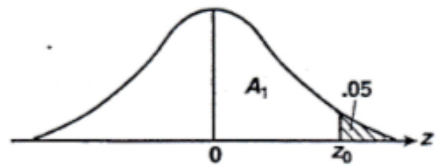
Homework #5 - Solutions

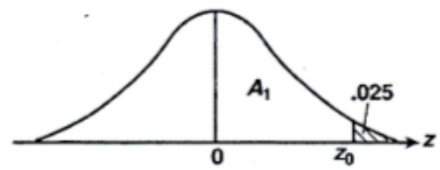
1. Sincich, 4.88

Using Table IV, Appendix B:

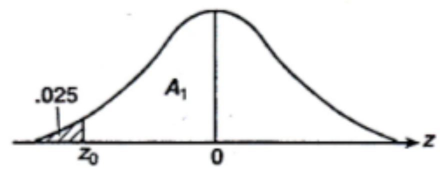
a. $P(z \geq z_0) = .05$
$A_1 = .5 - .05 = .4500$
Looking up the area .4500 in Table IV gives
$z_0 = 1.645$.



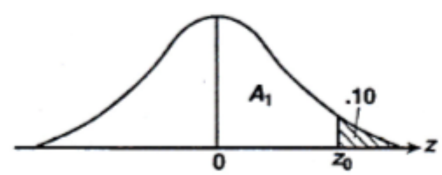
b. $P(z \geq z_0) = .025$
$A_1 = .5 - .025 = .4750$
Looking up the area .4750 in Table IV
gives $z_0 = 1.96$.



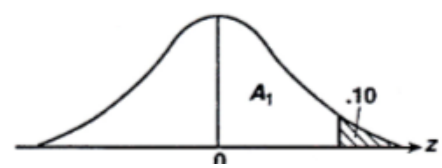
c. $P(z \leq z_0) = .025$
$A_1 = .5 - .025 = .4750$
Looking up the area .4750 in Table IV gives
$z = 1.96$. Since z_0 is to the left of 0, $z_0 = -1.96$.



d. $P(z \geq z_0) = .10$
$A_1 = .5 - .1 = .4$
Looking up the area .4000 in Table IV
gives $z_0 = 1.28$.



e. $P(z > z_0) = .10$
$A_1 = .5 - .1 = .4$
$z_0 = 1.28$ (same as in **d**)



2. Sincich, 4.94

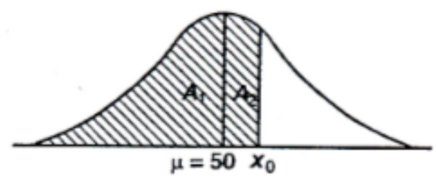
The random variable x has a normal distribution with $\mu = 50$ and $\sigma = 3$.

a. $P(x \le x_0) = .8413$

So, $A_1 + A_2 = .8413$

Since $A_1 = .5$, $A_2 = .8413 - .5 = .3413$.
Look up the area .3413 in the body of Table IV, Appendix B; $z_0 = 1.0$.

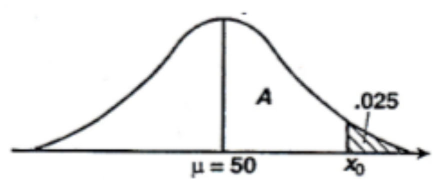
To find x_0, substitute all the values into the z-score formula:



$$z = \frac{x - \mu}{\sigma}$$
$$1.0 = \frac{x_0 - 50}{3}$$
$$x_0 = 50 + 3(1.0) = 53$$

b. $P(x > x_0) = .025$
So, $A = .5000 - .025 = .4750$

Look up the area .4750 in the body of Table IV, Appendix B; $z_0 = 1.96$.

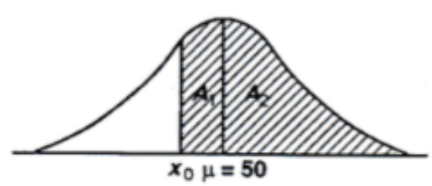


To find x_0, substitute all the values into the z-score formula:

$$z = \frac{x - \mu}{\sigma}$$
$$1.96 = \frac{x_0 - 50}{3}$$
$$x_0 = 50 + 3(1.96) = 55.88$$

c. $P(x > x_0) = .95$

So, $A_1 + A_2 = .95$. Since $A_2 = .5$, $A_1 = .95 - .5 = .4500$.
Look up the area .4500 in the body of Table IV, Appendix B; (since it is exactly between two values, average the z-scores). $z_0 \approx -1.645$.

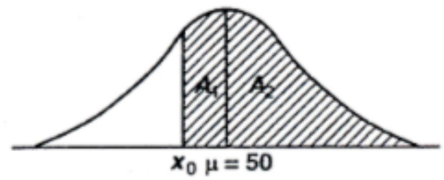


To find x_0, substitute into the z-score formula:

$$z = \frac{x - \mu}{\sigma}$$
$$-1.645 = \frac{x_0 - 50}{3}$$
$$x_0 = 50 - 3(1.645) = 45.065$$

d. $P(41 \le x < x_0) = .8630$

$z = \frac{x-\mu}{\sigma} = \frac{41-50}{3} = -3$



$A_1 = P(41 \le x \le \mu) = P(-3 \le z \le 0)$
$= P(0 \le z \le 3)$
$= .4987$

$A_1 + A_2 = .8630$, since $A_1 = .4987$, $A_2 = .8630 - .4987 = .3643$. Look up .3643 in the body of Table IV, Appendix B; $z_0 = 1.1$.
To find x_0, substitute into the z-score formula:

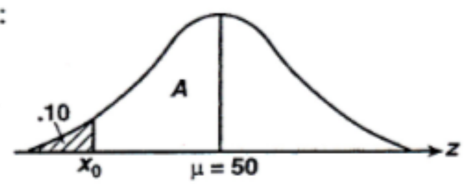
$$z = \frac{x-\mu}{\sigma}$$
$$1.1 = \frac{x_0 - 50}{3}$$
$$x_0 = 50 + 3(1.1) = 53.3$$

e. $P(x < x_0) = .10$

So $A = .5000 - .10 = .4000$

Look up area .4000 in the body of Table IV, Appendix B; $z_0 = 1.28$. Since z_0 is to the left of 0, $z_0 = -1.28$.

To find x_0, substitute all the values into the z-score formula:

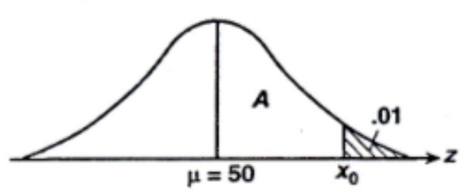


$$z = \frac{x-\mu}{\sigma}$$
$$-1.28 = \frac{x_0 - 50}{3}$$
$$x_0 = 50 - 1.28(3) = 46.16$$

f. $P(x > x_0) = .01$

So $A = .5000 - .01 = .4900$

Look up area .4900 in the body of Table IV, Appendix B; $z_0 = 2.33$.
To find x_0, substitute all the values into the z-score formula:



$$z = \frac{x-\mu}{\sigma}$$
$$2.33 = \frac{x_0 - 50}{3}$$
$$x_0 = 50 + 2.33(3) = 56.99$$

3. Sincich, 4.102. NHTSA crash safety tests.

Let x = driver's head injury rating. The random variable x has a normal distribution with $\mu = 605$ and $\sigma = 185$. Using Table IV, Appendix B,

a. $$P(500 < x < 700) = P\left(\frac{500-605}{185} < z < \frac{700-605}{185}\right) = P(-0.57 < z < 0.51)$$
$$= P(-0.57 < z < 0) + P(0 < z < 0.51) = .2157 + .1950 = .4107$$

b. $$P(400 < x < 500) = P\left(\frac{400-605}{185} < z < \frac{500-605}{185}\right) = P(-1.11 < z < -0.57)$$
$$= P(-1.11 < z < 0) - P(-0.57 < z < 0) = .3665 - .2157 = .1508$$

c. $$P(x < 850) = P\left(z < \frac{850-605}{185}\right) = P(z < 1.32) = .5 + P(0 < z < 1.32)$$
$$= .5 + .4066 = .9066$$

d. $$P(x > 1{,}000) = P\left(z > \frac{1{,}000-605}{185}\right) = P(z > 2.14) = .5 - P(0 < z < 2.14)$$
$$= .5 - .4838 = .0162$$

4. Sincich, 4.114. Industrial filling process.

a. Let x = quantity injected per container. The random variable x has a normal distribution with μ and $\sigma = .2$.

$$P(x < 10) = P\left(z < \frac{10-10}{.2}\right) = P(z < 0.0) = .5$$

$$P(x \geq 10) = P\left(z \geq \frac{10-10}{.2}\right) = P(z \geq 0.0) = .5$$

b. Since the container needed to be reprocessed, it cost \$10. Upon refilling, it contained 10.60 uni with a cost of 10.60(\$20) = \$212. Thus, the total cost for filling this container is \$10 + \$212 = Since the container sells for \$230, the profit is \$230 – \$222 = \$8.

c. Let x = quantity injected per container. The random variable x has a normal distribution with μ 10.10 and $\sigma = .2$. The expected value of x is $E(x) = \mu = 10.10$. The cost of a container with 10. units is 10.10(\$20) = \$202. Thus, the expected profit would be the selling price minus the cost \$230 – \$202 = \$28.

5. **Sincich, 5.18.**

a. $\mu_{\bar{x}} = \mu = 20, \sigma_{\bar{x}} = \sigma / \sqrt{n} = 16 / \sqrt{64} = 2$

b. By the Central Limit Theorem, the distribution of is approximately normal. In order for the Central Limit Theorem to apply, n must be sufficiently large. For this problem, $n = 64$ is sufficiently large.

c. $z = \dfrac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \dfrac{15.5 - 20}{2} = -2.25$

d. $z = \dfrac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \dfrac{23 - 20}{2} = 1.50$

6. **Sincich, 5.24. Salary of a travel management professional.**

(a) 96850
(b) 30000/√50 = 4242.641
(c) approximately normal
(d) z = (89500 - 96850) / (4242.641) = -1.73
(e) 0.0418

7. **Sincich, 5.30. Surface roughness of pipe.**

a. Since the sample size is small, we also have to assume that the distribution from which the sample was drawn is normal. $\mu_{\bar{x}} = \mu = 1.8$, $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{.5}{\sqrt{20}} = .1118$

$$P(\bar{x} \geq 1.85) = P\left(z \geq \frac{1.85 - 1.8}{.1118}\right) = P(z \geq 0.45) = .5 - .1736 = .3264$$

(using Table IV, Appendix B)

b. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Rough

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Rough	20	0	1.881	0.117	0.524	1.060	1.303	2.040	2.293	2.640

From this output, the value of $\bar{x}$ is 1.881.

c. For $\bar{x} = 1.881$:

$$P(\bar{x} \geq 1.881) = P\left(z \geq \frac{1.881 - 1.8}{.1118}\right) = P(z \geq 0.72) = .5 - .1736 = .3264$$

Since this probability is so high, observing a sample mean of $\bar{x} = 1.881$, is not unusual. The assumptions in part a appear to be valid.

8. A Pepsi machine in a Burger King store can be regulated so that it dispenses an average of μ ounces per cup. If the amount dispensed is normally distributed with standard deviation 0.2 ounces, what should be the setting for μ so that 8 ounce cups will overflow only 1% of the time?

Let X be the amount dispensed from the machine. We want to find μ such that $P(X > 8) = 0.01$. Thus,

$$
\begin{aligned}
P\left(\frac{X-\mu}{0.2} > \frac{8-\mu}{0.2}\right) &= 0.01 \\
P\left(Z > \frac{8-\mu}{0.2}\right) &= 0.01,
\end{aligned}
$$

so that

$$\frac{8-\mu}{0.2} = 2.3263,$$

and hence

$$
\begin{aligned}
\mu &= 8 - (0.2)(2.3263) \\
&= 7.53474.
\end{aligned}
$$

. . . .