

SICP: Ex. 2.13, p. 96

Multiplication (Interval Arithmetics, Δ_x as a Ratio of x):

$$c = ab \quad (1)$$

$$c \pm c\Delta_c = (a \pm a\Delta_a)(b \pm b\Delta_b) \quad | c = ab \quad (2)$$

$$ab \pm ab\Delta_c = (a \pm a\Delta_a)(b \pm b\Delta_b) \quad | - ab \quad (3)$$

$$\pm ab\Delta_c = (a \pm a\Delta_a)(b \pm b\Delta_b) - ab \quad | \div \pm ab \quad (4)$$

$$\Delta_c = \frac{(a \pm a\Delta_a)(b \pm b\Delta_b) - ab}{\pm ab} \quad | \text{ignore negative case} \quad (5)$$

$$\Delta_c = \frac{(a + a\Delta_a)(b + b\Delta_b) - ab}{ab} \quad | \text{expand} \quad (6)$$

$$\Delta_c = \frac{ab + ab\Delta_b + ab\Delta_a + ab\Delta_a\Delta_b - ab}{ab} \quad | \text{simplify} \quad (7)$$

$$\Delta_c = \frac{ab\Delta_b + ab\Delta_a + ab\Delta_a\Delta_b}{ab} \quad | \text{factor out } ab \quad (8)$$

$$\Delta_c = \frac{ab(\Delta_b + \Delta_a + \Delta_a\Delta_b)}{ab} \quad | \div ab \quad (9)$$

$$\Delta_c = \Delta_b + \Delta_a + \Delta_a\Delta_b \quad (10)$$

Example:

$$a = 2, \Delta_a = 0.10 \quad (11)$$

$$b = 4, \Delta_b = 0.05 \quad (12)$$

$$c = ab = 2 \times 4 = 8 \quad (13)$$

$$\Delta_c = \Delta_a + \Delta_b + \Delta_a\Delta_b = 0.10 + 0.05 + 0.10 \times 0.05 = 0.155 \quad (14)$$

$$c_l = c - c\Delta_c = 8 - 8 \times 0.155 = 6.76 \quad (15)$$

$$c_u = c + c\Delta_c = 8 + 8 \times 0.155 = 9.24 \quad (16)$$