Ps-5

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Abstract

This document contains results of ps_5. The code written to solve the problems could be found in my github: patrick612. Codes for solutions to all problems could be found in ps-5.py.

1

If we plot the integrand for different values of a = 2, 3, 4 as shown in Figure 1, we see that the curve starts from 0 then peaks then decays. Also as the parameter a increases the point at which the curve decays. Furthermore, by setting the first derivative of the integrand to zero, we can identify the extremum at which the integrand starts to decay as shown below.

$$\frac{d}{dx}(x^{a-1}e^{-x}) = e^{-x}x^{a-1}((a-1)x^{-1}-1) = 0$$

$$\Rightarrow x = a-1$$

Next, we see that at $x = c, z = 1/2$. This means that as we re parameterize the integral domain to [0,1], the mid point of the domain corresponds to c on the original domain. Therefore, to place the peak of the integrand in the middle of new domain, we should set $c = a-1$.

In order to write the gamma function using an integral, we first represent the integrand as $e^{(a-1)lnx-x}$. This reduces the error arising from multiplying a very large number by a very small number by first calculating the exponential factor via addition then calculating the exponent. Gamma function was written using the Gaussian quadrature method where the integral domain was changed to $[0,1]$ and the maximum of the curve brought to the middle by setting $c = a-1$. By calculating $\Gamma(3/2)$ we retrieve the value 0.8862285387850651. Furthermore, table below shows the solution for gamma functions of integers. When compared to the factorial value we see that the resulting values from the gamma function come close at the accuracy of order $10e-8$ of the calculated value.

Number	Factorial	Gamma Function(Number+1)
2	2	2.0000000039983
5	120	119.9999999983641
9	362880	362880.0001680839

Table 1: Quantum uncertainty of 5th energy level of a harmonic oscillator

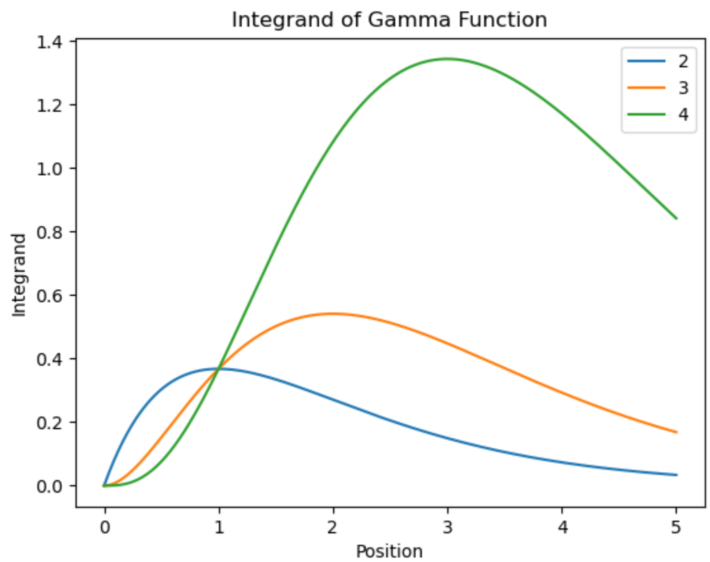

Figure 1: Graph of the integrand of Gamma function.

2

The data was first plotted as shown in Figure 2 below. Then singular value decomposition was used to find the best fit to the third power in polynomial. The resulting coefficients for each polynomial term is shown in Table 2 and the fitted curve in Figure 3. We notice that the scaling coefficients for the linear term is the greatest. This implies that the found fit is almost linear. The residuals which compares the difference in predicted data according to SVD and the actual data are shown in Figure 4. We can see that altough the SVD fitting removed the slight upward slant the fitting did a very poor job of representing the data since the variance of the residual is as big as the variance of the data. This means that out model has severely under fitted the actual data.

Next, I have tried fitting higher order polynomials using SVD. We see from figure 5 that although the fitted curve seems sinusoidal, there is an upward spike near the end due to the nature of polynomials. Also from Figure 6, the condition number increases with the polynomial degree and the condition number is of order 1e16. The increasing in trend is due to over fitting.

Finally, the signal data was fitted using SVD in sinusoidal basis. Specifically, sinusoids with frequencies corresponding to $\omega = n * 2 * \pi * \frac{1}{halftime}$ where $halftime = 0.5e8$ is the half of the total time of the raw data and n is an integer taken from 1 to 11. Figure 7 shows the curve corresponding to fitted data in comparison to the raw data. We see that the periodicity generally matches. Furthermore the condition number is 1.6429039500967997 which implies a much better fitting than the polynomial SVD

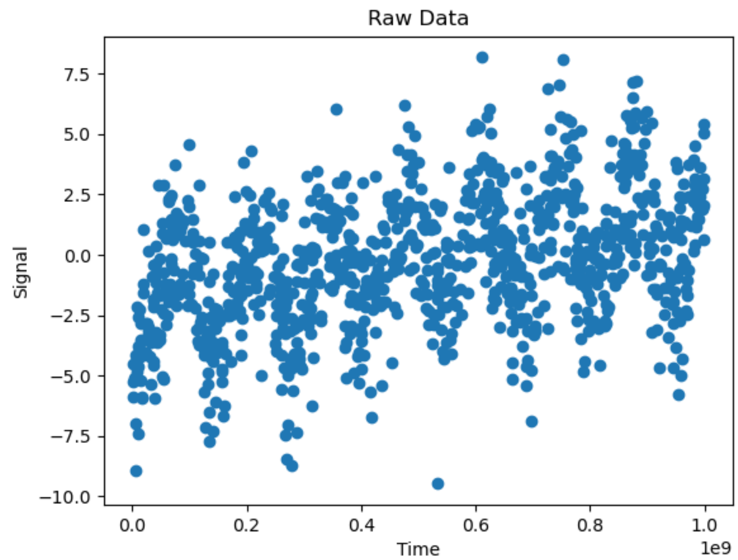

Figure 2: Plot of raw data.

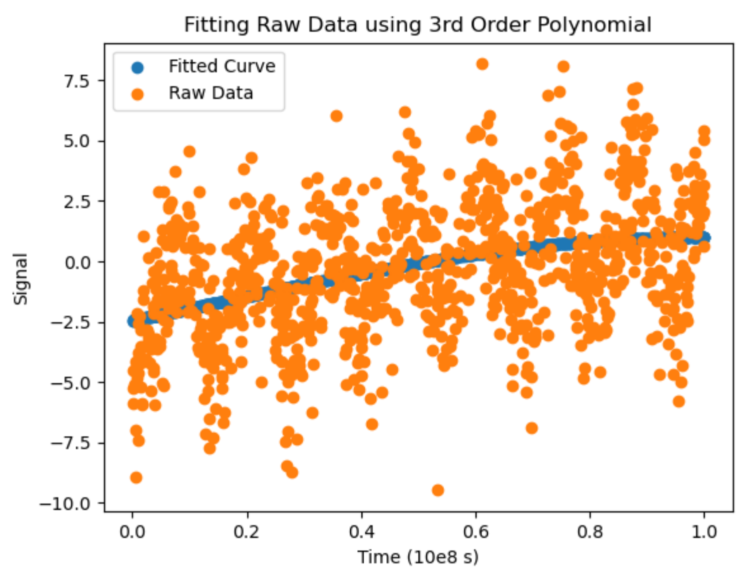

Figure 3: Raw signal and fitted curve using SVD on third order polynomial

Contant	Coeff of x	Coeff of s^2	Coeff of x^3
-2.44865508	4.94947851	0.73076754	-2.23973372

Table 2: SVD fitting of raw data to third order polynomial.

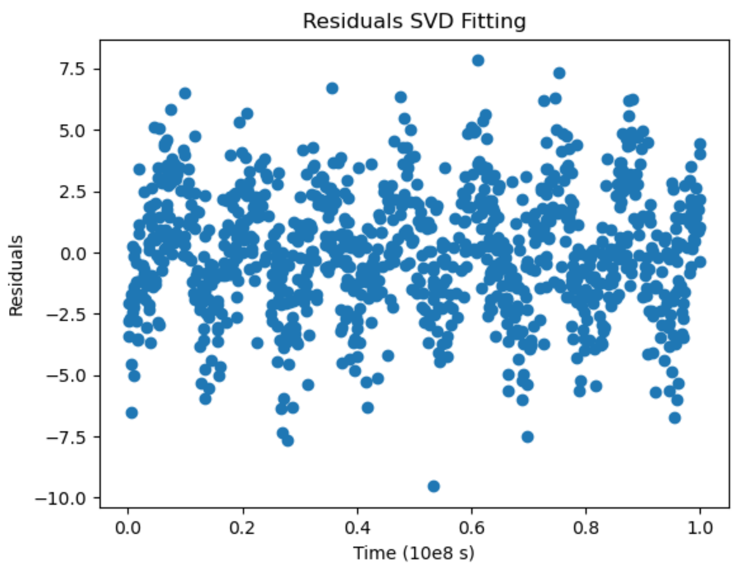

Figure 4: Residual for fitting using SVD up to third order in polynomial

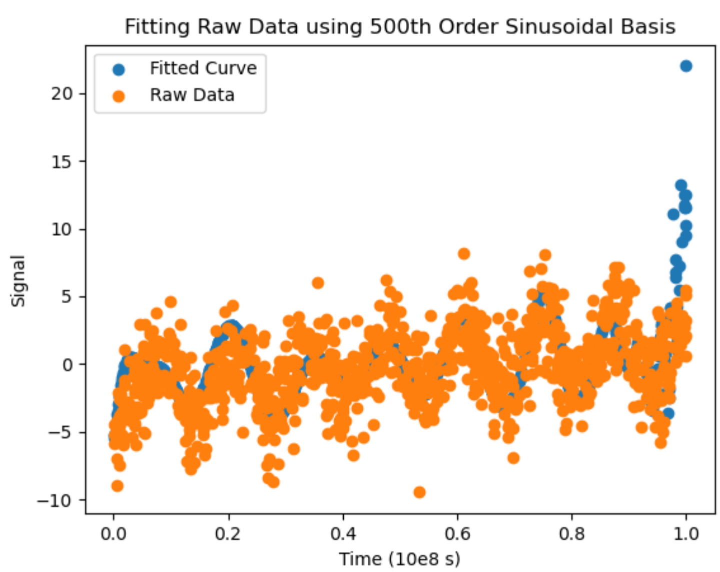

Figure 5: SVD fitting of Polynomial of degree 500

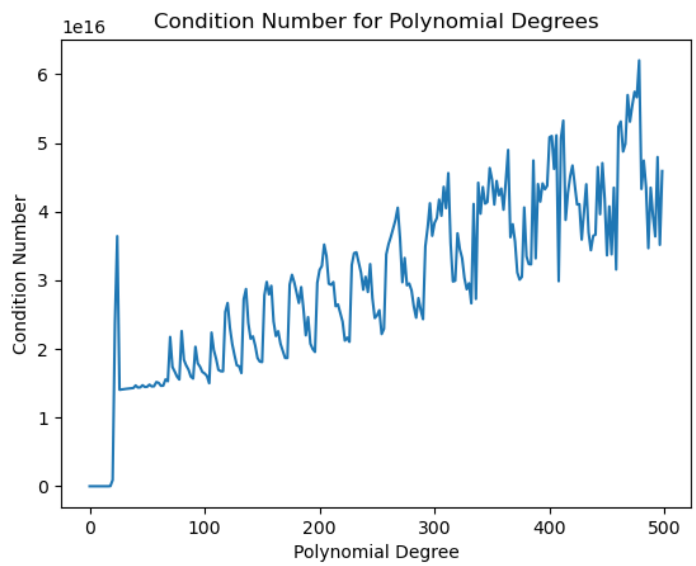

Figure 6: Condition Number for different polynomial degrees

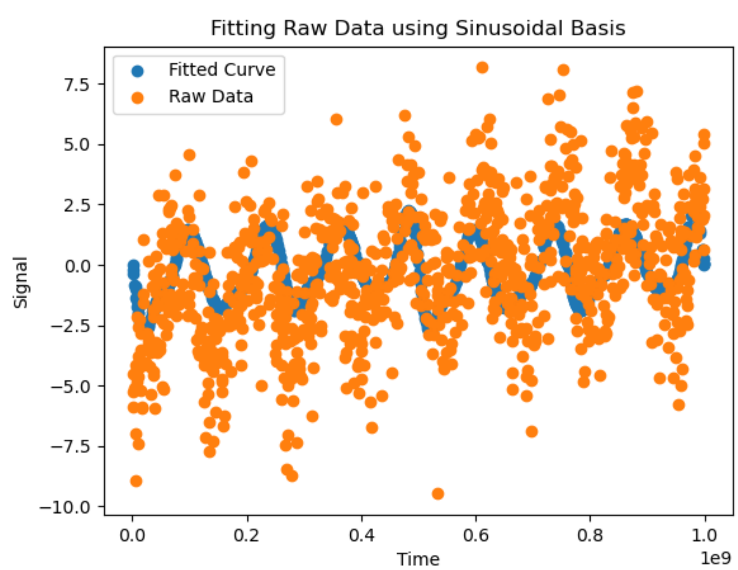

Figure 7: Raw data and fitted curve.