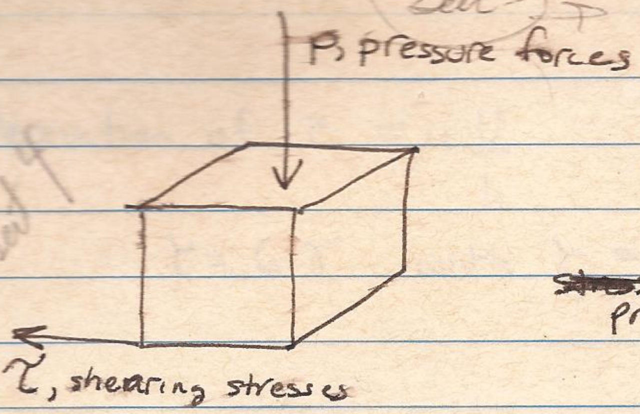
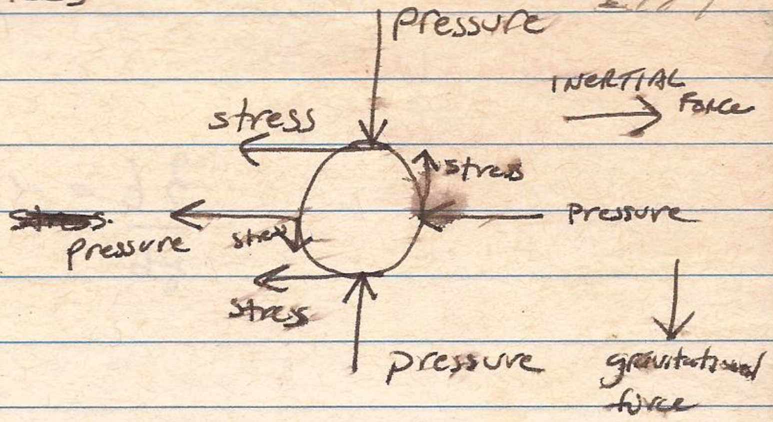
the Legal Pad by Tops

Ref sect- Encyclopedia of Physical Science & Technology Q 123 E497

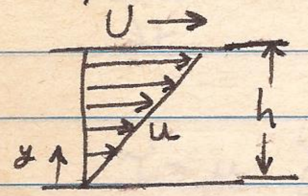
2/17 MAP 3302 Sect 4





$$F_{RESULTANT} = F_{INERTIAL} - F_{PRESSURE} - F_{STRESS} - F_{GRAVITATIONAL}$$

VELOCITY DISTRIBUTION:



$u(y) = \frac{y}{h} U$

u = fluid velocity in boundary layer
U = free stream velocity
h = boundary layer thickness

rate of change of velocity with respect to position y. $= \frac{du}{dy} = \frac{U}{h}$ ⇒ THIS IS ALSO KNOWN AS A VELOCITY GRADIENT.

$\therefore$ Frictional Force, $\tau = \mu \frac{du}{dy} = \frac{\mu U}{h}$ μ = is the viscosity of the fluid

τ units in $\frac{N}{m^2}$ μ units in $\frac{kg}{m \cdot sec}$

γ magnitude of the strain

$\frac{d\gamma}{dt}$ rate of change of strain

$\therefore$ ~~$\tau = \mu$~~ $\frac{d\gamma}{dt}$

Direvation of $\tau = \mu \frac{U}{h}$

G = modulus of shear
ξ = displacement

$$\tau = G\gamma \quad \text{with } \gamma = \frac{\partial \xi}{\partial y}$$

and $\xi = ut$

$$\therefore \quad \tau = \mu \frac{\partial \gamma}{\partial t} = \mu \frac{\partial}{\partial t}\left(\frac{\partial \xi}{\partial y}\right) = \mu \frac{\partial}{\partial y}\left(\frac{\partial \xi}{\partial t}\right)$$

with $\frac{\partial \xi}{\partial t} = u$, then

$$\tau = \mu \frac{\partial u}{\partial y}$$

with $u = \frac{y}{h} U$ and $\frac{\partial u}{\partial y} = \frac{U}{h}$

$$\therefore \quad \boxed{\tau = \mu \frac{U}{h}}$$

PROOF!

IMP TO ACKNOWLEDGE KINEMATIC VISCOSITY WHEN TAKING INTO ACCOUNT THE FRICTIONAL + INERTIAL FORCES.

$$\nu = \frac{\mu}{\rho} \quad (m^2/s)$$

ν = kinematic viscosity, μ = viscosity, ρ = density

$r_{REAL} = (91.37t - 48.41t^2)i + (1.218 - 4.9t^2)j$

$$S = (91.37)(0.25) - 48.41(.25)^2$$
$$= 22.84 - 3.026$$
$$= 19.81\,m \times 39.4\,in \times \frac{1}{12\,in} = \boxed{65.1\,ft}$$

$Q_{min} = 1.57 \text{ in}^3 \quad \delta = .02 \text{ in}$

$Q_{max} = 3.48 \text{ in}^3 \quad \delta = .009 \text{ in}$

0.18 ft/s

$C_q = \frac{V_0}{U} = \frac{.18}{300} = .0006$

$C_q \cdot A \cdot U_\infty$

$\text{ft}^2 \quad \text{ft/s}$

$A = 4\pi r^2$

$(.0006)(0.355866)(300) = .0640558 \text{ ft}^3/\text{s}$

$r = \frac{.34"}{12"/\text{ft}} = 0.0283$

0.35587 ft^2

$Q = .0640558 \text{ ft}^3/\text{s} \times 1728 \frac{\text{in}^3}{\text{ft}^3} = 110.69 \text{ in}^3/\text{s}$

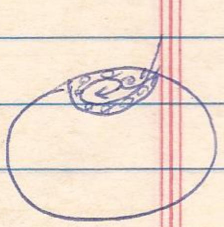
For a flight time of 0.24 s,

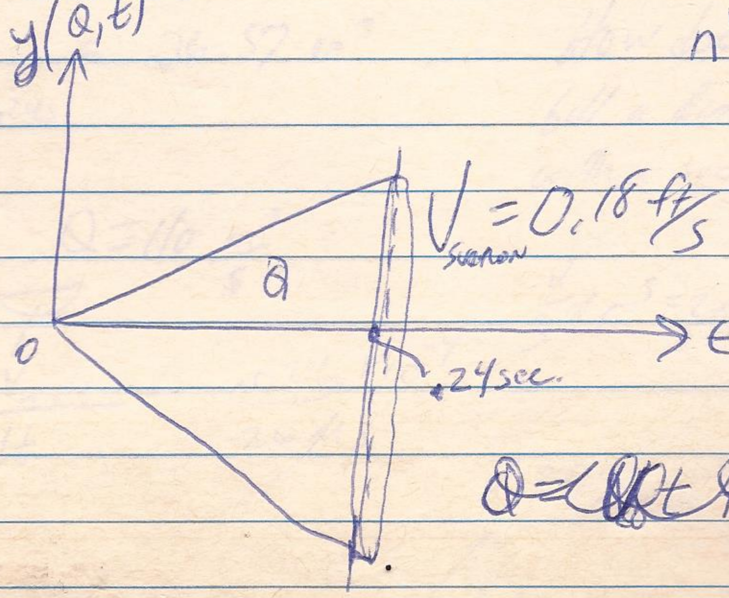
$Q = 26.56522 \text{ in}^3$

U_{suction}

U velocity of suction

$n V_{\text{dimple}}$





$V_{\text{suction}} = 0.18 \text{ ft/s} \times 12 \text{ in/ft} = 2.16 \text{ in/s}$

$Q = $ [illegible]

$$V = \frac{4}{3}\pi r^{3}$$

$$\frac{dV}{dt} = \frac{8}{3}\pi r = 110\ \frac{in^3}{s}$$ Size of Dimple - independent of laminar flow

$$\frac{dV}{dr} = 4\pi r^2$$

$$\cancel{\frac{dV}{dt} + V_0}$$

$$\frac{\frac{4}{3}\pi r^3}{2} \quad \frac{\frac{4}{3}}{2}\pi r^3 =$$

$$V = \dot{V}t + V_0 = 110t + \frac{4}{3}\pi r^3$$

$$\frac{d}{dr}\left(\frac{dV}{dt}\right) \qquad 4\pi r^2$$

$$Q\Big|_{t=.24s} = 26.57\ in^3$$

How does it take to fill a dimple of Volume V_0 with a suction velocity of U_0?

$$Q = 110\ \frac{in^3}{s}$$

$$\frac{dV_{dimple}}{dt} \qquad .026 \times 10^{-9} \quad 26\ \mu s$$

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \quad V = 2.882 \times 10^{-9}$$

$$1.377 \times 10^{-9}$$

$$\left(1.1125 \times 10^{-3}\right)^3 \qquad .0445$$

$d = .00223$

$r = .0011125$

$Q_{min} = 1.568\ in^3$ $8\ s=.02$

$Q_{max} = 31.362\ in^3$ $8\ s=.001$

1) CHOOSE ARBITRARY DIMPLE SIZE.

2) Compute IF # OF DIMPLES meets ~~optimum~~ optimum dimple S.A.

3) Compute VOLUME OF Dimples (TOTAL)

4) FROM MAX + MIN SUCTION VELOCITIES OBTAIN FROM OTHER SOURCES, COMPUTE TOTAL MAX + MIN SUCTION VOLUME

5) COMPUTE C_Q + boundary-layer thickness

6) COMPUTE NEW REYNOLDS NUMBER + COMPARE WITH ORIGINAL

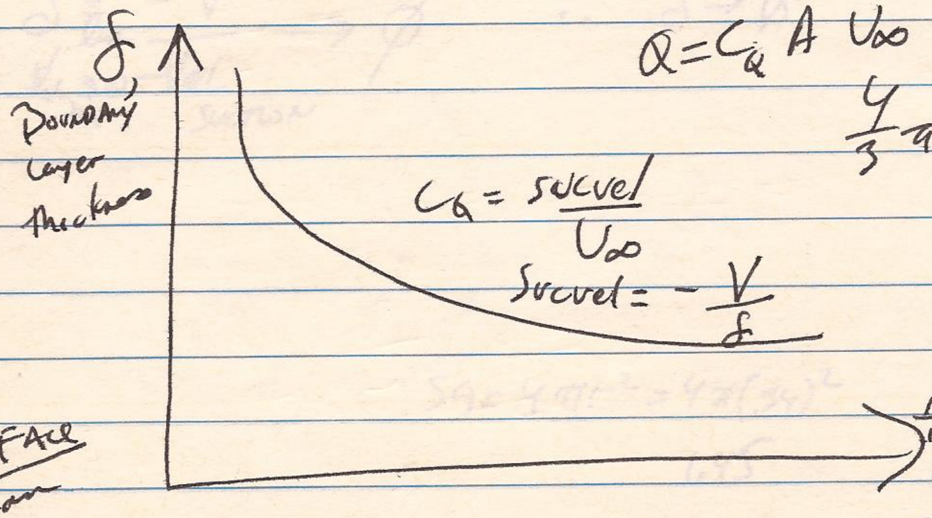
7) Compute ~~local coefficient of drag.~~ new SURFACE (Rep. then smooth ball due to hills + valleys) + get Drag equation.

8) Plot improved-ball trajectory.

$A = SA - A_{total\ dimples}$ wetted area int. BALL

$Q = C_Q A U_\infty$ $\qquad \frac{Q}{A U_\infty} = C_Q$

$\frac{4}{3}\pi r^2 = \frac{4}{3}\pi(.34)^2$



$C_Q = \frac{sucvel}{U_\infty}$

$Sucvel = -\frac{V}{\delta}$

$\frac{1.000}{.0068}$

$nD = \frac{SURFACE}{diam}$

.34
.34
136
102
.1156

.1156
3.14
4624
1156
3468

Suction volume

.362984
4
1.451936

12 in³

120
$25\overline{)3.00}$
25
50

.483
$3\overline{)1.45}$
12
25
24
10

2.0
$24\overline{)48.3}$

$2\ in^3/5$

$\frac{n V_{dimple}}{~~25~~} = ~~12\ in^3~~\ 3.\ in^3$

$V_{dimple} = \frac{3}{n}$

.483 in³
.24

$$C_Q = \lim_{A \to \emptyset} \frac{Q}{A U_\infty} \stackrel{C_Q}{=} \to \infty$$

$$Vel._{SUCTION} = \lim_{C_Q \to \infty} C_Q U_\infty \to \infty$$

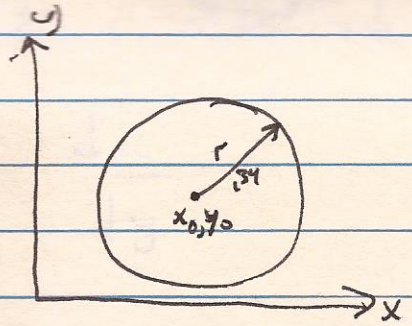
$$\delta = \lim_{Vel_{SUCTION} \to \infty} \frac{-V}{Vel_{SUCTION}} \to \emptyset \quad \therefore \ \delta \to \emptyset$$

$$SA = 4\pi r^2 = 4\pi(.34)^2$$

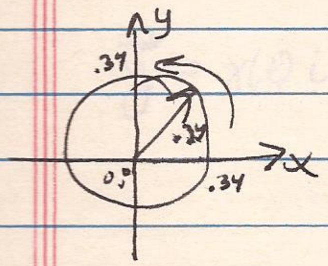
$$1.45$$

$$\begin{array}{r} .34 \\ .34 \\ \hline 6 \end{array}$$

$$\begin{array}{r} .1156 \\ \underline{3.14} \end{array}$$



$$\int_C x^2 + y^2 - r^2$$



$$\int_C x^2\,dy + y^2\,dx$$

$$\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA =$$

$\sqrt{r^2 - x^2}$

$$\int_0^{.34}\int_0^{0}\left(\frac{\partial (x^2)}{\partial x} - \frac{\partial}{\partial y}(y^2)\right) dA$$

$$\int_{0}^{.34}\int_{0}^{\sqrt{r^2-x^2}}\left(\frac{2x^{\cancel{3}}}{\cancel{3}} - \frac{2y^{\cancel{3}}}{\cancel{3}}\right) dy\,dx$$

$$\int_0^{.34} (2xy - y^2)\Big|_0^{\sqrt{r^2-x^2}} dx$$

$$\int_0^{.34}\left(2x\sqrt{r^2-x^2} - (r^2-x^2)\right) dx$$

$$\int 2x(r^2-x^2)^{1/2}dx - \int (r^2+x^2)\,dx$$

$u = r^2 - x^2$

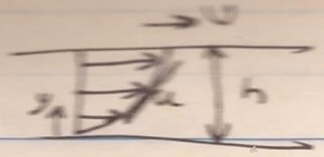
$du = -2x\,dx$

$$-\int u^{1/2}\,du - \left(xr^2 + \frac{x^3}{3}\right)$$

$$-\frac{2}{3}u^{3/2} = -\frac{2}{3}(r^2-x^2)^{3/2} - xr^2 - \frac{x^3}{3}\Big]_0^{.34}$$

$$-\frac{2}{3}\left(r^2 - (.34)^2\right)^{3/2} - .34r^2 - \frac{(.34)^3}{3}$$

$$\frac{-3(.34)^3 - (.34)^3}{3} = \frac{-4(.34)^3}{3} = \frac{-4(.039304)}{3} = -0.0524053$$



$W = F \cdot S = mas$

$\frac{du}{dy}$ $\frac{d\vec{u}}{d\vec{y}}$

$t^3 \; t^2_i + (t - t^2)_j$

$t^5 i \; t(t - t^2) j$

$\vec{y} = x(t) i + y(t) j$

Partial Derivatives

A function of two variables where the derivative of the function with respect to one variable.

Ex. $f(x,y) = 2x^3y^2 + 2y + 4x$

Find $F_x(1,2)$, $f_y(1,2)$

$$f_x(1,2) = 6x^2y^2 + 4 = 6(1)(2)^2 + 4 = 24 + 4 = 28$$

$$f_y(1,2) = 4x^3y + 2 = 4(1)(2) + 2 = 8 + 2 = 10$$

Other notation.

$$f_x = \frac{\partial f}{\partial x} \qquad f_y = \frac{\partial f}{\partial y}$$

Ex

$z = x^4 \sin(xy^3)$ find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial x} = 4x^3 \sin(xy^3) + x^4 \cos(xy^3)\, y^3$$

$$\frac{\partial z}{\partial y} = x^4 \cos(xy^3)\, 3xy^2 = 3x^5y^2 \cos(xy^3)$$

E of Diff Eq

1) $\frac{dy}{dx} + y = x+1$ — 1ST ORDER — Linear O.D.E

2) $\frac{d^2y}{dx^2} + xy^2\left(\frac{dy}{dx}\right)^3 = 0$ — 2ND ORDER — NO, because 1st derivative is raised to 3rd power + in the form it is not raise.

3) $\frac{d^5y}{dx^5} + \frac{dy}{dx} = x^2y$ $\Rightarrow$ Also written $\frac{d^5y}{dx^5} + \frac{dy}{dx} - x^2y = 0$ — 5TH ORDER — Linear O.D.E, where x^2 is $a_0(x)$ of the form

4) $\frac{\partial^2u}{\partial x^2} + \frac{\partial^2u}{\partial y^2} = 0$ — 2ND ORDER — NO

5) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ — 1ST ORDER. — NO

Differential Equation = is an equation containing derivatives
Ordinary Diff. Eq (O.D.E) = NO PARTIAL DERIVATIVES
Partial Diff. Eq (P.D.E) = CONTAINING ONLY PARTIAL DERIVATIVES

ORDER OF DIFF. EQ. determined be order of derivative

Linear O.D.E of order n in the dependent variable y + independent variable x is a Diff. Eq. OF THE FORM

$$a_n(x)\frac{d^ny}{dx} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$$

IF $a_n(x)$ is constant, then the above D.E. is a D.E. with constant coefficients

otherwise it has variable coefficients

Ex

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = \emptyset$$ is a D.E. w/ constant coefficients

SOLUTION OF D.E.

1) $y = x + 3e^{-x}$ is a solution of $\frac{dy}{dx} + y = x + 1$

An explicit solution of the D.E.

$$\frac{dy}{dx} = 1 - 3e^{-x}$$

$$\frac{dy}{dx} + y = 1 + x$$

OBJECTIVE:

Given the D.E. and determine the solution.

Given the relation $x^3 + 3xy^2 = 1$

cannot find y in terms of x, therefore must use implicit differentiation to find the D.E.,

$$2xy\left(\frac{dy}{dx}\right) + x^2 + y^2 = \emptyset$$

remember (this is an example of an implicit solution of the D.E.)

Def

A function $f(x)$ defined for all x in some interval I and having an nth derivative for all $x \in I$ is called an explicit solution of the D.E. $F\left[x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}\right] = \emptyset$

F.

1) $F[x, F(x), F'(x), f^{(2)}(x), \ldots, f^{n}(x)]$ is defined for all $x \in I$

2) $F[x, f(x), f'(x), \ldots f^{n}(x)] = \emptyset$ for all $x \in I$,

Def

$g(x,y) = 0$ is a implicit solution of the D.E. if

$f(x)$ is an explicit solution of the D.E.

Ex

$5x^2y^2 - 2x^3y^2 = 1$ is an implicit solution of the D.E.

$$x\frac{dy}{dx} + y = x^3y^3$$

$$\frac{6.334 \pm \sqrt{C^2\frac{A_s}{\pi}\left(40.12 + \frac{4.031\times10^7 V^3 A_s^2}{C\pi}\right)}}{432 V \cancel{C\pi}}$$

$$r_{0,1} = \frac{6.334 \pm \sqrt{40.12 + \frac{(4.031\times10^7) V^3 A_s^2}{C\pi}}}{432 V}$$

$$C = INT\left(\frac{A_s}{\pi r^2}\right)$$

$$INT\left(\frac{.0369}{\pi r^2}\right)$$

$$216 V A_s = \cancel{\pi r} (A_s/\cancel{\pi r^2})[216 V r - 6.334]$$

$$INT\left(\frac{.0117}{r^2}\right)$$

$$216 V A_s = \frac{A_s}{r}\left[216 V r - 6.334\right]$$

$$216 V A r = \underline{216 A_s V r - 6.334 A_s}$$

$$INT\left(\frac{5}{2}\right) = 2$$

$$.01852\times10^{-5}$$

$$(40.31\times10^7)(.0004595\times10^{-12})$$

$$INT\left(\frac{2}{3}\right) =$$

40.31×10^7 $.3375\times10^{-9}$ $.001362\times10^{-3}$

$$\frac{6.334 \pm \sqrt{40.12 + \frac{(40.31\times10^7)(.000150)^3(.0369)^2}{C(3.14)}}}{432(.000150)}$$

$$\lim_{r\to 0} \sqrt{INT\left(\frac{.0117}{r^2}\right)}$$

$$(150\times10^{-6})^3$$

$$40.12/3.14C$$

$$3375000\times10^{-18}$$

$$.3375\times10^{-9}$$

$$\frac{6.334 \pm \sqrt{12.777/C}}{}$$

$4\pi r^2$

$$6.334 \pm 3.5745 (C^{-1})^{1/2}$$

$.0648$

A $.0369$

$$\frac{1.451936 \text{ in}\cdot\text{ft}}{12 \text{ in}} = .121 \text{ ft}$$

$3\times$ ft

$$r_{01} = 90.745 \pm 51.21 C^{-1/2}$$

$Ax^2 + Bx + C = 0$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$C^{1/2} = INT\left(\frac{.0369}{r_1^2}\right)$

$C^{-1/2} = \frac{1}{INT\left(\frac{.0369}{r_1^2}\right)} =$

$(r_1 - 51.21C^{-1/2})^2 = (90.75)^2$

$(r_1 - 51.21C^{-1/2})(r_1 - 51.21C^{-1/2})$

$r_1^2 - 102.42C^{-1/2} + 2622.46C^{-1} = 8235.56$

$INT(5.5) =$

$r_1^2 - \frac{102.42}{C^{1/2}} + \frac{2622.46}{C} = 8235.56$

$C = INT\left(\frac{.0369}{r_1^2}\right)$

$Cr^2 - 102.42C^{1/2} + 2622.46 = 8235.56C$

$C^{1/2} = \left[INT\left(\frac{.0369}{r_1^2}\right)\right]^{1/2}$

$8235.56C - Cr^2 + 102.42\sqrt{C} - 2622.46 = 0$

$f(C, r) = 8235.56C - Cr^2 + 102.46C^{1/2} - 2622.46 = 0$

$\frac{\partial f}{\partial C} = 8235.56 - r^2 + \frac{1}{2}(102.46)C^{-1/2}$ $\qquad \frac{\partial f}{\partial r} = 2rC = 0$

$C = \phi$

$8235.86 - r^2 = 0$

$r^2 = 8235.56$

$D = \sqrt{C}$.

$r = 90.75$

$(8235.56 - r^2)D^2 + 102.46D - 2622.46 = 0$

$D =$

$C =$