① $\neg\exists x \neg R(x) \vdash \forall y (R(y))$

① $\neg\exists x \neg R(x)$
② $\neg(\forall y R(y))$
③ intro a: $\neg R(a)$
④ $\neg\neg R(a)$ (1)
⑤ $R(a)$

$\perp$

$\forall y R(y)$

é válido

② $\neg\forall x \neg R(x) \vdash \exists x R(x)$

① $\neg\forall x \neg R(x)$
② $\neg(\exists x R(x))$
③ intro a: $\neg\neg R(a)$ ①
④ $R(a)$
⑤ $\forall x \neg R(x)$
⑥ $\neg R(a)$

$\perp$

$\exists x R(x)$

é válido

③

$\neg\forall x R(x) \vdash \exists x \neg R(x)$

Por contradição

① $\neg\forall x R(x)$
② $\neg(\exists x \neg R(x)$
③ intro (a): $\neg Ra$
④ $\neg(\neg Ra)$
⑤ Ra

$\perp$

$\exists x \neg R(x)$

válido.

④ $\forall x P(i,x) \vee (\forall y P(x,y)) \vdash \exists x \exists y P(x,y)$:

Por contradição

① $\forall x P(i,x) \vee (\forall y P(x,y))$
② $\neg\exists x \exists y (P(x,y))$
③ intro i: $\neg P(i,i)$
④ $P(i,i) \vee P(i,i)$

$\perp$ $\perp$

$\exists x \exists y (P(x,y))$.

Válido.

5 ① $P(x) \to \exists u Q(u, z)$,

② $\neg P(x) \to \exists u Q(u, z)$

③ $\neg(\forall x \exists u Q(u, z))$ | ~~contradição~~

④ intro a:

$P(a) \to \exists u Q(u, z)$

⑤ intro b)

$\neg P(a) \to Q(b, z)$

⑤ intro(c)

$\neg Q(b, c)$

$\cdot P a \longrightarrow Q(b, c)$

$F \to V$

$\neg P(a)$ ④ $\quad$ $Q(b, c)$ ④

$\bot$

$\neg\neg P(a)$ ⑤ $\quad$ $Q(b, c)$

$\bot$

$\therefore \forall x \exists u Q(u, z)$

é válida.

$\forall z \exists w \forall x \exists y((P(x,y) \to P(y,w)) \land (P(y,w) \to \exists u\, Q(u,w)))$, $\forall x \forall z(\neg P(x,z) \to \exists u\, Q(u,z))$, $\exists y \exists x\, Q(x,y) \to \forall x\, R(x,x) \vdash \forall x \exists y\, R(x,y)$

I, II, III, IV

(6)

Por contradição

IV $\neg \forall x \exists y\, R(x,y)$

V. intro a: $\neg \exists y\, R(a,y)$ (4)

VI: $\exists w \forall x \exists y((P(x,y) \to P(y,w)) \land (P(y,w) \to \exists w\, Q(w,w)))$ (1)

VII: intro b: $\forall x \exists y((P(x,y) \to P(y,b)) \land (P(y,b) \to \exists u\, Q(u,b)))$ (6)

VIII: $\exists y((P(a,y) \to P(y,b)) \land (P(y,b) \to \exists w\, Q(w,b)))$ (7)

IX: intro c: $(P(a,c) \to P(c,b)) \land (P(c,b) \to \exists u\, Q(u,b))$ (8)

XII $(4,5) \to \neg R(a,a)$

X $P(a,c) \to P(c,b)$ XI $P(c,b) \to \exists w\, Q(u,b)$

$F \to V$

X $\neg P(c,b)$ $\exists w\, Q(w,b)$

X: $P(a,c) \to P(c,b)$ XI $P(c,b) \to \exists w\, Q(w,b)$

XII: $(4,5 \to \neg R(a,a))$

$F \to V$

IIXV: $\neg P(c,b)$ (11) IXV: $\exists u\, Q(u,b)$ (11)

$F \to V$

XV: $\neg P(a,c)$ (10) XVI: $P(c,b)$ intro d: $Q(b,d)$ (14)

$\perp$

$F \to V$ $F \to V$

$\neg \exists y \exists x\, Q x y$ $\forall x\, R x x$ $\neg \exists y \exists x\, Q(x,y)$ $\forall x\, R(x,x)$ (3)

$\forall z(\neg P(x,z) \to \exists u\, Q(u,z))$ (2) $(4,5)\ \neg R(a,a)$ $\neg \exists x\, Q(x,b)$ $R(a,a)$

X $\neg \exists Q(b,d)$ (14 X (4,5)

$\neg P(a,c) \to \exists u\, Q(u,c)$

$F \to V$ X

$\neg\neg P(a,c)$ $\exists u\, Q(u,c)$ ∴ é válida

$P(a,c)$ $Q(e,c)$

$\perp$ (15) $\neg \exists x\, Q(x,c)$

$\neg Q(e,c)$ $\perp$

(7) $\forall x \forall y ((P(x) \land Q(y)) \to (\exists z R z \land \forall w S(w)))$,

$\exists x \exists y (P(x) \land Q(y)) \vdash \exists z R(z)$

(1) $\forall x \forall y ((P(x) \land Q(y)) \to (\exists z R(z) \land \forall w S(w))$

(2) $\exists x \exists y (P(x) \land Q(y))$

(3) Por contradição $\neg \exists z R(z)$

(4) intro a: $\exists y (P(a) \land Q(y)))$

(5) intro b: $P(a) \land Q(B)$

(6) $P(a)$ $\land E_1$

$Q(B)$ $\land E_2$

(7)

(8) $(P(a) \land Q(B) \to (\exists z R(z) \land \forall w S(w)))$

9) $F \to V$

$\neg(P(a) \land Q(B))$

12) $\exists z R(z) \land \forall w S(w)$

10) $\neg P(a)$ $\perp$

11) $\neg Q(b)$ $\perp$

∴ É válida.

13) $\exists z R(z)$ X (3)

14) $\forall w S(w)$

$R(c)$ e $\neg R(c)$

13 3

$\perp$

(13) $\land$ (14) $= F \perp$

F ?

8) ① $\exists x\, P(x)$,

② $\forall x(F(x) \to (\neg G(x) \wedge R(x)))$, ③ $\forall x(P(x) \to G(x) \wedge F(x))$

④ $\forall x(P(x) \to Q(x)) \wedge \exists x(P(x) \wedge R(x))$

5) por contradição: $\neg \exists x (Q(x) \wedge P(x))$

6: intro a: $P(a)$,

7: $P(a) \to G(a) \wedge F(a)$

8) $F(a) \to (\neg G(a) \wedge R(a))$

9) (*)

F → V

10) $\neg P(a)$

$\perp$ 6

11) $G(a) \wedge F(a)$

12) $G(a)$

13) $F(a)$

$\perp_{(13,18)}$

(*) 18) $\neg F(a)$

F → V

14) $\neg G(a)$ 15) $R(a)$

16) $\neg G(a)$

17) $R(a)$

$\perp$ 12

R: É válida

9) ① $\forall x(P(x) \vee (Q(x \to \neg R(x)))$,

② $\forall x((Qx \to \neg Sx) \to (P(x) \wedge R(x))$

③ Por contradição $\forall x\, S(x)$

④ intro a°. Sa (3)

⑤ $(Q(a) \to \neg S(a)) \to (P(a) \wedge R(a))$ (2)

⑥ $(P(a) \vee (Q(a) \to \neg R(a)))$ (1)

⑤ ← ⑤

$F \to V$

$\neg(Q(a) \to \neg S(a))$

$F \to V$

$\neg Q(a)$ $\quad$ $\neg\neg S(a)$

$S(a)$

$(P(a) \wedge R(a))$

$P(a)$

$R(a)$

V $\quad$ F

$P(x)$ $\quad$ $\neg(Q(x) \to \neg R(X))$

F $\quad$ V

$F \to V$

$\neg\neg Q(x)$ $\quad$ $\neg\neg R(x)$

$Q(x)$ $\quad$ $R(x)$

$Q(a)$ $\quad$ $R(a)$

$\neg P(X)$

F

$Q(X) \to \neg R(X)$

$F \to V$

$\neg Q(X)$ $\quad$ $\neg R(a)$

$Q(a)$

F

V

Neste caso, o lado direito corresponde a tabela verdade, portanto a contradição está correta. Deste modo, pode-se concluir que é inválida.

$\forall x \exists y F(x,y)$ 1

10

2 $\forall x \exists y\, G(x,y)$

3 $\forall x \forall y ((F(x,y) \vee G(x,y)) \rightarrow \forall z ((F(y,z) \vee G(y,z)) \rightarrow H(x,z)))$

4 Por contradição: $\neg(\forall x \exists y\, H(x,y))$

5 intro a: $F(a,y)$ (1)

6: intro b: $G(a,b)$ (2) -

7: $F(a,b)$ (5) -

8: $F(a,b) \vee G(a,b) \rightarrow \forall z((F(b,z) \vee G(b,z)) \rightarrow H(a,z$

9) intro c:

$F(a,b) \vee G(a,b) \rightarrow (F(b,c) \vee G(b,c)) \rightarrow H(a,c))$

$F \rightarrow V$

$\neg(F(a,b) \vee G(a,b)$ $(F(b,c) \vee G(b,c)) \rightarrow H(a,c))$

F V $F \rightarrow V$

$\neg F(a,b)$ $\neg G(a,b)$ $\neg F(b,c)$ $\neg G(b,c)$ $\neg H a,c$

$F \vee F = F$ $F \vee F = F$ $\perp$ 4