Abstract Patterns and Features

AI Systems Implementation

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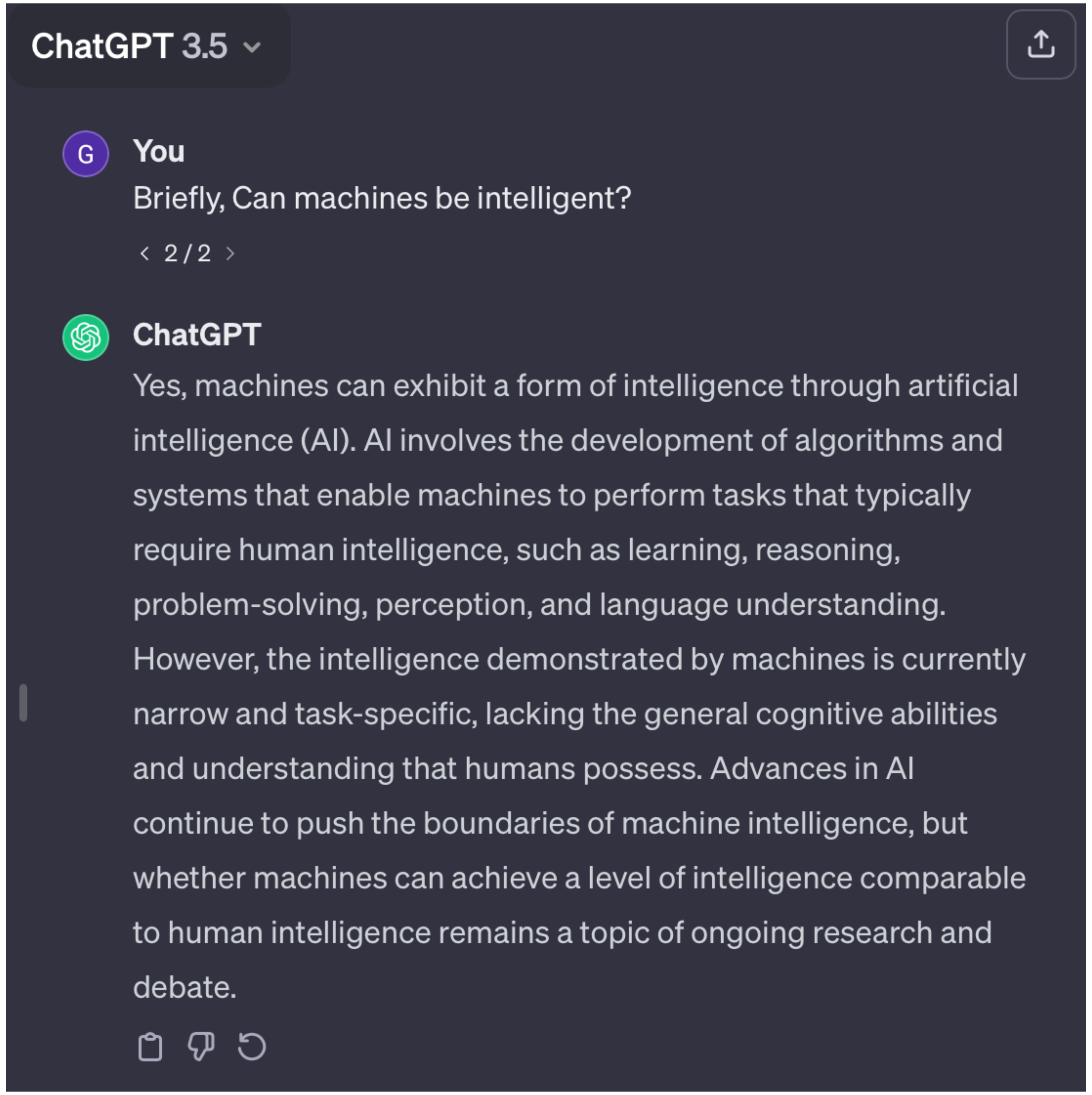
Based on slides by Gareth Howells

Can machines be intelligent?

- Is this intelligent?
    1. Yes!
    2. Not at the moment, but maybe when Machine Learning technology improves
    3. It'll only ever have a basic understanding of the world, it'll never compare to humans
    4. Don't be daft, it's made of plastic!
- Why?



Lecture Overview

- What are features?
- Choosing features
- Deep dive on Moments
  - What is a Moment?
  - How are they extracted?
- This week’s class work

BUT FIRST!

- How would you rate your AI knowledge?
  - I have a good understanding of the theory
  - I understand the principles, but not the practical details
  - This all sounds confusing, I'm absolutely terrified, what have I signed up for?
  - Please stop asking us questions and get on with it…
- Fine…

Features

- What is a feature?
- In essence, values that describe an object, task, or domain
  - Abstract away the raw data
  - Reduce the *dimensionality* of the problem!

Features

- Features could be:
  - Raw, quantitive data
    - Numerical data, boolean values, vectors, even images
  - Direct features
    - Edge detection, detected circles/ellipses, Spectrograms
  - Abstract features
    - Region textures, *Moments*

Features

Garbage in, Garbage out

- Problems with data = problems with classification
  - Insufficient training data
  - Unrepresentative training data
  - Irrelevant features
  - Poor quality data
- Aim for high quality, representative data, with relevant features
  - Easier said than done…

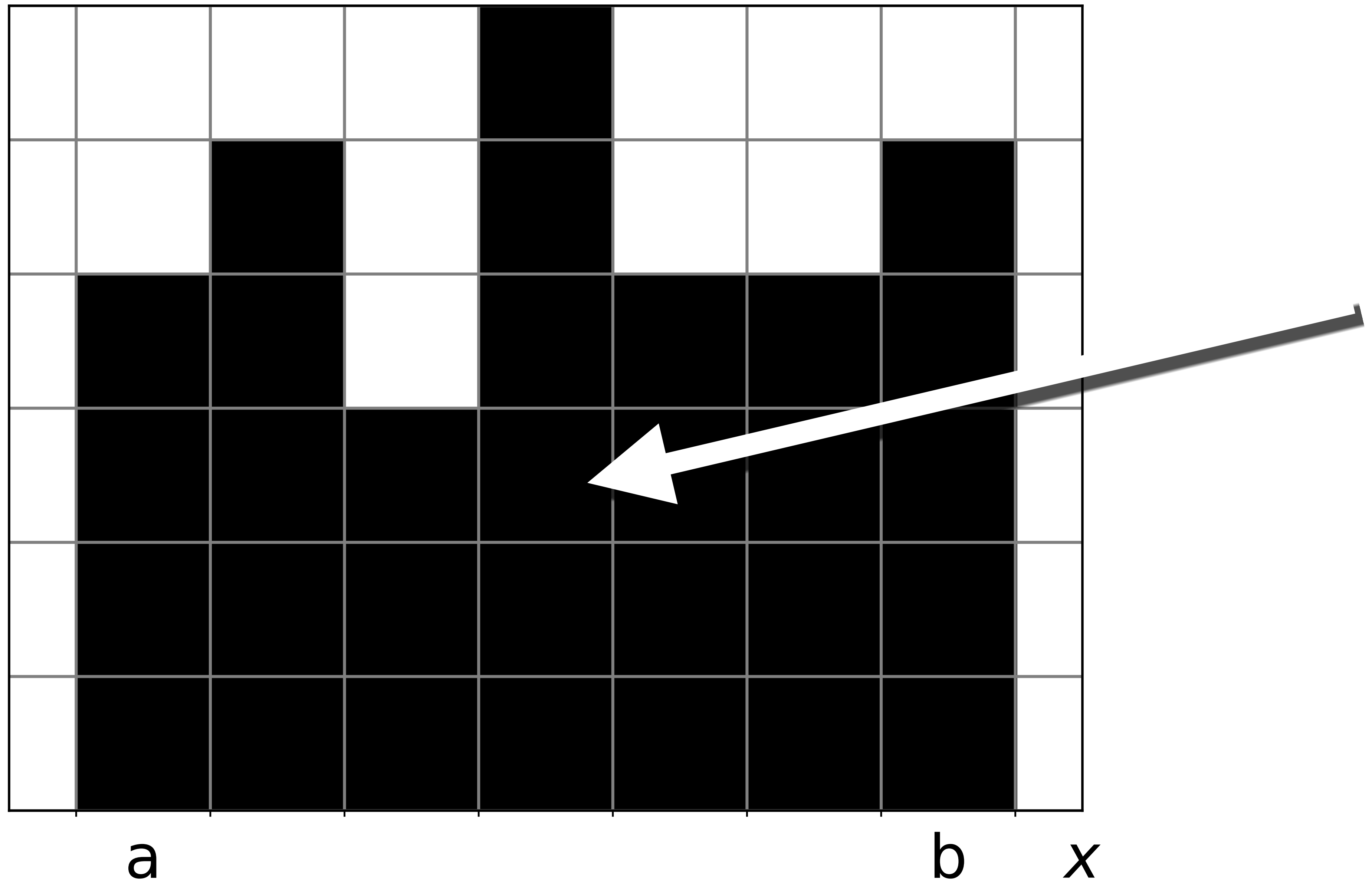
Moments

- Moments are an abstract feature that characterises the shape of objects and regions
- The $(p, q)^{th}$ moment of an image $f(x, y)$ is given by:

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) dx \,.\, dy$$

- Where $p, q = 0,1,2...\infty$
- $f(x, y)$is the image intensity (value) as point (x, y)

OK, that looked horrible



- If x is made from discrete intervals, then $\displaystyle\int_a^b f(x)dx$ becomes $\displaystyle\sum_a^b f(x)$
- Because the area is the sum of each column ($= 4 + 5 + 3 + 6 + 4 + 4 + 5 = 31$ in this case)

Order of a Moment

- The order of a moment is the sum of p and q: $(p + q)$
- Therefor there is just one zero order moment:
  - m_{00}
- Two first order moments
  - m_{10}, m_{01}
- Three second order moments
  - m_{20}, m_{02}, m_{11}
- And so on…

Calculating Moments

- Assume a binary image
  - Each pixel is either 0 (white) or 1 (black)
- Moment: $$m_{pq} = \sum_x \sum_y x^p y^q \,.\, f(x, y)$$
- But $f(x, y)$ is either 0 or 1
- If $f(x, y)$ is 0, then $x^p y^q \,.\, f(x, y) = 0$
- If $f(x, y)$ is 1, then $x^p y^q \,.\, f(x, y) = x^p y^q$

Calculating Moments

- Therefore, for a binary image, $$m_{pq} = \sum_x \sum_y x^p y^q$$
  - Where $\sum$ is taken over all black elements
- $m_{00} =$ Total number of black pixels in the image
  - Since x^0 is always 1 and y^0 is always 1 and $1 \times 1 = 1$
- $m_{10} = \sum x$ (the sum of all the x coordinates of the black pixels
  - Since x^1 is equal to x and y^0 is always 1 and $x \times 1 = x$
- $m_{01} = \sum y$ (the same of all the y coordinates of the black pixels)
  - Since x^0 is always 1 and y^1 is equal to y and $1 \times y = y$

Moments

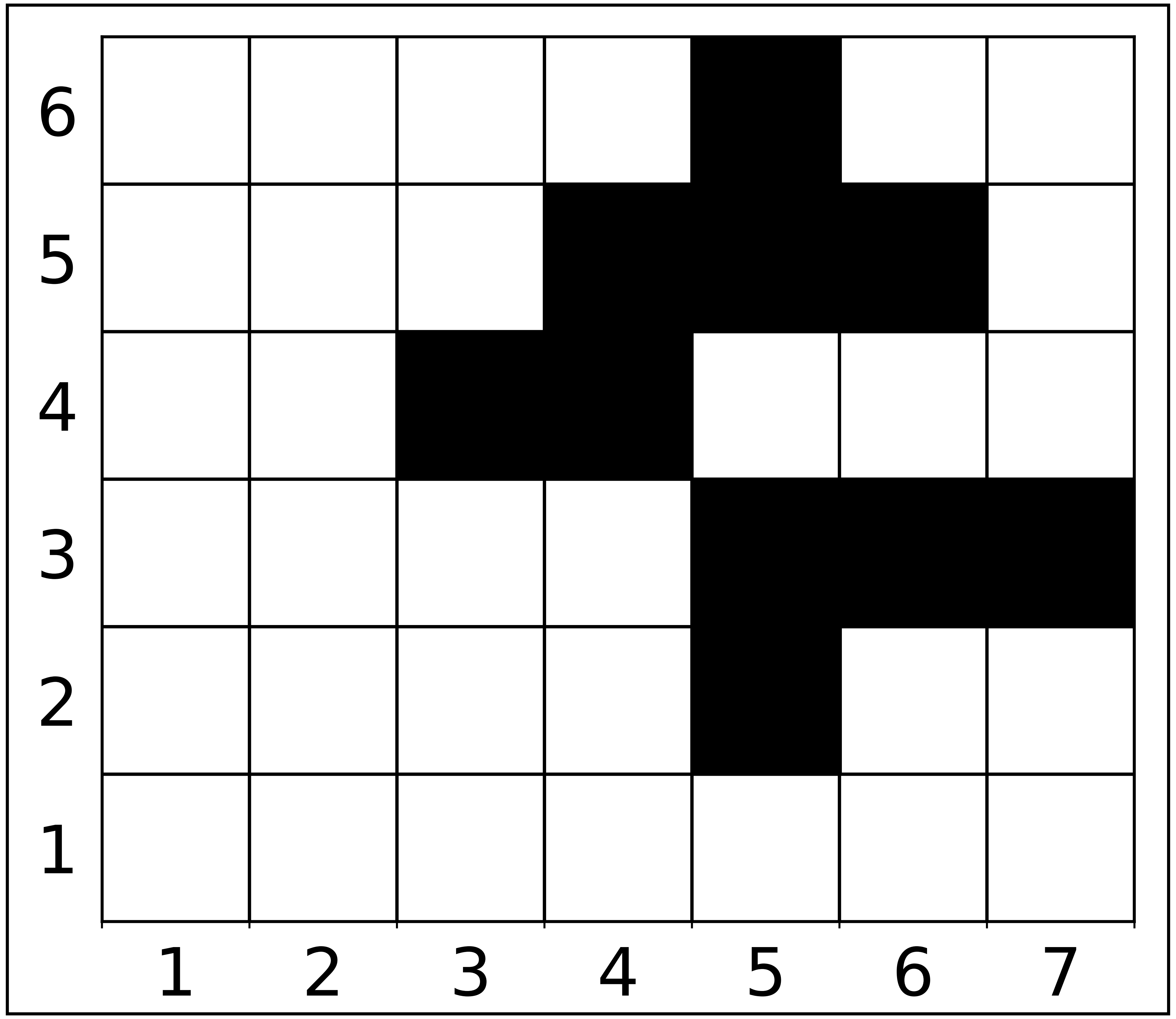
So, to recap…

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) dx \,.\, dy$$

- m is shorthand for moment
- m_{pq} means the pq^{th} moment
- $f(x, y)$ means the intensity of the pixel at (x, y)
- For a binary image, we can simplify to: $m_{pq} = \sum_x \sum_y x^p y^q$

Example

- A binary image has black pixels at the following coordinates



x	3	7	4	5	5	6	5	6	5	4
y	4	3	5	2	3	5	6	3	5	4

Example

x	3	7	4	5	5	6	5	6	5	4
y	4	3	5	2	3	5	6	3	5	4

$$m_{pq} = \sum_x \sum_y x^p y^q \qquad m_{11} = \sum_x \sum_y xy$$

- $= (3 \times 4) + (7 \times 3) + (4 \times 5) + (5 \times 2) + (5 \times 3)$
  $+(6 \times 5) + (5 \times 6) + (6 \times 3) + (5 \times 5) + (4 \times 4) = 197$

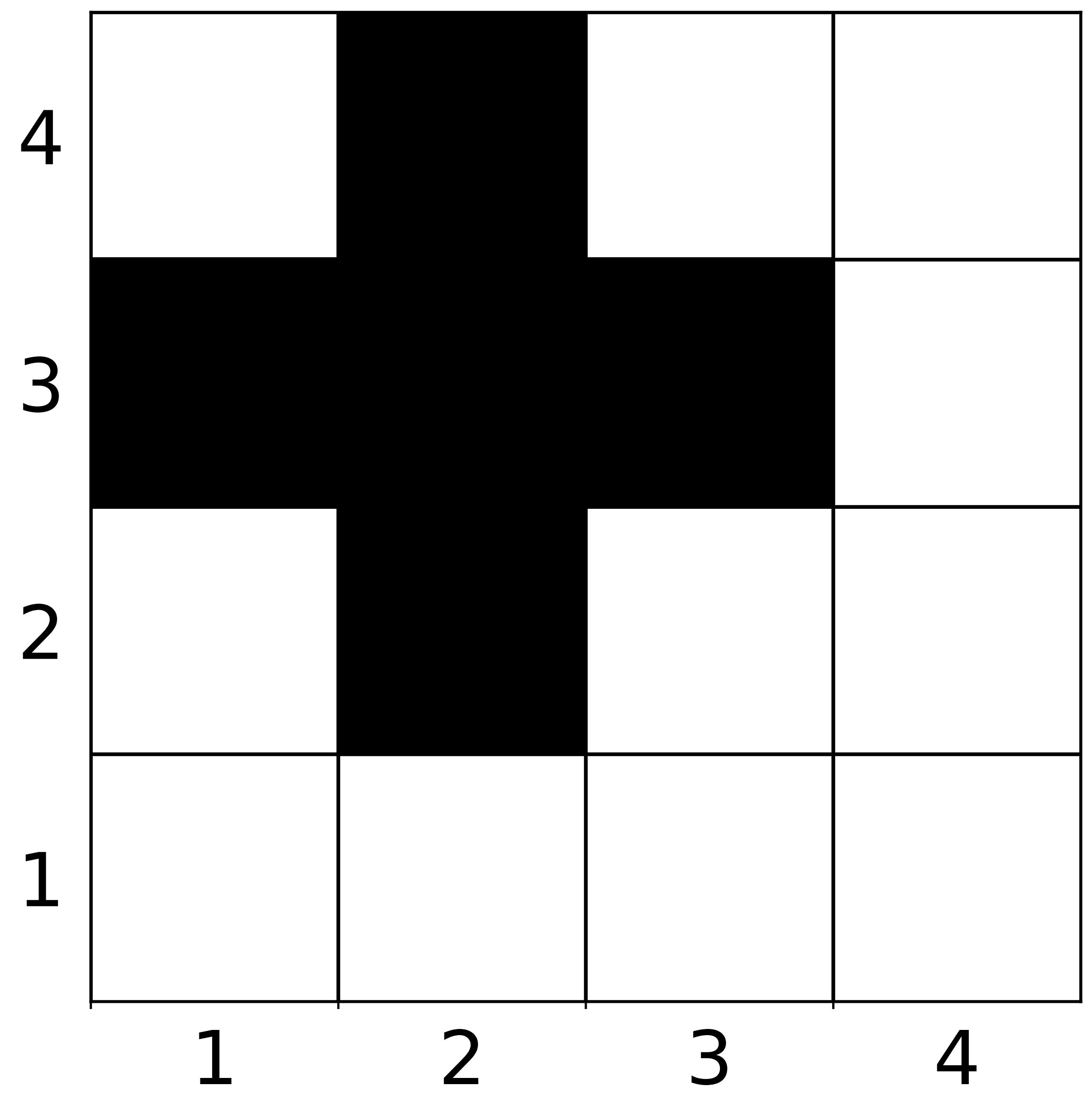
Example

Another One

$$m_{pq} = \sum_x \sum_y x^p y^q f(x, y)$$

$$\begin{aligned}
m_{20} &= ((1^2 \times 3^0) + (2^2 \times 2^0) + (2^2 \times 3^0) + (2^2 \times 4^0) + (3^2 \times 3^0)) \\
&= ((1^2 \times 1) + (2^2 \times 1) + (2^2 \times 1) + (2^2 \times 1) + (3^2 \times 1)) \\
&= (1^2 + 2^2 + 2^2 + 2^2 + 3^2) \\
&= (1 + 4 + 4 + 4 + 9) \\
&= 22
\end{aligned}$$

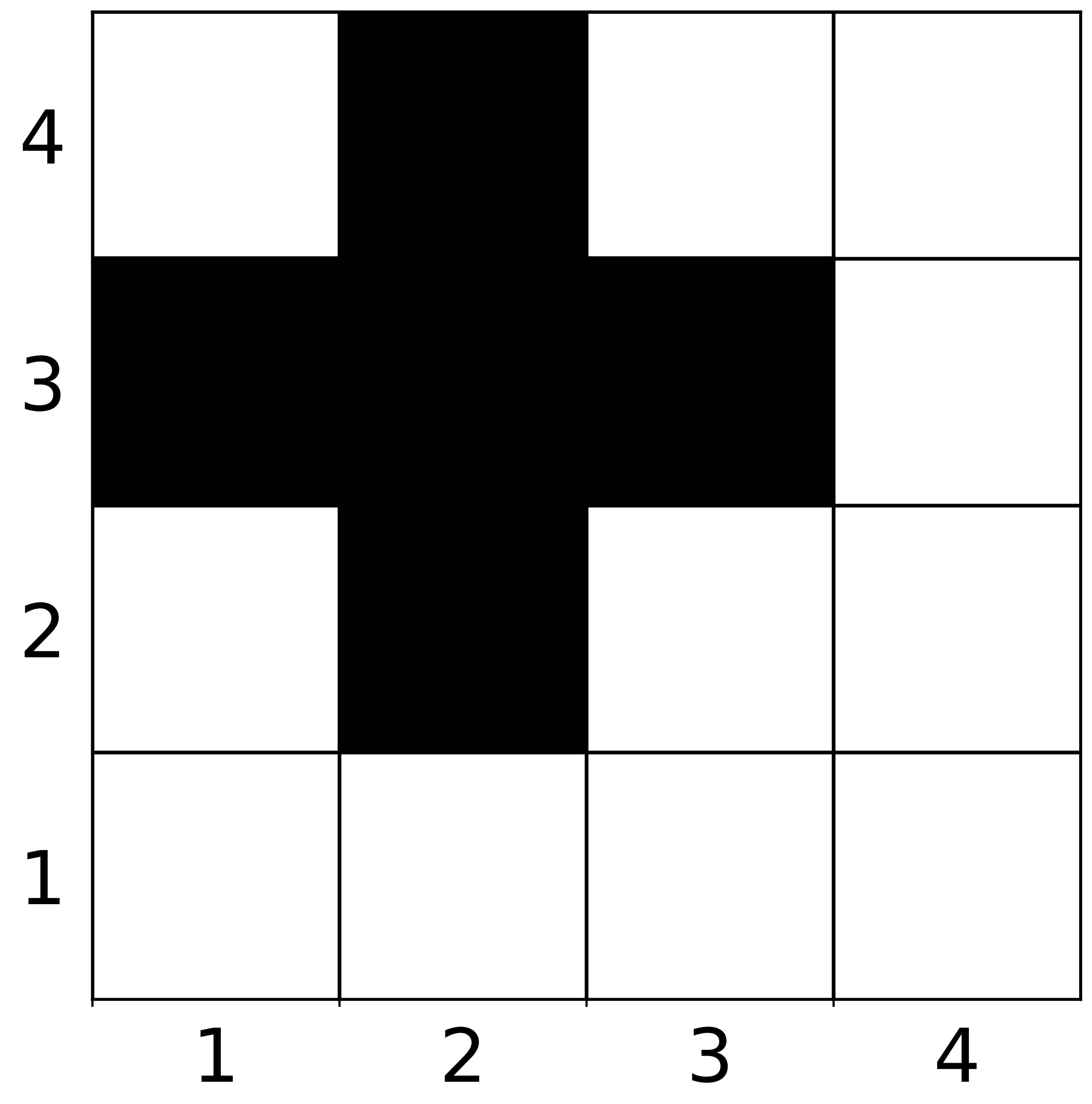
$$\begin{aligned}
m_{22} &= ((1^2 \times 3^2) + (2^2 \times 2^2) + (2^2 \times 3^2) + (2^2 \times 4^2) + (3^2 \times 3^2)) \\
&= ((1 \times 9) + (4 \times 4) + (4 \times 9) + (4 \times 16) + (9 \times 9)) \\
&= (9 + 16 + 36 + 64 + 81) \\
&= 206
\end{aligned}$$



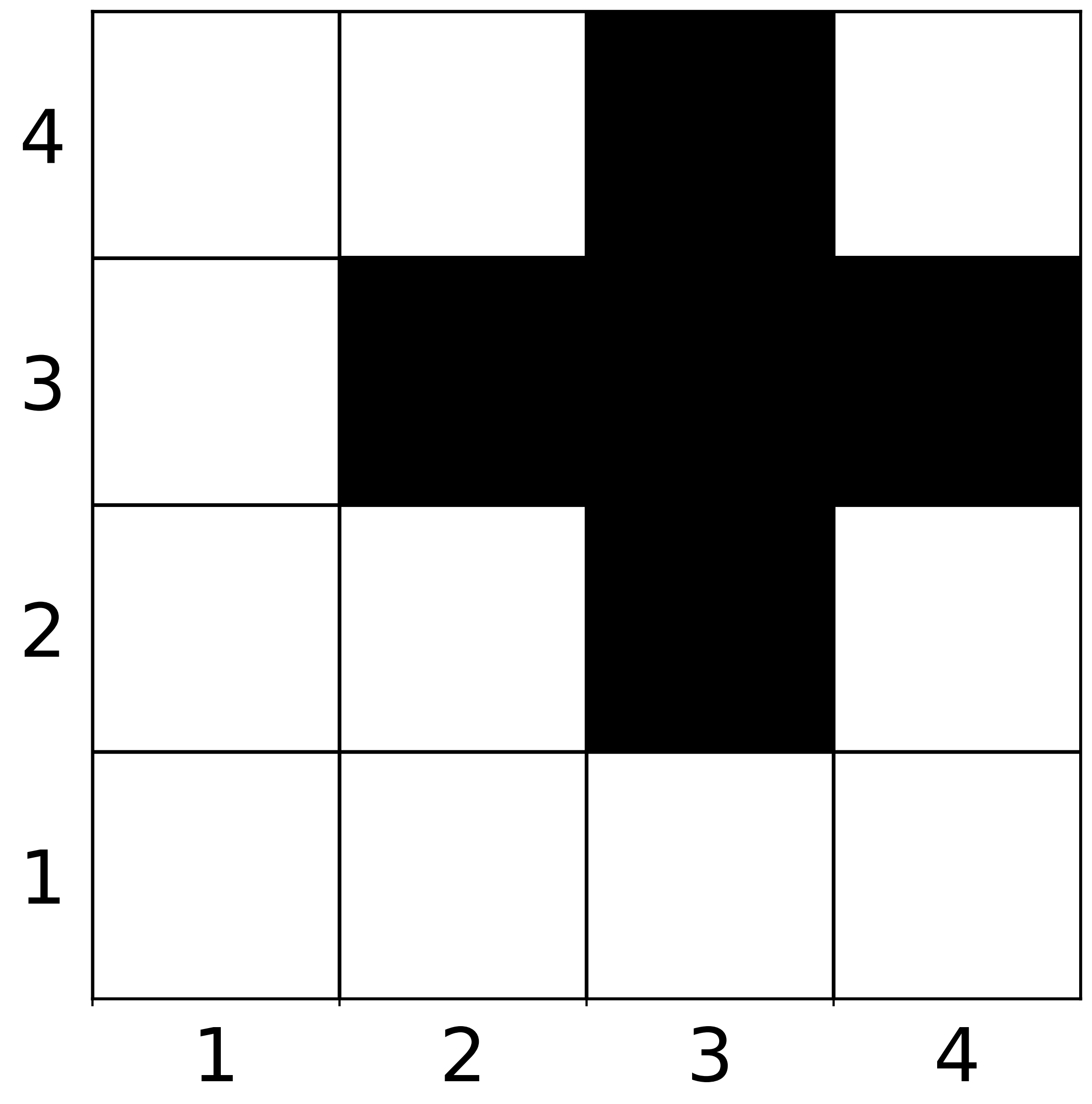
Normalised Example

Un-Normalised

- A : $m_{20} = (1^2 + 2^2 + 2^2 + 2^2 + 3^2)$
  $= 22$



- B: $m_{20} = (2^2 + 3^2 + 3^2 + 3^2 + 4^2)$
  $= 47$



- Same shapes, but different values!

Normalised Moments

- Currently, the moments we extract are relative to the location of an object…
- But what if we don't always know where the object will be in an image?

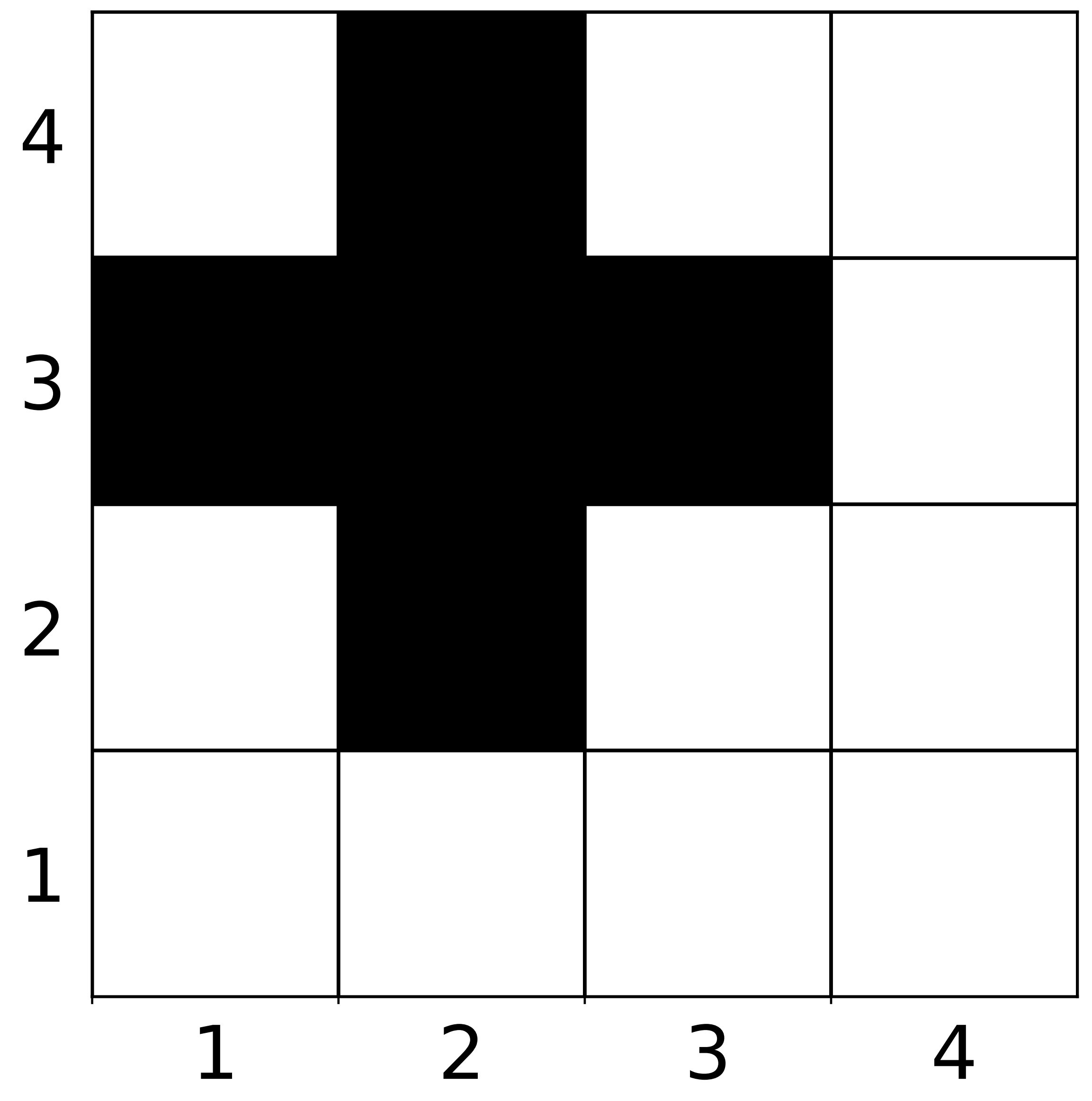
$$M_{pq} = \sum (x - \overline{x})^p (y - \overline{y})^q$$

- Where $\overline{x}$ is the average x coordinate of all black pixels, and $\overline{y}$ is the average y coordinate of black pixels

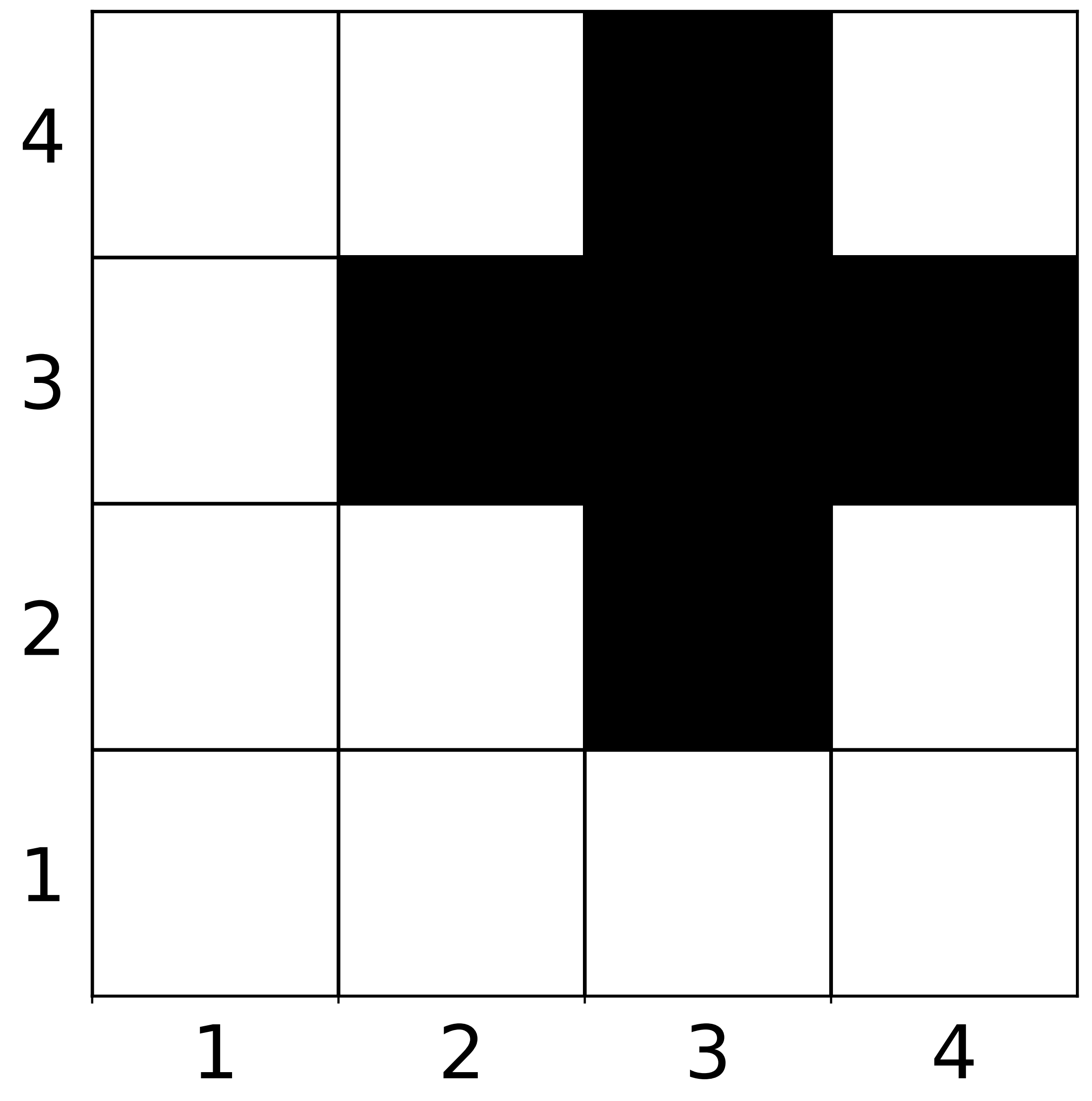
Normalised Example

Normalised

- A :
$$
\begin{aligned}
M_{20} &= ((1-2)^2 + (2-2)^2 + (2-2)^2 + (2-2)^2 + (3-2)^2) \\
&= (1+0+0+0+1) \\
&= 2
\end{aligned}
$$



- B:
$$
\begin{aligned}
M_{20} &= ((2-3)^2 + (3-3)^2 + (3-3)^2 + (3-3)^2 + (4-3)^2) \\
&= (1+0+0+0+1) \\
&= 2
\end{aligned}
$$



- Same shape, different locations = Same value!

Recap

- This: $m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) dx \,.\, dy$
- Becomes this: $m_{pq} = \sum_{x} \sum_{y} x^p y^q f(x, y)$
- And to normalise it, use this: $M_{pq} = \sum (x - \overline{x})^p (y - \overline{y})^q$
- Now… your turn to implement it!

Up next

- Classes on Thursday/Friday
  - Implementing Moments in Python (Jupyter), and performing classification with them.
- Next week's lecture
  - Classification!