INFO 4178 Software Engineering

Mathematical Modelling

Content

- What is a mathematical model?
- Why is important to do mathematical modelling
- Real life examples

I. What is a mathematical model?

A mathematical model is the abstract representation of real-world systems with the use of mathematical concepts and notations(language). A mathematical model can also be defined as the process of describing real world problems in mathematics terms.

The process of building such models is called mathematical modelling. They exist several concepts in mathematics that can be used to capture and represent most real-world systems such as algebra, probability, graph theory etc. The diagram below depicts the notion of mathematical modelling.

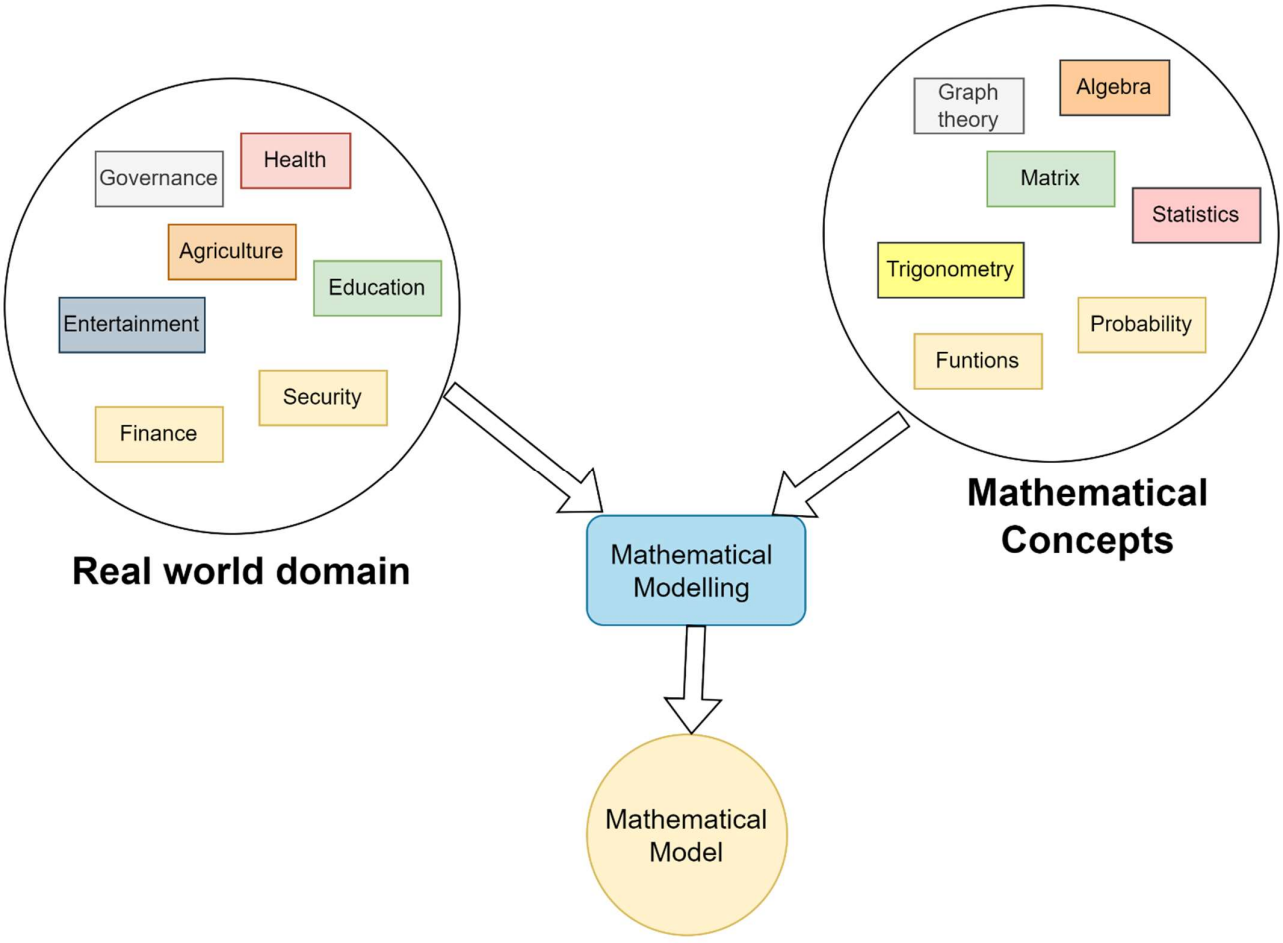

Figure 1: Mathematical Modelling Process

II. Why is it Important to do Mathematical Modelling?

Mathematical modelling is of vital importance and provides us with numerous advantages such as:

a. Very useful in capturing and representing your topic and most especially your problem statement during Masters II and PhD project writing and gives more weight to your research work.
b. Helps to ease communication amongst scientific community
c. Offers a means to have a standardized representation of a system
d. Gives us the ability to leverage the tools and algorithms already available in mathematics for the analysis of our system.
e. Permits us to try out new concepts and see the results without having to implement them physically.

III. Real life examples

The process of mathematical modeling is an open-ended activity and they is no one solution or best possible mathematical model for a given system, rather it depends on problem at hand and what the modelling expert will want to communicate and analyze from the system in question. Below are two real world examples and their corresponding mathematical models.

A. Client Data Reselling service on a Telecommunications network such as MTN, Orange, Camtel etc.

Research Problem: The Inefficient and non-flexible cost management of data bundle of a telecommunication network user.

This system will help Telecom users to efficiently and flexibly resell their extra data bundles which they will not or don't envisage to use before it expires. The price of the data bundle will vary depending on the time left to expire. Users will also have the possibility of buying data bundles at a cheaper cost than what is provided by the network service provider.

Below is the mathematical model for the given system.

Mathematical Model for Data Reselling over Telecommunication Network:

Let:

t = *time in hours*
V_t = *be the volume of internet data valid for time t*
R_0 = cost Rate offered by telecom company when buying data V_t
R_o = *(1,2,3, 4,...,o)*
R= cost rate when reselling the data on our platform

System Models

- $\text{SellModel}(V_t) = \begin{cases} 1 & iff,\ t \geq 1 \\ 0, & Otherwise \end{cases}$

- $RateModel(R_0, t) = \begin{cases} R = R_0 * 0.25 & if\ 1 \leq t \leq 2 \\ R = R_0 * 0.5 & if\ 2 < t \leq 3 \\ R = R_0 * 0.75 & if\ 3 < t \leq 4 \\ R = R_0 * 0.80 & if\ 4 < t \leq 5 \\ R = R_0 * 0.95 & Otherwise \end{cases}$

- $\text{CostModel } (V_t, R_j) = [V_1 V_2 .. V_t] \begin{bmatrix} R_1 \\ R_2 \\ . \\ . \\ R_j \end{bmatrix}$

$$= \begin{bmatrix} V_1R_1 & V_1R_2 & \cdots & V_1R_j \\ \vdots & & \ddots & \vdots \\ V_tR_1 & V_iR_2 & \cdots & V_tR_j \end{bmatrix}$$

B. E-commerce website for Agricultural products in a Rural-Urban area setting.

Research Problem: The main problem identified is the low visibility of farmers in rural areas and their products to potential buyers in urban areas.

The system will help to increase the visibility and map farmers and their products to potential buyers. The mathematical model for the system is presented below and it is worth noting that the mathematical concept used is ***Matrix***

Mathematical Model for Mapping farmers and their products to potential customers:

- For a farmer in a location (F)
  - Let ***F*** stand for ***i*** farmers who want to sell their Products

    Therefore F_i *= (1,2, 3…. i)*
- For a farmer's product (P)
  - Let ***P*** stand for ***j*** farmers Products

    Therefore F_j*= (1,2, 3…. j)*
- For 1st Customer (C)
  - Let C stand for K first customers

    Therefore C_k *= (1,2, 3…. k)*

❖ **Matrix for mapping farmers and product**

$$\textbf{Match } (\mathbf{F_i}, \mathbf{P_j}) = [F_1 F_2 \ldots F_i] \begin{bmatrix} P_1 \\ P_2 \\ . \\ . \\ P_j \end{bmatrix}$$

$$= \begin{bmatrix} F_1P_1 & F_1P_2 & \cdots & F_1P_J \\ \vdots & \ddots & & \vdots \\ F_iP_1 & F_iP_2 & \cdots & F_iP_j \end{bmatrix}$$

❖ **Matrix for mapping Customers to Farmers and their products**

$$\textbf{Match } (\mathbf{F_i}, \mathbf{P_j}, \mathbf{C_k}) = [C_1 C_2 \ldots C_k] \begin{bmatrix} F_1P_1 & F_1P_2 & \cdots & F_1P_J \\ \vdots & \ddots & & \vdots \\ F_iP_1 & F_iP_2 & \cdots & F_iP_j \end{bmatrix}$$

$$= \begin{bmatrix} C_1F_1P_1 & C_1F_1P_2 & \cdots & C_1F_1P_J \\ \vdots & \ddots & & \vdots \\ C_kF_iP_1 & C_kF_iP_2 & \cdots & C_kF_iP_j \end{bmatrix}$$

NB: in the cases above we have made use of matrix model but it's not always the case. Feel free to use any mathematical concept of your choice graph, functions etc. to model your system.