

COMP8760

Lecture 1

Solutions to Worksheet for Practice

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1. Sets: Membership, Subset, Cardinality

Let $A = \{1, 2, 3\}$. Fill in the blanks.

- (a) Is $1 \in A$? Yes (Yes/No)
- (b) Is $4 \in A$? No (Yes/No)
- (c) Is $4 \notin A$? Yes (Yes/No)
- (d) Is $100 \notin A$? Yes (Yes/No)
- (e) Is $\{1, 3\} \subseteq A$? Yes (Yes/No)
- (f) Is $\{4\} \subseteq A$? No (Yes/No)
- (g) Is $\{3, 4\} \subseteq A$? No (Yes/No)
- (h) What is $|A|$? 3
- (i) What is $|\{1, 2, 3, 4, \dots, 100\}|$? 100
- (j) What is $|\{2, 4, 6, 8, \dots, 100\}|$? 50
- (k) Is $A \subseteq \{2, 4, 6, 8, \dots, 100\} \subseteq \mathbb{Z}$? No (Yes/No) (error corrected)

2. Division Theorem

Fill in the blanks.

- (a) $99 = \underline{8} \times 12 + \underline{3}$
- (b) $199 = \underline{3} \times 53 + \underline{40}$
- (c) $9 = \underline{0} \times 12 + \underline{9}$
- (d) $0 = \underline{0} \times 53 + \underline{0}$
- (e) $-1 = \underline{-1} \times 12 + \underline{11}$
- (f) $-12 = \underline{-1} \times 12 + \underline{0}$

3. \mathbb{Z}_N : Set of All Remainders of N

Fill in the blanks.

- (a) $\mathbb{Z}_2 = \{\underline{0}, \underline{1}\}$
- (b) $\mathbb{Z}_5 = \{\underline{0}, \underline{1}, \underline{2}, \underline{3}, \underline{4}\}$
- (c) $\mathbb{Z}_{12} = \{\underline{0}, \underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6}, \underline{7}, \underline{8}, \underline{9}, \underline{10}, \underline{11}\}$
- (d) $\mathbb{Z}_{13} = \{\underline{0}, \underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6}, \underline{7}, \underline{8}, \underline{9}, \underline{10}, \underline{11}, \underline{12}\}$
- (e) $\mathbb{Z}_{1297} = \{\underline{0}, \underline{1}, \underline{2}, \dots, \underline{1296}\}$

4. Modulus operator: $a \bmod N$

Fill in the blanks.

- (a) $23 \bmod 11 = \underline{1}$
- (b) $22 \bmod 11 = \underline{0}$
- (c) $23 = 34 \bmod 11$; True (True/False)
- (d) $-3 = 8 \bmod 11$; True (True/False)
- (e) $-3 = -15 \bmod 11$; False (True/False)

5. Modular Arithmetic

Write the addition and multiplication tables for all elements in \mathbb{Z}_4 .

Answer:

We know that \mathbb{Z}_4 is the set of all remainders of 4. So,

$$\mathbb{Z}_4 = \{0, 1, 2, 3\}.$$

Addition (mod 4) in \mathbb{Z}_4

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Multiplication (mod 4) in \mathbb{Z}_4

	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

6. Prime Numbers

Fill in the blanks.

- (a) Is 63 a prime? No (Yes/No)
- (b) Is 67 a prime? Yes (Yes/No)
- (c) What is the prime factorisation of 1001? $1001 = 7 \times 11 \times 13$
- (d) Using the prime factorisation technique, find the GCD and LCM of the integer pair (539, 1001)?

$$539 = 7^2 \times 11^1;$$

$$1001 = 7^1 \times 11^1 \times 13^1;$$

So,

$$\text{lcm}(539, 1001) = 7^2 \times 11^1 \times 13^1 = 7007,$$

and

$$\text{gcd}(539, 1001) = 7^1 \times 11^1 \times 13^0 = 77.$$