1 A universal property for the pushout product

We would now like to generalize the framework of cyclic 2-fold double categories to cyclic 2-fold double multicategories, in order to incorporate the multivariable adjunctions of awfs defined in [Rie13].

[TODO: Review cyclic multicategories/double multicategories]

First, we will need to find a generalization of the universal property for arrow objects to (cyclic) double multicategories. In the theory of multivariable Quillen adjunctions, the lift of a multivariable adjunction to arrow categories is provided by the pushout/pullback product, so we will identify a universal property satisfied by this construction.

Define a cyclic double multicategory $\mathbb{J}$ as follows. The objects are A_i, B_i, for $i \in \{0,1,2\}$, and their duals. The horizontal 1-cells are $d_0^i, d_1^i \colon B_i \to A_i$. The vertical 1-cells are $F_i \colon (A_{i-1}, A_{i+1}) \to A_i^\bullet$ and $G_i \colon (B_{i-1}, B_{i+1}) \to B_i^\bullet$, which form two orbits under the cyclic action.

There are two types of 2-cells. There are

$$\begin{tikzcd} B_i \arrow[r, "d_1^i"] \arrow[d, "\mathrm{id}"'] \arrow[dr, phantom, "\Downarrow\alpha_i"] & A_i \arrow[d, "\mathrm{id}"] \\ B_i \arrow[r, "d_0^i"'] & A_i \end{tikzcd}$$

for each i. We will often draw these 2-cells globularly.

There are also 2-cells

$$\begin{tikzcd} B_{i+1}, B_{i-1} \arrow[r, "{d^{i+1}_{k_{i+1}}, d^{i-1}_{k_{i-1}}}"] \arrow[d, "G_i"'] \arrow[dr, phantom, "\Downarrow\lambda^i_{k_{i+1},k_{i-1},k_i}"] & A_{i+1}, A_{i-1} \arrow[d, "F_i"] \\ B_i^\bullet \arrow[r, "d^{i\bullet}_{k_i}"'] & A_i^\bullet \end{tikzcd}$$

for all choices of $(k_0, k_1, k_2) \in \{0,1\}^3$ except $(0,0,0)$.

Notice that there is at most one element of every hom-set, so all compositions and cyclic actions are uniquely defined. From now on, we will omit indices whenever doing so is unambiguous.

Remark 1.1. The cyclic double multicategory $\mathbb{J}$ is generated under composition by the α_i and the $\lambda^i_{k_{i+1},k_{i-1},k_i}$ with exactly one of k_0, k_1, k_2 equal to 1. These nine λ generators are further generated under the cyclic action by only three, though there are many choices of which three. These generators satisfy the relations

$$
\begin{tikzcd}
B_1, B_2 \arrow[r, "{d_1,d_0}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & A_1, A_2 \arrow[d, "F_0"] \\
B_0^\bullet \arrow[r, "d_0^\bullet"] \arrow[r, bend right=50, "d_1^\bullet"'] \arrow[r, phantom, shift right=3ex, "\Downarrow\alpha^\bullet"] & A_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
B_1, B_2 \arrow[r, "{d_0,d_0}"'] \arrow[r, bend left=50, "{d_1,d_0}"] \arrow[r, phantom, shift left=3ex, "{\Downarrow\alpha,\mathrm{id}}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & A_1, A_2 \arrow[d, "F_0"] \\
B_0^\bullet \arrow[r, "d_1^\bullet"'] & A_0^\bullet
\end{tikzcd}
$$

$$
\begin{tikzcd}
B_1, B_2 \arrow[r, "{d_0,d_1}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & A_1, A_2 \arrow[d, "F_0"] \\
B_0^\bullet \arrow[r, "d_0^\bullet"] \arrow[r, bend right=50, "d_1^\bullet"'] \arrow[r, phantom, shift right=3ex, "\Downarrow\alpha^\bullet"] & A_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
B_1, B_2 \arrow[r, "{d_0,d_0}"'] \arrow[r, bend left=50, "{d_0,d_1}"] \arrow[r, phantom, shift left=3ex, "{\Downarrow\mathrm{id},\alpha}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & A_1, A_2 \arrow[d, "F_0"] \\
B_0^\bullet \arrow[r, "d_1^\bullet"'] & A_0^\bullet
\end{tikzcd}
$$

$$
\begin{tikzcd}
B_1, B_2 \arrow[r, "{d_1,d_0}"'] \arrow[r, bend left=50, "{d_1,d_1}"] \arrow[r, phantom, shift left=3ex, "{\Downarrow\mathrm{id},\alpha}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & A_1, A_2 \arrow[d, "F_0"] \\
B_0^\bullet \arrow[r, "d_0^\bullet"'] & A_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
B_1, B_2 \arrow[r, "{d_0,d_1}"'] \arrow[r, bend left=50, "{d_1,d_1}"] \arrow[r, phantom, shift left=3ex, "{\Downarrow\alpha,\mathrm{id}}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & A_1, A_2 \arrow[d, "F_0"] \\
B_0^\bullet \arrow[r, "d_0^\bullet"'] & A_0^\bullet
\end{tikzcd}
$$

and their reflections under the cyclic action.

Example 1.2. Let $\mathbb{M}\mathbf{Adj}$ be the double cyclic multicategory of categories, functors, and multivariable right adjunctions. Any multivariable right adjunction $F_0\colon \mathscr{A}_1 \times \mathscr{A}_2 \to \mathscr{A}_0$ extends to a functor $\widehat{\mathbb{F}}\colon \mathbb{J} \to \mathbb{M}\mathbf{Adj}$ as follows.

- B_i is sent to $\mathscr{A}_i^{\mathbf{2}}$, the arrow category of $\mathscr{A}_i$.
- The d_1 are sent to the domain functors $\mathrm{dom}\colon \mathscr{A}_i^{\mathbf{2}} \to \mathscr{A}_i$ and the d_0 are sent to the codomain functors $\mathrm{cod}\colon \mathscr{A}_i^{\mathbf{2}} \to \mathscr{A}_i$.
- The α are sent to the canonical natural transformations $\mathrm{dom} \Rightarrow \mathrm{cod}$.
- The G_i are sent to functors $\hat{F}_i$. Given morphisms $f\colon A \to B \in \mathscr{A}_1$ and

$g\colon X \to Y \in \mathscr{A}_2$, $\hat{F}_0(f,g)$ is defined as in the diagram

$$\begin{tikzcd}
F_0(B,Y) \arrow[dr, dashed, "\hat{F}_0(f,g)"] \arrow[drr, bend left, "F_0(1,g)"] \arrow[ddr, bend right, "F_0(f,1)"'] & & \\
& F_0(A,Y) \prod_{F_0(A,X)} F_0(B,X) \arrow[r, "p_2"] \arrow[d, "p_1"'] & F_0(B,X) \arrow[d, "F_0(f,1)"] \\
& F_0(A,Y) \arrow[r, "F_0(1,g)"'] & F_0(A,X)
\end{tikzcd} \tag{1}$$

It is a standard fact that the $\hat{F}_i$ form a two-variable adjunction between the arrow categories.

- Looking at diagram 1,

$$\begin{aligned}
(\lambda^0_{1,0,0})_{f,g} &= p_1\colon \operatorname{cod} \hat{F}_0(f,g) \to F_0(\operatorname{dom} f, \operatorname{cod} g)\\
(\lambda^0_{0,1,0})_{f,g} &= p_2\colon \operatorname{cod} \hat{F}_0(f,g) \to F_0(\operatorname{cod} f, \operatorname{dom} g)\\
(\lambda^0_{0,0,1})_{f,g} &= \mathrm{id}\colon \operatorname{dom} \hat{F}_0(f,g) \to F_0(\operatorname{cod} f, \operatorname{cod} g).
\end{aligned}$$

The three relations (1)-(3) then correspond precisely to the commutativity of the three regions in diagram (1).

Exercise 1.1. Check that the mates of the morphism p_1 in diagram 1 are p_2 and id in the two similar diagrams defining $\hat{F}_1$ and $\hat{F}_2$.

Let $\mathbb{I}$ be the sub-category of $\mathbb{J}$ consisting of just the 1-cells F_i. Let **CDMCat** denote the 2-category of cyclic double multicategories, functors, and horizontal transformations.

Theorem 1.3. *Fix a functor $\mathbb{F}\colon \mathbb{I} \to \mathbb{M}\mathbf{Adj}$. Then the functor $\hat{\mathbb{F}}\colon \mathbb{J} \to \mathbb{M}\mathbf{Adj}$ constructed in example 1.2 is terminal in the category $\mathbf{CDMCat}_{\mathbb{F}}(\mathbb{J}, \mathbb{M}\mathbf{Adj})$ of functors on $\mathbb{J}$ restricting to $\mathbb{F}$ on $\mathbb{I}$.*

Proof. Concretely, the theorem says that given the data of a functor $\mathbb{J} \to \mathbb{M}\mathbf{Adj}$, there is a unique 2-cell

$$\begin{tikzcd}
\mathscr{B}_1, \mathscr{B}_2 \arrow[r, "{H_1,H_2}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\theta"] & \mathscr{A}_1^{\mathbf{2}}, \mathscr{A}_2^{\mathbf{2}} \arrow[d, "\hat{F}_0"] \\
\mathscr{B}_0^{\bullet} \arrow[r, "H_3^{\bullet}"'] & \mathscr{A}_0^{\bullet\mathbf{2}}
\end{tikzcd}$$

such that

$$
\begin{tikzcd}
\mathscr{B}_1,\mathscr{B}_2 \arrow[r, "H_1{,}H_2"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\theta"] & \mathscr{A}_1^{\mathbf{2}},\mathscr{A}_2^{\mathbf{2}} \arrow[r, "\mathrm{cod{,}cod}"] \arrow[d, "\hat{F}_0"] \arrow[dr, phantom, "\Downarrow\mathrm{id}"] & \mathscr{A}_1,\mathscr{A}_2 \arrow[d, "F_0"] \\
\mathscr{B}_0^\bullet \arrow[r, "H_3^\bullet"'] & \mathscr{A}_0^{\bullet\mathbf{2}} \arrow[r, "\mathrm{dom}^\bullet"'] & \mathscr{A}_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
\mathscr{B}_1,\mathscr{B}_2 \arrow[r, "d_0{,}d_0"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & \mathscr{A}_1,\mathscr{A}_2 \arrow[d, "F_0"] \\
\mathscr{B}_0^\bullet \arrow[r, "d_1^\bullet"'] & \mathscr{A}_0^\bullet
\end{tikzcd}
\tag{2}
$$

$$
\begin{tikzcd}
\mathscr{B}_1,\mathscr{B}_2 \arrow[r, "H_1{,}H_2"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\theta"] & \mathscr{A}_1^{\mathbf{2}},\mathscr{A}_2^{\mathbf{2}} \arrow[r, "\mathrm{dom{,}cod}"] \arrow[d, "\hat{F}_0"] \arrow[dr, phantom, "\Downarrow p_1"] & \mathscr{A}_1,\mathscr{A}_2 \arrow[d, "F_0"] \\
\mathscr{B}_0^\bullet \arrow[r, "H_3^\bullet"'] & \mathscr{A}_0^{\bullet\mathbf{2}} \arrow[r, "\mathrm{cod}^\bullet"'] & \mathscr{A}_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
\mathscr{B}_1,\mathscr{B}_2 \arrow[r, "d_1{,}d_0"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & \mathscr{A}_1,\mathscr{A}_2 \arrow[d, "F_0"] \\
\mathscr{B}_0^\bullet \arrow[r, "d_0^\bullet"'] & \mathscr{A}_0^\bullet
\end{tikzcd}
\tag{3}
$$

$$
\begin{tikzcd}
\mathscr{B}_1,\mathscr{B}_2 \arrow[r, "H_1{,}H_2"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\theta"] & \mathscr{A}_1^{\mathbf{2}},\mathscr{A}_2^{\mathbf{2}} \arrow[r, "\mathrm{cod{,}dom}"] \arrow[d, "\hat{F}_0"] \arrow[dr, phantom, "\Downarrow p_2"] & \mathscr{A}_1,\mathscr{A}_2 \arrow[d, "F_0"] \\
\mathscr{B}_0^\bullet \arrow[r, "H_3^\bullet"'] & \mathscr{A}_0^{\bullet\mathbf{2}} \arrow[r, "\mathrm{cod}^\bullet"'] & \mathscr{A}_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
\mathscr{B}_1,\mathscr{B}_2 \arrow[r, "d_0{,}d_1"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\lambda"] & \mathscr{A}_1,\mathscr{A}_2 \arrow[d, "F_0"] \\
\mathscr{B}_0^\bullet \arrow[r, "d_0^\bullet"'] & \mathscr{A}_0^\bullet
\end{tikzcd}
\tag{4}
$$

Fix objects $B_1 \in \mathscr{B}_1$, $B_2 \in \mathscr{B}_2$. The H_i are the functors sending B_i to $H_i(B_i) = \alpha_{B_i}\colon d_1 B_i \to d_0 B_i$. The component of θ at (B_1, B_2) is a square

$$
\begin{tikzcd}
d_1 G_0(B_1,B_2) \arrow[rr] \arrow[d] & & F_0(d_0 B_1, d_0 B_2) \arrow[d] \\
d_0 G_0(B_1,B_2) \arrow[r] & F_0(d_1 B_1, d_0 B_2) & \prod_{F_0(d_1 B_1, d_1 B_2)} & F_0(d_0 B_1, d_1 B_2)
\end{tikzcd}
$$

The top arrow is uniquely determined by equation (2), while the components of the bottom arrow are uniquely determined by equations (3) and (4). □

Now let $\mathbb{M}$ be any double multicategory. Given an object C of $\mathbb{M}$, an arrow object C^2 is an object together with a globular 2-cell $\kappa\colon \mathrm{dom} \Rightarrow \mathrm{cod}$ satisfying the same universal property as in Section **??** (this only involves the horizontal 2-category, so carries over unchanged).

Given a vertical 1-cell $F\colon (C_1, C_2) \to C_0^\bullet$, the lift to arrow objects $\hat{F}$ is a vertical 1-cell $\hat{F}$ together with 2-cells

$$
\begin{tikzcd}
C_1^2,C_2^2 \arrow[r, "\mathrm{cod{,}cod}"] \arrow[d, "\hat{F}"'] \arrow[dr, phantom, "\Downarrow\gamma_0"] & C_1,C_2 \arrow[d, "F"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{dom}^\bullet"'] & C_0^\bullet
\end{tikzcd}
\qquad
\begin{tikzcd}
C_1^2,C_2^2 \arrow[r, "\mathrm{dom{,}cod}"] \arrow[d, "\hat{F}"'] \arrow[dr, phantom, "\Downarrow\gamma_1"] & C_1,C_2 \arrow[d, "F"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{cod}^\bullet"'] & C_0^\bullet
\end{tikzcd}
\qquad
\begin{tikzcd}
C_1^2,C_2^2 \arrow[r, "\mathrm{cod{,}dom}"] \arrow[d, "\hat{F}"'] \arrow[dr, phantom, "\Downarrow\gamma_2"] & C_1,C_2 \arrow[d, "F"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{cod}^\bullet"'] & C_0^\bullet
\end{tikzcd}
$$

satisfying the equations

$$
\begin{tikzcd}
C_1^2, C_2^2 \arrow[r, "\mathrm{dom},\mathrm{cod}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\gamma_1"] & C_1, C_2 \arrow[d, "F_0"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{cod}^\bullet"] \arrow[r, bend right=50, "\mathrm{dom}^\bullet"'] \arrow[r, phantom, shift right=3ex, "\Downarrow\kappa^\bullet"] & C_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
C_1^2, C_2^2 \arrow[r, "\mathrm{cod},\mathrm{cod}"'] \arrow[r, bend left=50, "\mathrm{dom},\mathrm{cod}"] \arrow[r, phantom, shift left=3ex, "\Downarrow\kappa,\mathrm{id}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\gamma_0"] & C_1, C_2 \arrow[d, "F_0"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{dom}^\bullet"'] & C_0^\bullet
\end{tikzcd}
\tag{5}
$$

$$
\begin{tikzcd}
C_1^2, C_2^2 \arrow[r, "\mathrm{cod},\mathrm{dom}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\gamma_2"] & C_1, C_2 \arrow[d, "F_0"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{cod}^\bullet"] \arrow[r, bend right=50, "\mathrm{dom}^\bullet"'] \arrow[r, phantom, shift right=3ex, "\Downarrow\kappa^\bullet"] & C_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
C_1^2, C_2^2 \arrow[r, "\mathrm{cod},\mathrm{cod}"'] \arrow[r, bend left=50, "\mathrm{cod},\mathrm{dom}"] \arrow[r, phantom, shift left=3ex, "\Downarrow\mathrm{id},\kappa"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\gamma_0"] & C_1, C_2 \arrow[d, "F_0"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{dom}^\bullet"'] & C_0^\bullet
\end{tikzcd}
\tag{6}
$$

$$
\begin{tikzcd}
C_1^2, C_2^2 \arrow[r, "\mathrm{dom},\mathrm{cod}"'] \arrow[r, bend left=50, "\mathrm{dom},\mathrm{dom}"] \arrow[r, phantom, shift left=3ex, "\Downarrow\mathrm{id},\kappa"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\gamma_1"] & C_1, C_2 \arrow[d, "F_0"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{cod}^\bullet"'] & C_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
C_1^2, C_2^2 \arrow[r, "\mathrm{cod},\mathrm{dom}"'] \arrow[r, bend left=50, "\mathrm{dom},\mathrm{dom}"] \arrow[r, phantom, shift left=3ex, "\Downarrow\kappa,\mathrm{id}"] \arrow[d, "G_0"'] \arrow[dr, phantom, "\Downarrow\gamma_2"] & C_1, C_2 \arrow[d, "F_0"] \\
C_0^{\bullet 2} \arrow[r, "\mathrm{cod}^\bullet"'] & C_0^\bullet
\end{tikzcd}
\tag{7}
$$

and which is universal, meaning that given any objects X_0, X_1, X_2, horizontal 1-cells $d_{i,0}, d_{i,1} \colon X_i \to C_i$, a vertical 1-cell $G \colon X_1, X_2 \to X_0^\bullet$, globular 2-cells $\alpha_i \colon d_{i,1} \Rightarrow d_{i,0}$, and 2-cells

$$
\begin{tikzcd}
X_1, X_2 \arrow[r, "{d_{1,0}, d_{2,0}}"] \arrow[d, "G"'] \arrow[dr, phantom, "\Downarrow\lambda_0"] & C_1, C_2 \arrow[d, "F"] \\
X_0^\bullet \arrow[r, "d_{0,1}^\bullet"'] & C_0^\bullet
\end{tikzcd}
\qquad
\begin{tikzcd}
X_1, X_2 \arrow[r, "{d_{1,1}, d_{2,0}}"] \arrow[d, "G"'] \arrow[dr, phantom, "\Downarrow\lambda_0"] & C_1, C_2 \arrow[d, "F"] \\
X_0^\bullet \arrow[r, "d_{0,0}^\bullet"'] & C_0^\bullet
\end{tikzcd}
\qquad
\begin{tikzcd}
X_1, X_2 \arrow[r, "{d_{1,0}, d_{2,1}}"] \arrow[d, "G"'] \arrow[dr, phantom, "\Downarrow\lambda_2"] & C_1, C_2 \arrow[d, "F"] \\
X_0^\bullet \arrow[r, "d_{0,0}^\bullet"'] & C_0^\bullet
\end{tikzcd}
$$

satisfying the three equations analagous to (5)–(7), there exists a unique 2-cell

$$
\begin{tikzcd}
X_1, X_2 \arrow[r, "{\hat{\alpha}_1, \hat{\alpha}_2}"] \arrow[d, "G"'] \arrow[dr, phantom, "\Downarrow\theta"] & C_1^2, C_2^2 \arrow[d, "\hat{F}"] \\
X_0^\bullet \arrow[r, "\hat{\alpha}_0^\bullet"'] & C_0^{\bullet 2}
\end{tikzcd}
$$

(where $\hat{\alpha}_i$ is the 1-cell determined by α_i by the universal property of the arrow object C_i) such that

$$\begin{tikzcd}
X_1, X_2 \arrow[r, "\hat{\alpha}_1{,}\hat{\alpha}_2"] \arrow[d, "G"'] \arrow[dr, phantom, "\Downarrow\theta"] & C_1^2, C_2^2 \arrow[r] \arrow[d, "\hat{F}"] \arrow[dr, phantom, "\Downarrow\gamma_i"] & C_1, C_2 \arrow[d, "F"] \\
X_0^\bullet \arrow[r, "\hat{\alpha}_0^\bullet"'] & C_0^{\bullet 2} \arrow[r] & C_0^\bullet
\end{tikzcd}
=
\begin{tikzcd}
X_1, X_2 \arrow[r] \arrow[d, "G"'] \arrow[dr, phantom, "\Downarrow\lambda_i"] & C_1, C_2 \arrow[d, "F"] \\
X_0^\bullet \arrow[r] & C_0^\bullet
\end{tikzcd}$$

for each $i \in \{0, 1, 2\}$.

Similarly, we define the lift of a vertical 1-cell $F\colon (C_1, \ldots, C_n) \to C_0^\bullet$ to arrow objects to be a vertical 1-cell $\hat{F}$ together with $(n+1)$ 2-cells γ_i satisfying $(n+1)$ equations analagous to (5)–(7) and which is universal in the analagous way.

Definition 1.4. Let $\mathbb{M}$ be a double multicategory. We say $\mathbb{M}$ *has arrow objects* if for every object C there is an arrow object C^2, and if for every vertical 1-cell $F\colon (C_1, \ldots, C_n) \to C_0^\bullet$ there is a lift to arrow objects $\hat{F}$.

We have given the universal property of arrow objects and lifts of vertical 1-cells in ordinary double multicategories, but it is clear from the cyclical symmetry of the construction that a cyclic action respects arrow objects. Specifically, $(C^2)^\bullet = (C^\bullet)^2$ for any object C, and $\sigma(\hat{F}) = \widehat{\sigma F}$ for any vertical 1-cell F.

Bibliography

[Rie13] Emily Riehl. Monoidal algebraic model structures. *Journal of Pure and Applied Algebra*, 217:1069–1104, 2013.