Lecture 06—Dependencies

ECE 459: Programming for Performance

January 24, 2013

Last Time

Three-address code, volatile and restrict.

Race conditions:

- detecting them with helgrind.
- avoiding them via synchronization: mutexes, spinlocks, read-write locks, semaphores, barriers, lock-free algorithms.

This time: Dependencies

Dependencies are the main limitation to parallelization.

Example: computation must be evaulated as XY and not YX.

Not synchronization

Assume (for now) no synchronization problems.

Only trying to identify code that is safe to run in parallel.

Memory-carried Dependencies

Dependencies limit the amount of parallelization.

Can we execute these 2 lines in parallel?

```
\begin{array}{l} x \ = \ 42 \\ x \ = \ x \ + \ 1 \end{array}
```

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\begin{array}{l}
x = 42 \\
x = x + 1
\end{array}
```

No.

• Assume x initially 1. What are possible outcomes?

Memory-carried Dependencies

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Can we execute these 2 lines in parallel?

$$\begin{array}{ccc} x & = & 42 \\ x & = & x & + & 1 \end{array}$$

No.

• Assume x initially 1. What are possible outcomes? x = 43 or x = 42

Next, we'll classify dependencies.

Read After Read (RAR)

Can we execute these 2 lines in parallel? (initially x is 2)

```
\begin{vmatrix} y = x + 1 \\ z = x + 5 \end{vmatrix}
```

Read After Read (RAR)

Can we execute these 2 lines in parallel? (initially x is 2)

```
\begin{vmatrix}
y = x + 1 \\
z = x + 5
\end{vmatrix}
```

Yes.

- Variables y and z are independent.
- Variable x is only read.

RAR dependency allows parallelization.

Read After Write (RAW)

What about these 2 lines? (again, initially x is 2):

```
\begin{array}{c}
x = 37 \\
z = x + 5
\end{array}
```

Read After Write (RAW)

What about these 2 lines? (again, initially x is 2):

$$\begin{array}{ccc}
x &=& 37 \\
z &=& x &+& 5
\end{array}$$

No, z = 42 or z = 7.

RAW inhibits parallelization: can't change ordering. Also known as a true dependency.

Write After Read (WAR)

What if we change the order now? (again, initially x is 2)

```
\begin{vmatrix} z = x + 5 \\ x = 37 \end{vmatrix}
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Write After Read (WAR)

What if we change the order now? (again, initially x is 2)

$$\begin{bmatrix} z = x + 5 \\ x = 37 \end{bmatrix}$$

No. Again, z = 42 or z = 7.

- WAR is also known as a anti-dependency.
- But, we can modify this code to enable parallelization.

Removing Write After Read (WAR) Dependencies

Make a copy of the variable:

```
x_copy = x
z = x_copy + 5
x = 37
```

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We can now run the last 2 lines in parallel.

- Induced a true dependency (RAW) between first 2 lines.
- Isn't that bad?

Removing Write After Read (WAR) Dependencies

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x_copy = x
z = x_copy + 5
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```

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- Induced a true dependency (RAW) between first 2 lines.
- Isn't that bad?

Not always:

```
z = very_long_function(x) + 5
x = very_long_calculation()
```

Write After Write (WAW)

Can we run these lines in parallel? (initially x is 2)

```
\begin{array}{c}
z = x + 5 \\
z = x + 40
\end{array}
```

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Can we run these lines in parallel? (initially x is 2)

```
\begin{array}{c}
z = x + 5 \\
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Nope, z = 42 or z = 7.

- WAW is also known as an output dependency.
- We can remove this dependency (like WAR):

Write After Write (WAW)

Can we run these lines in parallel? (initially x is 2)

```
\begin{bmatrix} z = x + 5 \\ z = x + 40 \end{bmatrix}
```

Nope, z = 42 or z = 7.

- WAW is also known as an output dependency.
- We can remove this dependency (like WAR):

```
z_{copy} = x + 5

z = x + 40
```

Summary of Memory-carried Dependencies

		Second Access	
		Read	Write
First Access	Read	No Dependency Read After Read (RAR)	Anti-dependency Write After Read (WAR)
	Write	True Dependency Read After Write (RAW)	Output Dependency Write After Write (WAW)

Part II

Loop-carried Dependencies

Loop-carried Dependencies (1)

Can we run these lines in parallel? (initially a[0] and a[1] are 1)

```
a[4] = a[0] + 1
a[5] = a[1] + 2
```

Loop-carried Dependencies (1)

Can we run these lines in parallel? (initially a[0] and a[1] are 1)

```
a[4] = a[0] + 1
a[5] = a[1] + 2
```

Yes.

- There are no dependencies between these lines.
- However, this is not how we normally use arrays...

Loop-carried Dependencies (2)

What about this? (all elements initially 1)

```
for (int i = 1; i < 12; ++i) a[i] = a[i-1] + 1
```

Loop-carried Dependencies (2)

What about this? (all elements initially 1)

```
for (int i = 1; i < 12; ++i) a[i] = a[i-1] + 1
```

No,
$$a[2] = 3$$
 or $a[2] = 2$.

- Statements depend on previous loop iterations.
- An example of a loop-carried dependency.

Loop-carried Dependencies (3)

Can we parallelize this? (again, all elements initially 1)

```
for (int i = 4; i < 12; ++i)
 a[i] = a[i-4] + 1
```

Loop-carried Dependencies (3)

Can we parallelize this? (again, all elements initially 1)

```
for (int i = 4; i < 12; ++i) a[i] = a[i-4] + 1
```

Yes, to a degree.

- We can execute 4 statements in parallel:
 - \bullet a[4] = a[0] + 1, a[8] = a[4] + 1
 - ightharpoonup a[5] = a[1] + 1, a[9] = a[5] + 1
 - ightharpoonup a[6] = a[2] + 1, a[10] = a[6] + 1
 - ightharpoonup a[7] = a[3] + 1, a[11] = a[7] + 1

Loop-carried Dependencies (3)

Can we parallelize this? (again, all elements initially 1)

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 a[i] = a[i-4] + 1
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Yes, to a degree.

- We can execute 4 statements in parallel:
 - a[4] = a[0] + 1, a[8] = a[4] + 1
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 - ightharpoonup a[7] = a[3] + 1, a[11] = a[7] + 1

Always consider dependencies between iterations.

Larger example: Loop-carried Dependencies

```
// Repeatedly square input, return number of iterations before
// absolute value exceeds 4, or 1000, whichever is smaller.
int inMandelbrot(double x0, double y0) {
  int iterations = 0:
  double x = x0, y = y0, x2 = x*x, y2 = y*y;
  while ((x2+y2 < 4) \&\& (iterations < 1000)) {
   y = 2*x*y + y0;
   x = x2 - y2 + x0;
   x2 = x*x; y2 = y*y;
    iterations++;
  return iterations;
```

How can we parallelize this?

Larger example: Loop-carried Dependencies

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// Repeatedly square input, return number of iterations before
// absolute value exceeds 4, or 1000, whichever is smaller.
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  while ((x2+y2 < 4) \&\& (iterations < 1000)) {
   y = 2*x*y + y0;
   x = x2 - y2 + x0;
   x2 = x*x; y2 = y*y;
    iterations++;
  return iterations;
```

How can we parallelize this?

 Run inMandelbrot sequentially for each point, but parallelize different point computations.

Summary

Identifed memory-carried dependencies:

• 3 types of dependencies (RAW, WAR, WAW).

Removed output and anti-dependencies (at a price).

Identified loop-carried dependencies.