Conditional Probability 2

COR1-GB.1305 – Statistics and Data Analysis

The Birthday Problem

1. A class has 50 students. What is the probability that at least two students have the same birthday? Assume that each person in the class was assigned a random birthday between January 1 and December 31.

Independence

2. Suppose that you flip two fair coins. Let $A =$"the first coin shows Heads," $B =$ "The second coin shows Heads." Find the probability of getting Heads on both coins, i.e. find $P(A \cap B)$.

3. Suppose that you roll two dice. What is the probability of getting exactly one 6?

4. Suppose that you sell fire insurance policies to two different buildings in Manhattan, located in different neighborhoods. You estimate that the buildings have the following chances of being damaged by fire in the next 10 years: 5%, and 1%. Assume that fire damages to the three buildings are independent events. Compute the probability that exactly one building gets damaged by fire in the next 10 years.

5. Consider the following experiment. A hat contains two coins:

- one coin, the "fair" coin, has 50% chance of heads and 50% chance of tails on every flip;
- the other coin, the "heads" coin, has heads on both sides, so it always lands heads on every flip.

You reach into the hat and pull out a random coin, equally likely to get the fair coin or the heads coin. Then, you flip this coin twice.

Define events A and B as

$$\begin{aligned} A &= \text{the first flip lands heads} \\ B &= \text{the second flip lands heads.} \end{aligned}$$

(a) Based on your intuition, do you think that A and B independent events?

(b) Compute $P(A)$.

(c) Compute $P(A \cap B)$.

(d) Use your answers to parts (b) and (c) to either prove or disprove that A and B are independent.

Bayes' Rule

6. Every year in March there is a standardized exam for people who want to be licensed sheep herders. It happens that, with probability 0.45, a person will pass this exam. In the process of screening people, it turns out that among those who passed the exam, 60% had taken college courses in biology. It happens also that 30% of all those who take the exam had college courses in biology. Find the probability that a person with college courses in biology will pass the exam.

7. Amazon.com maintains a list of all registered customers, along with their email addresses. During July, they sent coupons to 20% of their customers. They recorded that 5% of their customers made purchases in July, and 40% of all purchases were made with coupons. In this problem we will compute the proportion of customers sent a coupon in July who made a purchase in that month. For simplicity, we will assume that customers either make 0 or 1 purchases in July.

   (a) Consider a random customer, and define two events:

   $$\text{Coupon} = \text{the customer received a coupon in July},$$
   $$\text{Purchase} = \text{the customer made a purchase in July}.$$

   Express all percentages given in the problem statement as probabilities or conditional probabilities of these two events. Example: $\text{P(Coupon)} = 0.20$.

   (b) Use Bayes' rule to compute the proportion of custoers sent a coupon in July who made a purchase that month.