Identifying the roles of race-based choice and chance in high school friendship network formation

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Homophily, the tendency of people to associate with others similar to themselves, is observed in many social networks, ranging from friendships to marriages to business relationships, and is based on a variety of characteristics, including race, age, gender, religion, and education. We present a technique for distinguishing two primary sources of homophily: biases in the preferences of individuals over the types of their friends and biases in the chances that people meet individuals of other types. We use this technique to analyze racial patterns in friendship networks in a set of American high schools from the Add Health dataset. Biases in preferences and biases in meeting rates are both highly significant in these data, and both types of biases differ significantly across races. Asians and Blacks are biased toward interacting with their own race at rates >7 times higher than Whites, whereas Hispanics exhibit an intermediate bias in meeting opportunities. Asians exhibit the least preference bias, valuing friendships with other types 90% as much as friendships with Asians, whereas Blacks and Hispanics value friendships with other types 55% and 65% as much as same-type friendships, respectively, and Whites fall in between, valuing other-type friendships 75% as much as friendships with Whites. Meetings are significantly more biased in large schools (>1,000 students) than in small schools (<1,000 students), and biases in preferences exhibit some significant variation with the median household income levels in the counties surrounding the schools.

friendships | high schools | homophily | segregation | social networks

Friendship networks from a sample of American high schools in the Add Health national survey[†] exhibit a strong pattern: students tend to form friendships with other students of their same ethnic group at rates that are substantially higher than their population shares (Fig. 1) (1–4). This feature, referred to as "homophily" in the sociological literature (5), is prevalent across many applications and can have important implications for behaviors (6–9). The widespread presence of homophily indicates that friendship formation differs substantially from a process of uniformly random assortment. Two key sources of homophily are (*i*) biases in individual preferences for which relationships they form and (*ii*) biases in the rates at which individuals meet each other. It is important to identify whether homophily is primarily due to just one of these biases or to both because, for instance, this can shape policies aimed at producing more integrated high schools. In this article, we present a technique for identifying these two biases, we apply this technique to the Add Health dataset, and we estimate how preference and meeting biases differ across races.

Although there is substantial evidence that race is a salient feature in how people view each other (10), such evidence does not sort out the sources of homophily, other than indicating that student preferences could be a factor. Without detailed and reliable data on the mechanics of which students meet which others, these questions are not answered by a direct analysis of friendship data. Moreover, surveys of students asking them about their racial attitudes may not reliably reflect the choices that they make. To this end, we use a technique that is well established in economics for estimating consumers' preferences: revealed preference theory (11). One infers preferences of the individual by careful observation of the choices that they make based on the opportunities that they have. We adapt these techniques for the analysis of social behavior and friendship formation. Here we infer students' preferences by observing how the number of friendships they have changes with the racial composition of their school. We employ the friendship formation model (3), here extended to allow for different biases across race in both preferences and opportunities. Using a parameterized version of this model, we are able to distinguish between the two primary sources of homophily (that is, preference bias and meeting bias), and to measure their relative magnitudes and how these differ across races. The results illustrate these techniques and the factors leading to homophily and racial segregation patterns in friendships.

Results and Discussion

There are two patterns of homophily in the Add Health data that are important to understand (Fig. 1). First, not only is there substantial homophily, but it follows a nonlinear and nonmonotone trend with respect to group size, with low levels of homophily for groups that form very large or small fractions of a school and higher levels of homophily for groups that form an intermediate sized fraction of a school. Second, homophily patterns differ significantly across races and by school size (see *SI Text*, Fig. S1, and Tables S1–S3 for a statistical analysis).

The model of friendship formation developed in ref. 3 showed how preference biases and meeting biases lead to different patterns in the numbers of friendships that people form and the resulting homophily. Thus, by taking advantage of those differences in patterns, one can identify preference and meeting biases. Here we enrich and extend the model in such a way to be able to identify race-by-race differences in these biases. Students enter the system and randomly meet friends, leaving the process when expected gains from new meetings are outweighed by the cost of searching. One key element is that students have preferences over the racial mix of their friends. Students can value a friendship with someone of their own race differently from a friendship with someone of another race. The second element is that students may end up meeting other students of their own race at a rate that is higher than what would occur if they were meeting other searching students uniformly at random. This bias may stem from the various ways in which the meeting process can depart from a uniform random process, including self-segregation through racially homogeneous activities, as well as meeting friends of friends, etc.

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[†]The National Longitudinal Study of Adolescent Health (Add Health) is a longitudinal study of a nationally representative sample of adolescents in grades 7–12 in the United States during the 1994–1995 school year. Data files are available from Add Health, Carolina Population Center (addhealth@unc.edu).

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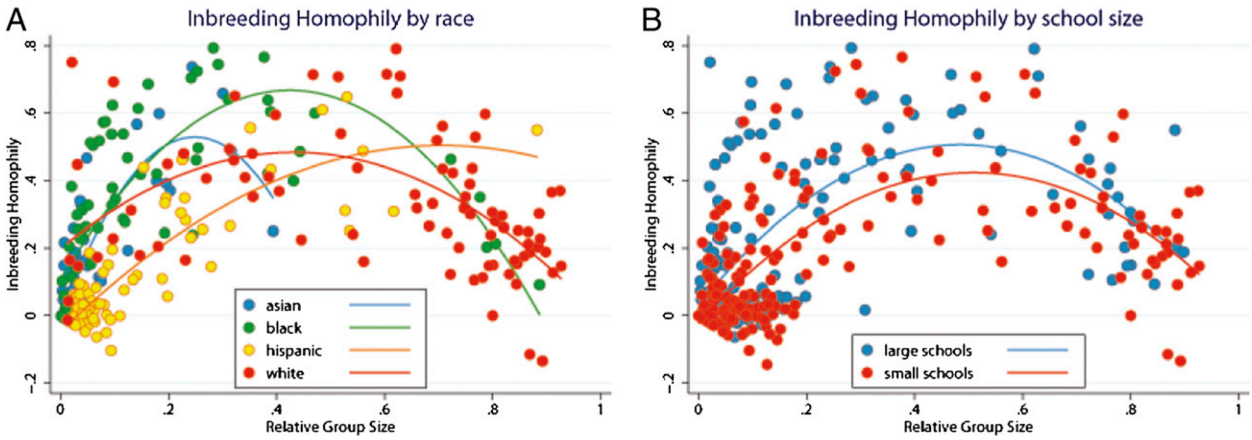

Fig. 1. Homophily as a function of the fraction of a school's population that a group comprises, differentiating by race (*A*) and school size (*B*). The homophily index, due to Coleman (12), is a normalized measure of the difference between the observed racial mix of friendships and the expected mix if friendships were formed uniformly at random. An index of 0 indicates that students in a given group (each datum is a racial group within 1 of 84 schools) have friendships distributed according to the racial mix of the society, whereas an index of 1 indicates that the students only form friendships with other students of their own race. Letting w_i be the fraction of race i in a given school, and q_i be the fraction of their friendships on average that are of own-type, the index is $(q_i - w_i)/(1 - w_i)$.

Details about the model and the way in which we estimate it from the data are found in *Material and Methods* and *SI Text*.

We find that all racial groups exhibit significant biases in both their preferences over friendships and in the rate at which they meet students of their own race. Moreover, there are significant differences across races in the relative biases.

Estimated biases in preferences range from one extreme where Blacks value friendships with students of other races 55% as much as those with other Blacks, to the other extreme where Asians value friendships with students of other races 90% as much as those with other Asians; Hispanics and Whites fall in between at 65% and 75%, respectively.

Estimated biases in meetings range from Whites meeting students without any bias, to Asians and Blacks exhibiting meeting biases of 7–7.5, and Hispanics at an intermediate rate of 2.5. To interpret the meeting biases: a meeting bias of 7 is such that >90% of the people that Asians meet are other Asians in a case where Asians comprise 50% of the population. The meeting bias of 2.5 is such that >70% of the people that Hispanics meet are other Hispanics, in a case where Hispanics comprise 50% of the population.

These results suggest that the differences across ethnic groups in both the homophily patterns and in the total number of friends (see *SI Text* for the statistical analysis of these differences) are explained by differences in both types of biases. For instance, the estimates suggest that Blacks' homophily stems from both significant meeting and preference biases, whereas Whites' homophily is more driven by preference bias.

An additional relevant issue is the potential influence of school size on observed behavior. Hypothetically, it could be that differences across races are driven by general differences in behavior in large versus small schools, which could correlate with the racial makeup of a school and therefore potentially drive our results. We therefore control for school size, splitting the sample in "large" and "small" schools (as described in more detail below). Although controlling for school size does not affect the conclusions discussed above, it is interesting to note that larger schools exhibit significantly higher biases in the rates at which students meet students of their own race. This is consistent with more opportunities to self-segregate in large schools, where academic tracking and the availability of specialized clubs, athletics, and extracurricular activities, and other mechanisms, could bias meetings (13–19). To the extent that more integrated friendship patterns are a goal for policy, the higher bias in meetings that is observed with larger schools provides some support for the opinion expressed in some of the recent sociology literature that smaller schools offer some advantages.[‡]

We also perform an analogous control for the income level surrounding a school. We split the sample into "high" and "low" income schools, where high corresponds to schools that are in counties with median household incomes above $30,000 in the 1990 census, and low corresponds to schools in counties below this level. Again, controlling for income levels does not change the basic patterns observed in the preference and meeting biases, but we do see significant differences in preference biases between high and low income schools, although the differences in meeting biases do not differ significantly. We see less preference bias for some races (Hispanics and Whites) in higher income schools and the reverse for Blacks.

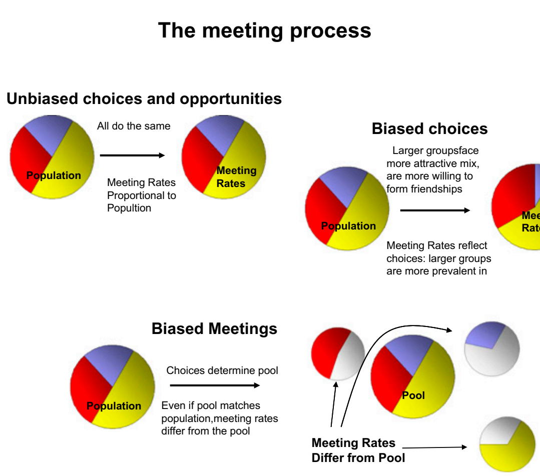

Fig. 2. The meeting process.

[‡]Larger schools also offer some advantages, as there may be economies of scale and it may also be easier to draw more racially balanced populations in larger districts. The significant increases in meeting biases in large schools, however, suggest that either one might want to create schools within schools, or understand the factors leading to increased meeting biases and homophily in larger schools.

Table 1. Estimation of preference and meeting biases

Preference parameter	α	γ_{Asians}	γ_{Blacks}	$\gamma_{Hispanics}$	γ_{Whites}	γ_{Others}
Estimated value	0.55	0.90	0.55	0.65	0.75	0.90
Meeting parameter		β_{Asians}	β_{Blacks}	$\beta_{Hispanics}$	β_{Whites}	β_{Others}
Estimated value		7	7.5	2.5	1	1

Preference bias on a grid of step 0.05, from 0.20 to 0.85 for α and from 0.40 to 1 for each of the γ; meeting bias on a grid of step 0.5 from 1 to 9.

Materials and Methods

Here we describe the model and empirical analysis, referring to *SI Text* for technical details.

In our model a population of agents is partitioned into K different groups, where group definitions delineate the relevant characteristics that agents care about in forming friendships. Let w_i denote the fraction of the population that is of type i. Agents have preferences over the number of friends that they have from their own group, henceforth referred to as "same-type" friends, and from other groups, henceforth referred to as "different-type" friends. Let s_i and d_i denote the number of same-type and different-type friends that a representative agent in group i has, and let $t_i = s_i + d_i$ be the agent's total number of friends. The agent's preferences are represented by a utility function,

$$U_i(s_i, d_i) = (s_i + \gamma_i d_i)^{\alpha}, \qquad [1]$$

where both γ_i and α lie between 0 and 1, so that U_i is increasing in both s_i and d_i. The function U_i measures the utility or satisfaction drawn from one's friendships. The parameter γ_i captures the bias in preferences, with $\gamma_i < 1$ indicating that different-type friends are valued less than same-type friends. The parameter α captures diminishing returns to friendships overall: doubling the number of each type of friends that an agent has results in less than double the utility. Finally, agents only distinguish between same-type and different-type friends, as is roughly consistent with empirical evidence (20–23).

Friendship formation takes place via a meeting process in which agents randomly meet potential friends, perhaps in a biased manner. The meeting process can be thought of as a sort of "party" where agents come to the party and randomly meet other agents and then eventually leave the party once the benefit from meeting more friends no longer exceeds the opportunity cost ($c > 0$) of time and resources of staying at the party. The relevant meeting parameter from an agent's decision perspective is the expected rate at which he/she will meet same-type versus different-type friends at the party. For type i, these are denoted by q_i and $1 - q_i$, respectively. If preferences are biased so that $\gamma_i < 1$ and same-type friendships yield higher marginal values than different-type friendships, then a higher matching rate q_i provides higher incentives to form additional friendships. In this case, groups that meet their own types at higher rates (face higher q_i) at the party will choose to stay at the party longer and thus form more friendships per capita than groups that meet their own types at lower rates. The meeting rates for different races and total numbers of friendships are both observable in the data, allowing us to identify the preference parameters. Note also that choice and chance feed back upon each other: given biased preferences groups accounting for a larger share of the population will choose to stay at the party longer (i.e., socialize more) and so end up forming an even greater portion of people at the party, thus making it even more attractive for their types and less attractive for other types. This feedback does not prevent us from identifying preference and meeting biases, as described below.

Solving the model requires determining the meeting probabilities. If meetings follow a uniform random process (in which case we talk of an "unbiased" meeting process), agents meet each other in proportion to their relative stocks (i.e., proportions at the party), so that $q_i = \frac{M_i}{\sum_k M_k}$, where $M_i = w_i t_i$ is the stock of agents of type i. Biases in meetings, such that agents meet same friends at higher rates than their relative stocks, are captured by

$$q_i = \left(\frac{M_i}{\sum_k M_k}\right)^{1/\beta_i}, \qquad [2]$$

where $\beta_i > 1$ is the bias that each type has toward itself in the meeting process, and $\beta_i = 1$ is the case of unbiased process. For instance, if a group comprises half of the meeting pool and has a bias of $\beta_i = 2$, then it would meet itself at a rate of $(0.5)^{1/2}$ or ∼0.7, while if $\beta_i = 3$ this rises to ∼0.8, and at $\beta_i = 6$ is ∼0.9. Given that the stocks of different types in the meeting process must sum to one, $\sum_i \frac{M_i}{\sum_k M_k} = 1$, it follows that

Table 2. *F*-statistics of constrained calibrations, compared with the unconstrained calibrations in which every race has a different parameter

	Preference bias, γ			Meeting bias, β		
	F-statistic	95%	99%	*F*-statistic	95%	99%
Asian = Black	9.93**	3.96	6.97	0.04	3.96	6.97
Asian = Hispanic	8.17**	"	"	4.95*	"	"
Asian = White	2.65	"	"	47.97**	"	"
Black = Hispanic	1.56	"	"	19.31**	"	"
Black = White	10.43**	"	"	124.5**	"	"
Hispanic = White	3.43	"	"	23.34**	"	"
All = 1	42.61**	2.33	3.26	220.6**	2.33	3.26
All =	6.10**	2.49	3.57	51.25**	2.49	3.57

*Significance above a 95 percent level; **significance above a 99 percent level.

$$\sum_i q_i^{\beta_i} = 1. \qquad [3]$$

We also impose conditions that relate meeting rates across races, since if a person of type i is meeting a person of type j then the converse is also true, as described in *SI Text*. The meeting process is illustrated in Fig. 2.

Using this model, we estimate the preference bias and meeting bias parameters from the data. If an agent forms t_i friendships when a fraction of q_i are of same-type, then the resulting utility (including costs of time in the meeting process) is $(t_i q_i + \gamma_i(1 - q_i)t_i)^{\alpha} - ct_i$. Thus, utility optimization (see *SI Text* for details) implies that for every type i the following first order necessary condition holds:

$$t_i = \left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} (\gamma_i + (1-\gamma_i)q_i)^{\frac{\alpha}{1-\alpha}}. \qquad [4]$$

From Eq. **4**, we see that γ_i dictates how sensitive the total number of friends of a given agent is to changes in the odds of meeting a same-type agent. If there is not much preference bias (i.e., a γ_i near 1), then the number of friendships formed is relatively insensitive to the rate at which same-type friends are met. In contrast, if preferences are heavily biased toward own-type (i.e., γ_i is lower), then the number of friendships formed will be very sensitive to the rate at which same-type friends are met. This is a key to the identification: the sensitivity of the number of friendships formed by a given type of agent to the same-type meeting rate identifies the bias in preferences. The identification of the meeting biases can then be found from Eq. **3**. To see why it identifies the meeting biases, note that it implicitly keeps track of the extent to which the relative fraction of types met (the q_is) differ from the relative stocks at the 'party' (the w_is weighted by the t_is).

As described in *SI Text*, we allow for individual idiosyncrasies in preferences and other perturbations, by allowing Eq. **4** to only hold up to an individual error term, and we also allow costs to vary by school. Applying condition **4** to any pair of types i and j, we can eliminate the cost term (which is unobserved in the data) and obtain the following equation:

$$t_i(\gamma_j + (1-\gamma_j)q_j)^{\frac{\alpha}{1-\alpha}} = t_j(\gamma_i + (1-\gamma_i)q_i)^{\frac{\alpha}{1-\alpha}}. \qquad [5]$$

Note that t_i, t_j, and q_i, q_j are available data, since t_i is the number of friendships on average by type i students, and $q_i = s_i/(s_i + d_i)$ is the average percent of friendships formed by type i students that are of the same-race. We thus estimate α and the γ_i's by minimizing the errors in Eq. **5**. Then, each choice of values for α, γ_{Asians}, γ_{Blacks}, $\gamma_{Hispanics}$, γ_{Whites}, and γ_{Others} leads to a difference between the right hand side and the left hand side of Eq. **5**. Weighting schools to correct for their size and hence the variance in errors which are coming from individual choices (as described in detail in *SI Text*) yields a sum of squared errors for each choice of α and γ's. The statistical analysis reported below is based on a presumption that the errors follow a Normal distribution, and in *SI Text*, Fig. S2, we verify that the realized distribution of errors does not differ significantly from a Normal distribution. We search over a grid to find parameters that minimize this weighted sum of squared errors. To estimate the β parameters, we follow the same technique based on Eq. **3**. Results are reported in Table 1.

The patterns across race differ significantly. Asian students are the least biased in their preferences over racial mixes, having the highest γ at 0.9, but

Table 3. Preference and meeting biases when allowed to vary by school size

	α	γ_a	γ_b	γ_h	γ_w	γ_o	RSS	*F*	95% thresh.	99% thresh.
Ignoring size	0.55	0.90	0.55	0.65	0.75	0.90	4704	—	—	—
Small schools	0.65	0.90	0.75	0.80	0.80	0.90	1685	—	—	—
Large schools	0.55	0.85	0.40	0.45	0.65	0.85	2531	—	—	—
	Total error small + large						4216	1.39	2.23	3.06
	β_a	β_b	β_h	β_w	β_o	—	RSS	*F*	95% thresh.	99% thresh.
Ignoring size	7.0	7.5	2.5	1.0	1.0	—	1.7265	—	—	—
Small schools	6.5	6.0	2.0	1.0	1.0	—	0.9406	—	—	—
Large schools	3.0	9.0	6.5	1.0	1.0	—	0.3688	—	—	—
	Total error small + large					—	1.3094	4.71**	2.34	3.28

Table 4. Preference and meeting biases when allowed to vary by the school's county median household income level (low is <30,000 dollars in 1990 census and high is above)—for 78 of the 84 schools for which we have income data

	α	γ_a	γ_b	γ_h	γ_w	γ_o	RSS	*F*	95% thresh.	99% thresh.
Ignoring income	0.55	0.95	0.55	0.65	0.75	0.90	4255	—	—	—
Low income schools	0.60	1.0	0.50	0.50	0.65	0.95	1541	—	—	—
High income schools	0.35	1.0	0.40	0.70	0.90	0.75	1703	—	—	—
	Total error low + high						3244	3.43**	2.23	3.06
	β_a	β_b	β_h	β_w	β_o	—	*RSS*	*F*	95% thresh.	99% thresh.
Ignoring income	7.5	7.5	2.5	1.0	1.0	—	1.652	—	—	—
Low income schools	2.0	8.0	3.0	1.0	1.0	—	0.8699	—	—	—
High income schools	5.0	4.5	4.0	1.0	1.0	—	0.7485	—	—	—
	Total error low + high					—	1.618	0.28	2.34	3.28

they face a very substantial meeting bias with a β parameter of 7. Blacks exhibit the greatest bias in both preferences and meetings, with a γ of 0.55 and a β of 7.5. Hispanics are intermediate in terms of both biases, while Whites have an intermediate preference bias and no meeting bias.

To check that these differences reflect a systematic heterogeneity and not the mere effect of randomness, we run a series of F-tests on the above calibrations, as described in *SI Text* and Tables S4–S7. A summary of the results appears in Table 2. The hypothesis that all of the races have unbiased preferences is rejected well above the 99% confidence level. Moreover, Asian and White preference biases do not differ significantly from each other, and the same holds for Blacks and Hispanics, but Blacks and Hispanics both differ significantly from both Asians and Whites. Similar patterns are found in the meeting biases, but this time with Asians and Blacks having indistinguishable meeting biases but both differing significantly from Hispanics and Whites.[§]

Beyond the analysis above, we also control for the effect of school size on the estimated parameters, as well as income. We analyze the effect of school size by re-estimating the model, when the schools are divided into a category of large schools (with >1000 students) and small schools (with <1000 students). We can then compare the estimated preference and meeting biases by school size. As reported in Table 3, larger schools exhibit significantly higher meeting biases (at the 99% level) than small schools, but differences in preference biases between small and large schools are insignificant (even at the 90% confidence level). In particular, meeting biases are greater for Blacks and Hispanics in large compared to small schools, but lesser for Asians in large compared to small schools. We also checked whether the patterns in biases are sensitive to the median household income level in the school's county, since it turns out that income shows very little correlation with school size in these data. Again, we divided schools into two categories, those in which the county level median household income level was above $30,000 in the 1990 census, and those for which it was below that level. As reported in Table 4, in this case preference biases differ significantly (at the 99% level) across the high and low income-level schools, but the meeting biases do not differ significantly (even at the 90% confidence level) across the high and low income-level schools. Here we see Hispanic and White exhibiting more bias in preferences (i.e., having lower γ's) in low income schools and there is more bias overall in low income schools (when estimating a single bias parameter), although Blacks exhibit slightly more preference bias in high income schools.

Further study is needed to understand the sources of preference and meeting biases, why they differ across races, and why they are correlated with school size and median income. Moreover, the racial categorization here is quite coarse,[¶] and many other attributes can also affect friendship formation. In addition, we note that our analysis might even underestimate preference biases, since meeting biases could incorporate some aspects of choice, and since meetings are partly endogenous given that students have some choice as to which clubs to join, which sports to participate in, which parties to go to, and so forth. Indeed, there is a positive correlation between inbreeding homophily and the number of clubs and athletic activities that a school has (a correlation of 0.26 which is significant at the 99% confidence level).[‖]

Beyond this, it would be interesting to extend the revealed preference sort of techniques used here to incorporate the sort of network induced homophily effects found by Kossinets and Watts (24, 25), who show the importance of proximity in an existing social network for the formation of new friendships.**

Finally, it is worth emphasizing that the "revealed preference" techniques applied here can be employed in many other settings; by specifying an

[§]In *SI Text* we also report estimation of these biases when restricting attention to groups with minimal weights in the population, as well as estimations of the meeting biases by estimating Eq. **2** on a race-by-race basis, rather than as a joint estimation procedure as we use under Eq. **3**. These provide biases that are similarly significantly above 1, but with some compression and variation in the differences across races.

[¶]The coarseness could lead the preference biases of some racial groups to be underestimated. For example, the "Asian" group includes Chinese, Japanese, Korean, Vietnamese, Indian, and many other populations under one umbrella. To the extent that their racial preferences differ by the finer ethnic categorizations, it could be that this leads their friendship patterns with other categories to look similar to the same group, and so leads to a higher γ parameter than would be estimated if data were available on finer categorizations. This might account for the significant differences between Asian preference bias and some of the other groups' biases.

[‖]We consider activities that have at least two student members and discard the few students who claimed to be involved in 20 or more clubs or sports with some having claimed to be involved in all possible activities.

**It is worth emphasizing that the terms "choice homophily" and "preferences," as used in the empirical literature on homophily, generally refer to some conditional likelihood, or log odds ratios, of forming ties based on some characteristics, rather than explicit modeling of a maximization of a utility function as we have done here. Thus, a hybrid of the sort of model analyzed here together with the network evolution and constraints that network structures place on the meeting process as analyzed in refs. 24 and 25 could be quite valuable.

appropriate preference structure and a process through which agents might meet each other, and then identifying various parameters through the co-movements of the fraction of agents of various types and their patterns of relationships.

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Supporting Information

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Description of the Add Health Data

Measuring Homophily. We begin with some simple definitions that are important in measuring homophily and also in presenting the model.

Let N_i denote the number of type i individuals in the population, and let $w_i = \frac{N_i}{N}$ be the relative fraction of type i in the population, where $N = \sum_k N_k$.

Let s_i denote the average number of friendships that agents of type i have with agents who are of the same type, and let d_i be the average number of friendships that type i agents form with agents of types different from i. Let $t_i = s_i + d_i$ be the average total number of friendships that type i agents form.

The homophily index H_i measures the fraction of the ties of individuals of type i that are with that same type.

Definition 1 The homophily index H_i is defined by

$$H_i = \frac{s_i}{s_i + d_i}.$$

The profile (s, d) exhibits **baseline homophily** for type i if $H_i = w_i$.

The profile (s, d) exhibits **inbreeding homophily** for type i if $H_i > w_i$.

Generally, there is a difficulty in simply measuring homophily according to H_i. For example, consider a group that comprises 95% of a population. Suppose that its same-type friendships are 96% of its friendships. Compare this to a group that comprises 5% of a population and has 96% of its friendships being same-type. Although both have the same homophily index, they are very different in terms of how homophilous they are relative to how homophilous they could be. Comparing the homophily index, H_i, to the baseline, w_i, provides some information, but even that does not fully capture the idea of how biased a group is compared to how biased it could potentially be. To take care of this we use the inbreeding homophily index introduced by Coleman [Coleman J (1958) *Hum Organ* 17:28–36] that normalizes the homophily index by the potential extent to which a group could be biased.

Definition 2 Coleman's inbreeding homophily index of type i is

$$IH_i = \frac{H_i - w_i}{1 - w_i}.$$

This index measures the amount of bias with respect to baseline homophily as it relates to the maximum possible bias (the term $1 - w_i$). It can be easily checked that we have inbreeding homophily for type i if and only if $IH_i > 0$, and inbreeding heterophily for type i if and only if $IH_i < 0$. The index of inbreeding homophily is 0 if there is pure baseline homophily, and 1 if a group completely inbreeds.[*†]

General Patterns of Homophily

The data from Add Health were collected over several years starting in 1994 from a carefully stratified sample of high schools and middle schools (to vary by size, location, include public and private, varied racial composition, and socio-economic backgrounds). There are behavioral and demographic data in the data set from 112 schools; here we use the data from 84 schools for which extensive network information was obtained. The data are based on student interviews. The friendship data were based on reports of friendships by each student. Student's were shown a list of all of the other students in the school and permitted to name up to five friends of each sex. Only 3% nominated 10 friends, and only 24% hit the constraint on one of the sexes, and so the constraints do not seem to impose a substantial measurement issue, although there are standard concerns about self-reported relationships and interview-based data. Here a tie is present if either student mentioned the other as a friend. Student's could also identify other students with whom they had romantic relations, which are not reported among friendships. The attribution of race is based on a self-reported classification.

In the analysis to follow, each observation refers to the average of a given racial group within a given high school. We have a total of 305 observations (all racial groups that are present in the sampled high schools).

Fig. 1 relates the total number of friendships (on average) held by each racial group to the relative size of that group. The average number of total friends is an increasing function of group size, with a mean of <6 friends for groups that are a small fraction of their school increasing up to >8 friends on average for a racial group that comprises most of a high school. Regressing the total number of friends t on relative group size w we find that (standard errors are in parenthesis)[‡]:

$$t = \underset{(.19)}{5.54} + \underset{(.31)}{2.27}\,w \qquad \textbf{[s1]}$$

Both the constant and the coefficient of group size are significant at a 99% confidence level (with t-statistics of 7.34 for w and 28.46 for the constant, and $R^2 = 0.076$).[§]

As explained above, it is informative to normalize groups' inbreeding relative to their inbreeding potential by dividing the difference between the observed index H and the relative group size w by a factor of one minus w, to obtain the Inbreeding Homophily Index. As shown in Fig. 1, this index varies nonmonotonically with relative group size, following a humped shape. Very small and very large groups tend to inbreed very little compared with their inbreeding potential, while, on average, middle sized groups inbreed the most. Because of the nonlinearity of the relation between IH and w, we regress the index IH on group size w and on the square of group size w^2 (higher order terms do not significantly improve the fit). We obtain the following relationship[¶]:

$$IH = \underset{(0.01)}{.032} + \underset{(0.13)}{2.15}\,w - \underset{(0.15)}{2.35}\,w^2 \qquad \textbf{[s2]}$$

Both coefficients and the constant term are significant at a 99% confidence level, with t-statistics of 2.16 for the intercept, of 24.8 for w and of 20.4 for w^2, and with $R^2 = 0.73$.

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*One could also define a heterophily index, which would be $\frac{\frac{s_i}{s_i+d_i} - w_i}{-w_i}$, reflecting the extent to which a group is outgoing. It would be 0 at baseline homophily and 1 if a group only formed different-type friendships.

†The measures H_i and IH_i have slight biases in small samples. For example, suppose that there was no bias in the friendship formation process so that we are in a "baseline" society. Then the fraction of other agents that are of type i is $\frac{N_i-1}{N-1}$. Thus, the expected value of $H_i - w_i$ in a baseline society is $-\frac{N-N_i}{N(N-1)}$, which vanishes as N becomes large. The expected value of IH_i is then $-\frac{1}{N-1}$, which is independent of i, vanishing in N, and slightly negative.

‡This is a weighted regression, since the relationship is a per-capita variable and small groups have only a few individuals and substantially higher variances. So the average value of a group of x students is weighted by x. Without weights, the average individual behavior of few students in a small racial group would affect results as much as the average behavior of hundreds of students in a large group, biasing the results in favor of small groups. Since total friends are a characteristic related to individual behavior of students, the student is the correct level of observation. For comparison, an unweighted regression gives similar results ($t = 5.86 + 1.86w$ with standard errors of 0.15 and 0.37, respectively), so it does not make much of a difference.

§The R^2 increases by a factor of 4 when we include racial information, below.

¶This is an unweighted regression since the relationship is a group-level one and the index is a normalized index.

A Closer Look at Data: Differences Across Schools and Races

A closer inspection of the data suggests that the observed relations between relative group size and friendship patterns result from the aggregation of seemingly different patterns for the various races. Significant differences are also associated with friendships patterns in schools of different sizes.

Inbreeding Homophily Index. We obtain a clear picture of the different trends followed by different races by running separate regressions of the Inbreeding Homophily Index for each race, and plotting the fitted curves in Fig. 1*A*.

To get an idea of the effect of school size, we run separate regressions of the inbreeding homophily index against group size and its square for small and large schools. We break the data into two parts, with a threshold that splits the data roughly in half: those schools with >1000 students and those with <1000 students. The separate fits are depicted in Fig. 1*B*.

The two quadratic fits suggest a general increase in the inbreeding of all groups in larger schools. Interestingly, this increase seems to be more substantial for smaller groups (and is statistically significant, as found by the significance of the intercept dummy variable for school size in the regression in Table S1).

To test the statistical significance of the above differences across races and schools, we run a regression of the Inbreeding Homophily Index against relative group size, controlling for the composition of the sample with respect to race and school size. The results of this regression are summarized in Table S1.

We control for the effect of race by means of race dummies for Black, Hispanic, and Asian groups, and slope dummies for these three races for both w and w^2; we control for school size by means of a dummy variable splitting the sample in large schools and small schools. We also control for the interaction effect of school size with both group size and the square of group size. We obtain qualitatively similar results for thresholds of school size other than the 1000 which splits the data roughly in half.

The parametric tests of Table S1 impose constraints on the functional forms. To further investigate the significance of the above differences, we run a nonparametric Mann–Whitney test on the difference of distributions from which observed data for the various races and schools are drawn. The null hypothesis is here that the observed Inbreeding Homophily Indices for the two races are drawn from the same distribution. The z-statistics are as follows:

Asian – Black: – 4.417***; *Asian – Hispanic*: 1.269; *Asian – White*: – 5.041***;
Black – Hispanic: 5.271***; *Black – White*: 0.549; *Hispanic – White*: – 6.036***.

The results of this test are in line with those found in the parametric tests, suggesting a systematic effect of race. There are very highly significant differences between races in all cases with the exception of Black-White and Asian-Hispanic. As this is a much weaker test than the parametric fits, it is not completely clear how to interpret this. Fitting the parametric regressions based on quadratics picks up a difference between Blacks and Whites and also for Asians and Hispanics (using confidence intervals on the dummies), while the nonparametric rank sum test does not (and is also weaker as it does not correct for school size effects). Fitting the model below will help in further sorting this out, as it provides a different angle altogether.

We can nonparametrically test the effect of school size by means of a Matt–Whitney test on the difference of distributions of the Inbreeding Homophily Index associated with large (>1,000 students) and small (<1,000 students) schools. The null hypothesis is that both samples are drawn from the same distribution. The null hypothesis is rejected at the 99% confidence level. Finally, we test whether the difference in distribution is driven by small racial groups. Interestingly, we obtain that the distributions from large and small schools are not statistically different for majority groups and are statistically different for minorities. In particular, for large groups we obtain a z-statistic of -1.703, with $\Pr > z$ equal to 0.089, while for small groups we obtain a z-statistic of -4.057, with $\Pr > z$ equal to 0.0001. We can rephrase this result by saying that the inbreeding behavior of students in groups that comprise small fractions of their school is "more affected" by school size than the behavior of groups that comprise large fractions of their school. A potential explanation for this, that will be consistent with the modeling below, is that small racial groups may find it easier to inbreed in larger rather than in smaller schools, possibly because of the presence of economies of scale in the formation of organized *loci* of activity that traditionally favor inbreeding behavior, such as clubs, societies, and other extracurricular activities and organizations.

Numbers of Friendships. We now turn to the pattern of how the average numbers of friendships varies as a function of group size. The bottom panel of Fig. S1 suggests that the relation between total number of friends and relative group size results from quite different patterns across races. Running separate regressions for separate races, we obtain the fitted lines pictured in the bottom panel of Fig. S1. As we see from Table S2, the relation between group size and total friends is significantly different from a flat line at 99% confidence level only for Whites and Hispanics, and also for Blacks if we relax to a 95% confidence level.[‖]

Finally, the effect of school size on total number of friends is such that total friendships slightly decrease in larger schools (as we see from Table S3, this difference is not statistically significant).

As we did for the Inbreeding Homophily Index, we run a single regression of the total number of friends against relative group size, controlling for racial composition and school size. Again, we control for the effect of race by means of racial dummies and racial slope dummies for w, and for the effect of school size by means of the dummy DS which splits the sample in schools with >1000 students and schools with <1000 students. The results are summarized in Table S3.

These results about both the number of friendships and the inbreeding homophily index point out effects of both race and school size on the relation between the relative size of racial groups and their pattern of friendship formation. Race significantly affects both total friendships and their racial mix, and in different ways across races. School size also affects the racial mix of friendships, strengthening the tendency to inbreed; but does not have a significant effect on total numbers of friendships. We again remark that although statistically significant, these results merely indicate an association between race and school size and some patterns of friendship formation, while no casual effect is implied.

A Preference- and Random Meeting-Based Model of Friendship Formation

Our model is such that friendship formation takes place via a meeting pool in which agents enter without any friends, are randomly matched to new possible friends and eventually exit the process after having formed some friendships. Heuristically, this can be thought of as being like a party that students attend, and they continue to form friendships while at the party and eventually decide to leave the party.

[‖]Again, these are weighted regressions. We note that the negative relationship for Hispanics seems driven by a single outlier. In that school, Hispanics form 89% of the population and yet form fewer than five friends per capita, and in this case they form <0.25 different-type friendships per capita. If we change the value of that outlier to the average value, then the relationship for Hispanics is positive (although not statistically significant).

Agents are characterized by race and generally we use the term *type*, as the model applies for all sorts of different categorizations, including things like age, gender, or combinations of such attributes. Agents have preferences over whom they are friends with, which are potentially sensitive to the racial (or type) mix of these friends. Each racial group i is characterized by its relative size w_i in the school.

We consider a steady state of the meeting process in which the flow of new agents into the matching balances those exiting. Three key elements of the model are: (*i*) the preferences and resulting choices of the agents of how many friendships to form given the meeting process; (*ii*) the random meeting process itself, which may be more or less biased in terms of the relative rates at which it matches types; and finally (*iii*) the steady-state requirements that require that friendships add up across agents and that people enter and exit at a similar rate so that the process is in equilibrium. We use the model to calibrate preferences and meeting rates across races, evaluating whether there are differences across races or according to school size. This approach complements the purely statistical one, since it allows us to infer which forces are affecting homophily and friendship numbers.

An Agent's Preferences. Each agent receives utility from the composition of the set of his or her friends.

Agents of the same type are characterized by their utility function, which may, however, differ across types. The total utility to an agent of type i who has s_i same-type friends and d_i different-type friends is given by Eq. **1**:

$$U_i(s_i, d_i) = (s_i + \gamma_i d_i)^{\alpha},$$

where both γ_i and α are between 0 and 1, so that U_i is increasing in both s_i and d_i.

This simple functional form for preferences has several features worth commenting on. First, the $\alpha(< 1)$ parameter captures the fact that there are diminishing marginal returns to friendships, so that although there are benefits from having more friends, those marginal benefits decrease as more friends are added. Second, the γ_i parameter captures the bias that an individual has in evaluating friendships of same type versus different type. A different-type friend is only worth γ_i as much as a same-type friend. Third, we allow γ_i to vary with type but keep α the same across types. We could extend the model to fuller generality, but at the risk of having too many free parameters and over-fitting the data. The critical difference that we are interested in exploring is racial attitudes toward cross-race friendships and so γ_i is a critical parameter to allow to vary, while the rate at which friendship value diminishes is less pertinent and so we hold that fixed across races.

The race-dependent parameter γ_i quantifies the bias toward own type in preferences: a value of $\gamma_i = 0$ indicates completely biased preferences which attributes no value to friendships with different types, a value of $\gamma_i = 1$ corresponds instead to preferences which are independent of types (in the economics terminology, this is a case of *perfect substitutes*).

There are costs to meeting people and forming friendships, both in time and energy, and that caps the numbers of friendships that agents form. In particular, an agent bears a cost $c > 0$, in terms of opportunity cost of time and resources, for each unit of time spent in the meeting process. We will see that the parameter c, although needed to close the model, does not end up playing a significant role in the calibration, and could in principle even be heterogeneous across schools.

The Meeting Process and Decision Problem. The way in which people meet to potentially form friendships is through a meeting process that is like a party. Agents begin by entering the process or party, and then once there they randomly meet other people. Agents can then choose to either form a friendship or not with each agent whom they meet. There is a cost to being in the meeting process (i.e., at the party), and so eventually people choose to leave when the benefits from meeting more people no longer exceeds the cost of staying. Given that preferences are increasing in friendships of both types, agents will accept whomever they meet as friends, and the main decision is simply when to leave.** Note that even though the only decision is when to exit the process, both biases in preferences and biases in the matching will affect that decision. The bias in preferences determines how the agent evaluates what the marginal return from staying in the meeting process is relative to its cost, while the bias in the meeting process will affect the mix of same versus different people that will be met and thus also the anticipated marginal return from being in the meeting process.

Thus, the relevant meeting parameter from an agent's decision perspective is the expected rate at which the agent will meet same versus different type friends. Because of the bias and potential heterogeneity in the actions of different types of agents, the relative rate at which agents meet their own type versus different types will not correspond directly to their relative fraction in the population w_i. This would only be true if all agents stayed in the meeting process for the same amount of time and the process operated completely uniformly at random and had no bias in matching. Otherwise, we need to keep track of the rate at which a type i agent meets other type i's, which we denote q_i. This parameter will be determined by the decisions and the steady-state.

Given the matching probabilities of same type of agents and different type agents of q_i and $1 - q_i$ per unit of time, respectively, if an agent of type i stays in the meeting process (at the party) for a time t_i, then he or she will end up with $(s_i, d_i) = (t_i q_i, t_i(1 - q_i))$ friends of same and different types, respectively. We solve the model in the case where the actual realizations of the matching are the expected numbers, so that q_i will be equal to the homophily index H_i (Definition 1).†† Thus, an agent of type i solves the following decision problem of how long to stay in the meeting process and thus how many friends to have:

$$\max_{t_i} U_i(q_i t_i, (1 - q_i)t_i) - c t_i. \qquad \textbf{[s3]}$$

Given the utility function described in Eq. **1**, this is a straightforward maximization problem and it has solution:

$$t_i = \left(\frac{\alpha}{c}\right)^{\frac{1}{1-\alpha}} (\gamma_i + (1 - \gamma_i) q_i)^{\frac{\alpha}{1-\alpha}}.$$

The Bias in Meeting Process and the Steady State. Solving the model requires determining the meeting rates. Clearly, there must be some conditions that relate meeting rates across races, since if a person of race i is meeting a person of race j then the converse is also true. Thus, there are cross conditions that constrain the potential configurations of q_i's.

The meeting process is described by an $n \times n$ matrix $\mathbf{q} \in [0, 1]^{n \times n}$, where q_{ij} is the fraction of i's meetings per unit of time that are with type j and the matrix is row stochastic so that $\sum_j q_{ij} = 1$.

Let M_i denote the relative stock of agents of type i in the meeting process at any time. In particular, $M_i = t_i w_i$, and we can also normalize the stocks, letting $m_i = \frac{M_i}{\sum_k M_k}$. The relative meeting probabilities (q_{ij}'s) depend on the stocks of agents in the society and how they bump into each other, which is captured by

**Allowing for satiation can lead agents to refuse friendships. We discuss an extension of the basic model to allow for satiation in the supplementary material to Currarini, Jackson and Pin [Currarini S, Jackson MO, Pin P (2009) *Econometrica* 77:1003–1045]. It significantly complicates the model as choices then become path-dependent. Thus, at some loss of generality we work with the simpler model as it still admits both sorts of biases.

††This can be justified by a limit process with an infinite number of agents and renders the analysis tractable.

a function F, where $\mathbf{q} = F(M_1, \ldots, M_n)$ is the matching that occurs as a function of the relative stocks of agents in the society, and of the utilities of the agents.

To be well defined, the meeting process needs to balance, so that the number of meetings where an i meets a j is the same as those where a j meets an i. A meeting process F is *balanced* at a given M if $\mathbf{q} = F(M_1, \ldots, M_n)$ is such that

$$m_{ij}q_{ij} = q_{ij}m_{ij}$$

for all i and j.

A canonical meeting process is one where agents meet each other in proportion to their relative stocks in the process (at the party). We call that the *unbiased meeting process*, and it is such that $q_{ij} = \frac{M_j}{\sum_k M_k}$. Given that agent's preferences only depend on own an other types, we let $q_i = q_{ii}$ and then $1 - q_i = \sum_{j \neq i} q_{ij}$.

We work with a parameterized version of the meeting process such that:

$$q_i = m_i^{1/\beta_i}, \qquad \textbf{[s4]}$$

where $\beta_i \geq 1$ is the bias that type i has toward itself in the meeting process. The case $\beta_i = 1$ in the unbiased process (uniformly random meetings), and the meeting bias of type i increases with $\beta_i > 1$.

If the meeting process were uniformly at random, then it would have to be that $\sum_i q_i = 1$. However, if there is a bias in the meeting process, agents can meet their own types at a rate which is greater than their relative mass in the meeting process. In that case $q_i > \frac{M_i}{\sum_k M_k}$ and so $\sum_i q_i > 1$.

The matching defined in s**4** will be balanced if, and only if, for each type i,[‡‡]

$$(1 - q_i)m_i \leq \sum_{j \neq i}(1 - q_j)m_j. \qquad \textbf{[s5]}$$

If there are only two types then the two inequalities from **s5** impose an additional equality, so that β_i is determined by β_j. As the number of types increase, as is in our 5-races case, then the inequalities from **s5** are less binding. With the β's that we calibrate, **s5** is satisfied for almost all of the cases in the data.

The parameterization of the meeting process **s4** implies Eq. **3**:

$$\sum_i q_i^{\beta_i} = 1.$$

Using this model, we can estimate the parameters from steady-state conditions. First, note that from the maximization of utility for the agents it follows that, for any pair of types i and j, Eq. **4** holds

$$t_i(\gamma_j + (1 - \gamma_j)q_j)^{\frac{\alpha}{1-\alpha}} = t_j(\gamma_i + (1 - \gamma_i)q_i)^{\frac{\alpha}{1-\alpha}}.$$

Next, note that t_i, t_j and q_i, q_j are available from the data since t_i is simply the total number of friends, and $q_i = s_i/(s_i + d_i)$ is the relative rate at which type i's meet themselves. Thus, we can estimate the preference parameters (α and the γ_i's) from the data by searching over values which come closest to satisfying Eq. **4**.

Finally, we can estimate the meeting bias parameters (the β_i's) directly by searching over values which come closest to satisfying Eq. **3**.

Estimation of the Model

Differences Across Races. Here we estimate the model described using the Add Health data.

The Add Health data actually have six (self-reported) ethnic categories: Asian, Black, Hispanic, White, Mixed, and Missing. For the sake of completeness and to avoid discrepancy with the empirical data, we use a category of *Others*: to include "Mixed" and "Missing" outcomes.

We estimate the model as follows. Index the schools by k. Let N_k denote the number of students in school k and let w_{ik} denote the fraction of students in school k who are of type i.

A student a of type i in school k faced with a rate q_{ik} of meeting own types solves

$$\max_{t_{ik}} (q_{ik}t_{ik} + \gamma_i(1 - q_{ik})t_{ik})^{\alpha} - c_k t_{ik}, \qquad \textbf{[s6]}$$

with solution

$$t_{ik} = \left(\frac{\alpha}{c_k}\right)^{\frac{1}{1-\alpha}} (\gamma_i + (1 - \gamma_i)q_{ik})^{\frac{\alpha}{1-\alpha}}. \qquad \textbf{[s7]}$$

We suppose that individual students are subject to independent idiosyncratic shocks on a student-by-student basis (which may be errors or individual preference differences or other idiosyncrasies), so that the solution to **s7** is subject to noise and hence,

$$t_{aik} = \left(\frac{\alpha}{c_k}\right)^{\frac{1}{1-\alpha}} (\gamma_i + (1 - \gamma_i)q_{ik})^{\frac{\alpha}{1-\alpha}} + \varepsilon_a, \qquad \textbf{[s8]}$$

where ε_a is the individual error that has 0 mean and variance σ^2 for every type in every school.

Eq. **s8** can be aggregated on a given type in a given school, so that

$$w_{ik}N_k t_{ik} = \left(\frac{\alpha}{c_k}\right)^{\frac{1}{1-\alpha}} w_{ik}N_k(\gamma_i + (1 - \gamma_i)q_{ik})^{\frac{\alpha}{1-\alpha}} + E_{ik}, \qquad \textbf{[s9]}$$

where E_{ik} has 0 mean and variance $w_{ik}N_k\sigma^2$; but can also be aggregated over all of the other types in the school, so that

$$\sum_{j \neq i} w_{jk}N_k t_{jk} = \left(\frac{\alpha}{c_k}\right)^{\frac{1}{1-\alpha}} \sum_{j \neq i}\left(w_{jk}N_k(\gamma_j + (1 - \gamma_j)q_{jk})^{\frac{\alpha}{1-\alpha}}\right) + E_{-i,k}, \qquad \textbf{[s10]}$$

where $E_{-i,\,k}$ has 0 mean and variance $\left(\sum_{j \neq i} w_{jk}\right)N_k\sigma^2$.

In principle, from our data, one could compare **s8** with **s9**, but this would take a very long time as we are running a calibration. What we can do is compare **s9** with **s10**.

Order the weights so that $w_{ik} \geq w_{jk}$ when $i < j$. Let

$$A_{ik} = (\gamma_{ik} + (1 - \gamma_i)q_{ik})^{\frac{\alpha}{1-\alpha}}. \qquad \textbf{[s11]}$$

Then, it follows from **s9** and **s10** that

$$\frac{w_{ik}N_k t_{ik} - E_{ik}}{w_{ik}N_k A_{ik}} = \frac{\left(\sum_{j \neq i} w_{jk}N_k t_{jk}\right) - E_{-i,k}}{\sum_{j \neq i} w_{jk}N_k A_{jk}}.$$

We obtain an error for school k

[‡‡]Condition **s5** is clearly necessary for balance. To see sufficiency, first note that this directly implies balance for two types, and so let us examine a case with three or more types. We describe one meeting process that works, but in most cases there are many others. For each type i, let $x_i^0 \equiv (1 - q_i)m_i$ and order types so that $x_1^0 \geq x_2^0 \cdots \geq x_n^0$. Start with type 1's. Have them meet the second largest group until the remaining part of the second group is equal in size to the third largest group, and then meet equally with those two groups until their remainders are equal to the fourth largest group, and so forth, until the type 1s are exhausted. Then iterate on this process with the remaining groups until there are no remainders. More formally, match $\lambda_j^0 x_j$ of the type js for $j > 1$ with the type 1s, where the $\lambda_j^0 \in [0, 1]$ for $j \geq 2$ are the unique scalars that satisfy $x_1 = \sum_{j \geq 2}\lambda_j^0 x_j$; and $(1 - \lambda_j^0)x_j \geq (1 - \lambda_{j+1}^0)x_{j+1}$ with equality whenever $\lambda_{j+1}^0 > 0$. Since $x_1^0 \leq \sum_{j \geq 2} x_j^0$, such a profile of λ_j^0s exists. Let $x_j^1 = (1 - \lambda_j^0)x_j^0$ and $x_1^1 = 0$. Note that by the constructions of the λ_j^0s the ordering is preserved and it is also straightforward to verify that $x_2^1 \leq \sum_{j \geq 3} x_j^1$, and so **s5** is still satisfied on the remaining parts of groups 2 to n, $(x_1^2, x_3^1, \ldots, x_n^1)$. Then repeat the process treating $(x_1^2, x_3^1, \ldots, x_n^1)$ as the starting point, until all groups are exhausted. Note that when we reach step $n - 3$, where $(x_{n-2}^{n-3}, x_{n-1}^{n-3}, x_n^{n-3})$ are remaining to be matched, then it must be that after that matching we are left with $x_{n-1}^{n-2}, x_n^{n-2})$ such that $x_{n-1}^{n-2} = x_n^{n-2}$, and so the final step is to match these two remainders together. To see that these two last groups must be equal in size, suppose the contrary so that $x_{n-1}^{n-2} > x_n^{n-2}$. By the definitions of the λs, it would have to be that $\lambda_n^{n-3} = 0$, and so it would have to be that $x_{n-2}^{n-3} = x_{n-1}^{n-3}$ (since $x_{n-2}^{n-3} \geq x_{n-1}^{n-3}$ and $x_{n-2}^{n-3} = \lambda_{n-1}^{n-3} x_{n-1}^{n-3}$ and $\lambda_{n-1}^{n-3} \in [0, 1]$). But this implies that $x_{n-1}^{n-2} = 0 > x_n^{n-2}$, which is a contradiction.

$$\Psi_k = w_{1k}N_k t_{1k}\Big(\sum_{j\neq 1} w_{jk}N_k A_{jk}\Big) - \Big(\sum_{j\neq 1} w_{jk}N_k t_{jk}\Big) w_{1k}N_k A_{1k.} \quad \textbf{[s12]}$$

$$= E_{1k}\Big(\sum_{j\neq 1} w_{jk}N_k A_{jk}\Big) - E_{-1,k}w_{1k}N_k A_{1k}. \quad \textbf{[s13]}$$

Ψ_k has mean 0 and variance $\sigma^2\phi_k$ where

$$\phi_k = N_k^3\Big(w_{1k}\Big(\sum_{j\neq 1} w_{jk}A_{jk}\Big)^2 + \Big(\sum_{j\neq 1} w_{jk}\Big) w_{1k}^2 A_{1k}^2\Big).$$

Normalizing the errors so that these are of equal variance across schools (required for an F-test), leads to setting $(Error_k)^2 = \frac{\Psi_k^2}{\phi_k}$. The total error is obtained aggregating across schools, which now have errors with zero mean and equal variances (of σ^2). Thus, the total error calculation is

$$TotalError = \sum_k \frac{\Psi_k^2}{\phi_k}.$$

We search over a grid of values for α and γs to find the one that minimizes this sum of squared errors.

The upper part of Table 1 reports the combination that minimizes the sum of squared errors as described above.

Our calibration of the model with respect to preference bias is consistent with the statistical evidence from Tables S1–S3 and the Mann–Whitney tests.

Next, we calibrate the meeting bias parameters based on Eq. **3**.

When we estimate the β's from Eq. **3**, there will be some error school by school, and so

$$\sum_i q_{ik}^{\beta_i} = 1 + \nu_k, \quad \textbf{[s14]}$$

where ν_k is an error for school k's matching that has mean 0 and a variance that is the same across schools. So, for any specification of βs we end up with a set of errors, one for each school. Thus, we sum the squared errors across schools and choose βs to minimize that sum. We first search over a grid of step 0.5, from 1 to 9. If we hit a corner then we refine the grid to a step of 0.1 and search again (now from 1 to 3). In this case we hit corners for Whites and Others, as we discuss in more detail below, but not for the remaining races. The lower part of Table 1 reports the combination that minimizes the sum of squared errors.

Alternative Estimations of the Meeting Biases. The result $\beta_{Whites} = 1$ identifies a corner solution. One reason for this is that the calculation ends up including some noisy observations which are those corresponding to groups that are very small fractions of their schools. For example, if a group is a few percent of a school, then it can end up just having a few students and their idiosyncratic behavior ends up influencing the error.[§§] We are considering averages across all of the students of the same type as a good indicator of their meeting opportunities, and this assumption relies implicitly on a law of large numbers. For this reason, as a check, we rerun the calibration excluding all those observed types i whose representative w_i in the school is smaller than a threshold τ. To do this we can define a "threshold" version of Eq. **3**, that accounts for this:

$$\sum_{i:w_i>\tau} q_i^{\beta_i} = \sum_{i:w_i>\tau} w_i \ . \quad \textbf{[s15]}$$

Here only a type i for which $w_i > \tau$ is considered. The sum of the biased opportunities $q_i^{\beta_i}$ of sufficiently represented types should now sum to 1 minus the fraction of those minorities we have excluded (clearly $\sum_{i:w_i>\tau} w_i = 1 - \sum_{i:w_i<\tau} w_i$).

We estimate Eq. **s15** by adopting a threshold $\tau = 0.06$ (which is the minimal value that avoids $\beta_{Whites} = 1$). In this way we consider 237 out of 389 observations (we discard 6 out of 83 Whites, 31 out of 70 Blacks, 41 out of 82 Hispanics, 58 out of 70 Asians and 16 out of 84 Others). The best fitting βs are:

$\beta_{Asian} = 4.5$, $\beta_{Black} = 6$, $\beta_{Hispanic} = 3$ $\beta_{White} = 1.1$ $\beta_{Other} = 1$.

The obtained result is no longer a corner solution for Whites and is consistent with the qualitative outcomes of Table 1.

When we estimate the meeting biases using the above techniques, we are jointly estimating the biases across all races. In principle, β_i could instead be inferred from **s4**, as $\beta_i = \frac{\log m_i}{\log q_i}$. This would, however, miss cases in which the logarithm is not defined, or where this expression is driven by small values. Of the 305 observations, $q_i = 0$ for 48 of them (and for those w_i has a mean of 0.011 and a maximum of 0.094, which happens in a school of 32 students). For the observations for which $q_i > 0$, the imputed β_i has a mean of 1.957, with a standard deviation of 0.095.

If we consider the size of the schools, then on large schools ($n >$ 1000, 118 observations) we have $\beta_{large} = 2.349$ (0.162), while on small schools ($n < 1000$, 139 observations) this bias is, as expected, lower: $\beta_{small} = 1.623$ (0.102).

Race by race we obtain the following by regressing $\log(q_i)$ on $\log(m_i)$ to obtain $1/\beta_i$ and then inverting,[¶¶] all of which are significant at the 99.9% level:

1. Asians (12 observations): $\beta_{asian} = 2.1$.
2. Blacks (39 observations): $\beta_{black} = 3.4$.
3. Hispanics (36 observations): $\beta_{hispanic} = 1.3$.
4. Whites (77 observations): $\beta_{white} = 2.1$.

The basic pattern is consistent with that obtained from the calibrations reported in Table 1. Although the patterns are somewhat similar to those estimated under the joint estimation procedure, the Whites' meeting bias has now jumped above that of the Hispanics to match the Asian bias, and the overall level of the biases is a bit attenuated. This fitting, however, ignores joint information which incorporates comovements in racial compositions and meeting rates that we estimate in the main article under Eq. **3** and so should be less accurate in estimating the βs.

Significance Tests. We consider the following F-statistic:

$$F = \frac{\frac{(SSR_{con} - SSR_{uncon})}{p_{uncon} - p_{con}}}{\frac{SSR_{uncon}}{n - p_{uncon}}}$$

where SSR stands for "sum of squared residuals" of the best fit calibration, while p is the number of parameters estimated in the various models, and n is the number of observations: 84. The subscript "con" stands for the constrained model under the null hypothesis that some of the γ's are equal to each other and/or take on some values. The subscript "uncon" stands for unconstrained model, where all parameters are fit as above.

To illustrate this, before presenting the full tables of all of the tests, we first test the null hypothesis that all γ parameters are equal to 1. This is a test of the hypothesis that preferences are not sensitive to race.

We examine a 99% confidence level, and look at the F-statistic with (5, 78) degrees of freedom. The threshold F level for a 99% level is 3.26. We obtain:

[§§]We cannot simply reweight observations, because we need to respect the structural equations from the model.

[¶¶]The regressions are done with a 0 intercept and considering only those above the weight threshold of 0.06, from above.

$$F = \frac{\frac{17554-4704}{6-1}}{\frac{4704}{84-6}} = 42.61^{**} > 3.26.$$

When considering small and large schools, we fit different parameters for the two cases, and then get a total error when we allow these parameters to vary. This becomes the unconstrained case for the F-test, and then we compare it to the error when we add the constraint that the parameters not vary with school size.

Significance of Differences Across Races. The first thing to note is that all of the preference bias parameters are lower than 1, as shown in Table S4. Thus all races exhibit some bias in preferences toward their own race. When we test whether the preference bias parameters are significantly different from 1, we find that they are significantly different well beyond the 99% confidence level (with an F-statistic of 42.61).

The second thing to note is that there are significant differences between the races. Blacks exhibit the strongest preference bias with a γ of 0.55, so that a friendship with another race provides only 55% of the utility of a friendship with another Black. Hispanics see values of 65% for the same parameter, Whites 75%, and Asians are the least biased with a parameter of 90%. Some of those parameters are significantly different from each other, basically with Blacks and Hispanics both being significantly different from both Asians and Whites, but with Blacks not significantly different from Hispanics and Asians not significantly different from Whites.

When we examine the meeting biases (see Table S5), we again see dramatic differences across races and significant biases for most of the races. Whites have nearly no meeting bias, while that of the other races are quite substantial, with Blacks seeing the largest meeting bias, Asians the second largest, and Hispanics the third. The bias parameter for Blacks is more than seven times that of Whites. The differences between each pair of races is highly significant, except between Asians and Blacks.

The previous interpretations are based on a model such that all students of a given race behave homogeneously. This is an important caveat, since significant differences may arise within groups, and possibly driving average data. Moreover, in deriving conclusions about norms and/or policies it is important to recall that we are not taking into account other socioeconomic factors that could be correlated with race and be driving some of the differences in calibration (for instance, a preference for friendships along some other dimension).

Estimation of Differences Due to School Size and Income Level. We now calibrate the model by school size, seeing what differences in biases exist between small and large schools. The threshold determining size is kept at 1000.

We find that large schools exhibit significantly higher biases in meetings, and although the preference biases are also higher in larger schools they are not significantly so. For example, when we examine the meeting biases by races and school size, Black meeting biases are 6 for small schools and 9 for large schools,[||||] Hispanics vary from 2 for small schools and 6.5 for large schools, where there is no change for Whites (unbiased in each case) and actually a decrease for Asians (from 6.5 for small to 3.0 for large). So again, we see different patterns for Blacks and Hispanics compared to Asians and Whites.

Table S6, below, shows the results. We find a larger meeting bias in larger compared to smaller schools, going from an average value of 2–2.5. We also check that these differences do not vanish once we control for differences across races. The lower part of Table S6 confirms that the net effect of size on biases remains significant at a 99% confidence level.[***]

We perform similar calculations by school income level, as discussed in the main article and reported in Table S7.

Normality of the Error Terms. The F-tests that we perform presume that the errors in **s12** and **s14** are Normally distributed. Plots of those errors appear in Fig. S2. To test that these distributions do not deviate significantly from Normal distributions, we employed Kolmogorov–Smirnov tests. The combined P values are 0.756 and 0.076, respectively, and so neither deviates significantly from a Normal distribution at a 95% confidence level. We also perform more specific tests on skewness and kurtosis separately using χ^2 tests. In those cases, neither distribution exhibits a skewness that deviates from a Normal distribution, but both show significant differences in kurtosis (in this case, more 'spiked' distributions around 0).

[||||] This last value is an upper bound on the range used in the estimations. A larger upper bound would further increase the F-statistic, which is already significant at the 99% level.

[***] We check that there is not excess correlation between population shares and school size. We find the following correlations: Asians: 0.0263 (0.8291), 0.0509199, 0.0463393; Black: -0.0081 (0.9469), 0.1835026, 0.1587189; Hispanic: 0.2727 (0.0132), 0.1614755, 0.1156209; White: -0.3245 (0.0028), 0.4976767, 0.6456013; where the first number is the correlation, the second number is the P value, the third and fourth numbers are the mean w_i for the large and small schools, respectively.

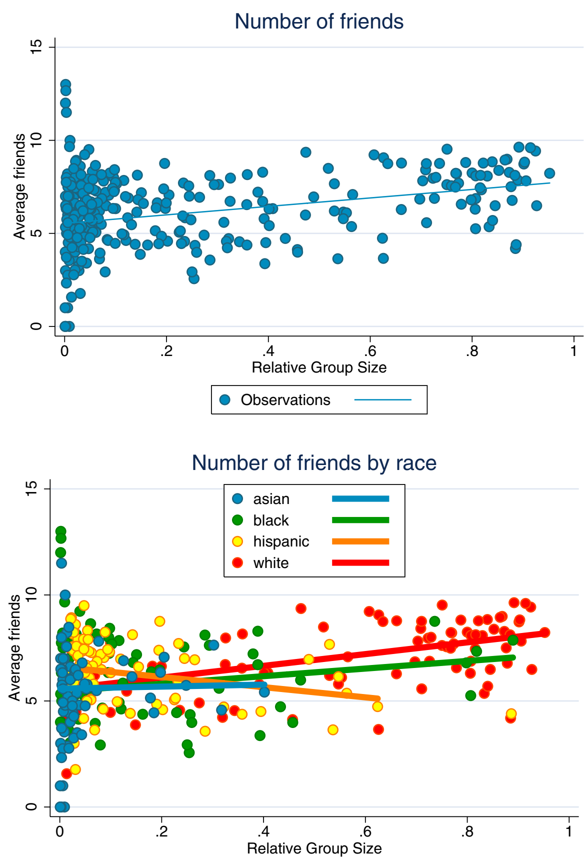

Fig. S1. Total number of friends by group size: all races (*Upper*) and by race (*Lower*).

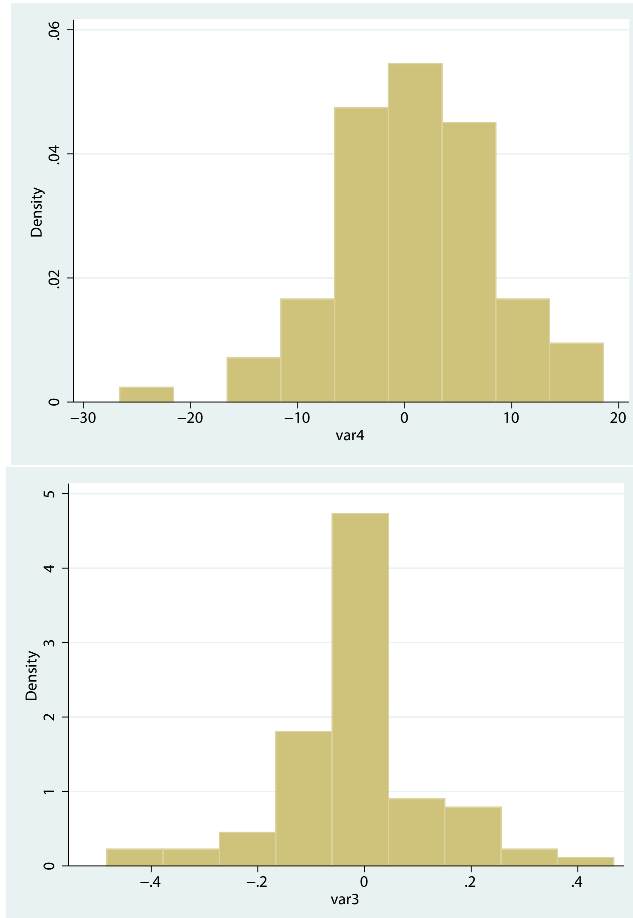

Fig. S2. Plot of residuals for estimation of γ and β.

Table S1. Inbreeding homophily index regressed on school size and racial variables

Inbreeding homophily index	Coefficient	SE	*t*-statistic	*P* value
Constant	0.135	0.047	2.83	0.005
Group size, *w*	1.44	0.23	6.18	0.000
Group size squared, w^2	−1.66	0.23	−7.13	0.000
School size dummy, *DS*	0.059	0.024	2.40	0.017
School size dummy times *w*	0.27	0.22	1.24	0.216
School size dummy times w^2	−0.40	0.27	−1.47	0.147
Dummy Black	−0.049	0.052	−0.95	0.344
Dummy Hispanic	−0.19	0.05	−3.73	0.000
Dummy Asian	−0.16	0.051	−3.23	0.001
Slope dummy Black times *w*	1.02	0.31	3.31	0.001
Slope dummy Hispanic times *w*	−0.16	0.34	−0.49	0.627
Slope dummy Asian times *w*	2.67	0.65	4.13	0.000
Slope dummy Black times w^2	−1.24	0.35	−3.53	0.000
Slope dummy Hispanic times w^2	0.92	0.44	2.09	0.038
Slope dummy Asian times w^2	−6.71	1.89	−3.55	0.000

Table S2. Number of friends against group size: regressions by race

Number of friends	Asian	Black	Hispanic	White
Constant	5.58	5.45	6.63	5.55
t-statistic	23.19	15.09	29.43	10.64
P value	0.000	0.000	0.000	0.000
Coefficient	0.47	1.80	−2.50	2.77
t-statistic	0.43	2.55	−5.26	3.82
P value	0.668	0.013	0.000	0.000

Table S3. Number of friends

Number of friends	Coefficient	SE	*t*-value	Pr > *t*-value
Constant	5.30	0.44	12.00	0.000
Group size, w	3.31	0.60	5.47	0.000
School size dummy	0.52	0.37	1.41	0.161
School size dummy times w	−1.14	0.58	−1.98	0.049
Dummy Asian	−0.087	0.69	−0.13	0.899
Dummy Black	−0.20	0.50	−0.40	0.687
Dummy Hispanic	0.96	0.48	1.98	0.048
Dummy Asian times w	−1.99	2.68	−0.75	0.457
Dummy Black times w	−0.74	0.84	−0.88	0.381
Dummy Hispanic times w	−4.90	0.84	−5.86	0.000

Table S4. Differences in preference biases across races

Preference parameter	α	γ_a	γ_b	γ_h	γ_w	γ_o	RSS_{con}	RSS_{uncon}	F	95%	99%
Unconstrained	0.55	0.90	0.55	0.65	0.75	0.90	—	4704	—	—	—
Asian = Black	0.70	0.80	0.80	0.80	0.85	0.95	5303	—	9.93**	3.963	6.971
Asian = Hispanic	0.65	0.75	0.70	0.75	0.80	0.95	5197	—	8.17**	"	"
Asian = White	0.65	0.85	0.70	0.75	0.85	0.95	4864	—	2.65	"	"
Black = Hispanic	0.65	0.90	0.70	0.70	0.80	0.95	4798	—	1.56	"	"
Black = White	0.55	0.80	0.65	0.60	0.65	0.90	5333	—	10.43**	"	"
Hispanic = White	0.60	0.90	0.55	0.70	0.70	0.90	4911	—	3.43	"	"
All = 1	0.20	1.00	1.00	1.00	1.00	1.00	17554	—	42.61**	2.332	3.261
All =	0.55	0.80	0.80	0.80	0.80	0.80	6175	—	6.10**	2.489	3.570

*Significance above a 95 percent level; **significance above a 99 percent level.

Table S5. Differences in meeting biases across races

Meeting parameters	β_a	β_b	β_h	β_w	β_o	RSS_{con}	RSS_{uncon}	F	95% thr.	99% thr.
Unconstrained	7.0	7.5	2.5	1.0	1.0	—	1.7265	—	—	—
Asian = Black	7.5	7.5	2.5	1.0	1.0	1.7274	—	0.04	3.962	6.967
Asian = Hispanic	3.5	7.5	3.5	1.0	1.0	1.8347	—	4.95*	"	"
Asian = White	1.5	6.5	3.5	1.5	1.0	2.7748	—	47.97**	"	"
Black = Hispanic	3.5	5.5	5.5	1.0	1.0	2.1486	—	19.31**	"	"
Black = White	9.0	3.0	1.0	3.0	1.0	4.4483	—	124.5**	"	"
Hispanic = White	8.5	7.0	1.5	1.5	1.0	2.2366	—	23.34**	"	"
All = 1	1.0	1.0	1.0	1.0	1.0	25.8836	—	220.64**	2.330	3.258
All =	2.0	2.0	2.0	2.0	2.0	6.2069	—	51.25**	2.487	3.566

*Significance above a 95 percent level; **significance above a 99 percent level.

Table S6. Preference and meeting biases when allowed to vary by school size

Preference parameters	α	γ_a	γ_b	γ_h	γ_w	γ_o	*RSS*	*F*	95% thr.	99% thr.
Ignoring size	0.55	0.90	0.55	0.65	0.75	0.90	4704	—	—	—
Small schools	0.65	0.90	0.75	0.80	0.80	0.90	1685	—	—	—
Large schools	0.55	0.85	0.40	0.45	0.65	0.85	2531	—	—	—
Total error small + large							4216	1.39	2.227	3.063
Small Schools, all =	0.70	0.90	0.90	0.90	0.90	0.90	1831	—	—	—
Large Schools, all =	0.55	0.80	0.80	0.80	0.80	0.80	4338	—	—	—
Total error small + large with all = within sml/lrg							6169	4.17**	2.070	2.769
Meeting parameters	β_a	β_b	β_h	β_w	β_o	—	*RSS*	*F*	95% thr.	99% thr.
Ignoring size	7.0	7.5	2.5	1.0	1.0	—	1.7265	—	—	—
Small schools	6.5	6.0	2.0	1.0	1.0	—	0.9406	—	—	—
Large schools	3.0	9.0	6.5	1.0	1.0	—	0.3688	—	—	—
Total error small + large						—	1.3094	4.71**	2.338	3.275
Small schools, all =	2.0	2.0	2.0	2.0	2.0	—	2.5607	—	—	—
Large schools, all =	2.5	2.5	2.5	2.5	2.5	—	2.8428	—	—	—
Total error small + large with all = within sml/lrg						—	5.4035	28.92**	2.066	2.762

**Significance above a 99 percent level.

Table S7. Preference biases when allowed to vary by the school's county median household income level (low is <30,000 dollars in 1990 census and high is above)—for 78 of the 84 schools for which we have income data

Preference parameters	α	γ_a	γ_b	γ_h	γ_w	γ_o	RSS	*F*	95% thr.	99% thr.
Ignoring income	0.55	0.95	0.55	0.65	0.75	0.90	4255	—	—	—
Low income schools	0.60	1.0	0.50	0.50	0.65	0.95	1541	—	—	—
High income schools	0.35	1.0	0.40	0.70	0.90	0.75	1703	—	—	—
Total error low + high							3244	3.43**	2.227	3.063
Low income schools, all =	0.55	0.75	0.75	0.75	0.75	0.75	2998	—	—	—
High income schools, all =	0.55	0.85	0.85	0.85	0.85	0.85	2076	—	—	—
Total error low + high with all = within low/high							5074	4.65**	2.070	2.769
Meeting parameters	β_a	β_b	β_h	β_w	β_o	—	RSS	*F*	95% thr.	99% thr.
Ignoring income	7.5	7.5	2.5	1.0	1.0	—	1.652	—	—	—
Low income schools	2.0	8.0	3.0	1.0	1.0	—	0.8699	—	—	—
High income schools	5.0	4.5	4.0	1.0	1.0	—	0.7485	—	—	—
Total error low + high						—	1.618	0.282	2.338	3.275
Low income schools, all =	2.0	2.0	2.0	2.0	2.0	—	2.6180	—	—	—
High income schools, all =	2.0	2.0	2.0	2.0	2.0	—	3.1506	—	—	—
Total error low + high with all = within low/high						—	5.7686	21.805**	2.066	2.756

**Significance above a 99 percent level.