Counterfactual Giving

[4,791 words]

Many of us have both personal and altruistic aims. We want a good life, but we also want the end of global poverty, the preservation of civil liberties, the flourishing of the arts, and so on. Some of us face risky processes whose outcomes vary dramatically along certain dimensions. To take one fanciful possibility, someone might pursue a career in academia (low hours, low pay) with the fallback option of working in corporate law (brutal hours, high pay).

In such cases, I argue, we can better advance both our personal and altruistic aims by planning to give "counterfactually." For example, one might commit, in advance, to donating more of one's salary should one be fortunate enough to secure a position in academia, in exchange for retaining more of one's salary should one have to practice law. Or vice-versa.

This note explains how such plans can serve both our personal and altruistic aims better than alternatives, such as the popular recommendation to donate 10% of one's income. §1 walks through some examples and §2 addresses potential pitfalls.

§I: Academia or Law

The underlying observation is that one's charitable contributions matter a great deal to one's own life but barely register in large-scale altruistic projects. For example, in the US in 2023 the median salary for philosophy professors was \$79,930¹ while the median salary for lawyers was

-

¹ https://www.bls.gov/oes/2023/may/oes251126.htm

\$145,760². Meanwhile, total charitable giving in the US in 2023 was \$557 billion.³ Thus, donating all of either median salary would change the total by less than a third of one percent of one percent. For most people, donating their annual salary would transform their lives but it would be a rounding error in the overall pool.⁴

Donations, like so much else, may have diminishing marginal value, but typical individual donations are such tiny drops in the overall bucket, we can reasonably assume that they enjoy constant marginal returns. For typical donors, doubling their donation would achieve about twice as much as their original donation and halving it would achieve about half as much. (I defer caveats to §2).

This leads to two important ways in which personal consumption, or spending on oneself, and charitable contributions differ. First, the marginal value of additional consumption drops far more quickly than the marginal value of additional charitable contributions. Owning a decent laptop (≈ \$1,000) is pretty nice, but owning a top-of-the-line

² https://www.bls.gov/oes/2023/may/oes231011.htm. This value, though, significantly underrepresents what corporate lawyers make for two reasons. First, it excludes profits from equity partnerships (https://www.americanbar.org/news/profile-legal-profession/wages/). Second, law has a bimodal salary distribution: many lawyers work at large law firms ("biglaw") and make much more (with 2023 starting salaries around \$215,000), while most lawyers work elsewhere and make far less (with 2023 starting salaries around \$70,000) (https://www.nalp.org/salarydistrib#2023). Since corporate lawyers tend to work in biglaw, the median salary of corporate lawyers is higher than the median for all lawyers.

³ https://philanthropy.indianapolis.iu.edu/news-events/news/ news/2024/giving-usa-us-charitable-giving-totaled-557.16-billion-in-2023.html

⁴ Things look much the same when we look at particular cause areas or well-known organizations. (§2 considers obscure organizations).

If we distinguish between different categories of charities and focus on the category that received the least funding ("environment and animals"), then donating all of either median salary would change the total by less than ten percent of one percent (ibid).

If we look at well-known organizations (e.g. the ACLU, Met Opera, NAACP, PETA, Red Cross), donating either median salary changes the total by less than a third of a percentage point (https://projects.propublica.org/nonprofits/organizations/136213516, https://projects.propublica.org/nonprofits/organizations/131624087, https://projects.propublica.org/nonprofits/organizations/530196605).

model (≈ \$4,000) is not four times as nice. By contrast, donating \$4,000 plausibly does do about four times as much good as donating \$1,000.

Second, some people are risk-averse towards lifetime personal spending but risk-neutral towards lifetime charitable contributions. Personally, I would prefer to guarantee myself a decent retirement rather than take a 70-30 gamble between one twice as good and none at all.⁵ But I have the opposite attitude towards my charitable contributions: I would prefer a 70-30 gamble between doubling the effects of my lifetime charitable contributions and voiding those effects to a guarantee that my contributions will have certain effects.⁶ Philosophers disagree about whether such attitudes can be rational or moral, but I believe they can be and that they are sufficiently widespread to warrant discussion in the analysis to come.⁷ Readers who disagree can skip those remarks.

Now imagine choosing between:

A: I%: fund o scholarships 9%: fund 10,000 schol B: I%: fund 1,000 scholarships 9%: fund 1,000 schol

9%: fund 10,000 scholarships 9%: fund 1,000 scholarships 90%: fund 0 scholarships 90%: fund 0 scholarships

C: 1%: fund o scholarships

9%: fund 10,000 scholarships

90%: fund 10,000 scholarships

D: 1%: fund 1,000 scholarships 9%: fund 1,000 scholarships

90%: fund 10,000 scholarships

Here, I would choose A over B and C over D.

⁵ It is hard to assess such colorless choices, but the Allais paradox offers more convincing evidence of risk-aversion towards retirement. Imagine choosing between:

A: 1%: retire with \$0

^{9%:} retire with \$10 million 90%: retire with \$0

B: 1%: retire with \$1 million 9%: retire with \$1 million 90%: retire with \$0

C: 1%: retire with \$0 9%: retire with \$10 million 90%: retire with \$10 million

D: 1%: retire with \$1 million 9%: retire with \$1 million 90%: retire with \$10 million

Most people choose A over B and choose D over C. These preferences conflict with risk-neutrality and support the thought that most people are risk-averse in such choices (Allais 1953, Buchak 2013: Chapter 2).

⁶ Again, it is hard to assess such descriptions. But imagine you could fund scholarships at a vocational school for a thousand students or make investments that, with 70% probability, would fund two thousand scholarships. I would choose the latter. Or consider the analogous Allais choices:

⁷ [Cite: Buchak 2023, Zhao 2023, Makins 2023, Thoma 2023, Wilkinson 2023].

§1.1: Example 1

It is time for an example. I am going to begin with a simple example, meant to illustrate the basic idea, before considering a more complex one. And I will compare counterfactual giving to the strategy of always giving 10% of one's income, which I stipulatively term "tithing." Tithing is simple, popular, and sensible, so it serves as a good benchmark.

Suppose you are a software engineer at Waymo, the self-driving car company privately owned by Alphabet. Much of your compensation is in stock options that unfortunately expire soon. These options are worthless if Waymo does not go public this year but worth quite a lot if it does. In particular, your base salary is \$100,000 and if the company goes public this year, then your options are worth another \$100,000.8 Forecasters estimate that Waymo has a 25% chance of going public this year: it is a strong candidate, but it also competes with Cruise, Tesla, and other car manufacturers building in-house alternatives and Alphabet may want to wait before deciding whether to spin it off. In short, there is a 25% chance that you earn \$200,000 this year and there is a 75% chance that you earn \$100,000 this year.

Money has diminishing marginal utility; you can only visit paradise so many times before the visits start to blend together. In particular, you derive \sqrt{X} units of wellbeing ("utiles") from spending \$X on personal consumption. But you also care about helping others. You especially value education and the contribution it may make to alleviating global poverty, and you know of a trustworthy charity that pays for the children of poor families to attend good schools. \$500 will cover a student's annual tuition. How should you spend your income?

Tithing prescribes:

⁸ Throughout, ignore taxes and inflation.

⁹ Similar remarks apply to any strictly concave utility function; see Fact 1 of the Appendix.

¹⁰ https://blogs.worldbank.org/en/developmenttalk/high-price-education-sub-saharan-africa

Tithing	Waymo Goes Public (25% chance)	Waymo Does Not Go Public (75% chance)
Income	\$200,000	\$100,000
Personal Consumption	\$180,000	\$90,000
Charitable Contributions	\$20,000	\$10,000
Individual Welfare	424	300
Scholarships Funded	40	20

On average, this strategy yields 331 utiles and funds 25 scholarships.

This is great and far better than what most people do.¹¹ But it is also a bit inefficient. On this plan, if Waymo goes public, then you spend \$180,000 on personal consumption, and if it does not, then you consume \$90,000. Because money has diminishing marginal utility, you would rather consume a little less in the world where Waymo goes public in exchange for consuming a little more in the world where it does not. The difference between consuming \$180,000 and consuming \$179,000 matters less than the difference between consuming \$91,000 and consuming \$90,000. So if you could trade between the version of yourself that lives in a world where Waymo goes public and the version that lives in a world where it does not, you would do it and reallocate some consumption from the former to the latter.¹²

But there is a way to do this! Simply adjust your charitable contributions so that you give more money in the former world and less money in the latter. This, in effect, transfers consumption from the world where Waymo goes public to the world where it does not. Such

^{II} On average, Americans donate less than 3% of their income (https://www.philanthropyroundtable.org/almanac/statistics-on-u-s-generosity/).

¹² Ord 2015 defines moral trade as trade that is "made possible by differences in the parties' moral views" (pg. 122). On this definition, the trade between 'you in the world where Waymo goes public' and 'you in the world where Waymo stays private' is not moral trade because those parties have the same moral views.

Nonetheless, we can view this example as moral trade by taking the parties to be egoistic and altruistic parts of you. Your inner egoist wants to move some money around, and your inner altruist does not mind.

strategies can outperform tithing by both personal and altruistic criteria. For example, suppose you decide to donate ≈ 20% of your salary if Waymo goes public and 5% otherwise:

	Waymo Goes Public (25% chance)	Waymo Does Not Go Public (75% chance)
Income	\$200,000	\$100,000
Personal Consumption	\$162,500	\$95,000
Charitable Contributions	\$37,500	\$5,000
Individual Welfare	403	308
Scholarships Funded	75	IO

On average, this strategy yields 332 utiles and funds 26 scholarships, outperforming tithing on both fronts. In addition, the range of possible values for individual welfare has shrunk, which will please the risk-averse.

Let counterfactual giving denote any strategy that donates different proportions of one's income in different states of the world. On this definition, there are a *lot* of ways to counterfactually give, many of which have nothing to recommend themselves. But in general, there are also ways to give counterfactually that do strictly better than tithing.¹³ This improved performance can be channeled into greater personal consumption, greater charitable contributions, or a mix of both. Different people will allocate the surplus in different ways. For illustration, we focus on the two extremal possibilities.

The first strategy allocates all of the surplus to personal consumption; it maximizes average individual welfare subject to the constraint that one's average donation remains at least as high as under tithing. Call this strategy Selfish. The second strategy allocates all of the

¹³ The Appendix proves: if utility depends solely on personal consumption, consumption has diminishing marginal utility, and one is not certain of one's future income, then some counterfactual giving strategy outperforms tithing on average for both personal and altruistic aims (Fact I).

surplus to charity; it maximizes average donations subject to the constraint that one's average individual welfare remains at least as high as under tithing. Call this strategy Altruistic.

Here is what Selfish and Altruistic choose in our example:¹⁴

Selfish	Waymo Goes Public (25% chance)	Waymo Does Not Go Public (75% chance)
Income	\$200,000	\$100,000
Personal Consumption	\$150,000	\$100,000
Charitable Contributions	\$50,000	\$o
Individual Welfare	387	316
Scholarships Funded	IOO	0

Altruistic	Waymo Goes Public (25% chance)	Waymo Does Not Go Public (75% chance)
Income	\$200,000	\$100,000
Personal Consumption	\$141,061	\$100,000
Charitable Contributions	\$58,939	\$o
Individual Welfare	376	316
Scholarships Funded	118	0

On average, Selfish yields 334 utiles and funds 25 scholarships, whereas Altruistic yields 331 utiles and funds 29 scholarships. Thus, on average, choosing Selfishness over Tithing does 1% better for ourselves and as much good for others, while choosing Altruism over Tithing does as much good for ourselves and 18% more good for others. In addition, both Selfish and Altruistic shrink the range of possible values for individual welfare, which suits the risk-averse.

¹⁴ See the Appendix for details. It may be surprising that both strategies give \$0 if Waymo does not go public, but this happens for a now familiar reason: money has diminishing marginal utility. So you prefer to move additional consumption from the world where Waymo goes public to the world where it does not, at least until you consume equally in the two worlds.

¹⁵ These numbers are not consistent because I round to the nearest whole number. See the Appendix for exact values.

I would not read too much into these particular percentages, which depend on details of the example. Nonetheless, there are some general morals to draw. Selfish and Altruistic typically outperform Tithing, but they only noticeably outperform when one's income, as here, varies starkly across different possibilities. Such cases are not common, but they are not exactly rare either. Anyone pursuing a big promotion, risky trading strategies, new business ventures, or a new profession may find themself in a similar position.

Even in such cases, one can typically do little to increase expected wellbeing; Selfish rarely improves individual welfare by more than one percent over Tithing. But Altruistic does better and often improves expected charitable contributions by more than seven percent over Tithing. And both strategies often benefit the risk-averse by reducing the variance of individual welfare.

§1.2: Example 2

The previous example varies only one kind of good, income. Here, we consider an example with multiple goods and more complex tradeoffs.

Suppose that you are a philosophy PhD student about to head into the job market. Based on your department's placement record and your advisors' frank assessments, you believe that you have a 50% chance of eventually landing a tenure-track job. For simplicity, suppose that such jobs invariably lead to tenure, involve about twenty hours a week of teaching and administrative duties, and pay \$100,000 a year in perpetuity. In addition, sleep takes another fifty-eight hours each week and household chores take twenty, leaving you with seventy hours a week of leisure, much of which you freely choose to spend on research.

If you strike out in academia, your next best option is to return to your old job in corporate law where the hours were brutal. For simplicity, suppose that you would spend about seventy hours a week litigating cases and earn \$250,000 a year in perpetuity. Again, you need to sleep and if you do your own housework, then you wind up with twenty hours a week of leisure.

Tithing recommends that you donate \$10,000 a year if you become a philosopher and donate \$25,000 a year if you become a lawyer. Again, this is a sensible recommendation that far exceeds what most people do. But we can do better.

Differences in people's tastes and values rationalize different reactions, so we need to specify your tastes and values. Suppose that you value leisure far more than income. Your strongest desire is to think and learn about whatever piques your interests while sharing what you find. Luxury travel, gourmet meals, the idyllic cottage upstate are wonderful, but you would not trade your time for them.

Further suppose that when you have a lot of leisure time, differences in consumption matter very little to you. With seventy hours (= 10 hours a day) of leisure, you have plenty of time to read, write, watch movies, listen to music, hang out with friends, garden, and so on. Earning an extra \$25,000 (≈ \$500 per week) would not change your lifestyle by much. And if it would, you have more than enough time to take up a side gig.

By contrast, when you have very little leisure time, differences in consumption matter a lot. With just twenty hours (≈ 3 hours a day) of leisure, you find yourself scrambling to relax, see friends, go out, exercise, and so on. Spending an extra \$500 each week could cover a weekend trip, some gourmet meals, a nice spa visit, or parterre seats at the opera. Or you could

put the money towards housekeeping, food delivery, or an early retirement, thereby buying back some leisure.

Money also has diminishing marginal utility, and you would make far more in law than in academia. So the question is how the difference between consuming \$90,000 and consuming \$65,000 within a leisurely philosophical life compares to the difference between consuming \$250,000 and \$225,000 within a harried lawyer's life.

Suppose that you, like me, think that the extra money actually goes further for the rich but time-poor lawyer. Then you should re-allocate your charitable giving accordingly. For example, you might plan to give \$35,000 if you get an academic job and to give nothing if you must return to law. On average, this Selfish strategy donates as much money as tithing while making you better off. It also reduces the variance of your future wellbeing, benefiting the risk-averse. (For simplicity, I omit consideration of the Altruistic strategy.)

Alternatively, you might think the extra money actually goes further for the academic who, after all, has more time to enjoy it. That money could go towards splashier summer travel, attending more conferences, purchasing rare books, and so on. You may even think that the difference between consuming \$100,000 and consuming \$90,000 as an academic matters more than the difference between consuming \$225,000 and \$215,000 as a lawyer. (Would the lawyer even notice the difference?)

You too can benefit from counterfactual giving. For example, you might plan to give nothing if you manage to secure an academic job and to give \$35,000 if you must return to law. As before, this Selfish strategy on average donates as much money as tithing while making you better off. But unlike before, this strategy also increases the variance of your future wellbeing

since you are sweetening the best-case scenario (academia) while worsening the worst-case scenario (law). The risk-averse may prefer to steer clear.

§2: Possible Pitfalls

These examples illustrate the potential of counterfactual giving, which in theory nearly always outperforms tithing. In practice, though, certain pitfalls make tithing often the wiser choice. Counterfactual giving (i) has implementation costs, (ii) requires special information, (iii) is vulnerable to insincerity, (iv) distorts incentives, and (v) assumes that donations have constant marginal value.

Accordingly, counterfactual giving should be used sparingly. It is most suitable in cases where agents (i) face a risk whose outcomes vary significantly, (ii) have good evidence about the possible outcomes, (iii) can credibly commit to a plan, (iv) will not distort incentives by adopting a plan, and (v) give relatively small contributions to a much larger pool. In what follows, I examine these pitfalls more closely.

First, counterfactual giving has implementation costs. It requires assessing the likelihoods of various outcomes and often requires considering how happy one would be in various lives, as in our second example. It is generally not worth spending the time to consider these questions for minor uncertainties, e.g. whether one gets the corner office. One reason is that frivolous commitments can easily do more harm than good. A casual commitment to donate more if one gets the corner office and less otherwise may lead to reneging if one actually does get the office and to donating less if not. Decision fatigue and the paradox of choice add to these costs. As a rule, we should only consider counterfactual giving for important decisions.

Second, counterfactual giving requires information about the likelihoods of various outcomes that can be costly or impossible to acquire. When I was eight, I could have considered the odds of becoming an inventor, janitor, astronaut, and so on and formulated an elaborate strategy of counterfactual giving in response. But my evidence for any of this would have been slim to none, so forming a plan on that basis would have been foolish. Moreover, when the evidence is slim, it is easy to distort the likelihoods in self-favoring ways; maybe my odds of becoming a billionaire are not so low after all, so my willingness to donate my counterfactual fortune actually counts for quite a lot! Furthermore, there is typically no harm in waiting and acquiring more information about the world or one's tastes, values, and prospects before committing to a plan. For these reasons, it is good to consider counterfactual giving only in cases where we have good evidence about the likelihoods of various outcomes, and we should often wait to acquire such evidence before committing.

Third and perhaps most importantly, counterfactual giving is vulnerable to insincerity. It is easy to promise to give more if something good happens and less otherwise. Talk is cheap. But this can easily lead to people reneging on the additional donations if the good event actually happens while happily donating less if it does not. So one must consider whether one can be trusted to follow through.

Commitment devices, such as publicly committing to the plan, asking friends to verify compliance, or automating aspects of the plan, can help. Alternatively, one can reduce the size of one's commitments; instead of committing to give an additional \$25,000 each year in perpetuity, one might commit to giving an additional \$10,000 for the next three years. This reduces the potential upside, but the gains in probable compliance can be worth it.

Fourth, counterfactual giving can distort one's incentives. ¹⁶ Suppose I commit to giving away all of my worldly assets should I receive a fellowship offer from All Souls. This might well affect my behavior; I might not apply for fear of getting an offer! Less fancifully, committing to give more of one's income if one earns more can affect one's motivation to earn more. Overtime is less enticing when much of the extra money goes to charity.

One can try to take these effects into account, but doing so complicates an already complicated deliberation. I suggest restricting counterfactual giving to cases where it does not affect one's incentives. These are not as uncommon as it might sound.

One such case arises when the risky event is largely independent of one's actions, as in our first example where the risk concerns whether Alphabet lists Waymo as a publicly traded company. That is not something an entry-level software engineer can do much to affect. Another such case arises when adopting the strategy leaves one's preferences largely unchanged, as in our second example where one strongly prefers academia to law both before and after adopting a course of counterfactual giving. In such cases, even after adopting the plan, one will continue to doggedly pursue one's favored option.

Fifth, the rationale for counterfactual giving assumes that personal consumption has diminishing marginal returns while donations have constant marginal returns: the last dollar spent on oneself produces less wellbeing than the first, but the last dollar donated produces as much good as the first. The idea behind the latter claim is that charities which typically receive tens or hundreds of millions of dollars in donations have standing plans for allocating those donations, and these plans are largely unchanged by receiving an extra twenty or thirty thousand dollars. A little more money, in expectation, just produces a little more of the same.

¹⁶ This is also true of tithing.

This is not plausible when donating massive sums or when donating to highly specific causes and small charities. The former complication applies primarily to foundations and the überwealthy, but the latter applies more broadly. You might care enormously about a cause that you can realistically affect, e.g. whether the local library can afford new computers. Such cases often involve minimums that must be reached for any good to result (e.g. the minimum number of computers that qualifies for an institutional discount) and maximums beyond which additional contributions do little. If the library needs \$20,000 to replace twenty old computers, then one may sensibly prefer to guarantee that they receive \$20,000 rather than to give them a 50-50 shot of getting \$50,000. After all, the library may have no comparably pressing use for the extra \$30,000. In such cases, determining the optimal counterfactual giving strategies becomes considerably more complicated.

Conclusion

Counterfactual giving can outperform tithing by both personal and altruistic criteria. But applying these strategies introduces new dangers that must be managed carefully and that require paying close attention to the details of a particular case.

It would be nice to have a general rule that is as simple as tithing yet outperforms it on average. One natural candidate is "variable tithing," where the percentage of income that one donates depends on one's income.¹⁷ For example, one could donate five percent of the portion of one's income below the average income and donate fifteen percent of the portion that exceeds it. If everyone did this, then people would collectively donate ten percent of all income

¹⁷ For precursors, see Sider 1977: 129 – 133's discussion of 'graduated tithing,' and Mesar 1982: 23's discussion of 'progressive tithing.' Singer 2009: 160 – 168 also proposes a particular variable tithing scheme, and his website https://www.thelifeyoucansave.org/ has recommendations for the appropriate percentages at different income levels. These authors do not, however, discuss how prior commitment to a particular scheme can be better for both oneself and others.

while being better off, on average, than if everyone tithed. Alternatively, in jurisdictions with progressive taxation, one can adopt a fixed rule (e.g. donate a third of one's tax liability) to similar effect. Whether any such rule proves superior, in general, to tithing remains to be seen.

Appendix

We prove Fact I and derive the Selfish and Altruistic prescriptions from Example I.

Fact 1

Let U: R \rightarrow R be strictly concave and let Pr: R \rightarrow [0, 1] be a probability function with finite support. If Pr(·) is non-trivial, then there exists a function g: R \rightarrow [0, 1] such that:

$$\sum_{r \in \mathbb{R}} \Pr(r) U((1 - g(r))r) > \sum_{r \in \mathbb{R}} \Pr(r) U(.9r)$$

$$\sum_{r \in \mathbb{R}} \Pr(r)g(r)r > \sum_{r \in \mathbb{R}} \Pr(r).1r$$

Proof

We construct such a $g(\cdot)$. Since $Pr(\cdot)$ is non-trivial, there exist $r_1 > r_2$ such that $Pr(r_1)$, $Pr(r_2) > o$. Intuitively, we want to give slightly more if we earn r_1 and slightly less if we earn r_2 , which increases our average utility because money has diminishing marginal utility. This allows us to then donate slightly more while still doing better for ourselves than under tithing.

Let
$$d = \min\{\Pr(r_1) \frac{.9(r_1 - r_2)}{2}, \Pr(r_2) \frac{.9(r_1 - r_2)}{2}\}$$
. Let $\epsilon_1 = \frac{d}{\Pr(r_1)}$ and $\epsilon_2 = \frac{d}{\Pr(r_2)}$.

Since U(·) is strictly concave and $.9r_1 - \epsilon_1 \ge .9r_2 + \epsilon_2$:

$$\Pr(r_1)U(.9r_1 - \epsilon_1) + \Pr(r_2)U(.9r_2 + \epsilon_2) > \Pr(r_1)U(.9r_1) + \Pr(r_2)U(.9r_2)$$

Since $U(\cdot)$ is continuous, there exists $\delta > 0$ such that:

$$\Pr(r_1)U(.9r_1 - \epsilon_1 - \delta) + \Pr(r_2)U(.9r_2 + \epsilon_2 - \delta) > \Pr(r_1)U(.9r_1) + \Pr(r_2)U(.9r_2)$$

One can then verify that
$$g(r) = \begin{cases} \frac{.1r_1 + \epsilon_1 + \delta}{r_1} & \text{if } r = r_1 \\ \frac{.1r_2 - \epsilon_2 + \delta}{r_2} & \text{if } r = r_2 \end{cases}$$
 satisfies the two inequalities.

Details for Example 1

Let c denote personal consumption and d denote donations (both in dollars). In Example 1:

$$Pr(Income = \$100,000) = .75$$

$$Pr(Income = \$200,000) = .25$$

In general, the agent chooses a strategy that specifies a bundle (c_y, d_y) for how much to consume and donate at each income level y. In our setting, the agent chooses what to do if she makes \$100,000 and if she makes \$200,000:

$$(c_{100,000}, d_{100,000})$$

$$(c_{200.000}, d_{200.000})$$

These choices must be feasible. In particular, the agent cannot spend more than their income:

$$100,000 \ge c_{100,000} + d_{100,000}$$

$$200,000 \ge c_{200,000} + d_{200,000}$$

and the agent cannot consume or donate negative amounts:

$$c_{100,000}, c_{200,000} \ge 0$$

$$d_{100,000}, d_{200,000} \ge 0$$

The agent's utility function is:

$$U(c,d) = \sqrt{c}$$

Let us now look at the different strategies.

In general, Tithing sets $(c_y, d_y) = (.9y, .1y)$. In our example,

$$(c_{100,000}^T, d_{100,000}^T) = (90000, 10000)$$

$$(c_{200,000}^T,d_{200,000}^T) = (180000,\!20000)$$

where the superscript *T* denotes that these prescriptions come from Tithing. This yields:

Tithing Expected Utility:
$$.75\sqrt{90,000} + .25\sqrt{180,000} \approx 331.07$$

Tithing Expected Donation:
$$.75(10,000) + .25(20,000) = 12,500$$

The Selfish strategy maximizes expected utility subject to the constraint that one donates, on average, no less than in Tithing. In other words, Selfish picks $(c_{100,000}, d_{100,000}), (c_{200,000}, d_{200,000})$ that maximize:

$$.75U(c_{100,000}) + .25U(c_{200,000})$$

subject to the constraint:

$$.75d_{100,000} + .25d_{200,000} \ge .75(.1 \times 100,000) + .25(.1 \times 200,000) = 12,500$$

along with the other constraints from before:

$$100,000 \ge c_{100,000} + d_{100,000}$$

$$200,\!000 \geq c_{200,000} + d_{200,000}$$

$$c_{100.000}, c_{200.000} \ge 0$$

$$d_{100,000}, d_{200,000} \ge 0$$

Solving this yields:

$$(c_{100,000}^S, d_{100,000}^S) = (100000,0)$$

$$(c_{200,000}^S, d_{200,000}^S) = (150000,50000)$$

where the superscript S denotes that these prescriptions come from Selfish. This yields:

Selfish Expected Utility:
$$.75\sqrt{100,000} + .25\sqrt{150,000} \approx 334$$

Selfish Expected Donation:
$$.75(0) + .25(50,000) = 12,500$$

The Altruistic strategy maximizes expected donations subject to the constraint that one is at least as well off, on average, as under Tithing. In other words, Selfish picks ($c_{100,000}$, $d_{100,000}$), ($c_{200,000}$, $d_{200,000}$) that maximize:

$$.75d_{100,000} + .25d_{200,000}$$

subject to the constraint:

$$.75U(c_{100.000}) + .25U(c_{200.000}) \ge .75U(.9 \times 100,000) + .25U(.9 \times 200,000) \approx 331.07$$

along with the other constraints from before:

$$100,000 \ge c_{100,000} + d_{100,000}$$

$$200,000 \ge c_{200,000} + d_{200,000}$$

$$c_{100,000}, c_{200,000} \geq 0$$

$$d_{100.000}, d_{200.000} \ge 0$$

Solving this yields:

$$(c_{100,000}^A, d_{100,000}^A) = (100000,0)$$

$$(c_{200,000}^A,d_{200,000}^A) = ((900 + \sqrt{180000} - 3\sqrt{100000})^2,200000 - (900 + \sqrt{180000} - 3\sqrt{100000})^2)$$

where the superscript *A* denotes that these prescriptions come from Altruistic. This yields:

Altruistic Expected Utility:
$$.75\sqrt{100,000} + .25\sqrt{(900 + \sqrt{180000} - 3\sqrt{100000})^2}$$

Altruistic Expected Donation:
$$.75(0) + .25(200000 - (900 + \sqrt{180000} - 3\sqrt{100000})^2)$$