Ps-3

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Abstract

This document contains results of ps_4. The code written to solve the problems could be found in my github: patrick612. Codes for solutions to all problems could be found in ps_4.py.

1

A function using Gaussian quadrature was defined where N sample points were found by approximating the roots of Legendre Polynomials. This allows us to find the integral by finding an exact integral up to 2N-1 order polynomial. The domain of integration was also made to fit the problem at hand. Finally, all the integration constants were fixed to unity during the integral then scaled up afterwards. Figure 1 shows how the specific heat varies with respect to temperature when number of sample points was held at N = 50. Furthermore, the integral of specific heat at $T = 50K$ was done at different number of sample points as shown in Figure 2. We see that the integral already converges at N = 10 with accuracy up to 10E-6 decimal point accuracy.

2

From $E = \frac{1}{2}m(\frac{dx}{dt})^2 + V(x)$ and $E = V(a)$, we see that $\frac{2(V(x)-V(a))}{m} = (\frac{dx}{dt})^2$. $\sqrt{\frac{2}{m}(V(a) - V(x))} = \frac{dx}{dt}$. $\int_0^{\frac{T}{4}} dt = \sqrt{\frac{m}{2}} \int_0^a \frac{dx}{\sqrt{V(a)-V(x)}}$. Thus, $T = \sqrt{8m} \int_0^a \frac{dx}{\sqrt{V(a)-V(x)}}$. Next, using the Gaussian Quadrature method, the integral of period was found at varying amplitudes with the potential of $V(z) = x^4$. In Figure 3, mass was set $m = 1$ and the unit was assumed as kg for mass and m for length.

Figure 3 shows that the period of oscillation decreases with increasing amplitude and also that the period of oscillation divergent 0 amplitude. When the potential has higher order dependence above quadratic, the anharmonic oscillator has non-linear dependence of restorative force on displacement. Therefore the period depends on the amplitude of oscillation. Furthermore, it is easy to see from the integral of period found in part a) that if the potential is of order 2 $V(x)\alpha x^2$, the integral is a constant regardless of choice of amplitude, where as higher order dependence will leave terms dependent on the amplitude. Since for $V(x)\alpha x^4$ the force on the particle increases exponentially with increase of amplitude, it takes less time to complete a cycle of oscillation. On the other hand, at very low amplitudes, the time of oscillation exponentially increases. Therefore when the amplitude approaches 0, the period diverges.

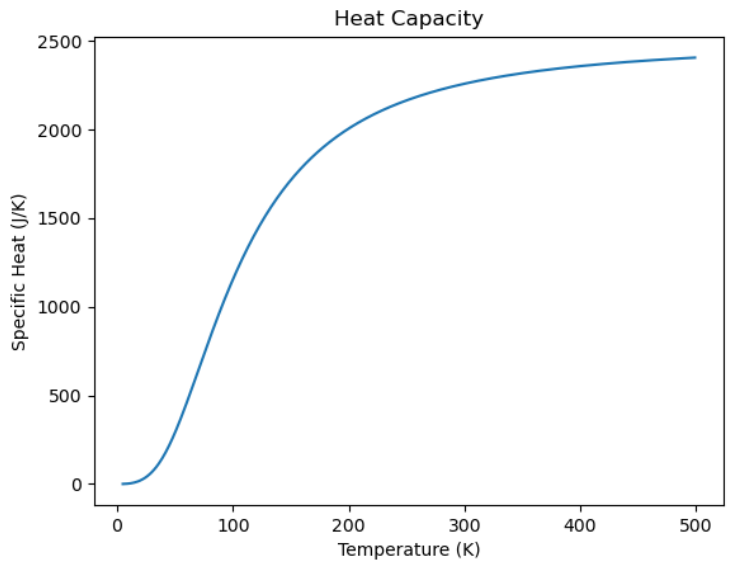

Figure 1: Specific heat found via integration using Gaussian Qaudrature, at different temperatures using N = 50.

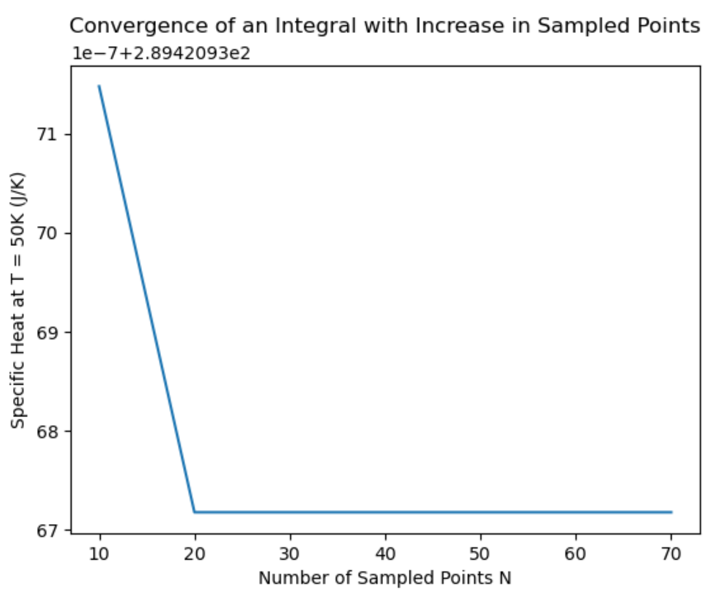

Figure 2: Specific heat at T = 50K for different number of sampled points.

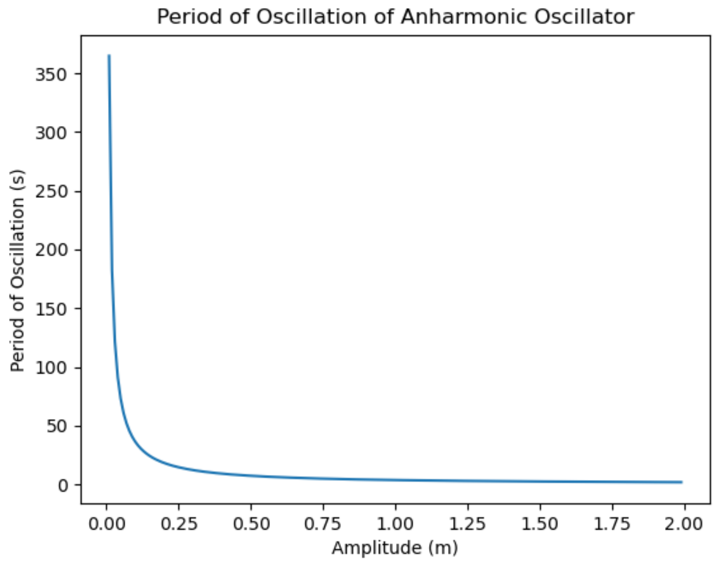

Figure 3: Period of anharmonic oscillator with V(x) = $v(x)$ = x^4 potential. Sample points were held at N = 20 and amplitudes were varied between [0,2]

Gaussian Quadrature	Gauss-Hermite Quadrature
2.3452078737858177	2.3452078799117135

Table 1: Quantum uncertainty of 5th energy level of a harmonic oscillator

3

Using the Hermite polynomial the first 4 probability distributions were found as shown in Figure 4. Furthermore, the uncertainty of the 5th eigenstate of the harmonic oscillator was found using both Gaussian quadrature and Gauss-Hermite quadrature as shown in Table 1. We could observe that the Gauss-Hermite quadrature converges to solution at a much lower number of sample points. This is because the Gauss-Hermite quadrature integrates the integral containing the weight function $f(x) = e^{x^2}$ up to polynomial degree $2n - 1$ for H_n

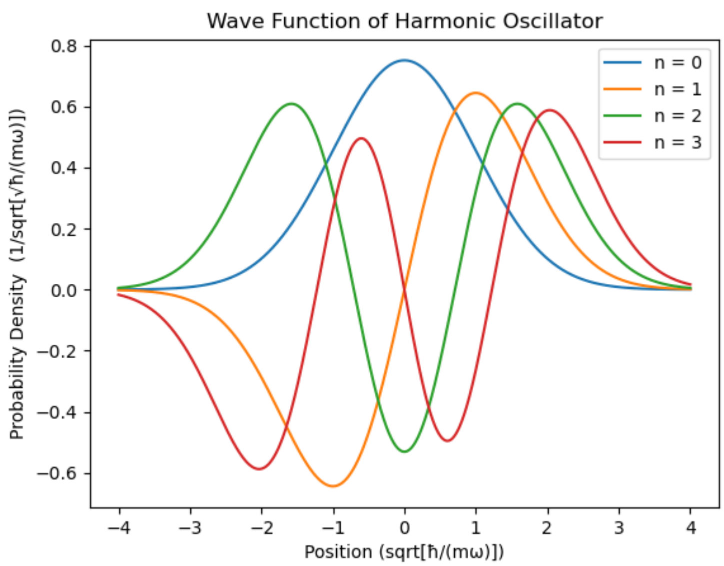

Figure 4: Probability density distribution of a quantum harmonic oscillator

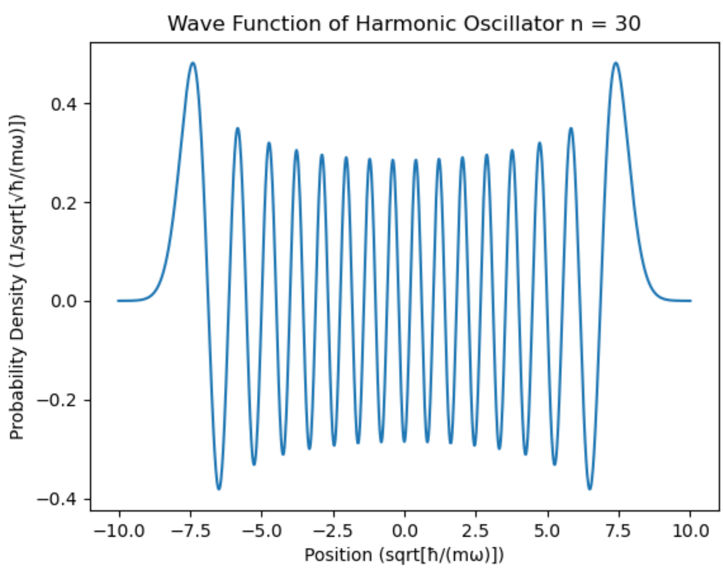

Figure 5: Enter Caption