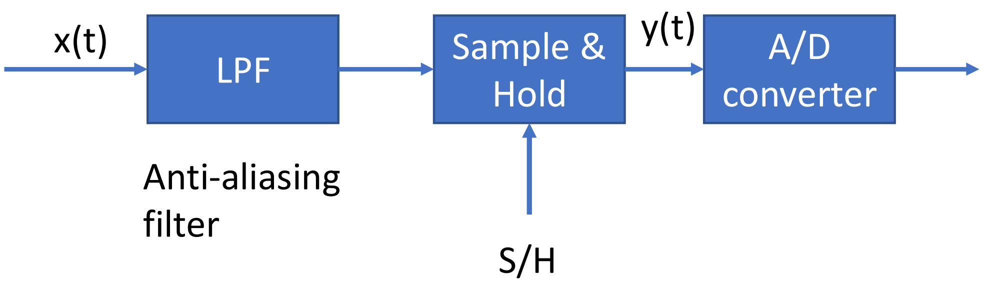
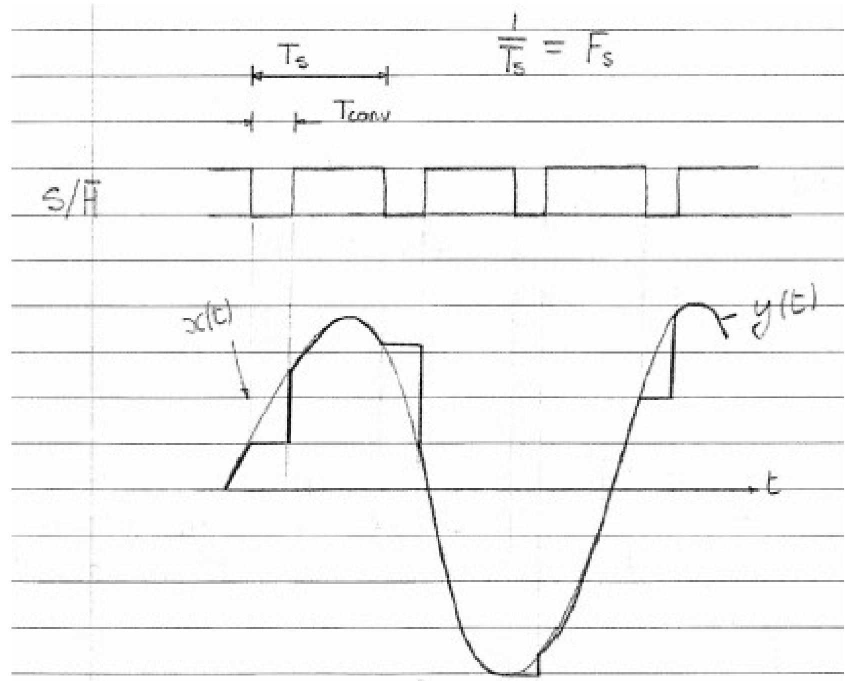
EE357 – Signals & Communications

Section 5 – Sampling and Quantisation

Quantisation (see Stremler section 9.4)





Quantisation

- The incoming signal is quantised to a certain number of voltage levels, and each voltage level is assigned a unique binary code by the ADC

$$No.of\ levels = 2^B$$

B = number of bits

e.g. $B\ =\ 8 \Rightarrow 256\ levels$

- Quantisation step size $\Delta = \frac{2A}{2^B - 1}$

Where A is the amplitude of a full scales sinusoid

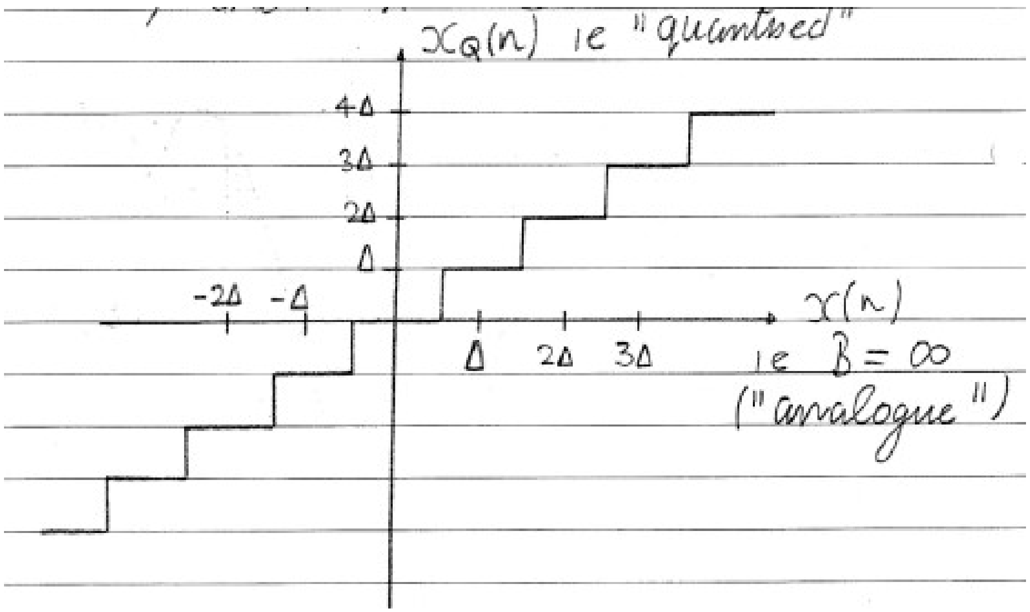
Quantisation - example

- If an ADC has an input range of $\pm 5V$, then A=5V
- $x_Q(n)$ takes on a value of $k\Delta$, if the input voltage is between $k\Delta - \frac{\Delta}{2}$ and $\mathrm{k}\Delta + \frac{\Delta}{2}$
- i.e. the “quantisation error” is

$$e(n) = |x(n) - x_Q(n)| \leq \frac{A}{2}$$

Or

$$-\frac{\Delta}{2} \leq e(n) \leq \frac{\Delta}{2}$$



Quantisation noise

- Assuming that the errors are equally likely to occur anywhere in this range, we can write a probability density function for $e(n)$ as

$$P[e(n)] = \frac{1}{\Delta}, \qquad -\frac{\Delta}{2} \leq e(n) \leq \frac{\Delta}{2}$$

$$\overline{e(n)} = 0$$

Variance:

$$\overline{e^2(n)} = \int_{-\infty}^{\infty} p[e]e^2 de - \overline{e(n)}$$

$$\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{1}{\Delta} e^2 de = \frac{\Delta^2}{12}$$

$$\Delta = \frac{2A}{2^B - 1} \Rightarrow \overline{e^2(n)} = \frac{4A^2}{12(2^B - 1)^2}$$

- Mean squared value of a full scale sinusoid is

$$\frac{A^2}{2}$$

- We can use this to calculate the Signal to Quantization Noise ratio…

Signal to Quantisation Noise

- "Signal to Quantisation Noise" ratio is

$$SQNR = \frac{\frac{A^2}{2}}{\frac{4A^2}{12(2^B-1)^2}}$$

$$= \frac{12(2^B-1)^2}{2.4}$$

$$= \frac{3}{2}(2^B-1)^2$$

- Take logs to get

$$SQNR_{dB} = 10\log_{10}\frac{3}{2} + 10\log_{10}(2^B-1)^2$$

$$= 10\log_{10}\frac{3}{2} + 20\log_{10}(2^B-1)$$

$$\approx 10\log_{10}\frac{3}{2} + 20\log_{10}2^B$$

$$= 6B + 1.8dB$$

e.g. 12 bit A/D converter has a theoretical SQNR of 73.7dB

Pulse Code Modulation (PCM)

- Samples of analogue signal are converted into binary numbers
- Widely used in telephony, where each binary number representing a sample is converted into serial form, and transmitted along a telephone line

$$Bit\ rate = B \times F_s$$

$$B - no.of\ bits/sample$$
$$F_s\ - sampling\ rate$$

- Telephony: $F_s = 8kHz$
- Where Δ is fixed, we have linear or uniform PCM
- Problem: SQNR decreases with decreasing signal level

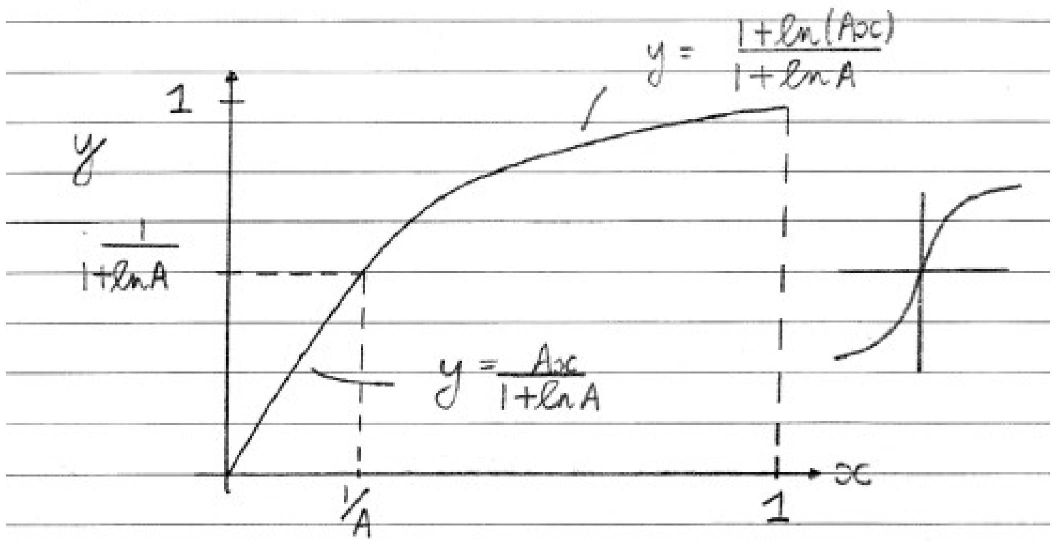
PCM – Problems with speech

- Problems
  - i) power is quite low compared to peak value (modulation index)
  - ii) variations in volumes between speakers
  - If linear uniform PCM is used, SQNR varies depending on volume
- Solution
  - Non-uniform quantisation i.e. small step sizes for low amplitudes, larger step sizes for larger amplitudes
  - Result: SQNR is roughly constant over entire range of signal

Log PCM

- Two variants:
  - A-Law (Europe)
  - µ-Law (USA)

- Note: x and y are normalised
- Typical values: A=87.6, B=8,Fs=8k
- ⇒64kbits/s "toll quality"
- Equivalent to 12 bit uniform PCM



Practical Implementation

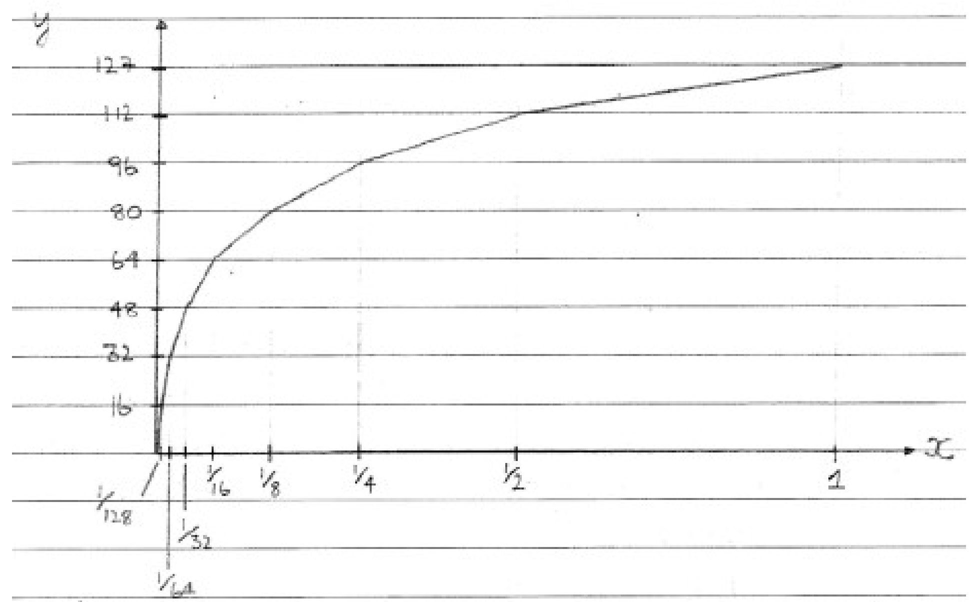
- In practice, a piecewise linear approximation is used for the compression equations
- Positive part: 8 linear segments
- Note: compression→expanding = “companding”

Practical Implementation

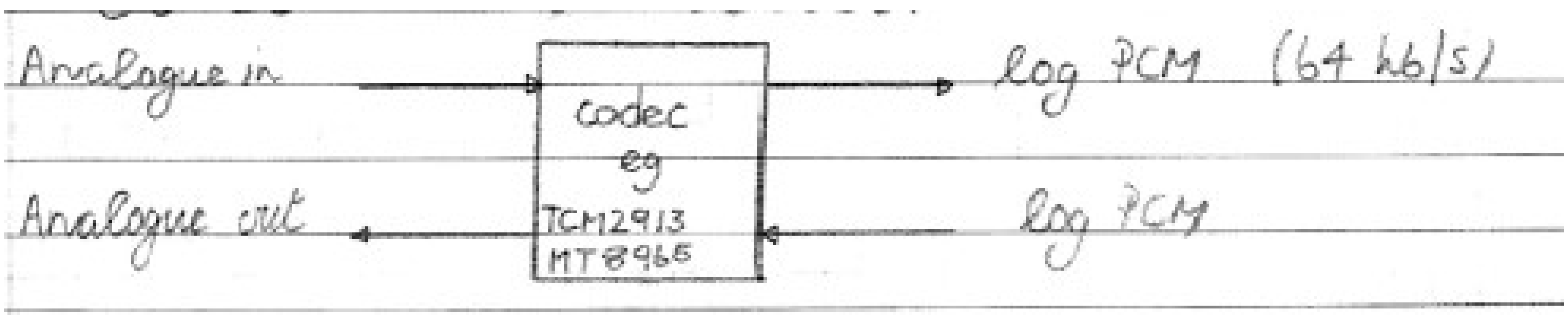
- First segment: codes 0 to 15 assigned uniformly to sample values between 0 and 1/128 of the max positive input
  - ⇒ equivalent to 12-bit linear PCM
- Second segment: codes 16 to 31 assigned to voltage between 1/128 and 1/64 of max input
  - ⇒ equivalent to 12-bit linear PCM

Etc…

- Codes 112 to 127 assigned uniformly to inputs between ½ and 1
  - ⇒ 6-bit linear PCM



“Codec” – coder/decoder



SQNR for arbitrary Signals

- Previously, we calculated the SQNR for a linear ADC, for the case of a signal consisting of a single sinewave (about as simple as it gets)
- We then looked at specific techniques for non-uniform quantisation of speech
- Here, we revisit uniform quantisation, but without placing any constraints on the nature of the signal
- We are interested in
  - Determining the SQNR for a given number of bits
  - Determining the required number of bits for a desired SQNR

SQNR for arbitrary Signals - analysis

- For a given number of bits, the SQNR is simply

$$SQNR = (V_{RMS})^2/\overline{e^2(n)}$$

Where V_{RMS} is the signal RMS value, and $\overline{e^2(n)}$ is the mean squared value of the quantisation noise, and is equal to $\frac{\Delta^2}{12}$

- Example:
  - Calculate SQNR for an ADC with B=12bits, an input voltage range of ±3V, and for an input signal with a peak value of 2.4V and a peak-to-RMS ratio (PRR) of 1.9

SQNR for arbitrary Signals - analysis

$$V_{RMS} = \frac{V_{peak}}{PRR} = \frac{2.4}{1.9} = 1.263V$$

$$\Delta = 2 \times \frac{3}{2^{12} - 1} = 1.46mV$$

$$\Rightarrow \overline{e^2(n)} = \frac{\Delta^2}{12} = 0.178\mu V^2$$

$$\Rightarrow \mathrm{SQNR} = \frac{(1.263)^2}{0.178 \times 10^{-6}} = 8.962 \times 10^6$$

$$= 69.5dB$$

SQNR for arbitrary Signals - design

- To calculate the number of bits required to achieve a certain minimum SQNR, we basically work backwards
- i.e. we need to determine the maximum permissible value of $\overline{e^2(n)}$, then determine Δ, and hence determine B

Example - design

- Example:
  - V_{peak}=2.3V, PRR=3.9, desired SQNR≥70dB
  - ADC input range = $\pm$2.5 (i.e. A=2.5)
- Step 1: Determine V_{RMS}

$$V_{RMS} = \frac{2.3}{3.9} = 0.59V$$

- Step 2: from V_{RMS}, and SQNR, determine $\overline{e^2(n)}$

$$SQNR = \frac{(V_{RMS})^2}{\overline{e^2(n)}} \geq 10^7 \; (i.e. 70dB)$$

$$\Rightarrow \frac{(0.59)^2}{e\text{^}2(n)} \geq 10^7$$

$$\overline{e^2(n)} \leq 0.0348 \times 10^{-6}$$

SQNR for arbitrary signals

- Step 3. Given $\overline{e^2(n)}$, determine upper limit on Δ

$$\overline{e^2(n)} = \frac{\Delta^2}{12} \leq 0.0348 \times 10^{-6}$$

$$\Rightarrow \Delta \leq 0.646mV$$

- Step 4. From Δ and the ADC input voltage range, determine B

$$\Delta = \frac{2A}{(2^B - 1)} \leq 0.646mV$$

$$\Rightarrow 2^B - 1 \geq \frac{2(2.5)}{0.646 \times 10^{-3}}$$

Solve to get $B \geq 12.91\ i.e. B = 13$