XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

A numerical comparison between the additive and multiplicative decomposition applied to large strain thermo-elastic models

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Abstract. In this work, we develop a numerical framework, using the finite element method, to perform two-dimensional analysis of solids with thermo-elastic constitutive model, subject to large displacements and large strains. Both the thermo-elastic model and the heat conduction equation are derived from the first and second laws of thermodynamics, where the latter is expressed by the Clausius-Duhem inequality. The formulation uses the concept of Helmholtz free energy, from which the stress and the entropy are derived. The elastic and thermal parts of deformations are distinguished by two main strategies: the additive decomposition of the Green-Lagrange strain tensor, and the multiplicative decomposition of the deformation gradient. For each, both the linear and exponential thermal expansion laws are considered, and a neo-Hookean model is applied for the elastic part. Numerical examples are proposed to show the differences and limitations of the applied models on moderate and large strain levels.

Keywords: Thermo-elasticity, Nonlinear analysis, Multiplicative decomposition, Finite Element Method

1 Introduction

The additive decomposition is already well established within the field of non-linear thermo-elasticity, as can be seen, for instance, in Holzapfel [1], Truesdell et al. [2] and Parkus [3]. The concept of multiplicative decomposition, on the other hand, emerged originally from elasto-plastic models, and was first introduced for the thermo-elastic case in Stojanović et al. [4]. The same idea was presented independently in the works of Lu and Pister [5] and Imam and Johnson [6], and since then developed and largely applied in finite deformation models (see, for instance, Mićunović [7] and Vujošević and Lubarda [8]). In this work, we present both the additive and multiplicative decompositions within a numerical scope, with the aid of the finite element method. The aim is to investigate the limitations of each model, and different thermal expansion laws, applied to cases with moderate and large strain levels. In order to do that, representative numerical examples are presented and discussed.

2 Thermo-elastic model

In this work, we apply a total Lagrangian description of the thermo-mechanical problem, i.e., we use as reference the initial undeformed configuration of the solid. Let $\mathbf{F}$ denote the deformation gradient from the initial to the deformed configuration. Then, the Green-Lagrange strain is given by $\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$, where $\mathbf{I}$ is the second order identity tensor. The work-conjugate of $\mathbf{E}$ is the Second Piola-Kirchhoff Stress, denoted here by $\mathbf{S}$.

The first and second laws of thermodynamics form the basis for a thermodynamically-based constitutive model. The first, also known as law of conservation of energy, can be written, using the principle of rate of work, as

$$\dot{\psi} + \rho_0 \left(T\dot{\eta} + \dot{T}\eta - R \right) = \mathbf{S} : \dot{\mathbf{E}} - \boldsymbol{\nabla}_0 \cdot \mathbf{q}_0, \tag{1}$$

where ψ denotes the Helmholtz free energy (per unit volume in the initial configuration), ρ_0 the density in the initial configuration, T the temperature, η the entropy per unity mass, R the internal heat produced per unit time and unit mass, $\mathbf{q}_0$ the heat flux in the initial configuration, and $\boldsymbol{\nabla}_0$ the gradient vector in the initial configuration.