Enhancing voluntary contributions in a public goods economy via a minimum individual contribution level

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Abstract

We propose and theoretically analyze a measure to encourage greater voluntary contributions to public goods. Our measure is a simple intervention that restricts individuals' strategy sets by imposing a minimum individual contribution level while still allowing for full free riding for those who do not want to contribute.

We show that for a well-chosen value of the minimum individual contribution level, this measure does not incentivize any additional free riding while strictly increasing the total contributions relative to the situation without the minimum contribution level. Our measure is appealing because it is nonintrusive and in line with the principle of "freedom of choice." It is easily implementable for many different public goods settings where more intrusive measures are less accepted. This approach has been implemented in practice in some applications, such as charities.

Keywords: Public goods, Voluntary contribution, Potential maximizer Nash equilibria, Minimum contribution level

JEL Classification: C72, H41

1 Introduction

Public goods are omnipresent in the economy and society. Such goods range from providing clean air and street lightning to ensuring national security. Due to the defining properties of nonrivalry and nonexludability, public goods tend to be underprovided without intervention because rational and selfish individuals have incentives to free ride on others' provisions (Samuelson, 1954). However, despite these incentives, there is evidence that people voluntarily contribute to public goods, sometimes in significant amounts. In many public goods settings, altruistic preferences or warm glow (Andreoni, 1989, 1990), a desire for fairness (Andreoni, 1999), or social norms (Reuben and Riedl, 2013) can mitigate free riding. Cooperation may also be sustained by the voluntary contributions of the involved individuals in the form of direct and indirect reciprocity (Rand and Nowak, 2013) or when individuals are sensitive to reputation (Akerlof, 1980; Bénabou and Tirole, 2006). Cooperation has been observed, for example, in the context of private donations to charity (Roberts, 1984; Young, 1982; Sugden, 1982), environmental attitudes (Milinski et al., 2006), or willingness to participate in medical research studies (Trauth et al., 2000).[1]

When relying on voluntary contributions only, the resulting total level of contribution often still falls short of the desired goals. Charities struggle to reach fundraising goals to achieve

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[1]We refer to Bergstrom et al. (1986), Kollock (1998) and Guido et al. (2019) for extensive reviews of the theoretical studies and applications on the topic of social dilemmas and the voluntary provision of public goods.

specific projects or initiatives (UNICEF, 2023). Similarly, the collective effort toward adopting responsible "green" attitudes, such as increasing recycling or promoting organic food, is still a long way from having a significant impact on mitigating the effects of climate change (Lee and Romero, 2023). Finally, in line with the literature in which information has been treated as a public good (McCain, 1988; Abowd and Schmutte, 2019), data collected for clinical research often fail to yield representative information about population health because of scarce contributions (Tolenen et al., 2017).

In this paper, we propose and theoretically analyze a measure to encourage greater voluntary contributions to public goods. Specifically, we investigate the effect of introducing a minimum contribution amount for those who decide to contribute, while we retain the option not to contribute at all. Such minimum contribution amounts are used in practice. For instance, charities often set a minimum donation amount (e.g., the price of a charity raffle ticket) to encourage voluntary donors to give more. By setting a minimum donation amount, they can ensure that the operational and administrative costs that arise from the processing of the donations are covered and that a significant amount is left for the cause they support. Moreover, they can reduce the administrative burden of dealing with a greater number of smaller contributions. This allows them to focus energy on increasing the donations of an existing set of engaged and committed donors and to make progress toward their goals more efficiently. However, when setting a minimum amount, charities run the risk of losing voluntary small contributions; moreover, the overall contribution may actually decrease if the minimum amount is not set optimally.

Setting a minimum contribution level can also improve the total provision of the public good in other areas where the provision is insufficient, such as global environmental issues, by encouraging greater participation in international agreements or medical research by encouraging patients to participate in feedback surveys on their treatment. With such a measure, no one is coerced to contribute, but the resulting total contribution can be larger than without a minimum contribution level. Therefore, this measure can be viewed as less intrusive than are more traditional policy measures—such as regulation, taxes or subsidies—because it leaves the liberal principle of "freedom of choice." That is, any contribution to the public good remains voluntary and is neither enforced nor punished nor rewarded.

In the literature, there are many examples of more intrusive measures available to achieve an efficient outcome. They typically use monetary incentives or other coercive methods. Monetary incentives can take the form of taxation (Clarke, 1971; Groves and Ledyard, 1977), tax-financed subsidies (Andreoni and Bergstrom, 1996), or subsidies for the contributions of other individuals (Varian, 1994). These are measures usually imposed by some central authority. More recently, it was shown that decentralized punishment of free riders can also increase voluntary contributions to public goods (Fehr and Gächter, 2000; Herrmann et al., 2008; Nikiforakis, 2008; Sigmund et al., 2001; Egas and Riedl, 2008). Moreover, voluntary contribution is observed under different kinds of assortative matching (Nax et al., 2014; Duca et al., 2018) or the implementation of a binary voting procedure for or against the provision of the public good (Ledyard and Palfrey, 1994; Tolenen et al., 2017).

Arguably, such measures may be undesirable from a liberal policy perspective because they undermine the principle of freedom of choice and might have negative political externalities in that policy-makers introducing such measures may be viewed as intrusive. The measure we propose does not have such an adverse effect. However, it is important to calibrate the minimum threshold carefully. When the threshold is too low, it may not have the desired effect, and when the threshold is too high, it may crowd out voluntary contributions of some people such that the overall effect is not beneficial.

In this paper, we investigate the effect of introducing a minimum contribution level as a measure to increase voluntary contributions in public goods economies.[2]

[2]Our proposal fundamentally differs from research on threshold public goods games (van de Kragt et al., 1983; Cadsby and Maynes, 1999; Palfrey and Rosenthal, 1984). In these studies instead, the researchers investigate the

As common in the literature (Bergstrom et al., 1986; Bramoullé and Kranton, 2007; Admati and Perry, 1991), we assume that contributions are continuous, that the profit function is concave in the total level of contributions and that the private costs of contributions are convex in individual contributions. Concavity implies a decreasing marginal benefit from consumption of the public good, while convexity of costs suggests that marginal costs are increasing, making it increasingly costly to contribute.

We analyze a public goods model in which we restrict the set of individual strategies by imposing a minimum contribution; we retain the possibility of not contributing. The restriction of the strategy set aims to encourage individuals to contribute more voluntarily. More specifically, it aims to encourage individuals who are already willing to contribute to do so more, and free riders remain free riders. The primary challenge arises from the potential for voluntary contributors to become free riders if the minimum contribution level is perceived as too high. Our results show that it is possible to set up a minimum contribution level that in equilibrium increases the total contribution while providing an equilibrium outcome where none of the individuals who contribute to the public good without the restricted strategy set have an incentive to free ride in the presence of a minimum contribution level. Specifically, there always exists a minimum contribution level such that, regardless of the possible multiplicity of Nash equilibria, there exists a unique potential maximizer equilibrium. At this equilibrium, all the contributors to the public good without the restricted strategy set still contribute. Moreover, in such an equilibrium, the total level of contribution is strictly greater than that in the game without a minimum contribution level.

It is important to stress that increasing the total contribution does not necessarily coincide with the maximization of social welfare. This issue has already been discussed in works on public goods about environmental cooperation, such as the paper by Courtois and Haeringer (2012). Similarly, our proposed solution can be interpreted as a second-best solution in terms of the social welfare of the individuals participating in the public goods game.

The remainder of this paper is structured as follows. In Section 2, we present our theoretical model. In Section 3, we first present the results of the model without the minimum contribution level; then, we investigate the game with the minimum contribution level. In the same section, we further detail the case of homogeneous contribution costs. Section 4 provides a discussion of possible applications and ways to implement our measure, and then, we draw conclusions. All proofs are relegated to the Appendix.

2 The Model

2.1 The public goods game

Our economy consists of a set $N = \{1, \dots, n\}$, $n \geq 2$ of individuals who may contribute to the provision of a public good. Each individual $i \in N$ has the same unit of wealth and contributes to the public good at a (normalized) level $\lambda_i \in [0, 1]$. We denote by $\boldsymbol{\lambda} = [\lambda_i]_{i \in N} \in [0, 1]^n$ the vector of contributions and by $G(N) = \sum_{i \in N} \lambda_i \in [0, n]$ the total contributions of the individuals of set N.

We assume that any contribution to the public good is voluntary. Individuals are endowed with utility, which is a function of the public good and of the cost of contributing to the public good. Formally, given her contribution λ_i and the contributions of all the other individuals (for which we use the standard notation $\boldsymbol{\lambda}_{-i}$), i receives utility

$$U_i(\lambda_i, \boldsymbol{\lambda}_{-i}) = h(G(N)) - p_i(\lambda_i). \tag{1}$$

impact of a minimum level for the total level of contributions or number of contributors for the public good to be provided.

The first component, $h(G(N))$, is the *public good utility*, i.e., the homogeneous utility that each individual equally experiences because of the total provision of the public good. We assume that $h : [0, n] \to \bar{\mathbb{R}}$ is twice continuously differentiable,[3] and that h is strictly increasing and strictly concave (i.e., the individuals experience positive and diminishing marginal utility from the provision of the public good). Moreover, we assume that the marginal utility becomes negligible for high contribution levels, i.e., $h'(x) \to 0$ when $x \to +\infty$. The second component $p_i : [0, 1] \to \mathbb{R}_+$ represents the *cost of contribution* of individual i. Cost functions are heterogeneous, i.e., they depend on the type of individual, and we assume that all p_is are twice continuously differentiable, nonnegative, increasing and strictly convex. Finally, we assume that for at least one $i \in N$, $h'(0) > p_i'(0)$. This assumption ensures that, in our model, at least one individual voluntarily provides a strictly positive contribution. This assumption is justified, given the extensive body of literature that illustrates how individuals frequently exhibit a voluntary inclination to contribute to public goods across various contexts (Sugden, 1982). As outlined in the introduction, our objective is to leverage this inherent inclination and to encourage non-free riders to increase their contributions.

Throughout our analysis, we assume that individuals can be ordered in such a way that an individual choosing λ has a greater marginal cost of contribution than the previous individuals choosing the same contribution level; this same ordering holds for any contribution level $\lambda \in [0, 1]$. Formally:

Assumption 1. *The costs of contribution are such that* $p_1'(\lambda) \leq \cdots \leq p_n'(\lambda)$ *for all* $\lambda \in [0, 1]$.

Given the set of individuals N and a vector of contributions $\boldsymbol{\lambda} \in [0, 1]^n$, we define the associated *social welfare* as

$$W(\boldsymbol{\lambda}) = \sum_{i \in N} U_i(\boldsymbol{\lambda}).$$

We represent individuals' strategic interaction as the noncooperative public goods game

$$\Gamma(N) = \langle N, [0, 1]^n, (U_i)_{i \in N} \rangle , \tag{2}$$

with a set of players N, strategy space $[0, 1]$ for each individual $i \in N$ and utility function U_i given by (1). We analyze the game $\Gamma(N)$ as a *complete information game* between individuals; i.e., we assume that the set of individuals, the strategy spaces and the utility functions are common knowledge. A *Nash equilibrium* (NE) in pure strategies of this game is a strategy profile $\boldsymbol{\lambda}^* \in [0, 1]^n$ satisfying

$$\lambda_i^* \in \arg\max_{\lambda_i \in [0,1]} U_i(\lambda_i, \boldsymbol{\lambda}_{-i}^*), \quad \forall i \in N.$$

Throughout our analysis, we always refer to NE (and refinements) in pure strategies, even when not explicitly specified. A NE $\boldsymbol{\lambda}^*$ is a *strict Nash equilibrium* (SNE), if, for each $i \in N$ and for each $\lambda_i \in [0, 1]$, $\lambda_i \neq \lambda_i^*$, it holds that

$$U_i(\lambda_i^*, \boldsymbol{\lambda}_{-i}^*) - U_i(\lambda_i, \boldsymbol{\lambda}_{-i}^*) > 0,$$

i.e., each individual strictly decreases her utility by deviating.

Importantly, we note that the game $\Gamma(N)$ under consideration is a potential game (Monderer and Shapley, 1996), with potential function $\Phi : [0, 1]^n \to \bar{\mathbb{R}}$, such that, for each $\boldsymbol{\lambda} \in [0, 1]^n$,

$$\Phi(\boldsymbol{\lambda}) = h(G(N)) - \sum_{j \in N} p_j(\lambda_j). \tag{3}$$

In potential games, the set of global maxima is a subset of the set of Nash equilibria and thus a refinement of the NE in terms of equilibrium prediction. Moreover, Monderer and Shapley

[3]We denote by $\bar{\mathbb{R}}$ the extended real number line $\mathbb{R} \cup \{-\infty, +\infty\}$.

(1996) argue that such a refinement is expected to accurately predict the results obtained through an experimental implementation of the model. In particular, they showed that this refinement accurately predicts the experimental results obtained by Van Huyck et al. (1990).[4] In the game under consideration, a *potential maximizer Nash equilibrium* (PMNE) is a vector $\boldsymbol{\lambda}^P \in [0,1]^n$ satisfying

$$\boldsymbol{\lambda}^P \in \arg\max_{\boldsymbol{\lambda} \in [0,1]^n} \Phi(\boldsymbol{\lambda}).$$

The potential function in (3) is concave, and the strategy sets are closed intervals of the real line. It follows that the set of Nash equilibria and the set of profiles that are maximizers of the potential function coincide. Exploiting the potential function for the game, we can perform a complete analysis of the game, and we can provide its equilibrium outcomes.

2.2 The public goods game with a minimum contribution level

We propose a variation of the public goods game $\Gamma(N)$. Our variation is simply based on a restriction of the individuals' strategy space to $\{0\} \cup [\eta, 1]$ for a given $\eta \in [0,1]$, which we call the *minimum contribution level.* As discussed in the introduction, this approach mirrors common practices in numerous applications. We investigate how to adjust the minimum contribution level to improve the total level of contribution.

To analyze the strategic interaction between individuals when introducing a minimum contribution level, we define the game $\Gamma(N,\eta) = \langle N, \big[\{0\} \cup [\eta,1]\big]^n, (U_i)_{i\in N}\rangle$, where the utility function U_i is still defined by (1) but is now restricted to the domain $\big[\{0\} \cup [\eta,1]\big]^n$. Note that the game $\Gamma(N)$ is a special case of this modified game $\Gamma(N,\eta)$ when $\eta = 0$. From now on, we suppose that $\eta \in (0,1]$. As we did for $\Gamma(N)$, we analyze the game $\Gamma(N,\eta)$ as a *complete information game* between individuals; i.e., we assume that the set of individuals, the action sets (in particular, the minimum contribution level η) and the utilities are common knowledge. A Nash equilibrium (in pure strategy) of the modified game $\Gamma(N,\eta)$ is a strategy profile $\boldsymbol{\lambda}^*(\eta) \in \big[\{0\} \cup [\eta,1]\big]^n$ satisfying

$$\lambda_i^*(\eta) \in \arg\max_{\lambda_i \in \{0\}\cup[\eta,1]} U_i(\lambda_i, \boldsymbol{\lambda}^*_{-i}(\eta)), \quad \forall i \in N.$$

The modified game $\Gamma(N,\eta)$ is a potential game with potential function Φ defined on the restricted domain $\big[\{0\} \cup [\eta,1]\big]^n$. In contrast to $\Gamma(N)$, the strategy sets of the modified game are not closed intervals of the real line. Thus, the set of global maxima of the potential function may not coincide with the set of Nash equilibria (but it still represents a refinement of it). As a consequence, we cannot directly exploit the potential function of the game to perform a complete analysis of the Nash equilibrium structure. However, given its strict concavity, the potential function Φ still has a unique global maximum when restricted to the smaller and convex domain $[\eta,1]^n$. We denote by $\boldsymbol{\lambda}^M(N,\eta) \in [\eta,1]^n$ such a maximum

$$\boldsymbol{\lambda}^M(N,\eta) = \arg\max_{\boldsymbol{\lambda} \in [\eta,1]^n} \Phi(\boldsymbol{\lambda}), \tag{4}$$

and $G^M(N,\eta)$ is the corresponding total level of contribution. In the following, we show how the vector $\boldsymbol{\lambda}^M(N,\eta)$ plays a fundamental role in the analysis of the Nash equilibrium structure of the game $\Gamma(N,\eta)$.

[4] As we discuss in the conclusions, an experimental validation of our model is the main direction for future work on the topic.

3 Results

3.1 Nash equilibria and total contribution

The game $\Gamma(N)$ has a unique NE $\boldsymbol{\lambda}^*$. Given the assumption that $h'(0) > p_i'(0)$ for at least one $i \in N$, this equilibrium is strict such that $\boldsymbol{\lambda}^* \neq (0, \dots, 0)$. That is, there exists a nonempty set of individuals who are not free rider individuals (i.e., non-free rider individuals). Moreover, if the costs of contribution satisfy Assumption 1, the equilibrium is such that $0 \leq \lambda_n^* \leq \cdots \leq \lambda_1^*$. This means that, as expected, the individual with the lowest marginal cost of contributing, namely, individual 1, contributes the most. Moreover, the contribution decreases as we progress toward individual n, who is the one with the highest marginal cost.

In the following, we use the notation $\boldsymbol{\lambda}^* = \boldsymbol{\lambda}^*(N)$ and $\lambda_i^* = \lambda_i^*(N)$ for each $i \in N$ to denote that this equilibrium depends on the specific identity of the agents in the set of individuals N (and, in particular, on their contribution costs).

Let us recall from the introduction that our purpose is to encourage non-free riders to contribute more. We note that when $\lambda_i^*(N) < 1$ for some $i \in N$, the resulting total level of contributions $G^*(N)$ in the unique Nash equilibrium falls short of the maximum possible contribution of n. We restrict our attention to those situations in which this is the case, i.e., those situations for which there is room for improvement in terms of total level of contribution.

3.2 Nash equilibria and increased total contribution with a minimum contribution level

First, enlarging the set of potential contributors to the public good may be beneficial for mitigating the underprovision problem. This approach is widely implemented in many contexts, including charitable giving campaigns, where the aim is often to reach a wider audience in the expectation of attracting a greater number of contributors. From a formal standpoint, at equilibrium, the individual contribution, the total level of contribution, the individual utility and the social welfare vary when a new individual enters the game. Given the game $\Gamma(N)$, if we assume that an additional $(n+1)$-th individual enters the game, as expected and according to the standard public goods literature,[5] the i-th individual's contribution to each $i \in N$ decreases, $\lambda_i^*(N \cup \{n+1\}) \leq \lambda_i^*(N)$. However, the total level of contribution increases, i.e., $G^*(N \cup \{n+1\}) \geq G^*(N)$, as does the individual utility and the social welfare: $U_i(\boldsymbol{\lambda}^*(N \cup \{n+1\})) \geq U_i(\boldsymbol{\lambda}^*(N))$ for all $i \in N$, and $W(\boldsymbol{\lambda}^*(N \cup \{n+1\}))) \geq W(\boldsymbol{\lambda}^*(N))$. These results hold without any ordering assumption and independently of whether the additional individual has a higher or lower marginal cost than does the remaining population.[6]

However, in many real-world applications, improving the equilibrium contribution level by enlarging the set of individuals is often infeasible, as certain objective constraints may prevent this solution. Objective constraints may be imposed by the fact that potential contributors are *a priori* limited (e.g., the set of participants involved in a given charity event is finite). Moreover, reaching a broader audience, even when feasible, can still be very costly.

Instead, we propose and illustrate a new and alternative way to increase the total contribution level while maintaining the fixed population of potential contributors.

In the following, we assume that the game $\Gamma(N)$ is such that there are no free riders. Formally:

Assumption 2. *The game $\Gamma(N)$ is such that at the unique NE $\boldsymbol{\lambda}^*$, $\lambda_i^* > 0$ for each $i \in N$.*

[5] These preliminary results, which are valid for a fairly classic nonlinear public goods model, translate into our specific context some well-known results already present in the literature (see, e.g., Bergstrom et al., 1986).

[6] However, the last three inequalities hold instead strictly under some specific assumptions; for instance, whenever the new individual is such that $h'(G^*(N \cup \{n+1\})) > p_{n+1}(\lambda_{n+1}^*(N))$ (i.e., if she is not a free rider once she enters the game) and, in particular, whenever Assumption 1 holds and $n+1$ is ranked in the ordering no higher than the last non-free rider of the original game $\Gamma(N)$. This second condition is sufficient but not necessary for the inequalities to hold strictly.

We observe that when we restrict our analysis to games that satisfy this assumption, we do not lose any generality. For any game $\Gamma(N)$ defined as in (2), we may consider an equivalent game reduced to the original non-free riders and with a utility function still defined as in (1) but up to an additive constant.[7] The non-free riders in the original game still contribute equally to the reduced game at equilibrium.

When choosing a minimum contribution level $\eta \in (\lambda_n^*, 1]$, vector $\boldsymbol{\lambda}^M(N,\eta)$ is necessarily on the boundary. [8] Then, the vector $\boldsymbol{\lambda}^M(N,\eta)$ is such that the individual with the highest marginal cost contributes exactly $\lambda_n^M(N,\eta) = \eta$. In this interval, we define the following minimum contribution level.

Definition 1. *Suppose that Assumptions 1 and 2 are satisfied. Suppose that there exists a minimum contribution level $\eta \in (\lambda_n^*, 1]$ such that the corresponding modified game $\Gamma(N,\eta)$ has a nonstrict Nash equilibrium[9] in which all the individuals are contributing. We define η^* as the smallest contribution level in $(\lambda_n^*, 1]$ at which such an equilibrium for the corresponding modified game exists. If such a minimum contribution level does not exist, we set η^* at the maximum, i.e., equal to* 1.[10]

For some choices of the minimum contribution level $\eta \in (\lambda_n^*, 1]$, there may exist multiple equilibria, and in these equilibria, some individuals may contribute zero. In Proposition 3.1, we show that, if we select a minimum contribution level that is not "too high", i.e., that is not larger than η^*, there always exists exactly one equilibrium where all the individuals are still contributing. This equilibrium coincides with the maximum of the potential function, as defined in (4).

Proposition 3.1. *Suppose that Assumptions 1 and 2 are satisfied. It holds that for any $\eta \in (0, \eta^*]$, every NE $\boldsymbol{\lambda}^*$ of $\Gamma(N,\eta)$ is such that, given $S \subseteq N$ the set of non-free riders at this equilibrium, the NE $\boldsymbol{\lambda}^*$ coincides with the unique global maximum of the potential function Φ when restricted to the individuals in S and on the domain $[\eta, 1]^s$, with $s = \mid S \mid$. In particular, $\boldsymbol{\lambda}^*(N,\eta) := \boldsymbol{\lambda}^M(N,\eta)$ is the unique NE of $\Gamma(N,\eta)$ with no free riders.*

It remains an open question whether the result in Proposition 3.1 holds for a value of η greater than this threshold, at least when subject to certain conditions.[11] In the final part of this section, we conjecture that this result never holds for a minimum contribution level larger than η^*. This reflects the fact that for excessively high minimum contribution levels, individuals no longer exhibits their inclination to contribute, as they perceive the effort of contributing as too important.

Proposition 3.1 allows us to establish the main result of this section. According to Theorem 3.2, by imposing a minimum contribution level, not only can all individuals still contribute, but it is also possible to strictly increase the total contribution when choosing $\eta \in [\lambda_n^*, \eta^*]$.

[7] The additive constant is given by the sum over i of the $p_i(0)$ with i being the original free riders. All our findings for such a reduced game may be generalized to the original game, simply by observing that when imposing a minimum contribution level, the free riders continue to use free riding. Therefore, our incentives can be calibrated on and intended for increasing the effort of individuals who were already contributors to the public good.

[8] A critical point of Φ on the domain $(\lambda_n^*, 1]^n$ is also critical on the full domain $[0,1]^n$, but we have shown that $\boldsymbol{\lambda}^*$ is the only critical point of Φ on this domain.

[9] This Nash equilibrium is such that individual n has exactly the same utility by contributing and by free riding.

[10] Formally, if there exists $\eta \in (\lambda_n^*, 1]$, such that, $h(G^M(N,\eta)) - h(G^M(N,\eta) - \eta) = p_n(\eta)$, we define the parameter η^* as the smallest $\eta \in (\lambda_n^*, 1]$ satisfying the former equation. If the condition is never fulfilled, this corresponds to the situation in which individual n is always better off by contributing than by free riding; then, $\eta^* = 1$, as $h(G^*(N,1)) - h(G^*(N,1) - 1) = h(n) - h(n-1) > p_n(1)$.

[11] However, this is not true, for example, in the special case when $|N| = 1$, for which the utility function is necessarily decreasing in $[\eta^*, 1]$. As η^* is the threshold at which the unique individual has exactly the same utility by contributing and by free riding, for each minimum contribution level strictly greater than η^*, she will be better off by free riding than by contributing.

Theorem 3.2. *Suppose that Assumptions 1 and 2 are satisfied. The total level of contribution $G^*(N, \eta)$ at equilibrium $\boldsymbol{\lambda}^*(N, \eta)$ of the modified game $\Gamma(N, \eta)$ is a nondecreasing function of $\eta \in [0, \eta^*]$ and, in particular, is strictly increasing in $[\lambda_n^*, \eta^*]$.*

It is important to recall that, without loss of generality, with Assumption 2, we considered an original game $\Gamma(N)$ without free riders. Under this assumption, we have shown that up to a certain value of the minimum contribution level, there exists a NE still without free riders in which the increase in the total contribution level is strictly positive and monotonic in the choice of parameter η. In the more general setting, this translates into the existence of a NE where all the individuals who were previously contributing do not start free riding because of the minimum contribution level. However, the introduction of a minimum contribution level may lead to the existence of multiple Nash equilibria, thereby necessitating to consider potential refinements.

3.3 Nash equilibria refinement

Even for a given minimum contribution level $\eta \in (\lambda_n^*, \eta^*]$, the uniqueness of the NE $\boldsymbol{\lambda}^*(N, \eta)$ is not assured. There may exist other suboptimal NEs with some free riders.

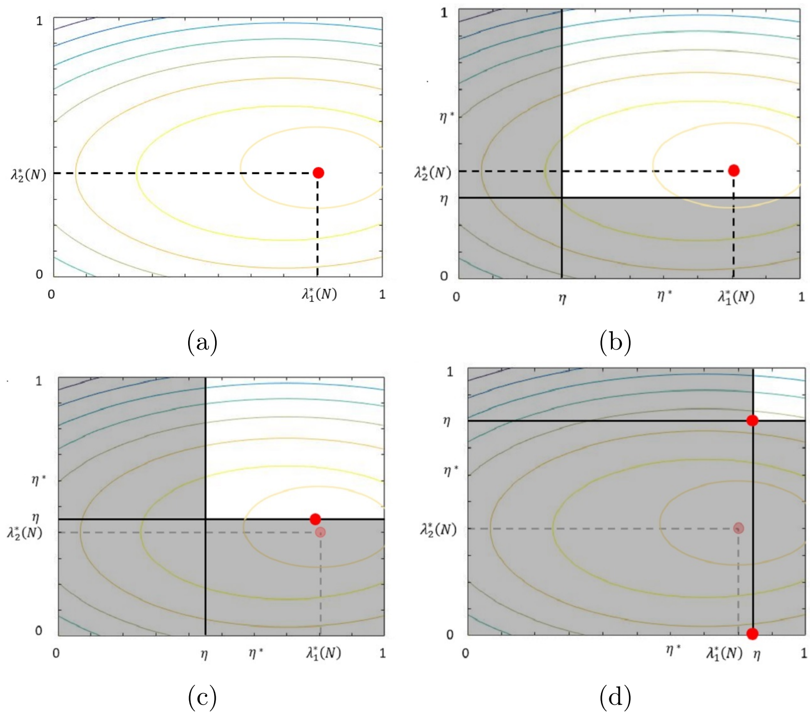

Figure 1: Level curve representation of the potential function of our public goods economy with two individuals. In (a), the original public goods game, with its unique NE (PMNE). In (b), the minimum contribution level is too low, i.e., less than λ_2^*, and the NE remains the same as that in (a). In (c), the minimum contribution level is effective but not too high; individual 2 contributes more, and as a response, individual 1 contributes less. The unique NE (PMNE) is characterized by a greater total level of contribution when compared to that of the equilibrium in (a). In (d), the minimum contribution level is too high, and the NE is no longer unique. In the figure, we highlight two different PMNEs. At the bottom equilibrium, one individual free rider is observed.

In Section 2, we have provided the definition of a Nash equilibrium refinement, the potential maximizer Nash equilibrium. The game under consideration admits such refinement because it is a potential game. Even when restricting our attention to PMNEs, uniqueness is not assured. For instance, multiple PMNEs appear when $\eta = \eta^*$. Indeed, when implementing the corresponding modified game, any strategy vector that corresponds to the PMNE, but such that one of the individuals who has the largest contribution cost deviates to zero, is still a PMNE.[12] This is because, by Definition 1, η^* is the contribution level such that not only her individual utility but also the potential function does not vary when individual n deviates. However, such a PMNE is inefficient in terms of total level of contribution, because $n-1$ individuals are contributing the same, and one individual who was contributing strictly more than zero is now free riding.

In Figure 1, we provide an intuitive graphical representation of how, for different choices of the minimum contribution level, it is possible to span through different scenarios. When the contribution level has no effect because it is too low, or when multiple PMNEs can occur because it is too high. Tuning the minimum contribution level accurately is crucial for avoiding these extreme cases. For these reasons, we investigate whether, for some choices of the minimum contribution level η, we can ensure the existence of a unique PMNE. We also investigate whether this solution coincides with the nonzero component NE of the previous subsection.

In Definition 2, we introduce a new parameter $\bar{\eta}$ that is less than or equal to η^* such that, as we see in Proposition 3.3, if we set a minimum contribution level strictly smaller than $\bar{\eta}$, the unique NE such that there are no free riders, is also the unique PMNE.

Definition 2. *Suppose that Assumptions 1 and 2 are satisfied. We define the parameter $\bar{\eta}$ as the smallest parameter in $(\lambda_n^*, \eta^*]$ at which the maximum value of the potential function remains unchanged when allowing individual n the choice of whether to contribute or not to contribute. If such a parameter does not exist, we set it as $\bar{\eta} = \eta^*$.*[13]

Proposition 3.3. *Suppose that Assumptions 1 and 2 are satisfied. For any $\eta \in (0, \bar{\eta})$, $\boldsymbol{\lambda}^*(N, \eta) = \boldsymbol{\lambda}^M(N, \eta)$ is the unique PMNE of the modified game $\Gamma(N, \eta)$.*

Proposition 3.3 enables us to adjust the minimum contribution level, thereby preventing the occurrence of situations such as those in Figure 1d, where multiple PMNEs may arise.

We provide now a concise overview of the course and key findings from our analysis:

1. In Proposition 3.1, we show that when introducing a minimum contribution level $\eta < \eta^*$, there exists a unique equilibrium such that no individual is free riding.

2. In Proposition 3.3, we show that with a minimum contribution level $\eta < \bar{\eta} \leq \eta^*$, such an equilibrium is the unique potential maximizer.

3. In Theorem 3.2, we show that with such a minimum contribution level $\eta < \bar{\eta} \leq \eta^*$, the total level of contribution at the PMNE of the modified game is strictly higher than that of the original game.

In summary, our results indicate that choosing a parameter η as close as possible to $\bar{\eta}$ is optimal for maximizing the total contribution level. Additionally, we conjecture, as it is still an open question, that for any choice of parameter exceeding η^*, the presence of free riders is inevitable. This means that there is no increase in the total contribution level.

In the next subsection, we present results on the public goods game with a minimum contribution level under the assumption of homogeneous contribution costs. In this special case, our aforementioned conjecture can be demonstrated.

[12] Formally, for each $j \in N$ such that $p'_j(\cdot) = p'_n(\cdot)$, there exists a NE $\boldsymbol{\nu}$ of $\Gamma(N, \eta^*)$ such that, $\nu_i = \lambda_i^*(N, \eta^*) = \lambda_i^*(N \setminus \{n\}, \eta^*)$ for each $i \neq j$ and $\nu_j = 0$. Such an equilibrium is such that $\Phi(\boldsymbol{\nu}) = \Phi(\boldsymbol{\lambda}^*(N, \eta^*))$.

[13] Formally, if there exists $\bar{\eta} \in (\lambda_n^*, \eta^*]$ such that $\Phi \mid_{\lambda_n = 0} (\boldsymbol{\lambda}^{M0}(N \setminus \{n\}, \eta)) = \Phi(\boldsymbol{\lambda}^M(N, \eta))$, where $\boldsymbol{\lambda}^{M0}(N \setminus \{n\}, \bar{\eta})$ is the unique global maximum of the potential function Φ defined on the restricted domain $[\bar{\eta}, 1]^{n-1} \cup \{0\}$ and $\boldsymbol{\lambda}^M(N, \bar{\eta})$ is the unique global maximum of Φ on $[\bar{\eta}, 1]^n$, we define the parameter $\bar{\eta}$ as the smallest $\bar{\eta} \in (\lambda_n^*, \eta^*]$ satisfying the former equation. Of course, for values of the minimum level of contribution in $[0, \lambda_n^*]$, the maximum value of the potential function is unique, and this condition is never satisfied.

3.4 Further Results for a Homogeneous Public Goods Game

The homogeneous case is of particular interest because it allows us to completely characterize the equilibrium structure when the minimum contribution level is high, i.e., when $\eta \in (\eta^*(n), 1]$.

Given the game $\Gamma(N)$, we now assume that individuals' utility is still defined by (1), but it is such that $p_i(\cdot) = p(\cdot)$ for each $i \in N$. When the game $\Gamma(N)$ is homogeneous, we denote it by $\Gamma(n)$, i.e., as dependent on the number of individuals and not on their identity, and similarly for all the other notation in our analysis.

In this special case, Assumptions 1 and 2 always hold. The game $\Gamma(n)$ has a unique NE $\boldsymbol{\lambda}^*(n)$ such that, $\lambda_i^*(n) = \lambda^*(n) > 0$ for each $i \in N$. In particular, the individual contribution $\lambda^*(n)$ is a nonincreasing function of the number n of individuals, such that $\lim_{n\to+\infty} \lambda^*(n) = 0$ and $G^*(n) = n\lambda^*(n)$ is an increasing function of the number n of individuals, and $\lim_{n\to+\infty} G^*(n) = +\infty$.[14] However, note that even if the total contribution goes to infinity when the number of individuals goes to infinity, it still grows slower than does the optimal total contribution n, as

$$\frac{n}{G^*(n)} = \frac{n}{n\lambda^*(n)} = \frac{1}{\lambda^*(n)} \longrightarrow +\infty.$$

Thus, while enlarging the set of potential contributors allows for an increase in the total contribution, the contribution becomes increasingly less efficient when compared to the maximum possible contribution.

As in the heterogeneous case, we now revisit the analysis that led to the results obtained in Theorem 3.2 and provide in Theorem 3.4 a detailed account of the NE structure in the homogeneous case. In particular, this allows us to prove the conjecture that when the minimum contribution level is greater than $\eta^*(n)$, in equilibrium, some individuals have an incentive to free ride.

Theorem 3.4. *Let the game $\Gamma(n)$ be homogeneous and let $\boldsymbol{\lambda}^*(n)$ be the unique Nash equilibrium of the original game $\Gamma(n)$.*

(i) *For any $\eta \in (0, \eta^*(n))$, $\Gamma(N, \eta)$ has a unique NE $\boldsymbol{\lambda}^*(n, \eta)$ such that $\lambda_i^*(n, \eta) > 0$ for each $i \in N$. This equilibrium is such that $\lambda_i^*(n, \eta) = \lambda^*(n, \eta)$ for each $i \in N$, with*

$$\lambda^*(n, \eta) = \begin{cases} \lambda^*(n) & \text{if } 0 \le \eta \le \lambda^*(n) \\ \eta & \text{if } \lambda^*(n) < \eta < \eta^*(n); \end{cases} \tag{5}$$

In particular, if $\eta \in (0, \bar{\eta}(n))$, such an equilibrium is the unique PMNE.

(ii) *For $\eta = \eta^*(n)$, $\Gamma(n, \eta^*(n))$ has at least $n + 1$ NE: An equilibrium $\boldsymbol{\lambda}^*(n, \eta^*(n))$ such that $\lambda_i^*(n, \eta^*(n)) = \eta^*(n)$ for each $i \in N$, and n equilibria $\boldsymbol{\nu}^1, \ldots, \boldsymbol{\nu}^n$ such that for each $j = 1 \ldots n$ $\nu_i^j = \eta^*(n)$ for each $i \neq j$ and $\nu_j^j = 0$. In particular, if $\boldsymbol{\lambda}^*(n, \eta^*(n))$ is a PMNE, then all $\boldsymbol{\nu}^j$ are PMNEs for each $j = 1 \ldots n$.*

(iii) *for any $\eta \in (\eta^*(n), 1]$, there does not exist a Nash equilibrium $\bar{\boldsymbol{\lambda}}$ of $\Gamma(n, \eta)$ such that $\bar{\lambda}_i > 0$ for each $i \in N$.*

Moreover, we assume that $\eta \le \eta^*(1)$, where $\eta^*(1)$ is defined as in Definition 1 but for the special case with $n = 1$, and we let m_η be such that $\eta \in (\eta^*(m_\eta + 1), \eta^*(m_\eta)]$. Then, for any profile such that m_η individuals contribute $\max(\lambda^*(m_\eta), \eta)$ and the rest give zero is a NE, there exists no NE with s individuals contributing positively to any $s > m$. Finally, if $\eta > \eta^*(1)$, the only equilibrium is $(0, \cdots, 0)$.

[14] In addition, we also observe that, in equilibrium, the individual utility and the social welfare are strictly larger when a new individual enters the game.

Theorem 3.4-*(i)* presents results analogous to those of the heterogeneous case for values of the minimum contribution level $\eta \in (0, \eta^*(n))$. For the homogeneous case, Theorem 3.4-*(ii)* shows that for $\eta = \eta^*(n)$, we may have multiple PMNEs. Finally, Theorem 3.4-*(iii)* proves our conjecture true in the homogeneous case. This means that when $\eta \in (\eta^*(n), 1]$, the presence of free riders is inevitable.[15]

Finally, we observe that the parameter $\eta^*(n)$ is a nonincreasing function of the number n of individuals, a decreasing function when $\eta^*(n) < 1$, and it approaches zero for an infinitely large number of individuals, as shown in $\lim_{n\to+\infty} \eta^*(n) = 0$. It follows that the parameter $\bar{\eta}(n)$ goes to zero when the number of individuals goes to infinity.

Like what we observed for the homogeneous case without a minimum contribution level, the total contribution still grows slower than does the optimal total contribution n, as it holds that

$$\frac{n}{n\lambda^*(n, \bar{\eta}(n))} = \frac{n}{n\bar{\eta}(n)} = \frac{1}{\bar{\eta}(n)} \longrightarrow +\infty.$$

However, the improvement can still be very significant.

4 Discussion and conclusions

In this paper, we propose a method to enhance voluntary contributions in a public goods game. This approach involves limiting individuals strategy set by implementing a minimum contribution level while permitting free riding. We show that this measure can strictly increase the total level of contributions at a unique potential maximizer Nash equilibrium, while not leading to any additional free riding.

In practice, measures that impose some form of minimum contribution are applied in areas such as charitable giving, in which voluntary benefactors are asked to contribute a minimum amount. We formalize the rationale behind such practices and analyse a methodology to guarantee efficacy. In particular, our theoretical results provide an operational way to choose the optimal minimum contribution level. Intuitively, our results show that setting the minimum contribution level too high may result in free riding and a decrease in the total contribution level relative to the situation without the minimum contribution level. Therefore, a minimum contribution level that is too high may explain excessive free riding. Our proposed measure proves valuable when other forms of measures (e.g., monetary) are not feasible. Importantly, our measure can be applied to a much broader set of public goods problems. In the following, we illustrate a few examples of how our results can be applied in different scenarios.

Our main illustration involves donations to charities, where the requirement of a minimum contribution level is already a common practice. Our findings indicate that a careful examination of past donation patterns is crucial in determining an appropriate minimum contribution level. This threshold is then set just above the smallest contributions, gradually increasing it until some initial contributors start free riding.

A second example is cooperation in the context of environmental protection. Historically, many effective measures in this domain have been of a monetary nature (Marron and Toder, 2014). Alternatively, environmental protection efforts, such as jointly deciding on the level of abatement, have focused on engaging actors in a collective agreement before action is taken by anyone (in the form of an *ex-ante* institution formation). These measures often take the form of multistage games and require, in an earlier stage, the formalization of a binding agreement between voluntary actors, who thus commit to adhering to the rules of the agreement in subsequent stages. In Courtois and Haeringer (2012), in the first stage, actors independently choose

[15]Formally, given a value of $\eta \in (\eta^*(m_\eta + 1), \eta^*(m_\eta)]$, one can at best obtain m_η individuals contributing $\eta^*(m_\eta)$ at a NE. That is, the maximal total contribution at a NE is $m\eta^*(m)$ for some $m \leq n$. Hence, in all cases where $m\eta^*(m)$ is an increasing function of m, one cannot improve in terms of total public good contribution by setting $\eta > \eta^*(n)$.

whether to ratify the agreement, thus committing to respecting a minimum contribution level in the form of an abatement level in the second stage. Similarly, in Kosfeld et al. (2009), in the first stage, actors independently choose whether to participate in an organization. The organization becomes effective if and only if all participants unanimously vote for its realization in a second phase, thus committing to providing the full endowment in the third and final phase. However, one may question the feasibility of such *ex-ante* agreements for contributions. For instance, some actors may be unwilling to make such commitments for political reasons because they do not want to openly align themselves with other actors. This is often the case for international environmental treaties between countries (see, e.g., Paris Agreement UN, 2015).[16] With the measure proposed in this paper, an actor (country) can be certified as involved in the agreement if the contribution is above a settled minimum contribution threshold, but this contribution is fully voluntary (thus not binding) and independent of the other actors' choice.

Data collection, e.g., in medical research, serves as another example. This is not a prototypical example of a public goods problem. Rather, data collection and the resulting information that can be derived play a fundamental societal role that can be naturally represented as a public good (Chessa et al., 2015b,a). Contributing personal data can be perceived as costly because one has to give up privacy, but the aggregate information inferred from these contributions can be beneficial to society, e.g., in terms of advancing medical research. However, individual partial contributions, when some questions in a survey are perceived as too intrusive with respect to privacy, can result in missing data, posing a problem that is prevalent in various types of data analyses. For a robust data analysis, it is necessary to minimize the number of unobserved variables. Our measure suggests a way to accurately calibrate the minimum information required for an individual to participate in a survey. When the requirements are not too demanding, volunteers are likely to accept this minimum contribution. Conversely, an excessively intrusive and tedious questionnaire characterized by a poorly calibrated and overly high minimum contribution level may result in a very low participation rate.

A natural follow-up of this paper involves validating and complementing the theoretical results through an experimental study. Historically, experimental economics has provided a platform for validating or questioning numerous theoretical findings concerning methods to enhance contribution levels in public goods (see, e.g., the paper by Bohm, 1972; Marwell and Ames, 1979; Holt and Laury, 1997; Zelmer, 2003; Lugovskyy et al., 2017, among others). On the one hand, experimental analysis has sustained an ethically based rule of behavior (McGinty and Milam, 2013), according to which individuals, in many settings, voluntarily contribute to public goods without intervention. Importantly, in the experimental literature, it has also been demonstrated that one cannot depend solely on voluntary contributors (Ledyard, 1995), as various factors can act as deterrents. In many settings, for example, contributions may tend to decline over time (Burlando and Guala, 2005; Oliveira et al., 2015).

Close to our setting, within the body of experimental literature, the impact of different strategy sets on public goods contributions has already been investigated. In this literature, the focus has, for example, been on whether the size of the action set can affect cooperation (Gangadharan and Nikiforakis, 2009) or on whether there is an effect of granularity (i.e., the degree of divisibility of the space of feasible contribution options) on participation and public goods provision (Arlegi et al., 2021). Some researchers have also focused on the effect of an all-or-nothing contribution in nonlinear public goods games (Zhang et al., 2013).[17]

In our particular case, experimental analysis may deepen our understanding by accounting for psychological factors. For example, contributors who seek social approval (Bursztyn and Jensen, 2017) may be more motivated by the presence of a minimum contribution level, and

[16] Iran, Libya, Yemen and Eritrea have not ratified the agreement. The United States withdrew from the agreement in 2020 but rejoined in 2021.

[17] All-or-nothing contributions have also been studied in other kinds of games, such as trust games (Schniter et al., 2015).

therefore, they may choose to contribute at the minimum level over free riding. In the case of donations to a charitable organization, a small contribution avoids being classified as a free rider and, consequently, as completely indifferent to the problem. When a minimum contribution level is reached, individuals may, however, prefer such a higher contribution rather than tarnish their social image. This psychological effect may even further push the theoretical threshold represented by the minimum contribution level. Furthermore, the presence of this threshold may be perceived by individuals as a form of "donation tiers," that has a similar effect as a default option. In the experimental literature, it has already been demonstrated that subjects are hesitant to deviate from a default option (Liu and Riyanto, 2017), and this effect can play a significant role in fostering cooperation. However, importantly, this may also have an adversarial effect, as the default option may induce those individuals who are willing to contribute more to a lower contribution closer to the minimum contribution level.

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A Proofs

A.1 Proof of the results of Section 3.1

Game $\Gamma(N)$ is a potential game with a concave potential function Φ (defined as in (3)), where the strategy sets are closed intervals of the real line. It follows that a strategy profile is a Nash equilibrium if and only if it maximizes the potential function. As the potential function Φ is strictly concave on the convex set on which it is defined, it has a unique global maximum $\boldsymbol{\lambda}^* \in [0,1]^n$ that coincides with the unique Nash equilibrium of $\Gamma(N)$. Such an equilibrium is strict. In particular, the equilibrium $\boldsymbol{\lambda}^*$ is such that for each $i \in N$, λ_i^* satisfies the following KKT conditions

$$\begin{cases} h'(G^*(N)) - p_i'(\lambda_i^*) + \psi_i^* - \phi_i^* = 0 \\ \psi_i^* \lambda_i^* = 0 \quad \phi_i^*(\lambda_i^* - 1) = 0, \quad \psi_i^*, \phi_i^* \geq 0, \end{cases} \tag{6}$$

where $G^*(N)$ denotes the total contribution level at equilibrium. We may observe by (6) that, owing to the assumption that $h'(0) > p_i'(0)$ for at least one $i \in N$, this equilibrium is $\boldsymbol{\lambda}^* \neq (0, \ldots, 0)$.

As the term $h'(G^*(N))$ is the same for each $i \in N$, it immediately follows that, if the p_is are such that $p_1'(\lambda) \leq ... \leq p_n'(\lambda)$, for each $\lambda \in [0, 1]$, then $\lambda_n^* \leq \ldots \leq \lambda_1^*$.

A.2 Proof of the results at the beginning of Section 3.2

Given the game $\Gamma(N)$, we suppose that an additional $(n+1)$-th individual enters the game.

First, to show that the i-th individual's contribution at equilibrium for each $i \in N$ is such that $\lambda_i^*(N \cup \{n+1\}) \leq \lambda_i^*(N)$, we suppose by contradiction that there exists $i \in N$ such that $0 \leq \lambda_i^*(N) < \lambda_i^*(N \cup \{n+1\}) \leq 1$. Because of the strict convexity of p_i,

$$p_i'(\lambda_i^*(N)) < p_i'(\lambda_i^*(N \cup \{n+1\})),$$

and, from the KKT conditions in (6), and as $\lambda_i^*(N) < 1$ and $\lambda_i^*(N \cup \{n+1\}) > 0$,

$$h'(G^*(N)) \leq p_i'(\lambda_i^*(N)) < p_i'(\lambda_i^*(N \cup \{n+1\})) \leq h'(G^*(N \cup \{n+1\})).$$

From the first and from the last term and because of the concavity of h, it follows that

$$G^*(N) > G^*(N \cup \{n+1\}). \tag{7}$$

If the total contribution without individual $n+1$ is strictly greater than that after her entrance, this implies that there exists at least one individual $j \in N$, which verifies the opposite inequality compared to i, i.e., such that $1 \geq \lambda_j^*(N) > \lambda_j^*(N \cup \{n+1\}) \geq 0$. Following the same reasoning as before, we conclude that $G^*(N) < G^*(N \cup \{n+1\})$, which contradicts (7).

Second, we show that the total level of contribution is such that $G^*(N \cup \{n+1\}) \geq G^*(N)$. We know that $\lambda_i^*(N \cup \{n+1\}) \leq \lambda_i^*(N)$ for each $i \in N$. In particular, if the equality holds for each $i \in N$, the thesis follows trivially, as $G^*(N \cup \{n+1\}) = G^*(N) + \lambda_{n+1}^*(N \cup \{n+1\})$, with $\lambda_{n+1}^*(N \cup \{n+1\}) \geq 0$. In contrast, if there exists $i \in N$ such that $\lambda_i^*(N \cup \{n+1\}) < \lambda_i^*(N)$, following the same reasoning as before, then we can conclude that the inequality holds strictly.

Finally, we show that individual utility and social welfare are $U_i(\boldsymbol{\lambda}^*(N \cup \{n+1\})) \geq U_i(\boldsymbol{\lambda}^*(N))$ for all $i \in N$ and $W(\boldsymbol{\lambda}^*(N \cup \{n+1\}))) \geq W(\boldsymbol{\lambda}^*(N))$, respectively. It is sufficient to observe that at equilibrium,

$$\begin{aligned} U_i^*(N \cup \{n+1\}) &= h(G^*(N \cup \{n+1\})) - p_i(\lambda_i^*(N \cup \{n+1\})) \\ &\geq h(G^*(N)) - p_i(\lambda_i^*(N)) \\ &= U_i^*(N). \end{aligned}$$

A.3 Proof of Proposition 3.1

We observe that the modified game $\Gamma(N, \eta)$ is still a potential game with potential function Φ but defined in the restricted domain $\left[\{0\} \cup [\eta, 1]\right]^n$. In contrast to $\Gamma(N)$, the strategy sets of the modified game are not closed intervals of the real line, and the equilibria may not coincide with the global maxima of the potential function.

For a better reading of the following of the proof, we present it organized in 5 steps.

Step 1 Given a subset of the individuals $S \subseteq N$, with $s = |S|$, and for each $\eta \in [0, 1]$, we define a new modified game $\Gamma'(S,\eta) = \langle S, [\eta,1]^s, (U_i)_{i\in S}\rangle$, where the utility function U_i is defined by (1) for each individual $i \in S$, but it is now restricted to the domain $[\eta, 1]$. $\Gamma'(S,\eta)$ is still a potential game, with potential function Φ, defined on the restricted domain $[\eta,1]^s$. As for the original game $\Gamma(N)$ and the modified game $\Gamma(N,\eta)$, we also observe that the game $\Gamma'(S,\eta)$ is a potential game with a concave potential function and where similar to $\Gamma(N)$ but different from $\Gamma(N,\eta)$, the strategy sets are closed intervals of the real line. It follows that a strategy profile is a Nash equilibrium if and only if it maximizes the potential function. Moreover, the potential function is strictly concave on the convex set on which it is defined; then, it has a unique global maximum $\boldsymbol{\lambda}^M(S,\eta) \in [\eta,1]^s$, which coincides with the unique Nash equilibrium of $\Gamma'(S,\eta)$. In particular, for each $\eta \in (0,1]$, $\boldsymbol{\lambda}^M(N,\eta)$ is the unique Nash equilibrium of $\Gamma'(N,\eta)$.

Step 2 We define η^* as in Definition 1. We show that $\boldsymbol{\lambda}^M(N,\eta^*)$ is a Nash equilibrium of $\Gamma(N,\eta^*)$. First, we observe that no individual has incentives to deviate to a quantity in $[\eta^*,1]$ because $\boldsymbol{\lambda}^M(N,\eta^*)$ is a Nash equilibrium of the game $\Gamma'(N,\eta^*)$. It remains to be shown that no individual has incentives to go to zero. Individual n does not have incentives by Definition 1. For any other individual $i \neq n$, such that $\lambda_i^M(N,\eta^*) = \eta^*$, if agent n, who is the most privacy concerned, does not have incentives to deviate from η^*, that is still valid for i. For any other agent $i \neq n$, such that $\lambda_i^M(N,\eta^*) > \eta^*$, as i does not have incentives to deviate to η^*, then, because of the concavity of the utility function, she cannot have incentives to deviate to 0.

Step 3 We observe that for each $\eta \in [0,\eta^*)$, $\boldsymbol{\lambda}^M(N,\eta)$ is a Nash equilibrium of $\Gamma(N,\eta)$. Moreover, when $\eta \in (\lambda_n^*,\eta^*)$, we can repeat the same reasoning as in Step 3, with the only difference being that individual n (and any other individual contributing η) is always strictly better off by contributing rather than free riding, and then, the inequality is always strict.

Step 4 We observe that for any $\eta \in (0,\eta^*]$, $\boldsymbol{\lambda}^M(N,\eta)$ is the unique Nash equilibrium of $\Gamma(N,\eta)$ such that each individual has a nonzero contribution, i.e., such that $\lambda_i^M(N,\eta) > 0$ for each $i \in N$. To show that, it is sufficient to observe that an equilibrium of $\Gamma(N,\eta)$ such that each individual has a nonzero contribution, is also an equilibrium of the game $\Gamma'(N,\eta)$ (as if an individual does not have incentives to deviate in $\{0\} \cup [\eta,1]$, she does not have incentives to deviate in the restricted strategy set $[\eta,1]$ either), and the equilibrium of $\Gamma'(N,\eta)$ is unique.

Step 5 We show that any other NE $\bar{\boldsymbol{\lambda}}$ of $\Gamma(N,\eta)$ is such that $\bar{\lambda}_i = \lambda_i^M(S,\eta)$ for each $i \in S$, where $S = \{i \in N \mid \bar{\lambda}_i \neq 0\}$. Let $\eta \in (\lambda_n^*,\eta^*]$, and we let $\bar{\boldsymbol{\lambda}} = \bar{\boldsymbol{\lambda}}(N)$ be a Nash equilibrium of $\Gamma(N,\eta)$ such that $\bar{\lambda}_i(N) = 0$ for at least one $i \in N$. We define S as the set of individuals who contribute nonzero values at this equilibrium. As, by definition, of NE, none of the individuals has incentives to deviate, and, in particular, none of the individuals in S has incentives to deviate in $[\eta,1]$. It follows that $\bar{\boldsymbol{\lambda}}(S)$, i.e., the vector of the nonzero elements of the vector $\bar{\boldsymbol{\lambda}}(N)$, is a Nash equilibrium of the game $\Gamma'(S,\eta)$ defined in step 2 of this proof. As we have seen, this equilibrium is unique and coincides with the unique Nash equilibrium $\boldsymbol{\lambda}^M(S,\eta)$ of the game $\Gamma(S,\eta)$ such that no individual is free riding. Trivially, we can conclude that $\boldsymbol{\lambda}^*(N,\eta)$ is the unique NE of $\Gamma(N,\eta)$ s.t., $\lambda_i^*(N,\eta) > 0$ for all $i \in N$.

A.4 Proof of Theorem 3.2

Let us recall that we assumed that the equilibrium contribution $\boldsymbol{\lambda}^*(N)$ of $\Gamma(N)$ is such that $\lambda_n^*(N) < 1$, i.e., it is such that the total level of contribution is not optimal.

When $\eta \in [0,\lambda_n^*(N)]$, $\boldsymbol{\lambda}^*(N,\eta) = \boldsymbol{\lambda}^*(N)$; consequently, G^* is constant, i.e., $G^*(N,\eta) = G^*(N)$.

When $\eta \in (\lambda_n^*(N),\eta^*]$, individual n contributes to equilibrium $\lambda_n^*(N,\eta) = \eta > \lambda_n^*(N)$. We now show that for $\eta^* \geq \eta_2 > \eta_1 \geq \lambda_n^*$, $G^*(N,\eta_2) > G^*(N,\eta_1)$. By contradiction, we assume that $G^*(N,\eta_2) \leq G^*(N,\eta_1)$. It follows that

$$h'(G^*(N,\eta_2)) \geq h'(G^*(N,\eta_1)), \tag{8}$$

because of the concavity of the public good utility function h. Moreover, as the total level of contribution with η_2 is less than or equal to the level of contribution with η_1, we know that individual n has a strictly larger level of contribution; it follows that there exists $i \in N$ who contributes strictly less, i.e., such that,

$$\eta_2 \leq \lambda_i^*(N, \eta_2) < \lambda_i^*(N, \eta_1) \leq 1. \tag{9}$$

By equation (9), because of the convexity of the cost of contribution function p_i and from the KKT conditions for the potential function of the modified game $\Gamma^*(N, \eta)$, it follows that

$$h'(G^*(N, \eta_2)) \leq p_i'(\lambda_i^*(N, \eta_2)) < p_i'(\lambda_i^*(N, \eta_1)) \leq h'(G^*(N, \eta_1))$$

and this contradicts equation (8).

A.5 Proof of the results at the beginning of Section 3.3

First, we observe that

$$h(G^*(N, \eta^*)) - p_j(\lambda_j^*(N, \eta^*)) \geq h(G^*(N, \eta^*) - \lambda_j^*(N, \eta^*)) - p_j(0) \quad \forall j \in N \tag{10}$$

because, as shown in Proposition 3.1, $\boldsymbol{\lambda}^*(N, \eta^*)$ is a NE of $\Gamma(N, \eta^*)$, and thus, no individual has incentives to go to zero. Now, we suppose by contradiction that the vector $\boldsymbol{\nu}$ such that, $\nu_i = \lambda_i^*(N, \eta^*) = \lambda_i^*(N \setminus \{n\}, \eta^*)$ for each $i \neq n$ and $\nu_n = 0$ is not a NE of $\Gamma(N, \eta^*)$. This means that there exists an individual $j \in N \setminus \{n\}$ for whom it is convenient to deviate, i.e., such that, it holds that

$$h(G^*(N, \eta^*) - \eta^*) - p_j(\lambda_j^*(N, \eta^*)) < h(G^*(N, \eta^*) - \eta^* - \lambda_j^*(N, \eta^*)) - p_j(0). \tag{11}$$

From Equations (10) and (11), it follows that

$$\begin{aligned} &h(G^*(N, \eta^*)) - h(G^*(N, \eta^*) - \lambda_j^*(N, \eta^*)) \geq p_j(\lambda_j^*(N, \eta^*)) - p_j(0) \\ &> h(G^*(N, \eta^*) - \eta^*) - h(G^*(N, \eta^*) - \eta^* - \lambda_j^*(N, \eta^*)) \end{aligned}$$

This cannot hold because of the concavity of h. It follows that $\boldsymbol{\nu}$ is a NE of $\Gamma(N, \eta^*)$.

Second, we observe that $\boldsymbol{\nu}$ provides the same value of the potential function as $\boldsymbol{\lambda}^*(N, \eta^*)$. Then, if the second one is a potential maximizer, the first one is as well.

A.6 Proof of Proposition 3.3

Let $\bar{\eta}$ be defined as in Definition 2, $\eta \in (\lambda_n^*, \bar{\eta})$, and $\boldsymbol{\lambda}^*(N, \eta) = \boldsymbol{\lambda}^M(N, \eta)$ be the unique nonzero component Nash equilibrium of the modified game $\Gamma(N, \eta)$, which is the unique maximizer of Φ on $[\eta, 1]^n$. As $\eta < \bar{\eta}$, it follows that

$$\Phi(\boldsymbol{\lambda}^M(N, \eta)) > \Phi\mid_{\lambda_n=0} (\boldsymbol{\lambda}^{M0}(N \setminus \{n\}), \eta). \tag{12}$$

As $\boldsymbol{\lambda}^{M0}(N \setminus \{n\})$ is the potential maximizer of function Φ restricted to domain $[\bar{\eta}, 1]^{n-1} \cup \{0\}$, it also holds that

$$\Phi\mid_{\lambda_n=0} (\boldsymbol{\lambda}^{M0}(N \setminus \{n\}), \eta) \geq \Phi\mid_{\lambda_i=0, \forall i \in N \setminus S} (\boldsymbol{\lambda}^{M0}(S, \eta)). \tag{13}$$

(12) and (13) show that $\boldsymbol{\lambda}^*(N, \eta) = \boldsymbol{\lambda}^M(N, \eta)$ is the unique PMNE of the modified game $\Gamma(N, \eta)$.

A.7 Proof of the results of Section 3.4

When the game $\Gamma(N)$ is homogeneous, the potential function Φ is a symmetric function on a symmetric domain. As a consequence, the unique maximum is also symmetric, i.e., $\lambda_i^* = \lambda^*$ for each $i \in N$. In particular, as the equilibrium vector is a nonzero vector, we have $\lambda^* > 0$.

From Equation (6), recalling that we are now in a homogeneous case, we observe that $\lambda^* > 0$ is the unique solution of the following fixed-point problem

$$\lambda = g(n, \lambda), \tag{14}$$

where the function $g : \mathbb{N}_+ \times (0, 1] \to [0, +\infty]$ is defined for each $\lambda \in (0, 1]$ and for each $n \in \mathbb{N}_+$ as

$$g(n, \lambda) = \min\left\{(p')^{-1}(h'(n\lambda)), 1\right\}. \tag{15}$$

We consider the problem with the parameter n defined on the real interval $[1, +\infty]$. For each $n \in [1, +\infty]$, g is continuous in λ. Moreover, the function g is monotonic and nonincreasing in n. Indeed,

$$\frac{\partial g}{\partial n} = \frac{\lambda}{p''((p')^{-1}(h'(n\lambda)))} h''(n\lambda) < 0$$

for each λ internal solution, and g is identically 1 otherwise. Applying Corollary 1 of Milgrom and Roberts (1994), the unique fixed point $\lambda^*(n) > 0$ is nonincreasing in n; consequently, $\lim_{n\to+\infty} \lambda^*(n) \geq 0$ is well defined.

Second, for each $\lambda > 0$,

$$\lim_{n\to+\infty} g(n, \lambda) = 0$$

pointwise. Indeed, $\lim_{x\to+\infty} h'(x) = 0$, $p'(0) \geq 0$ and p' are strictly monotonic.

If we suppose by contradiction that $\lim_{n\to+\infty} \lambda^*(n) = a > 0$, then, from Equation (14), it follows that

$$0 < \lim_{n\to+\infty} \lambda^*(n) = \lim_{n\to+\infty} g(n, \lambda^*(n)) = \lim_{n\to+\infty} g(n, a) = 0.$$

Now, we know that $G^*(n+1) \geq G^*(n)$ for each $n \in N$. In particular, we know that the inequality holds strictly at equilibrium in the homogeneous case where any new individuals are not free riders. We suppose by contradiction that $\lim_{n\to+\infty} G^*(n) < +\infty$. It follows that $\lim_{n\to+\infty} h'(n\lambda^*(n)) > 0$; then, by (15), the solution to the fixed-point problem is such that $\lim_{n\to+\infty} \lambda^*(n) > 0$, which contradicts the fact that we already know that this limit goes to zero.

A.8 Proof of Theorem 3.4

For each $\eta \in (0, 1]$, let $\boldsymbol{\lambda}^M(n, \eta)$ be the unique global maximum of Φ on $[\eta, 1]^n$. When the individuals are homogeneous, the potential function Φ is a symmetric function that we are maximizing on a symmetric domain. As a consequence, the maximum is $\lambda_i^M(n, \eta) = \lambda^M(n, \eta)$ for each $i \in N$, with

$$\lambda^M(n, \eta) = \begin{cases} \lambda^*(n) & \text{if } 0 < \eta \leq \lambda^*(n) \\ \eta & \text{if } \lambda^*(n) < \eta \leq 1. \end{cases} \tag{16}$$

(i) For each $\eta \in (0, \eta^*(n)]$, where $\eta^*(n)$ is defined as in Definition 1, $\boldsymbol{\lambda}^*(n, \eta) = \boldsymbol{\lambda}^M(n, \eta)$ is the unique NE of the modified game $\Gamma(n, \eta)$ such that no individual is free riding. As a particular case of Proposition 3.3, for each $\eta \in (0, \bar{\eta}(n)]$, where $\bar{\eta}(n)$ is defined as in Definition 2, $\boldsymbol{\lambda}^*(n, \eta)$ is the unique PMNE of the modified game $\Gamma(n, \eta)$.

(ii) This result follows Definition 1 for the homogeneous case.

(iii) For each $\eta \in (\eta^*(n), 1]$, we show that there does not exist a NE $\bar{\boldsymbol{\lambda}}$ of $\Gamma(n, \eta)$ such that $\bar{\lambda}_i > 0$ for each $i \in N$. We assume that $\eta^*(n) < 1$. First, we observe that, with reasoning similar to that in the proof of Proposition 3.1, A.3-Step 5, whenever we have a nonzero component NE of $\Gamma(n, \eta)$, this has to be a NE of the corresponding game Γ' and then a maximum of the potential function Φ restricted to the domain $[\eta, 1]^n$. Because of this symmetry, the only possible candidate nonzero component NE is the vector $\bar{\boldsymbol{\lambda}} = \boldsymbol{\eta} = (\eta, \dots, \eta)$. Second, we observe that by definition, $\eta^*(n)$ is the smallest solution of the following fixed-point problem:

$$p(\eta) = h(n\eta) - h((n-1)\eta) \tag{17}$$

or, equivalently, of

$$\frac{p(\eta)}{\eta} = \frac{h(n\eta) - h((n-1)\eta)}{\eta} \tag{18}$$

where n is a fixed parameter. Then, we show that $\eta^*(n)$ is, in fact, the unique solution of (18). Indeed, because of the concavity of h, we have that

$$\frac{h(y) - h(x)}{y - x}$$

is a decreasing function both in y and in x; in particular, the right term of (18) is decreasing in η. Moreover, because of the convexity of p, we have

$$\frac{p(y) - p(x)}{y - x}$$

is an increasing function both in y and in x; in particular, the left term of (18) is increasing in η (when $y = \eta$, $x = 0$ and $p(0)$ are constants). It follows that the fixed-point problem in (18) has at most one solution.

In particular, applying Corollary 1 of Milgrom and Roberts (1994), the unique (i.e., smallest) fixed point $\eta^*(n) > 0$ is nonincreasing in n. Moreover, if we assume that $\eta^*(n) = \eta^*(n-1) = \eta < 1$, we obtain from (17) that

$$h(n\eta) - h((n-1)\eta) = h((n-1)\eta) - h((n-2)\eta),$$

which contradicts the fact that h is strictly concave. Hence, we conclude that $\eta^*(n)$ is strictly decreasing in n.

Third, we know that $\boldsymbol{\lambda}^*(n)$ is an SNE of $\Gamma(n)$; i.e., we know that $p(\lambda^*(n)) < h(n\lambda^*(n)) - h((n-1)\lambda^*(n))$. Moreover, we observe that $\eta^*(n)$ verifies the equality by definition when $\eta^*(n) < 1$. As the solution of the previous fixed-point problem is unique when it exists, it follows that

$$p(\eta) > h(n\eta) - h((n-1)\eta) \tag{19}$$

for each $\eta \in (\eta^*(n), 1]$. Equation (19) translates to the fact that when individuals adopt the strategy $\boldsymbol{\eta} = (\eta, \dots, \eta)$, with $\eta \in (\eta^*(n), 1]$, each individual in N is strictly better able to go to zero, and then, the only candidate nonzero component NE cannot be a NE.

Now, we assume that $\eta \le \eta^*(1)$ and let $m_\eta \in \{1, \cdots, n-1\}$ be the integer such that $\eta \in (\eta^*(m_\eta + 1), \eta^*(m_\eta)]$. Such an integer exists and is unique because $\eta^*(n)$ is strictly decreasing in n.

Let us note that by definition of m_η, we see that it is possible that $\eta^*(m_\eta) = 1$ (in which case $\eta^*(m) = 1$ for all $m \le m_\eta$ but those values of m never correspond to any $m_{\eta'}$ for any η'); however, we necessarily have $\eta^*(m_\eta + 1) < 1$.

We assume that there exists a NE $\bar{\boldsymbol{\lambda}}$ of $\Gamma(N, \eta)$ where s individuals in set S contribute positively, i.e., $\bar{\lambda}_i > 0$ for all $i \in S$ and $\bar{\lambda}_i^* = 0$ for all $i \in N \setminus S$, where $s = |S|$. First, following arguments from the proof of Proposition 3.1, $(\bar{\lambda}_i)_{i \in S}$ is equal to $\boldsymbol{\lambda}^M(S, \eta)$ which is symmetric. We denote by $\bar{\lambda}$ the value of $\bar{\lambda}_i$ for $i \in S$ and we have $\bar{\lambda} = \lambda^*(s)$ is $\eta < \lambda^*(s)$ and $\bar{\lambda} = \eta$ otherwise.

Next, we observe that we must have $s \le m_\eta$. Indeed, we assume that $s > m_\eta$. Then, $\lambda^*(s) \le \eta^*(s) \le \eta^*(m_\eta + 1) < \eta$, so that $\bar{\lambda} = \eta$. In that case, an individual $i \in S$ switching from $\bar{\lambda}$ to zero contribution gains $p(\bar{\lambda}) - \big(h(s\bar{\lambda}) - h((s-1)\bar{\lambda})\big)$, which is strictly positive since, by using arguments similar to the ones introduced above, we can show:

$$\begin{aligned}
\frac{p(\bar{\lambda})}{\bar{\lambda}} = \frac{p(\eta)}{\eta} &> \frac{p(\eta^*(m+1))}{\eta^*(m+1)} \\
&= \frac{h((m+1)\eta^*(m+1)) - h(m\eta^*(m+1))}{\eta^*(m+1)} \\
&\ge \frac{h(s\eta^*(m+1)) - h((s-1)\eta^*(m+1))}{\eta^*(m+1)} \\
&\ge \frac{h(s\bar{\lambda}) - h((s-1)\bar{\lambda})}{\bar{\lambda}}.
\end{aligned}$$

This contradicts the fact that $\bar{\boldsymbol{\lambda}}$ is a NE.

We now show that for $s = m_\eta$, the profile $\bar{\boldsymbol{\lambda}}$ with $\bar{\lambda}_i = \max(\lambda^*(s), \eta) \in (\eta^*(m_\eta + 1), \eta^*(m_\eta)]$ for all $i \in S$ is indeed a NE. First, let us note that, by definition of $\eta^*(m_\eta + 1)$, individuals outside S (i.e., given zero) have no incentive to deviate. Indeed, if $i \in N \setminus S$ deviates to $\lambda' \ge \eta > \eta^*(m_\eta + 1)$, the gain is

$$\begin{aligned}
h((s+1)\lambda') - h(s\lambda') - p(\lambda') &= \lambda' \frac{h((s+1)\lambda') - h(s\lambda')}{\lambda'} - \lambda' \frac{p(\lambda')}{\lambda'} \\
&\le \lambda' \frac{h((s+1)\eta^*(m_\eta + 1)) - h(s\eta^*(m_\eta + 1))}{\eta^*(m_\eta + 1)} + \\
&\quad - \lambda' \frac{p(\eta^*(m_\eta + 1))}{\eta^*(m_\eta + 1)} \\
&= 0,
\end{aligned}$$

Here, we again use the convexity of p and concavity of h, as derived above. Similarly, individuals in S have no incentive to deviate to zero. Hence, $\bar{\boldsymbol{\lambda}}$ is a NE.

A.9 Proof of the additional results of Section 3.4

It remains to be observed that, because of the monotonicity, $\lim_{n \to +\infty} \eta^*(n) \ge 0$ is well defined. Indeed, $\lim_{x \to +\infty} h(nx) - h((n-1)x) = 0$, $p(0) \ge 0$ and p are strictly monotonic. If we suppose by contradiction that $\lim_{n \to +\infty} \eta^*(n) = a > 0$, then, from Equation (17), it follows that

$$\begin{aligned}
0 &< p(a) \\
&= \lim_{n \to +\infty} p(\eta^*(n)) \\
&= \lim_{n \to +\infty} h(n\eta^*(n)) - h((n-1)\eta^*(n)) \\
&= \lim_{n \to +\infty} h(na) - h((n-1)a) \\
&= 0.
\end{aligned}$$