DIPLOMARBEIT

HOW RISKY ARE REVERSE MORTGAGES? A PRICING AND DYNAMIC STOCHASTIC SIMULATION MODEL

Prof. Dr. Klaus Adam
Chair of International Economics
Department of Economics
University of Mannheim

Advisor:

Prof. Dr. Klaus Adam
Dr. Viktor Winschel

submitted by:

Patrick Böert
Feldzeugmeisterstr. 4
10557 Berlin
Matrikel 977031

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Chapter 1

Introduction

Whether to rent, lease or own an asset has no implications for the value of the consumption derived from an asset. This is true for most asset types. Housing, however, is different. The difference lies in the fact that people usually regard the housing experience in a self-owned house as superior to being a tenant. Both housing experiences are no perfect substitutes. One of the consequences is that an investment in owner-occupied housing is a consumption decision as well as an investment decision. To the extent that both housing experiences are no perfect substitutes, rental markets do not provide a solution to the household's problem of divorcing the level of its real estate asset holding and the level of its real estate consumption.

This choice interdependence has been coined by the economic literature as the dual-use restriction of owner-occupied housing. Since the household's ownership of residential real estate determines the level of its consumption of housing services, the household's demand for real estate is "overdetermined" in the sense that the level of real estate ownership which is optimal from the point of view of the consumption of housing services may differ from the optimal level of housing assets from a portfolio point of view.[1]

One important, real-life situation in which the dual-use restriction limits the optimal choice is when the household experiences a shock to its income *after* fixing its level of owner-occupied housing. Given standard convex preferences, the household wants to reduce its non-housing consumption as well as its housing consumption. One option to achieve the latter could be to sell the owner-occupied house and to buy a smaller property. Another option could be to sell and then move into a rented place.[2] However, both solutions are only second-best: the moving itself results in substantial transaction costs. These costs are especially high in real estate markets, both in monetary terms (e.g. agent fees, moving costs, taxes) as well as more intangible ones (e.g. emotional upset, loss

[1]The implications of the dual role of housing for tenure decisions of households were first analyzed by Henderson and Ioannides (1983). Flavin and Yamashita (2002) is a more recent of the many studies of how the consumption demand for housing constraints the household's optimal holdings of financial assets.

[2]Note that the change from being a home owner to being a tenant (by moving into a rented place which is paid for by the proceeds of the property) is not a true, welfare-equivalent alternative to the extent that both types of housing services are no perfect substitutes.

of neighboodhood). Venti and Wise (1984) find that it takes a 14% increase in utility to induce the average low-income renter to move to a new location. This value includes not only the utility value of out-of-pocket relocation expenses, but also the utility value of the loss of neighborhood. Haurin and Gill (2002) study the homeownership choice of military personnel. Their best estimate for the transaction costs of moving are 3% of house value and 4% of household earnings, with the last term representing the time cost of selling.

An alternative first-best solution would allow the household to reduce its real estate equity and reassign the proceeds to non-housing consumption without risking the ownership of its property. Without a financial market for such owner-occupied housing equity - and given a significant income shock - the household will reduce its non-housing consumption instead of its housing consumption, will miss its optimal choice and welfare will be lost.

The problem has a special relevance for the group of elderly homeowners. Many elderly homeowners - especially those with low incomes - have a significant part of their financial assets looked up in their real estate. Bucks et al. (2009) report from the 2007 Survey of Consumer Finances that for the average U.S. family the primary residence accounts for 38.1% of all financial and non-financial assets. However, for lower-income households it is even more important. In the less-than-20-percent income bracket, house value makes up 47.1% of total family assets. The value of all bonds, stocks, pooled investment funds and retirement accounts - in comparison - is about a quarter of total assets for an average U.S. household. For the group of households with 65-74 year-old household heads the median value of their primary residence was $200,000, in the age group 75+ years it was $150,000.

In 1987, the U.S. Congress decided to help create a U.S. market for home equity release by introducing the Home Equity Conversion Mortgage (HECM) Program for elderly homeowners. The HECM Program is a federal government program administered by the Federal Housing Administration (FHA) and offers a national insurance for private lenders of home equity release products specifically designed for elderly homeowners. A home equity release product - often called reverse mortgage - is a financial product designed to allow the borrower to stay in her residence while receiving cash advancements on the equity value of her property. By early 2010, reverse mortgages in the U.S. have grown in recent years to become a $65 billion business. Many financial institutions across the world have since started to evaluate strategies to offer home equity conversion for elderly homeowners in their local markets.

A major barrier to the introduction of home equity release products is the complexity of their risk management. Different from conventional mortgages, home equity conversion mortgages have an insurance component as well as a lending component. The HECM Program pioneered a design that disconnects both components and pools the insurance risk of private lenders' HECM contracts in a separate national insurance fund backed by the U.S. government as the lender of last resort. The U.S. government thereby does not act as a lender of loans, but reduces the complexity of reverse mortgage lending for private lenders to that of a commonly understood lending product. This aspect of the HECM Program has been one of the critical success factors for the U.S. market launch of home equity conversion backed by private lending institutions.

In other countries with rapidly-aging societies, stressed public pension systems and sophisticated financial markets, home equity conversion products could arguably provide a lot of value to elderly consumers as well. Very different from the U.S. federal government, however, other national governments have not taken any steps to introduce national insurance programs for home equity conversion loans. Without any public or private insurance available, private lending institutions in international markets face the challenge of taking the insurance risk of reverse mortgages on their own bank's balance sheet. A risk and feasibility assessment of this strategy requires a concurrent quantification of the pricing and the insurance problem. It also requires an understanding of the appropriate calibration of the risk management model for local markets and its effects on the properties of home equity contracts. Given the lack of experience with home equity conversion, market participants also want to evaluate the expected stability of an insurance fund for reasonable scenarios.

In this paper I implement a set of MATLAB classes to compute a generalized HECM Program pricing and simulation model for reverse mortgage contracts. The pricing model uses the HECM pricing methodology to capture the insurance risk and resulting features of the contract. It is generalized in the sense that it covers most of the payment features offered by the HECM Program and that it can be calibrated to fit the parameters of any market environment. Based on the pricing mechanism, I add a simulation model for portfolios of contracts that stochastically simulates the properties of portfolios across time. Portfolios are subject to interest rate risk, aggregate house price appreciation risk and idiosyncratic house price appreciation risk. I calibrate the model to fit the German real estate market. In a first step, I use the model to compute actuarially-fair reverse mortgage contracts for three German calibrations. In a second step, I run a Monte Carlo simulation for a portfolio to assess the volatility of the insurance fund over time.

The remainder of this thesis is organized as follows. Chapter 2 introduces the properties of reverse mortgages as a financial instrument and their history. Chapter 3 discusses the HECM Program's pricing mechanism and its computational implementation. Chapter 4 documents the simulation approach, its calibration, possible extensions and results. Chapter 5 concludes.

Chapter 2

Reverse Mortgage Fundamentals

Before we dive into the detailed technical hand-wringing about pricing and simulating reverse mortgage contracts, I would like to give some intuition to back up the claim that the approach taken here is interesting. I start out with what a reverse mortgage is and what important aspects of the product have been discussed in the literature and why. The goal for the reader is to understand the properties of reverse mortgages, their history and the institutional setting in which they are originated, insured and liquidated.

2.1 Properties of Reverse Mortgage Contracts

2.1.1 Definition

A reverse mortgage is a loan available to seniors and is typically used to release the home equity in the property while the homeowner remains the resident. The resident also remains the owner of the property during the time of a reverse mortgage. The homeowner's obligation to repay the loan is deferred until the owner dies, the home is sold, or the owner leaves (e.g., into aged care). I will refer to this event as a termination. A reverse mortgage's principal and interest are paid with homeowner's equity. In a reverse mortgage, the homeowner makes no payments and all interest is added to the lien on the property. With every payment that the owner receives from her available equity, the debt on the property increases.

The fundamental difference between a conventional mortgage and a reverse mortgage is therefore the rising-debt character of the latter. With a reverse mortgage, the borrower *receives* a stream of payments from the lender while with a conventional mortgage, the borrower *repays* to the lender the initial advance. In a reverse mortgage contract, the cumulated advances and accrued interest form a rising outstanding debt secured by the house as collateral.

2.1.2 Forms of Payment

Payment of the reverse mortgage can come from a line of credit, in a lump sum, in the form of monthly payments or in a combination of all three. The homeowner typically decides on the form of payment at the time of the origination of the reverse mortgage contract. With a line of credit, the homeowner can freely decide the time and the amount of the payment. She may take out money from her line of credit on several occasions up to the total value of the line of credit. A lump sum payment is a single up-front payment to the homeowner in the first time period of the contract. With monthly payments, the monthly payment schedule is set up either as a fixed-term or as a tenure payment. With a tenure payment scheme, monthly payments take place for the rest of the life of the homeowner, no matter how long the homeowner lives. With a fixed-term payment scheme, monthly payments end after a fixed, predetermined number of periods.

2.1.3 Eligibility

Eligibility for a reverse mortgage is usually regulated by the lender of the loan and hence differs by lender. However, HUD (U.S. Department of Housing and Urban Development)-insured reverse mortgages have standardized eligibility requirements. In order to be eligible for a HECM (Home Equity Conversion Mortgage) loan, the borrower must be 62 years of age or older. She must have a very low outstanding mortgage balance or own their home free and clear and have received HUD-approved reverse mortgage counseling to learn about the program. There are no minimum income or credit requirements. No regulation exists to limit the purpose of use of proceeds from the reverse mortgage. In case of lender insolvency, proceeds from the HECM are guaranteed by HUD.

2.1.4 Non-Recourse Debt Property

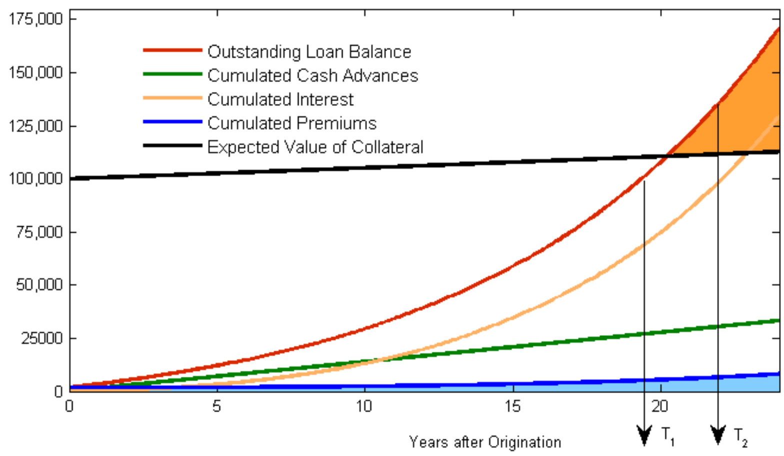
The borrower is not required to make payments on the note until it is due and payable. The note is not due or payable until the borrower dies, sells the house, or moves out of the house. At that point, the reverse mortgage can be paid off with the proceeds of the sale of the house, or if the borrower has died, the property can be refinanced by the heirs of the homeowner's estate (e.g. with a regular mortgage). If the proceeds exceed the loan amount including compounded interest and fees, the owner of the house receives the difference. If the owner has died, the heirs receive the difference. If the loan amount including compounded interest and fees exceeds the proceeds, the bank - or the insurance, which the bank has on the loan - absorbs the difference. Hence the debt of a reverse mortgage is non-recourse: the lender looks only to the value of the collateral for repayment and not to any other assets of the borrower or the borrower's estate.

2.1.5 Risk Factors

With the outstanding debt typically growing at a higher rate than the collateral, the risk for the lender lies in the possibility that the future outstanding debt outgrows the future collateral value. This risk is sometimes summarized as *crossover risk*. The individual components of the outstanding debt are 1) the cumulated advances paid out to the borrower, 2) the cumulated insurance premiums charged to cover the crossover risk and 3) the compounded interest on the outstanding debt. The crossover risk is a feature of reverse mortgages that is different from other conventional financial products and makes their risk management complex. It is a direct consequence of the non-recourse debt property paired with the reverse mortgage's rising-debt character.

Figure 2.1 shows the relevant, stylized time series for a reverse mortgage. The example was computed for a 75-year old female (at the time of the origination of the loan) with a property initially valued at 100,000 in a 10% interest rate environment and with a tenure payment plan. At time T_1 the outstanding loan balance of the loan outgrows the expected future value of the collateral. Because the lender's interest rate is higher than the appreciation rate of the collateral, this situation will occur systematically across a portfolio of reverse mortgages. If the termination of the reverse mortgage happens to take place before T_1, then the lender will sell the property or the heirs of the borrower will refinance it such that the lender can recover the full outstanding loan balance. If the termination of the reverse mortgage happens after T_1 - e.g. at time T_2 - then the insurer of the reverse mortgage will have to cover the loss at that time.[1] Due to the non-recourse debt property, the lender can look only to the value of the collateral for repayment and not to other financial assets of the borrower.

Figure 2.1: Stylized Time Series for a Reverse Mortgage Contract



[1]The expected loss over time is marked in Figure 2.1 as the orange-shaded area between the outstanding loan balance and the expected value of the collateral. The accumulated insurance premiums over time are shown as an area graph as the blue-shaded area at the bottom.

The losses which the insurer incurs have to be covered by insurance premiums which the insurer charges before covering for the loss. To stay within the logic of a reverse mortgage, the insurance premiums have to paid for by the lender and are charged to the borrower by adding them on the outstanding loan balance. The insurance premiums thereby implicitly reduce the advances paid out to the borrower, but there is no cash changing hands between the borrower and the insurer or the borrower and the lender. The borrower pays for the insurance through the reduced equity in her collateral. The payment of the insurance premium to the insurer only happens throughout the life of the loan and is therefore subject to the borrower not terminating the reverse mortgage in that period. Note that the lender and the insurer of the reverse mortgage could theoretically be the same institution, although this is not usually the case.[2] In the HECM Program the U.S. Department of Housing and Urban Development acts as the insurer of loan issued by a private lender.

2.1.6 Pricing

Pricing reverse mortgages means to use a risk management model to come up with a fair premium for the reverse mortgage insurance. Once a fair insurance premium has been computed, the lender then can derive the amount that can be paid out to the borrower. The valuation of the crossover risk therefore lies at the heart of the risk management and cash flow model of reverse mortgages.

The expected loss of a reverse mortgage depends on the future outstanding debt as well as the future value of the collateral. The *future outstanding debt* is a function of future interest rates, since higher interest rates lead to faster accumulation of compounded interest. It also depends on the timing and the amount of the advances as well as the timing and the amounts of the premiums. The former is chosen by the borrower through a payment scheme, the latter has to be exogenously set by the insurer. I will call the function which defines the timing and the amount of the premiums the premium structure. Finally, the *future collateral value* is determined by an unknown development of the house value.

The time and absolute amount of a loss for a specific reverse mortgage contract hence depend on a realized time series path of interest rates, the house value at the time of the termination of the loan and therefore also the time of the termination itself. The loan terminates when the borrower dies or moves out of the house. Longevity is a risk factor for the insurer: the longer the borrower lives, the higher the risk that the outstanding debt might outgrow the value of the collateral. This effect is especially obvious in a tenure payment scheme. By design, the borrower will receive guaranteed, lifelong payments - regardless of how long she lives and at what point in time the outstanding debt outgrows the collateral value. However, note that the crossover risk is not limited to tenure payment schemes. In other payment schemes compounding interest and pre-

[2]To act as both lender and insurer, the lender internally has to open two accounts for a single reverse mortgage: one for the actual lending, one for the insurance of the loan. Non-government insured, private lenders in the U.S. have taken this approach in competing with the government-insured HECM Program. According to Szymanoski (2008) they have found it hard to compete with the HECM Program except in the "jumbo" market for homes valued above the FHA loan limit.

miums will eventually make the outstanding balance outgrow the house value, even if advances to the borrower have stopped previously.

A pricing model for reverse mortgages has to account for all factors of the crossover risk - the longevity risk, interest rate risk and house price risk. Solving the insurance problem then implicitly solves the pricing problem. While the longevity risk can be diversified by holding a portfolio of contracts, the interest rate risk and house price risk are systematic in nature. An assessment of the quality of a pricing model for reverse mortgages therefore also requires an analysis of the sensitivity of the model to its assumptions.

2.2 The HECM Program

The United States is home to the most advanced market for home equity conversion. It is the only market that has reached a significant scale in converting home equity for elderlies. Government agencies, private lending institutions and regulators look back on 30 years of experience. This section presents a short history of the HECM Program, the theoretical and practical challenges during its implementation and how they were overcome.

2.2.1 The HECM Insurance Demonstration

First home equity conversion programs in the U.S. were established as small community projects as early as 1980 in San Francisco and 1981 in Buffalo. Almost all of the early contracts were fixed-term reverse mortgages (Nye and Archer (1987), Weinrobe (1988)). However, none of these projects reached any significant scale. The U.S. Department of Housing and Urban Development (1990) finds that prior to 1988 only about 2,500 reverse mortgages were issued by various public and private lenders.

The situation changed in 1987 when political supporters secured Congressional support for a federal reverse mortgage insurance demonstration. The HECM Insurance Demonstration was created and authorized HUD to insure 2,500 reverse mortgages through September 30, 1991. The first HECM loan was made in October 1989. The limit in absolute numbers of reverse mortgages that can be insured by HUD and the time limit have since been removed. The program that originally was meant to be just a demonstration became an official HUD Program in 1998.

The HECM Program is not the only option for elderlies in the U.S. to convert home equity. A number of other private and government-sponsored reverse mortgage programs exist which typically differ by costs (e.g. origination fees, closing cost), form of interest rate adjustment (fixed vs. adjustable) and absolute interest rate. Except for some private programs that are directed at the "jumbo" market for homes valued above the FHA loan limit, none of these programs have been especially successful. The HECM Program is the dominant program for home equity conversion and as of June 2008 represents about 95% of the U.S. reverse mortgage market (Szymanoski (2008)).

2.2.2 History of the FHA

The HECM Program is a government-sponsored program developed by the Federal Housing Administration (FHA), an operating unit of HUD. The FHA has a long history. It was created by the National Housing Act of 1934 after the Great Depression as part of an effort to restructure the federal banking system. During the banking crisis of the 1930's the U.S. mortgage market collapsed. Lenders retrieved due mortgages and refinancing was not available, such that borrowers - often unemployed - were unable to make mortgage payments and the resulting foreclosures caused the housing market to plummet.

To allow lower income Americans to borrow money for the purchase of a home in such a market environment, the FHA pioneered the concept of a mortgage insurance. FHA mortgage insurance is available for FHA-approved lenders, protecting them against homeowner mortgage default. The loans are insured through a combination of an upfront and a small monthly mortgage insurance premium. Since 1934, the FHA and HUD have insured over 34 million home mortgages and 47,205 multifamily project mortgages. Currently, the FHA has 4.8 million insured single family mortgages and 13,000 insured multifamily projects in its portfolio. Over time, private mortgage insurance companies adapted the concept and now also service mortgages of the conventional market. While the goal of the FHA is to be self-supporting, the FHA's liabilities as a government agency are eventually backed by the federal government.

2.2.3 Institutional Design

The HECM Program's logic is very similar to the one of the FHA mortgage insurance. The HECM Program provides an insurance framework for private lenders in the reverse mortgage industry: any lender authorized to make HUD-insured loans - such as banks, mortgage companies, and savings & loan associations - can participate in the HECM Program. Participants perform the function of originator, distributor and service provider of HECM loans. They counsel their clients, process documents, lend out the money, arrange payments (including payment of premiums to the FHA's insurance fund) and charge for their services through the interest rate of the reverse mortgage.

In a HECM loan contract the government does not lend out money, only private lenders do. However, the private lenders' crossover risk is in turn insured by the FHA insurance fund. This, in theory, eliminates collateral risk for the lender: As soon as the outstanding balance equals the maximum claim amount, the lender may assign the mortgage to the FHA.[3] The FHA then steps in as the new lender with all associated rights and obligations. The borrower is unaffected by the assignment of the loan.

[3]The maximum claim amount is a technical term developed by the FHA specifically for the HECM Program. It is a capped property value that was introduced in order to target borrowers with lower valued properties. Its definition has been changed several times and currently is the lesser of 1) the home's appraised value and 2) the national FHA 203(b) for HECM loans of $625,500.

2.2.4 Policy Objective

One of the three political objectives that Congress had in mind when it supported the HECM Insurance Demonstration (see Szymanoski (1994)) was to determine the extent of demand for home equity conversion. It also wanted to identify the types of home equity conversion mortgages that would best serve the needs of elderly households. It was not clear from the outset that elderly homeowners would be attracted to the idea of home equity conversion or even to the particular approach taken by the HECM Program. It was only assumed that elderly households face binding dual-use restrictions.

Such a demand for home equity conversion could be caused by several, different effects. First, elderlies are subject to higher idiosyncratic risks than other age groups: sudden health care spendings can require immediate liquidity that may not be available through other forms of income. Second, if financial assets saved for retirement serve as a source of income and take a hit due to turmoil in the financial markets, housing wealth can temporarily smooth out consumption. Third, inheritance as a major motivation for the preservation of home equity is not as important anymore as it once was: an increasing number of childless homeowners do not have immediate heirs. Those that do have children do not necessarily have children who want to live in their parent's house.

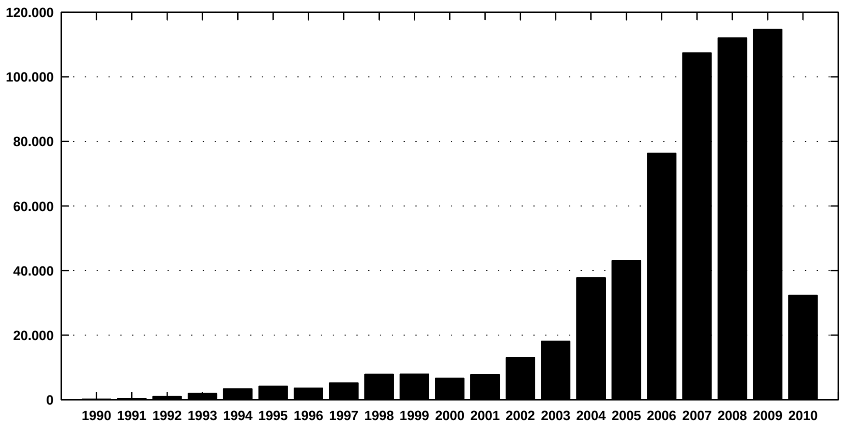
2.2.5 Evaluation of Demand Potential and Adoption

Whether or not these effects in fact mattered to household's financial decisions could only be confirmed empirically after the introduction of the HECM Insurance Demonstration. Some studies, however, provided an ex-ante estimate of the potential demand for home equity conversion programs by quantifying the target group of potential borrowers. Merrill et al. (1994) use the 1989 American Housing Survey and assume that the potential market for reverse mortgages is composed of households aged 70 or older, with annual incomes less than \$30,000, who have lived in their homes at least ten years, and who own fully paid-off houses valued between \$100,000 and \$200,000. They find that about 800,000 households (out of twelve million elderly homeowners aged 62 and older) in the United States meet those criteria, most of them in California and the Northeast.

Because Mayer and Simons (1994) look at the whole distribution of elderly households and consider debt as well as income, they find an even larger potential market based on the 1990 Survey of Income and Program Participation and Census population estimates. According to their findings over six million homeowners in the United States could increase their effective monthly income by at least 20% by using a reverse mortgage. Of these, more than 1.3 million have no children. Furthermore, a reverse mortgage would allow over 1.4 million poor elderly persons to raise their incomes above the poverty line.

Similar analysis has recently been performed for other international markets. For Germany, a computation by the Centre for European Economic Research (Lang (2008)) based on data of the 2003 Einkommens- und Verbrauchsstichprobe finds that about a million German elderly households could be interested in home equity conversion. These households occupy their own property valued at EUR 100,000 or more, have a household head 55 to 69 years old and a monthly income below that of the median household. The estimate implies a potential loan volume for reverse mortgages in Germany of around EUR 90.8 billion.

Figure 2.2: Number of HECM Contracts Endorsed for Insurance by HUD, by Fiscal Year*,**



* Data Source: HUD's FHA HECM Characteristics Report, January 2010
** Value for fiscal year 2010 is number of insured contracts between Oct. 2009 and Jan. 2010

For more than a decade it looked like the HECM Program would in fact not be a success. From 1990 to 2000, only about 42,000 loans were endorsed for insurance by HUD. However, since 2001 the HECM Program has seen exponential growth. 2009 set a new record with about 114,000 loans originated in that year alone (see Figure 2.2). The total outstanding principal balance of insured loans stood at $65.1 billion in January 2010.[4] Note that the recent financial crisis and the crash of U.S. home prices did not impact the overall volume of HECM contract origination.

[4]Number has been computed based on HUD data from table "HECM Cases Endorsed for Insurance by Fiscal Year" from January 31, 2010.

Chapter 3

Pricing Reverse Mortgages

The risk management model of a reverse mortgage is a modeling framework to find the fair value of an insurance contract that covers the crossover risk for the lender. If the insurer is risk-neutral and non-profit, then a fair-priced insurance contract will equalize the expected present value of losses and the expected present value of insurance premiums. We have called this problem before the pricing problem of reverse mortgages.

This chapter presents the technicalities of the HECM risk management model used throughout the paper. The HECM risk management model was previously developed by HUD exclusively for the HECM Program and is the most comprehensive and most advanced publicly-known model framework for home equity conversion mortgages. I will describe the logic of the model, its assumptions and my computational implementation. I then calibrate the model and compute prices for reverse mortgage contracts for the German market.

3.1 Risk Management Approach in the HECM Program

The HECM Program's risk analysis framework was developed by the Office of Economic Affairs (OEA), an organization within HUD. It is designed to be self-sufficient and includes neither a profit margin nor a subsidy. This section presents its equations and underlying logic. Most of the description is based on two technical documents by Szymanoski (1990, 1994). Both sources also contain more comprehensive explanations for the reasoning behind individual modeling decision.

3.1.1 The Insurance Equation

Every insurance program requires the contract to take a form such that the present value of expected losses to the insurer is less or equal the present value of expected premiums. Following Szymanoski (1990) I will write down the model in discrete time. This also simplifies the computational implementation. The fundamental insurance equation for reverse mortgages then becomes

$$\sum_{t=0}^{T} E(L_t)(1+i_t^d)^{-t} \leq \sum_{t=0}^{T} E(P_t)(1+i_t^d)^{-t} \tag{3.1}$$

where T denotes the calculation horizon of the insurer, t is the current month in the lifetime of the loan, L_t is the loss incurred by the insurer in month t, P_t are the insurance premiums collected in month t and i_t^d is the insurer's discount rate in month t. In the HECM Program and in the implementation of this paper, a period lasts one month. In the period of the origination of a loan $t = 0$.

A loss to the insurer only occurs when the outstanding loan balance exceeds the value of the collateral in the period of the termination of the loan. Filling in this definition of a loss and breaking up the expectation operators in equation (3.1), we get

$$\sum_{t=0}^{T} q_t^x \cdot |OB_t - C_t| \cdot \mathbf{1}(OB_t > C_t) \cdot (1+i_t^d)^{-t} \leq \sum_{t=0}^{T} p_t^x \cdot P_t \cdot (1+i_t^d)^{-t} \tag{3.2}$$

Here q_t^x denotes the probability that a loan originated by a borrower with initial age x terminates in period t, OB_t is the outstanding balance of the loan in period t and C_t is the value of the collateral in period t. The indicator function is expressed as $\mathbf{1}()$. The probability that a loan originated by a borrower with initial age x has not been terminated at time t is written as p_t^x.

Both equations (3.1) and (3.2) state in different form that the present value of expected losses needs to be less than the present value of expected premiums for the insurer in order for him to endorse a reverse mortgage. I will cover the variables involved in more detail. Note that we usually mean a time series vector of values - not a single value at one point in time - when I refer to a variable of the model.

3.1.2 The Premium Structure

When designing an insurance contract, the insurer decides through the structure of the model whether a variable of the contract's pricing model is exogenous or endogenous. For given exogenous variables, the insurer then sets the endogenous variables such that the insurance equation holds. A trivial endogenous variable for every insurer is the price of the insurance. In the context of equation (3.2), the price of the insurance is determined by the periodic insurance premiums P_t.

A standard pricing approach hence would be to set the periodic insurance premiums P_t endogenously such that the insurance equation holds in all possible cases. This, however, is not what the HECM Program does. The HECM Program instead sets an exogenous premium structure. HUD charges to the lender as insurance premium an upfront, one-time premium of α=2% of the initial, appraised value of the collateral and an additional yearly premium of β=0.5% of the outstanding loan balance during the lifetime of the loan. The yearly premium is charged on a monthly basis. We will see shortly why the exogenous premium structure is advantageous for the design of a reverse mortgage. For later reference, the premium structure takes the form

$$P_0 = \alpha H_0 \qquad \text{and} \qquad P_t = \frac{\beta}{12} \cdot OB_{t-1} \quad \forall t > 0 \tag{3.3}$$

Given the functional form of the premium structure (3.3), both expected losses on the left and expected premiums on the right of equation (3.2) now depend on the outstanding loan balance OB_t. Since the HECM Program contains no profit margin for the insurer, a fair insurance contract will imply a binding insurance equation (3.2). Any solution to the problem must equalize the present value of expected losses and the present value of expected premiums. To solve, we first need to understand how the outstanding balance evolves over time. We also need to find a variable that we want to solve for, that is an endogenous variable that can be used by the insurer to derive a fair insurance contract.

Section 3.2 will give more details about the modeling of the exogenous variables. The pricing model will assume future values for the discount rate i_t^d, the collateral C_t, actuarially-derived loan survival probabilities p_t^x and actuarially-derived loan termination probabilities q_t^x. Since all these variables are exogenous and future premiums depend only on the outstanding loan balance, only the outstanding loan balance is left to be manipulated in order to equalize expected losses and expected insurance premiums.

3.1.3 Limiting Cash Advances

The outstanding loan balance compounds over time due to the cash advances AD_t made to the borrower, the interest charged by the lender and the insurance premiums charged by the insurer. I assume that the cash advance is paid out at the beginning of each period. The outstanding balance in period t then is

$$OB_t = (OB_{t-1} + AD_t)(1 + r_t) + P_t \ , \forall t \tag{3.4}$$

Note that - given an exogenous loan interest rate r_t and an exogenous premium structure such as in equation (3.3) - the outstanding balance at any point in time depends only on the way cash advances are made to the borrower. The earlier cash advances are made and the higher the amount of the cash advance, the higher the resulting outstanding balance at any given moment. Once a payment scheme defines a time vector of cash advances AD_t for all t, we can calculate the outstanding balance for any period t. The outstanding balance then determines both sides of the insurance equation (3.2) and we can compute the present value of both expected losses and expected premiums.

Based on this result we have also identified an endogenous variable for the insurer that can be used to price the insurance. As seen before the only way to make equation (3.2) binding is to manipulate the outstanding balance. Since the outstanding balance can only be affected by changing the cash advances AD_t made to the borrower a limit to the cash advance must be the insurer's pricing strategy.

The risk for the insurer in this context is a cash advance that is too high: in a worst-case scenario the lender makes a huge initial cash advance to the borrower, the outstanding balance compounds very fast and the insurer can not cover the expected loss with expected premiums.[1] One way for the insurer to price a reverse mortgage contract then is finding the answer to the question: What is the limit to the maximum initial cash advance the lender should be allowed to make to the borrower, such that the insurer can still endorse the loan? Pricing an insurance contract by limiting the allowed cash advance of the lender to the borrower is a more indirect approach then setting an appropriate premium structure. In theory, both approaches are equivalent solutions to the same problem. In practice, limiting cash advances is the preferred option.

One important advantage is that the computation of a maximum initial cash advance to the borrower allows to generalize the pricing result to all other possible payment schemes. The cash advances of any payment scheme can be redefined as a one-time, lump-sum payment to the borrower in the first period. This is so because a payment stream of any complexity can easily be transformed in a one-time payment of equal present value - and vice versa.

The maximum, one-time, upfront payment to the borrower that still supports expected insurance premiums is one of the cornerstones of the HECM Program and is called the *initial principal limit*. The initial principal limit is defined as the maximum, one-time, upfront payment that the lender may make to the borrower such that the present value of expected losses to the insurer equals the present value of expected premiums. With any initial payment beyond the initial principal limit, the present value of expected losses of the contract would be higher than the present value of expected premiums.

The concept of the initial principal limit allows the lender to offer any form of payment scheme to the borrower without limitations imposed by the risk management framework. For the pricing of the insurance, the insurer focuses only on the computation of the initial principal limit and only endorses loans up to that maximum cash advance value. Based on this limit value, the lender then derives any fixed-term, tenure or other payment scheme. This division of labor is very helpful for the risk sharing and product management in a setup of two different institutions representing the insurer and the lender.

We will see later that the initial principal limit can not be determined analytically, it has to be computed numerically. In order to generalize the result to borrowers with different initial values of collateral, the HECM Program takes one additional step. The numerical computation of the initial principal limit will not use the initial principal limit itself. Instead, the derivative concept of the principal limit factor is used. The principal limit factor is defined as "the highest initial loan-to-value (LTV) ratio for which the insurance premium will cover expected losses from future claims" (Szymanoski (1994)), where the initial loan-to-value ratio refers to the initial principal limit divided by the initial value of the collateral.

[1]Strictly (mathematically) speaking, higher cash advances in the model do not only result in higher losses but - due to the form of the premium structure - also bring more premiums to the insurer. However, the increase in premiums can never offset the increased risk of a loss. It suffices to imagine very high insurance premiums which then themselves are charged on the outstanding loan balance and thereby increase the expected loss.

3.2 Modeling Assumptions

So far the model has assumed a number of exogenous variables as given - among them loan survival and loan termination probabilities, the lender's interest rate, the insurer's discount rate and the value of the collateral. All of these variables were necessary for deriving the solution to the pricing problem. We will see next how these factors are modeled in the HECM Program.

3.2.1 Loan Survival Probabilities

The importance of the timing of loan termination is one characteristic peculiarity of reverse mortgage contracts. The time of the termination has immediate cash flow consequences: advances to the borrower and premiums to the insurer are only paid during the lifetime of the loan; also, the outstanding loan balance and the value of collateral at the time of termination determine the insurer's loss.

The termination event of a reverse mortgage can be triggered by either the death of the borrower or the voluntary move-out with a subsequent sale of the property. Because reimbursement of the loan is deferred until termination, reverse mortgages partially hedge the borrower against his risk of longevity. The borrower's repayment of cash advances and interest is unaffected by his actual life-span. Instead, the insurer assumes the risk of longevity. A large pool of contracts can then effectively eliminate the longevity risk if probabilities are modeled accurately. Small gains from the majority of cases with below average losses can offset large losses on a small number of loans.[2]

The HECM Insurance Demonstration predicts loan termination probabilities through altered mortality tables and originally used the U.S. Decennial Life Tables for 1979-1981 for this purpose. Loan termination probabilities are assumed to be independent of house prices and interest rates.

In an attempt to account for voluntary move-outs unrelated to the actual death of the borrower, death probabilities derived from mortality tables are altered by a move-out factor of 1.3, implying a move-out rate of 30% of the age-specific death rate for all age groups. As there was no prior experience with reverse mortgages, this assumption for the move-out factor was, in fact, an educated guess. Data from the general elderly population known at the time (Jacobs (1988)) show that elderly homeowners have move-out rates that decline with advancing age when expressed as a percentage of the age-specific death rate. They are much higher than 30%, ranging between 519% for 65-69 year old homeowners down to 47% of mortality for the 85+ group. Szymanoski (1990)

[2] This statement will only be true if adverse selection effects are insignificant: only bad risks who think they get advantageous conditions on the loan might sign up. These borrowers might have private information about their life expectancy being higher than average, then resulting in higher than expected losses to the insurance fund, higher premiums and subsequent market breakdown. Several arguments have been presented for and against the existence of an adverse selection effect in reverse mortgage contracts (see e.g. Miceli and Sirmans (1994), Shiller and Weiss (2000), Davidoff and Welke (2007) and Szymanoski (1994)). Empirical evidence from the analysis of actual reverse mortgages insured by the HECM Program, however, suggests that adverse selection is not a serious problem because borrowers terminate loans rather quickly. Davidoff and Welke (2007) propose that advantageous selection combined with rapid price appreciation can explain this rapid termination.

justifies the assumption of a constant 30% move-out rate with the strong incentive that the HECM Program provides for borrowers to remain in their home. Given the empirical evidence of higher move-out rates, the 30% assumption is also a risk-minimizing, sufficiently conservative guess to protect the insurance fund.

Standard mortality tables usually provide yearly, age-specific death rates conditional on the attained age and different for males and females. Assume we derive d_x^{x+1} from a standard mortality table as the probability for a person initially aged x years to die before attaining age $x+1$ years. Hence $s_x^{x+1} = 1 - d_x^{x+1}$ is the probability for a person initially aged x to survive to attain age $x+1$.

In a first step we derive s_i^j as the probability that a person starting at initial age i will still be alive when turning age j. The values for s_i^j are computed for all possible combinations $i < j$, where $i = x_{min}, ..., T-1$, $j = x_{min}+1, ..., T$ and x_{min} is the allowed minimum age for a reverse mortgage borrower. Then s_i^j can be calculated by

$$s_i^j = \prod_{x=i}^{j-1} s_x^{x+1} \tag{3.5}$$

In a second step we need to transform yearly probabilities s_i^j into monthly loan survival probabilities p_t^x as in equation (3.2) by adjusting for the move-out factor and interpolating geometrically to monthly probabilities. Following Szymanoski (1990), the loan survival probability p_t^x that a loan originated by a borrower initially aged x years has not been terminated in or before period t can be computed as

$$p_t^x = (s_i^j (\frac{s_i^{j+1}}{s_i^j})^{\frac{r}{12}})^{1+m} \tag{3.6}$$

where i is the initial age in full years, j is the attained age in full years, $x = 12i$ is the initial age in months, $t = 12j + r$ is the attained age in months, r are the months between attained aged j and $j+1$ and m is the move-out rate as decimal.

By definition $p_0^x = 1$, because a loan can not have terminated yet in the month of the origination when $t = 0$. Since the mortality table has an upper age limit T beyond which no one survives, the loan survival probability will be zero when it hits the age limit and hence $p_T^x = 0$.

By splitting up the geometric interpolation in (3.6) we can also derive explicit expressions for the survival probabilities by the (non-)event of either move-out or death. This distinction will prove useful later in the simulation of portfolios to predict death and move-out events for individual contracts. Because death and move-out are independent events for a given period, we can write

$$pd_t^x = s_i^j \left(\frac{s_i^{j+1}}{s_i^j}\right)^{\frac{r}{12}} \tag{3.7}$$

$$pm_t^x = \left(s_i^j \left(\frac{s_i^{j+1}}{s_i^j}\right)^{\frac{r}{12}}\right)^m \tag{3.8}$$

Here pd_t^x is the monthly probability that a loan originated by a borrower initially aged x years has not been terminated due to the death of the borrower in or before period t and pm_t^x is the monthly probability that the loan has not been terminated due to the move-out of the borrower in or before period t.

Figure 3.1: Loan Survival Probabilities

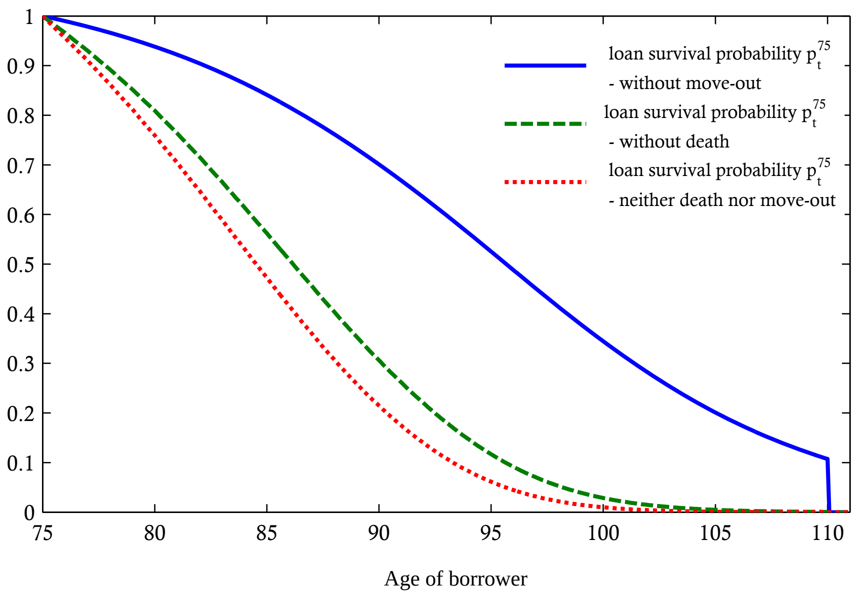


Figure 3.1 plots p_t^x, pd_t^x and pm_t^x for a 75-year old female borrower. The computations uses the U.S. Decennial Life Tables for 1979-1981.

3.2.2 Loan Termination Probabilities

To solve the insurance equation (3.2) for the principal limit factor, we also need explicit values for the loan termination probabilities q_t^x which weight the insurer's loss of each period. Monthly termination probabilities are easy to derive as the first-order difference of monthly loan survival probabilities:

$$d_t^x = p_t^x - p_{t+1}^x \tag{3.9}$$

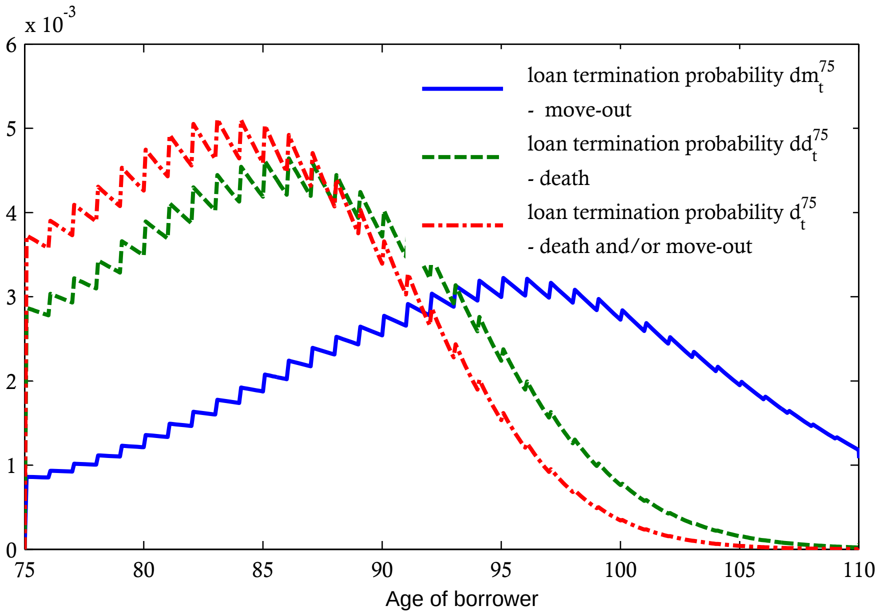
$$dd_t^x = pd_t^x - pd_{t+1}^x \tag{3.10}$$

$$dm_t^x = pm_t^x - pm_{t+1}^x \tag{3.11}$$

Here d_t^x is the joint monthly probability that a reverse mortgage loan terminates in period t due to death or move-out of the borrower. With probability dd_t^x the loan ends through death, with probability dm_t^x through move-out in period t.

Figure 3.2 plots the loan termination probabilities for a 75-year old female borrower, again using the U.S. Decennial Life Tables for 1979-1981. The hump shape is caused by the geometric interpolation of the monthly survival probabilities. Note that the model predicts most loans to terminate within the first years of loan origination.

Figure 3.2: Monthly Loan Termination Probabilities



With move-outs as a potential termination event, loans tend to end earlier than with death events only: d_t^x has more probability mass in the first years than dd_t^x while dd_t^x has more mass in later years. This demonstrates the conservative 30% move-out rate assumption by HUD: The higher the HECM model's assumed move-out rate, the lower the expected losses will be that the insurer accounts for in the principal limit factor. If the real move-out rate in the pool of insured contracts is higher than 30%, then the insurer will turn a profit.

3.2.3 Interest Rates

The interest rate risk is a systematic risk for the insurer that can not be diversified through holding a pool of insurances. In some realizations of future interest rates the lender of an adjustable-rate reverse mortgage (ARRM) will charge higher interest rates during the lifetime of the loan than previously anticipated by the insurer. These higher interest rates will result in outstanding balances outgrowing collateral values earlier and losses to the insurer will be higher.

Note that interest rate risk matters only for ARRMs. For fixed-rate reverse mortgages (FRRM), the insurer can project future outstanding balances with certainty and the interest rate risk is assumed by the lender. In the case of the HECM Program, however, FRRMs are of minor importance and their number remains small. The interest rate risk assumed by private lenders of FRRMs is very high since contracts by design can be expected to be in force for 15 and more years. The secondary market - crucial to the refinancing of HECM loans - is therefore reluctant to finance FRRMs.

Two different approaches exist to model uncertainty in future interest rates. The first approach assumes interest rates as certain, but adds a risk premium as a risk adjustment. The second approach explicitly models the uncertainty in future interest rates. Both approaches ultimately limit cash advances through the principal limit factor. The HECM Program uses the first approach. The risk-adjusted, fixed interest rate of the HECM Program for an ARRM is called the *expected rate* i_e and is defined in the HECM Program as the ten-year U.S. Treasury rate at the time of the origination of the loan plus a lender's margin. For the purpose of the principal limit factor iteration, r_t in equation (3.4) is assumed to equal the expected rate. The expected rate is a hypothetical value, only useful for the purpose of pricing the insurance and computed by the insurer in the moment the loan is endorsed based on the actual loan interest rate the borrower and the lender agreed on.

The loan interest rate r_t charged on the reverse mortgage by the lender is different from the expected rate. The expected rate is only used for the derivation of the principal limit factor. The loan interest rate is set by the lender, but re-defined for the HECM Program as the one-year U.S. Treasury rate plus the lender's margin. This lender margin is also used to compute the expected rate. In the typical upward sloping yield curve environment, the loan interest rate thus will be lower than the expected rate. Szymanoski (1994) rationalizes this design by interpreting the ten-year U.S. Treasury rate as "encompassing the market's best estimate of implied forward one-year U.S. Treasury rates and arguably, a liquidity premium.". For ARRMs, the expected rate is meant to be an estimation of the average loan interest rate over the first ten years of the loan.

The discount rate i_t^d of the HECM Program, like the expected rate, is a fixed rate that does not change over the lifetime of the loan. It is defined as the expected rate minus 0.5%. Since the interest rate of HECM loans is likely to be set by lenders at some margin above the ten-year U.S. Treasury rate, the discount rate can be considered to equal the ten-year U.S. Treasury rate plus a margin minus one-half percent.

3.2.4 Property Values

The house price risk contains two risk components and is only partially diversifiable. The first component is of regional character: recession followed by a local drop in property values. Whether or not pooling of reverse mortgage contracts eliminates this risk depends on the correlation of changes in regional house prices. Correlation will be lower the more properties are distributed across different regional real estate markets. The second component, however, is of systematic nature: a national economic recession and a subsequent drop in house prices are not diversifiable.

Geometric Brownian Motion (GBM) Process

The HECM Program models future property values based on a stochastic geometric Brownian motion (GBM) process. Such a process is chosen because it has the potential to reflect non-stationarity in time series, a feature commonly attributed to house prices. Non-stationarity is important because it leads to an increasing variation of property values around the projected expected value over time. In the context of the model, this means an increasing risk of default the further into the future the house price is projected.

The geometric Brownian motion is also often called a log-normal random walk. Log-normal random walk means that the asset's nominal return in each period is modeled as i.i.d., where the nominal return rate is normally distributed with constant mean μ and a constant standard deviation σ. The mean μ captures the expected price inflation while σ describes the deviation from expected inflation. Formally, the geometric Brownian motion process is described as

$$\frac{dH}{H} = \mu dt + \sigma dz \tag{3.12}$$

where z is a Brownian motion or Wiener process, or in other words the differential of a stochastic variable distributed $N(0,1)$. Period t lasts one year. The HECM Program assumes a yearly nominal house price appreciation rate $\mu = 4\%$ and standard deviation $\sigma = 10\%$, where the former is computed from past data.[3]

Properties of GBM Process

The process shows several important properties. One important aspect of the process is that cumulated return rates of the house price over time are normally distributed, but with linearly increasing mean μt and non-linearly increasing deviation $\sigma\sqrt{t}$. While the projected expected house price over time therefore trends with μ, the deviation from the projected expected house price increases over time.

Another implication of the random-walk specification is that the best predictor of future values is the current value trended by the mean cumulated appreciation rate. The process has no memory, so it is not possible to predict further future values based on current observations. This precludes adverse selection in property prices. It is implicitly assumed borrowers do not know more about the future value of the property than the insurer.[4]

[3] For more details on the reasoning behind the parameter values, see Szymanoski (1990,1994) and section 3.5.4.

[4] Specifically, adverse selection may occur due to autocorrelation and mean reversion if local house prices follow cyclical patterns and the borrower is informed better about these than the insurer. However, models of autocorrelation are probably more appropriate in the short-term where the asymmetric information advantage of the borrower may be more relevant. In the long run, when losses to the insurer peak, the i.i.d. assumption of return rates seems to be justified (Case and Shiller (1989), Gau (1987)).

Finally, the known distribution of appreciation rates allows to derive analytical expressions for the future house price distribution of pools of properties. Although point predictions for individual future values are not feasible, probabilities for the whole future distribution can be computed. This is especially useful to compute a probability of the future outstanding loan balance outgrowing the future collateral value and thereby the loss to the insurer on the left-hand side of insurance equation (3.1).

Computing Expected Losses

A loss occurs on loans that terminate at a time t when the outstanding balance OB_t exceeds the house value H_t. I have already computed loan termination probabilities q_t^x. What is still missing in the probabilistic setup is the probability of the occurrence of a loss $P(OB_t > H_t)$ and the absolute value of the loss $L_t = OB_t - E(H_t|(OB_t > H_t))$ in case of such a loss.

Remember that the cumulative appreciation rate (i.e. $ln(H_t/H_0)$) is normally distributed with mean μt and standard deviation $\sigma\sqrt{t}$. Using the density function of the normal distribution the probability of the occurrence of a loss in a given period t can be written as

$$P(OB_t > H_t) = \frac{1}{\sigma\sqrt{t}\sqrt{2\pi}} \int_{-\infty}^{\ln(\frac{OB_t}{H_0})} e^{-\frac{1}{2}(\frac{y-\mu t}{\sigma\sqrt{t}})^2} dy \tag{3.13}$$

Next we need to compute the conditional expected value of the property given a loss. To derive the value, we first find an expression for the unconditional expected value. It is given by[5]

$$E(H_t) = H_0 e^{\mu t + \frac{1}{2}\sigma^2 t} \tag{3.14}$$

The unconditional expected value defines the mean of the entire log-normal house price distribution across time. The conditional expected value, however, represents the mean of only the left-tail part of the distribution up to the value OB_t. Hence the conditional expected value will always be less than the unconditional expected value. In fact Szymanoski (1990) computes the conditional expected value by multiplying the unconditional expected value by a factor δ, where $0 < \delta < 1$. δ is defined as

$$\delta = \frac{1}{P(OB_t > H_t)} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(\ln(\frac{OB_t}{H_0})-\mu t)\cdot\frac{1}{\sigma\sqrt{t}}} e^{-\frac{1}{2}(y-\sigma\sqrt{t})^2} dy \tag{3.15}$$

and the conditional expected value then can be written as

$$E(H_t|OB_t > H_t) = \delta \cdot E(H_t) = \delta \cdot H_0 e^{\mu t + \frac{1}{2}\sigma^2 t} \tag{3.16}$$

[5]For the exact derivation of equations (3.14) and (3.15), see Appendix 1 of Szymanoski (1990).

When we plug the probability of the occurrence of a loss (3.13) and the conditional expected value (3.16) back into the left-hand side of the insurance equation (3.1), we have a final, closed-form expression for the expected losses to the insurer. The expected losses are

$$E(L_t) = q_t^x \cdot P(OB_t > H_t) \cdot (OB_t - E(H_t|OB_t > H_t)) \tag{3.17}$$

3.3 Deducing Payments

The preceding equations in this chapter define what Szymanoski (1990) calls the basic HECM payment model. Given the characteristics of the borrower (initial age, gender), the initial property value, the expected rate as the rate of interest on cash advances and an assumed pattern of cash advances, the insurer might underwrite a reverse mortgage loan as long as the insurance equation (3.1) holds. If equation (3.1) does not hold, then only the present value of cash advances can be reduced because expected premiums are fixed. Limiting the present value of cash advances either works through deferring payment into the future or by nominal reduction of payment amounts.

3.3.1 The Principal Limit Method

In principle the basic HECM payment model could be used to analyze any pattern of cash advances to the borrower: given any pattern one could always solve for the maximum payment such that equality exists in the insurance equation (3.1). However, the HECM Program limits the computational solution to only one pattern of cash advances: the initial principal limit. Remember that the initial principal limit is a cash advance pattern of a maximum, upfront, one-time-only payment in the first period of the loan.

If we express the initial principal limit as a fraction of the initial property value, this principal limit factor does not depend on the absolute value of the property. Hence, it is sufficient to compute the principal limit factor instead of initial principal limits for all possible initial property values. The principal limit factor, however, is a function of the initial age of the borrower, the gender of the borrower and the expected rate at the time of the origination of the loan. The principal limit factor depends on the initial age and gender of the borrower because loan survival and termination probabilities derived from mortality tables are computed based on both parameters. The principal limit factor also depends on the expected rate because higher compounded interest will result in higher expected losses. Higher losses have to be balanced out by a lower initial principal limit.

The HECM Program does not distinguish borrowers by gender, but instead assumes all borrowers to be female. This simplification implies a surplus to the insurance fund because females have a higher life expectancy than males. If a couple jointly enters a HECM reverse mortgage then the age of the younger borrower is used for the choice of the principal limit factor. There is no explicit modeling of couples' joint mortality rates.

Multiplying the principal limit factor by the initial property value gives the maximum upfront cash advance the borrower is allowed to receive in the first period.[6] For future periods, the maximum allowed cash advance can be projected by the principal limit. The principal limit is an increasing function of time. While in the first period it is equal to the initial principal limit, future values are computed as the future outstanding loan balance if the borrower initially receives the initial principal limit as a cash advance. We write the principal limit PL_t on a monthly basis as

$$PL_t = plf(x, i_e, g) \cdot MCA \cdot (1 + c)^t \tag{3.18}$$

where $plf(x, i_e, g)$ is the principal limit factor for a borrower initially aged x and of gender g with an expected rate at origination i_e. MCA is the maximum claim amount and c is the monthly compounding rate $c = i_e + \frac{\beta}{12}$. The principal limit represents an upper allowed bound of the loan balance at any point in time for all patterns of cash advances.

3.3.2 Term, Tenure and Line of Credit Payment Schemes

With the principal limit factor and the principal limit defined, we can turn to the calculation of cash advance patterns other than a one-time, upfront payment. In a term payment scheme the borrower receives fixed monthly payments for a fixed number of months. In a tenure payment scheme monthly payments are fixed as well, but paid until the termination of the loan. With a line of credit, the borrower is allowed to withdraw advances until his line of credit is exhausted. Note that the payment schemes differ only by the timing of cash advances. If the borrower chooses to draw the maximum amount - the initial principal limit - in the first period, then the outstanding balance over time grows only by accrued interest and mortgage insurance premiums. With term, tenure and line of credit payment schemes the outstanding balance grows faster - though from a lower level - by accrued interest, mortgage insurance premiums and cash advances.

To derive actual cash advances for other payment schemes it suffices to apply the principal limit's properties. The principal limit is at any point in time the upper bound of the outstanding loan balance for which expected premiums ex-ante still cover expected losses.

For term loans, this means that the monthly cash advance needs to be an amount such that the outstanding balance equals the principal limit at the end of the term, after which the outstanding balance grows like the principal limit.

For tenure loans, it means that the outstanding balance converges to the principal limit only at the maximum age of the mortality table employed. The outstanding balance is lower than the principal limit over the whole life of the loan because monthly payments do not stop. A tenure loan is therefore equivalent to a term loan with a term that lasts until the maximum period T.

[6] There is a caveat here for the HECM Program: strictly speaking the principal limit factor is multiplied by the maximum claim amount, not the initial property value. Both are different for properties valued above the national FHA loan limit of $625,500. HECM loans that are affected by the FHA loan limit contain an implicit surplus for the insurance fund because the actual initial property value is higher than the maximum claim amount used as initial property value for the principal limit factor computation.

In case of a line of credit, the borrower is allowed to draw cash advances until the outstanding loan balance reaches the principal limit. Once the principal limit has been reached the borrower may remain in his home, but is not allowed to draw any more money. By this logic the cash advance pattern of an initial principal limit is a special case of a line of credit loan.

Based on the definition of the principal limit, we can define the net principal limit NPL_t on a monthly basis as the maximum allowed cash advance for any period t during the lifetime of the loan as

$$NPL_t = \max(0, PL_t - OB_t(\bar{AD}, \bar{i^m})) \tag{3.19}$$

where $OB_t(\bar{AD}, \bar{i^m})$ is the actual outstanding loan balance at time t given a vector of cash advances $\bar{AD}$ and a vector of loan interest rates $\bar{i^m}$ that have been charged on the loan during its lifetime.[7] By taking the realization of interest rates $\bar{i^m}$ into account, the lender may allow the borrower a refinancing of the loan to a different payment scheme at any point in time during the lifetime of the loan.

Following Szymanoski (1990), we can then compute level monthly cash advances for term and tenure reverse mortgages. The maximum monthly advance beginning in month $t+1$ and lasting for m month is given by

$$AD_t^{max}(m) = NPL_t \cdot (1+c)^m \cdot \frac{c}{(1+c)^{m+1} - (1+c)} \tag{3.20}$$

where m is chosen by the borrower as $0 < m < (T - t)$ for a term loan and as $m = (T - t)$ for a tenure loan. The first part of the expression on the right-hand side of equation (3.20) represents the future value of the net principal limit at the end of the m months term. The second part is the expression for a standard sinking fund monthly contribution that will grow to $1 at the end of the term. As a product, both give the monthly cash advance which - with compound interest - will equal the future value of the net principal limit at the end of the term.

3.4 Source Code

The MATLAB source code of this paper implements a set of classes and methods to solve a generalized HECM Program pricing and simulation model for reverse mortgage contracts. Functions have been written in object-oriented programming (OOP) to reflect in the code the objects and the structure of the problem. Using the equations of this chapter, the code computes all properties of fair-priced reverse mortgage contracts based on configurable parameters and mortality table data, with the principal limit factor iteration as the core numerical operation.

[7] The HECM Program also accounts for set-asides from the principal limit that lower the net principal limit beyond the expression in equation (3.19). Set-asides are meant for repairs, first-year taxes and for future loan servicing fees not included in the interest rate (Szymanoski (1990)). See HUD Handbook 4235.1 for more detail. Set-asides have been incorporated in the computational implementation, but have been omitted here for clarity.

3.4.1 Implementation

The implementation specifically allows to compute the properties of fair-priced reverse mortgage contracts based on any available mortality table, for any payment scheme, for any set of parameters (α, β, μ, σ) and for any move-out rate. In a first step, the code iterates the principal limit factor matrix given the model's logic presented through the equations of this chapter. In a second step, actual cash flows for a reverse mortgage are computed based on the choice of the payment scheme and the principal limit factor.

The detailed look at the resulting data of the model on a loan-level basis has a number of advantages. First, the cash flow data allows to analyze the utilization level of the insurance premiums for specific cash flow pattern, for fractional utilization of the net principal limit and for fractional utilization of the collateral. Second - due to the generalized form of the model - the model can be calibrated to fit markets other than the U.S. to compute fair-priced cash advances of reverse mortgages for other markets. The flexible calibration also allows for comparative statics that captures the sensitivity of the solution to the model's assumptions. Finally, the OOP implementation on a loan-level basis makes possible the later extension of the code to a simulation of portfolios of contracts.

3.4.2 Class Descriptions

Model Class `model` defines the parameters and other configuration options of the risk management model and thereby separates the exogenous factors from the logic of the model's equations. It also stores the principal limit factors, because any distinct instance of class `model` will result in an associated, different principal limit factor matrix.

Contract Class `contract` encapsulates all loan-specific data of a single reverse mortgage contract and contains most of the data and functionalities of the code, including the numerical iteration of the principal limit factor. It requires as inputs an HECM model configuration, the borrower's characteristics, her payment scheme preferences, the initial collateral value and the expected rate.

Probability Class `probability` is used by and stored within class `contract`. It imports, transforms and interpolates yearly mortality table input data to compute monthly loan survival probabilities (see equation (3.6)) and monthly loan termination probabilities (see equation (3.9)). Class `probability` also stores the probability of the occurrence of a loss based on the geometric Brownian motion process (see equation (3.13)).

Account Instances of the class `account` are stored within class `contract` and contain all loan-related time series vectors (e.g. advances, interest, premiums, outstanding loan balance, principal limit) which depend on a specific cash flow pattern. `Account` also computes the present value of expected losses and the present value of expected premiums based on the specific cash flow pattern and returns the expected utilization rate u as the percentage ratio of both factors.

$$u = \frac{\sum_{t=0}^{T} E(L_t)(1+i_t^d)^{-t}}{\sum_{t=0}^{T} E(P_t)(1+i_t^d)^{-t}} \tag{3.21}$$

With $u = 100\%$, the insurance equation (3.1) holds with equality. Using the principal limit factor and the cash flow pattern of a maximum, one-time upfront payment, the expected utilization rate u will - by definition - equal 100%. With $u < 100\%$, the insurance fund will operate with an expected surplus for the loan given the specific assumption for the cash flow pattern.

By default, every instance of `contract` will store three different instances of the `account` class with every one of them assuming a different cash flow pattern: the first assumes a maximum, one-time upfront payment; the second assumes the cash flow pattern that was chosen by the borrower at the instantiation of the associated `contract` class; the third computes the time series vectors assuming a simulated lifetime history of the contract with a cash flow pattern chosen by the borrower in the associate instance of class `contract`.

3.4.3 Numerical Solution

The basic HECM payment model does not provide a closed-form analytical solution to finding the principal limit factor function $plf(x, i_e, g)$. Instead, we need to define a numerical solution that discretizes the continuous $plf(x, i_e, g)$ function to a principal limit factor matrix plf. Every distinct instance of the `model` class will have a different principal limit factor matrix. The matrix is of dimension $n \times m \times 2$, where n is the age range of the initial age x of the borrower and m is the number of values for the expected rate that we solve for.

The numerical problem is a simple root finding problem for a continuous, real-valued, non-linear function. With a binding insurance equation (3.1), the difference of the present value of expected premiums and the present value of expected losses will equal zero. This net present value of the insurance contract is the objective function for the root finding algorithm and computed by the `get_npv` method of class `contract`. MATLAB provides a built-in function `fzero` for root finding problems of continuous functions of one variable that is used in the implementation. `fzero` uses a combination of bisection, secant, and inverse quadratic interpolation methods and takes advantage of the fact that $0 < plf(x, i_e, g) < 1$.[8]

Every iteration of the root finding method takes a new guess at the principal limit factor. Based on this guess the time series payment vectors of a maximum, one-time upfront payment scheme are computed recursively, including the time series vector of insurance premiums. Using the modeling assumptions of the property value, the next step then calculates the time series vector of expected

[8] For a detailed description of all three root finding methods, see Chapter 3 of Miranda and Fackler (2004).

losses of the insurer given the outstanding loan balance. In a last step, the iteration run calculates the net present value of the insurance.

3.5 Calibration

The solution of a generalized HECM risk management model allows to calibrate the parameters to fit other, non-U.S. markets. In this paper I look at Germany and compute actuarially-fair reverse mortgage contracts and cash advances for the German market. This is a purely theoretical exercise to the extent to which the actual design of a German reverse mortgage program is different from the HECM Program's design. However, the analysis of the outcome of a calibrated model still provides a lot of value.

First, a calibrated model results in an upper-bound estimate of the maximum cash advance for any possible payment scheme of a German reverse mortgage program. The borrower's demand decision for reverse mortgages is at least partially influenced by the attractiveness of the cash advance. A computation of the actual income potential for elderly homeowners allows lenders to judge the demand prospectives of home equity products in Germany.

Second, the HECM Program's design arguably serves as a role model for the design of reverse mortgage programs in other local markets. Existing plans for the introduction of home equity products in Germany organize the risk sharing mechanism of the crossover risk in a way that is very similar to the HECM Program's approach. Their underlying risk management approach, however, is inferior.[9] A calibrated model shows how the risk management experience and flexibility of the HECM Program can be transfered to Germany and other local markets.

3.5.1 Mortality Table Data

The implementation includes five different mortality tables for Germany and the United States. The tables provided are two DAV 2004 R tables (Deutsche Aktuarsvereinigung (2005)), the 2005/2007 mortality table by the German Federal Statistical Office and the U.S. Decennial Lifetables for 1979-1981 (National Center for Health Statistics (1985)) and 1999-2001 (Arias et al. (2008)).

The DAV 2004 R table data is the de-facto standard for private German insurance companies in pricing longevity risk. Survival probabilities computed for the DAV 2004 R tables are based on large data sets of German annuity insurance contracts collected by reinsurance companies. Since the tables have been explicitly constructed for the risk management of longevity risk their maximum age is high (in the DAV 2004 R tables $T = 120$), they take account

[9] One recent effort to introduce home equity conversion products in Germany is the 'Förder-Immorente' product planed by the banking association of the German public sector banks (Bundesverband Öffentlicher Banken Deutschland (VÖB)). The product design proposes an insurance fund at the level of the banking association while member banks serve as lender. The risk management model, however, is inferior to the one of the HECM Program: it does only account for the collateral risk through a lump-sum deduction from the market value of the collateral, limits flexibility in payment options and breaks with the non-recourse debt property.

of future mortality trends and they weight the case data by the value of the annuity insurance contract.[10] The DAV 2004 R tables arguably also factor in self-selection effects to the extent to which the incentives of potential German reverse mortgage borrowers are aligned with those who own annuity insurances.

The implementation includes the DAV 2004 R basic table data of the first and second order. The basic first order table data is the best estimate of periodic mortality probabilities from the original case data. The basic second order table data discounts periodic mortality probabilities of the basic first order table to account for the risk of the deviation of statistical parameter estimates of the data construction technique from its true values and the model risk.

Since the DAV 2004 R table data is a specific-purpose mortality table based on pre-selected case data, I also include the 2005/2007 mortality table by the German Federal Statistical Office (FSO) for comparison. The FSO table data ($T = 100$) is a general-purpose mortality table computed based on the official reporting of death cases in Germany for the years 2005 to 2007. It does not make any adjustments to the original data.

I include the U.S. Decennial Lifetables for 1979-1981 ($T = 110$) to allow a verification of the model's results by comparing it with the data presented in Szymanoski (1994). The U.S. Decennial Lifetables for 1999-2001 ($T = 110$) are used to compare the results for the U.S. and Germany.

3.5.2 Move-Out Factor

Nobody at the time of the HECM Program's inception could foresee the pattern of loan termination probabilities in an actual portfolio of self-selected reverse mortgage borrowers. The existing HECM portfolio history now allows for a more comprehensive analysis of loan termination probabilities and the move-out factor assumption. While Chow et al. (2000) note that HUD cannot distinguish between mortality and move-out in actual HECM terminations due to the data collection method, observed loan termination probabilities can be checked for consistency with the original 0.3 assumption.

Chow et al. (2000), McConaghy (2004), and Rodda et al. (2004) construct multivariate statistical models of HECM termination probabilities and show that factors such as borrower type, house price appreciation at the metropolitan area level, and interest rates affect termination probabilities. Enriquez et al. (2007) present detailed results for a discrete-time hazard model that groups borrowers by initial age and status (couple, single female, single male) and estimates empirical hazard rates to compare them with mortality table data. The authors find that for most borrowers - especially for younger age groups - HUD's loan termination probability assumptions seem to underestimate actual hazard rates. For borrowers in the 64-66 years age group, payoffs occur at approximately 6 to 8 times the female mortality rate. Borrowers in their mid-70s at loan origination still terminate their loans at about 2 to 3 times the female mortality rates. For older age groups, the 30% move-out rate assumption about

[10] The annuity insurance case data suggests lower mortality for high-value annuity insurance contracts (see p. 71, Deutsche Aktuarsvereinigung (2005)). For an extensive explanation of the data construction techniques for the individual tables, see Deutsche Aktuarsvereinigung (2005).

fits the hazard rate.

Neither the modeling of move-out probabilities as a constant fraction of mortality rates nor the original value for the move-out rate assumption fit the HECM portfolio data particularly well. However, the assumption at least proved to be sufficiently conservative to the HECM insurance fund. Lacking experience and hence data, I use the same move-out rate assumption as the HECM Program for the calibration of the German scenario. An alternative would be to try to model German age-specific move-out rates, e.g. from micro-level household data such as the German Socio-Economic Panel. However, the added value probably does not justify the effort at this time because such a model would still not account for the potentially different behavior of self-selected reverse mortgage borrowers.

3.5.3 Premium Structure

The insurer may freely calibrate the premium structure parameters α, β in equation (3.3) without any risk to the insurance fund. Strictly speaking both parameters are no exogenous feature of the risk management model. The HECM Program's choice for the parameters follows the example of the FHA mortgage insurance premium. For the German scenario I use the same assumption as the HECM Program.

Note that every distinct parameterization of α, β will support a different initial principal limit: the higher expected premiums, the higher expected losses and cash advances that can be supported for the loan. By choice of a parameterization the insurer implicitly sets a cost structure for the reverse mortgage that relates the cost of the insurance to the equity fraction the borrower has access to.

3.5.4 Geometric Brownian Motion

The mean appreciation μ and the deviation from the mean σ are crucial assumptions for the geometric Brownian motion process that models collateral risk. Szymanoski (1990) argues that there exists an historically close relationship in the U.S. between the average annual house price appreciation measured by constant quality housing price indexes and the overall inflation rate measured by the CPI. For the years between 1977 and 1988 both average about 6%.

Szymanoski's argument at the time is supported by Case and Shiller (1987). They construct a housing index for four cities by a weighted repeat sales method for the time period 1970 to 1986 and also find that house price appreciation was greater than or equal to the CPI during the same period. Given the two very similar results assumption of a nominal mean appreciation rate of $\mu = 4\%$ for the HECM Program was considered a sufficiently conservative estimate.[11]

[11] When the value of existing houses depreciates at a rate of 1% per year, then an overall inflation of 5% percent would be necessary to support a mean appreciation rate of $\mu = 4\%$. Also, houses of elderly homeowners could appreciate at a lower rate due to locations in older neighboorhoods and moral hazard effects in the maintenance of the home. In an analysis of the question with national longitudinal sample data of the Annual Housing Surveys 1974-1983 Szymanoski (1990) does not, however, find any evidence for this effect.

While quality and availability of residential house price index data is good for the U.S., very few reliable data exists for Germany. Faced with the spillover effects of the U.S. housing market bubble, policymakers have taken notice and the German Federal Statistical Office is currently leading the German effort of a Eurostat initiative to develop a national house price index for self-owned property (see Dechent (2008)). The resulting data is meant to be used in the future by the ECB, Deutsche Bundesbank, Eurostat and others, but until now has only been computed back to 2000 and the data has not been published.

Other currently available data comes from a number of different sources with varying quality. Deutsche Bundesbank publishes a residential house price index that is computed based on data from private real estate company BulwienGesa AG. The index, however, only goes back to the year 1995. Other sources of house price data are the DEIX Deutscher Eigentums-Immobilien-Index (with 1989-2005 data), data from IFS Städtebauinstitut and from IPD Investment Property Database. However, much of this data has limited value because it is based on commercial real estate, imputes rents and/or covers a limited time range.

The most reliable data both in terms of data quality and coverage seems to come from BulwienGesa AG. BulwienGesa publishes real estate indexes for eight different categories - including residential real estate - in their internal RIWIS system. Individual house price data for the index comes from a number of sources, including certified real estate appraisers, surveys conducted by the company, data analysis of online real estate marketplaces and test purchases. Their yearly house price index 'Wohnen' covers the time range 1975-2008, with data from 1975-1989 for West Germany and since 1990 for unified Germany. The mean of the annualized house price index return rate as the best estimator for μ is 2.4%, which I will use for the German calibration. Lacking any empirical studies on the variability of German house price returns, I follow the HECM Program and set $\sigma = 10\%$ for the German scenarios.

3.6 Results

Scenarios Before we turn to the results, Table 3.1 presents the calibration of five different scenarios. Each scenario assumes specific mortality table data, a move-out factor m, premium structure parameters α, β and geometric Brownian motion parameters for drift μ and diffusion σ.

Table 3.1: Parameter Calibration of Scenarios

	new U.S.	**old U.S.**	**1st German**	**2nd German**	**FSO German**
mortality table	US Decennial Lifetable 1999-2001	US Decennial Lifetable 1979-1981	DAV 2004 R base table 1st order	DAV 2004 R base table 2nd order	German Federal Statistical Office 2005/2007
m	0.3	0.3	0.3	0.3	0.3
α	2%	2%	2%	2%	2%
β	0.5%	0.5%	0.5%	0.5%	0.5%
μ	4%	4%	2.4%	2.4%	2.4%
σ	10%	10%	10%	10%	10%

Principal Limit Factors With the fully calibrated model I am able to compute principal limit factor matrices. Matrices for male and female borrowers are presented in Figure 3.3. The bottom graphic of Figure 3.3 shows the difference of male and female principal limit factors. Values are based on the new U.S. scenario and graphed by expected rate and the age of the borrower at origination.

Note that males have higher principal limit factors than females due to their lower life expectancy. The difference becomes smaller as the conditional life expectancy converges with higher initial age of the borrower. As one would expect, principal limit factors strictly increase in age and strictly decrease in the expected rate.[12]

Maximum Cash Advances Given the principal limit factor $plf(x, i_e, g)$ we can derive maximum monthly cash advances to a borrower based on equation (3.20). Table 3.2 by way of example presents the maximum monthly cash advance for tenure, 10-year term and 20-year term payment schemes by scenario. Values have been computed with expected rate $i_e = 7.0\%$ and an initial collateral value $C_0 = 200,000$. To better compare the different scenarios, Table 3.3 shows the resulting maximum monthly cash advances again as %-fraction of the new U.S. case scenario.

Table 3.2: Maximum Monthly Cash Advances
$i_e = 7.0\%$, $C_0 = 200000$

female, 65-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment	613.27	624.79	378.58	400.33	449.78
10-year term payment	1127.37	1148.54	708.87	749.59	800.51
20-year term payment	765.11	779.48	481.09	508.37	543.28

female, 75-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment	795.08	806.76	548.71	579.93	690.89
10-year term payment	1407.70	1428.38	1011.09	1068.62	1137.86
20-year term payment	955.36	969.40	686.20	725.24	772.23

male, 65-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment	669.07	706.93	428.01	452.24	509.72
10-year term payment	1229.94	1299.55	801.43	846.80	907.19
20-year term payment	834.72	881.96	543.91	574.70	615.68

male, 75-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment	852.87	882.73	608.79	641.31	754.65
10-year term payment	1510.01	1562.89	1121.79	1181.73	1242.87
20-year term payment	1024.80	1060.69	761.33	802.00	843.50

Note that the cash advances for the German scenarios are substantially lower than those for the U.S. scenarios. The FSO scenario's maximum cash advances are 22% lower than in the new U.S. scenario if averaged across all cases. Cash advances for the DAV 2004 R tables are even less, with 28% lower payouts for

[12]Strict monotonicity in both variables can be confirmed by method `ismonotonic` of class `model`.

Table 3.3: Maximum Monthly Cash Advances (as % of new U.S. calibration); $i_e = 7.0\%$, $C_0 = 200000$

female, 65-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment	100%	102%	62%	65%	73%
10-year term payment	100%	102%	63%	66%	71%
20 year term payment	100%	102%	63%	66%	71%

female, 75-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment	100%	101%	69%	73%	87%
10-year term payment	100%	101%	72%	76%	81%
20 year term payment	100%	101%	72%	76%	81%

male, 65-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment	100%	106%	64%	68%	76%
10-year term payment	100%	106%	65%	69%	74%
20 year term payment	100%	106%	65%	69%	74%

male, 75-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment	100%	104%	71%	75%	88%
10-year term payment	100%	104%	74%	78%	82%
20 year term payment	100%	104%	74%	78%	82%

the second-order base mortality table and 32% lower payouts for the first-order base mortality table. The differences between the U.S. and German scenarios are smaller the higher the initial age of the borrower at the time of the origination of the loan.

To understand to what degree mortality table data and the drift parameter contribute to the lower payouts, I also computed maximum monthly payments for German scenarios, where $\mu = 4\%$ and all other parameters are as in Table 3.1. With this calibration only mortality tables are different across the scenarios. Maximum monthly payments are on average 1% higher in the FSO scenario than in the new U.S. scenario. They are on average 6% lower in the DAV 2004 R second order table scenario and 9% lower in the DAV 2004 R first order table scenario.

The results suggest that reverse mortgages are a substantially less attractive product in Germany than in the United States. A small part of the effect is due to a longer conditional life expectancy for elderlies in Germany. However, the main difference comes from the dynamic of real estate prices: the long-term trend justifies higher expectations for the nominal mean appreciation of real estate in the U.S. than in Germany.

Sensitivity of Maximum Cash Advances Maximum monthly cash advances vary substantially for different exogenous parameters. The results from Table 3.2 and the additional computation for German scenarios with $\mu = 4\%$ e.g. suggest a high sensitivity of maximum cash advances to a change in the drift. To give a better intuition for the sensitivity of the principal limit to the parameter values, Figure 3.4 shows maximum monthly 10-year term and tenure cash advances for a 65-year old female borrower with $i_e = 7.0\%$ and $C_0 = 200000$ by different exogenous parameters.

Sensitivity of Premium Utilization It is also important to understand how sensitive the net receivables of the insurance is to the assumptions of the pricing model. This is a very different question from the previous paragraph and Figure 3.4, were I computed maximum cash advances assuming different values for the exogenous parameters. Now I ask: how sensitive is the stability of the insurance fund to wrong assumptions about the parameters? Or in technical terms of the model: how does the expected utilization rate u change when the real world behaves like in the pricing model, but the true parameter is different from the one that I use to price the contract? If the change to the expected utilization rate u is large, then an imprecise calibration of a parameter might jeopardize the insurance fund.

We follow Szymanoski (1990) in the presentation of the comparative statics and compute by example the expected utilization rate u for a situation in which the assumed parameter value differs from the true parameter value of a baseline model. The baseline model scenario is 1st German. Parameters that I vary include the move-out factor m, the drift μ, the diffusion σ and the constant discount rate i_t^d. Figure 3.5 presents the results.

While I do not directly analyze the uncertainty in loan termination probabilities, a variation in the move-out factor scales periodic mortality table probabilities to different levels. This manipulation achieves a similar effect as the direct manipulation of loan termination probabilities.

Premium Utilization in the Principal Limit Method Note that the expected utilization rate u of the baseline scenario in Figure 3.5 does not always equal 100% as one might expect. In fact, most contracts do not have utilization rates of 100%. This might seem paradoxical at first. From the start the single purpose was to solve the model such that insurance equation (3.1) would be binding. However, this statement is only true for payment calculations in the basic HECM payment model when we assume a cash advance pattern of a maximum, one-time upfront payment. Most actual loans contain either term or tenure monthly cash advances computed by equation (3.20). The utilization factor u will be different from 100% to the extent that the timing of a monthly cash advance over time is different from the basic HECM payment model. The different timing changes the structure of the outstanding balance, which then implicitly drives variables in the model (e.g. premiums, expected loss).

The difference in the expected utilization rate between the HECM basic payment model and the principal limit method, however, is not large.[13] Table 3.4 shows expected utilization rates u for the previous examples. The table shows the expected utilization rate u for a 100%, 80% and 60% use of the initial principal limit. A full draw of the maximum monthly payment equals 100% and other, lower values represent a borrower who does not ask for the full amount of the maximum monthly payment. Compared to the impact of a wrong calibration of parameters on the stability of the insurance fund, the inaccuracy of the payment model's design is insignificant. However, if the borrower does not use his full initial principal limit, then this creates excess value for the insurer.

[13] Expected utilization rates u for the HECM basic payment model and the principal limit method are calculated and presented by method `compare` of class `contract`.

Premium Utilization by Collateral Value Table 3.5 presents a similar logic for the insurer's use of the market value of the collateral. The insurer might only insure loans up to a certain fraction of the property value. This equals a lump-sum deduction from the market value. With a lump-sum deduction the insurer creates significant excess value, which in Table 3.5 has been computed for a 100%, 90% and 80% use of the market value of the collateral.

Figure 3.3: Principal Limit Factor Matrix by Gender and Age, new U.S. Scenario

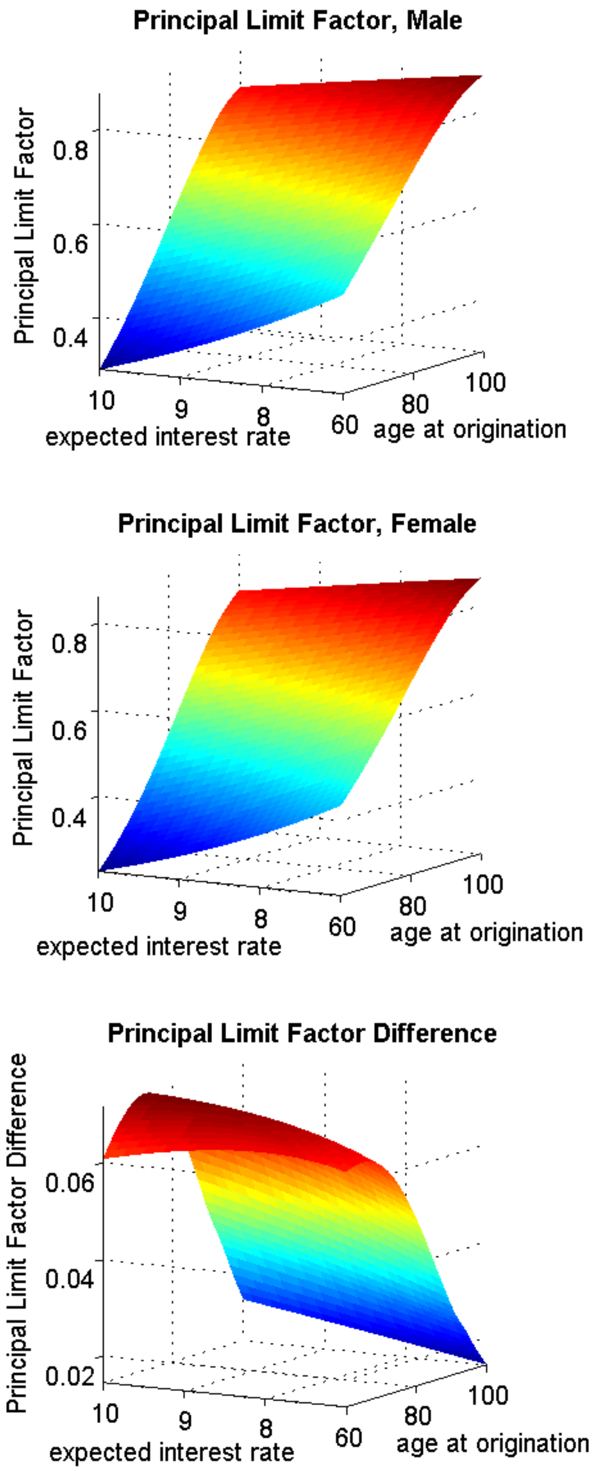


Figure 3.4: Parameter Sensitivity of Maximum Monthly Cash Advance to Variation of Calibrated Value of Exogenous Parameter

Computed for 65-year old female (except for 'by initial age of borrower', $i_e = 7.0\%$, $i^d = 5.0\%$, $C_0 = 200000$, scenario 1st German

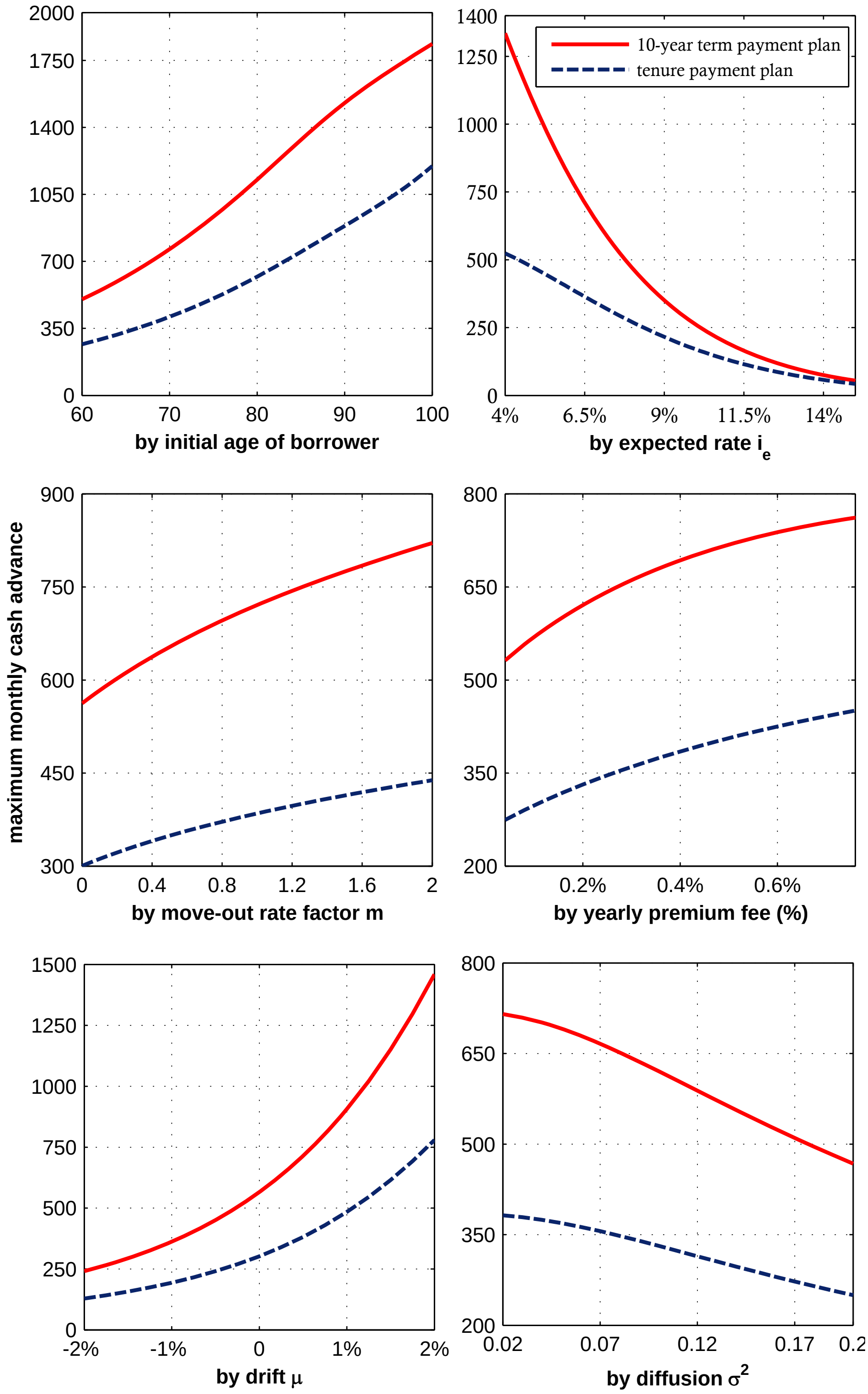


Figure 3.5: Sensitivity of Utilization Rate u to a Deviation of Parameter Value from True, Baseline Model Value (=red line)

Computed for 65-year old female, tenure payment scheme, $i_e = 7.0\%$, $i^d = 5.0\%$, $C_0 = 200000$, scenario 1st German

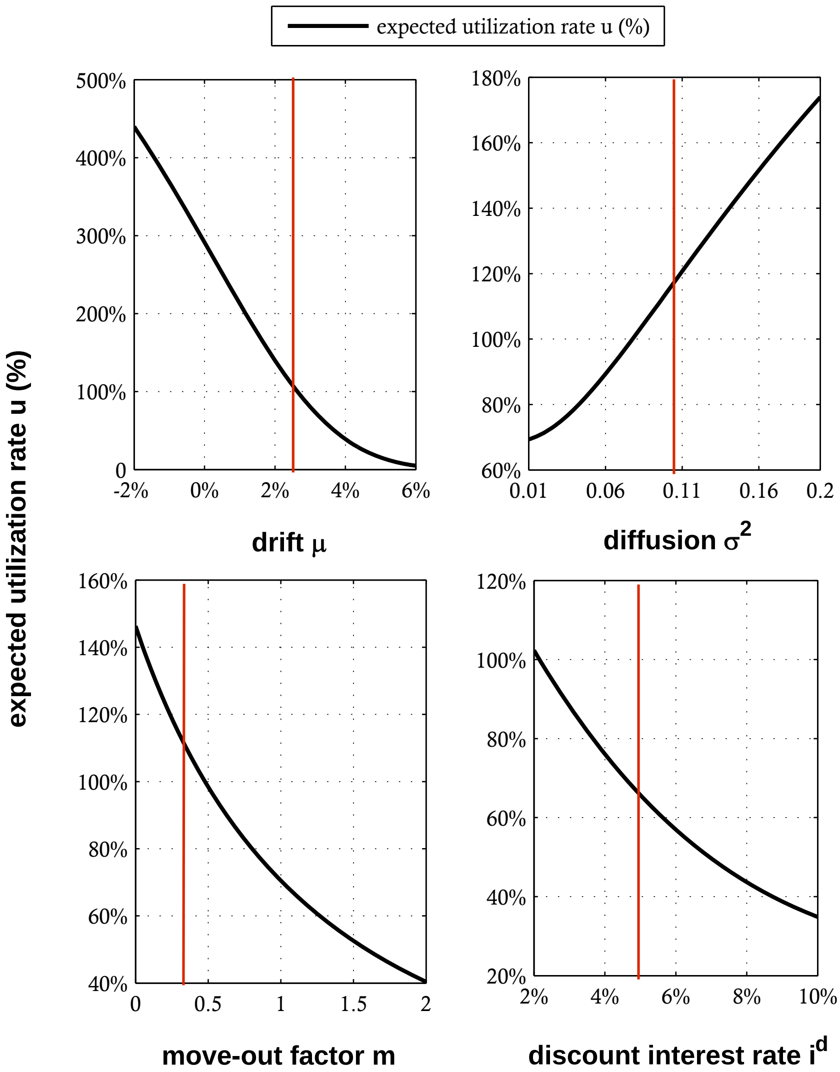


Table 3.4: Expected Utilization Rate u by %-Utilization Level of Cash Advance; $i_e = 7.0\%$, $C_0 = 200000$

female, 65-year old	**new U.S.**	**old U.S.**	**1st German**	**2nd German**	**FSO German**
tenure payment					
100%	98.8%	96.2%	111.0%	106.3%	112.8%
80%	54.2%	52.3%	67.2%	62.7%	65.4%
60%	22.4%	21.4%	32.4%	29.1%	29.4%
10-year term payment					
100%	130.2%	130.7%	129.5%	130.3%	131.3%
80%	72.1%	71.7%	79.7%	78.4%	76.9%
60%	29.9%	29.4%	39.1%	37.0%	34.7%
20-year term payment					
100%	135.7%	135.0%	139.9%	140.3%	140.0%
80%	76.2%	75.2%	85.8%	84.3%	82.2%
60%	31.9%	31.2%	41.8%	39.7%	37.1%

female, 75-year old	**new U.S.**	**old U.S.**	**1st German**	**2nd German**	**FSO German**
tenure payment					
100%	62.8%	59.7%	78.2%	69.0%	81.9%
80%	29.1%	27.6%	41.3%	34.9%	40.5%
60%	9.4%	9.0%	16.5%	13.1%	14.4%
10-year term payment					
100%	134.1%	133.6%	135.6%	136.6%	137.3%
80%	63.0%	62.1%	72.1%	70.1%	67.5%
60%	20.1%	19.6%	28.5%	26.1%	23.2%
20-year term payment					
100%	104.4%	100.1%	126.2%	118.9%	108.8%
80%	51.1%	48.7%	69.2%	63.2%	55.3%
60%	17.4%	16.5%	28.4%	24.7%	20.1%

male, 65-year old	**new U.S.**	**old U.S.**	**1st German**	**2nd German**	**FSO German**
tenure payment					
100%	85.7%	77.6%	102.0%	96.4%	102.3%
80%	44.9%	39.8%	59.5%	54.7%	58.0%
60%	17.4%	15.1%	27.2%	24.0%	25.3%
10-year term payment					
100%	132.5%	133.3%	131.0%	131.9%	133.0%
80%	69.8%	68.4%	77.7%	76.4%	75.4%
60%	27.0%	25.5%	36.0%	34.1%	32.7%
20-year term payment					
100%	129.6%	123.0%	139.4%	138.5%	135.7%
80%	70.2%	65.4%	83.0%	80.8%	78.1%
60%	27.9%	25.4%	38.5%	36.2%	34.4%

male, 75-year old	**new U.S.**	**old U.S.**	**1st German**	**2nd German**	**FSO German**
tenure payment					
100%	47.3%	41.3%	63.9%	54.8%	69.0%
80%	20.7%	17.9%	31.9%	26.1%	33.3%
60%	6.2%	5.3%	11.7%	9.0%	11.5%
10-year term payment					
100%	129.7%	124.9%	136.6%	136.5%	134.9%
80%	58.0%	55.0%	69.1%	66.7%	64.7%
60%	17.0%	15.8%	25.2%	22.8%	21.4%
20-year term payment					
100%	84.4%	74.7%	112.8%	102.7%	93.8%
80%	39.3%	34.4%	59.3%	52.2%	46.8%
60%	12.6%	10.9%	22.9%	19.2%	16.7%

Table 3.5: Expected Utilization Rate u by %-Utilization Level of Market Collateral Value C_0; $i_e = 7.0\%$, $C_0 = 200000$

female, 65-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment					
100%	98.8%	96.2%	111.0%	106.3%	112.8%
90%	77.9%	75.6%	91.2%	86.5%	91.3%
80%	58.4%	56.5%	72.1%	67.4%	70.7%
10-year term payment					
100%	130.2%	130.7%	129.5%	130.3%	131.3%
90%	102.2%	102.2%	106.5%	106.2%	105.9%
80%	76.3%	76.0%	84.2%	83.0%	81.6%
20-year term payment					
100%	135.7%	135.0%	139.9%	140.3%	140.0%
90%	107.9%	107.0%	115.5%	115.0%	113.8%
80%	81.6%	80.7%	91.6%	90.2%	88.3%

female, 75-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment					
100%	62.8%	59.7%	78.2%	69.0%	81.9%
90%	46.4%	44.1%	61.1%	53.0%	62.4%
80%	32.2%	30.6%	45.5%	38.6%	44.9%
10-year term payment					
100%	134.1%	133.6%	135.6%	136.6%	137.3%
90%	99.1%	98.4%	105.4%	104.8%	103.7%
80%	68.4%	67.6%	77.8%	75.9%	73.6%
20-year term payment					
100%	104.4%	100.1%	126.2%	118.9%	108.8%
90%	79.1%	75.7%	100.3%	93.4%	84.0%
80%	56.4%	53.9%	75.8%	69.6%	61.3%

male, 65-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment					
100%	85.7%	77.6%	102.0%	96.4%	102.3%
90%	66.4%	59.8%	82.8%	77.4%	82.4%
80%	48.9%	43.8%	64.4%	59.4%	63.4%
10-year term payment					
100%	132.5%	133.3%	131.0%	131.9%	133.0%
90%	102.2%	101.9%	106.4%	106.1%	106.2%
80%	74.6%	73.7%	82.7%	81.6%	81.0%
20-year term payment					
100%	129.6%	123.0%	139.4%	138.5%	135.7%
90%	101.9%	96.3%	114.0%	112.5%	109.9%
80%	76.0%	71.5%	89.4%	87.4%	85.1%

male, 75-year old	new U.S.	old U.S.	1st German	2nd German	FSO German
tenure payment					
100%	47.3%	41.3%	63.9%	54.8%	69.0%
90%	34.2%	29.8%	48.9%	41.1%	52.3%
80%	23.1%	20.2%	35.4%	29.1%	37.4%
10-year term payment					
100%	129.7%	124.9%	136.6%	136.5%	134.9%
90%	94.4%	90.6%	104.5%	103.1%	101.4%
80%	63.8%	61.0%	75.4%	73.1%	71.4%
20-year term payment					
100%	84.4%	74.7%	112.8%	102.7%	93.8%
90%	62.9%	55.5%	88.4%	79.4%	72.2%
80%	44.0%	38.8%	65.8%	58.2%	52.5%

Chapter 4

Stochastic Simulation of Portfolios

A lender or insurer of contracts is eventually more interested in the properties of a portfolio of contracts than in the expected properties of a single loan. More specifically, the insurer is interested in the variability of the net receivables of the insurance fund of a portfolio of contracts if interest rates and house price appreciation deviate from the constant assumptions of the model. If both variables fluctuate according to historical patterns than the resulting variability of the net receivables of the fund can be interpreted as a risk measure for the model itself. The distinction is between an expected loss and an unexpected loss: the expected loss is covered for by the insurance premiums under the model's assumptions while the unexpected loss may result from an inevitable discrepancy between the model's assumptions and future realizations of the real world.

4.1 Motivation

Quantifying the unexpected loss by stochastic simulation of portfolios is particularly interesting when taking into account the market environment in most non-US reverse mortgage markets. In no country other than the U.S. exists a government-backed, federal insurance similar to the HECM Program. Private lenders that originate reverse mortgages have to insure their loans themselves and act simultaneously as both lender and insurer of their loans. The usual assumption by those private lenders arguably has been that the potential of reverse mortgages does not justify the risks. Risk in this context is not the crossover risk, but the risk of a future negative scenario that might jeopardize the insurance fund.

A stochastic simulation of future scenarios offers a way to assess the stability characteristics of the analytical model and its consequences for the insurance fund. I use existing data of the HECM Program together with the computational implementation of this paper to quantify the risk of an unexpected loss. If the variance of the net receivables of the insurance fund is sufficiently low compared to the revenue potential of a lender, then any private lender should reconsider the assumption of the inherent riskiness of reverse mortgages.

4.2 Stochastic Modeling

The stochastic modeling of reverse mortgage contracts requires multiple steps. Every step and its underlying process is described in this section. The stochastic effects of each simulation trial run are multidimensional: stochastic variables include the house price appreciation μ, the loan interest rate r, the contract-specific time of loan termination and the contract-specific path of the collateral value C_t. I write the realized path of the stochastic simulation of each variable with a bar, e.g. $\bar{\mu}_n$ is the realized time series path of appreciation rates for simulation run n and $\bar{C}_n^m$ is the realized time series path of the collateral for loan m of simulation run n.

House price appreciation path $\bar{\mu}_n$ and loan interest rate path $\bar{r}_n$ affect all contracts in simulation run n equally. The time of loan termination $\bar{T}_n^m$ and the path of the collateral $\bar{C}_n^m$ are simulated separately for every loan in each trial run. The simulation documented in this chapter is done with $M = 500$ contracts and $N = 100$ trial runs. As calibration for the model's parameters I use the 1st German scenario from Table 3.1, except for the house price appreciation rate μ which I dampen to $\mu = 0\%$.

4.2.1 HECM Data

An available dataset of HECM contracts allows me to sample borrower characteristics from an existing reverse mortgage program to create more realistic contract features for the simulation. HUD provided to me a dataset of $390,626$ contracts with 42 variables. The most recent loan of the dataset was originated in March 2007. Unfortunately the data is not complete, e.g. for $129,435$ contracts there is no data of the birth date of the borrower. The dataset also misses the variable for the payment scheme choice.

Consistency checking and data validation leaves me with $258,051$ observations with valid data on the value of the appraised property, the initial age and the gender of the borrower. Figure 4.2 shows a 2D-kernel density estimate for appraised value and the initial age of the borrower in the dataset. Figure 4.3 shows separate 1D-kernel density estimates for initial age and appraised value. Note that the left part of the kernel density for the collateral value distribution looks like a typical Pareto distribution which is often observed for wealth. Also, very few borrowers enter their reverse mortgage contract when they are older than 90 years.

I sample 500 contracts from the dataset to determine the initial age, the gender and the appraised value of the collateral for the initial portfolio of the simulation. Different from the HECM model, the simulation actually uses the gender information to derive different principal limit factors for males and females. Since there is no payment scheme information, I choose payment schemes manually: 50% of the contracts are tenure loans, 50% are term loans. The term loan duration is set randomly between 5 and 20 years. Since I do not put line-of-credit loans into the portfolio (which typically have lower utilization levels because of long periods without new cash advances) I dampen the utilization level of the collateral value to 85% for all loans. This is in line with the utilization levels of the collateral value observed in the HECM Program[1].

The HECM Program's participants may be different from potential German reverse mortgage borrowers and the distribution of the appraised values might be as well. However, I still regard the sampling process as superior to a random generation of borrower characteristics, as it still captures some of the structure of the data.

4.2.2 Termination Event

For a set of contracts in the original portfolio I start out by simulating the time period of the loan termination $\bar{T}_n^m$ for each loan. Loan termination probabilities for the random draw are based on the probabilities of the pricing model in equations (3.9)-(3.11) and allow to distinguish between move-out and death as reasons for the termination of the loan. I assume that the insurer settles a potential loss immediately and has no sales expenses, that is she sells the collateral in the termination period at market price.

Since the termination event is determined based on the analytical probabilities of the pricing model, the simulation results will not reflect empirical loan termination probabilities of an actual portfolio of reverse mortgages. This will prove to be an important limitation of my approach. The difference between the analytical and empirical termination probabilities of HECM contracts is in fact large and has a significant impact on the simulation results. Enriquez et al. (2007) find that HECM loans terminate significantly faster across all age groups than the move-out factor of the HECM pricing model assumes. Davidoff and Welke (2007) explain this observation by favorable selection: HECM borrowers may be self-selected heavy discounters who prefer to move early to consume their profits from increased housing equity in a market environment of increasing house prices.

With the stochastic termination of loans based on smaller analytical termination probabilities, the simulation results will be biased toward a lower value of the insurance fund because more borrowers live longer and eventually outgrow their equity. A more realistic approach would explicitly model empirical loan termination probabilities (see below). However, the dataset of HECM contracts that is available to me unfortunately does not contain enough information to follow this strategy at this point. I might be able to explicitly model empirical loan probabilities at a later stage based on more complete HECM data. In the meantime absolute values for the net receivables will be biased, unless the move-out factor is set $m = 0$ and the reverse mortgage contract is redefined such that only more predictable death events result in termination.

One straightforward way to model empirical loan termination probabilities is a discrete-time hazard model (see Enriquez et al. (2007), Chow et al. (2000)). Rodda et al. (2004) e.g. estimate a multivariate discrete-time hazard model that

[1] The excess value of collateral in the HECM Program is caused by the definition of the maximum claim amount that limits cash advances beyond a threshold value of collateral.

also incorporates house prices and interest rates as regressors. This approach allows to drive empirical loan termination probabilities in a simulation model by the realized time series paths of appreciation rates $\bar{\mu}_n$ and loan interest rates $\bar{r}_n$. Their estimated multivariate hazard model performs better than the original 0.3 assumption of the move-out factor.

The lack of a model for empirical loan termination probabilities limits the confidence in the simulation's results of the absolute value of the insurance fund. However, the fluctuation of the net receivables around its mean will not be affected by this deficiency if the analytical loan termination probabilities are uniform across all simulation runs. Rodda et al. (2004) show that the loan interest rate is not significant in explaining loan termination. While the cumulated house price growth is negatively correlated with termination probabilities and significant at the 5%-level, its effect on loan termination is small. The feedback effect of the simulated paths on termination probabilities is hence small, so that the variance of the net receivables is not affected by the lack of a hazard rate model.

4.2.3 Aggregate Interest Rate Risk

At the core of the stochastic model is the simulation of loan interest rates and house price appreciation rates. The fluctuation of both variables over the long simulation period of 60 years drives the simulation model. The time series model underlying the stochastic simulation should reflect the historical patterns of both time series and incorporate their correlation.

The model follows a 2-step approach: First, I estimate a transition matrix for a Markov sequence for German 1-year interest rates based on past observations. For each simulation run I then feed the simulated interest rate path as an exogenous input into a previously estimated time series model of house price appreciation. House price appreciation is assumed to follow a linear autoregressive process with lag p ($ARX(p)$), where 1-year interest rates are an exogenous input to the model.

I considered other modeling options, but finally retreated to this approach. The estimation of a vector auto regression for both interest rates and house price appreciation - while feasible - can not prevent the interest rate from becoming negative. Short-term interest rate models[2] often are highly-dimensional and require the modeling of the complete term structure of interest rates. The additional explanatory value helps little in the effort to understand the comovement with house prices. Given the small number of available observations for the BulwienGesa house price index ($T = 33$), the Markov Chain / ARX(p) modeling approach seems to be a good compromise. A simulation for other countries with more complete data may allow higher-dimensional time series modeling.

Model Structure The interest rate that I model is the monthly German 1-year interest rate. As in the HECM Program I link the loan interest rate to

[2]For a discussion of popular short-term interest rate models and their empirical performance, see Chan et al. (1992).

the 1-year rate: the actual loan interest rate $\bar{r}_n$ is set at the simulated German 1-year interest rate plus the lender's margin. I set the lender's margin to a constant 1.5% for all loans, which is the observed average from the HECM dataset. The loan interest rate adjusts to the current rate on a monthly basis. Simulated contracts are all adjustable-rate reverse mortgages, since fixed-rate reverse mortgages are not subject to interest rate risk.

The monthly German 1-year interest rate is assumed to be a Markov sequence. The Markov sequence models the transition of a variable from one level to the next through a transition probability matrix. Each row of the matrix represents a level on interest rates and each column represents the amount of change in the interest rate for the following month. Given an initial start value, sampling from the transition matrix then is sufficient to simulate a time series sequence of interest rates that follows historical patterns.

Estimation To construct historical transition probabilities for the transition probability matrix I use past data of the German 1-year monthly interest rate from Deutsche Bundesbank for the period September 1972 - February 2009 ($\#obs = 438$). The interest rate data is a Deutsche Bundesbank estimate of the risk-free, 1-year interest rate based on a Svensson method estimate of the yield curve. The empirical conditional probability q_{ij} of a change j to the next period given the level i in the current period is estimated as

$$q_{ij} = \frac{S_{ij}}{\sum\limits_{i \in I} S_{ij}} \tag{4.1}$$

where S_{ij} is the number of times in the dataset that the interest rate changed by j after being in level i. To allow an estimation from the data, changes and levels of the interest rate are grouped into bandwidths of values. Table 4.1 presents a stylized, highly aggregated version of the transition probability matrix that was used for the simulation.

Table 4.1: Transition Matrix Estimates for the Markov Chain: Probability of a Change of the German 1-year Interest Rate, Given Current Level of Interest Rate*

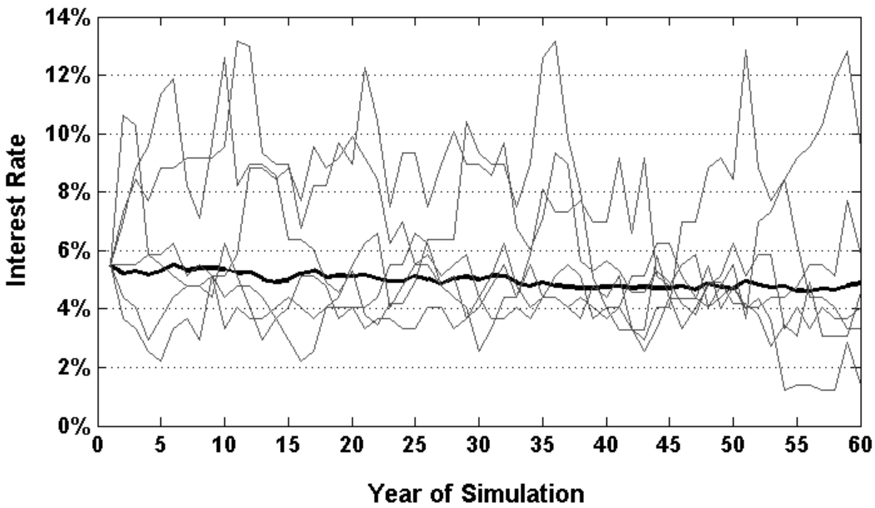
	Change of Interest Rate					
Level of Interest Rate	< -1.3%	-1.3 to -0.4%	-0.4 to +0.4%	+0.4 to +1.3%	+1.3 to +2.2%	> +2.2%
< 4.5%	0%	2%	97%	1%	0%	0%
4.5 to 6.2%	0%	5%	84%	11%	0%	0%
6.2 to 7.9%	0%	13%	75%	9%	4%	0%
7.9 to 9.7%	2%	15%	70%	10%	4%	0%
9.7 to 11.4%	0%	40%	53%	7%	0%	0%
> 11.4%	0%	29%	43%	29%	0%	0%

* Data Source: Deutsche Bundesbank Time Series WZ9808: Zinsstrukturkurve (Svensson-Methode), Börsennotierte Bundeswertpapiere, 1,0 Jahr(e) RLZ, Monatsendstand

Simulation To simulate interest rate paths I upgrade the model to a Monte Carlo Markov Chain. This essentially means sampling pseudo-random numbers from the conditional probability distribution in Table 4.1 to map the current interest rate level i_t into the next level i_{t+1}.[3] While interest rate values - starting from its initial value - move continuously across their domain, the shock j is always assumed to be the average value of its corresponding bandwidth. I set the initial starting value of the 1-year interest rate for all simulation runs to the long-term mean 5.50% of all observations. The interest rate paths circulate between the historical minimum of 1.05% and the historical maximum of 13.17% and - by definition of the matrix - show mean reversion.

The interest rate paths that result from the monthly random movements over a 60-year simulation period closely mimic the historical behavior of the 1-year interest rate. The range of the simulated interest rates covers almost completely the domain of historical rates: the minimum of the simulated values is 1.22%, the maximum 13.17%. The mean of all simulated values is 4.96%, which is slightly smaller than the observed historical average and corresponds to an average reverse mortgage loan rate of 6.46%. Figure 4.1 presents by example five simulated 1-year interest rate paths and the mean of the 100 paths of the simulation.

Figure 4.1: Five 1-year Simulated Interest Rate Paths and Mean of 100 Simulated Paths (in Bold)



While any given interest rate path is erratic in nature, a large number of equally-likely paths (in the simulation $n = 100$) reflect the probabilities of future high and low interest rate scenarios. Important for the fluctuation of the insurance fund is the accurate description of the tails of the probability distribution, because the tails entail the crossover risk. Minimizing a forecast error is not the objective of the model.

[3]Note that the level i_t is different from the actual value of the interest rate. For the sampling the routine uses the function `sampleDiscrete` from Kevin Murphy's 'pmtk3' toolbox to draw pseudo-random numbers from a discrete probability distribution.

4.2.4 Aggregate and Idiosyncratic House Price Risk

Model Structure The aggregate house price risk refers to the stochastic process of yearly house price returns. To model the univariate time series I use a simple autoregressive process with an exogenous input. An alternative strategy would have been to estimate a Markov transition matrix for house price returns as previously done for interest rates and then match the correlation of innovations between both. However, the small sample of annual house price returns from BulwienGesa index data ($T = 33$) makes this impossible. The autoregressive process has fewer parameters to estimate and still allows to drive simulated house price returns based on the stochastic realization of interest rates.

Estimation I estimate a univariate autoregressive process ARX(p) with the first-order difference of the (annualized) monthly German 1-year interest rates as exogenous input. The model's estimation is done with maximum likelihood. Maximum likelihood estimation is an alternative to OLS if the distribution of the process is Gaussian (Lütkepohl (2007)). The ARX(p) model that I estimate is

$$y_t = a_c + \sum_{i=1}^{p} ar_i y_{t-i} + br_{t-1} + u_t \tag{4.2}$$

where standard assumptions apply, that is the normally-distributed innovations have $E(u_t) = 0$, $Var(u_t) = \sigma_u^2$ and $E(u_t, u_s') = 0$ for $s \neq t$. I take log differences from the index data to remove the trend. Both the plot of the data and the plot of the sample autocorrelation suggest that the time series of return rates is stationary.[4]

For model selection of an appropriate number of lags p I follow the suggestion in Lütkepohl (2007). I look for the smallest possible ARX(p) order p such that $ar_p \neq 0$ and $ar_i = 0$ for $i > p$. I assume there is an upper bound M to p that I need to identify. I then test each restricted, smaller model (the null H_0) against the unrestricted, larger model (the alternative H_1) through a likelihood ratio test. The likelihood ratio test is based on the likelihood ratio test statistic which can be shown to have an asymptotic χ^2-distribution and degrees of freedom equal to the number of restrictions.[5] Test results are presented in Table 4.2. Each entry shows the binary for the rejection of the likelihood ratio test with the p-value in brackets.

The results suggest an optional number of lags $p = 4$. To see this, you have to take a conditional walk through the table. If the maximum number of lags is $M = 5$, then following the sequential testing of Lütkepohl (2007) we start in column $H_0 : p = 4$. While the null $H_0 : p = 4$ is accepted against $H_1 : p = 5$,

[4]The estimation and qualification of the model uses functions from the MATLAB Econometric Toolbox for time series analysis, including `vgxset` for specification of the model, `vgxvarx` for estimation, `vgxqual` for qualification and `lratiotest` for likelihood ratio hypothesis testing.

[5]I use MATLAB function *lratiotest* from the Econometrics Toolbox to compute the test statistics, which returns a binary for the rejection of the null as well as p-values for a specified significance level α.

Table 4.2: Likelihood Ratio Test Results for ARX(p) specification at significance level $\alpha = 0.01\%$

	H_0: $p=1$	H_0: $p=2$	H_0: $p=3$	H_0: $p=4$
H_1: $p=2$	1 (0.0000)	NaN	NaN	NaN
H_1: $p=3$	1 (0.0000)	1 (0.0027)	NaN	NaN
H_1: $p=4$	1 (0.0000)	1 (0.0003)	1 (0.0068)	NaN
H_1: $p=5$	1 (0.0000)	1 (0.0003)	1 (0.0064)	0 (0.0949)

the subsequent test of the null $H_0 : p = 3$ is rejected against $H_1 : p = 4$. I will hence use $p = 4$ as the order of the process.[6] Table 4.3 presents estimates and standard errors for the model.

Table 4.3: ARX(p=3) Model Estimates of (4.2) - Parameters and Standard Errors*

Parameter	Value	Std. Error	t-Statistic
a(1)	-1.13536	0.59071	-1.92203
b(1)	0.494183	0.151189	3.26864
AR(1)(1,1)	0.789708	0.196724	4.0143
AR(2)(1,1)	-0.191227	0.229252	-0.834136
AR(3)(1,1)	-0.135904	0.233972	-0.580855
AR(4)(1,1)	-0.168801	0.159093	-1.06102
Q(1,1)	0.861321		

* Model: 1-D VARMAX(4,0,1) with Additive Constant, Conditional mean is AR-stable, Standard errors without DoF adjustment (maximum likelihood); Series: Index 'Wohnen' ('IX 75-08 BRD IX') for 1975-2008 period, Source: BulwienGesa AG

Simulation The simulation generates 100 house price appreciation paths from model equation (4.2) for a 60-year period. Each trial uses another stochastic realization of the 1-year interest rate as exogenous input. The average return rate of the simulated values is 1.99%, with minimum -7.53% and maximum 12.09%. The average correlation coefficient of all trial runs for the simulated data is 0.65.

[6] There are additional heuristics to choose an optimal number of lags with different strength and weaknesses. The final prediction error (FPE) ranks the order selection by the forecast MSE of the process. Akaike's Information Criterion (AIC) behaves in a very similar way. The Hannan-Quinn criterion is a consistent order selection criterion for large samples. For a comprehensive discussion of the properties of different order ranking selection criteria, see (Lütkepohl (2007)). I also computed the Akaike's Information Criterion (AIC) for the model (4.2) but the resulting ranking was inconclusive because the criterion monotonically decreases in p and hence suggests an infinite p. The missing coherence of the two criteria for different processes has been observed in simulation studies (see. p.155, Lütkepohl (2007)) and may have been reinforced by the small sample size.

Idiosyncratic House Price Risk The simulated market appreciation rates make it possible in a last step to generate consistent stochastic realizations for the house price path of each contract. I use the geometric Brownian motion process of the analytical model described in subsection 3.2.4 to simulate house price paths that start from the initial appraised value. The simulating process takes the simulated market appreciation rates as parameter input, making the idiosyncratic house price risk consistent with the aggregate risk.[7]

This step is an important improvement over the simple projection of individual future property values with market appreciation rates. The Brownian motion process represents both risk types - aggregate and idiosyncratic - in one single step. It allows the individual house price to have its own dynamic, but links it to the realization of the market return.

4.3 Results

Figures 4.4-4.10 plot the mean as well as upper and lower 5% confidence intervals bounds for the 100 simulated paths. I present the values for a number of variables, including the nominal net receivables of the fund in Figure 4.4.

Before we turn to the interpretation of results, it is important to clarify what interpretations the model allows for. The simulation model in its present form is not a simulation study of a realistic portfolio of reverse mortgages and can not be used for e.g. a value-at-risk computation for a given portfolio. While I sample data from the HECM dataset, this is only meant to give some empirical basis for loan characteristics in the simulation. The code could easily be extended to analyze the value-at-risk, but the present limitation in the modeling of empirical loan termination probabilities prevents an interpretation along these lines.

However, what the simulation model can account for is the dynamic stability of the analytical model. The variance of the simulated paths around its mean can be interpreted as a measure for the stability of the analytical risk management framework when faced with real-life scenarios. It quantifies the risk of an unexpected loss to the insurer for bad scenarios and I can set the unexpected losses in relation to the revenue potential of the lender. In this sense the results from the simulation are a dynamic extension of the comparative statics results in Figure 3.5.

The mean nominal fund value at the end of the simulation period is negative. Only 9 runs end with a positive fund value and the average loss is about 7.3 million for the fund or about 14,000 for an average loan. The bandwidth of the net fund values is large with a range from -97,900 to +4,400 for the average loan. The negative mean value of the fund at the end of the simulation period suggests that the gap between the analytical and empirical loan termination probabilities for HECM-insured loans is very important to the stability of HUD's insurance fund.

The revenue potential measured by cumulated interest is very sensitive to

[7] The simulated process is a calibrated model of the form $dX_t = \mu(t)X_t dt + D(t, X_t)V_t dW_t$ and implemented in the MATLAB Econometrics Toolbox. It allows an approximate numerical solution (`simByEuler`) and approximate analytic solution (`simBySolution`), each of them invoked automatically based on the calibration of the model.

the stochastic events in the model. End-of-simulation values range from about 29,300 to 312,000 per loan, with the average being about 94,000. Since I did not explicitly model refinancing costs I can not compute the profit potential of a lender. However, it is clear from the comparison of the variation of the end-of-simulation fund value around its mean (range 102,000) and the average cumulated interest per loan (94,000) that the amount of insurance risk is significant when compared to the revenue potential of the loan lending. Any private lender that tries to insure her own reverse mortgages would take an enormous amount of risk on her balance sheet.

Figure 4.2: 2D-Kernel Joint Estimate of Age and Appraisal Value of Collateral from HECM Dataset ($n = 258,051$)

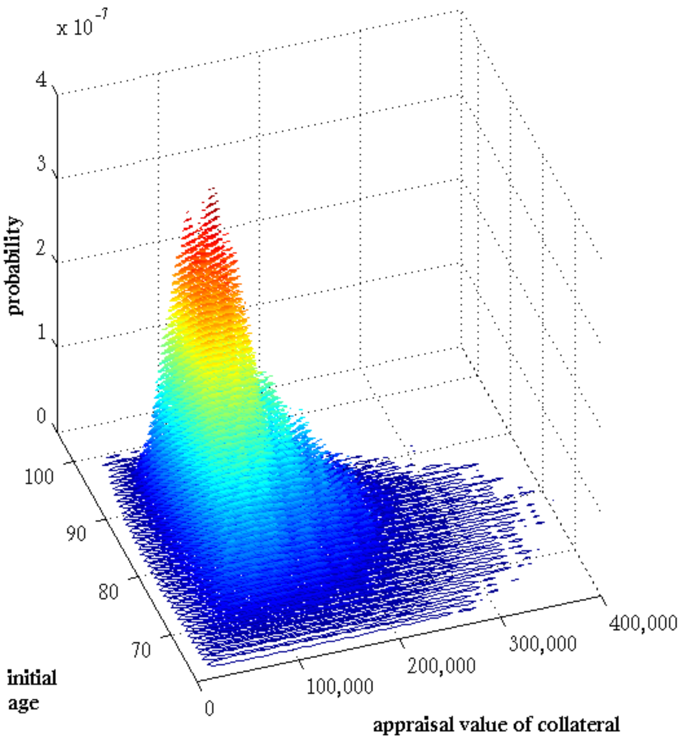


Figure 4.3: 1D-Kernel Estimates of Age and Appraisal Value of Collateral from HECM Dataset ($n = 258,051$)

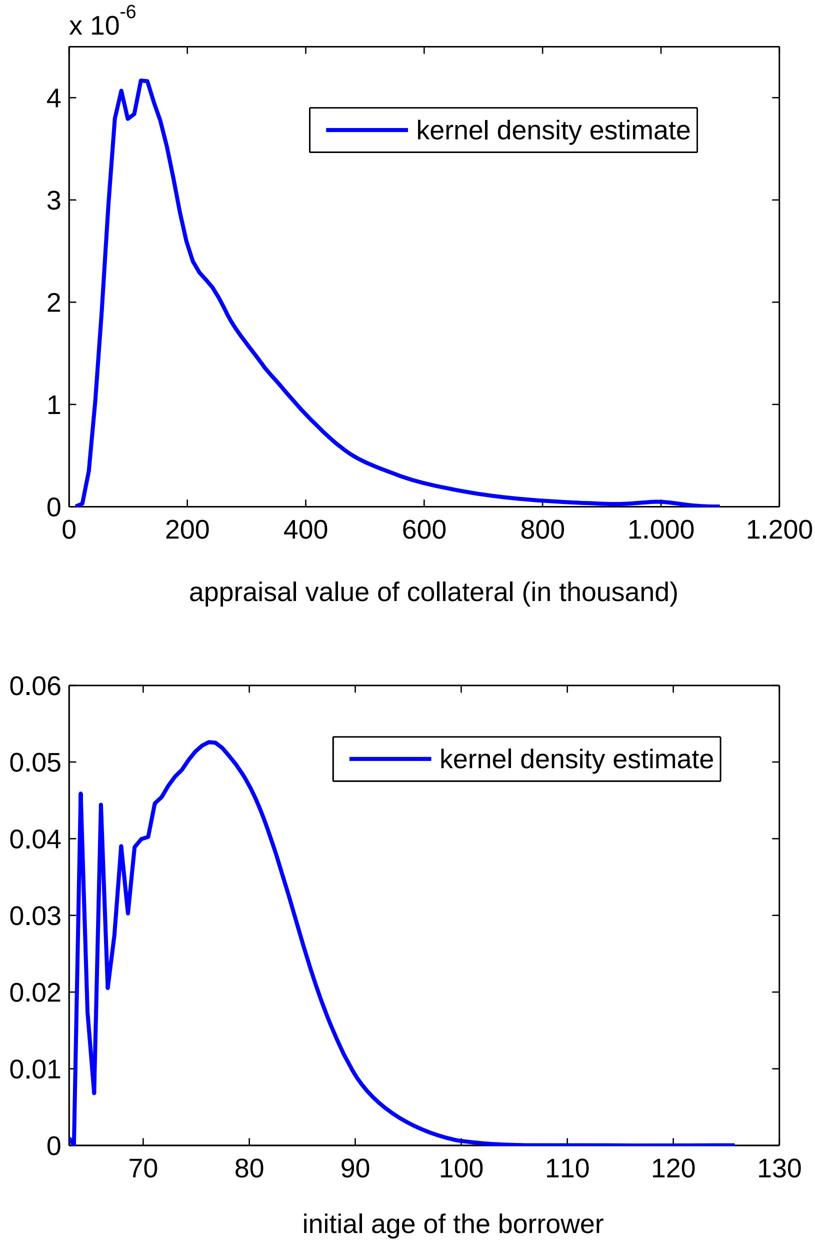


Figure 4.4: Net Receivables of the Insurance Fund: Mean and Interval Bounds

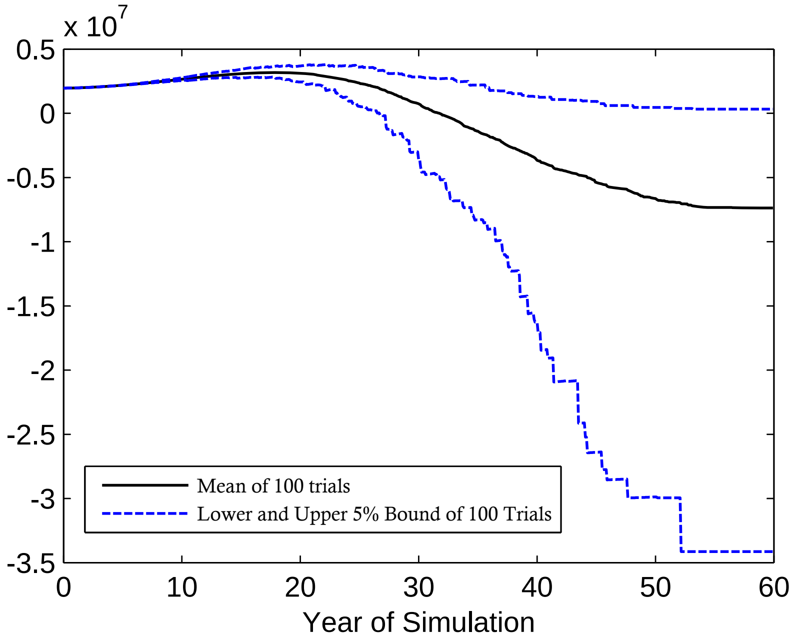


Figure 4.5: Cumulated Claims: Mean and Interval Bounds

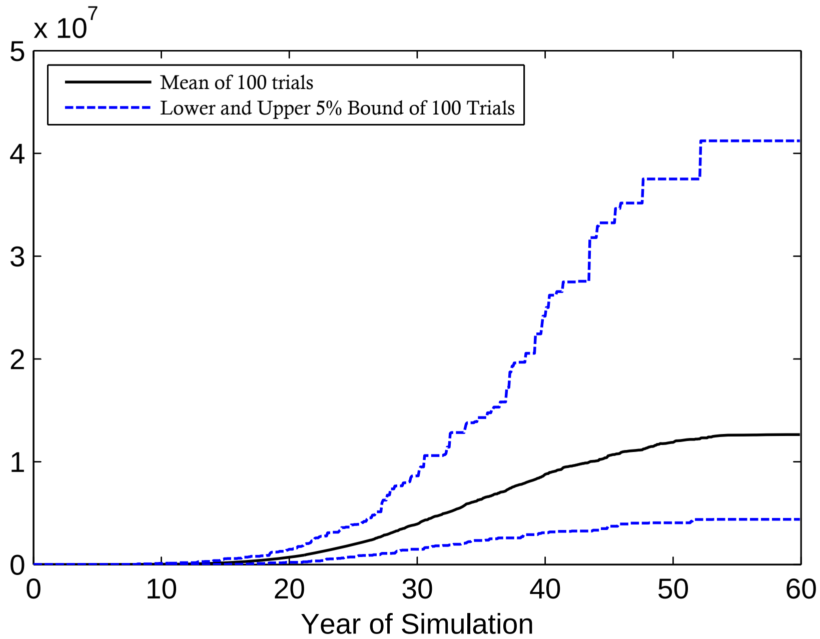


Figure 4.6: Outstanding Balance: Mean and Interval Bounds

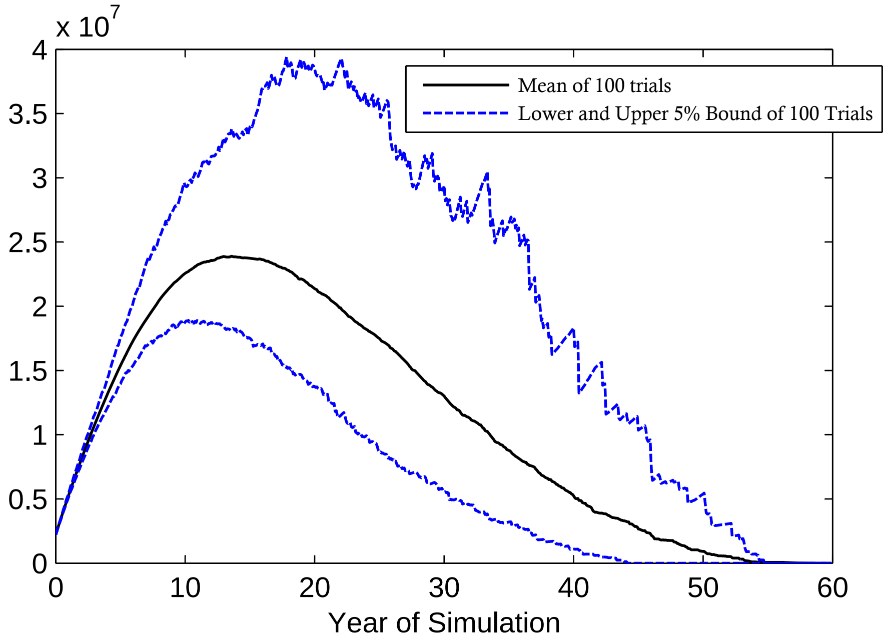


Figure 4.7: Cash Advances: Mean and Interval Bounds

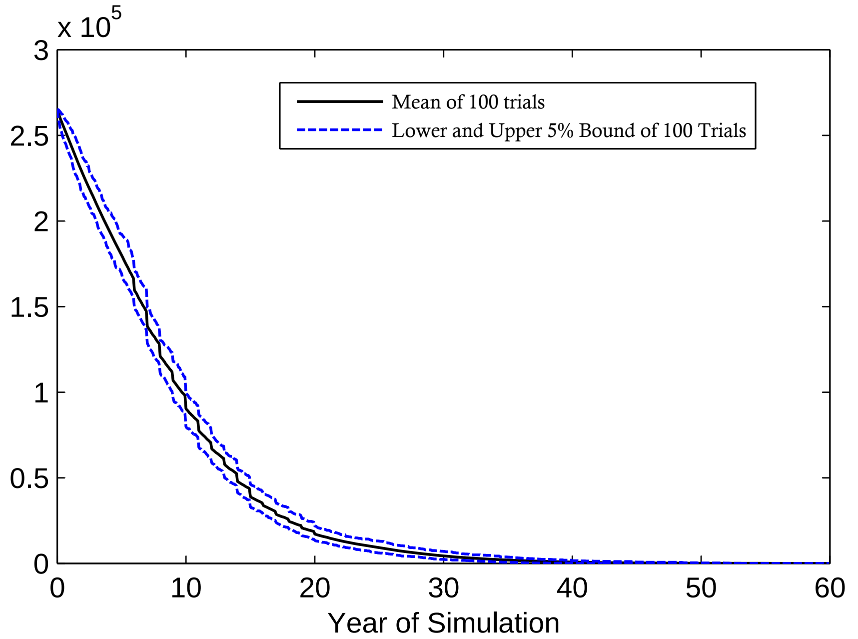


Figure 4.8: Cumulated Premiums: Mean and Interval Bounds

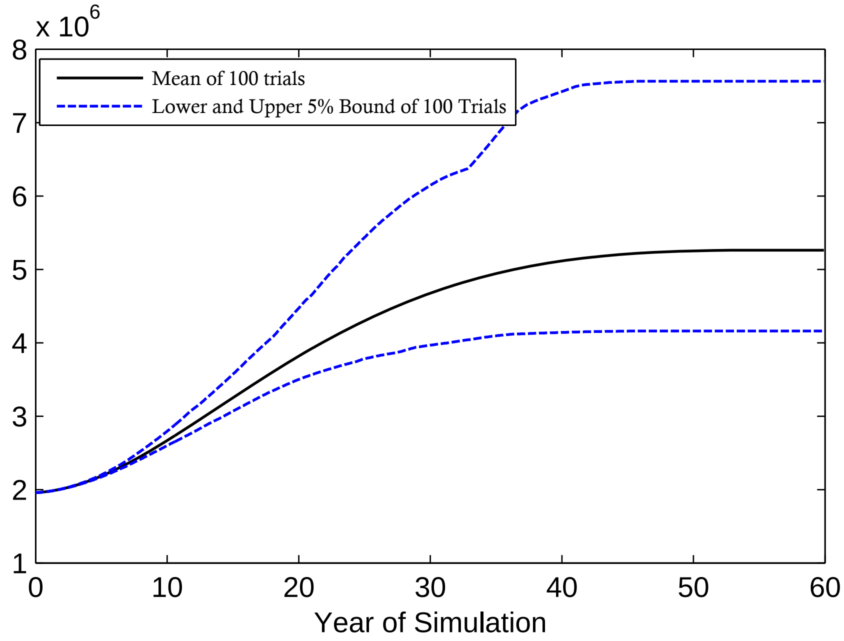


Figure 4.9: Cumulated Interest: Mean and Interval Bounds

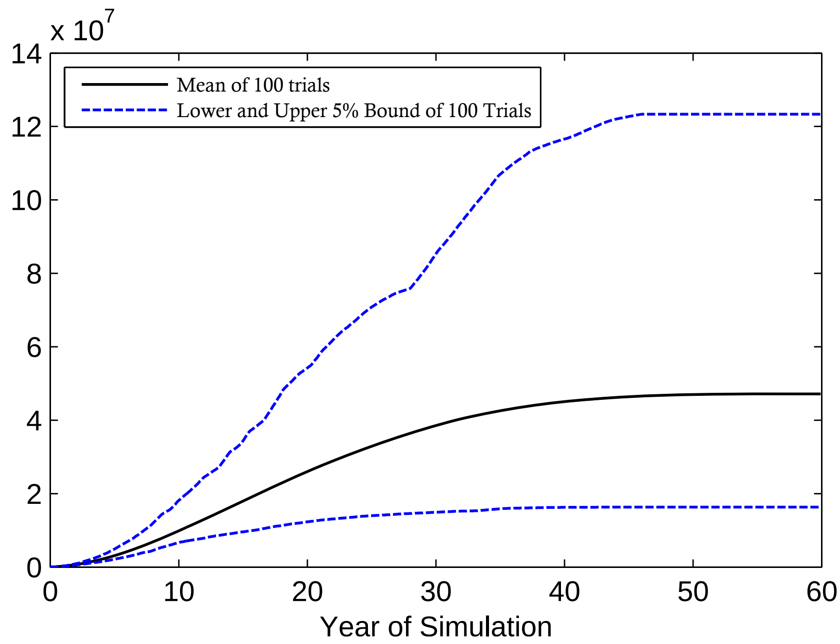
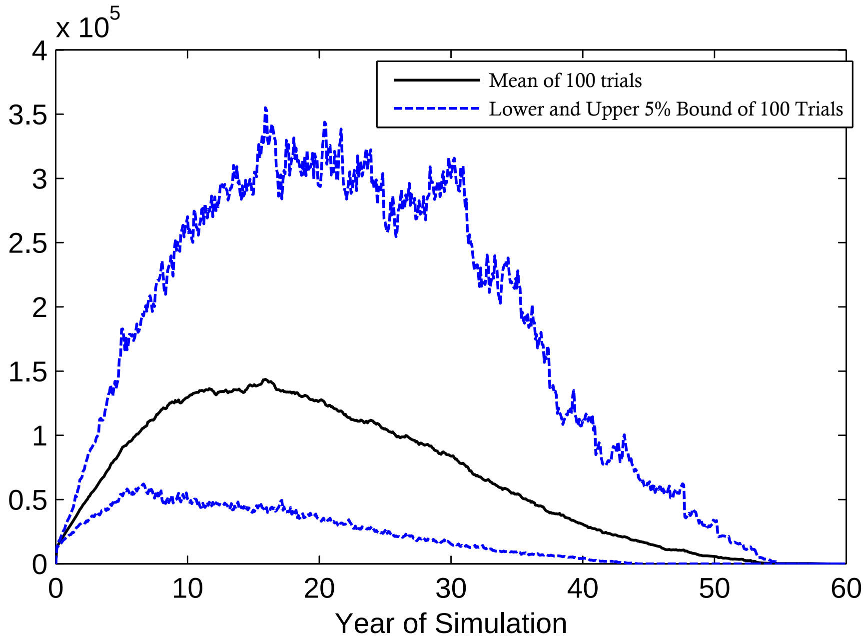


Figure 4.10: Interest: Mean and Interval Bounds



Chapter 5

Conclusion

Although the HECM Program has been very successful in recent years, its key features are hard to replicate in other international markets. Actuarially-fair maximum cash advances for Germany derived from the identical, calibrated HECM risk management framework are substantially lower than in the United States, limiting the attractiveness of home equity conversion products for potential borrowers.

The lack of a centralized, federal insurer for loans is a major obstacle for the competitive pricing of contracts. The alternative solution of a private lender that takes loans on her balance sheet and insures them herself is too risky to serve as a general solution for the problem, even when the revenue potential is taken into account.

The MATLAB classes developed for this paper provide a toolbox for the pricing and stochastic simulation of reverse mortgages. Potential future extensions of the functionality include the modeling of empirical loan termination probabilities based on hazard rates for the stochastic simulation of existing portfolios and Value-at-Risk modeling. A calibration for the U.S. market with HUD data could also help better understand HUD's risk position.

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Selbständigkeitserklärung

Hiermit versichere ich, Patrick Böert, geboren am 22.9.1983, dass die vorliegende Arbeit selbständig und ohne Benutzung anderer als der angegebenen Quellen und Hilfsmittel angefertigt wurde. Alle Stellen, die wörtlich oder sinngemäß anderen Quellen entnommen sind, wurden als solche gekennzeichnet.

Berlin, April 30, 2010 ________________________________

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