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Benford's Law as an Indicator of Fraud in Economics

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Abstract. *Contrary to intuition, first digits of randomly selected data are not uniformly distributed but follow a logarithmically declining pattern, known as Benford's law. This law is increasingly used as a 'doping check' for detecting fraudulent data in business and administration. Benford's law also applies to regression coefficients and standard errors in empirical economics. This article reviews Benford's law and examines its potential as an indicator of fraud in economic research. Evidence from a sample of recently published articles shows that a surprisingly large proportion of first digits, but not of second digits, contradicts Benford's law.*

JEL classification: *C*8, *C*52, *C*12.

Keywords: Benford's law; first digits; fraud control; regression coefficients; standard errors.

1. INTRODUCTION

The science system is not immune to fraud and dishonesty. It cannot be ruled out that research results are made up or tuned to obtain funding and to dodge publication pressure (Reulecke, 2006). Graber *et al.* (2008) provide evidence on the growing importance of publications at the time of the first appointment of economics professors in Austria, Germany and Switzerland and predict that publication pressure will increase further within the next years. In economics, the traditional control mechanisms are easily swamped when authors submit papers based on large datasets and complex econometric techniques. However, independent review of empirical research results is a cornerstone of science.

Benford's law, first discovered by Newcomb (1881), was rediscovered and empirically substantiated by Benford (1938). Nevertheless, for a long time, Benford's law was regarded as an anomaly, a curiosity and even a paradox (Székely, 1990). In recent years it has received increasing attention. A bibliography by Hürlimann (2006) lists 350 publications on Benford's law between 1881 and 2006, of which 166 appeared between 2000 and 2006. Today, Benford's law is successfully applied in many areas, from optimizing computer algorithms (Gent and Walsh, 2001) to testing eBay auctions (Giles, 2007). In business and administration it gains increasing importance as a form of 'doping control' for datasets. Auditors and tax inspectors use Benford's law to get hold of fraud and other forms of data manipulation in accounting and taxation (Nigrini, 1996). This article explores the potential of Benford's law as an indicator of fraud in empirical economics. Section 2 reviews some properties and tests of the Benford distribution. Section 3 provides evidence on Benford's law in econometric research and Section 4 concludes.

2. PROPERTIES OF BENFORD'S LAW

2.1. Distribution of first and second digits

At those times when calculations were done with the help of logarithmic tables, the American mathematician and astronomer Simon Newcomb (1881) came across a strange phenomenon: 'That the ten digits do not occur with equal frequency must be evident to any one making use of logarithmic tables, and noticing how much faster the first pages wear out than the last ones'. Apparently, logarithms of numbers beginning with 1 or 2 were looked up more often than those starting with 8 or 9. Newcomb (1881) postulated: 'The law of probability of the occurrence of numbers is such that all mantissae of their logarithms are equally probable'. Thus, denoting the mantissa of any number by X, Newcomb postulated that its logarithms are uniformly distributed in the interval $[0, 1)$. Ignoring leading zeros, the implied probability of the first digit d_1 $(= 1, 2, \ldots, 9)$ is

$$P(D_1 = d_1) = \log(1 + 1/d_1) \tag{1}$$

where log denotes base 10 logarithms. Thus, the probability of 1 (9) as a first digit is not 11.1% as under the uniform distribution but 30.1 (4.6)% (Figure 1).

The conditional probability of the second digit d_2 $(= 0, 1, 2, \ldots, 9)$ is $P(D_2 = d_2/D_1 = d_1) = \log[1 + 1/(10d_1 + d_2)]$. The unconditional probability of second digit d_2 is obtained by summing over all first digits. Table 1 shows the distribution of first and second digits. In the final columns their expectation $[E(d)]$ and variance $[\mathrm{Var}(d)]$ are reported. The probabilities of first and second digits decline monotonically. Newcomb's discovery, soon forgotten, was rediscovered by the American physicist Frank Benford (1938). He substantiated the law empirically with such diverse datasets as mortality tables, baseball statistics, newspaper articles and atomic weights of chemical compounds.

If there exists a universal law for first digits, Pinkham (1961) noted, then it must be *scale invariant*. That means, the distribution of first digits should not change if distances are expressed in miles instead of kilometres and values in dollars rather than euros. Hill (1995a) showed that scale invariance also implies *base invariance*. Thus, the distribution remains unchanged if numbers are expressed in a base other than 10. Benford's law is not invariant to

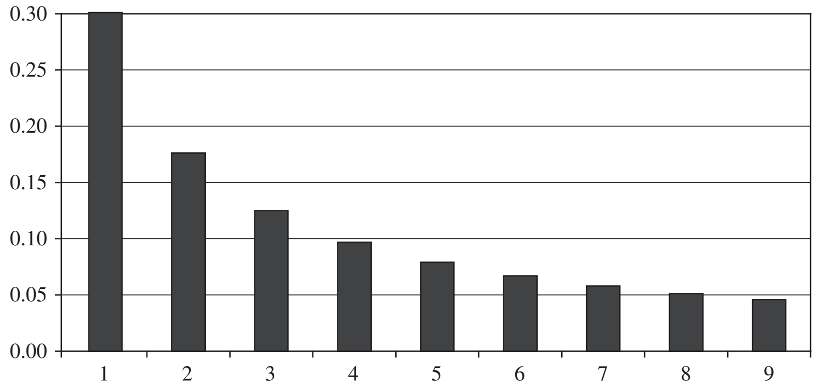

Figure 1 Benford distribution of first digits

Table 1 Benford distribution of first and second digits

d	0	1	2	3	4	5	6	7	8	9	$E(d)$	Var(d)
$P(d_1)$	n.a.	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046	3.440	6.057
$P(d_2)$	0.120	0.114	0.109	0.104	0.100	0.097	0.093	0.090	0.088	0.085	4.187	8.254

rounding, however. Newspapers and scientific journals often publish just two significant digits, sometimes only one. If the first digits obey Benford's law originally, this is no longer true for rounded figures. Rounding can even destroy the property of monotonically declining probabilities of first digits. For example, if figures are rounded to one significant digit, the probability of first digit 2 (0.222) is greater than the probability of first digit 1 (0.198).

2.2. Convergence to the Benford distribution

Scale and base invariances are mathematical properties of the Benford distribution, but, as Hill (1995b, p. 358) remarked, '... they hardly help explain the appearance of Benford's law empirically'. Neither Newcomb nor Benford had a really good explanation for the phenomenon discovered by them. Benford (1938, p. 552) collected 20 datasets with 20,229 observations in total according to the following criteria: 'The method of study consists of selecting any tabulation of data that is not too restricted in numerical range, or conditioned in some way too sharply'. As Benford pointed out, numbers following the law, he called them 'irregular numbers', should not be purely random (as lottery drawings) nor should they be too rigid (as dates or phone numbers) and they should be sufficiently volatile (unlike body heights). Recently, Durtschi *et al.* (2004) and Mochty (2002) sharpened those guidelines.

Empirically, Benford's law often appears to hold best when first digits are taken as a mixture from diverse datasets, such as from newspaper articles on share prices, exchange rates, election results, length of rivers, temperatures and the like. Benford's law is also frequently observed in datasets of natural and socioeconomic phenomena driven by multiplicative processes with different distributions. In fact, Hill (1995b, p. 354) has shown that random variables need not be *identically* distributed. He proved a 'central limit theorem for first digits': 'If distributions are selected at random (in any "unbiased" way) and random samples are taken from each of these distributions, then the significant-digit frequencies of the combined sample will converge to the logarithmic (Benford) distribution, even though the individual distributions selected may not closely follow the law'. Thus, Benford's law may be regarded as a 'distribution of distributions' of first digits, which turns up if random samples are taken independently from diverse datasets.

2.3. Testing Benford's law

Empirical distributions of first digits do not obey Benford's law exactly. To test whether deviations are statistically significant, the χ^2 goodness-of-fit test is often applied. It checks whether the sum of the squared deviations between observed relative frequencies (h_d) and probabilities under the null (p_d) are significantly different from zero. Denoting the sample size by N, the test statistic

$$Q = N \sum_{d=1(0)}^{9} \frac{(h_d - p_d)^2}{p_d} \tag{2}$$

has an approximate χ^2 distribution with 8 (9) degrees of freedom when testing the nine first digits (ten second digits). As an alternative test we consider the mean test. It checks whether the arithmetic mean of the observed first digits, defined as $\bar{d} = \sum_{d=1(0)}^{9} d \times h_d$, deviates from its expected value under the null hypothesis of Benford's law. The test statistic

$$M = N \frac{(\bar{d} - E(d))^2}{\mathrm{Var}(d)} \tag{3}$$

has an approximate χ^2 distribution with one degree of freedom, regardless of testing first or second digits, where $E(d)$ and $\mathrm{Var}(d)$ are given in Table 1. The test is responsive to violations of Benford's law that do not preserve the mean of the first digits. Since the mean test 'uses up' only one degree of freedom, it often has higher power than the Q-test.

Assume that a researcher deliberately modifies first digits such that the ratio p_d of first digit d changes to $h_d = p_d + \varphi_d$ ($0 \leq h_d \leq 1$, $\sum \varphi_d = 0$). From (2) and (3) we see that the null is rejected if the test statistics

$$Q = N \sum \frac{\varphi_d^2}{p_d}, \quad M = N \frac{(\sum d\, \varphi_d)^2}{\mathrm{Var}(d)} \tag{4}$$

exceed their critical values at the α-level (e.g. for $\alpha = 5\%$ these are 15.507 and 3.84, respectively). People appear to regard the uniform distribution of first digits as natural and most likely. Perhaps unconsciously, they may tend to leave a tampered dataset more evenly distributed than implied by Benford's law. Thus, the observed distribution may be approximated by a mixture $(0 \leq \psi \leq 1)$ of Benford's law and the uniform law, $h_d = (1-\psi)p_d + \psi(1/9)$. Thus, for $\psi = 0.1$, the observed frequency of first digit 1 would drop from 0.301 to 0.282. The manipulated distribution preserves monotonically declining probabilities but is flatter than Benford's law. Monte Carlo simulations (which can be obtained on request) for sample sizes $N = 50, 100, 200$ and different ratios ψ have shown that both tests have approximately the correct size. However, the Q-test was biased. In contrast, the M-test was found to be unbiased, with power exceeding that of the Q-test uniformly and substantially.

How many rejections of Benford's law can be expected in a sample of articles drawn from a population in which Ω $(0 \leq \Omega \leq 1)$ is the ratio of Benford law violations? The probability of rejecting the null hypothesis for some randomly selected article is

$$\theta = \alpha(1-\Omega) + (1-\beta)\Omega = \alpha + (1-\beta-\alpha)\Omega \tag{5}$$

The probability of a type II error (β) is a function of the sample size and the size of the test (N, α) and it depends on the pattern of manipulation. Provided the test is unbiased, the rejection probability increases linearly in Ω. If the population is clean ($\Omega = 0$), the null is rejected (erroneously in this case) with probability α. In the other extreme ($\Omega = 1$), the null is (correctly) rejected with probability $1-\beta$.

3. BENFORD'S LAW AS AN INDICATOR OF FRAUD

3.1. Fraud in science and research

Martinson *et al.* (2005) estimate that between 1% and 2% of scientists commit 'fabrication, falsification, or plagiarism'. In some cases this may be due to lack of knowledge, for example when the presence of a 'selection bias' is ignored. But often it may be the conviction 'I know my theory is right and I am entitled to cook up my data and/or results accordingly'. Scientists are only human and 'the world of science is just as fiercely competitive as the world of business and commerce', Hand (2007, p. 22) observes.

The traditional control mechanisms in peer-reviewed journals, such as anonymous refereeing, have difficulties handling papers that use large datasets and/or apply complex econometric procedures. But independent review of research is important because, 'Replication is the cornerstone of science. Research that cannot be replicated is not science, and cannot be trusted either as part of the profession's accumulated body of knowledge or as a basis for policy' (McCullough and Vinod, 2003, p. 888). However, in contrast to natural sciences, there is no distinct tradition of replication in social sciences (Hammermesh, 2007). In rare cases researchers try to replicate results obtained by others, such as Krämer *et al.* (1985) in a comprehensive meta-study. Only a few journals require from authors the deposition of data and software in an archive to facilitate replication. Even if that is the case, as with the *Journal of Money, Credit, and Banking*, reproduction of results often fails. McCullough *et al.* (2006) analysed more than 150 articles from that journal and were able to replicate the results in less than 10% of the cases.

If research outcomes obtained by others is replicated in rare instances only, indirect methods of fraud control like Benford's law become indispensable. Benford's law is already applied successfully in business and administration for detecting fraud in accounting, tax evasion and data manipulation in general (IDW, 2006). If first digits of balance-sheet data or tax declarations deviate from Benford's law, this evidence is taken by auditors and tax officers as a clue to check the records more carefully. In the United States, Nigrini (1992, 1996) was influential in establishing Benford's law as an indicator of fraud in finance and taxation. He has shown that data in tax declarations follow the Benford law, whereas tax data known to be fraudulent do not. Such evidence induced tax authorities in the United States and Europe to check tax declarations routinely for inconsistencies with the aid of Benford's law.

Researchers in economics increasingly use large sets of survey-based micro data, for example the enterprise surveys by the IFO institute or the socioeconomic household panel by the DIW. Survey data are not always free of errors and falsifications (Diekmann, 2002, p. 11). Interviewers have an incentive to manipulate survey results, by deviating from prescribed procedures or by 'fabricating' responses. Schäfer *et al.* (2005) investigated interview outcomes with Benford's law and were able to identify practically all (known as forged) responses. It is another advantage of Benford tests that forged interviews can be detected in the cross-section, without the need to wait for a second wave of data (Schräpler and Wagner, 2005). Benford's law can also be used to check economic models for plausibility. If the first digits of real data obey Benford's law, such as population data or share prices, then a model should be able to reproduce this pattern.

3.2. Analysis of first digits in econometric studies

Apart from manipulating input data, there are temptations to falsify research output as well. Diekmann (2007) investigated regression results published in the *American Journal of Sociology* and found that regression coefficients broadly agree with Benford's law. Günnel and Tödter (2009) evaluated regressions in economics drawn from *Empirica* (vols. 2003–06) and *Applied Economics Letters* (vol. 2006). Figures 2 and 3 show the distributions of first and second digits of regression coefficients in comparison with the Benford distribution from their dataset of about 30,000 observations and Figures 4 and 5 report these

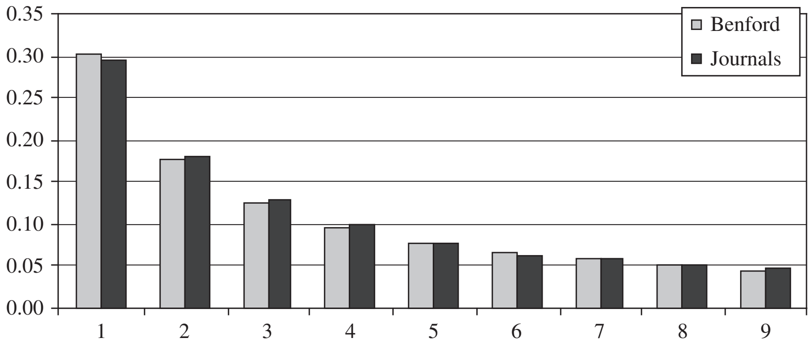

Figure 2 First digits of regression coefficients

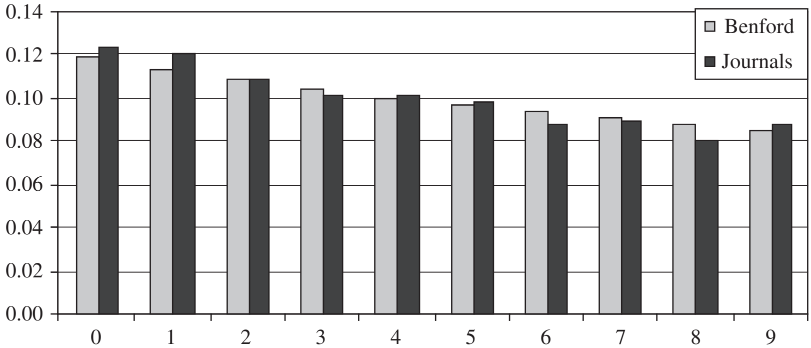

Figure 3 Second digits of regression coefficients

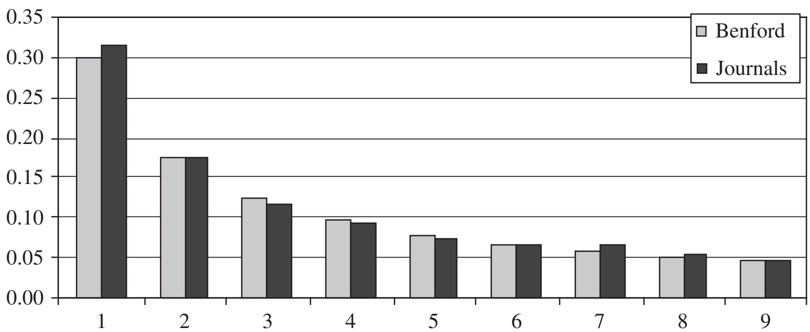

Figure 4 First digits of standard errors

distributions for the standard errors of estimated coefficients. They find that Benford's law holds in econometric research and conclude, 'Thus, our results suggest that Benford's law can serve as a tool to assess the reliability of econometric research outcomes'.

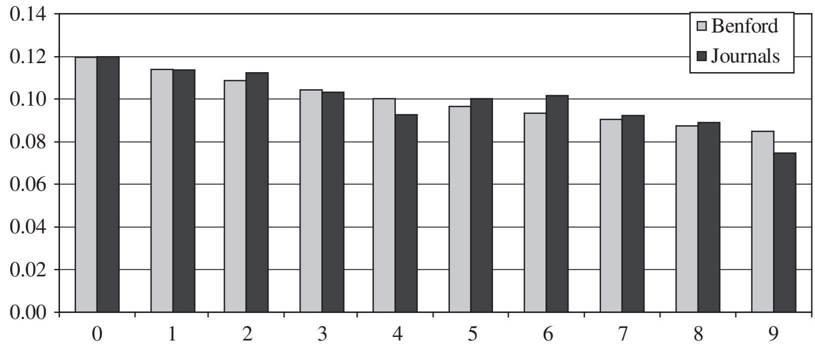

Figure 5 Second digits of standard errors

3.3. Analysing first digits in single articles

This section provides some evidence on single articles from the dataset of Günnel and Tödter (2009). These data contain observations on regression coefficients from 117 published articles. Since not always reported in the papers, only 90 articles include information on standard errors. On average, there are about 70 observations per article, but the standard deviations are large, as shown in Table 2. It was checked whether the statistics for the mean test are correlated with the number of observations in single articles. Regressions (not reported) have shown that the number of observations is insignificant in all four groups. A dummy variable for the journal (*Empirica* and *Applied Economics Letters*, respectively) was insignificant as well. Moreover, the bilateral correlations between the *M*-statistics in the four groups are very small.

We regard an article as doubtful if the mean test rejects the null hypothesis of Benford's law at the 5% level of significance ($M > 3.84$). According to this, admittedly arbitrary, criterion a surprisingly large proportion of articles is doubtful: 26% (first digits of regression coefficients) and 30% (first digits of standard errors), respectively. Concerning second digits of regression coefficients and standard errors, only 11% and 14%, respectively, of the articles investigated are dubious. Hill's (1995b) theorem explains that a considerable proportion of articles with Benford law violations is consistent with its validity in the aggregate.

Manipulating first digits apparently occurs more often because it is more 'effective', having a bigger impact on the results. The evidence on doubtful articles is reflected by the test statistics for the mean test. On average, the *M*-statistics are substantially larger (3.65, 3.88) when testing first digits compared with second digits (1.41, 1.91). In contrast, the differences between first digits (second digits) of regression coefficients and standard errors are relatively small. This is confirmed by tests for differences in mean (Hogg *et al.*, 2005). The difference between the average of the *M*-statistics for regression

Table 2 Benford analysis of single articles

	Regression coefficients		Standard errors	
	First digits	Second digits	First digits	Second digits
Total no. of observations	9,777	8,627	5,928	5,392
Observations per article	84	74	66	60
(Standard deviation)	(136)	(123)	(121)	(116)
No. of articles	117	117	90	90
of which doubtful[a]	30	13	27	13
of which doubtful (%)	26	11	30	14
Mean test (average)	3.65	1.41	3.88	1.91
Mean test (standard deviation)	(5.45)	(2.30)	(6.60)	(2.56)

Note:
[a] No. of articles for which the test statistic (3) exceeds 3.84.

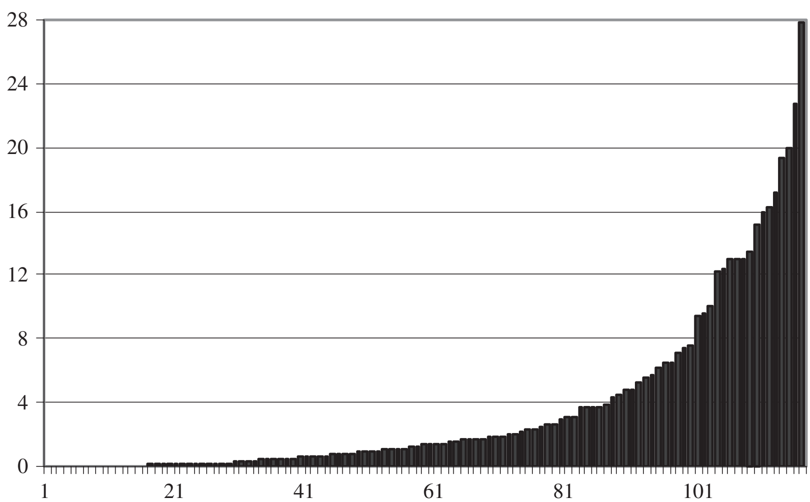

Figure 6 Mean test for first digits of regression coefficients

coefficients and standard errors is statistically insignificant at the 5% level, both for first digits and for second digits. In contrast, the differences between first and second digits are significantly different from zero, for regression coefficients and standard errors as well. Thus, manipulations for regression results pertain mostly to first digits. Figures 6 and 7 show the *M*-statistics for first and second digits of regression coefficients of all 117 articles in increasing order. Similar figures (not shown) are obtained for standard errors.

In a population of articles that obey Benford's law and are free of any type of manipulation ($\Omega = 0$), one expects to observe a ratio of doubtful articles (θ) that is not greater than the size of the test (i.e. $\alpha = 5\%$). But, as Table 2 shows,

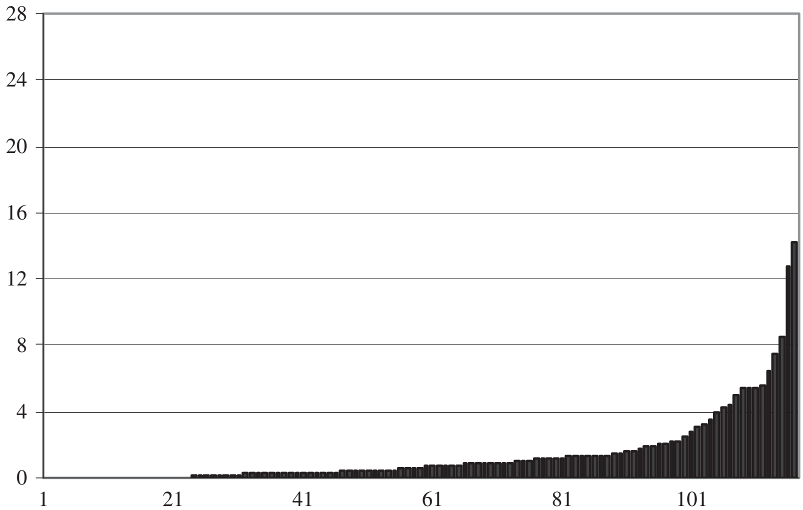

Figure 7 Mean test for second digits of regression coefficients

the observed rejection ratio is greater, in particular for first digits. Thus, we may safely conclude that the population is not free of manipulation of some sort ($\Omega > 0$). Given the rejection ratio (θ) observed in a sample of articles, equation (5) suggests that the proportion of articles in the population violating Benford's law is $\Omega = (\theta - \alpha)/(1 - \beta - \alpha)$. This formula is not operational because type II errors (β) are not known, neither for single articles nor on average for all articles sampled. Rather, β depends on the sample size and on the type and degree of contamination, all differing from paper to paper. In large and/or heavily contaminated samples β would be close to zero. Thus, a lower bound for the ratio of doubtful articles in the population can be provided:

$$\Omega \geq \frac{\theta - \alpha}{1 - \alpha} \tag{6}$$

From Table 2, $\Omega \geq 22\%$ is obtained for first digits of regression coefficients and $\Omega \geq 26\%$ for first digits of standard errors. For second digits of regression coefficients and standard errors we get $\Omega \geq 6\%$ and $\Omega \geq 9\%$, respectively. Since the figures for second digits are rather inconspicuous, contrary to those on first digits, we may indeed suspect that some sort of intervention has distorted the first digits. However, it needs to be emphasized that this evidence is not yet a proof of falsification. In every single case there may be other plausible reasons for deviations from Benford's law, such as insufficient variability of the underlying data, rounding effects or other irregularities. Nevertheless, anomalies revealed by Benford tests may be useful signals to initiate further investigation. Anyway, routinely checking Benford's law would raise the risk of cheaters being disclosed.

4. CONCLUSIONS

Benford's law is an empirical regularity for the distribution of first digits that is observed in many diverse datasets. Benford's law is successfully applied in business and administration to detect fraud in accounting and taxation. Repeatedly occurring spectacular cases of falsifications in the international research system demonstrate that science is not free of dishonesty and deception.

Against this backdrop, Benford's law is a potentially useful instrument to discover fraud and manipulation in quantitative economic research. About 100 recently published research articles in two economics journals were tested for violations of Benford's law. Testing second digits of regression coefficients and standard errors, the mean test rejected Benford's law in about 10% of the articles. However, when testing first digits, violations of Benford's law occurred in about 25% of the articles, far more often than could be expected in untampered samples.

In economics it is unusual and notoriously difficult to replicate empirical research. Therefore, checking for deviations from Benford's law is a simple and promising route to discover first hints of anomalies, manipulations and falsifications. Benford tests do not provide conclusive evidence, but they can help identify papers that need closer inspection and thus complement the control mechanisms already in place.

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