Journal of the Brazilian Society of Mechanical Sciences and Engineering (2020) 42:245
https://doi.org/10.1007/s40430-020-02344-z

TECHNICAL PAPER

Positional finite element formulation for two-dimensional analysis of elasto-plastic solids with contact applied to cold forming processes simulation

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Received: 11 July 2019 / Accepted: 8 April 2020 / Published online: 20 April 2020

Abstract

In this work, we develop a position-based finite element formulation for elasto-plastic solids under contact situation. The proposed positional formulation employs a total Lagrangian description and naturally considers geometric nonlinearities. The employed elasto-plastic model is derived from the dissipation inequality, using the thermodynamic conjugacy between the plastic strain rate and the so-called Mandel stress. The formulation is based on the Kröner–Lee decomposition, in which the deformation gradient is multiplicatively split into its elastic and plastic parts. We apply the backward Euler method to solve the plastic evolutions and von Mises yield criterion to define the elastic limit. The adopted kinematic hardening model is a finite strain generalization of the Armstrong–Frederick law, which uses the objective Jaumann derivative for the evolution equation and the concept of back stress tensor as an internal variable. For the elastic parcel of strains, we adopt a neo-Hookean constitutive law. With respect to the 2D application, plane strain and plane stress approximations are considered, where the latter is solved numerically by a local Newton–Raphson numerical procedure. Regarding the contact problem, a classical node-to-segment algorithm is applied, considering both frictionless and frictional cases, with the introduction of Lagrange multipliers in order to enforce contact constraints. Representative numerical examples are used to validate and show the possibilities of the proposed formulation in macroscale simulation of metal cold forming manufacturing processes.

Keywords Positional finite element method · Elasto-plasticity · Large strain · Contact · Cold forming

1 Introduction

The modeling of elasto-plastic materials subject to finite strains has a wide range of applications, specially in the context of metal forming, in which a contact analysis is also required between the machinery and the piece being manipulated. An adequate numerical simulation of common manufacturing processes can help predicting issues, like springback behavior, in a general and practical way without the need to spend labor in experimental procedures.

In this study, we develop a framework using the positional finite element method, as described in [5, 8–11]. The formulation is based on a total Lagrangian approach and is characterized by using positions as nodal parameters. Geometrical nonlinearities are naturally addressed, so problems involving finite displacements can be solved accordingly. In order to write the equilibrium equations in terms of positions, we make use of the energetic conjugacy between the Green–Lagrange strain tensor and the second Piola–Kirchhoff stress.

For solid finite elements and in terms of practical results, the position-based formulation is equivalent to the displacement-based one. However, position-based and displacement-based formulations derivations are conceptually different, so that the position-based results in a simple and more compact description. One can easily notice that Green strain tensor is more compact if expressed as a function of positions instead of displacements. The positional formulation is also

Technical Editor: João Marciano Laredo dos Reis.

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