

SICP: Ex. 2.9, p. 95

Addition

Given:

$$a = 1, w_a = 0.1, a = [0.9; 1.1] \quad (1)$$

$$b = 2, w_b = 0.2, b = [1.8; 2.2] \quad (2)$$

$$c_l = 0.9 + 1.8 = 2.7 \quad (3)$$

$$c_u = 1.1 + 2.2 = 3.3 \quad (4)$$

$$w_c = \frac{c_u - c_l}{2} = \frac{3.3 - 2.7}{2} = \frac{0.6}{2} = 0.3 \quad (5)$$

$$w_c = w_a + w_b = 0.1 + 0.2 = 0.3 \quad (6)$$

Same width, switched values:

$$p = 2, w_p = 0.1, p = [1.9; 2.1] \quad (7)$$

$$q = 1, w_q = 0.2, q = [0.8; 1.2] \quad (8)$$

$$r_l = 1.9 + 0.8 = 2.7 \quad (9)$$

$$r_u = 2.1 + 1.2 = 3.3 \quad (10)$$

$$w_r = \frac{r_u - r_l}{2} = \frac{3.3 - 2.7}{2} = \frac{0.6}{2} = 0.3 \quad (11)$$

$$w_r = w_p + w_q = 0.1 + 0.2 = 0.3 \quad (12)$$

For addition, only the width matters, not the actual numbers.

Multiplication

For multiplication, all four combinations have to be tried:

$$a = 1, w_a = 0.1, a = [0.9; 1.1] \quad (13)$$

$$b = 2, w_b = 0.2, b = [1.8; 2.2] \quad (14)$$

$$c_1 = a_l * b_l = 0.9 \times 1.8 = 1.62 \quad (15)$$

$$c_2 = a_l * b_u = 0.9 \times 2.2 = 1.98 \quad (16)$$

$$c_3 = a_u * b_l = 1.1 \times 1.8 = 1.98 \quad (17)$$

$$c_4 = a_u * b_u = 1.1 \times 2.2 = 2.42 \quad (18)$$

$$c_l = \min(c_1, c_2, c_3, c_4) = 1.62 \quad (19)$$

$$c_u = \max(c_1, c_2, c_3, c_4) = 2.42 \quad (20)$$

$$w_c = \frac{c_u - c_l}{2} = \frac{2.42 - 1.62}{2} = \frac{0.8}{2} = 0.4 \quad (21)$$

Same width, switched values:

$$p = 2, w_p = 0.1, p = [1.9; 2.1] \quad (22)$$

$$q = 1, w_q = 0.2, q = [0.8; 1.2] \quad (23)$$

$$r_1 = p_l * q_l = 1.9 \times 0.8 = 1.52 \quad (24)$$

$$r_2 = p_l * q_u = 1.9 \times 1.2 = 2.28 \quad (25)$$

$$r_3 = p_u * q_l = 2.1 \times 0.8 = 1.68 \quad (26)$$

$$r_4 = p_u * q_u = 2.1 \times 1.2 = 2.52 \quad (27)$$

$$r_l = \min(r_1, r_2, r_3, r_4) = 1.52 \quad (28)$$

$$r_u = \max(r_1, r_2, r_3, r_4) = 2.52 \quad (29)$$

$$w_r = \frac{r_u - r_l}{2} = \frac{2.52 - 1.52}{2} = \frac{1.0}{2} = 0.5 \quad (30)$$

The product's width cannot be determined by the factors width, because it also depends on the factors itself!