

Systematic Decision-Making. Let's start with a general process for making decisions.

1. List all choices or courses of action.
2. List all criteria (attributes) that will be used to evaluate the choices.
3. Compare the two lists and remove impractical choices.
4. Evaluate the advantages and disadvantages of each remaining choice according to all of the criteria.

This doesn't really say how to evaluate the advantages and disadvantages systematically, though.

Computational Decision-Making

Our goal is to quantify the decision-making process. We'll do so by assigning a numerical *weight* to each criterion, according to its importance, and a numerical *score* for each choice according to each criterion. Together, these allow us to compute the value of a payoff function for each choice.

Assume that there are m choices and n criteria.

1. Decide on the importance of each criterion, and assign a weight w_j to each criterion. Make sure that $\sum_{j=1}^n w_j = 1$ (or 100%). Larger weights are more important.
2. For each choice i and criterion j , assign a score p_{ij} which reflects the extent to which i satisfies j . This should be a number between 0 and 1.

This data will allow us to compute scores s_{ij} and a payoff function f_i for each alternative and to choose the alternative with the largest expected payoff:

$$s_{ij} = p_{ij}w_j; \quad f_i = \sum_{j=1}^n s_{ij} = \sum_{j=1}^n p_{ij}w_j$$

(which is just the dot product of the weights and the scores).

Choose the alternative with the largest payoff f_i .

Normalizing scores. As an alternative to choosing scores $p_{ij} \in [0, 1]$, you can instead normalize the scores, as follows.

1. Assign a real number c_{ij} for each alternative i and criterion j . Be sure to use the same units for all of the different alternatives of a single criterion. You can use different units for different criteria.
2. For each criterion, calculate

$$C_j = \max\{|c_{1j}|, |c_{2j}|, \dots, |c_{mj}|\},$$

so that

$$p_{ij} = \frac{c_{ij}}{C_j}.$$

3. The payoff function is then given by:

$$f_i = \sum_{j=1}^n p_{ij} w_j = \sum_{j=1}^n \frac{c_{ij}}{C_j} w_j.$$

Example

Let's consider the question: "How should I get to Montreal?"

The possible answers I'll consider are:

- Train
- Personal automobile
- Airplane

In Step 1, we will consider three criteria:

- Cost
- Travel time
- Schedule Flexibility (i.e. when can I leave?)

Let's assign weights to these criteria. Step 2:

- Pretend you have lots of money. Let's estimate $w_1 = 0.2$.
- Time is important, so $w_2 = 0.4$.
- Schedule Flexibility is important; let $w_3 = 0.4$.

(Yes, these are arbitrary. We'll see just how much these arbitrary decisions affect the outcome.)

Now, we need to decide how we'll assign scores. Step 3:

- Cost: dollars (lower = better);
- Time: hours (lower = better);
- Schedule Flexibility: qualitative assessment (higher = better)

All of the scores need to point the same way, so we'll invert our schedule flexibility score before computing.

Let's make up some raw data.

Criterion	Train	Car	Plane	Category Winner
Cost	\$102	\$206 ¹	\$191	Train
Time	7.7 hrs	6.5 hrs	3.5 hrs	Plane
Schedule Flexibility	0.5	1.0	0.9	Car

What factors do these data not capture?

Now we can build a decision matrix:

Criterion ($n = 3$)	Weight w_i (%)	Alternatives								
		Train			Car			Plane		
		c_{1j}	p_{1j}	s_{1j}	c_{2j}	p_{2j}	s_{2j}	c_{3j}	p_{3j}	s_{3j}
Cost	20	102	0.5		206	1.0		191	0.9	
Time	40	7.7	1.0		6.5	0.85		3.5	0.45	
Flexibility	40	0.5	0.5		0.0	0.0		0.1	0.1	
Totals f_j										

Note that, in this matrix, lower is better. It may make sense to make higher better, depending on how your criteria are set up.

Potential Flaws. Did we get an optimal decision? Let's look at threats to validity of the analysis to see how robust our decision is.

- (Not an issue here.) In some cases, your values might be stuck in a small subrange of the possible range, which affects the numbers you get out.
- The values for flexibility are subjective, and the way you assign numbers to the options changes the outcome.
- The cost depends on what you include; for instance, gas alone would be \$72.50, and it would be less if you carpooled.
- The weights are subjective.