Swiping Left and Right: Two-Sided Search in Swipe-Based Dating Platforms

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Abstract

In today's love market, swipe-based dating platforms (SBDP's) such as Tinder or Bumble have a well-established presence, but novel platform features can add significant complexities to the user's search problem in ways that have gone largely under-studied in existing literature. This paper formulates a game-theoretic model of two-sided search within SBDP's, along with an appropriate equilibrium refinement. Using numerical methods, I approximate equilibria at the steady-state and perform comparative statics on various model parameters that help explain stylized facts observed in aggregate Tinder data. Finally, agent-based simulations are used to analyse off-path dynamics and discuss how exogenous platform features can be set in a socially-efficient manner.

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1 Introduction

It is widely acknowledged that the search for love is a complex social phenomenon, but in today's world, swipe-based dating platforms (SBDP's) seem to only make it trickier. These platforms, exemplified by Tinder, Bumble, or Hinge, provide a gamified way of browsing through potential romantic partners by swiping through a stack of suggested candidates to indicate likes or dislikes for these, one profile at a time. In the search and matching literature, settings like these fall under the category of decentralized two-sided matching markets (Kanoria and Saban, 2021) and, despite broad differences from traditional dating sites featuring centralized static matching, SBDP's have come to dominate the modern love market, with Tinder alone boasting 75 million monthly active users and 9.6 million paid subscribers as of 2021 (Business of Apps, 2022).

From a theoretical standpoint, search within SBDP's induces several complexities stemming from platform-specific features, such as swiping caps, asynchronicity, and directed search algorithms. These impose non-trivial constraints on the way utility-maximising agents strategise their search, but they have been sparsely studied in the economics literature due to the relative novelty of these platforms. Overall, the prevalent role of SBDP's in shaping modern romantic interactions and their largely understudied nature motivates many different questions. Nevertheless, exploring these demands a fundamental understanding of how users make decisions in these platforms: to put it simply, *when should a utility-maximising user swipe right?*

This dissertation will explore the above within an SBDP setting, where agents with heterogeneous preferences on both sides of the market search simultaneously for multiple romantic partners. Crucially, I focus on explaining (what I refer to as) the 'Fast-Swiping Males' puzzle: that is, the empirical observation that men in SBDP's respond with significantly higher swipe rates and face considerably worse matching outcomes than women. This phenomenon has been both a subject of empirical research (Tyson et al., 2016) and a contentious discussion point within mainstream media (Vice News, 2016; The Washington Post, 2016), and yet a significant gap persists within the literature for a discussion of this through a theoretical lens [1]. Such analysis would add significant value since the potential causes of this phenomenon (user patience, differential preferences, and strategic dominance) are all systematically endogenous with one another, thus demanding a rigorous model that can isolate these individual effects and capture their propagation across the SBDP market. Fundamentally, I show how gender imbalances within the platform (which arise due to several exogenous factors) can explain the above disparities and, expanding on this, I model a possible intervention where the swiping cap ratio between sexes can be set in a socially-efficient manner.

[1] Among the surveyed literature, perhaps the only partial examination of this phenomenon is provided by Kanoria and Saban (2021)

This work presents two main contributions to existing literature on the topic. Firstly, it constitutes one of a handful of attempts to model the market configurations arising within SBDP's, which is unsurprising due to the novelty of these platforms, but important given their current social relevance. Furthermore, it distinguishes itself from these others by directly considering both the 'Fast-Swiping Males' puzzle as well as the impact of swiping caps both as a constraint to the agent's search problem and a potential market correction mechanism. Finally, this work provides an interesting case study for the use of computational methods within game theory, a field that has traditionally emphasised pure mathematical analysis. By pairing a rigorously-formulated model with numerical computations and agent-based simulations, this dissertation exemplifies how the two approaches, rather than being mutually exclusive, can be jointly applied to complicated questions, as computational methods enable quick explorations that can serve as a stepping stone towards formalising mathematical arguments.

The remainder of the paper is structured as follows. In Section 2, I outline the theoretical framework for the model developed in this paper, and derive necessary conditions for both the system steady state and the agent best-responses. In Section 3, I present a refined definition for the steady state equilibrium of the model and perform computational comparative statics on several parameters, with the aim of replicating stylized empirical facts and explaining the 'Fast-Swiping Males' phenomenon. In Section 4, I utilize agent-based simulation methods to analyse the convergence and dynamics of my model, and present a discussion on socially-efficient budget interventions. Finally, Section 5 presents concluding remarks and outlines potential avenues for future research.

1.1 Related Work

The present work draws inspiration from two key branches of economics literature: that of search and matching, which studies the decision-making process of agents who seek, for example, a job, a business partner, or a spouse, and that of mean-field game theory, which models complex dynamic games involving a large number of players. I discuss each of these in turn, and then contrast this work with the handful of papers that have focused on specifically analysing SBDP market configurations.

Despite the abundance of papers within the search and matching literature, which has been amply surveyed by Chade et al. (2017), I draw focus on works that consider the three defining features of SBDP markets: decentralised matching, two-sidedness, and non-transferable utility. A seminal paper at this intersection is that of Burdett and Coles (1997), which studies the marriage market for ex-ante heterogeneous agents under uniform random search, extending the work of Becker (1973) by showing that positive assortative matching can arise even in the absence of log-supermodularity. Several extensions follow from this, considering settings with idiosyncratic preferences (Burdett and

Wright, 1998), noisy attractiveness observations (Chade, 2006), and even convergence onto the set of stable matchings (Adachi, 2003). The framework outlined in this dissertation is perhaps most similar to that of Burdett and Wright (1998), with three major distinctions. Firstly, the model formulated in this paper extends the above by allowing for multiple partners within a user's lifetime, a feature which was probably not significant within the labour market context considered by Burdett and Wright (1998), but which is nevertheless quintessential of SBDP's given their role in fomenting casual relationships. Furthermore, I also extend the work of Burdett and Wright by allowing for sex-specific mass differences, as well as exogenous agent inflows; a point which of noted interest for the authors themselves, and which is fundamental when considering the effects of gender imbalances within the platform. Finally, my framework considers a discrete time framework, departing from Burdett and Wright (1998) and most of the matching literature. Although continuous-time models provide sharper analysis and more flexible empirical specifications (Burdett and Coles, 1999), this modelling choice lends itself naturally to the use of agent-based simulations, which are used to explore equilibria convergence and dynamics in a richer manner.

On the other hand, mean-field game theory focuses on dynamic games with a large number of agents, for which curses of dimensionality arise due to intractable state spaces (Light and Weintraub, 2022). Mean-field models tackle this issue by considering interactions with the *aggregate state* only, rather than interactions with all other players. This simplification is cemented with a *consistency check*, such that equilibria arises when rational play given an aggregate state maintains this same state as a fixed point. This approach, perhaps first considered by Jovanovic and Rosenthal (1988), has been successfully applied to settings such as network routing (Calderone and Sastry, 2017), dynamic auctions with learning (Iyer et al., 2014) and, perhaps most relevantly, online matching platforms (Kanoria and Saban, 2021; Immorlica et al., 2021). In this paper, I rely on mean-field assumptions to abstract from observability considerations: within SBDP's, the market-wide history is unobservable to players, and thus traditional concepts such as Perfect Bayesian Equilibrium would require beliefs over history spaces, and even beliefs over the beliefs other players may hold (a complication known as nested beliefs (Brandenburger and Dekel, 1993)). This yields two central problems: first, that equilibria become virtually impossible to compute, and, by extension, that the model assumes an unreasonable level of rationality on behalf of agents. Thus, by considering interactions with the platform state, the model presented in this paper characterizes equilibria that are both insightful and representative of real-life behaviour and dynamics.

Among the few papers specifically considering SBDP matching markets, Kanoria and Saban (2021) propose a two-sided dynamic matching model with vertically-differentiated agents, and show that platforms with unbalanced markets can improve welfare by forcing the short side to propose. Furthermore, Immorlica et al. (2021) focus on the problem of determining a directed search algorithm in SBDP's through type-contingent meeting rates for agents. Both of these papers present theoretical models with similar features, and these have largely influenced my work in several ways; for example, by embedding mean-field assumptions that simplify the SBDP market from a game theoretical perspective. Despite this, there are three main differences between my work and above that are worth discussing.

2 Theoretical Model

2.1 Setup

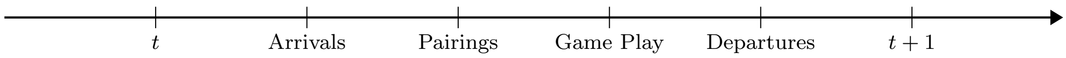
In this section, I establish the theoretical framework for the model developed throughout this paper. Fix a non-atomic continuum of male and female agents and consider the dynamic two-sided market formed by the SBDP, which agents can join to search for potential romantic partners. For ease of exposition, I assume that this market is heteronormative such that male agents search exclusively for female agents and vice-versa. Time is discrete and indexed by $t = 0, 1, 2, ...$ over an infinite horizon. Each period, masses $\lambda_m, \lambda_w > 0$ of new men and women enter the platform, where they are paired and presented a candidate partner from the opposite side of the market.

We model agents with heterogeneous preferences (capturing the notion that 'beauty lies in the eye of the beholder') and thus, after being paired, each agent observes an *idiosyncratic attractiveness value* $\theta \in \Theta := [0, 1]$ for their candidate. These values are i.i.d according to a pair of absolutely continuous CDF's F_m, F_w, with corresponding PDF's f_m, f_w. Female agents draw male candidate values from F_m and vice versa. Importantly, the value man i draws for woman j does not necessarily equal the value that j draws for i, and, for simplicity, these are modelled these as independent from one another.

After observing their candidate's attractiveness, agents then choose whether to swipe left (dislike) or right (like) on them, yielding an action space $\mathcal{A} = \{\text{swipe left}, \text{swipe right}\}$. If both agents swipe right on one another, they are said to have *matched* and both receive a matching payoff, however, if either agent swipes left, they both receive a payoff of zero. Contingent on swiping right on a candidate with attractiveness θ, a user earns a matching payoff $u(\theta)$, where $u(\cdot)$ is a continuous, strictly increasing function that satisfies $u(0) = 0$. This last property stems from the fact that, in most SBDP's, users are allowed to unmatch with each other, therefore matching with the least attractive individual on the other side of the market is weakly preferred to not matching.

After payoffs have been received, players are paired to different candidates and the stage interaction is repeated. Given the continuum of agents, I assume that interactions take place *anonymously* in the style of Jovanovic and Rosenthal (1988). Furthermore, to the agents' knowledge, pairings are determined in an unknown manner (since SBDP's are generally secretive regarding the algorithms they use), effectively making their problem one of uniform random search.

Figure 1: Sequence of events within each time period



Perhaps trivially, swiping right in the stage interaction described above is weakly dominant for all agents and yields a Pareto-optimal outcome, thus implying that, in a repeated interaction, the market equilibrium would have all agents exclusively swiping right. This becomes problematic since the main selling point of SBDP's is a reduction in searching costs for individuals seeking romantic encounters, and this is only accomplished if matches have a high likelihood of resulting in real-life romantic attraction. Because of this, SBDP's like Tinder place a cap on the total number of right swipes for each user, thus enabling this as a form of costly signalling. I refer to the total number of right-swipes a user has left as its *budget*, b_t, which evolves dynamically according to the law of motion:

$$b_{t+1} = b_t - a_t$$

where the budget caps for each sex, B_m and B_w, are determined exogenously. The budget sets for men and women are thus defined by $\mathcal{B}_s = \{b \in \mathbb{Z} : 1 \leq b \leq B_s\}$, for each sex $s = m, w$. Importantly, agents depart from the platform in one of two ways: they can leave *endogenously*, if they expend their swiping budget, or *exogenously* with probability $(1 - \delta)$. This admits to the interpretation of a geometrically distributed lifetime within the platform, parametrised by δ, and implies that users use this as a discounting factor for future payments.

Due to the infinitely-sized market, agents can assume that they seldom interact with the same candidate more than once throughout their lifetime. This, along with the mean-field assumption established in subsection 2.2, simplifies gameplay significantly by making it dependency on the candidate attractiveness and their own budget only. I therefore restrict focus to (pure) stationary strategies, which are both time and history-independent, denoted by $\mu : \Theta \times \mathcal{B}_m \to \mathcal{A}_m$ for men and $\omega : \Theta \times \mathcal{B}_w \to \mathcal{A}_w$ for women.

2.2 The Dating Market

Given the above framework, I now outline the platform state variables that make up the SBDP market. Let $N_{mt}(b), N_{wt}(b)$ denote the mass of male and female agents (respectively) with a budget of $b \in \mathcal{B}$ in a given time period t. Furthermore, since gender imbalances can leave some agents in the long side of the market unpaired, a pairings process must be fixed. Given fairness considerations as well the automated nature of

SBDP's, I assume an efficient matching technology and model pairings as a Bernoulli process parametrised by market tightness; thus, the probability of being paired with a candidate is defined for both sides as:

$$\tau_{mt} := \min\left\{\frac{\sum_{b\in\mathcal{B}_w} N_{wt}(b)}{\sum_{b\in\mathcal{B}_m} N_{mt}(b)}, 1\right\}, \quad \tau_{wt} := \left(\frac{\sum_{b\in\mathcal{B}_m} N_{mt}(b)}{\sum_{b\in\mathcal{B}_w} N_{wt}(b)}\right)\tau_{mt}$$

From the above, the platform state at time period t can be defined as $\Psi_t = (N_{mt}, N_{wt})$. For most of this paper, I focus on characterising user behaviour and its resulting implications in a stationary setting (which is denoted by omitted time subscripts), although some discussion of coupled strategy and market dynamics is provided in Section 4. As a necessary requirement, the market steady state $\Psi_t = \Psi_{t+1} = ... = \Psi$ must satisfy the balanced flow conditions [2] for our continuum model; these are presented below for the female agents, but they apply analogously to the male side of the market. Firstly, the entry flow of agents into the platform must equal the departure flow:

$$\lambda_w \;=\; \underbrace{(1-\delta)\sum_{b\in\mathcal{B}_w} N_w(b)}_{\text{Exogenous Outflow}} \;+\; \underbrace{N_w(1)\delta\tau_w\int_\Theta \omega(\theta,1)\,dF_m(\theta)}_{\text{Endogenous Outflow}} \tag{2.1}$$

Secondly, for both sides, the flow of agents into any particular budget level must equal the outflow of agents from that same level. Thus, for all $b \in \mathcal{B}_w$:

$$\underbrace{N_w(b+1)\delta\tau_w\int_\Theta \omega(\theta,b+1)\,dF_m(\theta)}_{\text{Inflow into } b} \;=\; \underbrace{N_w(b)\Big[(1-\delta) \;+\; \delta\tau_w\int_\Theta \omega(\theta,b)\,dF_m(\theta)\Big]}_{\text{Outflow from } b} \tag{2.2}$$

Finally, the entry flow of agents into the platform must equal the outflow from the top budget level, hence:

$$\lambda_w \;=\; \underbrace{N_w(B_w)\Big[(1-\delta) \;+\; \tau_w\delta\int_\Theta \omega(\theta,B_w)\,dF_m(\theta)\Big]}_{\text{Outflow from } B_w} \tag{2.3}$$

Importantly, the above conditions take strategy profile (μ,ω) as exogenously fixed. Over the next section, these are endogenously derived from the agents' best-responses, which themselves depends on the market state.

[2] Formally, these conditions rely on the exact law of large numbers, which has been rigorously developed for discrete-time settings by Duffie et al. (2018), but a technical discussion of this lies outside the scope of this paper.

2.3 The Search Problem

With the model framework and market dynamics outlined above, I now explore the decision problem faced by female agents in the market, with analogous results and implications for the male side. In the discussion below, I derive the female best-response function given a fixed, stationary market state Ψ and male strategy μ. To begin this analysis, consider a woman i who is paired with a man j in Tinder. The expected ex-interim payoff for this women, given that she observes attractiveness θ for candidate j and chooses action a, is the following:

$$U(\theta, a) = \Big(\overline{\mu}\,\mathbb{1}\{a = \text{swipe right}\}\Big)\, u(\theta), \quad \text{where} \quad \overline{\mu} \;= \sum_{b \in \mathcal{B}_m} \int_{\Theta} \frac{N_m(b)}{N_m}\, \mu(\theta', b)\, dF_w(\theta')$$

Here, $\overline{\mu}$ denotes candidate j's strategy averaged over the possible attractiveness that he may observe for i and his possible budget level, both of which are unknown to woman i. While traditional equilibrium concepts would consider candidate j's individual behaviour and, necesarily, beliefs over his possible budget level, the payoff function above imposes a mean-field assumption such that, conditional on the platform state Ψ, woman i considers only the empirical frequency with which she receives a right-swipe. This modelling choice has been employed by Immorlica et al. (2021) and Iyer et al. (2014) among others, as it simplifies the full-fledged dynamic game by collapsing it onto a pair of Markov Decision Processes (MDP's), where strategy ω is a best response for female agents if and only if it is an optimal policy for the corresponding MDP. Let the jump times of the realized pairing process for woman i be index by k. Given that, at the time of pairing, this women has a budget of b right swipes left, she then solves the constrained MDP presented below, captured by the value function $V_w(\theta, b)$:

$$\begin{aligned} V_w(\theta, b) = \max_{\{a_k\}_{k=0}^{\infty}} \quad & \mathbb{E}_\theta \left[\sum_{k=0}^{\infty} \delta^k U(\theta_k, a_k) \mid \theta_0 = \theta, b_0 = b \right] \\ \text{s.t.} \quad & b_{k+1} = b_k - a_k \\ & b_k \in \mathcal{B}_w \cup \{0\} \\ & a_k \in \mathcal{A} \end{aligned}$$

Importantly, the first two constraints along with the exogenous departure process make this problem non-trivial; by limiting woman i's right-swiping budget, the platform imposes an opportunity cost between swiping right on man j and foregoing potential future matches with more attractive men, whilst the exogenous departure process removes the possibility of simply waiting around to swipe right on the top B_w most attractive men. By standard dynamic programming arguments, this problem can be captured by two Bellman equations; one for when j is paired and another for when she isn't:

$$V_w^P(\theta, b) = \max\Big\{\, \overline{\mu}\, u(\theta) \;+\; \delta\tau_w\, \mathbb{E}_\theta\Big[V_w^P(\theta', b-1)\Big] \;+\; \delta(1-\tau)V_w^{NP}(b-1)\,, \\ \delta\tau_w\, \mathbb{E}_\theta\Big[V_w^P(\theta', b)\Big] \;+\; \delta(1-\tau_w)V_w^{NP}(b)\,\Big\} \tag{2.4}$$

$$V_w^{NP}(b) = \delta\tau_w\, \mathbb{E}_\theta\Big[V_w^P(\theta', b)\Big] \;+\; \delta(1-\tau_w)V_w^{NP}(b) \tag{2.5}$$

With some straightforward algebra, the above two equations can be merged into the full Bellman equation below. Note that, by imposing the swiping budget constraint from the above MDP, it must be the case that $V_w(\theta, 0) = 0,\ \forall\theta \in \Theta$, since agents who expend their budget must leave the platform and can't accumulate any additional payoffs:

$$V_w(\theta, b) \;=\; \max\Big\{\, \overline{\mu}\, u(\theta) + \alpha\, \mathbb{E}_\theta\Big[V_w(\theta', b-1)\Big]\,,\; \alpha\, \mathbb{E}_\theta\Big[V_w(\theta', b)\Big]\,\Big\} \tag{2.6}$$

Where α is the effective discount rate accounting for the exogenous possibilities of both departures and pairings, defined as:

$$\alpha := \frac{\tau_w\delta}{1-\delta(1-\tau_w)}$$

Upon inspection, it is clear that the value function is of a piecewise nature over Θ, in a similar manner to McCall (1970). This is more formally stated bellow:

Proposition 1. *Fix some $b \in \mathcal{B}_w$. Then the value function $V_w(\theta, b)$ for women admits the following piecewise form, with a flat portion, over Θ:*

$$V_w(\theta, b) = \begin{cases} \overline{\mu}u(\theta) + \alpha\, \mathbb{E}_\theta\Big[V_w(\theta', b-1)\Big], & \theta > \widetilde{\omega}_b \\ \alpha\, \mathbb{E}_\theta\Big[V_w(\theta', b)\Big], & \theta \leq \widetilde{\omega}_b \end{cases}$$

where $\widetilde{\omega}_b$ satisfies: $\quad \overline{\mu}u(\widetilde{\omega}_b) = \alpha\, \mathbb{E}_\theta\Big[\, V_w(\theta', b) - V_w(\theta', b-1)\,\Big]$

From this, the reservation attractiveness levels $\{\widetilde{\omega}\}_{b\in\mathcal{B}_w}$ must be such that woman i is indifferent between swiping left or right, thus creating the piecewise split in V_w. The above result helps determine a functional form for the optimal policy, in which women j swipes right for partners who exceed the reservation level for her current budget:

Corollary 1. *The following policy $\widetilde{\omega}$, parametrised by $\{\widetilde{\omega}\}_{b\in\mathcal{B}_w}$, attains $V_w(\theta, b)$:*

$$\widetilde{\omega}(\theta, b) = \begin{cases} \textit{swipe right}, & \theta \geq \widetilde{\omega}_b \\ \textit{swipe left}, & \theta < \widetilde{\omega}_b \end{cases}$$

Even though the reservation attractiveness levels can be computed using numerical methods, such as value or policy iteration, these are more explicitly characterized by the result below (with a corresponding proof included in Appendix A):

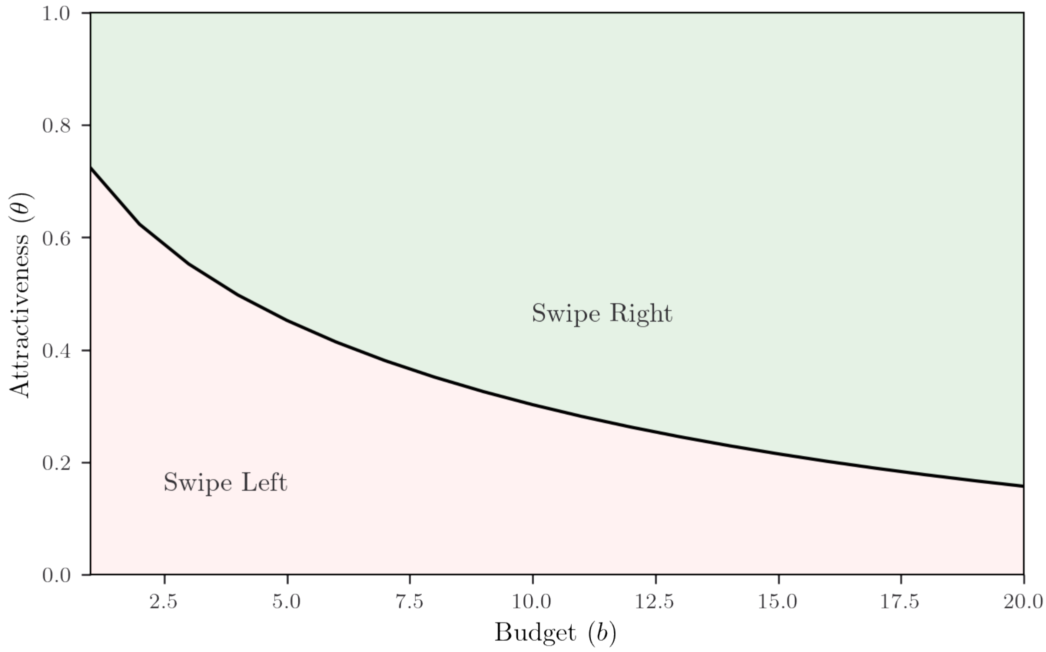
Proposition 2. *The set of reservation attractiveness levels for women, $\{\widetilde{\omega}_b\}_{b\in\mathcal{B}_w}$, uniquely satisfies the recurrence relation and initial condition below, over the budget set $\mathcal{B}_w$:*

$$u(\widetilde{\omega}_b) \;=\; \alpha u(\widetilde{\omega}_b)F_m(\widetilde{\omega}_b) \;+\; \alpha u(\widetilde{\omega}_{b-1})\Big[1 - F_m(\widetilde{\omega}_{b-1})\Big] \;+\; \int_{\widetilde{\omega}_b}^{\widetilde{\omega}_{b-1}} \alpha u(\theta')\, dF_m(\theta') \tag{2.7}$$

$$u(\widetilde{\omega}_1) \;=\; \alpha u(\widetilde{\omega}_1)F(\widetilde{\omega}_1) \;+\; \alpha \int_{\widetilde{\omega}_1}^{1} u(\theta')dF(\theta') \tag{2.8}$$

By inspecting the result in Proposition 2, it is evident that the aggregate behaviour of the opposite side of the market ($\overline{\mu}$) has no direct influence over female best responses. Instead, this influence happens indirectly through the steady state masses and their effect on how agents discount inter-temporal payoffs through α. Using the recurrence relation in Proposition 2, the agent-best response was computed for an arbitrary set of exogenous parameters, with results shown in Figure 2. As evidenced by these, the optimal policy (and best-response function) for agents is marked by a clear cut-off rule for swiping right. These cut-off values are decreasing in the agent's budget, which captures the notion that an agent's current swipe is more valuable than all preceding ones given the increasing opportunity cost.

Figure 2: The Optimal Swiping Rule



3 Equilibrium & Comparative Statics

3.1 Steady State Equilibrium and Computation

Using the framework and results above, I now present a refined definition for the steady state equilibrium of the market:

Definition 1. *A Steady State Equilibrium (SSE) is a triplet* $(\mu^*, \omega^*, \Psi^*)$ *such that:*

1. $\mu^*(\theta, b)$ *attains* $V_m(\theta, b)$, $\forall\, \theta, b \in \Theta \times \mathcal{B}_m$, *given* ω^*, Ψ^*
2. $\omega^*(\theta, b)$ *attains* $V_w(\theta, b)$, $\forall\, \theta, b \in \Theta \times \mathcal{B}_w$, *given* μ^*, Ψ^*
3. Ψ^* *satisfies Equations 2.1, 2.2, and 2.3 given the strategy profile* (μ^*, ω^*)

Intuitively, the above definition establishes two requirements that must be satisfied by an equilibrium market configuration. Firstly, it must be the case that μ^* and ω^* are mutual best responses given the platform state Ψ^* for which, as previously outlined, a necessary and sufficient condition would have them each solve the sex-specific MDP. These two conditions alone correspond to *partially rational expectations equilibria* (PREE), as per Burdett and Coles (1997), since they require agents to play optimally for some fixed steady state Ψ^*, imposing rationality on all game aspects other than the platform state dynamics. Furthermore, in line with mean-field game theory literature, a *consistency check* is imposed by the third condition, which requires that the platform steady state to which agents are best-responding with (μ^*, ω^*) is sustained as a fixed point, thus constraining the set of PREE to define full steady state equilibria.

Although formal proofs for the existence and uniqueness of steady-state equilibria are outside the scope of this paper, I rely on numerical procedures[3] to approximate equilibria under various exogenous settings. This approach has been frequently employed by related works (see Iyer et al., 2014; Gummadi et al., 2011) as it can help uncover insights provided by mean-field models. To compute model equilibria, I frame the recurrence relation presented in Proposition 2, as well as Equations 2.1, 2.2, and 2.3, as a system of $2(|\mathcal{B}_m| + |\mathcal{B}_w| + 1)$ non-linear equations, and solve this using a modified version of the hybrid Powell method, as implemented by the MINPACK 1 routine (Moré et al., 1980)

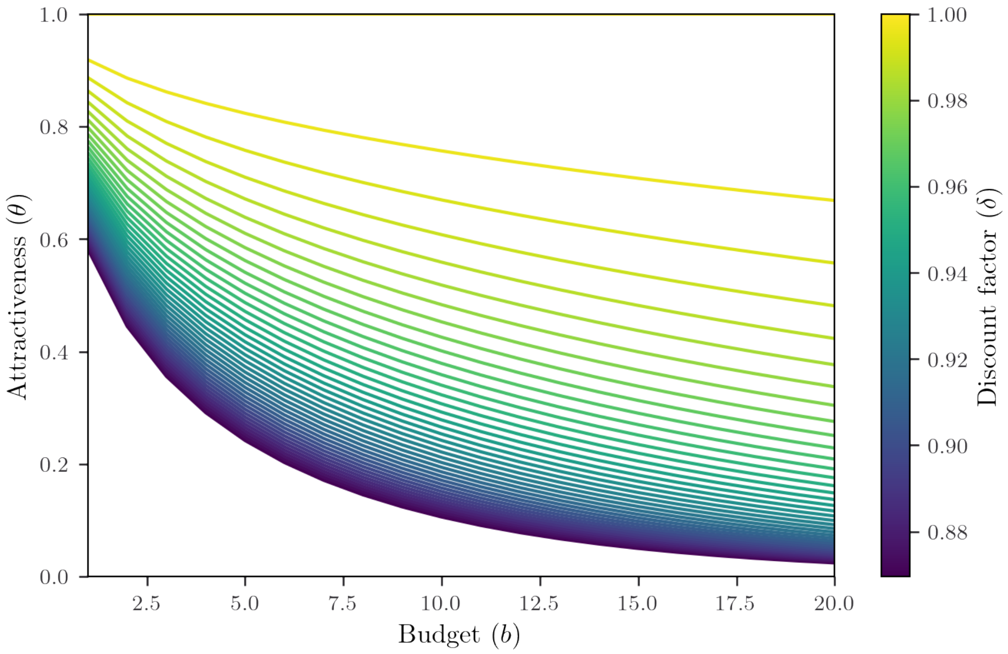
3.2 Best Response Analysis

Using the computational procedures outlined above, a number of insights can be uncovered related to how exogenous parameters affect an agent's best-response swiping strategy. The first parameter I analyse is the discount factor, which represents the probability of remaining inside the platform for an additional time period, but is often interpreted as the

[3] The code required to reproduce all analysis presented in this paper is fully accessible under the GitHub repository `patohdzs/project-tinder`

representative agent's patience level. To determine the effects of changes in the discount factor, I computed the best-response policy over a range of different values for δ (using an arbitrary set of exogenous parameters), with results shown in Figure 3. Evidently, as the agent becomes less patient, they 'lower their standards' for potential matches in the platform, shifting their swiping curve downwards.

Figure 3: Comparative Statics on the Discount Factor

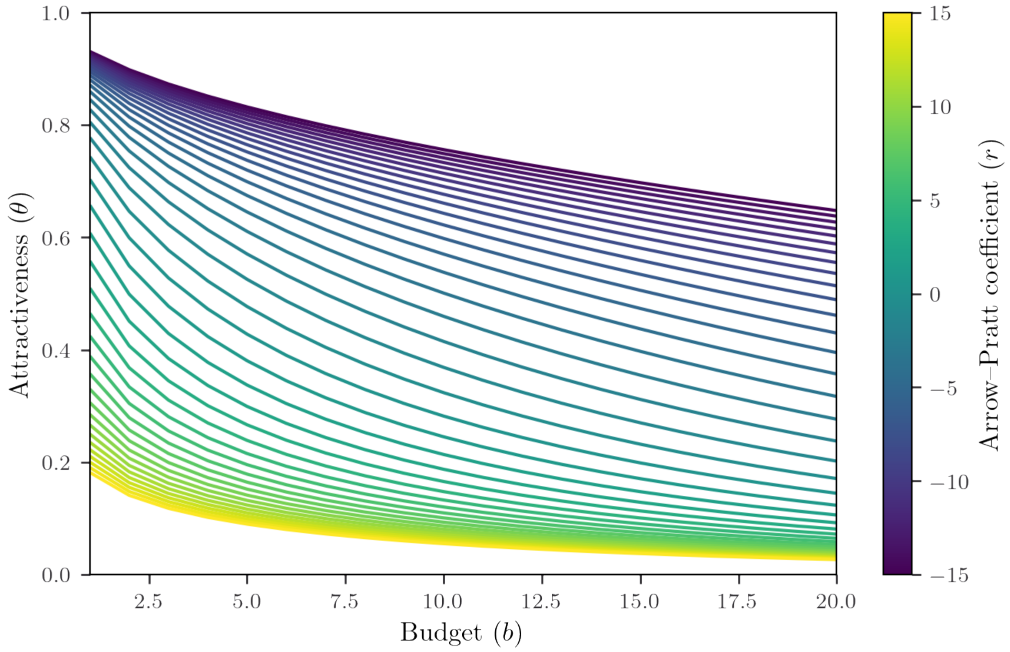


Another interesting parameter to examine is the absolute risk aversion of agents, which I choose to interpret as their 'desperateness' for matching. In the platform, risk-averse agents prefer a higher chance of matching (even if these yield relatively lower payoffs), whilst risk-loving agents prefer to wait around and save their swipes for high-yield candidates. To perform comparative statics on this parameter, I fix a CARA utility function for agents, with parameter r corresponding to the Arrow-Pratt coefficient for absolute risk aversion. I then compute the optimal swiping rule for various different values of r, with results for this shown on Figure 4. From here, it is evident that as agents become 'more desperate' for matches, implied by rising absolute risk aversion, they lower their standards for right-swiping on a candidate, thus shifting their swiping curve downwards.

3.3 Market Configuration Analysis

Finally, I perform comparative statics at the platform level to determine how different factors affect market configurations. This is especially important as it considers not only the effects on best-responses for one sex, but also how these propagate across the market through its aggregate state. More specifically, I focus the aforementioned 'Fast-Swiping

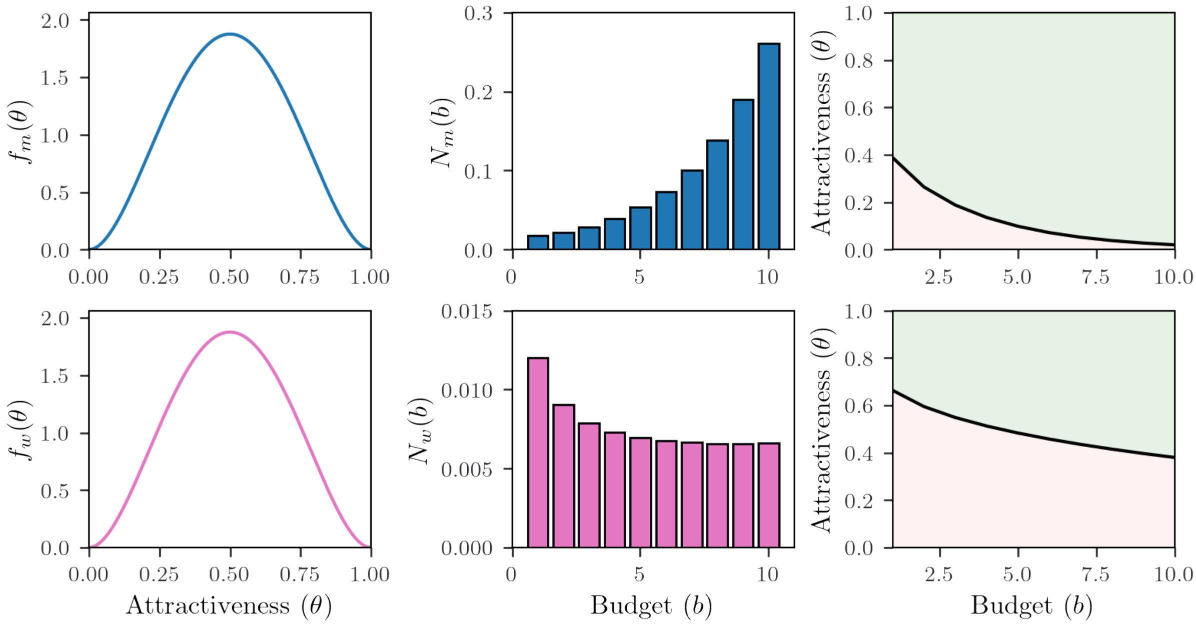
Figure 4: Comparative Statics on Absolute Risk Aversion



Males' puzzle, investigating the discrepancy in swiping rates and matching outcomes between men and women, and I present two possible explanations for how the model developed in this paper can replicate and explain this outcome. The first of these concerns differential agent inflows between men and women, which occur exogenously within my model but are in line with empirical findings, which place. To asses the market configurations arising from of this situation, I compute the model equilibria under a 6:1 ratio between arrival rates λ_m and λ_w. The results for this are shown in Figure 5, highlighting three main insights for this scenario.

Firstly, under the above scenario, the steady-state mass of male agents in the platform is around ten times greater than that of female agents (in line with empirical estimates), implying that male agents face a tight market and struggle to get paired with female candidates. This is further evidenced by the top-center plot within Figure 5, which shows that male agents are highly concentrated in the top budget levels. Due to the effect of market tightness on the effective discount rate, male agents are also more impatient than women on the platform, which makes sense intuitively as they also face considerably worse matching odds. This effect is captured by their best-response strategy, which sits considerably lower than the female swiping curve, effectively showing how a congested market lowers male patience and by extension, their standards, leading them to swipe right on most women. Ultimately, this explains the 'Fast-Swiping Males' puzzle given that, under this particular equilibrium, men receive right-swipes with probability $\overline{\omega} = 0.491$, compared to $\overline{\mu} = 0.988$ for women, thus replicating the observed phenomenon through a traceable shock on agent inflows.

Figure 5: Market Configuration Under Differential Agent Inflows



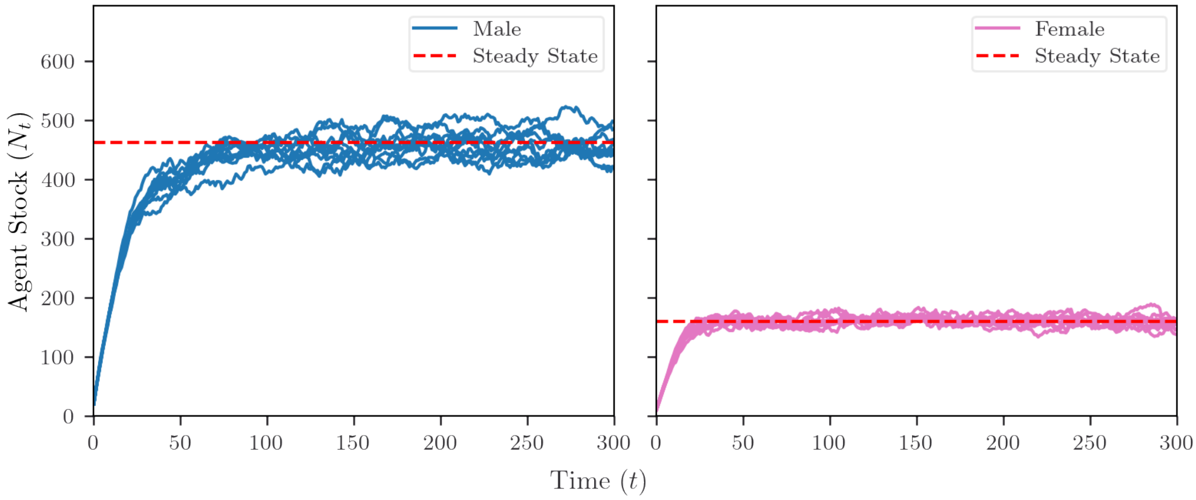
4 Agent-Based Simulations

4.1 Convergence and Dynamics

Given the lack of accessible SBDP user data, I developed on agent-based simulation environment to explore the evolution of both behavioural and market-level dynamics under the above theoretical foundation. Agent-based modelling (ABM) are used to study how *"macro phenomena emerges from micro level behaviour among a heterogeneous set of interacting agents"* (Janssen, 2005). As such, ABM methods aid in the analysis of complex dynamical processes through computational simulations, as opposed to determining closed-form solutions for these through heavily simplified assumptions, which can sometimes result in large differences between predicted and real-life trajectories for complex systems (such as markets). When combined with a micro-founded theoretical model, ABM can provide powerful insights by aiding in the identification of equilibrium states, which can be computationally expensive to approximate (and in some cases even non-existent).

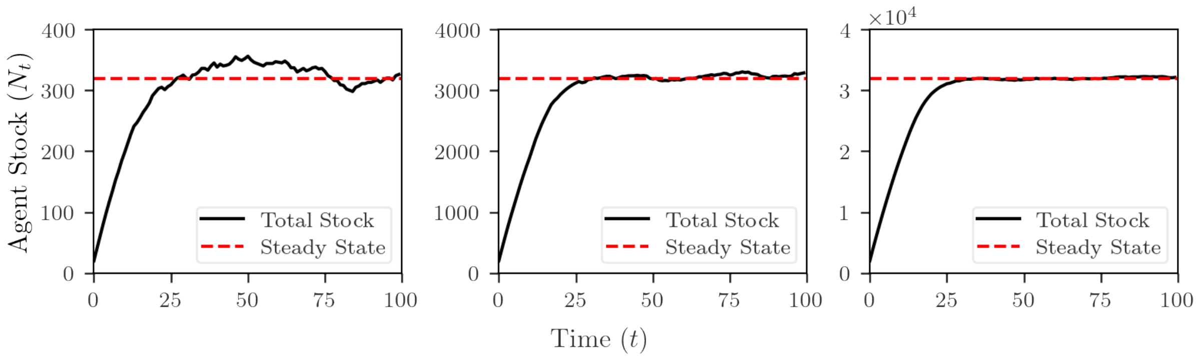
To start, I explore the convergence and stability of the SBDP market under various exogenous settings. In particular, Figure 6 shows the simulated evolution of (sex-specific) agent masses over 300 time periods, with a 2:1 ratio between male and female arrival flows. This simulation was conducted under PREE conditions; that is, with agents using optimal policies for some fixed steady state even when this is not equal to the current platform state. As evident from these results, this process converges onto the fixed steady state computed using the procedures in subsection 3.1. Furthermore, the ABM simulations

Figure 6: Agent-Based Simulation Convergence



show that the long side of the market (males) takes considerably longer to converge onto its steady-state level. One technical point worth noting is that the above simulations depict *agent stocks* as opposed to *agent masses*, as per our continuum model. Thus, because agent departures and pairings follow random processes, the above equilibrium acts as a *stochastic steady state*, with convergence occurring in the stationary sense rather than in typical deterministic fashion. Nevertheless, I examine the limiting case of these dynamics, with Figure 7 depicting how, by the law of large numbers, stationary deviations around the steady state level become negligible as the agent stocks tend to infinity.

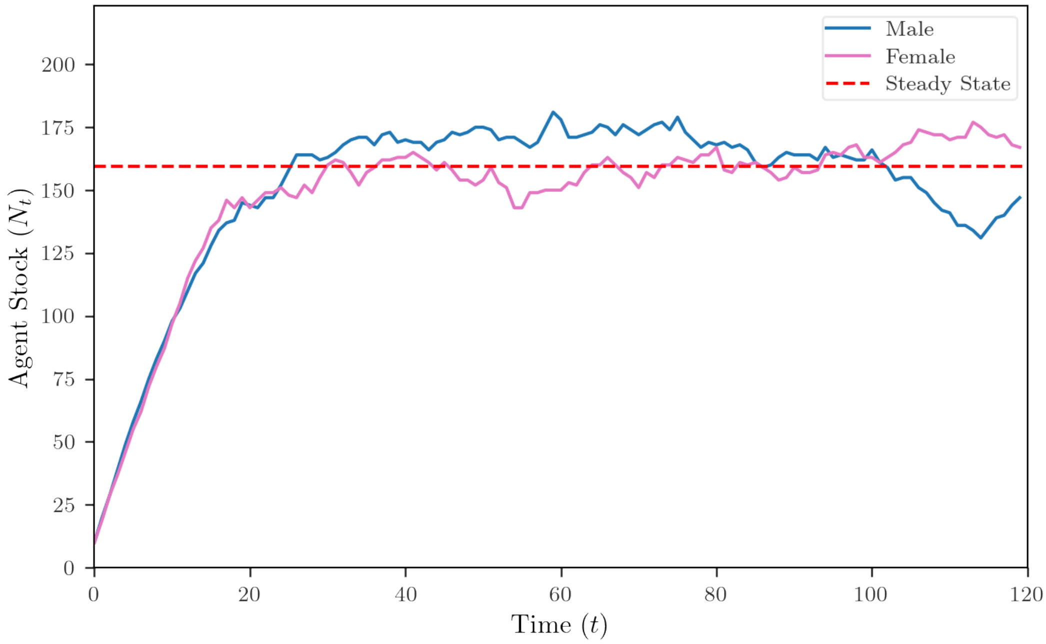
Figure 7: Agent-Based Simulation Convergence with Varying Sample Sizes



Finally, I simulate the SBDP market under myopic best-response dynamics to explore whether if steady state equilibria can be reached using a more robust process of gameplay. For this simulation, agents re-compute their optimal policies at the start of every time period given the current market state, unlike with PREE simulations where optimal policies are computed once with respect to the SSE for some given exogenous settings.

This process is *myopic* in the sense that optimal policies are computed accounting only for the current state but not for its dynamic evolution; yet it is still more robust than PREE as constant feedback between policies and states could create outward-spiralling dynamics that prevents convergence onto SSE. The results of these simulations over 120 time periods are presented in Figure 8, showing that, both in the case of balanced and unbalanced markets, the SSE can be attained using myopic best response dynamics.

Figure 8: Agent-Based Simulation Under Myopic Best Response Dynamics



5 Conclusion

Overall,

5.1 Future Work

There are several interesting avenues for future research concerning SBDP matching platforms. Firstly, although I present a proof for the existence and uniqueness of agent best-responses,

Firstly, more work could be done to explore alternative dynamics behind these platforms, both through ABM but

- Evolutionary dynamics? - Mixed preferences... - Model that allows for both multiple casual matches and long-term partnerships, after which agents leave the market

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A Proofs for subsection 2.3

To simplify notation for Appendix A, I denote the continuation value at budget b by:

$$K_b := \alpha\, \mathbb{E}_\theta \left[V_w(\theta', b)\right]$$

A.1 Proof for Proposition 1 and Corollary 1

Proof. To prove Proposition 1, fix any $b \in \mathcal{B}_w$. First, note that the difference between any two consecutive continuation values K_b and K_{b-1} must lie between 0 and $\overline{\mu}u(1)$. This is true since the value function denotes the expected lifetime sum of payoffs, and an additional right-swipe can provide an agent with, at most, an additional payoff of $\overline{\mu}u(1)$ and, at least, a payoff of 0. Now consider:

$$\begin{aligned} V_w(\theta, b) \;&=\; \max\left\{ \overline{\mu}\, u(\theta) + \alpha\, \mathbb{E}_\theta\Big[V_w(\theta', b-1)\Big] \,,\; \alpha\, \mathbb{E}_\theta\Big[V_w(\theta', b)\Big] \right\} \\ &=\; \max\left\{ \overline{\mu}\, u(\theta) + K_{b-1} \,,\; K_b \right\} \\ &=\; K_{b-1} + \max\left\{ \overline{\mu}\, u(\theta) \,,\; K_b - K_{b-1} \right\} \end{aligned}$$

Since $u(\theta)$ is, by assumption, continuous and increasing over Θ, then it must be that the second term on the RHS of the line above equals $K_b - K_{b-1}$ over $[0, \widetilde{\omega}_b]$, and $\overline{\mu}\, u(\theta)$ over $[\widetilde{\omega}_b, 1]$, where $\widetilde{\omega}_b$ satisfies:

$$\begin{aligned} \overline{\mu}u(\widetilde{\omega}_b) &= K_b - K_{b-1} \\ &= \alpha\, \mathbb{E}_\theta\Big[\, V_w(\theta', b) - V_w(\theta', b-1) \,\Big] \end{aligned}$$

Thus, by considering the above function over the intervals $[0, \widetilde{\omega}_b]$ and $[\widetilde{\omega}_b, 1]$ separately, and substituting back the expressions for K_b, K_{b-1}, we conclude that:

$$V_w(\theta, b) = \begin{cases} \overline{\mu}u(\theta) + \alpha\, \mathbb{E}_\theta\Big[V_w(\theta', b-1)\Big], & \theta > \widetilde{\omega}_b \\ \alpha\, \mathbb{E}_\theta\Big[V_w(\theta', b)\Big], & \theta \leq \widetilde{\omega}_b \end{cases}$$

Furthermore, Corollary 1 follows trivially from the above. □

A.2 Proof for Proposition 2

Proof. To prove Proposition 2, we rely on the two equations established by Proposition 1; the first of which expresses the agent's value function in piecewise form, and the second of which describes the necessary condition for all reservation attractiveness values:

$$V_w(\theta, b) = \begin{cases} \overline{\mu} u(\theta) + \alpha\, \mathbb{E}_\theta \Big[V_w(\theta', b-1) \Big], & \theta > \widetilde{\omega}_b \\ \alpha\, \mathbb{E}_\theta \Big[V_w(\theta', b) \Big], & \theta \leq \widetilde{\omega}_b \end{cases} \tag{A.1}$$

$$\overline{\mu} u(\widetilde{\omega}_b) = \alpha\, \mathbb{E}_\theta \Big[\, V_w(\theta', b) - V_w(\theta', b-1) \,\Big] \tag{A.2}$$

Starting out with Equation A.2 and expanding out the expectation operator, we can use A.1 to substitute in the piecewise definitions of $V_w(\theta, b)$ over the appropriate intervals:

$$\begin{aligned} \overline{\mu} u(\widetilde{\omega}_b) &= \alpha \int_0^1 V_w(\theta', b) - V_w(\theta', b-1)\, dF_m(\theta') \\ &= \alpha \int_0^{\widetilde{\omega}_b} K_b\, dF_m(\theta') \;+\; \alpha \int_{\widetilde{\omega}_b}^1 \overline{\mu} u(\theta') + K_{b-1}\, dF_m(\theta') \\ &\quad -\; \alpha \int_0^{\widetilde{\omega}_{b-1}} K_{b-1}\, dF_m(\theta') \;-\; \alpha \int_{\widetilde{\omega}_{b-1}}^1 \overline{\mu} u(\theta') + K_{b-2}\, dF_m(\theta') \end{aligned} \tag{A.3}$$

Furthermore, Equation A.2 implies that:

$$\overline{\mu} u(\widetilde{\omega}_b) + K_{b-1} = K_b$$

$$\overline{\mu} u(\widetilde{\omega}_{b-1}) + K_{b-2} = K_{b-1}$$

Then, by substituting these expressions into A.3, we arrive at A.4:

$$\begin{aligned} \overline{\mu} u(\widetilde{\omega}_b) = \alpha \int_0^{\widetilde{\omega}_b} \overline{\mu} u(\widetilde{\omega}_b) + K_{b-1}\, dF_m(\theta') \;+\; \alpha \int_{\widetilde{\omega}_b}^1 \overline{\mu} u(\theta') + K_{b-1}\, dF_m(\theta') \\ -\; \alpha \int_0^{\widetilde{\omega}_{b-1}} K_{b-1}\, dF_m(\theta') \;-\; \alpha \int_{\widetilde{\omega}_{b-1}}^1 \overline{\mu} u(\theta') + K_{b-1} - \overline{\mu} u(\widetilde{\omega}_{b-1})\, dF_m(\theta') \end{aligned} \tag{A.4}$$

With some algebra, this simplifies down to the recurrence relation in Equation 2.7:

$$u(\widetilde{\omega}_b) = \alpha u(\widetilde{\omega}_b) F_m(\widetilde{\omega}_b) \;+\; \alpha u(\widetilde{\omega}_{b-1}) \Big[1 - F_m(\widetilde{\omega}_{b-1}) \Big] \;+\; \alpha \int_{\widetilde{\omega}_b}^{\widetilde{\omega}_{b-1}} u(\theta')\, dF_m(\theta') \tag{A.5}$$

Furthermore, to obtain the initial condition for the above, note that the right-swiping budget constraint imposes $V_w(\theta, 0) = 0, \forall b \in \mathcal{B}_w$. Then, A.1 and A.2 simplify to:

$$V_w(\theta, 1) = \begin{cases} \overline{\mu} u(\theta), & \theta > \widetilde{\omega}_1 \\ \alpha\, \mathbb{E}_\theta \Big[V_w(\theta', 1) \Big], & \theta \leq \widetilde{\omega}_1 \end{cases} \tag{A.6}$$

$$\overline{\mu} u(\widetilde{\omega}_1) = \alpha\, \mathbb{E}_\theta \Big[\, V_w(\theta', 1) \,\Big] \tag{A.7}$$

Beginning with A.7, we simplify until arriving at Equation 2.8:

$$\begin{aligned} \overline{\mu} u(\widetilde{\omega}_1) &= \alpha\, \mathbb{E}_\theta \Big[\, V_w(\theta', 1) \,\Big] \\ &= \alpha \int_0^{\widetilde{\omega}_1} K_1 \, dF_m(\theta') + \alpha \int_{\widetilde{\omega}_1}^1 \overline{\mu} u(\theta') \, dF_m(\theta') \\ &= \alpha \int_0^{\widetilde{\omega}_1} \overline{\mu} u(\widetilde{\omega}_1) \, dF_m(\theta') + \alpha \int_{\widetilde{\omega}_1}^1 \overline{\mu} u(\theta') \, dF_m(\theta') \\ &= \alpha \overline{\mu} u(\widetilde{\omega}_1) F_m(\widetilde{\omega}_1) + \alpha \int_{\widetilde{\omega}_1}^1 \overline{\mu} u(\theta') \, dF_m(\theta') \\ \implies u(\widetilde{\omega}_1) &= \alpha u(\widetilde{\omega}_1) F_m(\widetilde{\omega}_1) + \alpha \int_{\widetilde{\omega}_1}^1 u(\theta') \, dF_m(\theta') \end{aligned}$$

To conclude the proof, note that the existence and uniqueness of some $\widetilde{\omega}_b$ that satisfies A.2 is guaranteed given the assumptions on $u(\theta)$ being continuous and strictly increasing. Since the difference between any two consecutive continuation values must lie strictly between 0 and $\overline{\mu} u(1)$, then, by the Intermediate Value Theorem, there exists one and only one root $\widetilde{\omega}_b$ satisfying A.2 and, by extension the above reoccurrence relation. $\square$