Random Variables
COR1-GB.1305 – Statistics and Data Analysis

Probability Distribution Function and Expectation

1. Consider the following game:
    1. You pay $6 to flip a coin.
    2. If the coin lands heads, you get $10; otherwise, you get nothing.

    (a) Would you play this game? Why or why not?

    (b) What is the random experiment involved in the game? What are the sample space? What are the probabilities of the sample points?

    (c) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the value of W for each of the sample points.

    (d) Describe W in terms of its probability distribution function (PDF).

    (e) What are your expected winnings? That is, what is μ, the expectation of W?

2. Suppose you flip two coins. Let X be the random variable which counts the number of heads on the two tosses.

   (a) List all of the sample points of the experiment, along with the corresponding values of X.

   (b) Compute the probability distribution function of X.

   (c) Compute the expectation of X.

   (d) What is the interpretation of the expectation of X?

3. Let X be a random variable describing the number of cups of coffee a randomly-chosen member of the class drinks on a typical day. There is a 22% chance that the student has one cup, a 16% chance that the student has two cups, a 16% chance that the student has three cups, an 11% chance that the student has four cups, and a 3% chance that the student has five cups. Also, there is a 32% chance that the student doesn't drink any coffee.

   (a) Let $p(x)$ be the probability distribution function of X. Fill in the following table:

x	0	1	2	3	4	5
$p(x)$						

   (b) Find $\mathrm{E}(X)$, the expectation of X.

   (c) What is the interpretation of the expectation of X?

Variance and Standard Deviation

4. This is a continuation of problem 3.

   (a) Find $\text{var}(X)$ and $\text{sd}(X)$, the variance and standard deviation of X, the number of cups of coffee that a random student from the class drinks on a typical day.

   (b) What is the interpretation of the standard deviation of X?

5. Consider the following game:

   1. You pay \$6 to pick a card from a standard 52-card deck.
   2. If the card is a diamond ($\diamondsuit$), you get \$22; if the card is a heart ($\heartsuit$), you get \$6; otherwise, you get nothing.

   Perform the following calculations to decide whether or not you would play this game.

   (a) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the PDF of W.

   (b) What are your expected winnings? That is, what is μ, the expectation of W?

   (c) What is the standard deviation of W?

   (d) What are the interpretations of the expectation and standard deviation of W?

Properties of Expectation

6. **Affine Transformations.** Let X be a random variable with expectation $\mu_X = 2$. What is the expectation of $5X + 2$?

7. **Sums of Independent Random Variables.** Let X and Y be random variables with $\mu_X = 1$, $\mu_Y = -5$. What is $\mathrm{E}(X + Y)$?

8. Let X and Y be random variables with $\mu_X = -2$, $\mu_Y = 3$.
   (a) Find the expectation of $-3X + 2$.

   (b) Find the expectation of $X + Y$.

9. You invite four people to go out to dinner on Friday night. The attendance probabilities for the four potential guests are 50%, 20%, 30%, and 90%.
   (a) Find the expected number of guests.

   (b) The dinner will be a *prix fixe* meal, costing \$50 per person. What is the expected total cost for yourself and your guests?

   (c) What is the interpretation of your answer to part (b)?