COMP8760

Lecture 2

Solutions to Worksheet for Practice

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1. Find the Prime Factorisation of 1001.
   Ans:
   $$\begin{array}{r|l} 7 & 1001 \\ \hline 11 & 143 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$
   Hence, $1001 = 7^1 \times 11^1 \times 13^1$.

2. Find the Prime Factorisation of 7497.
   Ans:
   $$\begin{array}{r|l} 3 & 7497 \\ \hline 3 & 2499 \\ \hline 7 & 833 \\ \hline 7 & 119 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$
   Hence, $7497 = 3^2 \times 7^2 \times 17^1$.

3. Using the prime factorisations, find the GCD and LCM of the integer pair $(1001, 7497)$?
   Ans:
   $$1001 = 7^1 \times 11^1 \times 13^1;$$
   $$7497 = 3^2 \times 7^2 \times 17^1;$$
   So,
   $$\begin{aligned} \text{lcm}(1001, 7497) &= 3^{max(0,2)} \times 7^{max(1,2)} \times 11^{max(1,0)} \times 13^{max(1,0)} \times 17^{max(0,1)} \\ &= 3^2 \times 7^2 \times 11^1 \times 13^1 \times 17^1 \\ &= 1072071. \end{aligned}$$
   and
   $$\begin{aligned} \text{gcd}(1001, 7497) &= 3^{min(0,2)} \times 7^{min(1,2)} \times 11^{min(1,0)} \times 13^{min(1,0)} \times 17^{min(0,1)} \\ &= 3^0 \times 7^1 \times 11^0 \times 13^0 \times 17^0 \\ &= 7. \end{aligned}$$

4. Find the GCD of the integer pair $(1001, 7497)$ using the Euclidean algorithm?
   Ans:

$$\begin{aligned}
\gcd(7497, 1001) &= \gcd(1001, 7497 \mod 1001) \\
&= \gcd(1001, 490) \\
&= \gcd(490, 1001 \mod 490) \\
&= \gcd(490, 21) \\
&= \gcd(21, 490 \mod 21) \\
&= \gcd(21, 7) \\
&= \gcd(7, 21 \mod 7) \\
&= \gcd(7, 0) \\
&= 7
\end{aligned}$$

5. Are the numbers $(1001, 749)$ mutually prime?
   Ans:
   To check if two numbers are mutually prime, we find their greatest common divisor. If the greatest common divisor is 1, then the numbers are mutually prime.
   Since $\gcd(1001, 749) > 1$, they are **not mutually prime**.

6. Are the numbers $(119, 143)$ mutually prime?
   Ans:
   As before, we find the greatest common divisor of the two numbers to check if it is 1.

$$\begin{aligned}
\gcd(143, 119) &= \gcd(119, 143 \mod 119) \\
&= \gcd(119, 24) \\
&= \gcd(24, 23) \\
&= \gcd(23, 1) \\
&= \gcd(1, 0) \\
&= 1.
\end{aligned}$$

   Hence, $(1001, 749)$ are mutually prime.

7. Find $\phi(441)$.
   Ans:
   We first find the prime factorisation of 441.

$$441 = 3^2 \times 7^2.$$

   Then, we use the formula for computing the value of $\phi(N)$.

$$\begin{aligned}
\phi(441) &= 441\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{7}\right) \\
&= 441\left(\frac{2}{3}\right)\left(\frac{6}{7}\right) \\
&= 252.
\end{aligned}$$