CSE 490 Q
Fall 2020
HW 7 part 2

Problem 2

Initial state	$\|0000\rangle$
Apply $Ry(\gamma)$ to bit 0	$\begin{bmatrix}\cos\frac{\gamma}{2} & -\sin\frac{\gamma}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\gamma}{2}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} \otimes \|000\rangle$
	$\left(\cos\frac{\gamma}{2}\|0\rangle + \sin\frac{\gamma}{2}\|1\rangle\right) \otimes \|000\rangle$
Apply CNOT to bit 1, controlled on bit 0	$\left(\cos\frac{\gamma}{2}\|00\rangle + \sin\frac{\gamma}{2}\|11\rangle\right) \otimes \|00\rangle$
Apply X to bit 0	$\left(\cos\frac{\gamma}{2}\|10\rangle + \sin\frac{\gamma}{2}\|01\rangle\right) \otimes \|00\rangle$
Apply X to bit 2	$\left(\cos\frac{\gamma}{2}\|10\rangle + \sin\frac{\gamma}{2}\|01\rangle\right) \otimes \|10\rangle$

$$|\psi(0.4\pi)\rangle = \left(\cos\frac{0.4\pi}{2}|10\rangle + \sin\frac{0.4\pi}{2}|01\rangle\right) \otimes |10\rangle = \boxed{(0.809\,|10\rangle + 0.588\,|01\rangle) \otimes |10\rangle}$$

$$|\psi(0.3\pi)\rangle = \left(\cos\frac{0.3\pi}{2}|10\rangle + \sin\frac{0.3\pi}{2}|01\rangle\right) \otimes |10\rangle = \boxed{(0.891\,|10\rangle + 0.454\,|01\rangle) \otimes |10\rangle}$$

$$|\psi(0)\rangle = \left(\cos\frac{0}{2}|10\rangle + \sin\frac{0}{2}|01\rangle\right) \otimes |10\rangle = (1\,|10\rangle + 0\,|01\rangle) \otimes |10\rangle = \boxed{|1010\rangle}$$

Problem 4

$$a_p^\dagger a_q^\dagger a_q a_p = a_p^\dagger (a_q^\dagger a_q) a_p$$

$$= a_p^\dagger \left(\left(\bigotimes_{l=0}^{q-1} Z_l \right) \otimes \left(\frac{X_q - iY_q}{2} \right) \otimes \left(\bigotimes_{l=q+1}^{n} I_l \right) \right) \left(\left(\bigotimes_{l=0}^{q-1} Z_l \right) \otimes \left(\frac{X_q + iY_q}{2} \right) \otimes \left(\bigotimes_{l=q+1}^{n} I_l \right) \right) a_p$$

$$= a_p^\dagger \left(\left(\bigotimes_{l=0}^{q-1} Z_l Z_l \right) \otimes \left(\frac{X_q - iY_q}{2} \cdot \frac{X_q + iY_q}{2} \right) \otimes \left(\bigotimes_{l=q+1}^{n} I_l I_l \right) \right) a_p$$

$$= a_p^\dagger \left(\left(\bigotimes_{l=0}^{q-1} I_l \right) \otimes \left(\frac{\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right)_q \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right)_q}{4} \right) \otimes \left(\bigotimes_{l=q+1}^{n} I_l \right) \right) a_p$$

$$= a_p^\dagger \left(\left(\bigotimes_{l=0}^{q-1} I_l \right) \otimes \left(\frac{\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}_q \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}_q}{4} \right) \otimes \left(\bigotimes_{l=q+1}^{n} I_l \right) \right) a_p$$

$$= a_p^\dagger \left(\left(\bigotimes_{l=0}^{q-1} I_l \right) \otimes \left(\frac{\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}_q}{4} \right) \otimes \left(\bigotimes_{l=q+1}^{n} I_l \right) \right) a_p$$

$$= a_p^\dagger \left(\left(\bigotimes_{l=0}^{q-1} I_l \right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_q \otimes \left(\bigotimes_{l=q+1}^{n} I_l \right) \right) a_p$$

$$= \left(\left(\bigotimes_{l=0}^{p-1} Z_l \right) \otimes \left(\frac{X_p - iY_p}{2} \right) \otimes \left(\bigotimes_{l=p+1}^{n} I_l \right) \right)$$
$$\cdot \left(\left(\bigotimes_{l=0}^{q-1} I_l \right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_q \otimes \left(\bigotimes_{l=q+1}^{n} I_l \right) \right)$$
$$\cdot \left(\left(\bigotimes_{l=0}^{p-1} Z_l \right) \otimes \left(\frac{X_p + iY_p}{2} \right) \otimes \left(\bigotimes_{l=p+1}^{n} I_l \right) \right)$$

This assumes that $p \neq q$.
From here we have two cases. In the first case, $p < q$

$$\begin{aligned}
a_p^\dagger a_q^\dagger a_q a_p = &\left(\left(\bigotimes_{l=0}^{p-1} Z_l\right) \otimes \left(\frac{X_p - iY_p}{2}\right) \otimes \left(\bigotimes_{l=p+1}^{n} I_l\right)\right)\\
&\cdot \left(\left(\bigotimes_{l=0}^{q-1} I_l\right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_q \otimes \left(\bigotimes_{l=q+1}^{n} I_l\right)\right)\\
&\cdot \left(\left(\bigotimes_{l=0}^{p-1} Z_l\right) \otimes \left(\frac{X_p + iY_p}{2}\right) \otimes \left(\bigotimes_{l=p+1}^{n} I_l\right)\right)
\end{aligned}$$

$$= \left(\bigotimes_{l=0}^{p-1} Z_l I_l Z_l\right) \otimes \left(\frac{X_p - iY_p}{2} I_p \frac{X_p + iY_p}{2}\right) \otimes \left(\bigotimes_{l=p+1}^{q-1} I_l I_l I_l\right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_q \left(\bigotimes_{l=q+1}^{n} I_l I_l I_l\right)$$

$$= \left(\bigotimes_{l=0}^{p-1} Z_l Z_l\right) \otimes \left(\frac{X_p - iY_p}{2} \frac{X_p + iY_p}{2}\right) \otimes \left(\bigotimes_{l=p+1}^{q-1} I_l\right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_q \left(\bigotimes_{l=q+1}^{n} I_l\right)$$

$$= \left(\bigotimes_{l=0}^{p-1} I_l\right) \otimes \left(\frac{\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}_p \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}_p}{4}\right) \otimes \left(\bigotimes_{l=p+1}^{q-1} I_l\right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_q \left(\bigotimes_{l=q+1}^{n} I_l\right)$$

$$= \left(\bigotimes_{l=0}^{p-1} I_l\right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_p \otimes \left(\bigotimes_{l=p+1}^{q-1} I_l\right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_q \left(\bigotimes_{l=q+1}^{n} I_l\right)$$

In the other case, where $p > q$, the math is the same but with p and q swapped.

$$a_p^\dagger a_q^\dagger a_q a_p = \left(\bigotimes_{l=0}^{q-1} I_l\right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_q \otimes \left(\bigotimes_{l=q+1}^{p-1} I_l\right) \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_p \left(\bigotimes_{l=p+1}^{n} I_l\right)$$