Transition to Advanced Mathematics

Fall 2021

Practically Perfect Proof

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Question 10.

Conjecture 1. Let S, T, X, and Y be subsets of some universal set. If all of the following:

$$\text{(i) } S \cup T \subseteq X \cup Y \qquad \text{(ii) } S \cap T = \emptyset \qquad \text{(iii) } X \subseteq S \tag{1}$$

then $T \subseteq Y$.

Proof. Let $a \in T$.

Then from (i), we have $a \in S \cup T$, so $a \in X \cup Y$.

It follows from (iii) that $X \subseteq S$, so $X \cup Y \subseteq S \cup Y$.

Then we have $a \in S \cup Y$.

Then from (ii), we know S shares no elements with T, so $a \in T \implies a \notin S$.

Then $a \in S \cup Y$ and $a \notin S$, thus $a \in Y$.

Hence $T \subseteq Y$.

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