Homework #4 – Due Monday, Oct. 20
COR1-GB.1305 – Statistics and Data Analysis

Problem 1

Find the probability that a standard normal random variable is:

(a) *Greater than zero*

$$P(Z > 0) = 0.5$$

(b) *Greater than* -1.5

$$P(Z > -1.5) = 0.93319$$

(c) *Less than* -0.3

$$P(Z < -0.3) = 0.3821$$

(d) *Between* -2 *and* 1

$$P(-2 \leq Z \leq 1) = 0.8419 - 0.02275 = 0.81915$$

(e) *Equal to* 1.

$$P(Z = 1) = 0$$

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Problem 2

Find a value of a standard normal random variable Z (call it z_0) such that

(a) $P(Z < z_0) = .20$

$$z_0 = -0.8416$$

(b) $P(Z > z_0) = .025$

$$z_0 = 1.96$$

(c) $P(-z_0 < Z < z_0) = .84$

$$z_0 = 1.4051$$

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Problem 3

Suppose that X is normally distributed with mean 11 and standard deviation 2. Find

(a) $P(10 < X < 12)$

$$\begin{aligned} P(10 < X < 12) &= P\left(\frac{10-11}{2} < \frac{X-11}{2} < \frac{12-11}{2}\right) \\ &= P(-0.5 < Z < 0.5) \\ &= 0.3829 \end{aligned}$$

(b) $P(X > 7.6)$.

$$\begin{aligned} P(X > 7.6) &= P\left(\frac{X-11}{2} > \frac{7.6-11}{2}\right) \\ &= P(Z > -1.7) \\ &= 0.04457 \end{aligned}$$

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Problem 4

A Pepsi machine in a Burger King store can be regulated so that it dispenses an average of μ ounces per cup. If the amount dispensed is normally distributed with standard deviation 0.2 *ounces, what should be the setting for μ so that* 8*-ounce cups will overflow only* 1% *of the time?*

Let X be the amount dispensed from the machine. We want to find μ such that $P(X > 8) = 0.01$. Thus,

$$\begin{aligned} P\left(\frac{X-\mu}{0.2} > \frac{8-\mu}{0.2}\right) &= 0.01 \\ P\left(Z > \frac{8-\mu}{0.2}\right) &= 0.01, \end{aligned}$$

so that

$$\frac{8-\mu}{0.2} = 2.3263,$$

and hence

$$\begin{aligned} \mu &= 8 - (0.2)(2.3263) \\ &= 7.53474. \end{aligned}$$

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Problem 5

Suppose that annual stock returns for a particular company are normally distributed with a mean of 16% *and a standard deviation of* 10%*. You are going to invest in this stock for one year. (Note: In reality, annual returns tend to be more nearly normally distributed than daily returns.) Find that the probability that your one-year return will exceed* 30%*.*

Let X be the annual return, in percent. This is a normal random variable with mean $\mu = 16$ and standard deviation $\sigma = 10$. The probability of interest is

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\begin{aligned}
P(X > 30) &= P\left(\frac{X-\mu}{\sigma} > \frac{30-\mu}{\sigma}\right) \\
&= P\left(Z > \frac{30-16}{10}\right) \\
&= P(Z > 1.4) \\
&= 0.08076.
\end{aligned}
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Problem 6

If the population standard deviation is 2.3 *and we take a random sample of size* 64*, what is* $\mathrm{sd}(\bar{X})$*? Note: this quantity is known as the "standard error of the mean."*

$$\mathrm{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{2.3}{\sqrt{64}} = 0.2875.$$

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Problem 7

Suppose that daily returns on a portfolio are independent, with a mean of 0.03% *and a standard deviation of* 1%*. Approximately what is the probability that the average daily return over the next 100 days will be between* 0.2% *and* 0.3%*?*

Let $\bar{X}$ denote the average return over the next 100 days, in percent. Then, by the central limit theorem, $\bar{X}$ is approximately normal with mean and standard deviation

$$\begin{aligned}
\mu_{\bar{X}} &= \mu = 0.03, \\
\sigma_{\bar{X}} &= \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1.
\end{aligned}$$

The probability of interest is

$$\begin{aligned}
P(0.2 < X < 0.3) &= P\left(\frac{0.2 - 0.03}{0.1} < \frac{X - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{0.3 - 0.03}{0.1}\right) \\
&= P(1.7 < Z < 2.7) \\
&= 0.996533 - 0.95543 \\
&= 0.041103.
\end{aligned}$$

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Problem 8

If we throw n dice where n is large, why can we think of the distribution of the sum as being approximately normal?

The sum is equal to $n\bar{X}$, where $\bar{X}$ is the average value of the n rules. By the central limit theorem, $\bar{X}$ is approximately normal if n is large. Further, if we scale a normal random variable by a constant (n), then we get a normal random variable. Thus, $n\bar{X}$, the sum, is approximately normal.

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Problem 9

Suppose that an auto factory has 10 *assembly lines, operating independently. For each assembly line, the number of autos produced per day has a mean of* 20 *and a standard deviation of* 3. *Approximately what is the probability that* 180 *or fewer autos will be produced tomorrow?*

Let $\bar{X}$ be the average number produced by the 10 assembly lines. Then, by the central limit theorem, $\bar{X}$ is approximately normal with mean and standard deviation

$$\begin{aligned} \mu_{\bar{X}} &= 20, \\ \sigma_{\bar{X}} &= \frac{3}{\sqrt{10}} = 0.9487. \end{aligned}$$

To produce a total of 180 or fewer autos, the 10 factories must produce an average of $180/10 = 18$ or fewer. The probability of interest is

$$\begin{aligned} P(\bar{X} < 18) &= P\left(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{18 - 20}{0.9487}\right) \\ &\approx P(Z < -2.1) \\ &= 0.01786. \end{aligned}$$

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