# SICP: Ex. 2.9, p. 95

## Addition

Given:

$$a = 1, w_a = 0.1, a = [0.9; 1.1] \tag{1}$$
$$b = 2, w_b = 0.2, b = [1.8; 2.2] \tag{2}$$
$$c_l = 0.9 + 1.8 = 2.7 \tag{3}$$
$$c_u = 1.1 + 2.2 = 3.3 \tag{4}$$
$$w_c = \frac{c_u - c_l}{2} = \frac{3.3 - 2.7}{2} = \frac{0.6}{2} = 0.3 \tag{5}$$
$$w_c = w_a + w_b = 0.1 + 0.2 = 0.3 \tag{6}$$

Same width, switched values:

$$p = 2, w_p = 0.1, p = [1.9; 2.1] \tag{7}$$
$$q = 1, w_q = 0.2, q = [0.8; 1.2] \tag{8}$$
$$r_l = 1.9 + 0.8 = 2.7 \tag{9}$$
$$r_u = 2.1 + 1.2 = 3.3 \tag{10}$$
$$w_r = \frac{r_u - r_l}{2} = \frac{3.3 - 2.7}{2} = \frac{0.6}{2} = 0.3 \tag{11}$$
$$w_r = w_p + w_q = 0.1 + 0.2 = 0.3 \tag{12}$$

For addition, only the width matters, not the actual numbers.

## Multiplication

For multiplication, all four combinations have to be tried:

$$a = 1, w_a = 0.1, a = [0.9; 1.1] \tag{13}$$
$$b = 2, w_b = 0.2, b = [1.8; 2.2] \tag{14}$$
$$c_1 = a_l * b_l = 0.9 \times 1.8 = 1.62 \tag{15}$$
$$c_2 = a_l * b_u = 0.9 \times 2.2 = 1.98 \tag{16}$$
$$c_3 = a_u * b_l = 1.1 \times 1.8 = 1.98 \tag{17}$$
$$c_4 = a_u * b_u = 1.1 \times 2.2 = 2.42 \tag{18}$$
$$c_l = \min(c_1, c_2, c_3, c_4) = 1.62 \tag{19}$$
$$c_u = \max(c_1, c_2, c_3, c_4) = 2.42 \tag{20}$$
$$w_c = \frac{c_u - c_l}{2} = \frac{2.42 - 1.62}{2} = \frac{0.8}{2} = 0.4 \tag{21}$$

Same width, switched values:

$$p = 2, w_p = 0.1, p = [1.9; 2.1] \tag{22}$$
$$q = 1, w_q = 0.2, q = [0.8; 1.2] \tag{23}$$
$$r_1 = p_l * q_l = 1.9 \times 0.8 = 1.52 \tag{24}$$
$$r_2 = p_l * q_u = 1.9 \times 1.2 = 2.28 \tag{25}$$
$$r_3 = p_u * q_l = 2.1 \times 0.8 = 1.68 \tag{26}$$
$$r_4 = p_u * q_u = 2.1 \times 1.2 = 2.52 \tag{27}$$
$$r_l = \min(r_1, r_2, r_3, r_4) = 1.52 \tag{28}$$
$$r_u = \max(r_1, r_2, r_3, r_4) = 2.52 \tag{29}$$
$$w_r = \frac{r_u - r_l}{2} = \frac{2.52 - 1.52}{2} = \frac{1.0}{2} = 0.5 \tag{30}$$

The product's width cannot be determined by the width of the two factors alone, because it also depends on the factors itself!