate effects that would help alleviate the above scenario. First, a larger budget set would allow women to stay in the platform for longer by shrinking the number of endogenous departures to zero (at the limit), thus leading to a larger steady state mass of female agents that would reduce market tightness for men. This is corroborated by the computed results, which show a 6:1 ratio between the male and female agent masses, down from 10:1. Furthermore, because optimal reservation values are decreasing in the agent’s budget, women in the top budget levels can now swipe right at considerably lower thresholds, thus improving matching outcomes for men. This is highlighted in Figure 6, which shows that female swiping policy quickly descends to a level comparable to its male counterpart since it is allowed to extend over a larger budget set. Furthermore, under this particular SE, men swipe right with probability  $\bar{\mu} = 0.906$ , compared to  $\bar{\omega} = 0.79$  for women, therefore highlighting the effectiveness of this intervention, which could be

of considerable importance given both its simplicity and its potential impact on user experience, as research by Kanoria and Saban (2021) identifies asymmetric selectiveness as a profound source of inefficiency in dynamic matching platforms. One potential draw-

Figure 6: Market Configuration Under Differential Swiping Caps



back of this intervention is that general selectiveness falls in the market, since the effect prompting women to swipe more is of greater magnitude than the one prompting men to swipe less. This could be easily avoided by lowering the swiping cap for men, instead of raising the cap for women, to achieve the desired counterbalancing ratio, but such intervention would be inevitably bounded as the swiping cap for men approaches zero.

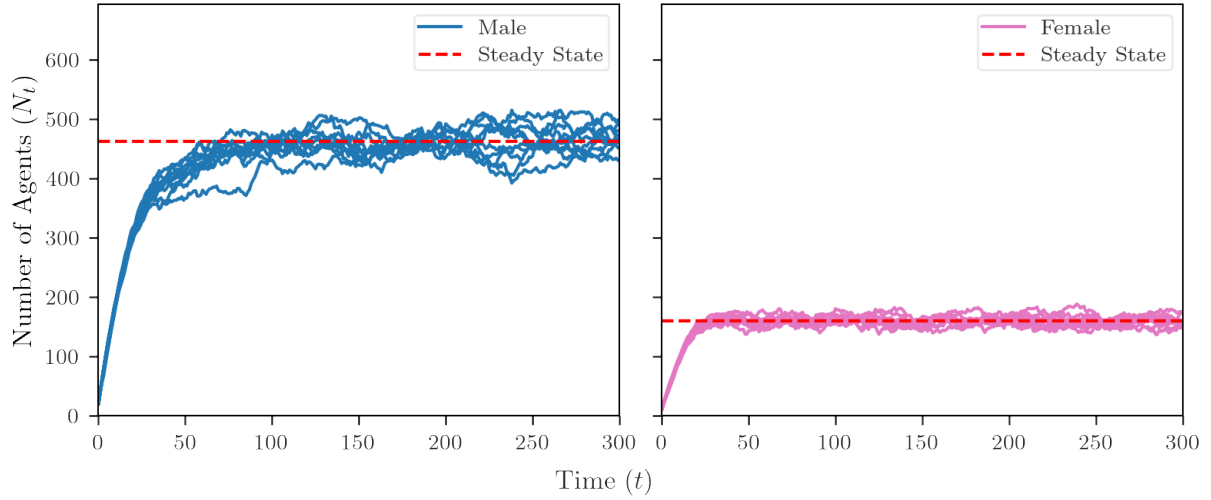
## 4 Agent-Based Simulations

### 4.1 Steady State Convergence

Given the lack of accessible SBDP user data, I developed an agent-based simulation environment to explore the evolution of both behavioural and market-level dynamics under the above theoretical foundation. Agent-based modelling (ABM) is used to study how “*macro phenomena emerges from micro level behaviour among a heterogeneous set of interacting agents*” (Janssen, 2005), and it has been successfully applied by recent work on matching platforms (Immorlica et al., 2021) to help identify and understand the structure of equilibria, which can be computationally expensive to approximate (and in some cases even non-existent). To start, I explore the convergence and stability of the SBDP market under arbitrary exogenous settings. In particular, Figure 7 shows

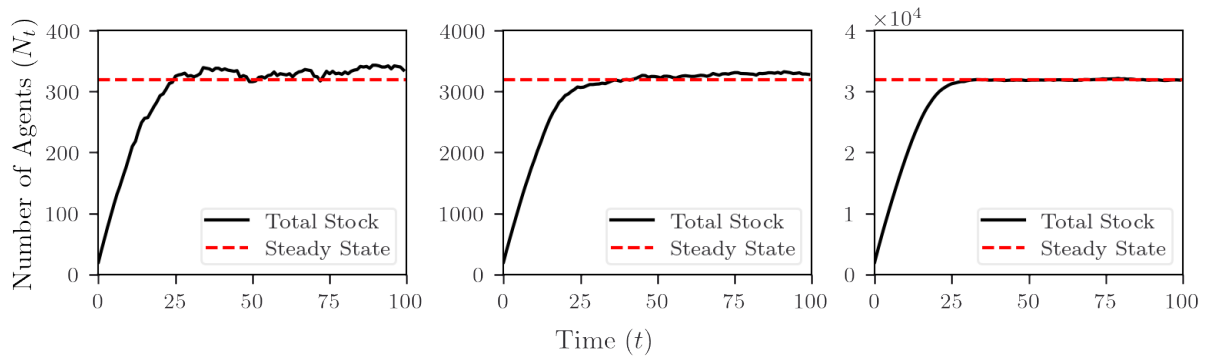


Figure 7: Agent-Based Simulation Convergence



the evolution of (sex-specific) agent masses for 10 independent simulation batches, over 300 time periods, and with a 2:1 ratio between male and female arrival flows. These simulations were conducted under partially rational expectation conditions; that is, with agents using optimal policies for some fixed steady state even when this is not actually the current platform state. As evident from these results, this process converges onto the SE computed using the procedures in subsection 3.1. Furthermore, the ABM simulations show that the long side of the market (males) takes considerably longer to converge onto its steady state level. One technical point worth noting is that the above simulations involve a finite number of agents as opposed to *agent masses*, as per our continuum model. Nevertheless, I examine the limiting case of these dynamics, with Figure 8 depicting how, by the law of large numbers, stationary deviations around the steady state level become negligible as the number of agents in the platform tends to infinity.

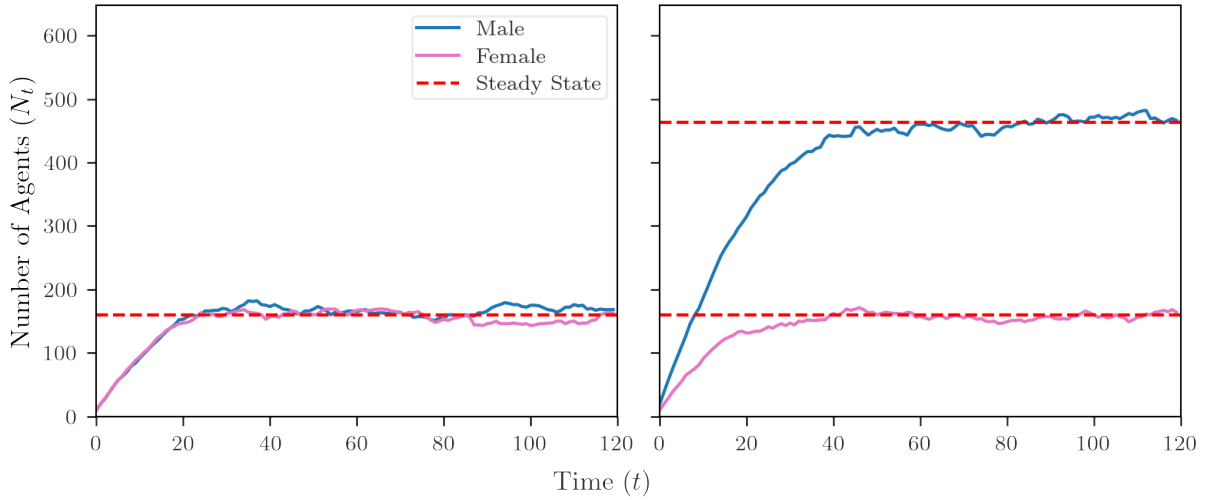
Figure 8: Agent-Based Simulation Convergence with Varying Sample Sizes



## 4.2 Myopic Best-Response Dynamics

Finally, I simulate the SBDP market under myopic best-response dynamics (Fudenberg et al., 1998) to explore whether if SE can be attained using a more robust process of gameplay. For this simulation, agents re-compute their optimal policies at the start of every time period given the current market state, unlike in the previous scenario where optimal policies are computed once with respect to the SE for some given exogenous settings. This process is *myopic* in the sense that agent policies account only for the current platform state but not for its dynamic evolution; yet it is still more robust than the previous experiment as echoing feedback between policy and state updates could create outward-spiralling dynamics that prevent convergence onto SE. The results of these simulations over 120 time periods are presented in Figure 9, showing that, both in the case of balanced and unbalanced markets, the SE can be attained using myopic best response dynamics.

Figure 9: Agent-Based Simulation Under Myopic Best Response Dynamics



## 5 Conclusion

This paper studied the strategic behaviour of users in SBDP markets by formulating a model of two-sided search for agents with heterogeneous preferences and intertemporal action constraints. Using mean-field assumptions that hold for large markets, I provided an explicit characterisation of agent best-responses and used computational procedures to approximate SSE, exploring the effects of different exogenous parameters on both individual behaviour and the aggregate SBDP market. Finally, I used ABM techniques to assess the convergence properties of my model, as well as its robustness under myopic best-response dynamics, to determine if equilibria can be attained under relaxed

gameplay conditions. I placed particular focus on explaining how the ‘Fast-Swiping Men’ phenomenon can arise in unbalanced markets due to the endogenous relation between an agent’s patience, their swiping behaviour, and the market steady state. Crucially, I identified that this puzzle is most likely the result of exogenous differences in sex-specific arrival flows, which can be counterbalanced with opposite differences in their respective swiping caps. Nevertheless, an interesting direction for future research could involve studying why these inflow differences occur in the first place, perhaps by considering an endogenous relation with competing SBDPs and alternative romantic search markets.

Overall, there are a number of actionable insights provided by my model that could allow SBDPs to enhance both their user experience and their profitability. The most direct application of these probably involves subscription pricing, since the main benefits that Tinder provides for its paid subscribers include: unlimited swipes, increased visibility, and the ability for an agent to observe profiles that have already liked them (Tinder, 2022). These benefits essentially equate to removing the three sources of search frictions explored by this paper: swiping constraints, market tightness, and strategic considerations, respectively. As such, the conclusions outlined above could provide guidance to future research, for example, by motivating subscription pricing models that discriminate based on market tightness. On a final note, one of the main benefits of the model presented in this paper is perhaps its flexibility, admitting to a number of potential extensions that could be used to study more focused aspects of SBDPs. One modification of particular interest would involve a richer action set that allows for both multiple casual matches and a single long-term match (after which agents leave the market permanently), which might uncover interesting insights on romantic incompatibility and associated search frictions in SBDPs.

## References

- Adachi, H. (2003). A search model of two-sided matching under nontransferable utility. *Journal of Economic Theory*, 113(2):182–198.
- Arnosti, N., Johari, R., and Kanoria, Y. (2021). Managing congestion in matching markets. *Manufacturing & Service Operations Management*, 23(3):620–636.
- Becker, G. S. (1973). A theory of marriage: Part i. *Journal of Political economy*, 81(4):813–846.
- Bellman, R. E. and Dreyfus, S. E. (2015). *Applied dynamic programming*. Princeton university press.
- Brandenburger, A. and Dekel, E. (1993). Hierarchies of beliefs and common knowledge. *Journal of Economic Theory*, 59(1):189–198.
- Burdett, K. and Coles, M. G. (1997). Marriage and class. *The Quarterly Journal of Economics*, 112(1):141–168.
- Burdett, K. and Coles, M. G. (1999). Long-term partnership formation: marriage and employment. *The Economic Journal*, 109(456):F307–F334.
- Burdett, K. and Wright, R. (1998). Two-sided search with nontransferable utility. *Review of Economic Dynamics*, 1(1):220–245.
- Business of Apps (2022). Tinder revenue and usage statistics. Last accessed 12 April 2022.
- Calderone, D. and Sastry, S. S. (2017). Markov decision process routing games. In *2017 ACM/IEEE 8th International Conference on Cyber-Physical Systems (ICCPS)*, pages 273–280. IEEE.
- Chade, H. (2006). Matching with noise and the acceptance curse. *Journal of Economic Theory*, 129(1):81–113.
- Chade, H., Eeckhout, J., and Smith, L. (2017). Sorting through search and matching models in economics. *Journal of Economic Literature*, 55(2):493–544.
- Dawkins, R. and Davis, N. (2017). *The selfish gene*. Macat Library.
- Duffie, D., Qiao, L., and Sun, Y. (2018). Dynamic directed random matching. *Journal of Economic Theory*, 174:124–183.
- Fudenberg, D., Drew, F., Levine, D. K., and Levine, D. K. (1998). *The theory of learning in games*, volume 2. MIT press.

- Gummadi, R., Key, P. B., and Proutiere, A. (2011). Optimal bidding strategies in dynamic auctions with budget constraints. In *2011 49th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 588–588. IEEE.
- Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, pages 1127–1150.
- Immorlica, N., Lucier, B., Manshadi, V., and Wei, A. (2021). Designing approximately optimal search on matching platforms. In *Proceedings of the 22nd ACM Conference on Economics and Computation*, pages 632–633.
- Iyer, K., Johari, R., and Sundararajan, M. (2014). Mean field equilibria of dynamic auctions with learning. *Management Science*, 60(12):2949–2970.
- Janssen, M. A. (2005). Agent-based modelling. *Modelling in ecological economics*, 155(1):172–181.
- Jovanovic, B. and Rosenthal, R. W. (1988). Anonymous sequential games. *Journal of Mathematical Economics*, 17(1):77–87.
- Kanoria, Y. and Saban, D. (2021). Facilitating the search for partners on matching platforms. *Management Science*, 67(10):5990–6029.
- Light, B. and Weintraub, G. Y. (2022). Mean field equilibrium: uniqueness, existence, and comparative statics. *Operations Research*, 70(1):585–605.
- Moré, J. J., Garbow, B. S., and Hillstom, K. E. (1980). User guide for minpack-1. Technical report, CM-P00068642.
- The New York Times (2018). Tinder, the fastest growing dating app, taps an age old truth. Last accessed 25 April 2022.
- The Washington Post (2016). Why everyone is miserable on tinder. Last accessed 15 April 2022.
- Tinder (2022). Tinder subscriptions. Last accessed 25 April 2022.
- Tyson, G., Perta, V. C., Haddadi, H., and Seto, M. C. (2016). A first look at user activity on tinder. In *2016 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining (ASONAM)*, pages 461–466. IEEE.
- Vice News (2016). Men on tinder explain why they swipe right on literally everyone. Last accessed 15 April 2022.

## A Mathematical Appendix

To simplify notation for Appendix A, I denote the continuation value at budget  $b$  by:

$$K_b := \alpha \mathbb{E}_\theta [V_w(\theta', b)]$$

### A.1 Proof for Proposition 1 and Corollary 1

*Proof.* Fix some  $b \in \mathcal{B}_w$  and, starting from Equation 2.6, consider the following:

$$\begin{aligned} V_w(\theta, b) &= \max \left\{ \bar{\mu} u(\theta) + \alpha \mathbb{E}_\theta [V_w(\theta', b-1)], \alpha \mathbb{E}_\theta [V_w(\theta', b)] \right\} \\ &= \max \{ \bar{\mu} u(\theta) + K_{b-1}, K_b \} \\ &= K_{b-1} + \max \{ \bar{\mu} u(\theta), K_b - K_{b-1} \} \end{aligned}$$

First, note that the difference between any two consecutive continuation values  $K_b$  and  $K_{b-1}$  must necessarily lie between 0 and  $\bar{\mu} u(1)$ . This is true since the value function denotes the expected lifetime sum of discounted payoffs, and an additional right-swipe can provide an agent with, at most, an additional expected payoff of  $\bar{\mu} u(1)$  and, at least, an additional payoff of 0. Furthermore, since  $u(\theta)$  is, by assumption, continuous and increasing over  $\Theta$  (and we assume  $\bar{\mu} > 0$  to prune out degenerate equilibria), then, by the Intermediate Value Theorem, there exists a unique root,  $\tilde{\omega}_b$ , satisfying:

$$\bar{\mu} u(\tilde{\omega}_b) = K_b - K_{b-1}$$

Consider now two cases. First, if  $\theta \leq \tilde{\omega}_b$ , then:

$$\begin{aligned} V_w(\theta, b) &= K_{b-1} + \max \{ \bar{\mu} u(\theta), K_b - K_{b-1} \} \\ &= K_{b-1} + K_b - K_{b-1} \\ &= K_b. \end{aligned}$$

Analogously, if  $\theta \geq \tilde{\omega}_b$ , then:

$$V_w(\theta, b) = \bar{\mu} u(\theta) + K_{b-1}.$$

Thus, by considering the above function over the intervals  $[0, \tilde{\omega}_b]$  and  $[\tilde{\omega}_b, 1]$  separately, and substituting back the expressions for  $K_b, K_{b-1}$ , we conclude that:

$$V_w(\theta, b) = \begin{cases} \bar{\mu} u(\theta) + \alpha \mathbb{E}_\theta [V_w(\theta', b-1)], & \theta \geq \tilde{\omega}_b \\ \alpha \mathbb{E}_\theta [V_w(\theta', b)], & \theta \leq \tilde{\omega}_b \end{cases}$$

Furthermore, Corollary 1 follows trivially from the above by considering a cutoff policy over the above intervals such that  $V_w(\theta, b)$  is attained.  $\square$

## A.2 Proof for Proposition 2

*Proof.* Fix some  $b \in \mathcal{B}_w$  and consider the result presented by Proposition 1, which guarantees the existence and uniqueness of some  $\tilde{\omega}_b$  satisfying:

$$V_w(\theta, b) = \begin{cases} \bar{\mu}u(\theta) + K_{b-1}, & \theta > \tilde{\omega}_b \\ K_b, & \theta \leq \tilde{\omega}_b \end{cases} \quad (\text{A.1})$$

$$\bar{\mu}u(\tilde{\omega}_b) = K_b - K_{b-1} \quad (\text{A.2})$$

Starting out with Equation A.2 and expanding out the expectation operator, we can use (A.1) to substitute in the piecewise definitions of  $V_w(\theta, b)$  over the appropriate intervals:

$$\begin{aligned} \bar{\mu}u(\tilde{\omega}_b) &= K_b - K_{b-1} \\ &= \alpha \int_0^1 V_w(\theta', b) - V_w(\theta', b-1) dF_m(\theta') \\ &= \alpha \int_0^{\tilde{\omega}_b} K_b dF_m(\theta') + \alpha \int_{\tilde{\omega}_b}^1 \bar{\mu}u(\theta') + K_{b-1} dF_m(\theta') \\ &\quad - \alpha \int_0^{\tilde{\omega}_{b-1}} K_{b-1} dF_m(\theta') - \alpha \int_{\tilde{\omega}_{b-1}}^1 \bar{\mu}u(\theta') + K_{b-2} dF_m(\theta') \end{aligned} \quad (\text{A.3})$$

Furthermore, Equation A.2 implies that:

$$\bar{\mu}u(\tilde{\omega}_b) + K_{b-1} = K_b$$

$$\bar{\mu}u(\tilde{\omega}_{b-1}) + K_{b-2} = K_{b-1}$$

Then, by substituting these expressions into (A.3), we arrive at (A.4):

$$\begin{aligned} \bar{\mu}u(\tilde{\omega}_b) &= \alpha \int_0^{\tilde{\omega}_b} \bar{\mu}u(\tilde{\omega}_b) + K_{b-1} dF_m(\theta') + \alpha \int_{\tilde{\omega}_b}^1 \bar{\mu}u(\theta') + K_{b-1} dF_m(\theta') \\ &\quad - \alpha \int_0^{\tilde{\omega}_{b-1}} K_{b-1} dF_m(\theta') - \alpha \int_{\tilde{\omega}_{b-1}}^1 \bar{\mu}u(\theta') + K_{b-1} - \bar{\mu}u(\tilde{\omega}_{b-1}) dF_m(\theta') \end{aligned} \quad (\text{A.4})$$

With some algebra, this simplifies down to the recurrence relation in Equation 2.7:

$$u(\tilde{\omega}_b) = \alpha u(\tilde{\omega}_b) F_m(\tilde{\omega}_b) + \alpha u(\tilde{\omega}_{b-1}) [1 - F_m(\tilde{\omega}_{b-1})] + \alpha \int_{\tilde{\omega}_b}^{\tilde{\omega}_{b-1}} u(\theta') dF_m(\theta') \quad (\text{A.5})$$

Furthermore, to obtain the initial condition for the above, note that the right-swiping budget constraint imposes  $V_w(\theta, 0) = 0, \forall \theta \in \mathcal{B}_w$ . Then, (A.1) and (A.2) simplify to:

$$V_w(\theta, 1) = \begin{cases} \bar{\mu}u(\theta), & \theta > \tilde{\omega}_1 \\ K_1, & \theta \leq \tilde{\omega}_1 \end{cases} \quad (\text{A.6})$$

$$\bar{\mu}u(\tilde{\omega}_1) = K_1 \quad (\text{A.7})$$

Beginning with Equation A.7, we simplify until arriving at Equation 2.8:

$$\begin{aligned} \bar{\mu}u(\tilde{\omega}_1) &= \alpha \mathbb{E}_\theta [V_w(\theta', 1)] \\ &= \alpha \int_0^{\tilde{\omega}_1} K_1 dF_m(\theta') + \alpha \int_{\tilde{\omega}_1}^1 \bar{\mu}u(\theta') dF_m(\theta') \\ &= \alpha \int_0^{\tilde{\omega}_1} \bar{\mu}u(\tilde{\omega}_1) dF_m(\theta') + \alpha \int_{\tilde{\omega}_1}^1 \bar{\mu}u(\theta') dF_m(\theta') \\ &= \alpha \bar{\mu}u(\tilde{\omega}_1) F_m(\tilde{\omega}_1) + \alpha \int_{\tilde{\omega}_1}^1 \bar{\mu}u(\theta') dF_m(\theta') \\ \implies u(\tilde{\omega}_1) &= \alpha u(\tilde{\omega}_1) F_m(\tilde{\omega}_1) + \alpha \int_{\tilde{\omega}_1}^1 u(\theta') dF_m(\theta') \end{aligned}$$

□

## B Exogenous Model Specifications

Table 1: Exogenous Model Specifications

Model Reference	$\lambda_m : \lambda_w$	$B_m, B_w$	$F_m, F_w$	$\delta$	$u(\theta)$
Figure 2	—	20	Uniform(0,1)	0.95	Linear
Figure 3	—	20	Uniform(0,1)	(0.87, 1)	Linear
Figure 4	—	20	Uniform(0,1)	0.95	CARA, with $r \in [-15, 15]$
Figure 5	6:1	10	Beta(2,2)	0.97	Logarithmic
Figure 6	6:1	10,40	Beta(2,2)	0.97	Logarithmic
Figure 7	2:1	10	Beta(2,2)	0.97	Logarithmic
Figure 8	1:1	10	Beta(2,2)	0.97	Logarithmic
Figure 9	2:1, 1:1	10	Beta(2,2)	0.97	Logarithmic