Trigonometrie

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1 Werte und Eigenschaften von Sinus und Kosinus

1.1 Werte

$$
\begin{aligned}
&\sin 0 = 0 && \cos 0 = 1 && \tan 0 = 0 \\
&\sin \frac{\pi}{6} = \sin 30^\circ = \frac{1}{2} && \cos \frac{\pi}{6} = \cos 30^\circ = \frac{\sqrt{3}}{2} && \tan \frac{\pi}{6} = \tan 30^\circ = \frac{\sqrt{3}}{3} \\
&\sin \frac{\pi}{4} = \sin 45^\circ = \frac{\sqrt{2}}{2} && \cos \frac{\pi}{4} = \cos 45^\circ = \frac{\sqrt{2}}{2} && \tan \frac{\pi}{4} = \tan 45^\circ = 1 \\
&\sin \frac{\pi}{3} = \sin 60^\circ = \frac{\sqrt{3}}{2} && \cos \frac{\pi}{3} = \cos 60^\circ = \frac{1}{2} && \tan \frac{\pi}{3} = \tan 60^\circ = \sqrt{3} \\
&\sin \frac{\pi}{2} = \sin 90^\circ = 1 && \cos \frac{\pi}{2} = \cos 90^\circ = 0 && \tan \frac{\pi}{2} = \tan 90^\circ = \emptyset
\end{aligned}
$$

1.2 Eigenschaften

$$
\begin{aligned}
&\sin(\pi - \alpha) = \sin \alpha && \cos(\pi - \alpha) = -\cos \alpha \\
&\sin \alpha = \sin(\pi - \alpha) && \cos \alpha = \cos(2\pi - \alpha) \\
&\sin -\alpha = -\sin \alpha && \cos -\alpha = \cos \alpha
\end{aligned}
$$

$$
\begin{aligned}
\cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\
&= 2\cos^2 \alpha - 1 \\
&= \frac{1}{2}(1 + \cos 2\alpha) \\
&= 1 - 2\sin^2 \alpha
\end{aligned}
$$

$$
\begin{aligned}
\sin 2\alpha &= 2 \sin \alpha \cos \alpha \\
\sin^2 \alpha &= \frac{1}{2}(1 - \cos 2\alpha)
\end{aligned}
$$

2 Sinus- und Kosinussatz

2.1 Sinussatz

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

2.2 Kosinussatz

$$\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos\alpha \\
b^2 &= a^2 + c^2 - 2ac\cos\beta \\
c^2 &= a^2 + b^2 - 2ab\cos\gamma
\end{aligned}$$

3 Additionstheoreme

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \sin\beta\cos\alpha \\
\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \sin\beta\cos\alpha \\
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \\
\tan(\alpha+\beta) &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \\
\tan(\alpha-\beta) &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}
\end{aligned}$$