

$$\textcircled{1} \neg \exists x \neg R(x) \vdash \forall y (R(y))$$

$$\textcircled{1} \neg \exists x \neg R(x)$$

$$\textcircled{2} \neg (\forall y R(y))$$

$$\textcircled{3} \text{ intro } a: \neg R(a)$$

$$\textcircled{4} \neg \neg R(a) \text{ (1)}$$

$$\textcircled{5} R(a)$$

\perp

$$\forall y R(y)$$

é válido

$\textcircled{3}$

$$\neg \forall x R(x) \vdash \exists x \neg R(x)$$

Por contradição

$$\textcircled{1} \neg \forall x R(x)$$

$$\textcircled{2} \neg (\exists x \neg R(x))$$

$$\textcircled{3} \text{ intro } a: \neg R a$$

$$\textcircled{4} \neg (\neg R a)$$

$$\textcircled{5} R a$$

\perp

$$\exists x \neg R(x)$$

válido.

$$\textcircled{2} \neg \forall x \neg R(x) \vdash \exists x R(x)$$

$$\textcircled{1} \neg \forall x \neg R(x)$$

$$\textcircled{2} \neg (\exists x R(x))$$

$$\textcircled{3} \text{ intro } a: \neg \neg R(a) \textcircled{1}$$

$$\textcircled{4} R(a)$$

$$\textcircled{5} \forall x \neg R(x)$$

$$\textcircled{6} \neg R(a)$$

\perp

$$\exists x R(x)$$

é válido

$$\textcircled{4} \forall x P(i, x) \vdash (\forall y P(x, y)) \vdash$$

$$\exists x \exists y P(x, y)$$

Por contradição:

$$\textcircled{1} \forall x P(i, x) \vdash (\forall y P(x, y))$$

$$\textcircled{2} \neg \exists x \exists y (P(x, y))$$

$$\textcircled{3} \text{ intro } i: \neg P(i, i)$$

$$\textcircled{4} P(i, i) \vee P(i, i)$$

\perp

\perp

$$\exists x \exists y (P(x, y))$$

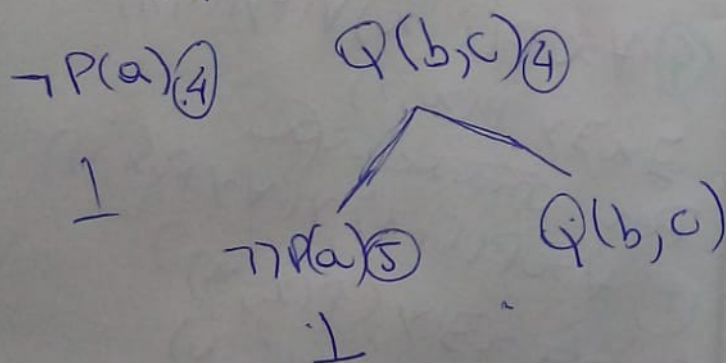
Válido.

- ⑤ ① $P(x) \rightarrow \exists u Q(u, z)$
 ② $\neg P(x) \rightarrow \exists u Q(u, z)$
 ③ $\neg (\forall x \exists u Q(u, z))$ ~~conclusion~~

④ intro a :
 $P(a) \rightarrow \exists u Q(u, z)$

⑤ intro b :
 $\neg P(a) \rightarrow Q(b, z)$

⑤ intro c :
 $\neg Q(b, c)$
 $P(a) \rightarrow Q(b, c)$
 $\neg \rightarrow \vee$



$\therefore \forall x \exists u Q(u, z)$
 \exists rule.

Per contradiction

$$\textcircled{\text{IV}} \neg \forall x \exists y \mathcal{P}(x, y)$$

Y. intro \hat{a}_i : $\neg \exists y R(a, y) \quad (4)$

$$\text{VII: } \exists w \forall x \exists y ((P(x, y) \rightarrow P(y, w)) \wedge (P(y, w) \rightarrow \exists w Q(w, w))) \quad (1)$$

VII: unter b^* : $\forall x \exists y ((P(x, y) \rightarrow P(y, b)) \wedge (P(y, b) \rightarrow \exists u Q(u, b)))$ (6)

$$\text{II X: } \exists y ((P(a, y) \rightarrow P(y, b)) \wedge (P(y, b) \rightarrow \exists w Q(w, b))) (\exists)$$

IX: intro c: $(P(a, c) \rightarrow P(c, b) \vee (P(c, b) \rightarrow \exists u Q(u, b))) @$

xii $(4,5) \rightarrow \neg R(a,a)^*$

$$\text{II } P(a, c) \rightarrow P(c, b)$$

$$\text{XI } P(c, b) \rightarrow \exists u Q(u, b)$$

$$x \quad \neg P(c, b)$$

$$E \cup Q(u, v)$$

$$x: P(a, c) \rightarrow P(c, b)$$

$$\exists x (P(x, b) \rightarrow \exists u Q(u, b))$$

x11: $(4,5 \rightarrow \neg R(a,a))$

$$11XV: \neg P(c, b)(11)$$

$$(m(q, n)QnE : \forall x)$$

$$xv: \neg P(a, c) \text{ (10)} \quad xvi: P(c, b)$$

entre d: $Q(b, d) \quad (14)$

73y3x Qxy

$$Y \times R^{++}$$

$$A_2 = (-1)^{2 \times 2} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1(1 - 4) = -3$$

$$\gamma_P(a, 0) \rightarrow \exists c \gamma_P(a, c)$$

$$37 \rho(a, c)$$

$\rho(a, c)$

\perp (15)

$f(u, v, c)$

$Q(e, c)$

$$\neg \exists x (Q(x) \wedge \neg P(x))$$

$$\neg Q(e, c) \perp$$

$\exists x \exists y (x \neq y)$
 $\exists x (x \neq x)$
 $\exists x (x \neq 0)$
 $\exists x (x \neq 1)$
 $\exists x (x \neq 2)$
 $\exists x (x \neq 3)$
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 $\exists x (x \neq 100)$

é vinda

7) $\forall x \forall y ((p(x) \vee q(y)) \rightarrow (\exists z R(z) \vee \forall w S(w)))$
 $\exists x \exists y (p(x) \wedge q(y) \wedge \neg (\exists z R(z) \vee \forall w S(w)))$

1) $\forall x \forall y ((p(x) \vee q(y)) \rightarrow (\exists z R(z) \vee \forall w S(w)))$

2) $\exists x \exists y (p(x) \wedge q(y) \wedge \neg (\exists z R(z) \vee \forall w S(w)))$

3) Per contradiction $\neg \exists z R(z)$

4) intro a : $\exists y (p(a) \wedge q(y))$

5) intro b : $p(a) \wedge q(b)$

6) $p(a)$ $\wedge E_1$
 $q(b)$ $\wedge E_2$

7) $(p(a) \wedge q(b)) \rightarrow (\exists z R(z) \vee \forall w S(w))$
 8) $\neg (\exists z R(z) \vee \forall w S(w))$

9) $\neg (p(a) \wedge q(b))$ $\neg \rightarrow \vee$ $\exists z R(z) \vee \forall w S(w)$

10) $\neg p(a)$ \perp
 11) $\neg q(b)$ \perp

$\therefore E$ valida-

13) $\exists z R(z)$ $X(3)$
 14) $\forall w S(w)$
 $R(c) \in \neg R(c)$
 13 3

\perp
 13) \wedge 14) $? = F \perp$
 F

8) ~~1~~ $\exists x P(x)$

(2) $\forall x (F(x) \rightarrow (\neg G(x) \wedge R(x)))$, $\forall x (P(x) \rightarrow G(x) \wedge F(x))$ (3)

(4) $\forall x (P(x) \rightarrow Q(x) \wedge \exists x (P(x) \wedge R(x)))$

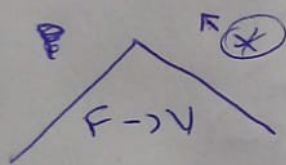
5) per contradiction: $\neg \exists x (Q(x) \wedge P(x))$

6: intro a: $P(a)$

7: $P(a) \rightarrow G(a) \wedge F(a)$

8) $F(a) \rightarrow (\neg G(a) \wedge R(a))$

9)



10)

$\neg P(a)$

\perp (6)

11) $G(a) \wedge F(a)$

12) $G(a)$

13) $F(a)$

\perp (13, 18)

(*) $\neg F(a)$ 18)



14) $\neg G(a)$ 15) $R(a)$

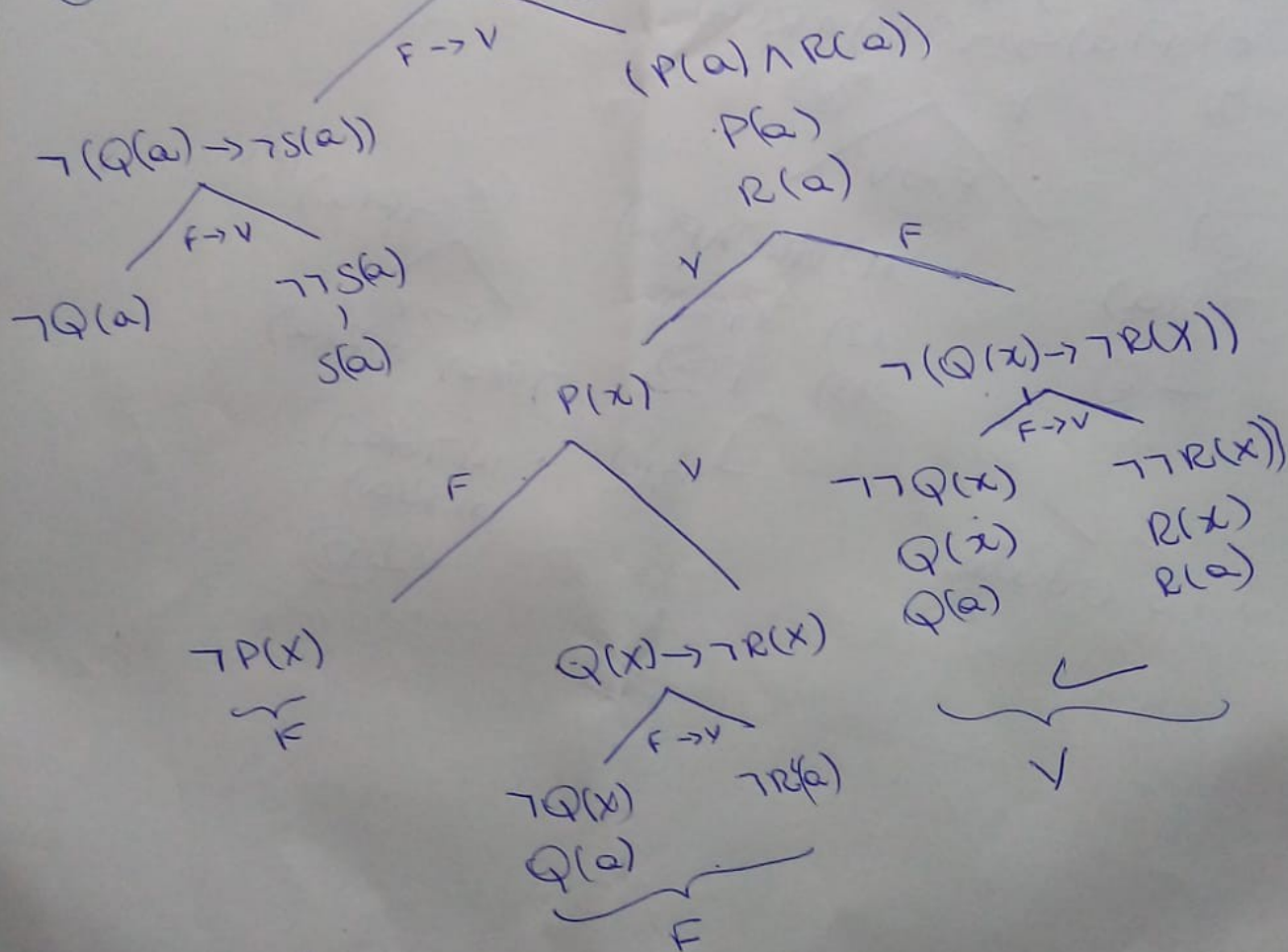
16) $\neg G(a)$

17) $R(a)$

\perp (12)

R: E valida

- 9) ① $\forall x (P(x) \vee (Q(x) \rightarrow \neg R(x)))$,
 ② $\forall x ((Q(x) \rightarrow \neg S(x)) \rightarrow (P(x) \wedge R(x)))$
 ③ Por contradição $\forall x S(x)$
 ④ Introd. a. $S(a)$ (3)
 ⑤ $(Q(a) \rightarrow \neg S(a)) \rightarrow (P(a) \wedge R(a))$ (2)
 ⑥ $(P(a) \vee (Q(a) \rightarrow \neg R(a)))$ (1)



Neste caso, o lado direito
 corresponde a Tabela verdade,
 por tanto a contradição
 está correta. Desta maneira,
 pode-se concluir que a é inválida.

- $\forall x \exists y F(x, y)$ ①
 ② $\forall x \exists y G(x, y)$
 ③ $\forall x \forall y ((F(x, y) \vee G(x, y)) \rightarrow \forall z ((F(y, z) \vee G(y, z)) \rightarrow H(x, z))$
 ④ For contradiction: $\neg \forall x \exists y H(x, y)$
 ⑤ Introduce a : $F(a, y)$ (1)
 ⑥ Introduce b : $G(a, b)$ (2) -
 \neg : $F(a, b)$ (5) -
 ⑦: $F(a, b) \vee G(a, b) \rightarrow \forall z ((F(b, z) \vee G(b, z)) \rightarrow H(a, z))$
 ⑧ Introduce c :
 $F(a, b) \vee G(a, b) \rightarrow (F(b, c) \vee G(b, c)) \rightarrow H(a, c)$

