Multiple Regression (Review)

1. Consider the dataset of 147 movies from 2013. Here is the result of fitting a linear regression model to predict the base-10 logarithm of the total gross (`Log10Gross`) using Rotten Tomatoes audience and critics scores, along with the base-10 logarithm of the budget (`Log10Budget`) as predictors:

```
Analysis of Variance

Source                              DF   Adj SS  Adj MS  F-Value  P-Value
Regression                           3  18.8920  6.2973    55.70    0.000
  Rotten Tomatoes Audience Score     1   3.3973  3.3973    30.05    0.000
  Rotten Tomatoes Critics Score      1   0.1526  0.1526     1.35    0.247
  Log10Budget                        1   9.5855  9.5855    84.78    0.000
Error                              143  16.1676  0.1131
Total                              146  35.0595

Model Summary

       S    R-sq  R-sq(adj)  R-sq(pred)
0.336244  53.89%     52.92%      51.28%

Coefficients

Term                                 Coef  SE Coef  T-Value  P-Value   VIF
Constant                            3.175    0.397     8.00    0.000
Rotten Tomatoes Audience Score    0.01388  0.00253     5.48    0.000  2.53
Rotten Tomatoes Critics Score    -0.00191  0.00164    -1.16    0.247  2.50
Log10Budget                       0.4934   0.0536     9.21    0.000  1.07

Regression Equation

Log10Gross = 3.175 + 0.01388 Rotten Tomatoes Audience Score
             - 0.00191 Rotten Tomatoes Critics Score + 0.4934 Log10Budget
```

   (a) Based on the ANOVA F test, is there evidence that the model is useful?

   Solution: The p-value for this test (reported in the first line of the "Analysis of Variance" table is reported as $p = 0.000$; there is very strong evidence that at least one of the true regression coefficients is nonzero. Thus, there is very strong evidence that the model is useful.

   (b) What is the interpretation of the R^2?

   Solution: The regression model explains 53.89% of the variability in `Log10Gross`.

   (c) In the fitted model, what is the interpretation of s?

   Solution: $s = 0.336244$ is the standard deviation of the error; based on the empirical rule, 95% of the `Log10Gross` values will be within $2(0.336244) = 0.672488$ of their

predicted (expected) value. (Note: $10^0.67 = 4.7$; thus, 95% of the `Gross` values will be within a factor of 4.7 of their predicted value.)

(d) In the fitted model, what is the interpretation of the coefficient of "Rotten Tomatoes Audience Score"?

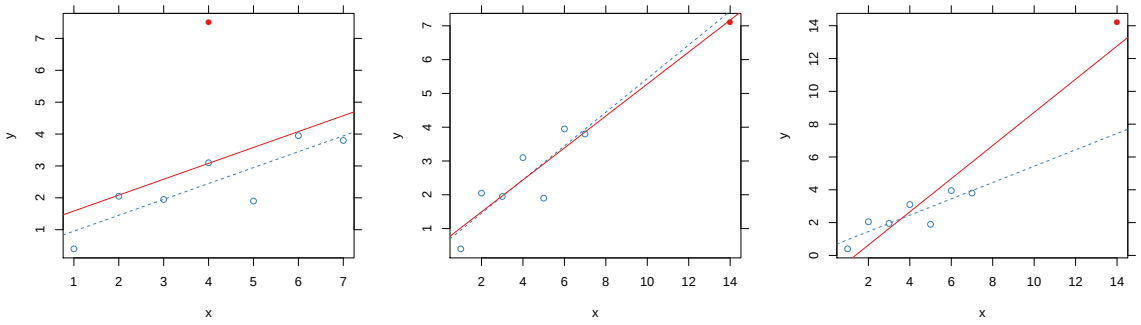
Solution: If we increase "Rotten Tomatoes Audience Score" by 1 unit while holding "Rotten Tomatoes Critics Score" and `Log10Budget` fixed, the expected value of `Log10Gross` increases by 0.01388 units. (Since $10^{0.013888} \approx 1.02$, the predicted value for `Gross` increases by 2%.)

(e) Based on the coefficient t tests, which predictor(s) would you remove from the model? What is the interpretation of the p-value for this predictor?

Solution: I would remove "Rotten Tomatoes Critics Score". The p-value for this predictor is $p = 0.247$; if this predictor were not useful after adjusting for "Rotten Tomatoes Audience Score" and `Log10Budget`, then there would be a 24.7% chance of seeing data like we observed.

Extreme Points

2. Each of the following scatterplots show two regression lines: the solid line is fitted to all of the points, and the dashed line is fitted to just the hollow points.



(a) (b) (c)

(a) For each of the three cases, when the solid point is added to the dataset, is its residual from the least squares line large or small?

Solution: (a) large; (b) small; (c) small

(b) Is the x value of the solid point close to $\bar{x}$ or far away from $\bar{x}$?

Solution: (a) close to $\bar{x}$; (b) far from $\bar{x}$; (c) far from $\bar{x}$.

(c) What affect does adding the solid point have on $\hat{\beta}_0$, $\hat{\beta}_1$, and R^2?

Solution: (a) Adding the point has very little affect on $\hat{\beta}_1$, but it changes $\hat{\beta}_0$ slightly and drastically reduces R^2.
(b) Adding the point has very little affect on $\hat{\beta}_0$, $\hat{\beta}_1$ and R^2. This is because the point is consistent with the trend of the other points.
(c) Adding the point has a huge affect on $\hat{\beta}_0$, $\hat{\beta}_1$ and R^2. This is because the point has high influence and it is not consistent with the trend of the other points.

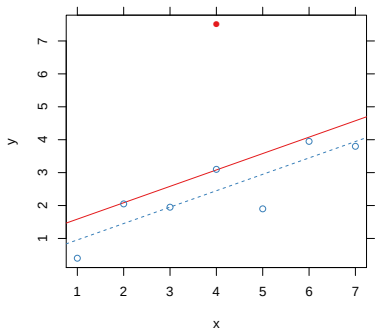
(d) Should we include the solid point in the regression analysis? If not, what should we do with it?

Solution: (a) Since the point has a big influence on the regression fit, we should not include it in the fit. We should discuss the point separately. We should *not* just delete the point from the dataset.
(b) Since the point doesn't have much influence on the regression, we should include it in the analysis.

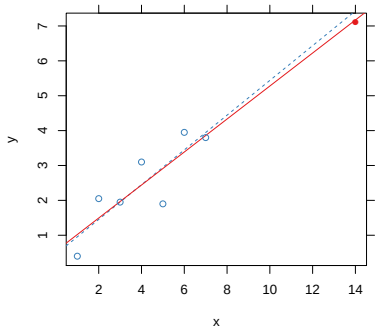
(c) Since the point has a high influence on the regression, we should not include it in the analysis. We should discuss the point separately.

Outliers, leverage, and influence

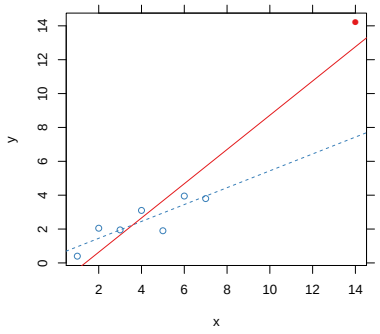
3. The following tables gives the observation number (i), the standardized residual (r_i), the leverage (h_i), and Cook's distance (C_i) for each data point. The solid point is obervation 8.



Obs.	Std. Resid.	Leverage	Cook's Dist.
1	-0.78	0.45	2×10^{-1}
2	-0.02	0.27	7×10^{-5}
3	-0.34	0.16	1×10^{-2}
4	0.01	0.12	7×10^{-6}
5	-0.90	0.16	8×10^{-2}
6	-0.07	0.27	1×10^{-3}
7	-0.51	0.45	1×10^{-1}
8	2.32	0.12	4×10^{-1}



Obs.	Std. Resid.	Leverage	Cook's Dist.
1	-1.14	0.28	3×10^{-1}
2	0.98	0.22	1×10^{-1}
3	-0.03	0.17	8×10^{-5}
4	1.11	0.14	1×10^{-1}
5	-1.68	0.13	2×10^{-1}
6	0.94	0.13	7×10^{-2}
7	-0.10	0.15	9×10^{-4}
8	-0.24	0.79	1×10^{-1}



Obs.	Std. Resid.	Leverage	Cook's Dist.
1	0.64	0.28	0.081
2	1.12	0.22	0.174
3	0.24	0.17	0.006
4	0.34	0.14	0.009
5	-1.33	0.13	0.126
6	-0.55	0.13	0.022
7	-1.44	0.15	0.185
8	2.19	0.79	8.892

In each of the three cases are any of the standardized residual, leverage, or Cook's distance large for observation 8? What counts as "large" for these diagnostics?

Solution: (a) The standardized residual (2.32) is large; Cook's distance (0.4) is moderate. We say that the standardized residual is large if $|r_i| > 2$; the leverage is large if $h_i > \frac{2}{n}$; Cook's distance is large if $C_i > 1$. For this problem, $n = 8$, so $\frac{2}{n} = .25$ and $\frac{4}{n} = .5$.

(b) Only the leverage (0.79) is large.

(c) Both the leverage (0.79) and Cook's distance (8.892) are large.

Summary

4. Should an outlier or a point with high leverage always be removed from a regression analysis?

> **Solution:** No. It should only be removed if the fit changes substantially.

5. If we decide to remove a point from an analysis, what should we do with the point?

> **Solution:** We should discuss the point separately. We should *not* just ignore the point and never discuss it.

6. Does a leverage point always have a high Cook's Distance?

> **Solution:** No. In Problem 2(b) the point has high leverage but low Cook's Distance.

7. Can a point have low leverage and high Cook's Distance?

> **Solution:** Yes. This is the case in Problem 2(a).