COMP8760 Lecture

Hash Functions

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qctxre

Outline

Study Material for Hash Functions

Book 1	*Cryptography Made Simple*
Author	Nigel P. Smart.
	Link to eBook

Section 1.4.2 Birthday Paradox
Section 1.5 Big Numbers

Chapter 14 Hash Functions
Section 14.1 Collision Resistance
Section 14.2 Padding
Section 14.3 The Merkle–Damgård construction
Section 14.4.3 SHA-256

Book Sections

Part 1 Preliminaries

- Birthday Paradox (Section 1.4.2)
- Big Numbers (Section 1.5)

Part 2 Cryptographic Hash Function

- The three security properties (Section 14.1)
- Padding (Section 14.2)
- The Merkle–Damgård construction (Section 14.3)
- SHA-256 (Section 14.4.3)

Outline

As people keep entering a room

What is the probability that none of them have the same birthday?

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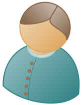

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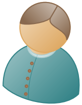

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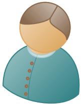


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 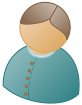 

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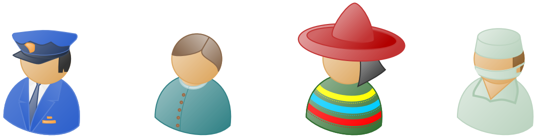

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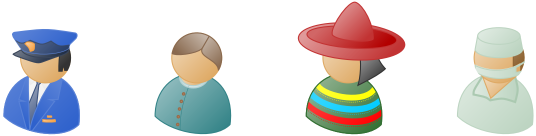

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$$\Pr[\text{None have same birthday}] = \frac{m\cdot(m-1)\cdot(m-2)\cdots(m-k+1)}{m^k} = \frac{{}^{m}P_k}{m^k}$$

$$\Pr[\text{At least one pair collides}] = 1 - \frac{m\cdot(m-1)\cdot(m-2)\cdots(m-k+1)}{m^k} = 1 - \frac{{}^{m}P_k}{m^k}$$

Birthday Paradox: Probability of collision ≈ 1 for $k \ll 365$

What is the probability that none of them have the same birthday?

$\frac{365}{365}$ $\frac{365-1}{365}$ $\frac{365-2}{365}$ $\frac{365-3}{365}$ $\frac{365-4}{365}$

- With 23 people in a room, the probability of a collision is
$$1 - \frac{{}^{365}P_{23}}{365^{23}} = 0.507.$$
- With 30 people in a room, the probability of a collision is
$$1 - \frac{{}^{365}P_{30}}{365^{30}} = 0.706.$$
- With 100 people in a room, the probability of a collision is
$$1 - \frac{{}^{365}P_{100}}{365^{100}} > 0.999.$$

Birthday Bound

The probability of collision in choosing n elements from m is at most

$$\frac{n^2}{2 \cdot m}.$$

Outline

Computational Difficulty

How big can a number be
before it is impossible for someone
to perform that many (classical) computer operations.

- $2^{10} \approx 10^3$.
- 3GHz processor can execute $3,000,000,000$ instructions per second.
- 3GHz octa-core processor can execute 24×10^9 instructions per second.
- Let $\mathcal{C}$ be an advanced classical computer that performs $10^{12} \approx 2^{40}$ instructions per second.
  - For 2^{64} operations:
    $\mathcal{C}$ would take $\frac{2^{64}}{2^{40}}$ seconds or 194 days;
    or one day for 194 $\mathcal{C}$s.
  - For 2^{80} operations ($> 10^{23}$):
    $\mathcal{C}$ would take $\frac{2^{80}}{2^{40}}$ seconds or 34,900 years;
    or 2 years for 15,000 $\mathcal{C}$s.
    Still imaginable!
  - For 2^{128} operations ($> 10^{37}$):
    $\mathcal{C}$ would take $\frac{2^{128}}{2^{40}}$ seconds or $9 \cdot 10^{18}$ years.
    Unimaginable!

Outline

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Cryptographic Hash Function

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A function that
takes arbitrary length bit strings as input and
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The output is often called digest, hashcode or hash value.

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Cryptographic Hash Function

... is a hash function that is one-way.

In other words,
given $H : D \to C$, and $y \in C$,
it is computationally infeasible to find any value $x \in D$ such that

$$y = H(x).$$

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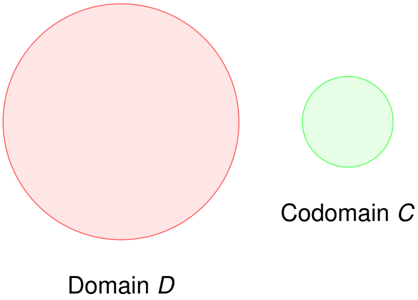

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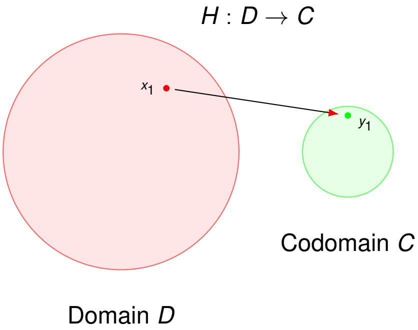

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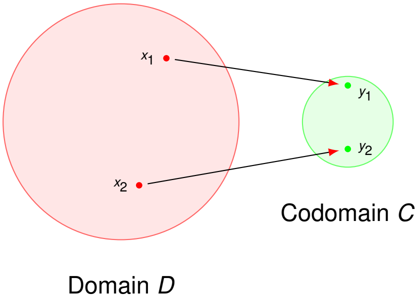

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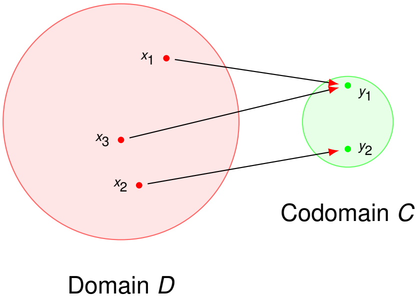
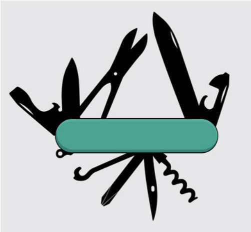


Applications



- Integrity checks: if the hashes do not match, the message must have been altered
- Authentication: through cryptographic signatures
- Key derivation functions: to generate keys from various inputs
- Proof-of-work: blockchains, mitigating spamming
- Password: stored in place of the plaintext password
- Indexing for search: in databases, other file systems
- Perceptual hashing: proximity search for text/image/video
- et cetera ...

Outline

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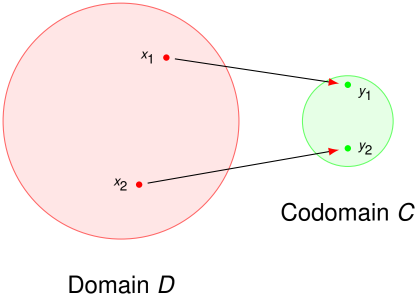
1. Preimage resistance

For a hash function H which produces outputs of t bits, finding preimages should require $O(2^t)$ time.

$$H : D \rightarrow C$$



Preimage Resistance $\equiv$ One-Way Function

Challenger

Adversary

Advantage of Adversary = Pr[Adversary wins the game]

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H $\longrightarrow$

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Preimage Resistance $\equiv$ One-Way Function

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$H \longrightarrow$

$m \leftarrow D$

$h \leftarrow H(m) \longrightarrow$

$m' \longleftarrow$

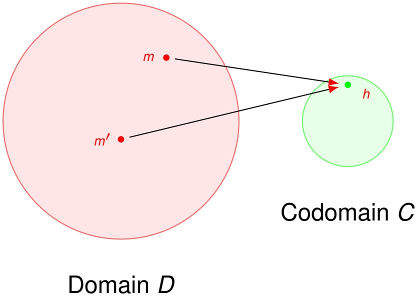
Win if $H(m') = h$

Advantage of Adversary = Pr[Adversary wins the game]

2. Second-preimage resistance

Given m, it should be hard to find an $m' \neq m$ with $H(m') = H(m)$.

$H : D \to C$



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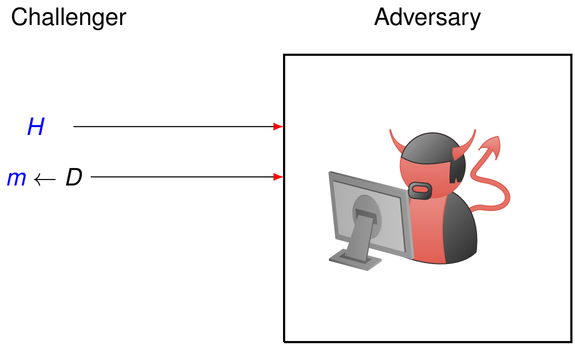

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Second-Preimage Resistance

Challenger | Adversary

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$m \leftarrow D \longrightarrow$

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Second-Preimage Resistance

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$m \leftarrow D$ $\longrightarrow$

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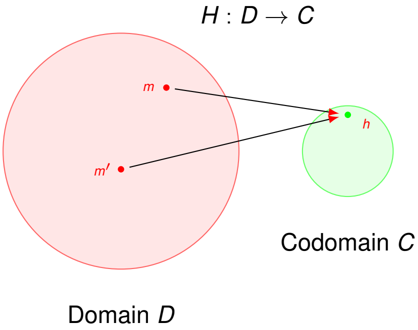
Win if $H(m) = H(m')$
and $m \neq m'$

Advantage of Adversary = Pr[Adversary wins the game]

3. Collision resistance

It should be hard to find a pair $m' \neq m$ with $H(m') = H(m)$.



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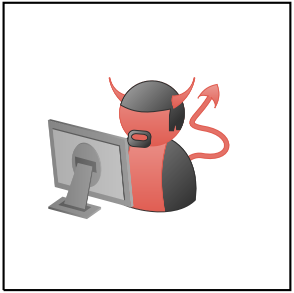

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Collision Resistance

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Assumption: Collision Resistance by Human Ignorance

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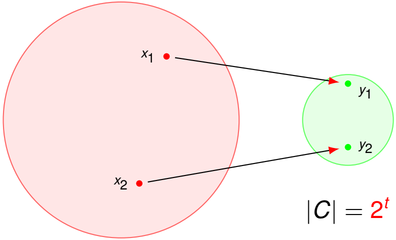
In other words, it is infeasible to find two elements $m, m' \in D$ such that

$$H(m) = H(m').$$

How many queries to find a preimage?

$$H : D \to C$$

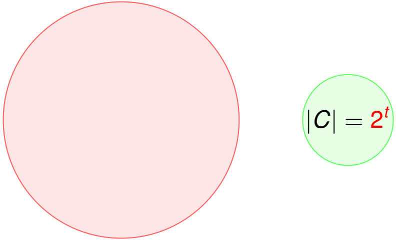


For a “well-designed hash function”, the preimage of an element would be one of 2^t elements of D.
So, the number of queries would be approximately

$$q_{\text{finding preimage}} = 2^t.$$

How many queries to find a collision?

$$H : D \to C$$



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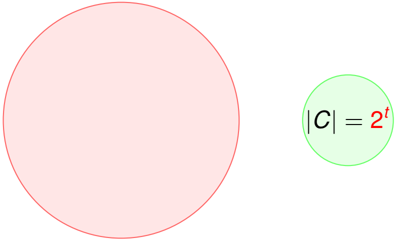

By the birthday bound, after q queries, the probability of collision is at most

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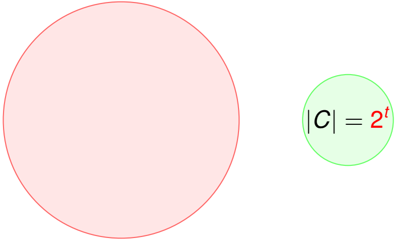

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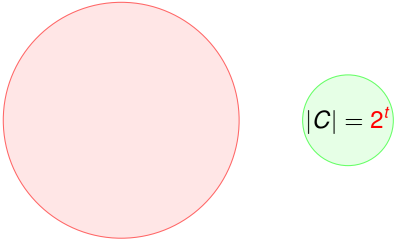

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So, if we require 128-bit security of a hash function, we would want the output to be of size at least 256 bits.

Collision Resistance -> Preimage Resistance (Reduction)

Lemma 14.2

If a function H is preimage resistant for every element of the range of H, it will be collision resistant and second-preimage resistant.

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Since D is a huge set with many preimages of h, hence it is highly likely that $x \neq x'$.

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If there is an efficient $\mathcal{O}_{preimage}$, then there is an efficient $\mathcal{A}_{collision}$.
By contrapositivity, since there is no efficient $\mathcal{A}_{collision}$, hence there is no efficient $\mathcal{O}_{preimage}$.

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Similarly for second-preimage resistance.

Hence, Preimage Resistance is a weaker assumption
than Collision Resistance or Second-Preimage Resistance.

Collision Resistance -> Second-Preimage Resistance (Reduction)

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- Let $\mathcal{O}_{second-preimage}$ be an oracle that given an input $x \in D$, provides a second-preimage $x' \in D$.

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By contrapositivity, since there is no efficient $\mathcal{A}_{collision}$, hence there is no efficient $\mathcal{O}_{second-preimage}$.

Hence, Second-Preimage Resistance is a weaker assumption than Collision Resistance.

Outline

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- Hash Functions: Preliminaries
  - Birthday Bound
  - “Big” Numbers
- Cryptographic Hash Function
  - Hash Function: Security Properties
- SHA-256
  - Padding
  - The Merkle–Damgård construction
  - SHA-256

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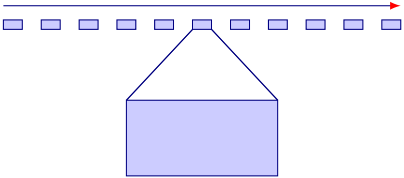

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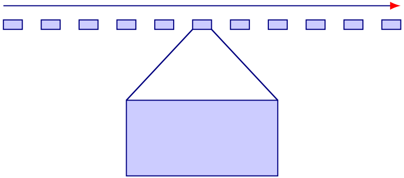

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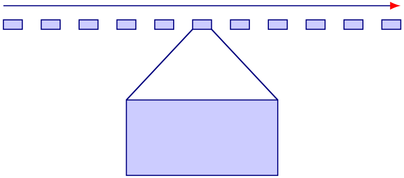

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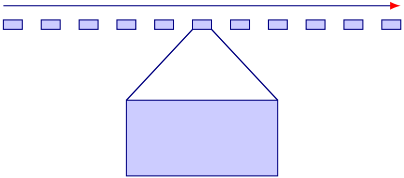

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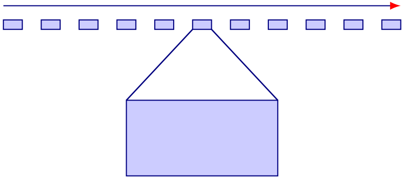

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  - The manner of adding bits should allow the decryption algorithm to separate the portion of the original plaintext from the padded part of the plaintext block.

Types of Padding

m	$pad_0(\|m\|, b)$

Also written as: $\underbrace{m || pad_0(|m|, b)}_{\text{Concatenation}}$

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Types of Padding

m	$pad_1(\lvert m\rvert, b)$

Also written as: $\underbrace{m \| pad_1(|m|, b)}_{\text{Concatenation}}$

Method 1:

Let $v \leftarrow b - \underbrace{(|m| + 1) \pmod{b}}_{\text{message bits } +1 \text{ in last block}}$. Then, the padded message is

$$m \| \underbrace{pad_1(|m|, b)}_{\text{padding bits}} = m \| 1 \underbrace{0 \ldots 0}_{v \text{ bits}}.$$

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Types of Padding

m	$pad_2(\|m\|, b)$

Also written as: $\underbrace{m || pad_2(|m|, b)}_{\text{Concatenation}}$

Method 2:

Let $v \leftarrow b - \underbrace{(|m| + 64 + 1) \pmod b}_{\text{message bits } +65 \text{ in last block}}$. Then, the padded message is

$$m || \underbrace{pad_2(|m|, b)}_{\text{padding bits}} = m || 1 \underbrace{0 \ldots 0}_{v \text{ bits}} || \underbrace{|m|}_{64 \text{ bits}} .$$

Types of Padding

Also written as: $\underbrace{m || pad_3(|m|, b)}_{\text{Concatenation}}$

Method 3:

Let $v \leftarrow b - \underbrace{(|m| + 64) \pmod b}_{\text{message bits } +64 \text{ in last block}}$. Then, the padded message is

$$m || \underbrace{pad_3(|m|, b)}_{\text{padding bits}} = m || \underbrace{0 \ldots 0}_{v \text{ bits}} || \underbrace{|m|}_{64 \text{ bits}} .$$

Types of Padding

m	$pad_4(\|m\|, b)$

Also written as: $\underbrace{m \| pad_4(|m|, b)}_{\text{Concatenation}}$

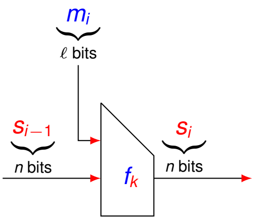
Method 4:

Let $v \leftarrow b - \underbrace{(|m| + 2) \pmod{b}}_{\text{message bits } +2 \text{ in last block}}$. Then, the padded message is

$$m \| \underbrace{pad_4(|m|, b)}_{\text{padding bits}} = m \| 1 \underbrace{0 \ldots 0}_{v \text{ bits}} 1.$$

Outline

The Merkle–Damgård construction



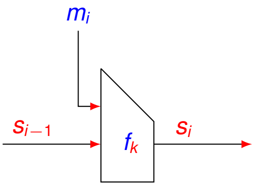
The Building Block: A Compression Function

- A compression function is a hash function taking inputs of a fixed length:

$$f_k : \{0,1\}^{\ell+n} \to \{0,1\}^n.$$

- Map $(\ell + n)$-bit inputs to n-bit outputs.
- When applied iteratively, the variable s_i works as an internal state.

The Merkle–Damgård construction



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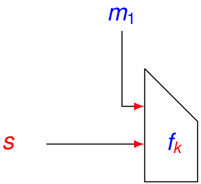

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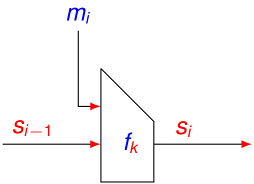

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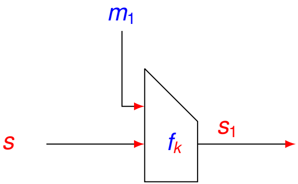

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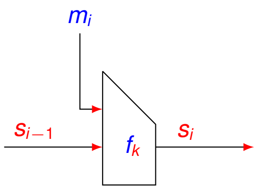

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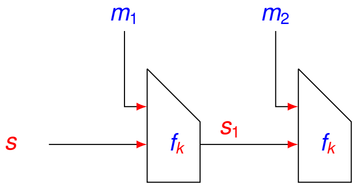

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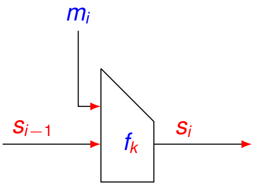

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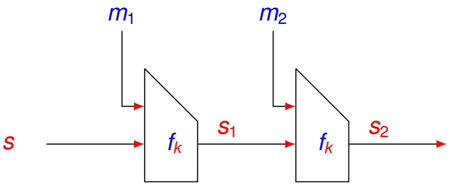

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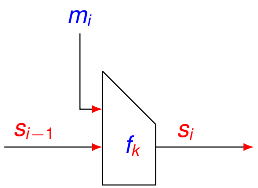

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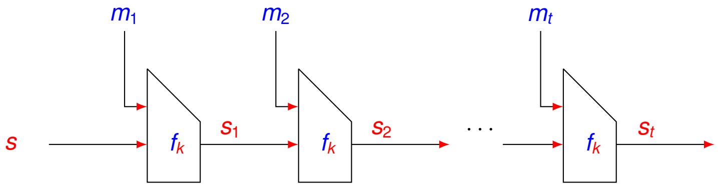

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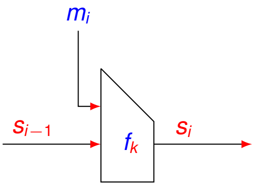

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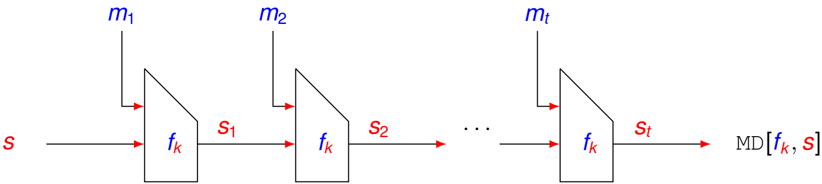

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The Algorithm

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Step 5: return s_t

The Merkle–Damgård construction

Padding Scheme determines Collision Resistance

Let $n = \ell = 4$. Consider the messages

$$m_1 = 0b0 \text{ and } m_2 = 0b00.$$

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Theorem 14.4

Let

$$H_{k,s}(m) = \mathrm{MD}[f_{k,s}](m)$$

denote the keyed hash function constructed using the Merkle–Damgård method from the keyed compression function $f_k(x)$ and Padding Method 2.
If $f_k(x)$ is collision resistant, then $H_{k,s}$ is collision resistant as well.

Common Hash functions

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  - SHA-256: outputs 256-bit hashes
  - SHA-384: outputs 384-bit hashes
  - SHA-512: outputs 512-bit hashes

Outline

SHA-256

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SHA-256

- Input: message m of length $0 \leq |m| < 2^{64}$ to be divided into t blocks

$$\underbrace{\underbrace{m_1}_{\text{512 bits}},\ \underbrace{m_2}_{\text{512 bits}},\ldots,\ \underbrace{m_t}_{\text{512 bits}}}_{\leq 2^{64} \approx 16 \times 10^{24} \text{ bits}}$$

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each of length $\ell = 512$ bits.
For each m_i, the $\ell = 512$ bits are further sub-divided into 16 words

$$\underbrace{\underbrace{X_1}_{\text{32 bits}}, \underbrace{X_2}_{\text{32 bits}}, \underbrace{X_2}_{\text{32 bits}}, \ldots, \underbrace{X_{16}}_{\text{32 bits}},}_{\text{512 bits}}$$

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SHA-256

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- Output: hash of length $n = 256$ bits

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- Padding: Method 2 (to make sure $|m|$ is a multiple of $\ell = 512$)
- Output: hash of length $n = 256$ bits
- Compression function

$$f : \{0,1\}^{\ell+n} \to \{0,1\}^n$$

  (64 rounds, each of one step)

SHA-256: 32-bit word functions

For 32-bit inputs u, v, w, x, we define the six functions:

Here, $\ggg$ denotes right rotate and $\gg$ denotes right shift

$$f'(u, v, w) \quad = (u \wedge v) \oplus ((\neg u) \wedge w),$$

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\sigma_0(x) &= (x \ggg 7) \oplus (x \ggg 18) \oplus (x \gg 3), \text{ and}
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SHA-256: 32-bit word constants $K_0, \ldots, K_{63}$

We define 64 constant words: $K_0, \ldots, K_{63}$.
Each K_i is the first 32 bits of the fractional parts of the cube roots of the first 64 prime numbers.

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$\vdots$	$\vdots$
K_{63}	first 32 bits of the fractional part of $\sqrt[3]{p_{63}}$

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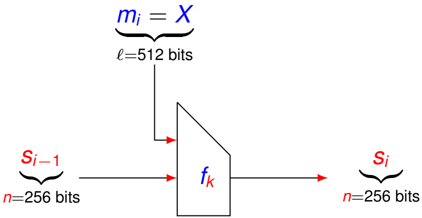
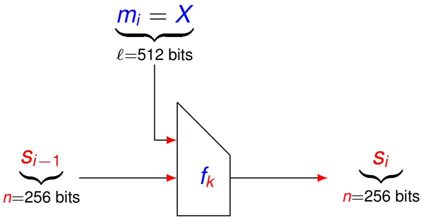
SHA-256: Internal State s_i



The Building Block: A Compression Function $f : \{0,1\}^{\ell+n} \rightarrow \{0,1\}^n$

SHA-256: Internal State s_i



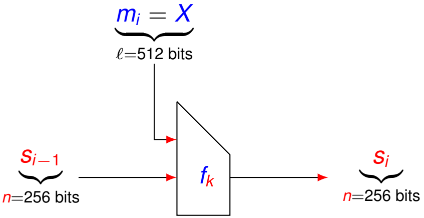
The Building Block: A Compression Function $f : \{0,1\}^{\ell+n} \to \{0,1\}^n$

The internal state s_i is a set of 8 values, each of size 32 bits.

$$H = \underbrace{(\underbrace{H_1}_{32 \text{ bits}}, \underbrace{H_2}_{32 \text{ bits}}, \underbrace{H_3}_{32 \text{ bits}}, \underbrace{H_4}_{32 \text{ bits}}, \underbrace{H_5}_{32 \text{ bits}}, \underbrace{H_6}_{32 \text{ bits}}, \underbrace{H_7}_{32 \text{ bits}}, \underbrace{H_8}_{32 \text{ bits}})}_{256 \text{ bits}}$$

This also corresponds to the 256 bits of input key s_0 / output s_t

SHA-256: Initial State s_0



$$H = \underbrace{(\underbrace{H_1}_{32 \text{ bits}}, \underbrace{H_2}_{32 \text{ bits}}, \underbrace{H_3}_{32 \text{ bits}}, \underbrace{H_4}_{32 \text{ bits}}, \underbrace{H_5}_{32 \text{ bits}}, \underbrace{H_6}_{32 \text{ bits}}, \underbrace{H_7}_{32 \text{ bits}}, \underbrace{H_8}_{32 \text{ bits}})}_{256 \text{ bits}}$$

For the initial state s_0, we assign:

$$
\begin{array}{ll}
H_1 \leftarrow 0x6A09E667, & H_2 \leftarrow 0xBB67AE85, \\
H_3 \leftarrow 0x3C6EF372, & H_4 \leftarrow 0xA54FF53A, \\
H_5 \leftarrow 0x510E527F, & H_6 \leftarrow 0x9B05688C, \\
H_7 \leftarrow 0x1F83D9AB, \text{ and} & H_8 \leftarrow 0x5BE0CD19.
\end{array}
$$

SHA-256: The Compression Function

SHA-256 compression function $f(X||s_{i-1})$

Initialisation: $(Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8) \leftarrow (H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8)$

Expansion: for $j = 17$ to 64 do

$$X_i \leftarrow (\sigma_1(X_{i-2}) + X_{i-7} + \sigma_0(X_{i-15}) + X_{i-16})$$

Rounds: for $j = 1$ to 64 do

$$t_1 \leftarrow (Y_8 + \sum_1(Y_5) + f'(Y_5, Y_6, Y_7) + K_i + X_i)$$
$$t_2 \leftarrow (\sum_0(Y_1) + g'(Y_1, Y_2, Y_3))$$
$$Y_1 \leftarrow (t_1 + t_2)$$
$$Y_2, Y_3, Y_4 \leftarrow Y_1, Y_2, Y_3$$
$$Y_5 \leftarrow Y_4 + t_1$$
$$Y_6, Y_7, Y_8 \leftarrow Y_5, Y_6, Y_7$$

for $k = 1$ to 8

$$H_k \leftarrow Y_k + H_k$$

$$s_i \leftarrow (H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8)$$

Popular Hash Functions $H(n)$

Check this link: `https://www.browserling.com/tools/all-hashes`

Check the following SHA-256 hashes:

SHA256("WISDOM IS NOWHERE") =
"7db96cca6d0160496161297d522425ffab681f25c6bd57c85bf976a8b7d7372d"

SHA256("WISDOM IS NOW HERE") =
"eae0bc219689024e359b23a1a3e81be946423be9a7f2332c430f7a17c1a315e7".

Note: Each character in a hash is the
hexadecimal encoding of 4 binary digits.

Hex	Binary	Hex	Binary	Hex	Binary	Hex	Binary
0	0000	4	0100	8	1000	c	1100
1	0001	5	0101	9	1001	d	1101
2	0010	6	0110	a	1010	e	1110
3	0011	7	0111	b	1011	f	1111

Note: A "slight" change in the input, completely changes the hash value!

Very useful for implementing integrity checks!

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Thank you for your kind attention!

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