# COMP8760

Lecture 1

Worksheet for Practice

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This is a set of practice problems for you to solve. You will **NOT have to submit your solutions** for this Worksheet as they will **not be graded**. The sample solutions will be provided before Lecture 2 for you to verify your attempt yourself.

1. **Sets: Membership, Subset, Cardinality**
   Let $A = \{1, 2, 3\}$. Fill in the blanks.
   (a) Is $1 \in A$? _____ (Yes/No)
   (b) Is $4 \in A$? _____ (Yes/No)
   (c) Is $4 \notin A$? _____ (Yes/No)
   (d) Is $100 \notin A$? _____ (Yes/No)
   (e) Is $\{1, 3\} \subseteq A$? _____ (Yes/No)
   (f) Is $\{4\} \subseteq A$? _____ (Yes/No)
   (g) Is $\{3, 4\} \subseteq A$? _____ (Yes/No)
   (h) What is $|A|$? _____
   (i) What is $|\{1, 2, 3, 4, \ldots, 100\}|$? _____
   (j) What is $|\{2, 4, 6, 8, \ldots, 100\}|$? _____
   (k) Is $A \subseteq \{2, 4, 6, 8, \ldots, 100\} \subseteq \mathbb{Z}$? _____ (Yes/No)

2. **Division Theorem**
   Fill in the blanks.
   (a) $99 =$ _____ $\times 12 +$ _____
   (b) $199 =$ _____ $\times 53 +$ _____
   (c) $9 =$ _____ $\times 12 +$ _____
   (d) $0 =$ _____ $\times 53 +$ _____
   (e) $-1 =$ _____ $\times 12 +$ _____
   (f) $-12 =$ _____ $\times 12 +$ _____

3. $\mathbb{Z}_N$**: Set of All Remainders of** $N$
   Fill in the blanks.
   (a) $\mathbb{Z}_2 = \{$_____$\}$
   (b) $\mathbb{Z}_5 = \{$_____$\}$

(c) $\mathbb{Z}_{12} = \{\_\_\_\_\}$

(d) $\mathbb{Z}_{13} = \{\_\_\_\_\}$

(e) $\mathbb{Z}_{1297} = \{\_\_\_\_\}$

4. **Modulus operator:** $a \mod N$
   Fill in the blanks.

   (a) $23 \mod 11 = \_\_\_\_\_$

   (b) $22 \mod 11 = \_\_\_\_\_$

   (c) $23 = 34 \mod 11; \_\_\_\_\_$ (True/False)

   (d) $-3 = 8 \mod 11; \_\_\_\_\_$ (True/False)

   (e) $-3 = -15 \mod 11; \_\_\_\_\_$ (True/False)

5. **Modular Arithmetic**
   Write the addition and multiplication tables for all elements in $\mathbb{Z}_4$.

6. **Prime Numbers**
   Fill in the blanks.

   (a) Is 63 a prime? _____ (Yes/No)

   (b) Is 67 a prime? _____ (Yes/No)

(c) What is the prime factorisation of 1001? _____

(d) Using the prime factorisation technique, find the GCD and LCM of the integer pair $(539, 1001)$?