# Homework #3 - Solutions

1. **Sincich 3.49.**

a. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.1}{.2} = .5$

b. $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.1}{.4} = .25$

c. Events A and B are said to be independent if $P(A|B) = P(A)$. In this case, $P(A|B) = .5$ and $P(A) = .4$. Thus, A and B are not independent.

2. **Sincich 3.53.**

a. $$\begin{aligned} P(A) &= P(E_1) + P(E_2) + P(E_3) \\ &= .2 + .3 + .3 \\ &= .8 \end{aligned}$$

$$\begin{aligned} P(B) &= P(E_2) + P(E_3) + P(E_5) \\ &= .3 + .3 + .1 \\ &= .7 \end{aligned}$$

$$\begin{aligned} P(A \cap B) &= P(E_2) + P(E_3) \\ &= .3 + .3 \\ &= .6 \end{aligned}$$

b. $$P(E_1 \mid A) = \frac{P(E_1 \cap A)}{P(A)} = \frac{P(E_1)}{P(A)} = \frac{.2}{.8} = .25$$

$$P(E_2 \mid A) = \frac{P(E_2 \cap A)}{P(A)} = \frac{P(E_2)}{P(A)} = \frac{.3}{.8} = .375$$

$$P(E_3 \mid A) = \frac{P(E_3 \cap A)}{P(A)} = \frac{P(E_3)}{P(A)} = \frac{.3}{.8} = .375$$

The original sample point probabilities are in the proportion .2 to .3 to .3 or 2 to 3 to 3.

The conditional probabilities for these sample points are in the proportion .25 to .375 to .375 or 2 to 3 to 3.

c. (1) $$\begin{aligned} P(B \mid A) &= P(E_2 \mid A) + P(E_3 \mid A) \\ &= .375 + .375 \quad \text{(from part } \mathbf{b}\text{)} \\ &= .75 \end{aligned}$$

(2) $$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{.6}{.8} = .75 \quad \text{(from part } \mathbf{a}\text{)}$$

The two methods do yield the same result.

d. If $A$ and $B$ are independent events, $P(B \mid A) = P(B)$.

From part **c**, $P(B \mid A) = .75$. From part **a**, $P(B) = .7$.

Since $.75 \neq .7$, $A$ and $B$ are not independent events.

## 3. Sincich 3.76: Software defects in NASA spacecraft instrument code.

Define the following events:

$A$: {Algorithm predicts defects}

$B$: {Module has defects}

$C$: {Algorithm is correct}

a. $Acccuracy = P(C) = P(A \cap B) + P(A^c \cap B^c) = \frac{a}{a+b+c+d} + \frac{d}{a+b+c+d} = \frac{a+d}{a+b+c+d}$

b. $Detection\ rate = P(A \mid B) = \frac{d}{b+d}$

c. $False\ alarm = P(A \mid B^c) = \frac{c}{a+c}$

d. $Precision = P(B \mid A) = \frac{d}{c+d}$

e. From the SWDEFECTS file the table is:

| | | *Module has Defects* | |
|---|---|---|---|
| | | False | True |
| *Algorithm Predicts Defects* | No | 400 | 29 |
| | Yes | 49 | 20 |

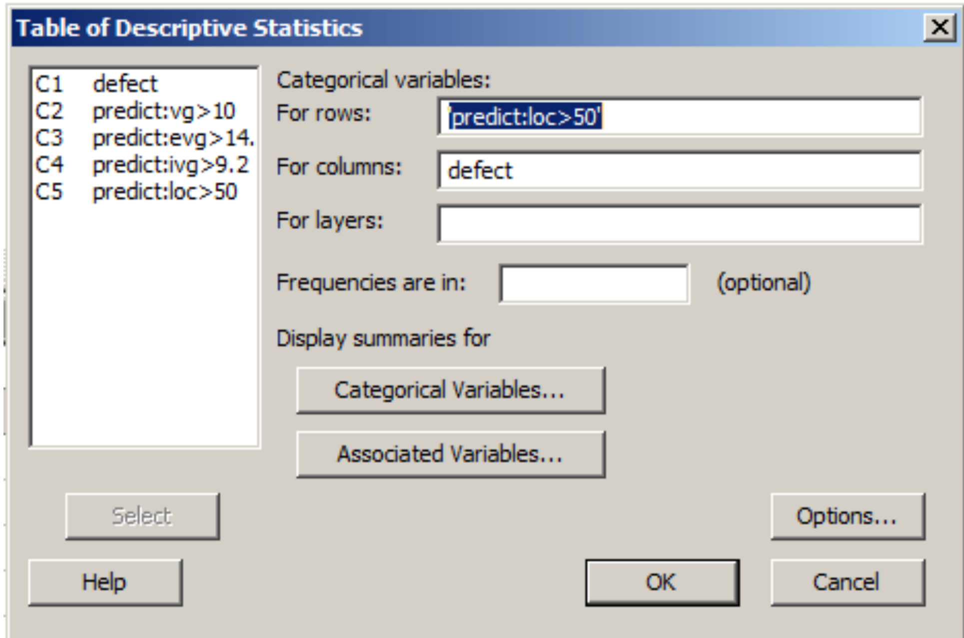
$$Acccuracy = P(C) = P(A \cap B) + P(A^c \cap B^c)$$
$$= \frac{a}{a+b+c+d} + \frac{d}{a+b+c+d} = \frac{a+d}{a+b+c+d} = \frac{400+20}{400+29+49+20} = \frac{420}{498} = .843$$

$$Detection\ rate = P(A \mid B) = \frac{d}{b+d} = \frac{20}{29+20} = \frac{20}{49} = .408$$

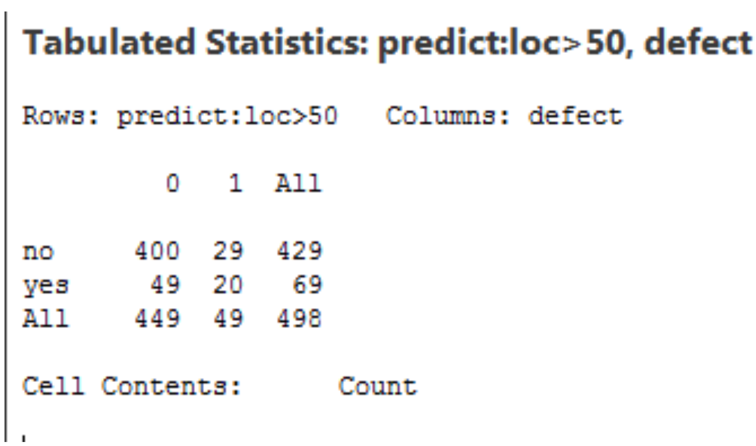
$$False\ alarm = P(A \mid B^c) = \frac{c}{a+c} = \frac{49}{400+49} = \frac{49}{449} = .109$$

$$Precision = P(B \mid A) = \frac{d}{c+d} = \frac{20}{49+20} = \frac{20}{69} = .290$$

To make the table in Minitab, enter use the Stat ⇒ Tables ⇒ Descriptive Statistics command as in the following screenshot:




The Minitab output should be as follows:



**Tabulated Statistics: predict:loc>50, defect**

```
Rows: predict:loc>50   Columns: defect

          0   1  All

no      400  29  429
yes      49  20   69
All     449  49  498

Cell Contents:      Count
```

## 4. Sincich 3.87: Purchasing microchips.

$$P(S_6 \mid D) = \frac{P(S_6 \cap D)}{P(D)} = \frac{P(D \mid S_6)P(S_6)}{P(D)} = \frac{.0002(.20)}{.001679} = \frac{.00004}{.001679} = .0238$$

$$P(S_7 \mid D) = \frac{P(S_7 \cap D)}{P(D)} = \frac{P(D \mid S_7)P(S_7)}{P(D)} = \frac{.001(.18)}{.001679} = \frac{.00018}{.001679} = .1072$$

Of these probabilities, .7147 is the largest. This implies that if a failure is observed, supplier number 4 was most likely responsible.

b. If the seven suppliers all produce defective chips at the same rate of .0005, then $P(D|S_i)$ =.0005 for all $i = 1, 2, 3, \ldots 7$ and $P(D) = .0005$.

For any supplier i, $P(S_i \cap D) = P(D \mid S_i)P(S_i) = .0005P(S_i)$ and

$$P(S_i \mid D) = \frac{P(S_i \cap D)}{P(D)} = \frac{P(D \mid S_i)P(S_i)}{.0005} = \frac{.0005P(S_i)}{.0005} = P(S_i)$$

Thus, if a defective is observed, then it most likely came from the supplier with the largest proportion of sales (probability). In this case, the most likely supplier would be either supplier 4 or supplier 6. Both of these have probabilities of .20.

Define the following event:

$D$: {Chip is defective}

From the Exercise, $P(S_1) = .15$, $P(S_2) = .05$, $P(S_3) = .10$, $P(S_4) = .20$, $P(S_5) = .12$, $P(S_6) = .20$, and $P(S_7) = .18$. Also, $P(D|S_1) = .001$, $P(D|S_2) = .0003$, $P(D|S_3) = .0007$, $P(D|S_4) = .006$, $P(D|S_5) = .0002$, $P(D|S_6) = .0002$, and $P(D|S_7) = .001$.

a. We must find the probability of each supplier given a defective chip.

$$P(S_1 \mid D) = \frac{P(S_1 \cap D)}{P(D)} =$$

$$\frac{P(D \mid S_1)P(S_1)}{P(D \mid S_1)P(S_1) + P(D \mid S_2)P(S_2) + P(D \mid S_3)P(S_3) + P(D \mid S_4)P(S_4) + P(D \mid S_5)P(S_5) + P(D \mid S_6)P(S_6) + P(D \mid S_7)P(S_7)}$$

$$= \frac{.001(.15)}{.001(.15) + .0003(.05) + .0007(.10) + .006(.20) + .0002(.12) + .0002(.02) + .001(.18)}$$

$$= \frac{.00015}{.00015 + .000015 + .00007 + .0012 + .000024 + .00004 + .00018} = \frac{.00015}{.001679} = .0893$$

$$P(S_2 \mid D) = \frac{P(S_2 \cap D)}{P(D)} = \frac{P(D \mid S_2)P(S_2)}{P(D)} = \frac{.0003(.05)}{.001679} = \frac{.000015}{.001679} = .0089$$

$$P(S_3 \mid D) = \frac{P(S_3 \cap D)}{P(D)} = \frac{P(D \mid S_3)P(S_3)}{P(D)} = \frac{.0007(.10)}{.001679} = \frac{.00007}{.001679} = .0417$$

$$P(S_4 \mid D) = \frac{P(S_4 \cap D)}{P(D)} = \frac{P(D \mid S_4)P(S_4)}{P(D)} = \frac{.006(.20)}{.001679} = \frac{.0012}{.001679} = .7147$$

$$P(S_5 \mid D) = \frac{P(S_5 \cap D)}{P(D)} = \frac{P(D \mid S_5)P(S_5)}{P(D)} = \frac{.0002(.12)}{.001679} = \frac{.000024}{.001679} = .0143$$

**5. A survey of workers in the two plants of a manufacturing firm includes the question "How effective is management in responding to legitimate grievances of workers?" In plant 1, 48 of 192 workers respond "poor"; in plant 2, 80 of 248 workers respond "poor." An employee of the manufacturing firm is to be selected randomly.**

**Let *A* be the event "worker comes from plant 1" and let *B* be the event "response is 'poor'."**

**(a) Find *P(A)*, *P(B)*, and *P(A|B)*.**

$$P(A) = 192/(192 + 248) = 192/440 = 0.4363$$

$$P(B) = (48 + 80)/(192 + 248) = 128/440 = 0.2909$$

$$P(A|B) = 48/(48 + 80) = 48/128 = 0.3750$$

Alternative solution, using Bayes' Rule:

$$P(A|B) = P(B|A)\, P(A) / P(B) = (48/192)\,(192/440) / (128/440) = 48/128 = 0.3750$$

**(b) Are the events *A* and *B* independent?**

No, since *P(A)* is not equal to *P(A|B).*

**(c) Find *P(B|A)* and *P(B|A$^c$)*. Are they equal?**

$$P(B|A) = 48/192 = 0.2500$$

$$P(B|A^c) = 80/248 = 0.3226$$

These quantities are unequal.

**(d) Show that $P(B^c) \neq P(B^c|A^c)$**

$$P(B^c) = 1 - P(B) = 312/440 = 0.7091$$

$$P(B^c|A^c) = 1 - P(B|A^c) = 168/248 = 0.6774$$

These quantities are unequal.