How Strong Is Your Tinder Game? Two-Sided Search in Swipe-Based Dating Applications

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Abstract

In today's love market, swipe-based dating apps such as Tinder or Bumble have a well-established presence, but novel platform features can add significant complexities to the user's search problem in ways that have not been studied in existing literature. This paper formulates a game-theoretic model of two-sided search within swipe-based dating apps, along with an appropriate equilibrium refinement. Using numerical methods, I approximate equilibria at the steady-state and perform comparative statics on various model parameters that help explain stylized facts observed in aggregate Tinder data. Finally, agent-based simulations are used to analyse off-path dynamics and discuss how exogenous platform features can be set in a socially-efficient manner.

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1 Introduction

It is widely acknowledged that the search for love is a deeply relevant, personal, and complex social phenomenon, but in today's world, swipe-based dating applications (SBDA's) seem to only make it trickier. These platforms, exemplified by Tinder, Bumble, or Hinge, provide a gamified way of browsing through potential romantic partners by swiping through a stack of suggestions to indicate likes or dislikes, one profile at a time. In the search and matching literature, settings like these fall under the category of decentralized two-sided matching markets with online search (Kanoria and Saban, 2021), emphasising three main characteristics. Firstly, that both sides of the market are comprised of decision-making agents undertaking a process of search. Secondly, that matches occur as outcomes of independently-determined search decisions, rather than as outcomes of a centralised procedure. Thirdly, that romantic suggestions are presented in an online, or *sequential*, manner to users, stressing the importance of sequential rationality within the search process. These apps differ widely from traditional dating sites where users are centrally and statically matched (such as OkCupid), but have come to dominate the modern love market, with Tinder alone boasting 75 million monthly active users and 9.6 million paid subscribers as of 2021 (Business of Apps, 2022).

From a theoretical standpoint, search within SBDA's encompasses many complexities that, due to the novelty of the platforms, have been sparsely studied in the economics literature. On one hand, platform-specific characteristics, such as swiping caps, asynchronicity, and the suggestion algorithms used, matching technologies, pose significant constraints to the way utility-maximising agents strategise their search process. On the other hand, the general problem of search in a two-sided setting is non-trivial in and of itself, as a simple stage interaction (to swipe or not to swipe on a romantic suggestion) can become increasingly complex when repeated over an infinite horizon, admitting to problems such as intractable strategy spaces. Overall, the prevalent role of SBDA's in shaping modern romantic interactions, the theoretical complexities they induce, and their largely understudied nature motivate many different questions. Nevertheless, answering these requires a fundamental understanding of how users make decisions in these platforms: to put it simply, *when should a utility-maximising user swipe right?*

This paper will explore the above within the setting of a swipe-based dating platform where agents on both formulating a game-theoretic model of two-sided search within these platforms along with a corresponding definition of equilibrium. Using numerical methods, I approximate the steady-state equilibria and perform comparative statics

This work presents three main contributions to existing literature on the topic. Firstly, it constitutes one of a handful of attempts to model market configurations arising within SBDA's, which is unsurprising due to the novelty of these platforms, but important given their current social relevance. Furthermore, this work distinguishes itself from other works by Finally, this work provides a marginal side-contribution as a methodological example for the use of computational methods within game theory, a field that has traditionally emphasised pure mathematical analysis. In order to explore the above questions, this paper relies on a rigorously-formulated model, but also on numerical approximation algorithms and agent-based simulations, which can be used to provide quick solutions and perform visually-intuitive comparative statics. As such, it shows that the two approaches, rather than being mutually exclusive, can be used jointly to explore complicated questions, as computational methods can enable quick intuitive explorations before formalising maths arguments

- Why is it different? - We focus on asymmetric pickiness - This is different from what others focus on eg. the algorithm utility or this or that - Only Kanoria focuses on this, but we explain it differently - Emphasis on budgets as a solution factor - Similar conclusions to other papers, but different solution - Side contribution on computational methods within game theory - agent-based simulations & numerical solving - Can be used as a tool rather than an alternative to rigorous analysis

1.1 Related Work

The present work draws inspiration from two key branches of economics literature: that of search and matching, which studies the decision-making process of agents who seek, for example, a job, a business partner, or a spouse, and also that of mean field game theory, which has been employed to study complex dynamic games involving a large number of players. I discuss each of these in turn, and then contrast this work with the handful of papers that have focused on specifically analysing SBDA market configurations.

Within the search and matching literature there is an abundance of different theoretical models, amply summarised by Chade et al. (2017), with several extensions considered to study a variety of different settings. As previously noted, three defining features of SBDA markets are decentralised matching, two-sidedness, and sequential interaction, and one of the most prominent works on matching markets at this intersection is that of Burdett and Coles (1997), which studies the marriage market with heterogeneous agents and non-transferable utility (NTU). The seminal paper models a context of uniform random search where agents receive marriage proposals from the other side of the market according to a continuous-time process, and must choose whether or not to accept these given the observable 'pizazz' of the proposing agent. Several extensions followed this work, considering cases such as noisy observations of 'pizazz' (Chade, 2006), idiosyncratic preferences Burdett and Wright (1998), directed search, and so on. Even though, this work proves how positive-assortative matching can arise as a steady-state equilibrium, extending the result of Becker (1973) to a NTU context. S

Most crucially, this paper outlines a frequently used approach for unifying both sides of the market via endogenous

On the other hand, mean field game theory focuses on dynamic games with a large number of agents, for which curses of dimensionality often arise thus making solution concepts such as Markov Perfect Equilibria intractable Maskin and Tirole (2001). To deal with this, mean field models consider individual interactions with the *aggregate system state*, ie the distributions over states and strategies within the game, rather than interactions with all other players. This abstraction is coupled with the notion of a *consistency check*, such that equilibrium arises when rational play given an aggregate state maintains this same state as a fixed point. The approach, first considered in the works of Jovanovic and Rosenthal (1988) and Hopenhayn (1992), greatly simplifies strategic settings with the aforementioned problem and has been successfully applied to settings such as network routing Calderone and Sastry (2017), auctions with learning Iyer et al. (2014). In this paper, we rely on mean-field considerations to abstract from considerations on observability; within SBDA's, the market-wide history and opponent state are unobservable to players, and thus traditional equilibrium concepts would demand beliefs over uncountable history spaces, and even beliefs over the beliefs other players may hold (a problem known in the literature as nested beliefs Brandenburger and Dekel (1993)). This yields two central problems: first, that equilibria become impossible to compute, and, by extension, that the model assumes an unreasonable level of rationality on behalf of agents, especially given that these rarely interact with the same individual twice amongst millions of other users. Thus, by considering interactions with the aggregate state of the platform, the model presented is able to better characterize an equilibrium that is both insightful and representative of real-life dynamics.

Among the select few papers that have specifically considered SBDA matching markets, one that stands out is the recent work by Kanoria and Saban (2021), which models.

- Main differences

- Trait: Decentralised - What it means: Matches result from the decision-making process of searching agents, with textbook examples involving agents who seek a job, a university, or a spouse. This differs from centralised matching context where matches are computed by a centralised authority who is fully informed of the preferences of all agents on both side of the market. Search models allow for, and characterize imperfect matches, but stable matches are still a nice theoretical benchmark - Top works: Gale and Shapley (1962) - How they differ

- Trait: Two-sided - What it means: Outcomes depend on the search process undertaken by both sides of the market; model aggregates tons of decision and information.

- Top works - What they found - How they differ

- Trait: Non-transferable utility - What it means: Payoffs are not transferable amongst agents. This contrasts with PTU where a bargaining process must be undertaken to divide the within-match surplus. - Top works: Roth and Sotomayor (1990). - What they found - How they differ

- Trait: heterogeneous preferences versus common ordinal preferences.

- Trait - What it means - Top works - What they found - How they differ

- What is Tinder? (brief)
  - When was it started?
  - What is swiping?
  - How popular it is?
- Why does Tinder pose an interesting economic problem?
  - Stage interaction
  - Platform features: budgets, observability, directed search, asynchronicity
  - Repeated games: curse of dimensionality, beliefs and meta-beliefs
- What and how does this paper study?
  - Model of two-sided search with strategic considerations
  - Equilibrium refinement, computation, and analysis
  - Planner considerations on directed search and budget setting
- What does this paper contribute?
  - First model to address budgeted search in Tinder?
  - First model to combine idiosyncracy and pizzaz
  - Case study for the use of computational techniques in

- Searching and Matching
  - Gale and Shapley (1962), Roth and Sotomayor (1992)
  - Two-sided: Burdett and Wright (1998), Chade (2006), Smith, Adachi
  - **Does not consider budgets**
    - ... important as this is a way for planners to influence outcomes
- Mean-Field Game Theory: Iyer et al. (2014), Gummadi et al. (2013), Jovanovic and Rosenthal (1988)
  - No models on MFG for Tinder
- Modern Dating Apps: Olmeda (2021), Kanoria and Saban (2021)
  - Not models where behaviour is derived from rational utility-maximizing assumptions

2 Theoretical Model

2.1 Setup

Consider the two-sided search market formed by the Tinder platform with both male and female agents looking for potential partners. For ease of exposition, I assume that this market is heteronormative such that male agents search exclusively for female agents and vice-versa. Time is discrete and indexed $t = 0, 1, ...$ over an infinite horizon. At every time period, agents from each sex are paired up and presented a suggested partner from the opposite side of the market. Each agent has an attractiveness type $\theta \in \Theta := [0, 1]$ which is unknown to them but observable to their suggestion, and it is common knowledge that this is the case. After being paired, agents observe the suggested partner's attractiveness and can then choose whether to swipe left (dislike) or right (like) on their suggestion, yielding an action space of $\mathcal{A} = \{\text{left, right}\}$. If both agents swipe right on one another, they are said to have *matched* and both receive a matching payoff, however, if either agent swipes left, they both receive a payoff of zero. Contingent on swiping right on a suggestion of attractiveness θ, a user earns a matching payoff $u(\theta)$, where $u(\cdot)$ is a strictly increasing, concave function that satisfies $u(0) = 0$. This last property stems from the fact that, in Tinder, users are allowed to unmatch with each other, thus implying that matching with the least attractive individual on the other side of the market is weakly preferred to not matching. After payoffs have been received, players are then paired with a different suggestion and the above stage interaction is repeated. Given the large number of agents in SBDA platforms, I assume that agents are paired *anonymously* in the style of Jovanovic and Rosenthal (1988), thus abstracting from history-related complexities. Furthermore, to the agents' knowledge, pairings are decided in an unknown manner (since SBDA's are generally secretive regarding the algorithms used), effectively making their problem on of random search.

Considering the above, it is evident that swiping right in the stage interaction is both weakly dominant for all agents and yields a Pareto-optimal outcome, thus implying that, in a repeated interaction, the market equilibrium would have all agents exclusively swiping right. Since that the main selling point of SBDA's is a reduction in searching costs, which is accomplished when matches have a high likelihood of resulting in real-life romantic attraction, Tinder places a cap on the total number of right swipes for each user, thus making it a form of costly signalling. I refer to the total number of right-swipes a user has left as its *budget*, b_t, which evolves dynamically according to the law of motion:

$$b_{t+1} = b_t - a_t$$

where the starting budgets for each sex, B_m and B_w, are determined exogenously. The budget sets for men and women are thus defined by $\mathcal{B}_i = \{b \in \mathbb{Z} : 0 \leq b \geq B_i\}$, with $i = m, W$ respectively. Each period, λ_m new men and λ_w new women enter the platform, with their attractiveness drawn i.i.d from distributions with cumulative distribution functions F_m and F_w, respectively. Importantly, agents depart from the platform in one of two ways: they can leave *endogenously*, if they expend their swiping budget, or *exogenously* with probability $(1 - \delta)$. This admits to the interpretation of a geometrically distributed lifetime within the platform, parametrized by δ, and implies that users use this as a discounting factor for future payments.

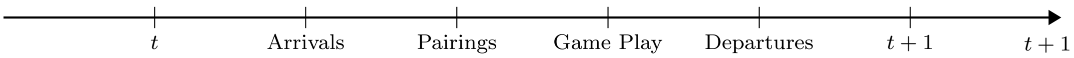

Figure 1: Sequence of events within each time period

Given the above, a number of simplifications to the explored setting are possible.

- 1. An agent's decision on any given time period depends fundamentally on the attractiveness of the suggested partner and their own budget

I restrict attention to stationary Markov strategies, defined by $\sigma_m : \Theta \times \mathcal{B}_m \to \Delta\mathcal{A}_m$ for men and $\sigma_w : \Theta \times \mathcal{B}_w \to \Delta\mathcal{A}_w$ for women, where ΔS denotes the probability simplex over set S.

2.2 The Dating Market

Given the sequence of events described in the stage interaction above, I now outline the aggregate market variables that make up the Tinder market, as these must be considered within the model given their endogenous relation with strategic search behaviour. At any given time t, the masses of men and women on Tinder are denoted by N_{mt} and N_{wt}, respectively. Furthermore, let $\pi_{it} : \mathcal{B}_i \to [0, 1], \quad i = m, w$ be the probability mass function over agent budgets. These are endogenously determined since the flow of agents into lower budget levels and eventually out of the platform depends on their swiping decisions. All in all, by considering aggregate variables on both sides of the platform, the Tinder market at time period t is defined as $\Psi_t = (N_{mt}, N_{wt}, \pi_{mt}, \pi_{wt})$. Finally, since gender imbalances can exist, resulting with unpaired agents in the long side of the market, a pairings process must be fixed. Given fairness considerations as well the efficient, automated nature of SBDA platforms, this paper assumes a frictionless matching technology, and models pairings as a Bernoulli process parametrized by market tightness, ie. the probability of receiving a suggestion on each side:

$$\tau_t = \min\left\{\frac{N_{wt}}{N_{mt}}, 1\right\}$$

For most of this paper, I focus on characterizing user behaviour and its resulting implications in a stationary setting, although some discussion of coupled strategy and market dynamics is provided in **Section 4**. Given this, it is firstly important to characterize the market steady state, which again arises as a result of exogenous pairing and departure processes as well as endogenous search behaviour. When doing so, time subscripts are omitted, thus denoting the steady state as the Tinder market $\Psi(\sigma) = (N_m(\sigma), N_w(\sigma), \pi_m(\cdot \mid \sigma), \pi_w(\cdot \mid \sigma))$ such that $\Psi_t(\sigma) = \Psi_{t+1}(\sigma) = ... = \Psi(\sigma)$. This market steady state must satisfy the balanced flow equations. Firstly, the entry flow of agents into the platform must equal the departure flow, thus for $i = m, w$:

$$\lambda_i \;=\; \underbrace{N_i(1-\delta)}_{\text{Exogenous Deaths}} \;+ \underbrace{N_i\pi_i(1)\delta\tau_i \int_{\Theta} \sigma_i(\theta, 1)\, dF_j(\theta)}_{\text{Expended Budgets}} \tag{1}$$

Secondly, the flow of agents into any particular budget level must equal the outflow of agents from that same level:

$$\underbrace{N_i\pi_i(b+1)\delta\tau_i \int_{\Theta} \sigma_i(\theta, b+1)\, dF_j(\theta)}_{\text{Inflow into } b} \;=\; \underbrace{N_i\pi_i(b)(1-\delta) + N_i\pi_i(b)\delta\tau_i \int_{\Theta} \sigma_i(\theta, b)\, dF_j(\theta)}_{\text{Outflow from } b} \tag{2}$$

Finally, the entry flow of agents into the platform must equal the outflow from the top budget level:

$$\lambda_i \;=\; \underbrace{N_i\pi_i(B_i)(1-\delta)}_{\text{Exogenous outflow from B}} \;+ \underbrace{N_i\pi_i(B_i)\delta \int_{\Theta} \sigma_i(\theta, b)\, dF_j(\theta)}_{\text{Endogenous outflow from B}} \tag{3}$$

Theorem 1. *Fix a profile σ of measurable strategies. The steady state of the market is given by:*

- Entry flows
- Leaves (including geometric lifetime)
- Masses
- Distribution
- Steady State

2.3 The Search Problem

Average swipe-receiving rate:

$$\overline{\sigma_j} = \sum_{b \in \mathcal{B}_j} \int_{\Theta} \sigma_j(\theta, b)\, P_j(b)\, dF_i(\theta) \tag{4}$$

$$U(\theta, a) = \overline{\sigma_j} a u(\theta) \tag{5}$$

Agent's problem:

$$
\begin{aligned}
&\max_{\{a_t\}_{t=0}^{\infty}} \quad \mathbb{E}_\theta \left[\sum_{t=0}^{\infty} \delta^t U(\theta_t, a_t) \right] \\
&\text{s.t.} \quad b_{t+1} = b_t - a_t \\
&\qquad\quad b_t \in \mathcal{B}_i \\
&\qquad\quad a_t \in \{0, 1\}
\end{aligned}
\tag{6}
$$

Bellman equation when paired:

$$
\begin{aligned}
V_i^P(\theta, b) = \max \Big\{ &\overline{\sigma_j}\, u(\theta) \; + \; \delta\tau\, \mathbb{E}\Big[V_i^P(\theta', b-1) \Big] \; + \; \delta(1-\tau) V_i^{NP}(b-1)\,, \\
&\delta\tau\, \mathbb{E}\Big[V_i^P(\theta', b) \Big] \; + \; \delta(1-\tau) V_i^{NP}(b) \Big\}
\end{aligned}
\tag{7}
$$

Bellman equation when not paired

$$
V_i^{NP}(b) = \delta\tau\, \mathbb{E}\Big[V_i^P(\theta', b) \Big] \; + \; \delta(1-\tau) V_i^{NP}(b) \tag{8}
$$

With some straightforward algebra, we can combine the above two equations into the cohesive Bellman equation. Define the effective discount rate α:

$$
\alpha := \frac{\tau\delta}{1 - \delta(1-\tau)}
$$

Through α, the agent discounts with consideration for both the departure and pairings processes, yielding the cohesive Bellman equation bellow:

$$
V_i(\theta, b) \; = \; \max \Big\{ \overline{\sigma_j}\, u(\theta) + \alpha\, \mathbb{E}\Big[V_i(\theta', b-1) \Big]\,, \; \alpha\, \mathbb{E}\Big[V_i(\theta', b) \Big] \Big\} \tag{9}
$$

Optimal policy can be parametrised by a reservation attractiveness due to the piecewise nature of the value function: Agent swipes right when current period reward is greater than the discounted loss in expected value from a unit decrease in budget.

$$
\sigma_i(\theta, b) = \begin{cases} 1, & \theta \geq \widetilde{\sigma}_b^i \\ 0, & \theta < \widetilde{\sigma}_b^i \end{cases} \tag{10}
$$

$$
u(\widetilde{\sigma}_b^i) = \alpha\, \mathbb{E}\Big[\, V(\theta', b) - V(\theta', b-1) \, \Big] \tag{11}
$$

$$
V(\theta, b) = \begin{cases} u(\theta) + \alpha\, \mathbb{E}_\Psi \Big[V(\theta', b-1) \Big], & \theta > \widetilde{\mu}_b \\[2ex] \alpha\, \mathbb{E}_\Psi \Big[V(\theta', b) \Big], & \theta \leq \widetilde{\mu}_b \end{cases} \tag{12}
$$

$$
u(\widetilde{\sigma}_b^i) = \alpha u(\widetilde{\sigma}_b^i) F_j(\widetilde{\sigma}_b^i) + \alpha u(\widetilde{\sigma}_{b-1}^i) \Big(1 - F_j(\widetilde{\mu}_{b-1}) \Big) + \int_{\widetilde{\sigma}_b^i}^{\widetilde{\mu}_{b-1}} \alpha u(\theta')\, dF_j(\theta') \tag{13}
$$

- Present case for women, then say case for men follows
- Condition on male strategy and steady state

- Present Ex-interim utility maximization
  - Show it reduces to a constant
- Present sequence problem
- Derive Bellman equation
- Prove uniqueness of value function and solution
- Derive solution

3 Equilibrium

3.1 Steady-State Equilibrium

Definition 1. *A steady-state equilibrium is defined by a triplet* $(\sigma_m^*, \sigma_w^*, \Psi)$ *such that:*

1. $\mu_b^* = \widetilde{\mu}(b \,|\, \Psi^*, \omega^*), \quad \forall b \in \mathcal{B}_m$
2. $\mu_b^* = \widetilde{\mu}(b \,|\, \Psi^*, \omega^*), \quad \forall b \in \mathcal{B}_w$
3. $\Psi^* = \Psi(\mu^*, \omega^*)$

- Define and explain concept of SSE
- Explain computation via least-squares
- Explain main properties (eg. ESS & uniqueness)

3.2 Comparative Statics

- Present CS on individual factors and explain intuitively
- These include: patience, risk aversion, distributions
- Present case of gender disbalance... why is it that men always swipe right?
-

4 Agent-Based Simulations

4.1 Convergence and Dynamics

- Check Mass convergence
- Check distribution convergence
- Relate to ESS
- What about Dynamics??? BR

4.2 Directed Search

- try page rank
- try elo rating
- try v simple RW algo
- Do any of these converge onto GS (note... define gale shapley mathcings)?

4.3 Social Efficiency

5 Conclusion

In this chapter we shall do a reference to an entry in the bibliography, `bibliography.bib`.

What we know of the invention of the flux capacitor is that Dr. Emmett Brown thought of this when hanging a clock in the bathroom. He was standing on his porcelain sink and slipped because it was wet, the resulting hit on the head was apparently a cause to this invention Olmeda (2021).

5.1 Future Work

The corresponding sketch made on this day has been attached in appendix B.

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A Uniqueness and Existence of Search Problem

B Notation

- Male types μ
- Female types ω
- Strategies $s = (s_m, s_w)$
- CDF's $M(\mu, b)$, $W(\omega, b)$
- Densities $m(\mu, b)$, $w(\omega, b)$
- Discount δ
- Population CDF's F_m, F_w
- Masses N_m, N_w
- Entry Flows λ_m, λ_w
- Tightness $\tau = \min\{\frac{N_w}{N_m}, 1\}$
- Effective discount α