Software Testing, Quality Assurance and Maintenance	Winter 2025
Lecture 5c — January 28, 2025	
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Here's a proof of semantic equivalence between $s_1 = \textsf{skip} \,;\, S$ and $s_2 = S$. From page 11 of the L05 notes, we have:

s_1 and s_2 are semantically equivalent if $\forall q.\ \langle s_1, q\rangle \Downarrow q_1$ and $\langle s_2, q\rangle \Downarrow q_2$ implies $q_1 = q_2$.

So, if we take any q, and let q' satisfy $\langle S, q\rangle \Downarrow q'$ for that q', then we have to show that $\langle \textsf{skip} \ ;\ S, q\rangle \Downarrow q'$ also. (If q' doesn't exist, because S doesn't terminate, then there is no proof obligation).

The composition rule says:

$$\frac{\langle s_1, q\rangle \Downarrow q'' \qquad \langle s_2, q''\rangle \Downarrow q'}{\langle s_1 \ ;\ s_2, q\rangle \Downarrow q'}$$

and the skip rule says:

$$\frac{}{\langle \textsf{skip}, q\rangle \Downarrow q}$$

Applying the rule for $\textsf{skip}$ gives us $\langle \textsf{skip}, q\rangle \Downarrow q$. We observe that the premises for the composition rule hold, if we substitute $s_1 = \textsf{skip}$, $q'' = q$, and $s_2 = S$. We thus have the conclusion

$$\langle \textsf{skip} \ ;\ S, q\rangle \Downarrow q',$$

as required.