Natural Language Processing Sequence Labeling and Hidden Markov Models

Felipe Bravo-Marquez

June 26, 2023

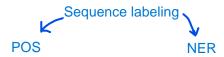
Overview

- The Sequence Labeling (or Tagging) Problem
- Generative models, and the noisy-channel model, for supervised learning
- Hidden Markov Model (HMM) taggers
 - Basic definitions
 - Parameter estimation
 - The Viterbi algorithm

This slides are based on the course material by Michael Collins: http://www.cs.columbia.edu/~mcollins/cs4705-spring2019/slides/tagging.pdf

Sequence Labeling or Tagging Tasks

- Sequence Labeling or Tagging is a task in NLP different from document classification.
- Here the goal is to map a sentence represented as a sequence of tokens x_1, x_2, \ldots, x_n into a sequence of tags or labels y_1, y_2, \ldots, y_n .
- Well known examples of this task are Part-of-Speech (POS) tagging and Named Entity Recognition (NER) to be presented next.



Part-of-Speech Tagging

INPUT: Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.

OUTPUT: Profits/N soared/V at/P Boeing/N Co./N ,/, easily/ADV topping/V forecasts/N on/P Wall/N Street/N ,/, as/P their/POSS CEO/N Alan/N Mulally/N announced/V first/ADJ guarter/N results/N ./.

- N = Noun
- V = Verb
- P = Preposition
- Adv = Adverb
- Adj = Adjective
- ..

Part-of-Speech Tag Descriptions

	Tag	Description	Example
Open Class	ADJ	Adjective: noun modifiers describing properties	red, young, awesome
	ADV	Adverb: verb modifiers of time, place, manner	very, slowly, home, yesterday
	NOUN	words for persons, places, things, etc.	algorithm, cat, mango, beauty
Sen	VERB	words for actions and processes	draw, provide, go
O	PROPN	Proper noun: name of a person, organization, place, etc	Regina, IBM, Colorado
	INTJ	Interjection: exclamation, greeting, yes/no response, etc.	oh, um, yes, hello
	ADP	Adposition (Preposition/Postposition): marks a noun's	in, on, by, under
S.		spacial, temporal, or other relation	
ord	AUX	Auxiliary: helping verb marking tense, aspect, mood, etc.,	can, may, should, are
1	CCONJ	Coordinating Conjunction: joins two phrases/clauses	and, or, but
ass	DET	Determiner: marks noun phrase properties	a, an, the, this
D D	NUM	Numeral	one, two, first, second
Closed Class Words	PART	Particle: a function word that must be associated with another word	's, not, (infinitive) to
	PRON	Pronoun: a shorthand for referring to an entity or event	she, who, I, others
	SCONJ	Subordinating Conjunction: joins a main clause with a	that, which
		subordinate clause such as a sentential complement	
22	PUNCT	Punctuation	;,()
Other	SYM	Symbols like \$ or emoji	\$, %
	X	Other	asdf, qwfg

Source: [Jurafsky and Martin, 2008]

Named Entity Recognition (NER)

A **named entity** is, roughly speaking, anything that can be referred to with a named entity proper name: a person, a location, an organization.

- INPUT: Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.
- **OUTPUT:** Profits soared at [Company Boeing Co.], easily topping forecasts on [Location Wall Street], as their CEO [Person Alan Mulally] announced first quarter results.
 - Since entities can span multiple words (i.e., a span recognition problem), we can
 use BIO tagging [Ramshaw and Marcus, 1999] to turn the problem into a
 sequence labeling problem.
 - BIO tagging: use tags that capture both the boundary and the named entity type.

BIO tagging: NER as Sequence Labeling

INPUT: Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.

OUTPUT: Profits/O soared/O at/O Boeing/B-C Co./I-C ,/O easily/O topping/O forecasts/O on/O Wall/B-L Street/I-L ,/O as/O their/O CEO/O Alan/B-P Mulally/I-P announced/O first/O guarter/O results/O ./O

- O = Outside (no entity)
- B-C = Begin Company
- I-C = Inside Company
- B-L = Begin Location
- I-L = Inside Location
- B-P = Begin Person
- I-P = Inside Person

NER BIO TAGGING

Our Goal

Training set:

- Pierre/NNP Vinken/NNP ,/, 61/CD years/NNS old/JJ ,/, will/MD join/VB the/DT board/NN as/IN a/DT nonexecutive/JJ director/NN Nov./NNP 29/CD ./.
- Mr./NNP Vinken/NNP is/VBZ chairman/NN of/IN Elsevier/NNP N.V./NNP ,/, the/DT Dutch/NNP publishing/VBG group/NN ./.
- Rudolph/NNP Agnew/NNP ,/, 55/CD years/NNS old/JJ and/CC chairman/NN of/IN Consolidated/NNP Gold/NNP Fields/NNP PLC/NNP ,/, was/VBD named/VBN a/DT nonexecutive/JJ director/NN of/IN this/DT British/JJ industrial/JJ conglomerate/NN ./.
- 4. ...

Our Goal: From the training set, induce a function/algorithm that maps new sentences to their tag sequences.

Two Types of Constraints

Influential/JJ members/NNS of/IN the/DT House/NNP Ways/NNP and/CC Means/NNP Committee/NNP introduced/VBD legislation/NN that/WDT would/MD restrict/VB how/WRB the/DT new/JJ savings-and-loan/NN bailout/NN agency/NN can/MD raise/VB capital/NN ./.

"Local":

 e.g., "can" is more likely to be a modal verb MD rather than a noun NN

"Contextual":

 e.g., a noun is much more likely than a verb to follow a determiner

Sometimes these preferences are in conflict:

• The trash can is in the garage

Sequence Labeling as Supervised Learning

- We have a sequence of inputs $x = (x_1, x_2, ..., x_n)$ and corresponding labels $y = (y_1, y_2, ..., y_n)$.
- Task is to learn a function f that maps input sequences to label sequences: $f(x_1, x_2, ..., x_n) = y_1, y_2, ..., y_n$.
- We have a training set of labeled sequences: $\{(x^{(1)}, v^{(1)}), (x^{(2)}, v^{(2)}), \dots, (x^{(m)}, v^{(m)})\}.$

Generative Approach for Sequence Labeling

- Generative models such as Naive Bayes was used for classification can also be used for sequence labeling tasks in NLP.
- Approach:
 - Training: Learn the joint distribution $p(x_1, x_2, ..., x_n, y_1, y_2, ..., y_n)$ of input sequences.
 - Decoding: Use the learned distribution to predict label sequences for new input sequences.
- Decoding in sequence labeling involves finding the label sequence with the highest joint probability: arg maxy1,y2,...,yn p(x1, x2,...,xn, y1, y2,...,yn).

Hidden Markov Models

- Hidden Markov Models (HMMs) provide a principled way to handle sequence labeling problems using generative modeling and efficient decoding algorithms.
- We have an input sentence $x = x_1, x_2, ..., x_n$ (x_i is the i-th word in the sentence).
- We have a tag sequence $y = y_1, y_2, \dots, y_n$ (y_i is the *i*-th tag in the sentence).
- We'll use an HMM to define $p(x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n)$ for any sentence x_1, \dots, x_n and tag sequence y_1, \dots, y_n of the same length. [Kupiec, 1992]
- Then, the most likely tag sequence for x is:

$$\arg\max_{y_1,\ldots,y_n}p(x_1,\ldots,x_n,y_1,\ldots,y_n)$$

Trigram Hidden Markov Models (Trigram HMMs)

For any sentence x_1, \ldots, x_n where $x_i \in V$ for $i = 1, \ldots, n$, and any tag sequence y_1, \ldots, y_{n+1} where $y_i \in S$ for $i = 1, \ldots, n$, and $y_{n+1} = STOP$, the joint probability of the sentence and tag sequence is:

$$p(x_1,\ldots,x_n,y_1,\ldots,y_{n+1})=\prod_{i=1}^{n+1}q(y_i|y_{i-2},y_{i-1})\prod_{i=1}^ne(x_i|y_i)$$

where we have assumed that $x_0 = x_{-1} = *$.

Parameters of the Model



- lacksq (s|u,v) for any $s \in S \cup \{STOP\}, u,v \in S \cup \{*\}$
 - The value for q(s|u, v) can be interpreted as the probability of seeing the tag s immediately after the bigram of tags (u, v).
- \bullet e(x|s) for any $s \in S$, $x \in V$
 - The value for e(x|s) can be interpreted as the probability of seeing observation x paired with state s.

An Example

If we have n = 3, x_1 , x_2 , x_3 equal to the sentence "the dog laughs", and y_1 , y_2 , y_3 , y_4 equal to the tag sequence "D N V STOP", then:

$$p(x_1, ..., x_n, y_1, ..., y_{n+1}) = q(D|*, *) \times q(N|*, D)$$

$$\times q(V|D, N) \times q(STOP|N, V)$$

$$\times e(\text{the}|D) \times e(\text{dog}|N) \times e(\text{laughs}|V)$$

- STOP is a special tag that terminates the sequence.
- We take $y_0 = y_{-1} = *$, where * is a special "padding" symbol.

Independence Assumptions in Trigram HMMs

- Trigram Hidden Markov Models (HMMs) are derived by making specific independence assumptions in the model.
- Consider two sequences of random variables: X_1, \ldots, X_n and Y_1, \ldots, Y_n , where n is the length of the sequences.
- Each X_i can take any value in a finite set V of words, and each Y_i can take any value in a finite set K of possible tags (e.g., $K = \{D, N, V \dots \}$).
- Our goal is to model the joint probability:

$$P(x_{1}, x_{2},...,x_{n},y_{1},...,y_{n})$$

$$= p(y_{1}) \times p(y_{2}|y_{1})$$

$$\times ...$$

$$\times p(y_{n}|y_{n-1}, y_{n-2},...y_{1})$$

$$\times p(x_{1}|y_{n}, y_{n-1},...y_{1})$$

$$\times p(x_{2}|x_{1}, y_{n}, y_{n-1},...y_{1})$$

$$\times p(x_{n}|x_{n-1},...,x_{1}, y_{n}, y_{n-1},...y_{1})$$

• We define an additional random variable Y_{n+1} that always takes the value "STOP."

Independence Assumptions in Trigram HMMs

The key idea in HMMs is the factorization of the joint probability:

$$P(X_1 = x_1, ..., X_n = x_n, Y_1 = y_1, ..., Y_{n+1} = y_{n+1})$$

$$= \prod_{i=1}^{n+1} P(Y_i = y_i | Y_{i-2} = y_{i-2}, Y_{i-1} = y_{i-1}) \times \prod_{i=1}^n P(X_i = x_i | Y_i = y_i)$$

We first assume that:

$$P(Y_i = y_i | Y_{i-2} = y_{i-2}, Y_{i-1} = y_{i-1}) = q(y_i | y_{i-2}, y_{i-1})$$



- This assumes that the sequence Y₁,..., Y_{n+1} is a second-order Markov sequence, where each state depends only on the previous two states.
- And we also assume that:

$$P(X_i = x_i | Y_i = y_i) = e(x_i | y_i)$$

- This assumes that the value of the random variable X_i depends only on the value of Y_i.
- These independence assumptions allow for the derivation of the joint probability equation.

Why the Name?

$$p(x_1, \dots, x_n, y_1, \dots, y_n) = q(\text{STOP}|y_{n-1}, y_n)$$

$$\times \prod_{j=1}^n q(y_j|y_{j-2}, y_{j-1})$$

$$\times \prod_{j=1}^n e(x_j|y_j)$$

• Markov Chain component:

$$q(STOP|y_{n-1}, y_n) \times \prod_{i=1}^{n} q(y_i|y_{j-2}, y_{j-1})$$

These transitions are not directly observed for a given sequence of words (x_1, \ldots, x_n) , hence the name "hidden".

Observed component:

$$\prod_{j=1}^n e(x_j|y_j)$$

The observed component of HMMs models the emission probabilities of observed symbols (ν 's) conditioned on the corresponding hidden states (ν 's).

Smoothed Estimation

$$\begin{split} q(\mathit{Vt}|\mathit{DT},\mathit{JJ}) = & \lambda_1 \times \frac{\mathsf{Count}(\mathit{Dt},\mathit{JJ},\mathit{Vt})}{\mathsf{Count}(\mathit{Dt},\mathit{JJ})} \\ & + \lambda_2 \times \frac{\mathsf{Count}(\mathit{JJ},\mathit{Vt})}{\mathsf{Count}(\mathit{JJ})} \\ & + \lambda_3 \times \frac{\mathsf{Count}(\mathit{Vt})}{\mathsf{Count}()} \end{split}$$
 where $\lambda_1 + \lambda_2 + \lambda_3 = 1$, and for all $i, \lambda_i > 0$.

$$e(\mathsf{base}|Vt) = \frac{\mathsf{Count}(Vt,\mathsf{base})}{\mathsf{Count}(Vt)}$$

Dealing with Low-Frequency Words

A common method is as follows:

- Step 1: Split vocabulary into two sets
 - Frequent words = words occurring ≥ 5 times in training
 - Low frequency words = all other words
- Step 2: Map low frequency words into a small, finite set, depending on prefixes, suffixes, etc.

Dealing with Low-Frequency Words: An Example

Below is an example of word classes for named entity recognition [Bikel et al., 1999]:

Word class	Example	Intuition
twoDigitNum	90	Two-digit year
fourDigitNum	1990 "	Four-digit year
containsDigitAndAlpha	<i>A</i> 8956 – 67	Product code
containsDigitAndDash	09 — 96	Date
containsDigitAndSlash	11/9/89	Date
containsDigitAndComma	23,000.00	Monetary amount
containsDigitAndPeriod	1.00	Monetary amount, percentage
othernum	456789	Other number
allCaps	BBN	Organization
capPeriod	M.	Person name initial
firstWord	First word of sentence	No useful capitalization information
initCap	Sally	Capitalized word
lowercase	can	Uncapitalized word
other	,	Punctuation marks, all other words

Dealing with Low-Frequency Words: An Example

Original Sentence: Profits/O soared/O at/O Boeing/B-C Co./I-C ,/O easily/O topping/O forecasts/O on/O Wall/B-L Street/I-L ,/O as/O their/O CEO/O Alan/B-P Mulally/I-P announced/O first/O quarter/O results/O ./O

Transformed Sentence:

firstword/O soared/O at/O initCap/B-C Co./I-C ,/O easily/O lowercase/O forecasts/O on/O initCap/B-L Street/I-L ,/O as/O their/O CEO/O Alan/B-P initCap/I-P announced/O first/O quarter/O results/O ./O

- O = Outside (no entity)
- B-C = Begin Company
- I-C = Inside Company
- B-L = Begin Location
- I-L = Inside Location
- B-P = Begin Person
- I-P = Inside Person

Decoding Problem



Decoding Problem: For an input $x_1 \dots x_n$, find

$$\arg\max_{y_1...y_{n+1}} p(x_1...x_n, y_1...y_{n+1})$$

where the arg max is taken over all sequences $y_1 ldots y_{n+1}$ such that $y_i \in S$ for i = 1 ldots n, and $y_{n+1} = STOP$. We assume that p takes the form:

$$p(x_1 \ldots x_n, y_1 \ldots y_{n+1}) = \prod_{i=1}^{n+1} q(y_i|y_{i-2}, y_{i-1}) \prod_{i=1}^n e(x_i|y_i)$$

Recall that we have assumed in this definition that $y_0 = y_{-1} = *$, and $y_{n+1} = STOP$.

Naive Brute Force Method

The naive, brute force method for finding the highest scoring tag sequence is to enumerate all possible tag sequences y_1, \ldots, y_{n+1} , score them under the function p, and select the sequence with the highest score.

- Example:
 - Input sentence: the dog barks
 - Set of possible tags: $K = \{D, N, V\}$
- Enumerate all possible tag sequences:
 - D D D STOP
 - D D N STOP
 - D D V STOP
 - D N D STOP
 - D N N STOP
 - D N V STOP
 - ...

decoding problem

finding highest scoring tag sequence

naive brute force method

Naive Brute Force Method

- In this case, there are $3^3 = 27$ possible sequences.
- However, for longer sentences, this method becomes inefficient.
- For an input sentence of length n, there are $|K|^n$ possible tag sequences.
- The exponential growth makes brute-force search infeasible for reasonable length sentences.

Viterbi Decoding Dynamic Programming

decoding problem -> find highest tag sequence -> viterbi algorithm

- The algorithm used by HMMs to perform efficient decoding is called Viterbi decoding.
- Viterbi decoding uses dynamic programming.
- Dynamic programming is a technique for solving optimization problems by breaking them down into overlapping subproblems.
- It stores the solutions to these subproblems in a table so that they do not have to be recalculated
- Dynamic programming can greatly improve the efficiency of algorithms.
- Next, we show how dynamic programming works with two examples: Factorial and Fibonacci

Factorial

Recursive implementation:

```
def recur_factorial(n):
      # Base case
       if n == 1:
           return n
      else:
           return n * recur_factorial(n-1)

    Dynamic programming implementation:

  def dynamic_factorial(n):
       table = [0 \text{ for } i \text{ in } range(0, n+1)]
      # Base case
       table[0] = 1
      for i in range(1, len(table)):
           table[i] = i * table[i-1]
       return table[n]
```

Fibonacci

Recursive implementation:

```
def recur_fibonacci(n):
       if n == 1 or n == 0:
           return 1
      else:
           return recur_fibonacci(n-1) + recur_fibonacci(n-2)

    Dynamic programming implementation:

  def dynamic_fibonacci(n):
       table = [0 \text{ for } i \text{ in range}(0, n+1)]
      # Base case
       table[0] = 1
       table[1] = 1
       for i in range(2, len(table)):
           table[i] = table[i-1] + table[i-2]
       return table[n]
```

Complexity

- In recursive implementations, the complexity can be quite high due to repeated calculations of the same subproblems.
- However, dynamic programming can significantly reduce the complexity by storing the solutions to subproblems in a table or array and reusing them when needed.
- This approach eliminates the redundant calculations and allows for a more efficient computation.
- For the case of Fibonacci the complexity is reduced from exponential to linear.

The Viterbi Algorithm

The Viterbi algorithm efficiently computes the maximum probability of a tag sequence by using dynamic programming.

Definitions:

- Define *n* as the length of the sentence.
- Define S_k for $k = -1 \dots n$ as the set of possible tags at position k: $S_{-1} = S_0 = \{*\}, S_k = S \text{ for } k \in \{1 \dots n\}.$
- Define a truncated version of the probability encoded by the HMM until position k, r(y₋₁, y₀, y₁,..., y_k) as:

$$r(y_{-1}, y_0, y_1, \dots, y_k) = \prod_{i=1}^k q(y_i|y_{i-2}, y_{i-1})$$

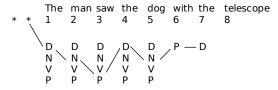
 Define a dynamic programming table π(k, u, v) as the maximum probability of a tag sequence ending in tags u, v at position k:

$$\pi(k, u, v) = \max_{y_{-1}, y_0, y_1, \dots, y_k : y_{k-1} = u, y_k = v} r(y_{-1}, y_0, y_1, \dots, y_k)$$

An Example

Recall that $\pi(k,u,v)$ is maximum probability of a tag sequence ending in tags u,v at position k

$$S = \{D, N, P, V\}$$



- There are many possible sequences of tags.
- ullet Each of them has a probability calculated from the parameters q and e.
- π(7, P, D) is the maximum probability that one of these tag sequences ends in P
 D at position 7.
- The path represents the sequence with the maximum probability.

A Recursive Definition

Base case:

$$\pi(0,*,*)=1$$

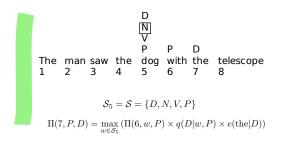
Recursive definition: For any $k \in \{1 ... n\}$, for any $u \in S_{k-1}$ and $v \in S_k$:

$$\pi(k, u, v) = \max_{w \in \mathcal{S}_{k-2}} (\pi(k-1, w, u) \times q(v|w, u) \times e(x_k|v))$$

Justification for the Recursive Definition

For any $k \in \{1 \dots n\}$, for any $u \in S_{k-1}$ and $v \in S_k$:

$$\pi(k, u, v) = \max_{w \in \mathcal{S}_{k-2}} (\pi(k-1, w, u) \times q(v|w, u) \times e(x_k|v))$$



- Let's consider an arbitrary tag sequence that ends with tags P and D at position
 7.
- It must contain some tag at position 5.
- We are basically searching for the tag that maximizes the probability at position
 5.

The Viterbi Algorithm

Algorithm 1: Viterbi Algorithm

Input: a sentence $x_1 ldots x_n$, parameters q(s|u,v) and e(x|s) **Initialization:** Set $\pi(0,*,*)=1$; $S_{-1}=S_0=\{*\}$, $S_k=S$ for $k\in\{1\ldots n\}$.

$$\begin{array}{c|c} \text{for } k=1 \text{ to } n \text{ do} \\ & \text{for } u \in S_{k-1}, v \in S_k \text{ do} \\ & & \\ & \pi(k,u,v) = \max_{w \in S_{k-2}} (\pi(k-1,w,u) \times q(v|w,u) \times e(x_k|v)) \\ & \text{end} \\ & \text{end} \end{array}$$

 $\texttt{return}\,(\mathsf{max}_{u \in \mathcal{S}_{n-1}, v \in \mathcal{S}_n}(\pi(n, u, v) \times q(STOP|u, v)))$

The Viterbi Algorithm with Backpointers

Algorithm 2: Viterbi Algorithm with Backpointers

```
Input: a sentence x_1 \dots x_n, parameters q(s|u,v) and e(x|s)
Initialization: Set \pi(0,*,*) = 1; S_{-1} = S_0 = \{*\}, S_k = S for k \in \{1 \dots n\}.
for k = 1 to n do
     for u \in S_{k-1}, v \in S_k do
                   \pi(k, u, v) = \max_{w \in S_{k-2}} (\pi(k-1, w, u) \times q(v|w, u) \times e(x_k|v))
                bp(k, u, v) = arg \max_{w \in S_{k-2}} (\pi(k-1, w, u) \times q(v|w, u) \times e(x_k|v))
     end
end
(y_{n-1}, y_n) = \operatorname{arg} \max_{(u,v)} (\pi(n, u, v) \times q(STOP|u, v));
                                                                     // Find maximum
 probability and corresponding tags
for k = (n-2) to 1 do
    y_k = bp(k+2, y_{k+1}, y_{k+2});
                                              // Retrieve tag seguence using
      backpointers
end
return (the tag sequence y_1 \dots y_n); // Return the final tag sequence
```

The Viterbi Algorithm: Running Time

- $O(n|S|^3)$ time to calculate $q(s|u,v) \times e(x_k|s)$ for all k, s, u, v.
- $n|S|^2$ entries in π to be filled in.
- O(|S|) time to fill in one entry.
- $\Rightarrow O(n|S|^3)$ time in total.

Pros and Cons

- Hidden Markov Model (HMM) taggers are simple to train (compile counts from training corpus).
- They perform relatively well (over 90% performance on named entity recognition).
- Main difficulty is modeling e(word|tag), which can be very complex if "words" are complex.

Questions?

Thanks for your Attention!

References I



Bikel, D. M., Schwartz, R. M., and Weischedel, R. M. (1999). An algorithm that learns what's in a name. *Mach. Learn.*, 34(1-3):211–231.



Jurafsky, D. and Martin, J. H. (2008). Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition. Prentice Hall, Upper Saddle River, NJ, USA, 2nd edition.



Kupiec, J. (1992). Robust part-of-speech tagging using a hidden markov model. Computer speech & language, 6(3):225–242.



Ramshaw, L. A. and Marcus, M. P. (1999). *Text Chunking Using Transformation-Based Learning*, pages 157–176. Springer Netherlands, Dordrecht.