

**Homework 6 – Due Tuesday, Apr. 14**  
STAT-GB.2302, STAT-UB.0018: Forecasting Time Series Data

The data sets chaos1 and chaos2 (available on the course website) were generated with  $n = 50$  by iterating the “tent map”,

$$f(x) = \begin{cases} x/0.6 & \text{if } 0 \leq x \leq 0.6, \\ (1-x)/0.4 & \text{if } 0.6 < x \leq 1. \end{cases}$$

We used  $x_0 = 0.500$  for chaos1 and  $x_0 = 0.501$  for chaos2.

### Problem 1

Check that  $x_1 = f(x_0)$ , where  $\{x_t\}$  is the series of observations on chaos1, and  $f$  is the function defined above.

### Problem 2

Plot chaos1 and chaos2 in separate plots. Do the series look random? Are they in fact random? Do the series look stationary?

### Problem 3

Plot the ACF and PACF for chaos1. Based on these, suggest an ARMA model. Would this model provide the best possible forecasts?

### Problem 4

Plot both chaos1 and chaos2 on the same plot. Do the paths look similar? Should they look similar when  $t$  is close to 1? What should happen if chaos1 and chaos2 happen to get very close together at some later time? Use the plot to help justify your answer.

### Problem 5

Plot  $x_2, \dots, x_{50}$  versus  $x_1, \dots, x_{49}$ , where  $\{x_t\}$  is the series of observations on chaos1. Does this plot reveal the map (in other words, the function  $f$ ) which generated the data? Do you see why this  $f$  is called the tent map? Does this plot help us to see that  $\{x_t\}$  is not an AR(1) series? How?