312-avoiding Permutations

Stuart Paton

January 24, 2013

1 Introduction

A 312-avoiding permutation is a permutation $\sigma \in \mathfrak{S}_n$ in which there does not exist elements $i < j < k$ such that $\sigma_k < \sigma_i < \sigma_j$.

2 312-avoiding permutations are Catalan Structures

2.1 The form of a 312-avoiding permutation

Consider the 312-avoiding permutation $\sigma = a_1a_2...a_n$ is of the form $\alpha n \beta$ where β is a subpermutation $b_1b_2...b_m$ of the form γm.
Let $n = max\{a_1a_2...a_n\}$
Let $m = max\{b_1b_2...b_n\}$
From this we know that n is the largest element of σ and for the subpermutation β which consists of every element after n, m is the largest element.
For σ to be 312-avoiding, $\gamma \neq \emptyset$. That is, γ must not contain no elements and since $m < n$, σ is 312-avoiding.

2.2 Proof that 312-avoiding permutations are Catalan structures

Since permutation $\sigma \in \mathfrak{S}_3$ and $|S_n(\sigma) = C_n|$ as proven in [1] then 312-avoiding permutations are Catalan structures.

3 Bijection from 312-avoiding permutations to 231-avoiding permutations

Theorem: We can convert a 312-avoiding permutation in to a 231-avoiding permutation.
Proof: Given a permutation $\sigma = \sigma_1\sigma_2...\sigma_n$, we create a new permutation τ such that $\tau = (n+1) - \sigma_n...(n+1) - \sigma_2(n+1) - \sigma_1$.[2]

3.1 Examples

Example 1: Let $\sigma = 1342 \Rightarrow \tau = 3124$
Example 2: Let Let $\sigma = 2431 \Rightarrow \tau = 4213$

References

[1] Mikos Bona, *Combinatorics of permutations*, Discrete Mathematics and Its Applications (Boca Ranton), Chapman & Hall, CRC, Boca Ranton, FL, 2004.

[2] Richard P. Stanley, *Enumerative combinatorics*, Cambridge Studies in Advanced Mathematics, vol. 2, Cambridge University Press, 1999.