Software Testing, Quality Assurance & Maintenance—Lecture 11

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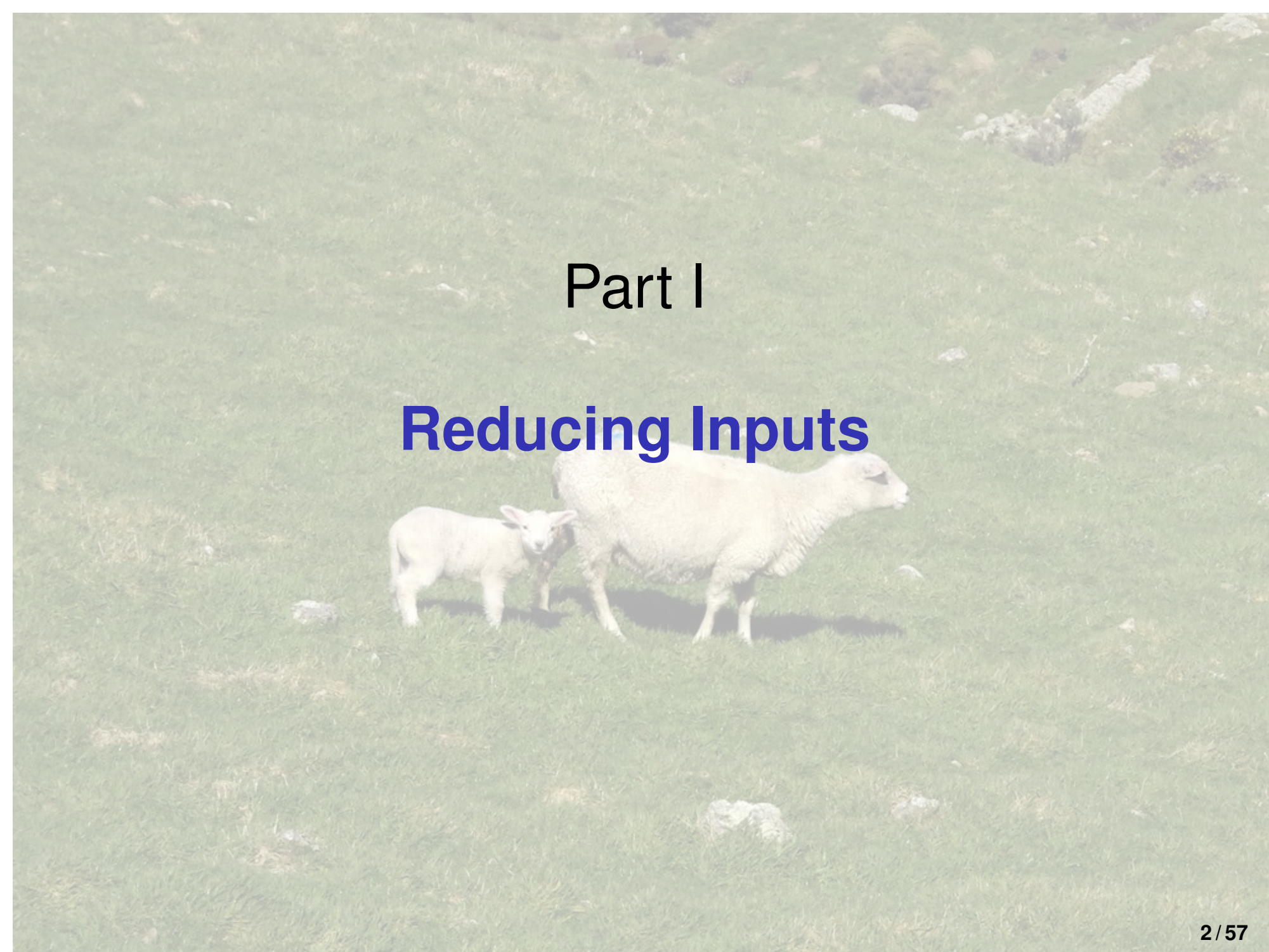


Part I

Reducing Inputs

LaTeX errors

```
) (/usr/share/texlive/texmf-dist/tex/latex/epstopdf-pkg/epstopdf-base.sty
(/usr/share/texlive/texmf-dist/tex/latex/latexconfig/epstopdf-sys.cfg))
(/usr/share/texlive/texmf-dist/tex/latex/upquote/upquote.sty)
(/usr/share/texlive/texmf-dist/tex/latex/inconsolata/ot1zi4.fd)
Overfull \hbox (3.22516pt too wide) in paragraph at lines 84--84
[][]\OT1/cmr/m/n/9 Discussion: $\OT1/zi4/m/n/9 https : / / tex . meta . stackex
change . com / questions / 6255 / why-[]does-[]tex-[]require-[]such-[]elaborate
-[]mwes$|

! Package tikz Error: Giving up on this path. Did you forget a semicolon?.

See the tikz package documentation for explanation.
Type  H <return>  for immediate help.
 ...

l.97 \end{tikzpicture}

? 
```

To ask for help on StackExchange, need a Minimal Working Example.

Fixing a bug

1. need to reproduce the bug, so need a working example;
2. better yet: a *minimal* working example is easier to deal with.

MWEs and Fuzzing

Fuzzers produce large inputs.

When input contains extraneous context,
hard to understand what's happening.

Reducing an input

We'll show a way to reduce a failing input:

> "to identify those circumstances of a failure that are relevant for the failure to occur, and to *omit* (if possible) those parts that are not"

A Mystery

```
class MysteryRunner(Runner):
    def run(self, inp: str) -> Tuple[str, Outcome]:
        x = inp.find(chr(0o17 + 0o31))
        y = inp.find(chr(0o27 + 0o22))
        if x >= 0 and y >= 0 and x < y:
            return (inp, Runner.FAIL)
        else:
            return (inp, Runner.PASS)
```

Fails on some inputs. Can use `RandomFuzzer` to find a failure.

Fuzzing a Failure

```
def fuzz_mystery_runner():
    mystery = MysteryRunner()
    random_fuzzer = RandomFuzzer()
    while True:
        inp = random_fuzzer.fuzz()
        result, outcome = mystery.run(inp)
        if outcome == mystery.FAIL:
            break
    print (result)
```

This works and eventually finds a failing input.
(Manually, took me 6 tries.)

```
$ python3 mystery_runner.py
(%*50  1)-&7,;49:4?%:43*(-.
```

But why?

Part II

Manual Input Reduction

Divide and Conquer

Kernighan and Pike recommend:

> Proceed by binary search. Throw away half the input and see if the output is still wrong; if not, go back to the previous state and discard the other half of the input.

Does this work?

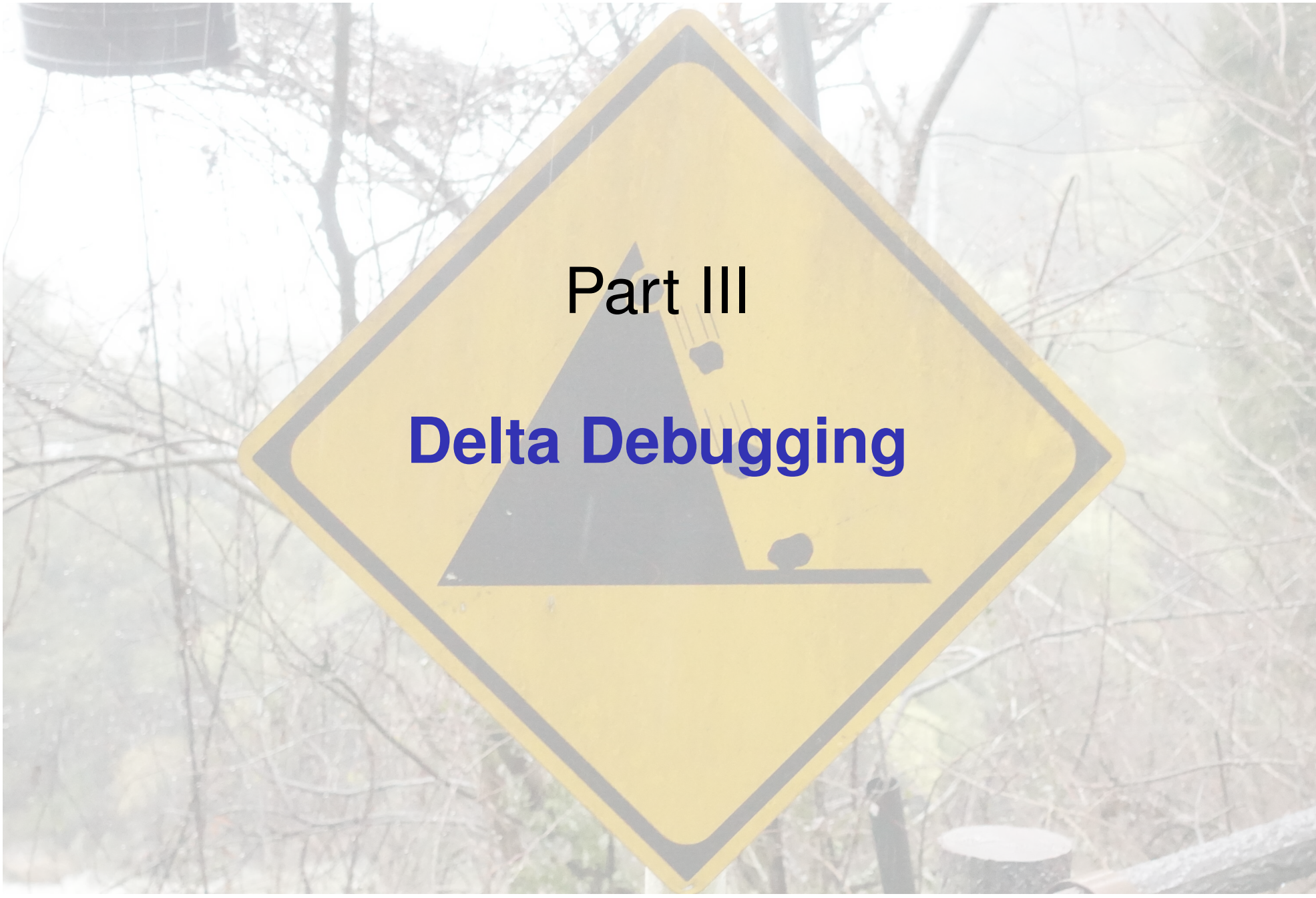
```
>>> from mystery_runner import *
>>> failing_input = "(%*50  1)-&7,;49:4?%:43*(-."
>>> mystery = MysteryRunner()
>>> mystery.run(failing_input)
('(%*50  1)-&7,;49:4?%:43*(-.', 'FAIL')
>>> half_length = len(failing_input) // 2 # integer division
>>> first_half = failing_input[:half_length]
>>> mystery.run(first_half)
('(%*50  1)-&7,', 'FAIL')
```

Progress! Halved the original input.

Failing at Failing

```
>>> quarter_length = len(first_half) // 2
>>> first_quarter = first_half[quarter_length:]
>>> mystery.run(first_quarter)
(' 1)-&7,', 'PASS')
>>> second_quarter = first_half[:quarter_length]
>>> mystery.run(second_quarter)
('(%*50 ', 'PASS')
```

Same trick doesn't work again.
The code says it's looking for two characters, but in our test case, the characters aren't in the same quarter.



Part III

Delta Debugging

A Change in Perspective

We tried direct binary search—didn't work.

Delta debugging is another way.

We instead *remove* smaller and smaller parts of the input,
and see if it still fails.

Intuitively: more likely to keep the brokenness.

Example: Removing Quarters 1

Let's start with the first quarter.

```
>>> quarter_length=len(failing_input)//4
>>> input_without_first_quarter=failing_input[
                                    quarter_length:]
>>> mystery.run(input_without_first_quarter)
(' 1)-&7,;49:4?%:43*(-.', 'PASS')
```

PASS, so must keep 1st quarter.

Example: Removing Quarters 2

Now for the second quarter.

```
>>> input_without_second_quarter=failing_input[:
                                  quarter_length]+
                                  failing_input[
                                  quarter_length*2:]
>>> mystery.run(input_without_second_quarter)
('(%*50 ,;49:4?%:43*(-.', 'PASS')
```

We knew this already:

must keep the first half.

We also know we can discard the second half, but let's see.

Example: Removing Quarters 3, 4

```
>>> input_without_3rd_quarter=
     failing_input[:quarter_length*2]+failing_input[
                                        quarter_length*3:]
>>> mystery.run(input_without_3rd_quarter)
('(%*50  1)-&7?%:43*(-.', 'FAIL')
>>> input_without_4th_quarter=failing_input[:quarter_length*3]
>>> mystery.run(input_without_4th_quarter)
('(%*50  1)-&7,;49:4', 'FAIL')
```

This doesn't tell us anything new,
but we're sort of following the algorithm.

(The algorithm doesn't actually quite work like this.)

Infrastructure for Reducing

An abstract base class that doesn't really do anything.

```
class Reducer:
    def __init__(self, runner: Runner, log_test: bool = False)
                                                -> None:
        # ...

    def test(self, inp: str) -> Outcome:
        # ...

    def reduce(self, inp: str) -> str:
        # here, non-real (abstract) impl
```

Caching

We can cache results:

```
class CachingReducer(Reducer):
    def test(self, inp):
        if inp in self.cache:
            return self.cache[inp]

        outcome = super().test(inp)
        self.cache[inp] = outcome
        return outcome
```

Actually reducing 1

Here is the outer loop for delta debugging.

```
class DeltaDebuggingReducer(CachingReducer):
  def reduce(self, inp: str) -> str:
    self.reset()
    assert self.test(inp) != Runner.PASS

    n = 2     # Initial granularity
    while len(inp) >= 2:
      start = 0.0
      subset_length = len(inp) / n
      some_complement_is_failing = False
      # inner loop goes here
```

We initialize `n` to specify that we divide the input into halves at first.

Also, we set the subset length to the current input length, divided by `n`.

Actually reducing 2

Now the inner loop:

```
    # remove chunks of size len(inp)/n
    while start < len(inp):
        complement = inp[:int(start)] + \
            inp[int(start + subset_length):]
        if self.test(complement) == Runner.FAIL:
            # save the failing test, decrease n
            inp = complement
            n = max(n - 1, 2)
            some_complement_is_failing = True
            break
        start += subset_length
    if not some_complement_is_failing:
        # all subtests pass, get half-as-small chunks.
        if n == len(inp):
            break
        n = min(n * 2, len(inp))
return inp
```

Running the delta debugger

```
dd_reducer = DeltaDebuggingReducer(mystery, log_test=True)
dd_reducer.reduce(failing_input)
```

and there is an example run in the *Fuzzing Book*, which I'll show excerpts from:

```
Test #1 ' 7:,>((/$$-/->.;.=;(.%!:50#7*8=$&&=$9!%6(4=&69\':\'<3+0-3.24#7=!&
Test #2 '\'<3+0-3.24#7=!&60)2/+";+<7+1<2!4$>92+$1<(3%&5\'\'>#' 49 PASS
Test #3 " 7:,>((/$$-/->.;.=;(.%!:50#7*8=$&&=$9!%6(4=&69':" 48 PASS
Test #4 '50#7*8=$&&=$9!%6(4=&69\':\'<3+0-3.24#7=!&60)2/+";+<7+1<2!4$>92+$1
Test #5 "50#7*8=$&&=$9!%6(4=&69':<7+1<2!4$>92+$1<(3%&5''>#" 49 PASS
Test #6 '50#7*8=$&&=$9!%6(4=&69\':\'<3+0-3.24#7=!&60)2/+";+' 48 FAIL
...
Test #23 '(460)' 5 FAIL
Test #24 '460)' 4 PASS
Test #25 '(0)' 3 FAIL
Test #26 '0)' 2 PASS
Test #27 '(' 1 PASS
Test #28 '()' 2 FAIL
Test #29 ')' 1 PASS
'()'
```

Solving the mystery

Answer: system fails on an input with a (and then a).

Delta debugger commentary

Wouldn't want to do this manually on this input:

random input is harder to understand than a human-generated one.

Assuming that the system is deterministic, we can run the algorithm and get the answer.

Also assume: test cases can run quickly enough that we can afford dozens of iterations.

These are the same conditions as for fuzzing to work well.

Commentary continued

Implementation checks that the initial test case does fail.

Delta debugging is best-case $O(\log n)$ and worst-case $O(n^2)$.

Minimality

We get a 1-minimal test case: removing any character is guaranteed to not fail.

In the example, we see that the single-paren cases pass.

This is a local minimum: might be some other smaller test case that one would reach with different choices.

Advantages of minimality

- reduces cognitive load for the programmer: no irrelevant details, easier to understand what's happening.
- easier to communicate: "`MysteryRunner` fails on "`()`"" vs "`MysteryRunner` fails on 4100-character input (attached)"
- helps identifying duplicates (to some extent— assuming failure has a single cause).

Part IV

Reducing Inputs, but With Grammars

Motivation

Grammar-Based Reduction: GRABR

Another application of grammars.

Structured input languages,
e.g. conforming to a grammar,
may not work well with delta debugging.

Idea: Use the grammar to drive reductions.

Delta Debugging & Expressions

```
>>> expr_input = "1 + (2 * 3)"
>>> dd_reducer = DeltaDebuggingReducer(mystery, log_test=True)
>>> dd_reducer.reduce(expr_input)
```

This works, but almost all of the substrings aren't valid expressions.

```
Test #2 '2 * 3)' 6 PASS
Test #3 '1 + (' 5 PASS
...
Test #13 '()' 2 FAIL
Test #14 ')' 1 PASS
Test #15 '(' 1 PASS
```

Modelling a Program

Situation: program immediately rejects invalid expressions.

We model this with a contrived `Runner`:

```
class EvalMysteryRunner(MysteryRunner):
  def __init__(self) -> None:
      self.parser = EarleyParser(EXPR_GRAMMAR)

  def run(self, inp: str) -> Tuple[str, Outcome]:
      try:
          tree, *_ = self.parser.parse(inp)
      except SyntaxError:
          return (inp, Runner.UNRESOLVED)
      return super().run(inp)
```

i.e. we get no information from an expression that doesn't parse.

Delta Debugging Utterly Fails

```
>>> expr_input = "1 + (2 * 3)"
>>> dd_reducer = DeltaDebuggingReducer(eval_mystery, l
... dd_reducer.reduce(expr_input)
Test #1 '1 + (2 * 3)' 11 FAIL
Test #2 '2 * 3)' 6 UNRESOLVED
Test #3 '1 + (' 5 UNRESOLVED
...
Test #20 '1 + (2 * )' 10 UNRESOLVED
Test #21 '1 + (2 * 3' 10 UNRESOLVED
'1 + (2 * 3)'
```

Zero of 20 attempts to reduce parse.
Delta debugging gives us nothing.

Delta Debugging and Validity Constraints

`EvalMysteryRunner` imposed a reasonable validity constraint on inputs—an input must be a valid expression.

Invalid inputs are valuable for fuzzing, but not useful for reducing inputs.

For reducing: want to give the runner something it can usually work with.

Delta Debugging and Validity Constraints

`EvalMysteryRunner` imposed a reasonable validity constraint on inputs—an input must be a valid expression.

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Overview: Grammar-based Reduction

Main idea: reduce by replacing subtrees with smaller subtrees.

Implication: have a syntactically valid tree at all times.

Implementing Grammar-based Reduction

Two ideas:

- "Substitution by subtrees": e.g. replace an `<expr>` with a smaller `<expr>`.
- "Simplifying by replacing subtrees": e.g. replace a `<term>` which is `<factor> * <term>` by its alternative `<factor>`.

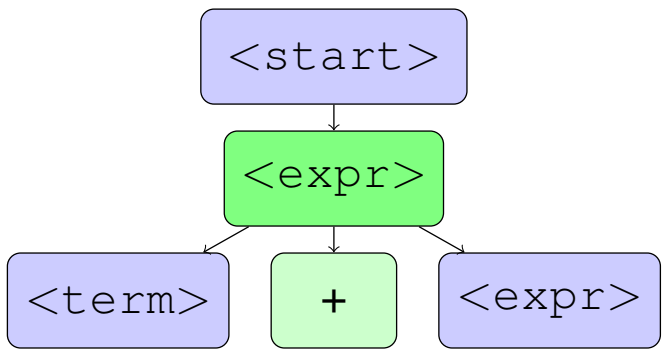
(1) Substitution by subtrees

This is a like-for-like substitution.

e.g.

$$\texttt{1 + (2 * 3)} \Rightarrow \texttt{(2 * 3)}$$

Replace the dark-green `<expr>` by smaller `<expr>`:



Substitution code

```
import copy
expr_input = "1 + (2 * 3)"
derivation_tree, *_ = EarleyParser(EXPR_GRAMMAR).parse(
                                        expr_input)
new_derivation_tree = copy.deepcopy(derivation_tree)
sub_expr_tree = new_derivation_tree[1][0][1][2]
new_derivation_tree[1][0] = sub_expr_tree
all_terminals(new_derivation_tree)
```

Other substitution by subtrees

We only substitute by subtrees already in the tree.

The two allowed substitutions for `<expr>`:

`1 + (2 * 3)` $\Rightarrow$ `(2 * 3)`

`1 + (2 * 3)` $\Rightarrow$ `2 * 3`

Arbitrary replacement of `<expr>` with non-`<expr>` likely violates the grammar.

EXPR Grammar Reminder

```
    "<term>":
        ["<factor> * <term>", "<factor> / <term>", "<factor>"],
```

(2) Simplifying by Replacing Subtrees

More complicated than substitution.
e.g. have `<term> = 2 * 3`
which is `<factor> * <term>`.

`<term>` can be just `<factor>`.
Remove the `<factor> * <term>` and put in `<factor> = 3`.

`1 + (2 * 3)` $\Rightarrow$ `1 + (3)`

Carrying out GRABR

Goal: replace derivation subtrees by smaller subtrees,
and alternatives with smaller subtrees,
thus simplifying the input.

But: which strategy—(1) or (2)—when?

About the implementation

```
class GrammarReducer(CachingReducer):
    def __init__(self, runner: Runner, parser: Parser, *,
                 log_test: bool = False, log_reduce: bool = False):
        # ...

    def subtrees_with_symbol(self, tree: DerivationTree,
                             symbol: str, depth: int = -1,
                             ignore_root: bool = True)
                    -> List[DerivationTree]:
        # straightforward tree traversal
```

Using `subtrees_with_symbol()`

`subtrees_with_symbol` is the main part of the substitution strategy and also used in simplifying.

Here's all the `<term>`s in our expression:

```
>>> expr_input = "1 + (2 * 3)"
>>> derivation_tree, *_ = EarleyParser(EXPR_GRAMMAR).parse(
                                    expr_input)
>>> grammar_reducer = GrammarReducer(
      mystery,
      EarleyParser(EXPR_GRAMMAR),
      log_reduce=True)
>>> [all_terminals(t) for t in grammar_reducer.
                                    subtrees_with_symbol(
      derivation_tree, "<term>")]
['1', '(2 * 3)', '2 * 3', '3']
```

We use it to find candidates to replace with.

How `alternate_reductions()` works

Say we have <x>, and grammar says.

$$\texttt{<x>} ::= Y \mid Z$$

Then: construct Ys and Zs using candidates we find in the tree; return the shortest Y and the shortest Z we can construct.

Using `alternate_reductions()`

Here we print all alternate reductions for `<term>`:

```
>>> grammar_reducer.try_all_combinations = True
>>> print([all_terminals(t)
       for t in grammar_reducer.alternate_reductions
                          (derivation_tree, "<term>")])
['1', '2', '3', '1 * 1', '1 * 3', '2 * 1', '2 * 3', '3 * 1', '3
                                  * 3', '(2 * 3)', '1 * 2 * 3',
                                 '2 * 2 * 3', '3 * 2 * 3', '1 *
                                 (2 * 3)', '(2 * 3) * 1', '(2 *
                                 3) * 3', '2 * (2 * 3)', '3 * (2
                                  * 3)']
```

Includes: all possible `<digit>`s in the tree, plus `<factor> * <term>`, with the `<factor>`s and `<term>`s already in the tree.

Without `try_all_combinations`

Returns the shortest `<factor>` and the shortest `<factor> * <term>`.

There is no `<factor> / <term>` in the tree.

```
>>> grammar_reducer.try_all_combinations = False
>>> print([all_terminals(t)
       for t in grammar_reducer.alternate_reductions
                          (derivation_tree, "<term>")])
['1', '1 * 1']
```

Combining Strategies

Why not both?

We collect results from substitution and simplifying, combine, and deduplicate.

```
    def symbol_reductions(self, tree: DerivationTree,
                          symbol: str,
                          depth: int = -1):
        reductions = (self.subtrees_with_symbol(tree, symbol,
                                                depth=depth)
                      + self.alternate_reductions(tree, symbol,
                                                  depth=depth))
        # Filter duplicates and put into unique_reductions
                                                  [... omitted]
        return unique_reductions
```

Running the Code: <expr>

Recall: <expr> is either "<term> + <expr>", or "<term>".

```
>>> print ([all_terminals(t) for t in grammar_reducer.
                                    subtrees_with_symbol(
                                    derivation_tree, "<expr
                                    >")])
['1 + (2 * 3)', '(2 * 3)', '2 * 3']
>>> print ([all_terminals(t) for t in grammar_reducer.
                                    alternate_reductions(
                                    derivation_tree, "<expr
                                    >")])
['1', '1 + (2 * 3)']
>>> print ([all_terminals(t) for t in grammar_reducer.
                                    symbol_reductions(
                                    derivation_tree, "<expr
                                    >")])
['1 + (2 * 3)', '(2 * 3)', '2 * 3', '1']
```

Running the Code: <term>

```
>>> print ([all_terminals(t) for t in grammar_reducer.
                                    symbol_reductions(
                                    derivation_tree, "<term>")])
['1', '(2 * 3)', '2 * 3', '3', '1 * 1']
```

The `1 * 1` is an alternate expansion for `<term>`.

Implementation structure: `reduce_subtree`

For each child:

1. collect the set of `symbol_reductions()`,
2. check that a reduction is indeed smaller,
3. replace the child with the reduction.

If the reduction fails (as desired), continue with reduced tree.

Recursively reduce children.

GRABR works

We can call the `reduce()` method, which uses some helper functions:

```
>>>  grammar_reducer = GrammarReducer(
       eval_mystery,
       EarleyParser(EXPR_GRAMMAR),
       log_test=True)
>>> grammar_reducer.reduce(expr_input)
Test #1 '(2 * 3)' 7 FAIL
Test #2 '2 * 3' 5 PASS
Test #3 '3' 1 PASS
Test #4 '2' 1 PASS
Test #5 '(3)' 3 FAIL
(3)
```

We get valid output `(3)` at the end, in 5 steps.

No `UNRESOLVED` tests along the way.

Removing less: using depth

So far: replace subtrees with smallest possible subtrees, e.g. `2 * 3` $\rightarrow$ `2` and then `3`.

Delta debugging tries to remove "sensible" chunk sizes. For trees: replace with larger subtrees rather than smaller subtrees.

Where depth comes in

Subtrees closer to root (smaller depth) are by definition bigger.

`subtrees_with_symbol()` takes `depth` parameter—only return subtrees at the given depth.

Search starting with depth 0 and increase;
hence, preferentially start with bigger subtrees.

Depth at work

```
>>>  grammar_reducer = GrammarReducer(
       eval_mystery,
       EarleyParser(EXPR_GRAMMAR),
       log_test=True)
>>> grammar_reducer.reduce_tree = grammar_reducer.
                                  reduce_tree_with_depth
>>> grammar_reducer.reduce(expr_input)
Test #1 '(2 * 3)' 7 FAIL
Test #2 '(3)' 3 FAIL
Test #3 '3' 1 PASS

'(3)'
```

Does not try subtrees `2 * 3` or single-digits.
Replaces top `<expr>` with `(2*3)` as before.
Then, replaces `<term>` by the `<factor>` "3".
`(3)` is as reduced as possible; 3 steps, not 5.

Grammar-based vs delta debugging

```
long_expr_input = GrammarFuzzer(EXPR_GRAMMAR, min_nonterminals=
                                100).fuzz()
```

Use the grammar reducer:

```
grammar_reducer = GrammarReducer(eval_mystery, EarleyParser(
                                 EXPR_GRAMMAR))
with Timer() as grammar_time:
    print(grammar_reducer.reduce(long_expr_input))
```

I ran it and it needed 9 tests, finishing in 0.033s.

Now the delta debugger:

```
dd_reducer = DeltaDebuggingReducer(eval_mystery)
with Timer() as dd_time:
    print(dd_reducer.reduce(long_expr_input))
```

This took 618 tests and took 0.503s.