Homework #2 – Solutions

COR1-GB.1305 – Statistics and Data Analysis

Problem 1

Suppose that in the packaged-cereals industry, 29% of all vice presidents hold MBA degrees, 24% hold undergraduate business degrees, and 8% hold both. A vice president is to be selected at random.

(a) What is the probability that the vice president holds either an MBA or an undergraduate business degree (or both)?

Define the events

$$\begin{aligned} M &= \text{vice president holds an MBA} \\ U &= \text{vice president holds an undergraduate business degree.} \end{aligned}$$

We are told that

$$\begin{aligned} P(M) &= .29 \\ P(U) &= .24 \\ P(M \cap U) &= .08 \end{aligned}$$

The probability that the vice president holds either degree is

$$\begin{aligned} P(M \cup U) &= P(M) + P(U) - P(M \cap U) \\ &= (.29) + (.24) - (.08) \\ &= .45. \end{aligned}$$

(b) What is the probability that the vice president holds neither degree?

Using the answer from the previous part, the probability of holding neither degree is

$$\begin{aligned} P\big((M \cup U)^c\big) &= 1 - P(M \cup U) \\ &= 1 - (.45) \\ &= .55. \end{aligned}$$

(c) What is the probability that the vice president holds an MBA degree but not an undergraduate business degree?

If the vice president holds an MBA degree, then there are two mutually exclusive possibilities: they either hold and undergraduate degree or they do not. Thus,

$$P(M) = P(M \cap U) + P(M \cap U^c).$$

From this equation, it follows that

$$\begin{aligned} P(M \cap U^c) &= P(M) - P(M \cap U) \\ &= (.29) - (.08) \\ &= .21. \end{aligned}$$

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Problem 2

A space agency estimates that the chance of a "critical-item failure" (that is, a catastrophic failure) in their space shuttle's main engines is 2 in 126 for each mission.

(a) What is the probability that at least one of the six shuttle missions scheduled in the next two years results in a critical-item failure?

First, use the complement rule to write

$$P(\text{at least one the six missions fails}) = 1 - P(\text{all six succeed}).$$

Define the events

$$\begin{aligned} S_1 &= \text{first mission succeeds} \\ S_2 &= \text{second mission succeeds} \\ &\vdots \\ S_6 &= \text{sixth mission succeeds.} \end{aligned}$$

Then,

$$P(\text{all six succeed}) = P(S_1 \cap S_2 \cap \cdots \cap S_6).$$

If the failures are independent, then

$$\begin{aligned} P(S_1 \cap S_2 \cap \cdots \cap S_6) &= P(S_1)P(S_2)\cdots P(S_6) \\ &= (\tfrac{124}{126})^6 \\ &= 0.9085. \end{aligned}$$

Thus,

$$\begin{aligned} P(\text{all six succeed} &= 1 - 0.9085 \\ &= 0.0915. \end{aligned}$$

(b) What is the probability that at least one of the 10 shuttle missions scheduled over the next three years results in a critical-item failure?

$$\begin{aligned} P(\text{all 10 succeed}) &= 1 - (\tfrac{124}{126})^{10} \\ &= 0.1479 \end{aligned}$$

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Problem 3

Suppose that the probability that a child born in the US in 2014 will survive past age 80 is 40%, and the probability that he or she will survive past age 90 is 20%. For an 80-year-old who was born in the US in 2014, what is the probability of surviving past age 90?

We are told that

$$P(\text{lifetime} \geq 80) = 0.40,$$
$$P(\text{lifetime} \geq 90) = 0.20.$$

The question is asking for

$$P(\text{lifetime} \geq 90 \mid \text{lifetime} \geq 80).$$

We apply the definition of conditional probability to get

$$\begin{aligned} P(\text{lifetime} \geq 90 \mid \text{lifetime} \geq 80) &= \frac{P(\text{lifetime} \geq 90 \textbf{ and } \text{lifetime} \geq 80)}{P(\text{lifetime} \geq 80)} \\ &= \frac{P(\text{lifetime} \geq 90)}{P(\text{lifetime} \geq 80)} \\ &= \frac{0.20}{0.40} \\ &= 0.50. \end{aligned}$$

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Problem 4

In an article in *The Journal of Portfolio Management*, Goetzmann and Ibbotson selected 1556 mutual funds and recorded their current and past performances. They produced the following table:

	Current Winner	Current Loser	Total
Past Winner	482	296	778
Past Loser	285	493	778
Total	767	789	1556

Here, "Past Loser" indicates that the fund lost value in the distant past, and "Current Loser" indicates that the fund lost value in the recent past. Suppose that you select a random fund from Goetzmann and Ibbotson's study. Let $A = \{\text{Past Loser}\}$, and $B = \{\text{Current Loser}\}$. Based on these data, calculate the following probabilities, and write in words what these probabilities represent.

(a) Find $P(A)$, $P(B)$ and $P(A \cap B)$.

$$\begin{aligned} P(A) &= \frac{778}{1556} \approx 0.5000 \\ P(B) &= \frac{789}{1556} \approx 0.5071 \\ P(A \cap B) &= \frac{493}{1556} \approx 0.3168. \end{aligned}$$

(b) Are the events A and B independent?

We check this by computing the product

$$\begin{aligned} P(A)P(B) &= \frac{778}{1556}\frac{789}{1556} \\ &= \frac{613842}{2421136} \\ &\approx 0.2535. \end{aligned}$$

Since $P(A)P(B) \neq P(A \cap B)$, the events are not independent.

(c) Find $P(B \mid A)$ and $P(B \mid A^c)$. Are they equal?

$$\begin{aligned} P(B \mid A) &= \frac{493}{778} \approx 0.6337 \\ P(B \mid A^c) &= \frac{296}{778} \approx 0.3805. \end{aligned}$$

These quantities are not equal.

(d) Show that $P(B^c) \neq P(B^c \mid A^c)$.

$$P(B^c) = 1 - P(B) = \frac{767}{1556} \approx 0.4929$$
$$P(B^c \mid A^c) = 1 - P(B \mid A^c) = \frac{482}{778} \approx 0.6195.$$

Clearly, $P(B^c) \neq P(B^c \mid A^c)$.

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Problem 5

A survey of workers in the two plants of a manufacturing firm includes the question "How effective is management in responding to legitimate grievances of workers?" In plant 1, 48 of 192 workers respond "poor"; in plant 2, 80 of 248 workers respond "poor". An employee of the manufacturing firm is to be selected randomly.

Let A be the event "worker comes from plant 1" and let B be the event "response is poor."

(a) Find $P(A)$, $P(B)$, and $P(A \mid B)$.

$$\begin{aligned} P(A) &= 192/(192+248) \\ &= 192/440 \\ &= 0.4363, \end{aligned}$$

$$\begin{aligned} P(B) &= (48+80)/(192+248) \\ &= 128/440 \\ &= 0.2909, \end{aligned}$$

$$\begin{aligned} P(A \mid B) &= 48/(48+80) \\ &= 48/128 \\ &= 0.3750. \end{aligned}$$

Alternative solution, using Bayes' Rule:

$$\begin{aligned} P(A \mid B) &= P(B \mid A)P(A)/P(B) \\ &= (48/192)(192/440)/(128/440) \\ &= 48/128 \\ &= 0.3750. \end{aligned}$$

(b) Are the events A and B independent?

No, since $P(A) \neq P(A \mid B)$.

(c) Find $P(B \mid A)$ and $P(B \mid A^c)$. Are they equal?

$$\begin{aligned} P(B \mid A) &= 48/192 \\ &= 0.2500 \end{aligned}$$

$$\begin{aligned} P(B \mid A^c) &= 80/248 \\ &= 0.3226. \end{aligned}$$

These quantities are unequal.

(d) Show that $P(B^c) \neq P(B^c \mid A^c)$.

$$\begin{aligned} P(B^c) &= 1 - P(B) \\ &= 312/440 \\ &= 0.7091, \end{aligned}$$

$$\begin{aligned} P(B^c \mid A^c) &= 1 - P(B \mid A^c) \\ &= 168/248 \\ &= 0.6774. \end{aligned}$$

These quantities are unequal.

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