# Homework #6 – Solutions

COR1-GB.1305 – Statistics and Data Analysis

## Problem 1

*Consider (again) the time it takes for a call center to answer its calls. The call center claims that the mean time to answer a call is* 3 *minutes. In a random sample of* 7 *calls, the average time for the call center to answer was* 191 *seconds, with a sample standard deviation of* 11.4 *seconds.*

(a) *What is the interpretation of the population mean, $\mu$?*

The expected time it takes for the call center to answer a call, in seconds. (Equivalently: the average of all of the times it takes the call center to answer its calls.)

(b) *Provide the null and alternative hypotheses for testing the call center's claim.*

$$H_0 : \mu = 180$$
$$H_a : \mu \neq 180.$$

(Note: we did not cover one-sided alternatives in class, but if you know about them, it would be acceptable to use $H_a : \mu > 180$. This will change your answer for the $p$-value in part (d).)

(c) *Compute the test statistic.*

The sample size is $n = 7$. The sample mean and standard deviation are $\bar{x} = 191$ and $s = 11.4$. The hypothesized null mean is $\mu_0 = 180$.

$$\begin{aligned} t &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{191 - 180}{11.4/\sqrt{7}} \\ &= 2.55 \end{aligned}$$

(d) *Compute the p-value.*

We will approximate the $p$-value using a $z$ table:

$$\begin{aligned} p &\approx P(|Z| > 2.55) \\ &= 0.009322 \end{aligned}$$

Note that the value $z = 2.55$ is not in the $Z$-table, so we use the value for $z = 2.6$ instead. It would be acceptable to use $z = 2.5$ instead, in which case you would get $p \approx 0.01242$.

(A more accurate solution would use a $t$ distribution with $n - 1 = 6$ degrees of freedom; this gives a $p$-value of $p = 0.04349$.)

(e) *Test the call center's claim, at the 1% level of significance.*

Since $p \geq 0.01$, we do not reject $H_0$. The data is consistent with the claim.

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## Problem 2

*Recall the study of* 80 *students who used a private tutor to help them improve their* SAT *scores. Their score on the mathematical section improved by an average of* 11 *points, with a sample standard deviation of* 65 *points.*

(a) *Is there evidence, at the 5% level of significance, that tutoring affects the math score?*

Let $\mu$ be the expected increase in SAT math scores for those students who receive the tutoring services. The null and alternative hypotheses are

$$\begin{aligned} H_0 &: \mu = 0 \\ H_0 &: \mu \neq 0 \end{aligned}$$

Here, the hypothesized null mean is $\mu_0 = 0$, which corresponds to no change in expected score (expected increase of 0).

The same is the scores of the $n = 80$ students. The sample mean and standard deviation are $\bar{x} = 11$, $s = 65$. The test statistic is

$$\begin{aligned} t &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{11 - 0}{65/\sqrt{80}} \\ &= 1.51 \end{aligned}$$

Since $|t| < 1.96$, the $p$-value is above 0.05. We do not reject the null hypothesis; there is no evidence that tutoring affects the math score.

(b) *Compute the p-value corresponding to the hypothesis test. Interpret the p-value. What does it tell us about whether tutoring affects the math score?*

$$\begin{aligned} p &\approx P(|Z| > 1.51) \\ &\approx 0.1336 \end{aligned}$$

If tutoring affected the math score, then the chance of seeing data like we observe would be 13.4%. In other words, the data is consistent with the hypothesis that the tutoring does not have an affect.

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## Problem 3

*Use the data in `Market.CSV` to test whether IBM has a different mean return than the market. To do this, first use Calculator to create a variable called `IBMedge`, defined as `IBMRet - MarketReturn`.*

(a) *What is the sample?*

The $n = 11537$ differences in daily returns between IBM and the market between July 1, 1963 and April 30, 2009.

(b) *What is the population?*

All (past and future) differences in daily returns between IBM and the market.

(c) *What is the interpretation of the population mean, $\mu$?*

The long run average difference in the daily return between IBM and the market.

(d) *Formulate the null and alternative hypotheses.*

$$H_0 : \mu = 0$$
$$H_a : \mu \neq 0$$

(e) *Run descriptive statistics for `IBMedge`. Use the information provided to compute the $t$-statistic and the $p$-value.*

Here are the descriptive statistics:

| Variable | N | N* | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 | Maximum |
|---|---|---|---|---|---|---|---|---|---|---|
| IBMedge | 11537 | 0 | 0.0078 | 0.0120 | 1.2893 | -14.8278 | -0.6420 | -0.0189 | 0.6179 | 12.0165 |

The $t$-statistic is

$$\begin{aligned} t &= \frac{0.0078 - 0}{1.2893/\sqrt{11537}} \\ &= 0.655 \end{aligned}$$

The $p$-value is

$$\begin{aligned} p &\approx P(|Z| > 0.655) \\ &\approx P(|Z| > 0.7) \\ &= .4839 \end{aligned}$$

(It would also be OK to approximate this as $P(|Z| > 0.6) = .5485$)

(f) *What is the result of the test, at the 5% level of significance?*

We do not reject $H_0$.

(g) *Interpret the p-value in the context of the question as to whether IBM beats the market.*

If the long run average difference in returns between IBM and the market were equal to zero, then there would be a 48.4% chance of seeing data like what we observed. In other words, there is no evidence that IBM beats the market (or differs from the market) in the long run.

(h) *Now, let Minitab do the hypothesis test using* Stat ⇒ Basic Statistics ⇒ 1-Sample T*, with options* One or more samples, each in a column: `IBMedge`*, and specifying* perform hypothesis test. *(You need to specify the null hypothesis, and you may need to use options to set the confidence level [*100(1 − *significance level*)*], and the alternative hypothesis.) Get the p-value from Minitab. Compare with your answer in (e).*

Here is the Minitab output:

```
Test of mu = 0 vs not = 0


Variable      N    Mean   StDev  SE Mean         95% CI            T      P
IBMedge   11537  0.0078  1.2893   0.0120  (-0.0157, 0.0314)  0.65  0.515
```

The $p$-value is slightly more accurate than the one we computed above, because Minitab does not have to rely on the $Z$-table.

(i) *Does the fact that stock returns are not normally distributed have any impact on the validity of the t-test and corresponding p-value? Explain.*

Since the sample size is large ($n \geq 30$), we do not need the stock returns to be normally distributed for the test to be valid.

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## Problem 4

*Recall the data set* `NormTemp.CSV`*. The first column (*`Temp`*) contains the body temperatures of 130 randomly selected subjects. Use Minitab's one-sample t to get the p-value corresponding to the null hypothesis that the mean temperature is* 98.6 *degrees Fahrenheit. Interpret the p-value. In the end, does the* 98.6 *"normal temperature" seem to be folklore or fact?*

Here is the Minitab output. Note that we have to specify 98.6 when we ask for the $t$-test:

```
Test of mu = 98.6 vs not = 98.6


Variable    N     Mean   StDev  SE Mean         95% CI              T      P
Temp      130  98.2492  0.7332   0.0643  (98.1220, 98.3765)  -5.45  0.000
```

If the average body temperature in the population were equal to 98.6, then it would be very unusual to see data like we observed (this would happen less than 0.1% of the time). The "normal temperature" seems to be folklore (at least for the population that the sample was drawn from).

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