Transition to Advanced Mathematics

Fall 2021

Practically Perfect Proof

Patrick May

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Question 1.

Theorem 1. *If a and b are positive integers, then $a^2(b+1)+b^2(a+1) \geq 4ab$.*

Proof. Assume a and b are positive integers.

Observe that a and b are positive integers and thus greater than or equal to 1, $(a-b)^2$ is positive or equal to zero.

ab is positive because $a, b \geq 1$.

$(a+b-2)$ is positive or equal to zero because $a, b \geq 1$.

Therefore $(a-b)^2 + ab(a+b-2)$ cannot be negative and is at least equal to 0.

Then $(a-b)^2 + ab(a+b-2) \geq 0$ is true.

Then,

$$
\begin{aligned}
&(a-b)^2 + ab(a+b-2) \geq 0 \\
&\implies (a-b)(a-b) + ab(a-2+b) \geq 0 \\
&\implies a^2 - 2ab + b^2 + a^2b - 2ab + b^2a \geq 0 \\
&\implies a^2b + a^2 + b^2a + b^2 - 4ab \geq 0 \\
&\implies a^2b + a^2 + b^2a + b^2 \geq 4ab \\
&\implies a^2(b+1) + b^2(a+1) \geq 4ab
\end{aligned}
$$

Thus $a^2(b+1) + b^2(a+1) \geq 4ab$ is true. $\square$