Warm-up Exercises for HW2 and HW3
STAT-GB.2302, STAT-UB.0018: Forecasting Time Series Data

Please do these warm-up exercises, which will help you to do HW3, as well as the last problem in HW2. You *should not* hand in your solutions to these warmup exercises. Use the basic facts on the first page of the handout "Linear Prediction of a Random Variable" to prove the following identities.

1. If $E[X] = E[Y] = \mu$, show that $E[X + 2Y] = 3\mu$.

2. If $E[X] = 0$, show that $\mathrm{var}(X) = E[X^2]$.

3. If $\mathrm{var}(X) = \mathrm{var}(Y) = 1$ and $\mathrm{cov}(X, Y) = \frac{1}{2}$, show that $\mathrm{var}(X+Y) = 3$ and $\mathrm{var}[(X+Y)/2] = \frac{3}{4}$.

4. Show that $\mathrm{var}(X - X) = 0$ using two different methods. Method 1: Note that $X - X = 0$. Method 2: Use the general formula for $\mathrm{var}(X + Y) = \mathrm{var}(X) + \mathrm{var}(Y) + 2\,\mathrm{cov}(X, Y)$.

5. Read the proof of formula (1) of the handout "White Noise and Moving Average Models." Make sure that you understand every step. (This is the warm-up for the last problem in HW2.)