

### Problem 1: Sincich 6.1

- a. For  $\alpha = .10$ ,  $\alpha/2 = .10/2 = .05$ .  $z_{\alpha/2} = z_{.05}$  is the z-score with .05 of the area to the right of it. The area between 0 and  $z_{.05}$  is  $.5 - .05 = .4500$ . Using Table IV, Appendix B,  $z_{.05} = 1.645$ .
- b. For  $\alpha = .01$ ,  $\alpha/2 = .01/2 = .005$ .  $z_{\alpha/2} = z_{.005}$  is the z-score with .005 of the area to the right of it. The area between 0 and  $z_{.005}$  is  $.5 - .005 = .4950$ . Using Table IV, Appendix B,  $z_{.005} = 2.575$ .
- c. For  $\alpha = .05$ ,  $\alpha/2 = .05/2 = .025$ .  $z_{\alpha/2} = z_{.025}$  is the z-score with .025 of the area to the right of it. The area between 0 and  $z_{.025}$  is  $.5 - .025 = .4750$ . Using Table IV, Appendix B,  $z_{.025} = 1.96$ .
- d. For  $\alpha = .20$ ,  $\alpha/2 = .20/2 = .10$ .  $z_{\alpha/2} = z_{.10}$  is the z-score with .10 of the area to the right of it. The area between 0 and  $z_{.10}$  is  $.5 - .10 = .4000$ . Using Table IV, Appendix B,  $z_{.10} = 1.28$ .

### Problem 2: Sincich 6.4

- a. For confidence coefficient .95,  $\alpha = .05$  and  $\alpha/2 = .05/2 = .025$ . From Table IV, Appendix B,  $z_{.025} = 1.96$ . The confidence interval is:

$$\bar{x} \pm z_{.025} \frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 1.96 \frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .56 \Rightarrow (25.34, 26.46)$$

- b. For confidence coefficient .90,  $\alpha = .10$  and  $\alpha/2 = .10/2 = .05$ . From Table IV, Appendix B,  $z_{.05} = 1.645$ . The confidence interval is:

$$\bar{x} \pm z_{.05} \frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 1.645 \frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .47 \Rightarrow (25.43, 26.37)$$

- c. For confidence coefficient .99,  $\alpha = .01$  and  $\alpha/2 = .01/2 = .005$ . From Table IV, Appendix B,  $z_{.005} = 2.58$ . The confidence interval is:

$$\bar{x} \pm z_{.005} \frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 2.58 \frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .73 \Rightarrow (25.17, 26.63)$$

**Problem 3: Sincich, 6.9**

Yes. As long as the sample size is sufficiently large, the Central Limit Theorem says the distribution of  $\bar{x}$  is approximately normal regardless of the original distribution.

**Problem 4: Sincich, 6.14**

- a. (1.6711, 2.1989)
- b. With 95% confidence, the mean wear-out failure time of all used colored display panels is between 1.67 years and 2.20 years
- c. 95%

### Problem 5: Sincich, 6.17

- a. The mean 2011 salary of all 500 CEOs
- b. (Minitab; everyone will get a different sample)
- c. Here is the Minitab output:

#### Descriptive Statistics: Sample

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Sample	49	1	9.63	1.27	8.87	0.04	3.84	6.36	14.84	38.94

The sample mean in my sample is 9.63 (everyone will get a different answer here).

- d. Here is the output:

#### Descriptive Statistics: 1-Year Pay (\$mil)

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
1-Year Pay (\$mil)	478	57	9.247	0.450	9.842	0.000	3.413	6.100	11.346	101.965

We can see StDev = 9.842

- e. My sample size is  $n=49$  since there is one missing value. My 99% confidence interval for the population mean is

$$(9.63) \pm (2.576) (9.842) / \sqrt{(49)} = 9.63 \pm 3.62 = (6.01, 13.25)$$

- f. With 99% confidence the mean salary of all 500 CEOs is in the range (6.01, 13.25).

- g. The true mean is 9.247. This is in the confidence interval. (This will happen for 99% of all confidence intervals)

## Problem 6: Sincich 6.27

First, we must compute  $\bar{x}$  and  $s$ .

$$\bar{x} = \frac{\sum x}{n} = \frac{30}{6} = 5$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{176 - \frac{(30)^2}{6}}{6-1} = \frac{26}{5} = 5.2$$

$$s = \sqrt{5.2} = 2.2804$$

- a. For confidence coefficient .90,  $\alpha = 1 - .90 = .10$  and  $\alpha/2 = .10/2 = .05$ . From Table V, Appendix B, with  $df = n - 1 = 6 - 1 = 5$ ,  $t_{.05} = 2.015$ . The 90% confidence interval is:

$$\bar{x} \pm t_{.05} \frac{s}{\sqrt{n}} \Rightarrow 5 \pm 2.015 \frac{2.2804}{\sqrt{6}} \Rightarrow 5 \pm 1.88 \Rightarrow (3.12, 6.88)$$

- b. For confidence coefficient .95,  $\alpha = 1 - .95 = .05$  and  $\alpha/2 = .05/2 = .025$ . From Table V, Appendix B, with  $df = n - 1 = 6 - 1 = 5$ ,  $t_{.025} = 2.571$ . The 95% confidence interval is:

$$\bar{x} \pm t_{.025} \frac{s}{\sqrt{n}} \Rightarrow 5 \pm 2.571 \frac{2.2804}{\sqrt{6}} \Rightarrow 5 \pm 2.39 \Rightarrow (2.61, 7.39)$$

- c. For confidence coefficient .99,  $\alpha = 1 - .99 = .01$  and  $\alpha/2 = .01/2 = .005$ . From Table V, Appendix B, with  $df = n - 1 = 6 - 1 = 5$ ,  $t_{.005} = 4.032$ . The 99% confidence interval is:

$$\bar{x} \pm t_{.005} \frac{s}{\sqrt{n}} \Rightarrow 5 \pm 4.032 \frac{2.2804}{\sqrt{6}} \Rightarrow 5 \pm 3.75 \Rightarrow (1.25, 8.75)$$

- d. a) For confidence coefficient .90,  $\alpha = 1 - .90 = .10$  and  $\alpha/2 = .10/2 = .05$ . From Table V, Appendix B, with  $df = n - 1 = 25 - 1 = 24$ ,  $t_{.05} = 1.711$ . The 90% confidence interval is:

$$\bar{x} \pm t_{.05} \frac{s}{\sqrt{n}} \Rightarrow 5 \pm 1.711 \frac{2.2804}{\sqrt{25}} \Rightarrow 5 \pm .78 \Rightarrow (4.22, 5.78)$$

- b) For confidence coefficient .95,  $\alpha = 1 - .95 = .05$  and  $\alpha/2 = .05/2 = .025$ . From Table V, Appendix B, with  $df = n - 1 = 25 - 1 = 24$ ,  $t_{.025} = 2.064$ . The 95% confidence interval is:

$$\bar{x} \pm t_{.025} \frac{s}{\sqrt{n}} \Rightarrow 5 \pm 2.064 \frac{2.2804}{\sqrt{25}} \Rightarrow 5 \pm .94 \Rightarrow (4.06, 5.94)$$

- c) For confidence coefficient .99,  $\alpha = 1 - .99 = .01$  and  $\alpha/2 = .01/2 = .005$ . From Table V, Appendix B, with  $df = n - 1 = 25 - 1 = 24$ ,  $t_{.005} = 2.797$ . The 99% confidence interval is:

$$\bar{x} \pm t_{.005} \frac{s}{\sqrt{n}} \Rightarrow 5 \pm 2.797 \frac{2.2804}{\sqrt{25}} \Rightarrow 5 \pm 1.28 \Rightarrow (3.72, 6.28)$$

Increasing the sample size decreases the width of the confidence interval.

### Problem 7: Sincich, 6.32

- a.  $(3.8) \pm (1.729) (1.2) / \sqrt{(20)} = 3.8 \pm 0.5 = (3.3, 4.3)$
- b. With 90% confidence, the mean LOS for all hospitals in the state is between 3.3 and 4.3 days.
- c. If we were to sample another 20 hospitals from the same state and construct a confidence interval in the same way, then there would be a 90% chance that the new interval would contain the true population mean.

### Problem 8

- a. The body temperatures of all humans.
- b. Minitab output:

### One-Sample T: Temp

Variable	N	Mean	StDev	SE Mean	95% CI
Temp	130	98.2492	0.7332	0.0643	(98.1220, 98.3765)

- c. We have independent samples from the population (i.e., that our sample is unbiased).
- d. It is somewhat surprising that 98.6 is not in the confidence interval. However, not every confidence interval contains its parameter: 5% of all 95% confidence intervals do not contain their parameters.