Homework #4 - Solutions

1. **Sincich, 4.18**

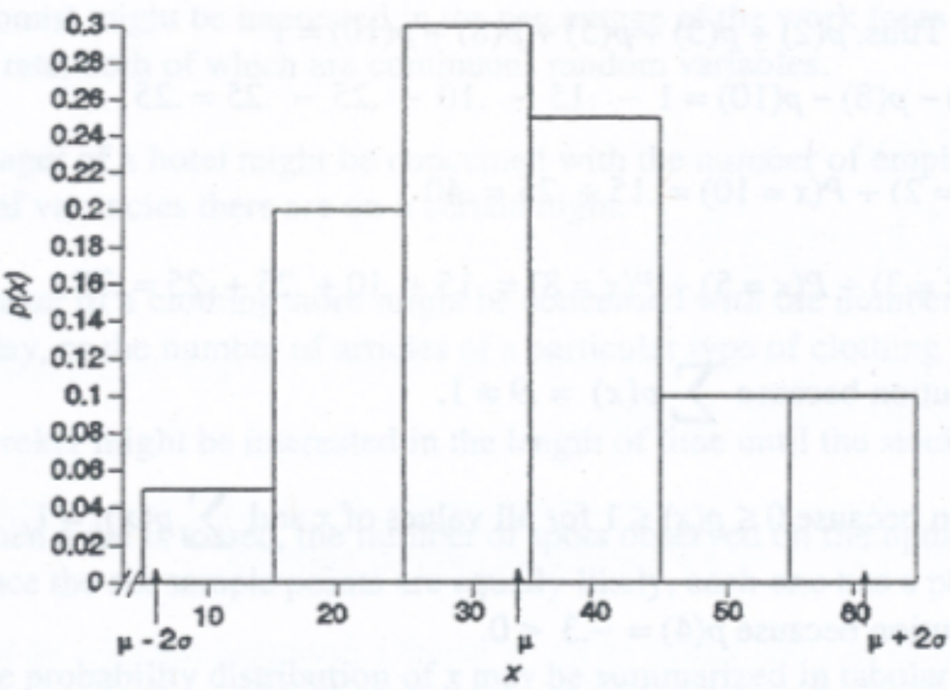
a. $\mu = E(x) = \sum xp(x)$

$$= 10(.05) + 20(.20) + 30(.30) + 40(.25) + 50(.10) + 60(.10)$$
$$= .5 + 4 + 9 + 10 + 5 + 6 = 34.5$$

$$\sigma^2 = E(x-\mu)^2 = \sum (x-\mu)^2 p(x)$$
$$= (10 - 34.5)^2(.05) + (20 - 34.5)^2(.20) + (30 - 34.5)^2(.30) + (40 - 34.5)^2(.25) + (50 - 34.5)^2(.10) + (60 - 34.5)^2(.10)$$
$$= 30.0125 + 42.05 + 6.075 + 7.5625 + 24.025 + 65.025 = 174.75$$

$$\sigma = \sqrt{174.75} = 13.219$$

b.



c. $\mu \pm 2\sigma \Rightarrow 34.5 \pm 2(13.219) \Rightarrow 34.5 \pm 26.438 \Rightarrow (8.062, 60.938)$

$$P(8.062 < x < 60.938) = p(10) + p(20) + p(30) + p(40) + p(50) + p(60)$$
$$= .05 + .20 + .30 + .25 + .10 + .10 = 1.00$$

2. **Sincich, 4.36**

Let x = winnings in the Florida lottery. The probability distribution for x is:

x	$p(x)$
–\$1	22,999,999/23,000,000
\$6,999,999	1/23,000,000

The expected net winnings would be:

$\mu = E(x) = (-1)(22{,}999{,}999/23{,}000{,}000) + 6{,}999{,}999(1/23{,}000{,}000) = -\$.70$

The average winnings of all those who play the lottery is –\$.70.

3. **Sincich, 4.47**

   a. A “success” is a person who does not work during summer vacation.
   b. The sampled individuals are approximately independent, each with the same probability of success.
   c. p = 0.35
   d.

$$P(X=3) = \binom{10}{3}(.35)^3(.65)^7 = 0.252$$

   e.

$$P(X \leq 2) = \binom{10}{0}(.35)^0(.65)^{10} + \binom{10}{1}(.35)^1(.65)^9 + \binom{10}{2}(.35)^2(.65)^8 = 0.262$$

6. **Sincich, 4.81**

   a. This is a poisson random variable with λ=10:

$$P(X=24) = \frac{(10)^{24}}{24!}e^{-10} = 0.000073$$

   b.

$$P(X=23) = \frac{(10)^{23}}{23!}e^{-10} = 0.000176$$

c. Yes, these probabilities are good approximations for the probability of “fire” and “theft.” The researchers estimated these probabilities to be .0001, indicating that these would be extremely rate events. Our probabilities are very close to .0001.

4. **A multiple-choice quiz has 15 questions. Each question has five possible answers, of which only one is correct.**
   (a) What is the probability that sheer guesswork will yield at least 12 correct answers?

$$P(X \geq 12) = \binom{15}{12}(.2)^{12}(.8)^3 + \binom{15}{13}(.2)^{13}(.8)^2 + \binom{15}{14}(.2)^{14}(.8)^1 + \binom{15}{15}(.2)^{15}(.8)^0 = 0.000001$$

   (b) What is the expected number of correct answers by sheer guesswork?

$$(15)(.2) = 3$$