Lecture 1: The L'Hopital's Rule

Contents

- The Quotients rule for limits.
- Indeterminate forms

$$0/0, \infty/\infty$$

  and L'Hospital rule

Objectives

- Identify and apply intermediate forms and L'Hospital rule.
- Distinguish the Quotients rule for limits and L'Hospital rule.

Materials in the textbook: Section 4.5 (Examples 1-3)

Recall:

Theorem: The quotient rule for limits

If L, M and c are real numbers and $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M},$$

provided $M \neq 0$.

Example 1

Find the limit of $\lim_{x\to 0} \dfrac{\sin(x)}{x+1}$.

Example 2

Find $\lim_{x\to 1} \dfrac{3x-3}{3x-1}$.

L'Hospital's rule

L'Hospital's rule

For functions f and g which are differentiable on an open interval I except possibly at a point c contained in I, if

- $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$,
- $g'(x) \neq 0$ for all $x \in I$ with $x \neq a$, and
- $\lim_{x\to a} \dfrac{f'(x)}{g'(x)}$ exists,

then $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Now applying L'Hospital's rule.

Example 3

Find $\lim_{x\to\infty} \dfrac{4x^3 - 6x^2 + 1}{2x^3 - 10x + 3}$

Example 4

Find the limit of $\lim_{x \to 0} \dfrac{2(1-\cos(x))}{x^2}$.

Example 5

Find the limit of $\lim_{x \to 0} \frac{\sin(x)}{x^2}$.

L'Hospital's rule uses derivatives to help evaluate limits involving indeterminate forms:

$$0/0 \quad \infty/\infty, \quad 0\times\infty, \quad \infty-\infty, \quad 0^0, \quad 1^\infty \text{ and } \infty^0.$$

Remark

The expression $1/0$ is not commonly regarded as an indeterminate form.

Self-quiz problems