QUESTÃO 1

a) Base monomial $\{1, x, x^2, x^3\}$

$P(-1) = c_0 - c_1 + c_2 - c_3 = 1$
$P(0) = c_0 \qquad\qquad = 1$
$P(1) = c_0 + c_1 + c_2 + c_3 = 2$
$P(2) = c_0 + 2c_1 + 4c_2 + 8c_3 = 0$

Substituindo $c_0 = 1$

$1 - c_1 + c_2 - c_3 = 1 \Rightarrow -c_1 + c_2 - c_3 = 0$
$1 + c_1 + c_2 + c_3 = 2 \Rightarrow c_1 + c_2 + c_3 = 1$
$1 + 2c_1 + 4c_2 + 8c_3 = 0 \Rightarrow 2c_1 + 4c_2 + 8c_3 = -1$

$-c_1 + c_2 - c_3 = 0 \quad (I)$
$c_1 + c_2 + c_3 = 1 \quad (II)$
$2c_1 + 4c_2 + 8c_3 = -1 \quad (III)$

Somando (I) + (II):

$c_1 + c_2 + c_3 - c_1 + c_2 - c_3 = 1 \Rightarrow 2c_2 = 1 \Rightarrow c_2 = 1/2.$

Substituindo $c_2 = 1/2$

$-c_1 + \frac{1}{2} - c_3 = 0 \Rightarrow -c_1 - c_3 = -\frac{1}{2} \Rightarrow c_1 + c_3 = 1/2$

$c_1 + \frac{1}{2} + c_3 = 1 \Rightarrow c_1 + c_3 = 1/2.$

Substituindo $c_2 = 1/2$ na terceira equação.

$2c_1 + 4\left(\frac{1}{2}\right) + 8c_3 = -1 \Rightarrow 2c_1 + 2 + 8c_3 = -1 \Rightarrow$
$2c_1 + 8c_3 = -3.$

USANDO $c_1 + c_3 = 1/2$

$2\left(\frac{1}{2} - c_3\right) + 8c_3 = -3 \Rightarrow 1 - 2c_3 + 8c_3 = -3$

USANDO $c_1 + c_3 = 1/2$

$2\left(\frac{1}{2} - c_3\right) + 8c_3 = -3 \Rightarrow 1 - 2c_3 + 8c_3 = -3$

$\Rightarrow 6c_3 = -4 \Rightarrow c_3 = -\frac{2}{3},$

$c_1 = \frac{1}{2} - \left(-\frac{2}{3}\right) = \frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$

ENTÃO, OS COEFICIENTES SÃO:

$c_0 = 1, c_1 = 7/6, c_2 = 1/2, c_3 = -\frac{2}{3}$

PORTANTO, O POLINÔMIO É:

$$\boxed{P(x) = 1 + \frac{7}{6}x + \frac{1}{2}x^2 - \frac{2}{3}x^3}$$

2. BASE DE LAGRANGE

USANDO A BASE DE LAGRANGE, TEMOS:

$P(x) = \sum_{i=0}^{3} y_i L_i(x)$ ONDE $L_i(x)$ SÃO OS

POLINÔMIOS DE LAGRANGE:

$$L_0(x) = \frac{(x-0)(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} = \frac{(x)(x-1)(x-2)}{-1 \cdot -2 \cdot -3} =$$

$$= \frac{(x)(x-1)(x-2)}{-6},$$

$$L_1(x) = \frac{(x+1)(x-1)(x-2)}{(0+1)(0-1)(0-2)} = \frac{(x+1)(x-1)(x-2)}{1 \cdot -1 \cdot -2}$$

$$= \frac{(x+1)(x-1)(x-2)}{2}$$

$$L_2(x) = \frac{(x+1)(x-0)(x-2)}{(1+1)(1-0)(1-2)} = \frac{(x+1)(x)(x-2)}{2 \cdot 1 \cdot -1} =$$

$$= \frac{(x+1)(x)(x-2)}{-2},$$

$$L_3(x) = \frac{(x+1)(x-0)(x-1)}{(2+1)(2-0)(2-1)} = \frac{(x+1)(x)(x-1)}{3 \cdot 2 \cdot 1} =$$

$$= \frac{(x+1)(x)(x-1)}{6}$$

CALCULANDO O POLINÔMIO:

$$P(x) = 1 \cdot \frac{(x)(x-1)(x-2)}{-6} + 1 \cdot \frac{(x+1)(x-1)(x-2)}{2}$$

$$+ 2 \cdot \frac{(x+1)(x)(x-2)}{-2} + 0 \cdot \frac{(x+1)(x)(x-1)}{6}$$

SIMPLIFICANDO:

$$P(x) = 1 \cdot \frac{(x)(x-1)(x-2)}{-6} + 1 \cdot \frac{(x+1)(x-1)(x-2)}{2} +$$

$$2 \cdot \frac{(x+1)(x)(x-2)}{-2}$$

$$= \frac{x(x-1)(x-2)}{-6} + \frac{(x+1)(x-1)(x-2)}{2}$$

$$- (x+1)(x)(x-2)$$

EXPANDINDO CADA TERMO

1.) $-\frac{x(x-1)(x-2)}{6} = -\frac{x(x^2-3x+2)}{6} =$

$$= -\frac{x^3-3x^2+2x}{6} = \frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{3}x$$

2.) $\frac{(x+1)(x-1)(x-2)}{2} =$

$$\frac{(x^2-1)(x-2)}{2} = \frac{x^3-2x^2-x+2}{2} = \frac{1}{2}x^3 - x^2 - \frac{1}{2}x + 1$$

3.) $-(x+1)(x)(x-2) =$

$$(x+1)(x-2) = (x^2 - 2x + x - 2) \cdot (x)$$

$$= -x^3 + 2x^2 - x^2 + 2x = -x^3 + x^2 + 2x$$

$$P(x) = -\frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{3}x + \frac{1}{2}x^3 - x^2 - \frac{1}{2}x + 1 - x^3 + x^2 + 2x,$$

$$= \left(-\frac{1}{6} + \frac{1}{2} - 1\right)x^3 + \left(\frac{1}{2} - 1 + 1\right)x^2 + \left(-\frac{1}{3} - \frac{1}{2} + 2\right)x + 1$$

$$= \left(-\frac{1}{6} + \frac{3}{6} - \frac{6}{6}\right)x^3 + \left(\frac{1}{2}\right)x^2 + \left(\frac{7}{6}\right)x + 1,$$

$$= -\frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{3}x + 1 - x^3 + 2x^2 - x,$$

$$= \left(-\frac{2}{3}\right)x^3 + \left(\frac{1}{2}\right)x^2 - \left(\frac{7}{6}\right)x + 1,$$

$$\boxed{P(x) = -\frac{2}{3}x^3 + \frac{1}{2}x^2 + \frac{7}{6}x + 1,}$$

USANDO NEWTON:

$P_N(x) = C_0 + C_1(x-x_0) + C_2(x-x_0)(x-x_1) + \dots$
$\dots + C_N(x-x_0)\dots(x-x_{N-1})$

Para calcular $P_0(x)$ $(-1,1),(0,1),(1,2),(2,0)$

$P_0(x = \overset{x_0}{-1}) = C_0 = f(x_0) = y_0 = \boxed{1 = C_0}$ $\boxed{P_i(x) = y_i \text{ POR DEFINIÇÃO}}$

$P_1(x = \overset{x_1}{0}) = 1 + C_1(x_1 - x_0) = y_1 = 1 \Rightarrow$
$\Rightarrow 1 + C_1(0+1) = 1 \Rightarrow \boxed{C_1 = 0}$

$P_2(x=1) = C_0 + C_1(x-x_0) + C_2(x-x_0)(x-x_1)$
$= 1 + 0(1+1) + C_2(1+1)(1-0)$
$= 1 + 2 \cdot C_2 = y_2 = 2$
$= 2C_2 = 2-1 \Rightarrow \boxed{C_2 = 1/2}$

$P_3(x=2) = C_0 + C_1(x-x_0) + C_2(x-x_0)(x-x_1) +$
$C_3(x-x_0)(x-x_1)(x-x_2)$
$= 1 + \cancel{0(2+1)} + \frac{1}{\cancel{2}}\overset{3}{(2+1)}\overset{2}{(2-0)} +$
$C_3\overset{3}{(2+1)}\overset{2}{(2-0)}(2-1)$
$= 1 + 3 + C_3 6$
$= C_3 = \frac{-4}{6} = -\frac{2}{3}$

$C_0 = 1,\ C_1 = 0,\ C_2 = \frac{1}{2},\ C_3 = -\frac{2}{3}$

$(x_i, y_i) = (-1,1), (0,1), (1,2)$ e $(2,0)$

$P(x) = C_0 + C_1(x - x_0) + C_2(x - x_0)(x - x_1)$
$\quad + C_3(x - x_0)(x - x_1)(x - x_2)$

$P(x) = C_0 + C_1(x+1) + C_2(x+1)(x-0)$
$\quad + C_3(x+1)(x-0)(x-1)$

$P(x) = C_0 + C_1(x+1) + C_2(x^2+x) + C_3(x^3 - x)$

SUBSTITUINDO

$P(x) = 1 + \cancel{0(x+1)} + \frac{1}{2}(x^2+x) + \left(-\frac{2}{3}\right)(x^3 - x)$

$P(x) = 1 + \frac{1}{2}x^2 + \frac{1}{2}x - \frac{2}{3}x^3 + \frac{2}{3}x$

$\boxed{P(x) = 1 + \frac{7}{6}x + \frac{1}{2}x^2 - \frac{2}{3}x^3}$

D) Com as 3 BASES chegamos no mesmo polinômio $P(x) = 1 + \frac{7}{6}x + \frac{1}{2}x^2 - \frac{2}{3}x^3$ USANDO CONTAS.
ISSO exemplifica SUA UNICIDADE