How Strong Is Your Tinder Game? Two-Sided Search in Swipe-Based Dating Applications

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Abstract

In today's love market, swipe-based dating apps such as Tinder or Bumble have a well-established presence, but novel platform features can add significant complexities to the user's search problem in ways that have not been studied in existing literature. This paper formulates a game-theoretic model of two-sided search within swipe-based dating apps, along with an appropriate equilibrium refinement. Using numerical methods, I approximate equilibria at the steady-state and perform comparative statics on various model parameters that help explain stylized facts observed in aggregate Tinder data. Finally, agent-based simulations are used to analyse off-path dynamics and discuss how exogenous platform features can be set in a socially-efficient manner.

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1 Introduction

It is widely acknowledged that the search for love is a deeply relevant, personal, and complex social phenomenon, but in today's world, swipe-based dating applications (SBDA's) seem to only make it trickier. These platforms, exemplified by Tinder, Bumble, or Hinge, provide a gamified way of browsing through potential romantic partners by swiping through a stack of suggestions to indicate likes or dislikes, one profile at a time. In the search and matching literature, settings like these fall under the category of decentralized two-sided matching markets with online search (Kanoria and Saban, 2021), emphasising three main characteristics. Firstly, that both sides of the market are comprised of decision-making agents undertaking a process of search. Secondly, that matches occur as outcomes of independently-determined search decisions, rather than through a centralised algorithm. Thirdly, that romantic suggestions are presented in an online manner to users, stressing the importance of sequential rationality within the search process given that it is not possible to interact with the same candidate twice. These apps differ widely from traditional dating sites where users are centrally and statically matched (such as match.com or eHarmony), but have come to dominate the modern love market, with Tinder alone boasting 75 million monthly active users and 9.6 million paid subscribers as of 2021 (Business of Apps, 2022).

From a theoretical standpoint, search within SBDA's encompasses many complexities that, due to the novelty of the platforms, have been sparsely studied in the economics literature. On one hand, platform-specific characteristics, such as swiping caps, asynchronicity, and the suggestion algorithms used impose non-trivial constraints to the way utility-maximising agents strategise their search process. On the other hand, the general problem of search in a two-sided setting is interesting in and of itself, as a simple stage interaction (to swipe or not to swipe on a romantic suggestion) can become increasingly complex when repeated over an infinite horizon, admitting to problems such as intractable strategy spaces. Overall, the prevalent role of SBDA's in shaping modern romantic interactions, the theoretical complexities they induce, and their largely understudied nature motivates many different questions. Nevertheless, answering these requires a fundamental understanding of how users make decisions in these platforms: to put it simply, *when should a utility-maximising user swipe right?*

This paper will explore the above within the setting of a swipe-based dating platform where agents on both sides of the market with heterogeneous preferences search simultaneously for multiple romantic partners. I present an appropriate refinement for the mean-field equilibria of my model and approximate these at steady-state using computational methods. Crucially, I find that gender imbalances can explain match rate disparities within the platform, a stylized fact that has been observed empirically and analysed theoretically in previous work. Furthermore, I model a possible intervention where the swiping cap ratio between sexes can be set in a socially-efficient manner in order to compensate for welfare losses due to gender imbalances. This work presents three main contributions to existing literature on the topic. Firstly, it constitutes one of a handful of attempts to model market configurations arising within SBDA's, which is unsurprising due to the novelty of these platforms, but important given their current social relevance. Furthermore, this work distinguishes itself by considering the impact of swiping caps in the market, both as a constraint within the agent's search problem and a potential market correction mechanism. Finally, this work provides a marginal side-contribution as a methodological example for the use of computational methods within game theory, a field that has traditionally emphasised pure mathematical analysis. To explore the above questions, this paper relies on a rigorously-formulated model, but also on numerical approximation algorithms and agent-based simulations, which can be used to derive quick solutions and perform visually-intuitive comparative statics. As such, it shows that the two approaches, rather than being mutually exclusive, can be jointly employed to explore complicated questions, as computational methods enable quick intuitive explorations that can serve as a stepping stone towards formalising rigorous mathematical arguments.

The remainder of the paper is structured as follows: Section 2

1.1 Related Work

The present work draws inspiration from two key branches of economics literature: that of search and matching, which studies the decision-making process of agents who seek, for example, a job, a business partner, or a spouse, and also that of mean field game theory, which has been employed to study complex dynamic games involving a large number of players. I discuss each of these in turn, and then contrast this work with the handful of papers that have focused on specifically analysing SBDA market configurations.

Within the search and matching literature there is an abundance of different theoretical models, amply summarised by Chade et al. (2017), with several extensions studying a wide variety of different settings. As previously noted, three defining features of SBDA markets are decentralised matching, two-sidedness, and sequential interaction; one of the most prominent works on matching markets at this intersection is that of Burdett and Coles (1997), which studies a setting of uniform random search where agents receive marriage proposals from the other side of the market according to a continuous-time process, and must choose whether or not to accept these given the observable 'pizazz' of the proposing agent. Several extensions followed this work, considering cases with noisy observations of 'pizazz' (Chade, 2006), idiosyncratic preferences (Burdett and Wright, 1998), directed search, and so on. Some papers have even studied the convergence of decentralised two-sided models like these onto the set of stable matchings, which can serve as a socially-efficient centralised benchmark (Adachi, 2003). This line of work served as a great inspiration for the different 'flavours' of two-sided matching models that could best represent the SBDA market, and there are several key modelling aspects that I apply within this paper such as the endogenous flow-based approach used to unify search in both sides of the market. Despite this, the main difference between my model and others within this line of research occurs since agents in SBDA platforms search not only for spouses but also for casual relationships, thus demanding a framework that allows them to accumulate multiple matches (something that been largely understudied in preceding works due to the focus on marriage).

On the other hand, mean field game theory focuses on dynamic games with a large number of agents, for which curses of dimensionality often arise, making solution concepts such as Markov Perfect Equilibria intractable (Maskin and Tirole, 2001). To deal with this, mean field models consider individual interactions with the *aggregate system state*, ie. the distributions over states and strategies within the game, rather than interactions with all other players. This abstraction is coupled with the notion of a *consistency check*, such that equilibrium arises when rational play given an aggregate state maintains this same state as a fixed point. The approach, first considered in the work of Jovanovic and Rosenthal (1988), greatly simplifies strategic settings with the aforementioned problem and has been successfully applied to settings such as network routing (Calderone and Sastry, 2017), auctions with learning (Iyer et al., 2014). In this paper, we rely on mean-field considerations to abstract from considerations on observability; within SBDA's, the market-wide history and opponent state are unobservable to players, and thus traditional equilibrium concepts would demand beliefs over uncountable history spaces, and even beliefs over the beliefs other players may hold (a complication known in the literature as nested beliefs (Brandenburger and Dekel, 1993)). This yields two central problems: first, that equilibria become virtually impossible to compute, and, by extension, that the model assumes an unreasonable level of rationality on behalf of agents, especially given that these rarely interact with the same individual twice amongst millions of other users. Thus, by considering interactions with the aggregate state of the platform, the model presented is able to better characterize equilibria that is both insightful and representative of real-life dynamics.

Among the few papers that specifically consider SBDA matching markets, one that stands out is the recent work by Kanoria and Saban (2021), which postulates a two-sided dynamic matching model with vertically-differentiated agents, and finds that platforms with unbalanced markets can improve welfare by forcing the short side to propose. There are several modelling choices distinguishing this work and mine, but I identify two main differences worth discussing. Firstly, the action space in Kanoria and Saban (2021) is far richer as it allows agents to both issue and receive proposals to the other side. Whilst this permits a focused study of platforms such as Bumble or Coffee Meets Bagel, with interaction mechanisms that allow for this, the model does not adjust naturally to mechanisms such as the one in Tinder, where agents do not know ex-ante if the other agent has swiped on them and must factor this within the strategic cost/benefit analysis of deciding to 'spend a swipe'. This is important since one of the main selling points of Tinder paid subscriptions is the ability to observe which users have already swiped right on you, thus providing a strategic advantage. Furthermore, the two key platform interventions studied, in line with their elaborate action space, involve restricting one side of the market from proposing or hiding information regarding the quality types of agents. On the other hand, this work explores mainly the impact of budget caps as a platform intervention in a way that is not directly modelled in the above, which is important given that this mechanism is more widely applicable to a broad range of SBDA's. Another exemplary model for SBDA markets is the one presented by Immorlica et al. (2021), which presents a two-sided model with a finite number of agent types, and focuses on the problem of guiding the search process through type-contingent meeting rates for agents. This paper differs greatly with my work both in terms of its main research focus and on several modelling choices, but

2 Theoretical Model

2.1 Setup

Throughout this section, I establish the theoretical framework for the model considered in this paper. Fix a non-atomic continuum of male and female agents [1] and consider the dynamic two-sided market formed by the Tinder platform, where agents can join in to search for potential romantic partners. For ease of exposition, I assume that this market

[1] More specifically, let $(I, \mathcal{I}, \alpha)$ be the non-atomic measurable space of agents.

is heteronormative such that male agents search exclusively for female agents and vice-versa. Time is discrete and indexed $t = ..., -1, 0, 1, ...$ over an infinite horizon. At every time period, agents from each sex are paired and presented a candidate partner from the opposite side of the market. We model agents with heterogeneous preferences (capturing the conventional notion that 'beauty lies in the eye of the beholder') and thus, after being paired, each agent observes an *idiosyncratic attractiveness value* $\theta \in \Theta := [0, 1]$ for their candidate. These values are drawn i.i.d from distributions with CDF's F_m, F_w, with female agents drawing male candidate values from F_m and vice versa. Importantly, the value men i draws for women j does not necessarily equal the value that j draws for i, and we model these independently. After observing their candidate's attractiveness, agents then choose whether to swipe left (dislike) or right (like) on them, yielding an action space of $\mathcal{A} = \{\text{left, right}\}$. If both agents swipe right on one another, they are said to have *matched* and both receive a matching payoff, however, if either agent swipes left, they both receive a payoff of zero. Contingent on swiping right on a candidate with attractiveness θ, a user earns a matching payoff $u(\theta)$, where $u(\cdot)$ is a strictly increasing, concave function that satisfies $u(0) = 0$. This last property stems from the fact that, in Tinder, users are allowed to unmatch with each other, thus implying that matching with the least attractive individual on the other side of the market is weakly preferred to not matching. After payoffs have been received, players are then paired with a different candidate and the above stage interaction is repeated. Given the continuum of agents, I assume that interactions take place *anonymously* in the style of Jovanovic and Rosenthal (1988), thus abstracting from history-related complexities. Furthermore, to the agents' knowledge, pairings are determined in an unknown manner (since SBDA's are generally secretive regarding the algorithms used), effectively making their problem one of uniform random search.

Considering the above, it is evident that swiping right in the stage interaction is both weakly dominant for all agents and yields a Pareto-optimal outcome, thus implying that, in a repeated interaction, the market equilibrium would have all agents exclusively swiping right. Since that the main selling point of SBDA's is a reduction in searching costs, which is accomplished when matches have a high likelihood of resulting in real-life romantic attraction, Tinder places a cap on the total number of right swipes for each user, thus making it a form of costly signalling. I refer to the total number of right-swipes a user has left as its *budget*, b_t, which evolves dynamically according to the law of motion:

$$b_{t+1} = b_t - a_t$$

where the starting budgets for each sex, B_m and B_w, are determined exogenously. The budget sets for men and women are thus defined by $\mathcal{B}_s = \{b \in \mathbb{Z} : 0 \leq b \leq B_s\}$, for each sex $s = m, w$. Each period, new men and women enter the platform at rates $\lambda_m, \lambda_w > 0$. Importantly, agents depart from the platform in one of two ways: they can leave *endogenously*, if they expend their swiping budget, or *exogenously* with probability $(1 - \delta)$. This admits to the interpretation of a geometrically distributed lifetime within the platform, parametrized by δ, and implies that users use this as a discounting factor for future payments.

Figure 1: Sequence of events within each time period

It can be shown that an agent's decision on any given time period depends fundamentally on the attractiveness of the candidate and their own budget. The paper's focus is therefore restricted to stationary Markov strategies, defined by $\mu : \Theta \times \mathcal{B}_m \to \Delta\mathcal{A}_m$ for men and $\omega : \Theta \times \mathcal{B}_w \to \Delta\mathcal{A}_w$ for women, where ΔS denotes the probability simplex over set S.

2.2 The Dating Market

Given the sequence of events described in the stage interaction above, I now outline the system state variables that make up the Tinder market, as these must be considered within the model given their endogenous relation with strategic search behaviour. At any given time t, the masses of men and women on Tinder are denoted by N_{mt} and N_{wt}, respectively. Furthermore, let π_{mt}, π_{wt} be the measures over agent budgets on either side, thus π_{mt} denotes the mass of male agents with budgets in $B \subseteq \mathcal{B}_m$; in a slight abuse of notation, we denote by π_{mt}^b the masses over singleton elements $b \in \mathcal{B}_m$. These are endogenously determined since the flow of agents into lower budget levels and eventually out of the platform depends on their swiping decisions. All in all, by considering aggregate variables on both sides of the platform, the Tinder market at time period t is defined as $\Psi_t = (N_{mt}, N_{wt}, \pi_{mt}, \pi_{wt})$. Finally, since gender imbalances can exist, resulting with unpaired agents in the long side of the market, a pairings process must be fixed. Given fairness considerations as well the automated nature of SBDA platforms, I assume an efficient matching technology and model pairings as a Bernoulli process parametrized by market tightness, defining the probability of being paired with a candidate on each side:

$$\tau_{mt} = \min\left\{\frac{N_{wt}}{N_{mt}}, 1\right\}, \quad \tau_{wt} = \frac{N_{mt}}{N_{wt}}\tau_{mt}$$

For most of this paper, I focus on characterizing user behaviour and its resulting implications in a stationary setting, although some discussion of coupled strategy and market dynamics is provided in **Section 4**. As a necessary requirement, the market steady state $\Psi_t = \Psi_{t+1} = ... = \Psi$ must satisfy the balanced flow conditions for our continuum model; these are presented below for the female agents, but they apply analogously to the male side of the market. Firstly, the entry flow of agents into the platform must equal the departure flow:

$$\lambda_w \;=\; \underbrace{N_w(1-\delta)}_{\text{Exogenous Deaths}} + \underbrace{\pi_w^1 \delta \tau_w \int_\Theta \omega(\theta, 1)\, dF_m(\theta)}_{\text{Expended Budgets}} \tag{2.1}$$

Secondly, for both sides, the flow of agents into any particular budget level must equal the outflow of agents from that same level:

$$\underbrace{\pi_w^{b+1} \delta \tau_w \int_\Theta \omega(\theta, b+1)\, dF_m(\theta)}_{\text{Inflow into } b} = \underbrace{\pi_w^b(1-\delta) + \pi_w^b \delta \tau_w \int_\Theta \omega(\theta, b)\, dF_m(\theta)}_{\text{Outflow from } b} \tag{2.2}$$

Finally, the entry flow of agents into the platform must equal the outflow from the top budget level:

$$\lambda_w \;=\; \underbrace{\pi_w^B(1-\delta)} \;+\; \underbrace{\pi_w^B \tau_w \delta \int_\Theta \omega(\theta, b)\, dF_m(\theta)}_{\text{Endogenous outflow from B}} \tag{2.3}$$

2.3 The Search Problem

With the model framework and market dynamics outlined above, I now explore the decision problem faced by female agents in the market, with results carrying analogously for the male side. In the discussion below, I derive the female best-response function given a fixed, stationary market state Ψ and male strategy μ. To begin this analysis, consider a woman i who is paired with a man j in Tinder. The expected ex-interim payoff for this women, given that she observes attractiveness θ for j and chooses action a is the following:

$$U(\theta, a) = (\mathbb{1}\{a = \text{right}\}\, \overline{\mu})\, u(\theta) \overline{\mu} = \sum_{b \in \mathcal{B}_m} \int_\Theta \mu(\theta', b')\, P_m(b')\, dF_w(\theta')$$

Where $\overline{\mu}$ is j's strategy averaged over the possible attractiveness that he may observe for i and the his budget level, both of which are unknown to woman i at the time. Let $\mathcal{K} = \{t :\}$Given that, at the time of pairing, this women has a budget of b right swipes left, then she solves the constrained inter-temporal maximization problem below, capture by the value function $V_w(\theta, b)$:

$$\begin{aligned} V_w(\theta, b) = \max_{\{a_t\}_{t=0}^\infty} \quad & \mathbb{E}_\theta \left[\sum_{t=0}^\infty \delta^t U(\theta_t, a_t) \mid \theta_0 = \theta, b_0 = b \right] \\ \text{s.t.} \quad & b_{t+1} = b_t - a_t \\ & b_t \in \mathcal{B}_w \\ & a_t \in \mathcal{A} \end{aligned} \tag{2.4}$$

Importantly, the first two constraints within this problem make it non-trivial; by limiting woman i's right-swiping budget, Tinder poses a trade-off where swiping right on man j implies foregoing a potential match with another more attractive man in the future, but the exogenous departure process also means that she is not patient enough to wait around for only the top B_w most attractive men. By standard dynamic programming arguments, the above problem can be captured by two Bellman equations; one for when j is paired and another for when she isn't:

$$\begin{aligned} V_w^P(\theta, b) = \max \Big\{ & \overline{\mu}\, u(\theta) \;+\; \delta\tau\, \mathbb{E}\Big[V_w^P(\theta', b-1)\Big] \;+\; \delta(1-\tau) V_w^{NP}(b-1)\,, \\ & \delta\tau\, \mathbb{E}\Big[V_w^P(\theta', b)\Big] \;+\; \delta(1-\tau) V_w^{NP}(b) \Big\} \end{aligned} \tag{2.5}$$

$$V_w^{NP}(b) = \delta\tau\, \mathbb{E}\Big[V_w^P(\theta', b)\Big] \;+\; \delta(1-\tau) V_w^{NP}(b) \tag{2.6}$$

With some straightforward algebra, we can combine the above two equations into the full Bellman equation:

$$V_w(\theta, b) \;=\; \max \Big\{ \overline{\mu}\, u(\theta) + \alpha\, \mathbb{E}\Big[V_w(\theta', b-1)\Big]\,,\; \alpha\, \mathbb{E}\Big[V_w(\theta', b)\Big] \Big\} \tag{2.7}$$

Where α is the effective discount rate accounting for both departures and pairings processes:

$$\alpha := \frac{\tau\delta}{1-\delta(1-\tau)}$$

By inspection, it is clear that the value function is piecewise in θ, meaning that the optimal policy can be parametrised by a set of reservation attractiveness levels, $\{\tilde{\omega}\}_{b\in\mathcal{B}_w}$, where women j swipes right for partners with attractiveness above the reservation level for her budget.:

$$\omega(\theta, b) = \begin{cases} 1, & \theta \geq \widetilde{\omega}_b \\ 0, & \theta < \widetilde{\omega}_b \end{cases}$$

Where the reservation attractiveness levels satisfy:

$$\overline{\mu}u(\widetilde{\omega}_b) = \alpha\,\mathbb{E}_\theta\Big[\,V(\theta', b) - V(\theta', b-1)\,\Big]$$

These reservation attractiveness levels can be computed using the result presented below (with a proof contained in A), effectively allowing use to characterize the female best-response function.

Proposition 1. *The set of reservation attractiveness levels for women, $\{\tilde{\omega}\}_{b\in\mathcal{B}_w}$, satisfies the recurrence relation defined by:*

$$u(\widetilde{\omega}_b) = \alpha u(\widetilde{\omega}_b)F_m(\widetilde{\omega}_b) + \alpha u(\widetilde{\omega}_{b-1})\Big(1 - F_m(\widetilde{\omega}_{b-1})\Big) + \int_{\widetilde{\omega}_b}^{\widetilde{\omega}_{b-1}} \alpha u(\theta')\, dF_m(\theta')$$

along with the initial condition:

$$u(\widetilde{\omega}_1) = \delta u(\widetilde{\omega}_1)F(\widetilde{\omega}_1) + \delta\int_{\widetilde{\omega}_1}^{1} u(\theta')dF(\theta')$$

3 Equilibrium

3.1 Steady-State Equilibrium

Definition 1. *A steady-state equilibrium is defined by a triplet $(\sigma^*_m, \sigma^*_w, \Psi)$ such that:*

1. $\mu^*_b = \widetilde{\mu}(b\,|\,\Psi^*, \omega^*), \quad \forall b \in \mathcal{B}_m$
2. $\mu^*_b = \widetilde{\mu}(b\,|\,\Psi^*, \omega^*), \quad \forall b \in \mathcal{B}_w$
3. $\Psi^* = \Psi(\mu^*, \omega^*)$

- Define and explain concept of SSE
- Explain computation via least-squares
- Explain main properties (eg. ESS & uniqueness)

3.2 Comparative Statics

- Present CS on individual factors and explain intuitively
- These include: patience, risk aversion, distributions
- Present case of gender disbalance... why is it that men always swipe right?
-

4 Agent-Based Simulations

4.1 Convergence and Dynamics

- Check Mass convergence
- Check distribution convergence
- Relate to ESS
- What about Dynamics??? BR

4.2 Social Efficiency

5 Conclusion

In this chapter we shall do a reference to an entry in the bibliography, `bibliography.bib`.

What we know of the invention of the flux capacitor is that Dr. Emmett Brown thought of this when hanging a clock in the bathroom. He was standing on his porcelain sink and slipped because it was wet, the resulting hit on the head was apparently a cause to this invention Olmeda (2021).

5.1 Future Work

The corresponding sketch made on this day has been attached in appendix B.

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A Search Problem

To show the above proposition, we rely the following two equations; the first of which describes the agent's value function in piecewise form, and the second of which describes the necessary condition for all reservation attractiveness values:

$$V(\theta, b) = \begin{cases} u(\theta) + \alpha\, \mathbb{E}_\theta\Big[V(\theta', b-1)\Big], & \theta > \widetilde{\mu}_b \\ \alpha\, \mathbb{E}_\theta\Big[V(\theta', b)\Big], & \theta \le \widetilde{\mu}_b \end{cases} \tag{A.1}$$

$$\overline{\mu} u(\widetilde{\omega}_b) = \alpha\, \mathbb{E}_\theta\Big[\, V(\theta', b) - V(\theta', b-1)\,\Big] \tag{A.2}$$

Now define the continuation value at budget b:

$$\begin{aligned} &K_b := \alpha\, \mathbb{E}_{\theta'}\left[V(\theta', b)\right] \\ \implies\ & V(\theta, b) = \begin{cases} u(\theta) + K_{b-1}, & \theta > \widetilde{\omega}_b \\ K_b, \quad \theta \le \widetilde{\omega}_b \end{cases} \end{aligned}$$

Starting out with equation (A.2) and expanding out the expectation, we can use (A.1) to substitute in the piecewise definitions of $V(\theta, b)$ over the appropriate intervals:

$$\begin{aligned} \overline{\mu} u(\widetilde{\omega}_b) &= \alpha \int_0^1 V(\theta', b) - V(\theta', b-1)\, dF_m(\theta') \\ &= \alpha \int_0^{\widetilde{\omega}_b} K_b\, dF_m(\theta') \ +\ \alpha \int_{\widetilde{\omega}_b}^1 \overline{\mu} u(\theta) + K_{b-1}\, dF_m(\theta') \\ &-\ \alpha \int_0^{\widetilde{\omega}_{b-1}} K_{b-1}\, dF_m(\theta') \ -\ \alpha \int_{\widetilde{\omega}_{b-1}}^1 \overline{\mu} u(\theta) + K_{b-2}\, dF_m(\theta') \end{aligned}$$

Equation (A.2) implies that:

$$\overline{\mu} u(\widetilde{\omega}_b) + K_{b-1} = K_b$$

$$\overline{\mu} u(\widetilde{\omega}_{b-1}) + K_{b-2} = K_{b-1}$$

So by substituting these values into the expression above, cancelling out terms, and dividing across by $\overline{\mu}$:

$$\begin{aligned} &= \alpha \int_0^{\widetilde{\omega}_b} \overline{\mu} u(\widetilde{\omega}_b) + K_{b-1}\, dF_m(\theta') \ +\ \alpha \int_{\widetilde{\omega}_b}^1 \overline{\mu} u(\theta) + K_{b-1}\, dF_m(\theta') \\ &\quad -\ \alpha \int_0^{\widetilde{\omega}_{b-1}} K_{b-1}\, dF_m(\theta') \ -\ \alpha \int_{\widetilde{\omega}_{b-1}}^1 \overline{\mu} u(\theta) + K_{b-1} - \overline{\mu} u(\widetilde{\omega}_{b-1})\, dF_m(\theta') \\ &= \alpha\left(\overline{\mu} u(\widetilde{\omega}_b) \ +\ K_{b-1}\right) F_m(\widetilde{\omega}_b) \ +\ \alpha K_{b-1}\Big(1 - F_m(\widetilde{\omega}_b)\Big) \ +\ \alpha \int_{\widetilde{\omega}_b}^1 \overline{\mu} u(\theta)\, dF_m(\theta') \\ &\quad -\ \alpha K_{b-1} F_m(\widetilde{\omega}_{b-1}) \ -\ \alpha\Big(K_{b-1} - \overline{\mu} u(\widetilde{\omega}_{b-1})\Big)\Big(1 - F_m(\widetilde{\omega}_{b-1})\Big) - \alpha \int_{\widetilde{\omega}_{b-1}}^1 \overline{\mu} u(\theta)\, dF_m(\theta') \\ &= \alpha u(\widetilde{\omega}_b) F_m(\widetilde{\omega}_b) \ +\ \alpha \int_{\widetilde{\omega}_b}^1 u(\theta)\, dF_m(\theta') \ +\ \alpha u(\widetilde{\omega}_{b-1})\Big(1 - F_m(\widetilde{\omega}_{b-1})\Big) \ -\ \alpha \int_{\widetilde{\omega}_{b-1}}^1 u(\theta)\, dF_m(\theta') \end{aligned}$$

- Merging the two integrals:

$$\therefore \quad u(\widetilde{\omega}_b) = \alpha u(\widetilde{\omega}_b) F_m(\widetilde{\omega}_b) \; + \; \alpha u(\widetilde{\omega}_{b-1}) \Big(1 - F_m(\widetilde{\omega}_{b-1})\Big) \; + \; \alpha \int_{\widetilde{\omega}_b}^{\widetilde{\omega}_{b-1}} u(\theta) \, dF_m(\theta')$$

B Notation

- Male types μ
- Female types ω
- Strategies $s = (s_m, s_w)$
- CDF's $M(\mu, b)$, $W(\omega, b)$
- Densities $m(\mu, b)$, $w(\omega, b)$
- Discount δ
- Population CDF's F_m, F_w
- Masses N_m, N_w
- Entry Flows λ_m, λ_w
- Tightness $\tau = \min\{\frac{N_w}{N_m}, 1\}$
- Effective discount α