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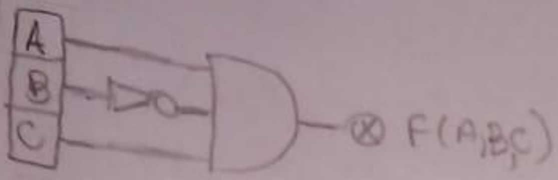
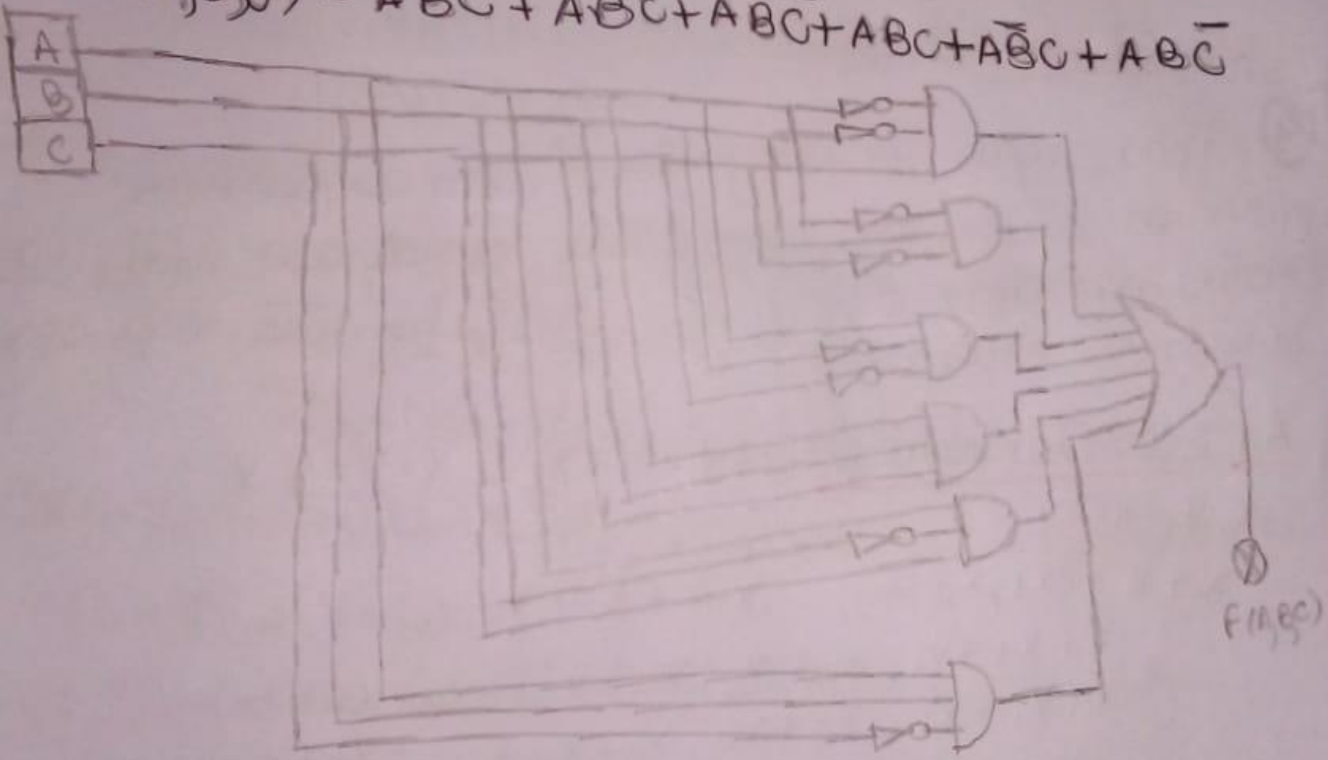
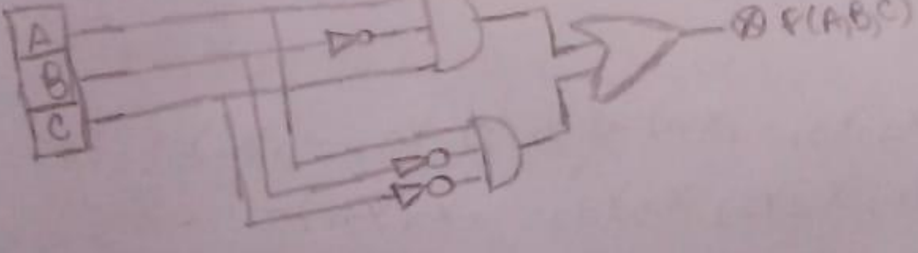
(A) $F(A,B,C) = A\bar{B}C$



(B) $F(A,B,C) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC + A\bar{B}C + AB\bar{C}$

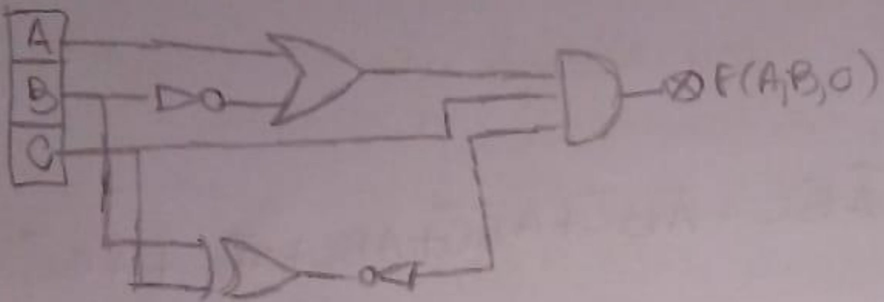


C) $F(A,B,C) = A\bar{B}C + A\bar{B}\bar{C}$



② $F(A,B,C) = (ABC) + (\bar{A} \oplus C)$

③



④ Para que o conjunto B^n, com as operações mais os elementos 0 e 1, seja considerado uma álgebra booleana, é necessário que ele satisfaça aos seguintes axiomas:

A1: Comutatividade: $x + y = y + x$ e $x \cdot y = y \cdot x$

$x, y \in B^n$:

$$x + y = (x_1, x_2, x_3, \dots, x_n) + (y_1, y_2, y_3, \dots, y_n) =$$
$$= (x_1 + y_1, x_2 + y_2, x_3 + y_3 + \dots + x_n + y_n) =$$
$$= (y_1 + x_1, y_2 + x_2; y_3 + x_3 + \dots + y_n + x_n) =$$
$$= (y_1, y_2, y_3, \dots, y_n) + (x_1, x_2, x_3, \dots, x_n) =$$
$$= y + x$$

$x, y \in B^n$:

$$x \cdot y = (x_1, x_2, x_3, \dots, x_n) \cdot (y_1, y_2, y_3, \dots, y_n) =$$
$$= (x_1 y_1, x_2 y_2, x_3 y_3, \dots, x_n y_n) =$$
$$= (y_1 x_1, y_2 x_2, y_3 x_3, \dots, y_n x_n) =$$
$$= (y_1, y_2, y_3 \dots y_n) \cdot (x_1, x_2, x_3, \dots x_n) =$$
$$= y \cdot x$$

A_2: Distributiva: $X(y+z) = (X y) + (X z)$ e
$x + (y z) = (x+y) \cdot (x+z)$

$x, y, z \in B^n$

$X \cdot (y+z) = (x_1, x_2, x_3, \dots, x_n) \cdot ((y_1, y_2, y_3, \dots, y_n) + (z_1, z_2, z_3, \dots, z_n))$
$= (x_1, x_2, x_3, \dots, x_n) \cdot ((y_1+z_1), (y_2+z_2), \dots, (y_n+z_n))$
$= (x_1 \cdot (y_1+z_1), x_2(y_2+z_2), x_3(y_3+z_3), \dots, x_n(y_n+z_n))$
$= ((x_1 y_1) + (x_1 z_1), (x_2 y_2) + (x_2 z_2), (x_3 y_3) + (x_3 z_3), \dots, (x_n y_n) + (x_n z_n))$
$= (xy) + (xz).$

$X, Y, Z \in B^n$

$X + (y z) = (x_1, x_2, x_3, \dots, x_n) + ((y_1, y_2, y_3, \dots, y_n) \cdot (z_1, z_2, z_3, \dots, z_n))$
$= (x_1, x_2, x_3, \dots, x_n) + ((y_1 z_1), (y_2 z_2), (y_3 z_3), \dots, (y_n z_n))$
$= (x_1 + (y_1 z_1), x_2 + (y_2 z_2), x_3 + (y_3 z_3), \dots, x_n + (y_n z_n))$
$= ((x_1 + y_1) \cdot (y_1 + z_1), (x_2 + y_2) \cdot (x_2 + z_2), (x_3 + y_3) \cdot (x_3 + z_3), \dots, (x_n + y_n) \cdot (x_n + z_n))$
$= (X + Y) \cdot (X + Z).$

A_3: Elementos identidades: $X + 0 = X$ e $X \cdot 1 = X$

$x \in B^n$:

$X + 0 = (x_1, x_2, x_3, \dots, x_n) + (0_1, 0_2, 0_3, \dots, 0_n) =$
$= (x_1 + 0, x_2 + 0 + x_3 + 0, \dots, x_n + 0) =$
$= (x_1, x_2, x_3, \dots, x_n) = X$

$x \in B^n$

A_4: Complemento: $X+\bar{X}=1$ e $X \cdot \bar{X}=0$

$X \in B^n, \exists \bar{X} \in B^n /$ p/ 0 e $1 \in B$, temos:

$X+\bar{X}=(x_1,x_2,x_3,\dots,x_n)+(\bar{x}_1,\bar{x}_2,\bar{x}_3,\dots,\bar{x}_n)=$

$=(x_1+\bar{x}_1, x_2+\bar{x}_2, x_3+\bar{x}_3,\dots,x_n+\bar{x}_n)$

$=(x_n,\bar{x}_n)=B^n$

EXPLICAÇÃO: Pode-se compreender pela lógica da complementariedade dos conjuntos que o $\bar{X}_n$ é todo o elemento $\in B^n$ que falta para que X seja igual a B^n e vice-versa $\therefore$ a soma $X_n+\bar{X}_n$ sempre dará 1, visto que 1 representa todo o conjunto B^n.

Exemplo:

Para 0 e $1 \in B^n / \exists\, \bar{0} \wedge \bar{1} \in B^n$ e $0 \longleftrightarrow \{\}$, temos:

$0+\bar{0}=(0,0,\dots,0)+(1,1,\dots,1)=(1,1,\dots,1)=1$

Sendo $X=1$:

$1+\bar{1}=(1,1,\dots,1)+(0,0,0,\dots,0)=$

$(1+0, 1+0, \dots, 1+0)=1.$

na lógica dos conjuntos equivale a união ($\cup$) ou $E(\cap)$

⑤ $\overline{X \cdot y} = \bar{x} + \bar{y}$. Prove (segunda igualdade de De Morgan).

$\overline{X \cdot y} = \bar{X} + \bar{y} \Rightarrow \overline{\overline{X \cdot y}} = \overline{\bar{X} + \bar{y}} \Rightarrow X \cdot y = \bar{x} + \bar{y}$.

Portanto $\bar{X} + \bar{y} + xy = 1$ e $(\bar{x} + \bar{y})\, xy = 0$

$(\bar{X} + \bar{y}) + xy \overset{A2}{=} (\bar{x} + \bar{y})\, x + (\bar{x} + \bar{y})\, y = \underbrace{\cancel{x\bar{x}}}_{0} + x\bar{y} + \bar{x}y + \underbrace{\cancel{y\bar{y}}}_{0}$

$= (x\bar{y}) + (\bar{x}y) = (x\bar{x}) + (y\bar{y}) \overset{A4}{=} 0$

$(\bar{x} + \bar{y})\, xy = xy\bar{x} + xy\bar{y} = 0$ (pois $yy' = 0$)