INFO 4178 Software Engineering

1. Analytical hierarchical Process (AHP)

1. What AHP is
2. Where can AHP be applied?
3. Steps involved in implementing AHP
4. Example: application of AHP in deciding which Laptop to buy for the upcoming academic year

1. **What AHP is:** AHP is a structured multicriteria decision making technique for organizing and analyzing complex decisions based on Mathematics and Psychology. It was developed by Thomas L. Saaty in the 1970's. It is one of the main Mathematical models used to support decision theory.

2. **Where can AHP be applied?** It can be applied to any problem that involves making a decision from a set of alternatives. Eg.
    - Choosing which phone to buy from a list of alternatives
    - Choosing which school to attend

Exercise: Come up with three (3) situations each to which AHP can be applied to the following areas

- Fire disaster situation
- Library management
- Software Testing
- Software Architecture

3. **Steps involved in implementing AHP**
    - **i.** Come up with hierarchical model of decision problem
    - **ii.** Come up with a criteria relative scale of preference
    - **iii.** Derive a pair wise comparison matrix
    - **iv.** Normalized the pair wise matrix
    - **v.** Calculate criteria weights
    - **vi.** Checking for consistency
        - **a.** Calculate criteria weighted sum
        - **b.** Calculate λ_i and λ_{max}
        - **c.** Calculate consistency index (CI)
        - **d.** Calculate consistency Ration (CR)
    - **vii.** Synthesis of Results.

4. **Example: application of AHP in deciding which Laptop to buy for the upcoming academic year**

Problem: You are faced with a decision problem of choosing which laptop to buy for the upcoming academic year that suites your taste, but you have a set of alternative laptops to choose from ASUS, DELL and LENOVO with each having its specifications. How can you use AHP to decide on which laptop is best for you?

Solution:

i. **Come up with hierarchical model of decision problem**

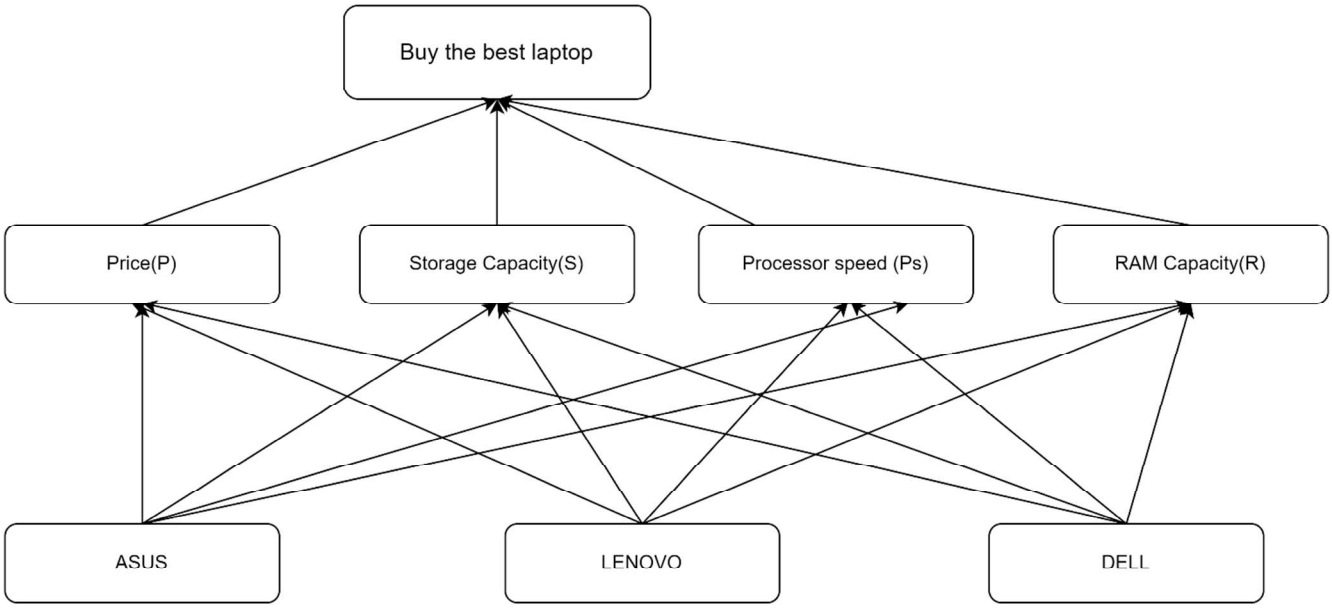

Figure 1: AHP Hierarchical model

ii. **Come up with a criteria relative scale of preference**

1 – Equal importance
3 – Moderate importance
5 – Strong importance
7 – Very strong importance
9 – Extreme importance
2,4,6,8 intermediate values
1/3,1/5,1/7,1/9 Inverse values

P – Price

S – Storage

Ps – Processor speed

R – Ram Capacity

iii. Derive a pair wise comparison matrix

This matrix gives the relative importance of each criterion with respect to the goal.

We ask the person buying the laptop the following question.

- How important is the **Price** with respect to the **storage?**

If we get the following response for example

- **Price** is of a **strong importance** than **storage**

NB: strong importance has a scale of 5 in our scale of preference

⇨ Storage space = X value

Price/storage = 5X value

Ie row element/ column element = 5X/X = 5

We keep on asking questions about scale of preference till we fill up the entire matrix

	P	**S**	**Ps**	**R**
P	1	5	4	7
S	1/5	1	2	3
Ps	1/4	1/2	1	3
R	1/7	1/3	1/3	1
SUM	1.59286	6.83333	7.33333	14

iv. Normalized the pair wise matrix

Normalization is done by dividing each cell by the corresponding column sum ie cell **PxP** we get **1/1.59286**

	P	**S**	**Ps**	**R**
P	0.6278	0.73171	0.54545	0.5
S	0.12556	0.14634	0.27273	0.21429
Ps	0.15695	0.07317	0.13636	0.21429
R	0.08969	0.04878	0.04545	0.07143
SUM	1	1	1	1

v. Calculate criteria weights

Criteria weights is calculated by averaging row values

i.e for row P we have (0.6278 + 0.73171+ 0.54545+ 0.5)/4 = **0.601241138**

	P	S	Ps	R	Weighted Criteria
P	0.6278	0.73171	0.54545	0.5	**0.601241138**
S	0.12556	0.14634	0.27273	0.21429	**0.189728747**
Ps	0.15695	0.07317	0.13636	0.21429	**0.145192689**
R	0.08969	0.04878	0.04545	0.07143	**0.063837426**
SUM	1	1	1	1	

vi. Checking for consistency

Multiply each cell value of the un-normalized pair-wise matrix with the criteria weight

	P	S	Ps	R	Weighted Criteria
P	1*0.601241138	5*0.189728747	4*0.145192689	7*0.063837426	**0.601241138**
S	(1/5)*0.601241138	1*0.189728747	2*0.145192689	3*0.063837426	**0.189728747**
Ps	(1/4)*0.601241138	(1/2)*0.189728747	1*0.145192689	3*0.063837426	**0.145192689**
R	(1/7)*0.601241138	(1/3)*0.189728747	(1/3)*0.145192689	1*0.063837426	**0.063837426**
SUM	1	1	1	1	

a. Calculate criteria weighted sum

This is done by summing up all values in a row

	P	S	Ps	R	Criteria Weighted Sum
P	1*0.601241138	5*0.189728747	4*0.145192689	7*0.063837426	2.57751761
S	(1/5)*0.601241138	1*0.189728747	2*0.145192689	3*0.063837426	0.79187463
Ps	(1/4)*0.601241138	(1/2)*0.189728747	1*0.145192689	3*0.063837426	0.581879624
R	(1/7)*0.601241138	(1/3)*0.189728747	(1/3)*0.145192689	1*0.063837426	0.261369496
SUM	1	1	1	1	

b. Calculate λ_i and λ_{max}

λ_i = (criteria weighted sum) / weighted Criteria

Criteria	Criteria Weighted Sum	*Weighted Criteria*	λi
P	2.57751761	0.601241138	4.286995
S	0.79187463	0.189728747	4.17372
Ps	0.581879624	0.145192689	4.007637
R	0.261369496	0.063837426	4.094299

λ_{max} = Average (λ_i)

λ_{max} = (4.286995+4.17372+4.007637+4.094299)/4
= **4.14066**

c. Calculate consistency index (CI)

CI = ($\lambda_{max} - n$)/(n-1)
Where n= number of criteria
in our case n=4

CI = (**4.14066-4**)/3 = 0.046888

d. Calculate consistency Ration (CR)

CR= CI/RI

Where RI is the randomly generated index which are randomly computed index for given number of criteria the RI for 10 criteria is as follows

Randomly Generated Index										
n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
R.I	**0**	**0**	**0.58**	**0.9**	**1.12**	**1.24**	**1.32**	**1.41**	**1.45**	**1.49**

CR (with n=4) = 0.046888/4 = **0.052097**

Since CR< 0.10 this implies we are consistent with our decision-making process hence we can conclude to synthesis.

vii. Synthesis of Results.

Let's consider our 3 laptops with the following specifications

	P(fcfa)	S(GB)	Ps(Hz)	R(GB)
ASUS	350000	1000	2.1	16
LENOVO	275000	500	3.2	16
DELL	375000	1000	2.7	8

To evaluate the overall weight of each alternative by its criteria weight.

	P(fcfa)	S(GB)	Ps (Hz)	R(GB)	Total item Weight
ASUS	210434.4	189.728747	0.3049	1.0214	**210625.5**
LENOVO	165341.31	94.8643736	0.46462	1.0214	**165437.7**
DELL	225465.43	189.728747	0.39202	0.5107	**225656.1**

From the synthesis we can se that **Dell laptop** is the best option because it has an overall item weight

Assignment:

Your friend is hospitalized and you wish to pay him a visit. In order to do so you decide to go to the market and buy some fruits for him. You are to choose from Pineapple, Banana, Oranges and Apples but you are confused on which he would love. Given that your friend has the following scale of preference

	Pineapple	Banana	Orange	Apple
Pineapple	1	3	2	7
Banana	1/3	1	5	3
Orange	1/2	1/5	1	3
Apple	1/7	1/3	1/3	1

Using AHP determine

- ⇨ If his/her scale of preference is consistent and if **YES**
- ⇨ which are the top two (2) fruits your friend would love