Skeleton Moment demonstrator

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The notenook provides code samples to explore the effectiveness of moment nased features for handwritten character recognition. The code is deliberately designed to be basic skeleton so as to allow you to experiment with it. For that reason, many parameters are set via global "constants" in the cell below and little in the form of parameter or error testing or handling is provided. There are also commented out **print** "debugging" commands which you may uncomment to display information if you wish as you explore (or you can use the debugger).

The following cell provides imports and constant values used by the system. For simplicity in providing a user interface, variations in performance can be explored by amending the appropraite value of each constants. It is recommended that only the constants under "values to explore" are edited.

Task 1: add Logic to moment function

In [1]:

```
import numpy as np
import pandas as pd
from sklearn import tree
from sklearn.tree import DecisionTreeClassifier
import matplotlib.pyplot as plt
import matplotlib.image as pltimg
import time

###############################################
## CHANGE THE FOLLOWING PATH BEFORE STARTING ##
###############################################
# change the following to the location where you have uploaded the 
# there should be two folders within this folder called "train" and
EXAMPLES_LOCATION = "digits/"

# Normally, you should not need to alter the following four values
LOCATION_SUFFIX = ".norm"
ROWS = 24 # number of rows in the image (moment value y)
COLS = 16 # number of columns in the image (moment value x)

# Values to explore===change the following as you proceed with your
NUMBER_TRAINING_EXAMPLES = 10    # change this to alter the number o
PMAX = 2 # you can increae the p values investigated
QMAX = 2 # you can independently change the q values investigated

MAX_EXAMPLES = NUMBER_TRAINING_EXAMPLES * 10 # maximum number of tr
```

moment calculator

The following function calculates the the pq'th moment value m_{pq} for a given image. It is thus the fundamental component of this notebook

1. add logic to this function

In [2]:
```
def moment(p,q,image):
    dim = image.shape # find dimensions of the image

    sum = 0

    # PUT YOUR CODE HERE
    """
    Calculate the (p, q)-order moment for a binary image.

    Parameters:
    - p (int): Order of x in the moment calculation.
    - q (int): Order of y in the moment calculation.
    - image (numpy.ndarray): Binary image where 0 represents the ob

    Returns:
    - float: The computed moment value.
    """
    # Version 1: calculate by np
    # Find the indices of black pixels (value 1)
    # black_pixels = np.argwhere(image == 1)

    # # Calculate the moment as the sum of (x^p * y^q)
    # sum = np.sum(
    #     (black_pixels[:, 1] ** p) * (black_pixels[:, 0] ** q)
    # )

    # return float(sum)

    # Version 2: calculate by iterating through pixels
    # Iterate through each pixel in the image
    for y in range(dim[0]):  # Rows (y-coordinate)
        for x in range(dim[1]):  # Columns (x-coordinate)
            if image[y, x] == 1:  # Only consider object pixels
                sum += (x ** p) * (y ** q)

    return float(sum)
```

Reading images from the example files

set of images from a given file. For simplicity, they are tailored to the file format supplied and hence not generic. They would require modification to read in differing sile fomats although the remainder of the notebook should be applicable with relatively little difficulty.

The first function **read_image** reads a single image into position ^^example** in the **images** array. Note that, in the file format supplied, there is an end of line character at the end of each row of the image which is read in and discarded.

Note that subsequently images follow immediately after the given image, there is no form of image separator in the file (you can open the file with a simple text editor like **notepad** to see it consists of only *0* and *1* characters

The second function **read_images** governs the opening and closing of the files together with reading a set of images from the given file. Note that the location of the files is governed by the constants names and a varaible with the name of the class as a numeral. "0", "1", "2" etc.

In [3]:
```
def read_image(f,example,images):

    for x in range(ROWS):
        for y in range(COLS):
            images[example,x,y]=(int(f.read(1))) # read either the (
        f.read(1) # skip eol character

    return(images)


def read_images(dir_name, examples, class_name,images):

    location= EXAMPLES_LOCATION + dir_name + str(class_name) + LOCAT

    with open(location, 'r') as f:
        for example in range(examples):
            read_image(f, example, images)

    return images
```

Display pattern

Another support function to display a range of patterns from an array using **matplotlib**.

It displays the first **no** images from the array so requires modification to display a specific image.

In [4]:
```
def show_patterns(images,no):

    for nxt_img in range(no):
        # print (nxt_img)
        # imgplot = plt.imshow(images[nxt_img])
        # plt.show()
        #time.sleep(2) # unblock this and change delay to watch imag
        print(f"Displaying image {nxt_img + 1}/{no}")
        plt.imshow(images[nxt_img], cmap="gray")  # Use grayscale fo
        plt.title(f"Image {nxt_img + 1}")
        plt.axis("off")  # Turn off axis for a cleaner display
        plt.show()
```

Generate moments for the given image

To generate all the required moments for a particular image up to the order governed by **pmax** and **qmax**.

The results are stored in a dictionary named **results** where the calculated moment is added to the end of the list of values for all images associated with the named moment m_{pq}.

In [5]:
```
def gen_image_moments(pmax,qmax, image, results):

    for p in range(pmax):
        for q in range(qmax):

            m_name = "m" + str(p) + str (q) # generate the "name" o

            m = moment(p,q,image) # calculate the moment value m_pq

            print(m_name + " is "+ str(m) +"\n") # uncomment to see

            results[m_name].append(m) # add the calculated moment to

            print ("Moments for " + m_name + " is " + str(results[m_

    return(results)
```

Generate all Training Feature values

This function generates all the required moment values for training by repeatedly calling the **gen_image_moments** function to generate a dictionary containing all the required training values for a set of patterns. They are stored sequencially via a list associated with each dictionary item. i.e. the value of the moment m_{pq} for the n'th training pattern will be the n'th entry in the list associated by the dictionary entry associted with m_{pq}.

The class identifier **class_name** of each entry (i.e. the actual class of the training pattern whose values lie at the n'th value of each dictionary entry is added after the moments have been calculated as dictionary entry named **class_id** in the corresponding n'th entry in the list associted with this dictionary item. It will form the target for the Dicision Tree during training.

In [6]:
```
def gen_moment_features(class_id,no_training_pats,images,train_featu

    for pattern in range(no_training_pats):
        train_features = gen_image_moments(PMAX,QMAX,images[pattern

        train_features["class_id"].append(class_id) # add the true c

    # print("Raw Trained Feature values")
    # print(train_features) # uncomment here if you want to see how

    return(train_features)
```

Read in and calculate the feature values for the test image set

test_image generates the moment feature values for a given image and appends it to the list of feature values returning a list of all the feature values for the image

test_images drives the process by reading in a number of images given in **no_pats** for a given class given by **class_id**. Note that the file name is contructed using this variable also. The list of feature values is added to the list of lists **test_feature_vals** where each component list consists od the moment features for a given image.

NOTE: these function produce a list of lists which each entry in the outer list comprising a list of moment features for a given pattern

In [7]:
```
def test_image(image,pmax,qmax):

    # calculate list of moment values for the image and add it to t

    test_feature_val = [ moment(p,q,image) for p in range(pmax) for

    return(test_feature_val)


def test_images(class_id,no_pats):

    images = np.zeros((MAX_EXAMPLES,ROWS,COLS)) # empty array to st

    images = read_images("test/bs/",no_pats,class_id,images) # read

    show_patterns(images,no_pats) # uncomment this statement to see

    # Generate the list of lists of test image---each entry is a li
    test_feature_vals = [test_image(images[pat],PMAX,QMAX) for pat 

    print(" Test vals ") # uncomment here to see the test images us
    # print(test_feature_vals)

    return(test_feature_vals)
```

Another relatively simple function **generate_feature_dict** which generates an empty Python Dictionary with the names of the moment features paired with a list of the associted values for each pattern together with an additional entry **class_id** whcih contains a list of the id's for each pattern in the order they occur, e.g. the first entry contained the class identifier for the first entry in the correcponding lists of all the feature values. For the following example therefore, the **class_id** would contain a list identifying the class from which each row derives.

- 214.0 2759.0 1699.0 21463.0
- 167.0 2056.0 1336.0 14440.0
- 187.0 2371.0 1554.0 18152.0

In [8]:
```
def generate_feature_dict(feature_names):

    feature_dict = dict()

    for nxt_feature in feature_names:
        feature_dict[nxt_feature] = list()

    feature_dict["class_id"] = list()

    return(feature_dict)
```

Training

This function drives the generation the training for the Decision Tree **test_tree** passed in as a parameter.

The images are read into an array called **images**.

The feature names are stored in a dictionary with the following format {"m00" : list(), "m01":list(), "m10":list(), "m11": list(), "class_id" : list()}

Training works as follows

1. generate a list of the names of the moment features using the p and q numbers tagged onto the end of the character "m"
2. generate an empty dictinbary to store the generated values
3. for each class, read in the required number of training images from the file, generate the moment features, placing the results in the dictionary
4. load the training data nto a Pandas DataFrame object
5. divide the data frame into the training features and the target class **class_id**
6. train the decision tree

The above comments are repeated prior to the Python statements associted with them below.

It is possible to print out the decision tree generated.This can be quite insightful in gaining an understanding of what is being deduced from the training patterns.

In [9]:
```
def train_classifier(test_tree, no_classes, no_training_examples):
    images = np.zeros((MAX_EXAMPLES,ROWS,COLS))

    # generate a list of the names of the moment features using the
    feature_names =  ["m" + str(p) + str (q) for p in range(PMAX) fo

    print(feature_names) # uncomment this statement to see the gene

    # generate an empty dictinbary
    panel_cols = generate_feature_dict(feature_names)

    #print("cols : " + str(panel_cols)) # ncomment this statement to

    # for each class, read in the required number of training image
    # placing the results in the dictionary

    for file_name in range(no_classes):
        images= read_images("train/br/",no_training_examples,file_na

        show_patterns(images,no_training_examples) # uncomment this

        features = gen_moment_features(file_name,no_training_exampl

    #load the training data nto a Pandas DataFrame object:

    df = pd.DataFrame(panel_cols)

    print(df) # uncomment here to see the data frame

    # Divide the into the training features and the target class cl
    features = df[feature_names]

    # print( "FEATURES")
    print(features) # uncomment here to see the actual dataframe

    target=df['class_id']
    test_tree = test_tree.fit(features,target) # train the decision

    tree.plot_tree(test_tree) # uncomment here to see the actual de
    # plt.show()                 # gaining an understanding of what

    return(test_tree)
```

A simple function to test your moment calculator

Use the following function initially to test your moment generator.

The answer should be: {'m00': [214.0], 'm01': [2759.0], 'm10': [1699.0], 'm11': [21463.0], 'class_id': []}

In [10]:
```
def moment_tester():

    image = np.zeros((1,ROWS,COLS)) # space for one image

    feature_names = ["m" + str(p) + str (q) for p in range(2) for q

    panel_cols = generate_feature_dict(feature_names) # create empty

    read_images("train/br/", 1, 0,image) # read 1 image from the cl

    features = gen_image_moments(2,2,image[0],panel_cols) # calcula

    print( "FEATURES")
    print(features)
```

Main / Driver Function

Main driver function for the system. Requested how many training classes and repeats the test for a given class and number of examples.

Feel free to edit this function to loop and ask for as many training samples as you require or test all classes. Also include error checking. Additionally you could explore changing the order of the moments by reading in the PMAX and QMAX values,

In [11]:
```
def driver():

    test_tree = DecisionTreeClassifier() # untrained decision tree


    no_classes = int(input("How many training classes?"))
    test_tree = train_classifier(test_tree, no_classes, NUMBER_TRAI

    # loop classes and class numbers until user manually stops exec
    while (True):
        class_name = input("Test class name>")
        no_examples = int(input("how many examples?"))

        test_features = test_images(class_name,no_examples)

        print("tree features") # uncomment here to see the feature
        print(test_features)

        # the results contained as a list of numerals showing the p
        print("I think the test patterns are: " + str(test_tree.pre
```

2. Test moment function with this function

In [12]:
```
moment_tester() # use this function first to test your moment calcu
```

```
m00 is 214.0

Moments for m00 is [214.0]
m01 is 2759.0

Moments for m01 is [2759.0]
m10 is 1699.0

Moments for m10 is [1699.0]
m11 is 21463.0

Moments for m11 is [21463.0]
FEATURES
{'m00': [214.0], 'm01': [2759.0], 'm10': [1699.0], 'm11': [21463.
0], 'class_id': []}
```

In [13]:
```
# driver() # uncomment here to tun main decision tree classifier.
```

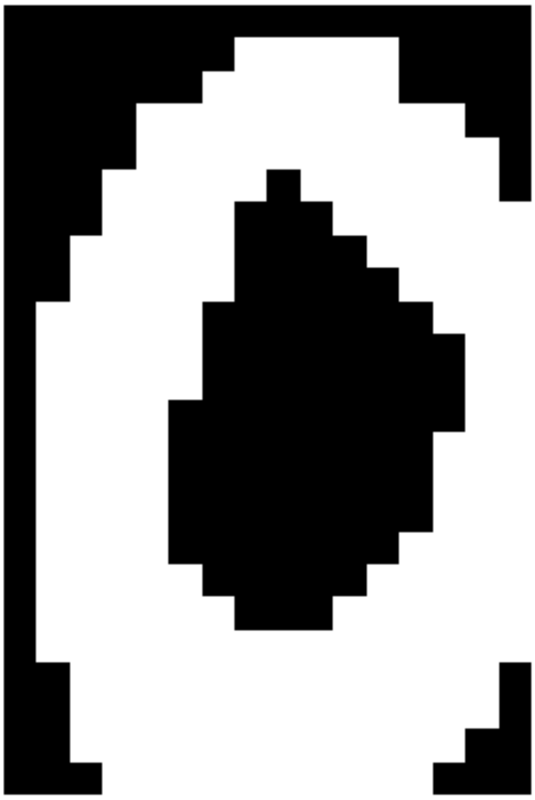
Custom functions to solve the tasks (without using driver())

Task 2: to observe the difference between between 2 classes "0" and "1"

In [14]:
```
def two_class_test():
    """
    Train and test the classifier for a two-class problem ("0" and '
    """
    # Train the classifier
    test_tree = DecisionTreeClassifier()
    print("Training the classifier for classes 0 and 1...")
    test_tree = train_classifier(test_tree, 2, 10)  # Two classes, .

    # Test the classifier
    print("\nTesting the classifier...")
    test_features = test_images(0, 10)  # Test 5 examples from clas.
    predictions = test_tree.predict(test_features)
    print(f"Predicted classes for class 0 test patterns: {predictio

    test_features = test_images(1, 10)  # Test 5 examples from clas.
    predictions = test_tree.predict(test_features)
    print(f"Predicted classes for class 1 test patterns: {predictio

# Run the test
two_class_test()
```

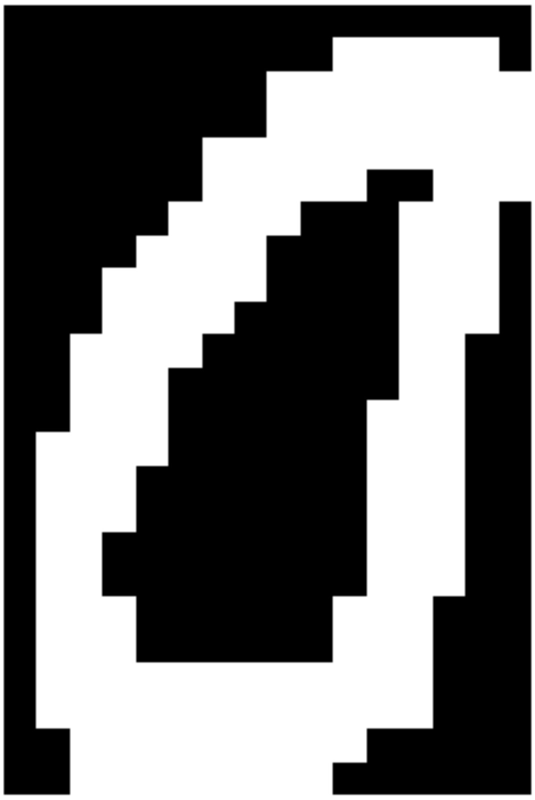
```
Training the classifier for classes 0 and 1...
['m00', 'm01', 'm10', 'm11']
Displaying image 1/10
```

Image 1



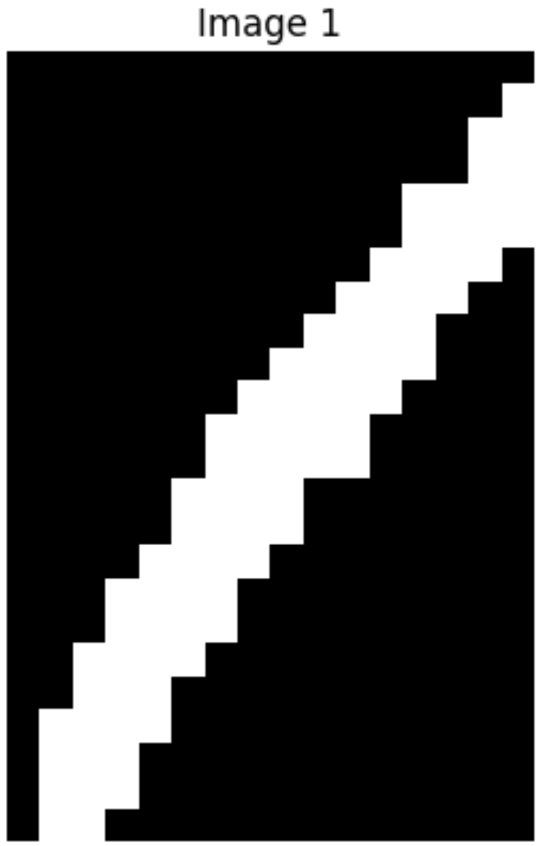
```
Displaying image 2/10
```

Image 2



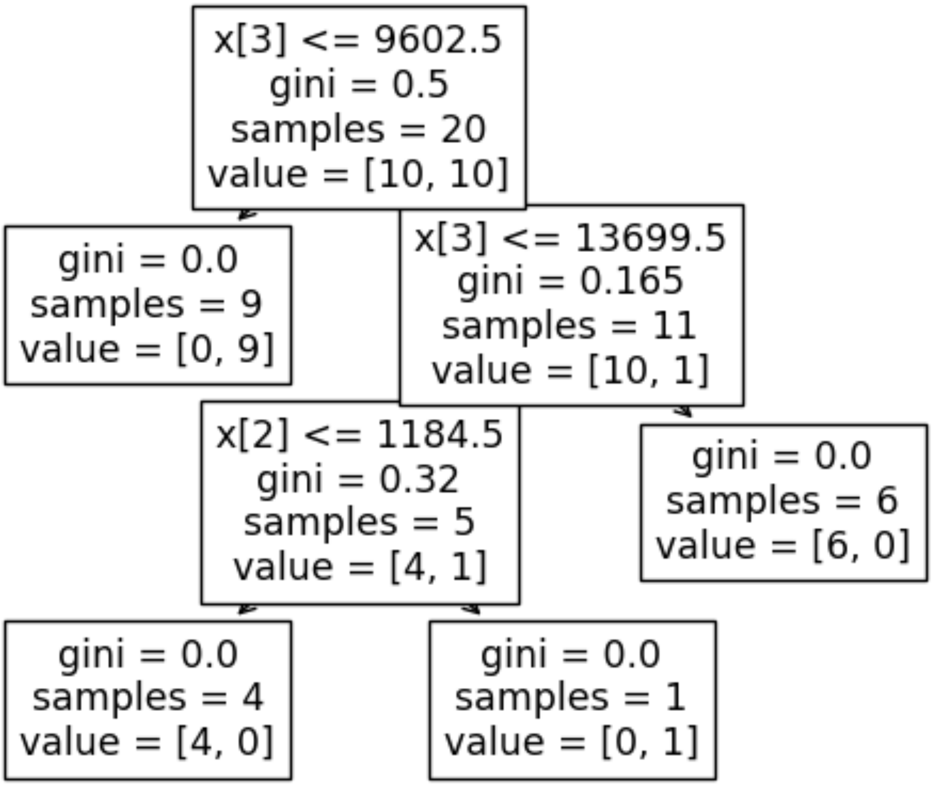
```
Displaying image 3/10
```

```
Moments for m11 is [21463.0, 14440.0, 18152.0, 15854.0, 13970.0, 97
62.0, 14855.0, 9833.0, 11643.0]
m00 is 134.0

Moments for m00 is [214.0, 167.0, 187.0, 176.0, 169.0, 109.0, 155.
0, 103.0, 142.0, 134.0]
m01 is 1690.0

Moments for m01 is [2759.0, 2056.0, 2371.0, 2067.0, 2074.0, 1289.0,
1966.0, 1360.0, 1802.0, 1690.0]
m10 is 1023.0

Moments for m10 is [1699.0, 1336.0, 1554.0, 1410.0, 1340.0, 843.0,
1230.0, 815.0, 1068.0, 1023.0]
m11 is 10458.0

Moments for m11 is [21463.0, 14440.0, 18152.0, 15854.0, 13970.0, 97
62.0, 14855.0, 9833.0, 11643.0, 10458.0]
Displaying image 1/10
```




```
Displaying image 2/10
```

```
17  105.0  1327.0   891.0   8994.0          1
18  101.0  1245.0   911.0   8936.0          1
19  106.0  1217.0   863.0   7374.0          1
      m00     m01     m10      m11
0   214.0  2759.0  1699.0  21463.0
1   167.0  2056.0  1336.0  14440.0
2   187.0  2371.0  1554.0  18152.0
3   176.0  2067.0  1410.0  15854.0
4   169.0  2074.0  1340.0  13970.0
5   109.0  1289.0   843.0   9762.0
6   155.0  1966.0  1230.0  14855.0
7   103.0  1360.0   815.0   9833.0
8   142.0  1802.0  1068.0  11643.0
9   134.0  1690.0  1023.0  10458.0
10   84.0  1025.0   652.0   6000.0
11  161.0  2039.0  1301.0  13429.0
12   96.0  1281.0   824.0   8841.0
13  111.0  1376.0   946.0   9087.0
14   54.0   660.0   440.0   3906.0
15  102.0  1262.0   809.0   7731.0
16  129.0  1566.0   967.0   9443.0
17  105.0  1327.0   891.0   8994.0
18  101.0  1245.0   911.0   8936.0
19  106.0  1217.0   863.0   7374.0

Visualizing the decision tree...
```

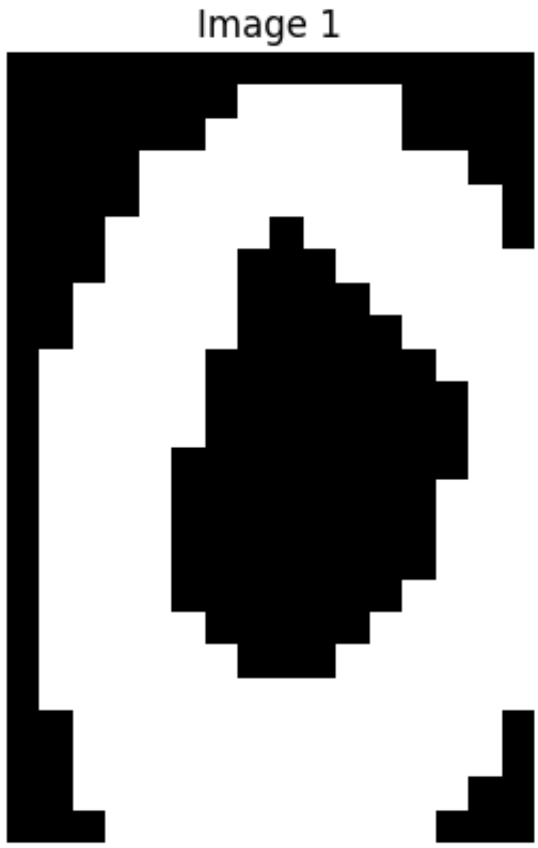


Task 4 trying to predict 5 classes with 10 training examples

In [16]:

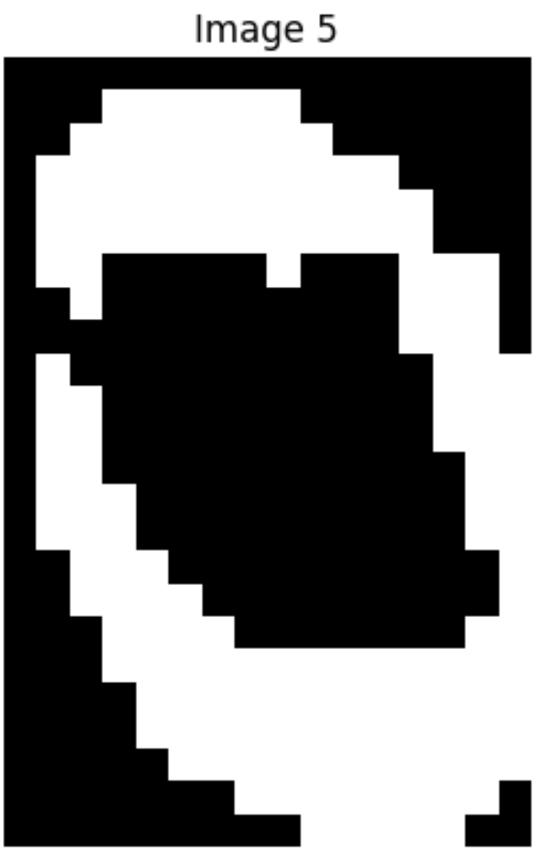
```python
def multi_class_test():
    """
    Train and test the classifier for multiple classes.
    """
    # Train the classifier for 5 classes
    test_tree = DecisionTreeClassifier()
    print("Training the classifier for 5 classes...")
    test_tree = train_classifier(test_tree, 5, 10)  # Five classes,

    # Test the classifier
    print("\nTesting the classifier for 5 classes...")
    for class_id in range(5):
        test_features = test_images(class_id, 5)
        predictions = test_tree.predict(test_features)
        print(f"Predicted classes for class {class_id} test pattern

# Run the multi-class test
multi_class_test()
```

```
Training the classifier for 5 classes...
['m00', 'm01', 'm10', 'm11']
Displaying image 1/10
```




```
Displaying image 2/10
```

```
Moments for m10 is [1699.0, 1336.0, 1554.0]
m11 is 18152.0

Moments for m11 is [21463.0, 14440.0, 18152.0]
m00 is 176.0

Moments for m00 is [214.0, 167.0, 187.0, 176.0]
m01 is 2067.0

Moments for m01 is [2759.0, 2056.0, 2371.0, 2067.0]
m10 is 1410.0

Moments for m10 is [1699.0, 1336.0, 1554.0, 1410.0]
m11 is 15854.0

Moments for m11 is [21463.0, 14440.0, 18152.0, 15854.0]
m00 is 169.0

Moments for m00 is [214.0, 167.0, 187.0, 176.0, 169.0]
m01 is 2074.0

Moments for m01 is [2759.0, 2056.0, 2371.0, 2067.0, 2074.0]
m10 is 1340.0

Moments for m10 is [1699.0, 1336.0, 1554.0, 1410.0, 1340.0]
m11 is 13970.0

Moments for m11 is [21463.0, 14440.0, 18152.0, 15854.0, 13970.0]
m00 is 109.0

Moments for m00 is [214.0, 167.0, 187.0, 176.0, 169.0, 109.0]
m01 is 1289.0

Moments for m01 is [2759.0, 2056.0, 2371.0, 2067.0, 2074.0, 1289.0]
m10 is 843.0

Moments for m10 is [1699.0, 1336.0, 1554.0, 1410.0, 1340.0, 843.0]
m11 is 9762.0

Moments for m11 is [21463.0, 14440.0, 18152.0, 15854.0, 13970.0, 97
62.0]
m00 is 155.0

Moments for m00 is [214.0, 167.0, 187.0, 176.0, 169.0, 109.0, 155.
0]
m01 is 1966.0

Moments for m01 is [2759.0, 2056.0, 2371.0, 2067.0, 2074.0, 1289.0,
1966.0]
m10 is 1230.0

Moments for m10 is [1699.0, 1336.0, 1554.0, 1410.0, 1340.0, 843.0,
1230.0]
m11 is 14855.0

Moments for m11 is [21463.0, 14440.0, 18152.0, 15854.0, 13970.0, 97
```




```
 Test vals
Predicted classes for class 0 test patterns: [0 0 0 0 0]
```

```
/opt/jupyterhub/pyvenv/lib/python3.10/site-packages/sklearn/base.p
y:465: UserWarning: X does not have valid feature names, but Decisi
onTreeClassifier was fitted with feature names
  warnings.warn(
```

Appendix B

1. Observations

As the problem complexity increases:

- **More Features:** Higher-order moments (e.g., m20, m21) or normalized moments are necessary.
- **Training Data:** A larger and more representative training set is required.
- **Classifier Choice:** Decision Trees are transparent but may not scale well with complex data. Naïve Bayes, or even more advanced models like SVM or Neural Networks, could be explored for better results.

Mornalised Moment

In [18]:
```
def normalized_moment(p, q, image):
    """
    Calculate the normalized central moment (p, q) for a binary imag

    Parameters:
    - p (int): Order of x in the moment calculation.
    - q (int): Order of y in the moment calculation.
    - image (numpy.ndarray): Binary image.

    Returns:
    - float: Normalized central moment.
    """
    # Calculate raw moments
    m00 = moment(0, 0, image)
    m10 = moment(1, 0, image)
    m01 = moment(0, 1, image)

    if m00 == 0:
        return 0  # Avoid division by zero

    # Calculate centroid
    x_bar = m10 / m00
    y_bar = m01 / m00

    # Central moment
    central_moment = 0
    for y in range(image.shape[0]):
        for x in range(image.shape[1]):
            if image[y, x] == 1:
                central_moment += ((x - x_bar) ** p) * ((y - y_bar)

    # Normalize
    gamma = 1 + (p + q) / 2
    normalized_value = central_moment / (m00 ** gamma)

    return normalized_value
```

In [19]:
```
def gen_image_moments(pmax,qmax, image, results):

    for p in range(pmax):
        for q in range(qmax):

            m_name = "m" + str(p) + str (q) # generate the "name" o

            m = normalized_moment(p,q,image) # calculate the moment

            print(m_name + " is "+ str(m) +"\n") # uncomment to see

            results[m_name].append(m) # add the calculated moment to

            print ("Moments for " + m_name + " is " + str(results[m_

    return(results)

def normalized_moment_tester():

    image = np.zeros((1,ROWS,COLS)) # space for one image

    feature_names = ["m" + str(p) + str (q) for p in range(2) for q

    panel_cols = generate_feature_dict(feature_names) # create empty

    read_images("train/br/", 1, 0,image) # read 1 image from the cl

    features = gen_image_moments(2,2,image[0],panel_cols) # calcula

    print( "FEATURES")
    print(features)

normalized_moment_tester()
```

```
m00 is 1.0

Moments for m00 is [1.0]
m01 is -1.4889269236129753e-15

Moments for m01 is [-1.4889269236129753e-15]
m10 is 6.241690609658053e-18

Moments for m10 is [6.241690609658053e-18]
m11 is -0.009638335144154119

Moments for m11 is [-0.009638335144154119]
FEATURES
{'m00': [1.0], 'm01': [-1.4889269236129753e-15], 'm10': [6.24169060
9658053e-18], 'm11': [-0.009638335144154119], 'class_id': []}
```

Alternative Classifier: Naïve Bayes

In [20]:
```
from sklearn.naive_bayes import GaussianNB

def train_naive_bayes(num_classes, num_training_examples):
    """
    Train a Naïve Bayes classifier with moment features.

    Returns:
    - GaussianNB: Trained classifier.
    """
    images = np.zeros((MAX_EXAMPLES, ROWS, COLS))
    feature_names = [f"m{p}{q}" for p in range(PMAX) for q in range
    feature_dict = generate_feature_dict(feature_names)

    for class_id in range(num_classes):
        images = read_images("train/br/", num_training_examples, cl
        feature_dict = gen_moment_features(class_id, num_training_e

    df = pd.DataFrame(feature_dict)
    features = df[feature_names]
    target = df["class_id"]

    model = GaussianNB()
    model.fit(features, target)
    return model
```

In [21]:
```
def test_naive_bayes(model, num_test_examples):
    """
    Test the Naïve Bayes model on test data.

    Parameters:
    - model: Trained Naïve Bayes model.
    - num_test_examples (int): Number of test examples per class.

    Returns:
    - None
    """
    for class_id in range(2):  # Assuming 2 classes
        # Generate test features for each class
        test_features = test_images(class_id, num_test_examples)

        # Predict classes and probabilities
        predictions = model.predict(test_features)
        probabilities = model.predict_proba(test_features)

        print(f"\nClass {class_id} Test Results:")
        for i, probs in enumerate(probabilities):
            print(f"Test {i + 1}: Prediction={predictions[i]}, Proba
```

In [22]:
```
# Train a Naïve Bayes classifier
num_classes = 2  # Number of classes (e.g., "0" and "1")
num_training_examples = 10  # Number of training examples per class
nb_model = train_naive_bayes(num_classes, num_training_examples)

print("Naïve Bayes classifier trained successfully.")
```

```
m00 is 1.0

Moments for m00 is [1.0]
m01 is -1.4889269236129753e-15

Moments for m01 is [-1.4889269236129753e-15]
m10 is 6.241690609658053e-18

Moments for m10 is [6.241690609658053e-18]
m11 is -0.009638335144154119

Moments for m11 is [-0.009638335144154119]
m00 is 1.0

Moments for m00 is [1.0, 1.0]
m01 is -6.568381921532107e-16

Moments for m01 is [-1.4889269236129753e-15, -6.568381921532107e-1
6]
m10 is 0.0

Moments for m10 is [6.241690609658053e-18, 0.0]
m11 is -0.07199971314855315

Moments for m11 is [-0.009638335144154119, -0.07199971314855315]
m00 is 1.0

Moments for m00 is [1.0, 1.0, 1.0]
m01 is 6.196303148022578e-16

Moments for m01 is [-1.4889269236129753e-15, -6.568381921532107e-1
6, 6.196303148022578e-16]
m10 is -1.3059473002558796e-16

Moments for m10 is [6.241690609658053e-18, 0.0, -1.3059473002558796
e-16]
m11 is -0.044364733745075643

Moments for m11 is [-0.009638335144154119, -0.07199971314855315, -
0.044364733745075643]
m00 is 1.0

Moments for m00 is [1.0, 1.0, 1.0, 1.0]
m01 is 2.358429532957636e-16

Moments for m01 is [-1.4889269236129753e-15, -6.568381921532107e-1
6, 6.196303148022578e-16, 2.358429532957636e-16]
m10 is -1.2172539524942638e-16

Moments for m10 is [6.241690609658053e-18, 0.0, -1.3059473002558796
e-16, -1.2172539524942638e-16]
m11 is -0.022775330461119465

Moments for m11 is [-0.009638335144154119, -0.07199971314855315, -
0.044364733745075643, -0.022775330461119465]
m00 is 1.0
```