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Dissecting demand response mechanisms: The role of consumption forecasts and personalized offers

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ABSTRACT

Demand-Response (DR) programs, whereby users of an electricity network are encouraged by economic incentives to re-arrange their consumption in order to reduce production costs, are envisioned to be a key feature of the smart grid paradigm. Several recent works proposed DR mechanisms and used analytical models to derive optimal incentives. Most of these works, however, rely on a macroscopic description of the population that does not model individual choices of users.

In this paper, we conduct a detailed analysis of those models and we argue that the macroscopic descriptions hide important assumptions that can jeopardize the mechanisms' implementation (such as the ability to make personalized offers and to perfectly estimate the demand that is moved from a timeslot to another). Then, we start from a microscopic description that explicitly models each user's decision. We introduce four DR mechanisms with various assumptions on the provider's capabilities. Contrarily to previous studies, we find that the optimization problems that result from our mechanisms are complex and can be solved numerically only through a heuristic. We present numerical simulations that compare the different mechanisms and their sensitivity to forecast errors. At a high level, our results show that the performance of DR mechanisms under reasonable assumptions on the provider's capabilities are significantly lower than those suggested by previous studies, but that the gap reduces when the population's flexibility increases.

1. Introduction

Demand Response (DR hereinafter) programs are envisioned to be a key feature of the Smart Grid paradigm [1]. By means of economic incentives (discounts or penalties), DR schemes encourage users to rearrange their consumption in response to the network state, thus mitigating the grid overload and driving wholesale prices down.

Several analytical models are available in the literature, which describe and quantify the effects of DR mechanisms. Whatever their specifics are, these schemes need to model how users react to the incentives. Ideally the models should capture the most realistic features of a practical DR mechanism while maintaining tractability.

Among these contributions, the authors of [2] study how an energy provider should select time-dependent discounts to minimize its production costs. They assume that the percentage of users who shift their consumption from slot i to slot j is a decreasing function of the temporal distance between slots i and j and a concave and increasing function of the discount offered in slot j (R_j), *independent* from discounts in other slots. The paper claims that, under these assumptions along with the requirement of piecewise linearity of energy production costs, the problem of finding the set of discounts that minimize the provider's cost is convex and therefore simple to solve. Under similar modeling assumptions, however, we find that the optimization problem can be non-convex even in such a simple scenario (see Section 4). The same user's model as in [2] is adopted also in [3], where the optimization problem is extended in order to account for battery storages and distributed renewable sources available into a specific microgrid. Authors of [4] propose a day ahead pricing scheme which maximizes the provider's profitability and capacity utilization. Users are assumed to reschedule their consumption by comparing the utility v_i they get by scheduling a task in each timeslot i; therefore they allocate their consumption proportionally to these utilities,

i.e., they consume a fraction $\frac{v_i}{\sum_{j=1}^{T} v_j}$ of their total energy demand in timeslot i. The resulting optimization problem is non convex but some relaxation techniques are introduced, which allow one to calculate a solution within a reasonable amount of time. In [5], a more realistic model is proposed where each user first calculates the welfare (defined as utility minus time-dependent cost) she gets from consuming electricity in each of the possible timeslots, and then allocates all the consumption to the slot returning the largest welfare. As we show below (see Section 4.4) this model can lead to a much more complex optimization problem than the one presented in [5]. Finally, the authors of [6] propose a full-fledged game theoretical model, but their results hold only if users experience a large number of interactions without any change in the system.

We claim that these studies rely on too strong assumptions, which jeopardize their usability for practical purposes. Interestingly, we observe that the assumptions are sometimes hidden in the macroscopic models the papers start from. In particular in this paper we focus on [2] and show that its model requires personalized offers and a very precise forecast of the baseline consumption of each user. The implementation of these features may require potentially significant costs in terms of communication, measurement and computation infrastructure. Besides highlighting these implicit requirements in the analytical framework in [2] (and then also in [3]), we explore their potentials considering four DR mechanisms with different levels of complexity:

1. the *base* mechanism corresponds to an optimization problem similar to the one considered in [2], it requires personalized offers and individual consumption forecasts; the energy production cost is optimized over the discount values, each of which is offered to a given fraction of the population,

2. the *optimized* mechanism takes full advantage of personalized offers and consumption forecasts by minimizing the cost over both the discount values and the population fractions to which the discounts are offered,

3. the *robust* mechanism relies on personalized offers, but does not need individual consumption forecasts,

4. finally the *broadcast* mechanism (analogous to that in [5]) needs neither of the two features.

Interestingly, contrarily to prior studies, we find that the cost-minimization problems resulting from our DR mechanisms are not convex (even for the base mechanism). Nevertheless, simple heuristics can identify (potential) minima in a reasonable amount of time in realistic scenarios. Then, our numerical results show that the simpler robust and broadcast mechanisms achieve significantly lower cost reductions than the optimized mechanism, which is difficult to implement, but that the gap reduces when the population's flexibility increases.

The paper is organized as follows. In Section 2 we discuss how the macroscopic models considered in [2–4] hide some implicit assumptions about the user rationality or about the interactions between the provider and the user. We define our microscopic model in Section 3 and then describe different DR mechanisms and their corresponding optimization problems in Section 4. We evaluate their performance numerically in a realistic scenario in Section 5. Finally in Section 6 we discuss how our models can be tuned and which other psychological and social insights should be taken into account to explain users' decisions.

2. Pitfalls when starting from macroscopic models

In this section, we describe in more detail the macroscopic models proposed in the literature for day-ahead price optimization.

Consider a finite time horizon discretized in a set $\mathcal{T}$ of N timeslots and a large population $\mathcal{S}$ of users. The baseline aggregate energy consumption in slot j is denoted by E_j^0.

The energy provider charges a flat rate B, but it can offer discount rates to incentivize the users to move some of their consumption so as to reduce the energy production cost. Due to consumption shifts, the actual aggregate consumption in time slot j is E_j^1. Observe that a usual assumption in the literature (including the papers mentioned above) is that the introduction of a DR scheme neither reduces nor increases users' demand; it merely rearranges users' consumption in a more cost effective way, so that

$$\sum_{j=1}^{N} E_j^0 = \sum_{j=1}^{N} E_j^1. \qquad (1)$$

We denote the amount of consumption shifted from slot j to slot $i \neq j$ as $E_{j \to i}$, and the amount of consumption the users refuse to shift away from j as $E_{j \to j}$. Then we have

$$E_i^1 = E_i^0 + \sum_{z=1}^{N} E_{z \to i} - \sum_{k=1}^{N} E_{i \to k}.$$

We now start to further detail the model considering some specific assumptions made in previous works. In [2] and [3], the electricity provider offers an energy price discount $R_i \geq 0$ in each slot i. The users are assumed to react to these incentives by shifting a fraction of their baseline consumption from slot j to slot i ($|j - i|$ slots away) according to the following formula:

$$E_{j \to i} = E_j^0 S_j(R_i, |j - i|). \qquad (2)$$

$S_j(R_i, |j - i|)$ is called the aggregate sensitivity function and is increasing in the discount R_i and decreasing in the temporal shift $|j - i|$, in order to take into account the user discomfort.

The provider selects the vector of discounts $\mathbf{R}$ in order to minimize its total cost, equal to the sum of the electricity generation costs and the loss of revenues due to the discounts. In particular the optimization problem considered in [2] is the following:

$$\min_{\mathbf{R}} \sum_i \sum_{j \neq i} R_i E_{j \to i} + \sum_i c_i\left(E_i^1\right) \qquad (3)$$

$$\text{s.t. } 0 \leq R_i \leq B \quad \forall i = 1, \ldots, N, \qquad (4)$$

where $c_i(\cdot)$ is the cost of electricity production in slot i. Eq. (4) guarantees that discounts $\mathbf{R}$ are non negative and smaller than the flat rate B, so that the money stream goes toward the provider.

As it often happens, the devil is hidden in the details, and in this case in Eqs. (2) and (3). Our first remark is that the cost of lost revenues $\sum_i \sum_{j \neq i} R_i E_{j \to i}$ in Eq. (3) implicitly assumes the possibility to reward only the consumption actually shifted from j to i, i.e., $E_{j \to i}$, but this quantity cannot be directly measured. The actual consumption E_i^1 can be measured, and then $E_{j \to i}$ can be quantified provided that we have good estimates of the sensitivity function $S_j(R_i, |j - i|)$ and of the baseline consumption E_i^0. Let us assume for a moment that $S_j(R_i, |j-i|)$ is known from historical data and that the aggregate baseline consumption may be predicted with a reasonably high level of accuracy on a large set of users. Then it seems possible to solve the macroscopic problem in Eqs. (3) and (4), but we need to consider also what should happen at the microscopic scale. While the estimates for the aggregate baseline consumption can be adequately precise, finally the billing is done at the user's granularity and each user expects to receive the price discount corresponding to the energy consumption she actually moved. If the energy bill's reduction does not correspond to her forecast, the user is likely to opt out of the program (in particular if she has experienced underpayments) or to reduce her efforts and

milk occasional discounts. It appears then that Eq. (3) implicitly requires very precise predictions of *individual* consumptions.

We now observe that the form of the sensitivity function $S_j(R_i, |j - i|)$ in Eq. (2) indicates that the amount of energy shifted from j to i depends on the discount R_i but not on the other discounts. We can then ask ourselves which individual decisions may lead to this aggregate behavior, an issue ignored both in [2] and [3]. As long as a rational individual is offered two different discounts R_i and R_k, it seems natural that her decision to move some consumption from j to i or from j to k or to keep it in j will take into account both the discounts. To stress the point, consider a case when both $S_j(R_i, |j - i|)$ and $S_j(R_k, |j - k|)$ are positive, but moving the consumption from i to k is both less inconvenient (i.e., $|j - k| < |j - i|$) and more rewarding (i.e., $R_k > R_i$). There is then no reason why the user would move consumption to i. The conclusion is the same for all the users and then we should have $E_{j \to i} = 0$ at the aggregate level, in contradiction with Eq. (2). We can then conclude that the expression of the sensitivity function in Eq. (2) is not suited to model the situation when a user is offered two or more rewards, but it can capture the case when the user decides between moving from j to i in exchange of a discount R_i or staying in j. If every user is offered a single discount to move to a given slot, but different users can receive different offers, then Eq. (2) can reasonably describe the macroscopic effect of such personalized offers. The details are described in Section 4.1, here we only highlight that Eq. (2) requires then that the electricity provider (i) calculates an offer for each user, (ii) communicates individually to the user, (iii) considers the individual offer when billing the user. This is clearly more demanding than simply advertising to the whole population the same set of discounts.

We observe that the equivalent sensitivity function considered in [4] poses similar problems. Using our notation, we have $E_{j \to i} = \frac{v_i}{\sum_{k \in \mathcal{T}} v_k} E_j^0$, where v_i is the net utility a user gets by consuming electricity in slot i and can be a function of the timeslot itself and of the discount R_i. This formula tries to capture the effect of the whole set of discounts, but it is not clear again what is the underlying user's model: if slot i has a larger utility than slot k ($v_i > v_k$), why should the user consume in k?

3. Starting from a microscopic model

In the previous section we made the point that, while aggregate population models may be convenient, it is necessary to explicitly consider the microscopic level: how the user takes the decisions and how the provider and the user are supposed to interact. In this paper we follow the opposite path in comparison to the existing works mentioned: we move from the microscopic level to the macroscopic one. In particular, in this section, we start from a clear model of rationality for the single user and then move to describe how aggregate quantities can be derived.

Each user $u \in \mathcal{S}$ has a baseline energy consumption

$$\{e_j^{0u}\}_{j=1,\dots,N}, \tag{5}$$

that leads to the aggregate baseline consumption $E_j^0 = \sum_{u \in \mathcal{S}} e_j^{0u}$ for $j = 1, \dots, N$. This baseline consumption corresponds to the consumptions without any incentive mechanism and is completely arbitrary. We assume in what follows that users are homogeneous, i.e.,

$$\forall u, \quad e_j^{0u} = e_j^0 \quad j = 1, \dots, N. \tag{6}$$

In Section 5.1 we show how the DR mechanisms perform when this assumption does not hold.

User u is characterized by a private type $\underline{D}^u = \{d_{j \to i}^u\}_{j,i=1,\dots,N}$ where $d_{j \to i}^u$ indicates the discomfort due to shifting one unit of consumption from timeslot j to timeslot i. We assume that discomforts

are expressed in monetary units; and that, $\forall u \in \mathcal{S}$,

$$d_{j \to j}^u = 0 \quad \text{and} \quad d_{j \to i}^u > 0, \quad \forall j, i \neq j, \tag{7}$$

i.e., there is a strictly positive discomfort if and only if consumption is shifted from its original timeslot. The provider does not know the private type $\underline{D}^u$ of each user u: from its point of view, each discomfort $d_{j \to i}^u$ is drawn from a known, continuous distribution $F_{j \to i}$ on $[0, \alpha_{j,i}]$ (where possibly $\alpha_{j,i} = +\infty$). Discomforts of distinct users are independent but note that we do not assume that, for a given user, the discomforts $\{d_{j \to i}^u\}_{j,i=1,\dots,N}$ are mutually independent.

3.1. Rational users

We assume that a user simply chooses the option that maximizes her utility. In particular let $\mathcal{T}_j^u$ be a set of timeslots the user could move the baseline consumption e_j^{0u} to in exchange for different discounts $\mathcal{R}_j^u = \{R_{j \to k}^u \geq 0, k \in \mathcal{T}_j^u\}$. The set pair $(\mathcal{T}_j^u, \mathcal{R}_j^u)$ defines the offer user u receives for timeslot j. The set of options includes the possibility to keep the consumption in j, i.e., $j \in \mathcal{T}_j^u$. A rational user maximizes her utility by scheduling her consumption e_j^{0u} to a timeslot

$$i^* \in \underset{k \in \mathcal{T}_j^u}{\operatorname{argmax}} \left\{ R_{j \to k}^u - d_{j \to k}^u \right\}. \tag{8}$$

We assume that if two or more timeslots are equally palatable, the whole consumption is shifted to only one of them, picked at random with equal probability.

3.2. Aggregation

We observe that the quantities $d_{j \to k}^u$ in Eq. (8) are random variables, then two different users could take different decisions while confronted with the same offers. The aggregate consumption E_i^1, for $i \in \mathcal{T}$, would then be a random variable. Here we assume (as it is implicit in the other works) that we always work with large sets of the population so that the variability can be neglected by approximating actual random quantities with their expected values. In particular, if a subset $\mathcal{Q}$ containing a fraction q of the population receives an offer $(\mathcal{T}_j, \mathcal{R}_j)$, the corresponding consumption shifted from j to a time slot i, denoted as $E_{j \to i}^{\mathcal{Q}}$ will be

$$E_{j \to i}^{\mathcal{Q}} = q E_j^0 \operatorname{Prob} \left(i \in \underset{k \in \mathcal{T}_j}{\operatorname{argmax}} \left\{ R_{j \to k}^u - d_{j \to k}^u \right\} \right), \tag{9}$$

if the probability that a user has two or more equally palatable timeslots is zero. When discomforts are continuous random variables (as we consider in this paper), this is always the case if each user receives only one offer (the first three mechanisms introduced below) or if the discomforts $\{d_{j \to i}^u\}_{j,i=1,\dots,N}$ are mutually independent. In Section 4.4, we discuss how Eq. (9) should be modified if this probability is not zero. We denote $\operatorname{Prob} \left(i \in \operatorname{argmax}_{k \in \mathcal{T}_j} \left\{ R_{j \to k} - d_{j \to k}^u \right\} \right)$ simply as $P_{j \to i}(\mathcal{R}_j^u, \mathcal{T}_j)$.

4. DR mechanisms

Under different assumptions on the provider's capabilities, we introduce different demand response mechanisms based on the microscopic model above, which are therefore practically implementable. We introduce and study the corresponding optimization problems. We start by the base mechanism that leads to the same aggregate optimization problem considered in [2,3].

4.1. Base mechanism

This mechanism requires that the energy provider can manage personalized offers to its customers and moreover that it has

perfect knowledge (or very precise estimates) of the baseline consumption of each user.

The population is segmented into N^2 disjoint subsets $\mathcal{Q}_{j\to i}$, for $j, i \in \mathcal{T}$, respectively including a fixed fraction $q_{j\to i}$ of the population. Each user in $\mathcal{Q}_{j\to i}$ is simply offered to move her baseline consumption in slot j (e_j^0) to slot i in exchange for a price discount R_i.

The total consumption that is shifted from j to i is then

$$E_{j\to i} = q_{j\to i} E_j^0 \, \text{Prob}\left(R_i - d_{j\to i}^u > 0\right)$$

as it can be obtained from Eq. (9), taking into account that in this case $\mathcal{T}_j = \{j, i\}$ and $R_j - d_{j\to j}^u = 0$. We observe that the probability appearing on the right-hand side only depends on the reward R_i and on the random variable $d_{j\to i}^u$. If the discomfort is only a function of the temporal distance $|j - i|$, then the sensitivity function (the ratio of people who move from j to i) has the same properties than in [2], in particular:

$$S_j(R_i, |i - j|) = q_{j\to i} P_{j\to i}(R_i),$$

where for $P_{j\to i}(\cdot)$ we have made explicit the only variable it depends from.

As we discussed in Section 2, because the provider knows exactly the consumption shifted from each user, it can formulate the optimization problem (3)–(4). In [2] it is stated that the problem is convex if (i) the productions costs $c_j(\cdot)$ are continuous piecewise linear and increasing and (ii) the discomfort distributions $F_{j\to i}(\cdot)$ are continuous and concave. We show in Appendix A that this is not the case by providing a counterexample. Stronger hypotheses are required for the problem to be concave, as for example the linearity of the discomfort functions.

In particular in [2] the numerical evaluation considers

$$S_j(R_i, |i - j|) = \frac{1}{\sum_{k=1}^{N} \frac{1}{(|k-j|+1)}} \frac{R_i}{B \cdot (|i - j| + 1)},$$

that leads us to consider

$$q_{j\to i} = \frac{\frac{1}{|i-j|+1}}{\sum_{k=0}^{N} \frac{1}{|k-j|+1}}, \quad P_{j\to i}(R_i) = \frac{R_i}{B}. \tag{10}$$

This particular expression for $P_{j\to i}$ can be obtained if $d_{j\to i}^u$ is a uniform random variable with support in $[0, B]$. The numerical results for the base mechanism in Section 5 are obtained considering the same expression for the fractions $q_{j\to i}$.

Due to the non-convexity of the optimization problem (3)–(4) we cannot use one of the classic algorithms for convex optimization. For the results shown in Section 5 we have adopted instead a multi start approach: we have generated random starting points uniformly distributed in the problem domain and we have run per each point a descendent algorithm which converged on a local minimum; the optimal offers are therefore those returning the smallest cost among these minimizers. This approach does not guarantee convergence to the global optimum but its reliability can be improved by increasing the number of starting points.

4.2. Optimized mechanism

We have now understood which DR mechanism can lead to the optimization problem (3), but now that we look at its implementation at the microscopic level and the need for personalized offers, some specifics of the base mechanism look arbitrary and unjustified. For example, given that discounts are not broadcast but each user receives an individual offer, why should the discounts offered to the two disjoint sets of users $\mathcal{Q}_{j\to i}$ and $\mathcal{Q}_{k\to i}$ be equal to the same value R_i? It is clear that the energy provider can further reduce the cost if it can independently choose $R_{j\to i}$ and $R_{k\to i}$. Moreover, there is no reason to think that the size of the sets $\{\mathcal{Q}_{j\to i}\}$ should be fixed, the fractions $\{q_{j\to i}\}$ can also be optimization variables.

We allow the provider to take advantage of these additional degrees of freedom that – we repeat – do not impose any additional requirement to the system. We call this new DR mechanism optimized. The load $E_{j\to i}(R_{j\to i}, q_{j\to i})$ rescheduled from j to i is now

$$E_{j\to i}(R_{j\to i}, q_{j\to i}) = q_{j\to i} P_{j\to i}(R_{j\to i}) E_j^0 \tag{11}$$

and the cost minimization problem becomes:

$$\min_{\mathbf{R},\mathbf{q}} \quad \text{cost}_{\text{opt.}}(\mathbf{R}, \mathbf{q}) = \sum_i \sum_{z \neq i} R_{z\to i} E_{z\to i} + \sum_i c_i(E_i^1) \tag{12}$$

$$\text{s.t.} \qquad 0 \leq R_{z\to i} \leq B \quad \forall z, i = 1, \dots, N \tag{13}$$

$$0 \leq q_{z\to i} \leq 1, \quad z, i = 1, \dots, N \tag{14}$$

$$\sum_i q_{z\to i} \leq 1, \quad \forall z = 1 \dots N. \tag{15}$$

Eq. (13) guarantees that discounts $\mathbf{R}$ are non negative and smaller than the flat rate B, Eq. (15) is a consequence of the fact that each user receives at most one offer for its baseline consumption in a given slot.

The optimization problem (12)–(15) can be solved with the same heuristic proposed for problem (3)–(4).

4.3. Robust mechanism

The optimization problems (3)–(4) and (12)–(15) assume that the provider has perfect knowledge of each user's baseline consumption, so that it can correctly identify the consumption shifted and reduce accordingly the energy bill. This assumption is probably unrealistic. If the provider does not have such capability, then it can offer the user a discount for all the consumption in a given timeslot i and not just for the consumption moved to i. The population is then divided into N subsets $\mathcal{Q}_i$, each containing a fraction q_i of the users. All users in $\mathcal{Q}_i$ receive one and only one offer: they are encouraged to shift their consumption from any timeslot in the time horizon to timeslot i and they get the discount R_i for all the electricity consumed in i, including the one originally in i.

We call this scheme robust, because it does not rely on estimates of individual consumption. It is clearly simpler than the previous two, because the provider needs only to measure the amount of consumption in i for the users who got the offer and to bill them accordingly.

The load $E_{j\to i}(R_i, q_i)$ shifted from j to i is $E_{j\to i}(R_i, q_i) = q_i P_{j\to i}(R_i) E_j^0$. Note that users in $\mathcal{Q}_i$ have no interest to move their baseline consumption away from i, then $E_{i\to i} = q_i E_i^0$. The robust mechanisms leads to the following optimization problem:

$$\min_{\mathbf{R},\mathbf{q}} \quad \text{cost}_{\text{rob.}}(\mathbf{R}, \mathbf{q}) = \sum_{i,z} R_i E_{z\to i} + \sum_i c_i(E_i^1) \tag{16}$$

$$\text{s.t.} \qquad 0 \leq R_i \leq B \quad i = 1, \dots, N \tag{17}$$

$$0 \leq q_i \leq 1, \quad i = 1, \dots, N \tag{18}$$

$$\sum_{i=1}^{N} q_i \leq 1. \tag{19}$$

Note that in Eq. (16) the first sum includes also $E_{i\to i}$ because all the final consumption in i from the users in $\mathcal{Q}_i$ is paid at a discounted price. The term does not appear in Eqs. (3) and (12).

The optimization problem (16)–(19) can be solved with the same heuristic proposed for problem (3)–(4).

4.4. Broadcast mechanism

In the three mechanisms introduced above, the provider makes personalized offers to users in selected fractions of the population. This may not always be possible (due to the complexity it

introduces for instance in billing) or desirable (for perceived fairness issues). Our last mechanism, which is the simplest (in its definition), does not assume personalized offers. The provider selects a single vector **R** of discounts for every time slot and broadcasts these discounts to all users (hence the name broadcast mechanism). Users then re-arrange their demand and pay the discounted price for their demand in each slot (hence this mechanism also does not rely on the need to estimate shifted demand).

As explained in Section 3.1, each individual user moves her demand from slot j to a slot (potentially j itself) that maximizes her net utility (discount minus discomfort). Recall that if several slots give equal net utility, the user chooses one of them randomly.

Until now, we have not made any assumption on the possible correlation of a given user's discomforts. This is because, in the previous three mechanisms, each user was receiving only one offer. In the broadcast mechanism, each user has several offers to compare to decide on his new demand schedule, we therefore need to describe the discomfort correlations.

Let us consider now the particular case when two slots, say h and k may appear equally attractive to a user, i.e., $R_h - d_{j \to h} = R_k - d_{j \to k}$. If we assumed that, for each user, the discomforts $\{d_{j \to i}\}_{\{i,j=1,\dots,N\}}$ were mutually independent, this event would have probability zero according to our assumption on $F_{i \to j}$, and therefore it would not appear at the aggregate level. As a result, the aggregate demand moved from j to i would be

$$E_{j \to i}(\mathbf{R}) = P_{j \to i}(\mathbf{R})E_j^0, \tag{20}$$

where

$$P_{j \to i}(\mathbf{R}) = \Pr\left(R_i - d_{j \to i} \geq \max_{k \neq i}\left\{R_k - d_{j \to k}\right\}\right). \tag{21}$$

However, rather than making the above independence assumption, we prefer to assume that the discomforts have the form $d_{j \to i} = \beta_j|i-j|^{t_j}$, where t_j is a constant independent of the user and β_j is a random variable with concave Cumulative Distribution Function (CDF) $F_j(\cdot)$. This model describes a symmetric delay sensitivity of users (users are indifferent between moving two hours earlier or two hours later) while keeping the flexibility of users having a different flexibility of demand of different times (since β and t are indexed by the origin timeslot j); but it also introduces correlations between the discomforts of a user. As a result, the fraction of demand shifting from j to i is

$$P_{j \to i}(\mathbf{R}) = \frac{\Pr\left(R_i - d_{j \to i} \geq \max_{k \neq i}\left\{R_k - d_{j \to k}\right\}\right)}{1 + \mathbf{1}_{R_{2j-i}=R_i}}, \tag{22}$$

rather than (21). The denominator in (22) accounts for cases when a slot other than i (which has to be $2j - i$) gives equal net utility *for all users*. The broadcast mechanism then leads to the following optimization problem:

$$\min_{\mathbf{R}} \quad \text{cost}_{\text{brd.}}(\mathbf{R}) = \sum_z \sum_i R_i E_{z \to i} + \sum_i c_i\left(E_i^1\right) \tag{23}$$

$$\text{s.t.} \quad 0 \leq R_i \leq B \quad \forall i = 1, \dots, N. \tag{24}$$

Unfortunately, due to indicator function in Eq. (22), the cost function (23) of the broadcast mechanism is not continuous, even in very simple scenarios with continuous production costs; see Appendix B for details. Discontinuity arises also in the macroscopic model in [5], but it seems to have been ignored.

In practice, we solve problem (23)–(24) using the same heuristic proposed for the previous problems, but we work on a continuous and smooth approximation of the cost function.

4.5. Ranking DR mechanisms

Under certain assumptions, the DR mechanisms introduced above can be partially ranked in terms of the cost savings they generate.

Proposition 4.1. *Assume that the discomforts have the form $d_{j \to i} = \beta_j|i-j|^{t_j}$, where t_j is a constant independent of the user and β_j is a random variable with concave CDF $F_j(\cdot)$; and assume that the production costs $c_j(\cdot)$ are piecewise linear, continuous and increasing. Then, for a given initial demand E^0, the final cost generated by the optimized scheme is smaller than the cost of the robust and base mechanism.*

The proof of Proposition 4.1 is provided in Appendix C. As we show in Appendix C, the ranking cannot be extended further. In particular, the broadcast and robust mechanisms cannot be compared (one or the other is more efficient depending on the scenario).

5. Numerical results

In this section we evaluate the performance of the different DR mechanisms in the realistic scenario considered in [2] and based on energy data about the Ontario province in Canada. We extracted from [2] the baseline consumption $\mathbf{E_0}$, reported in Fig. 1, the flat rate $B = 110\$/\text{MWh}$ and the timeslot-independent cost function $c(\cdot)$ is piecewise linear with derivative:

$$c'(E) = \begin{cases} \$10 & E \leq C_1, \\ \$72.46 & E \in (C_1, C_2), \\ \$91 & E \geq C_2, \end{cases} \tag{25}$$

where

$$\begin{cases} C_1 = 16.3 \text{ GWh}, \\ C_2 = 17.9 \text{ GWh}, \end{cases} \tag{26}$$

represent respectively the base to intermediate load capacity and intermediate to peak load capacity. Baseline consumption $\mathbf{E_0}$ and the cost functions are estimated from the IESO energy portfolio [7], consisting of nuclear plans, hydro gas powered stations and renewable and from typical costs associated to these energy sources. We assume that discomforts take the form:

$$d_{j \to i} = \beta_j|i-j|, \tag{27}$$

where β_j is an exponential random variable with CDF

$$F_j(\beta) = 1 - e^{-\frac{\beta}{\mu}}, \tag{28}$$

where μ is a parameter representing the population's flexibility. The larger it is, the smaller (in a stochastic order sense) the discomfort of the users to shift their consumption.

In Figs. 2 and 3 we respectively show final load distributions $\mathbf{E_1}$ and optimal discounts and segment sizes of the four DR mechanisms, for the flexibility parameter $\mu = \frac{1}{10}$. Since production costs $c(\cdot)$ are constant across the horizon, provider's costs are cut by flattening the load; this is mostly clear in the optimized scheme where the peak of 19 GWh in $\mathbf{E_0}$ is reduced to 17.9 GWh which is exactly the intermediate to peak capacity C_2, as can be seen in Fig. 2.

In Fig. 4, we report the cost savings of the DR schemes, normalized to the initial cost, for four different values of the μ parameters: $\frac{1}{10}, \frac{1}{6}, \frac{1}{3}, 1$. The dashed line represents the saving which could be achieved if users' demand could be rearranged at the provider's will without providing any discount (we indicate it as the dictatorial solution). Consistently with Proposition 4.1, the optimized mechanism returns larger savings than the robust and the base ones. Interestingly, the robust mechanism performs consistently better than the base one despite the fact that it does not require the ability to estimate the demand shifted and it therefore "wastes" some discount by giving it to demand that was already scheduled in a given timeslot in the baseline demand. Moreover, as the population flexibility increases, the savings gap between the
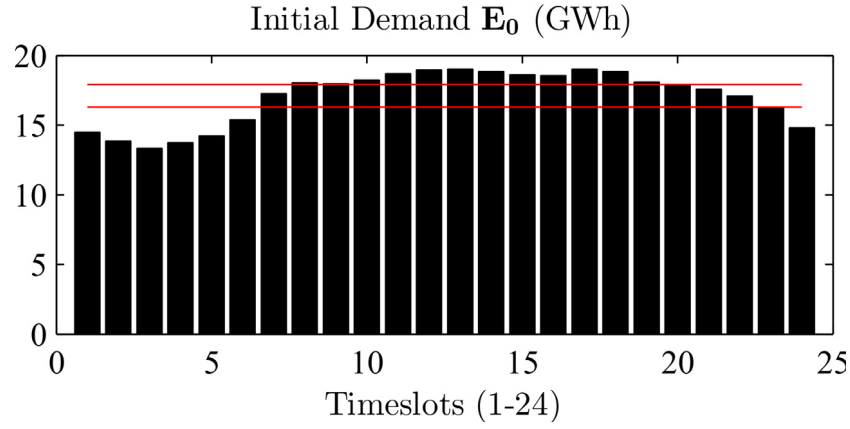
Fig. 1. Initial consumption $\mathbf{E_0}$ extracted from [2]. The red lines represent the capacities C_1 and C_2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
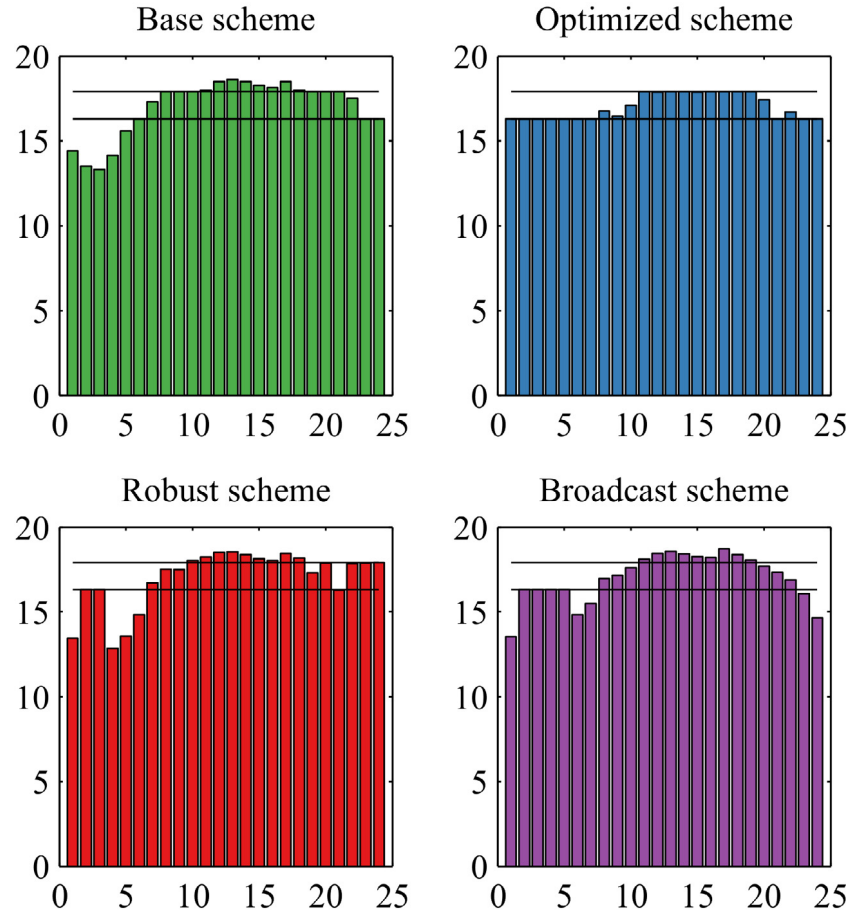

Fig. 2. Final consumption in GWh for the four DR mechanisms ($\mu = \frac{1}{10}$). The black lines represents respectively C_1 and C_2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

optimized scheme and the robust mechanism reduces, the latter being effectively close to exploiting all the population's flexibility.

In Fig. 5, we focus on the case $\mu = \frac{1}{3}$ and analyze the components of the cost for each DR mechanism. Fig. 5 confirms that the optimized scheme provides the largest savings as it can minimize the production cost while paying the smallest amount of discounts. We indicate with wasted discounts the amount of discounts paid to consumption that would in any case have been scheduled in that timeslot. The base and optimized mechanisms do not waste any discount, while the robust mechanism and the broadcast scheme

do, as they provide the discount R_i to all the electricity consumed in i, including the part of E_i^0 that remains in i.

5.1. The effect of noise

In this section we evaluate how sensitive the DR mechanisms are to the following two different sources of randomness: (1) the number of users who accept an offer is a random variable and (2) baseline consumption forecasts can be more or less accurate.
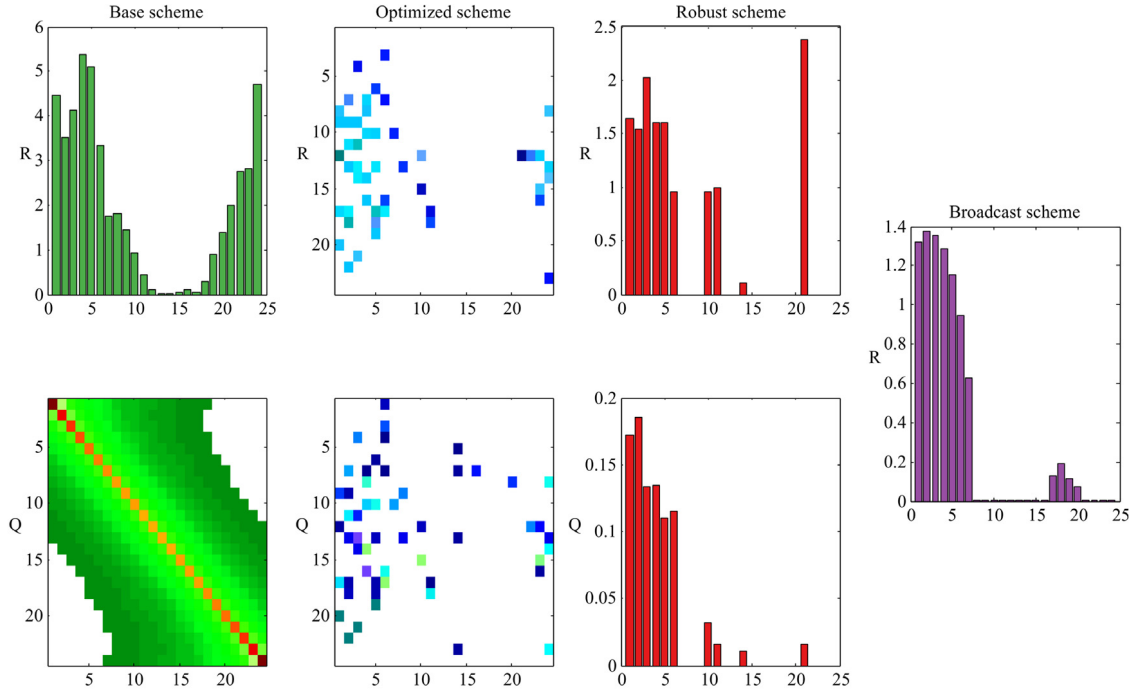
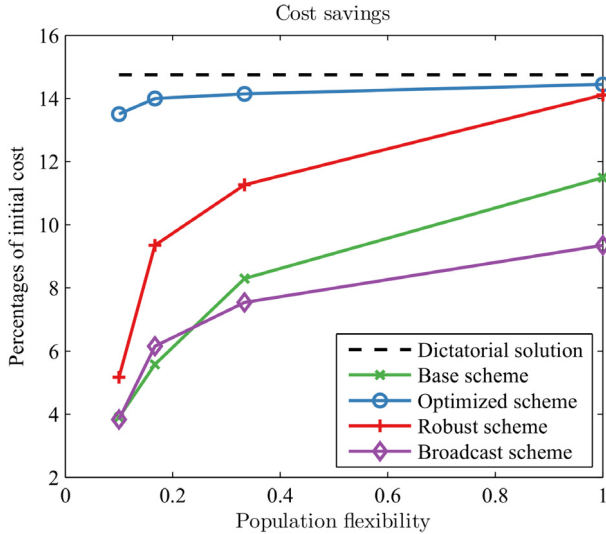**Fig. 3.** Minimizers for the four DR mechanisms ($\mu = \frac{1}{10}$).

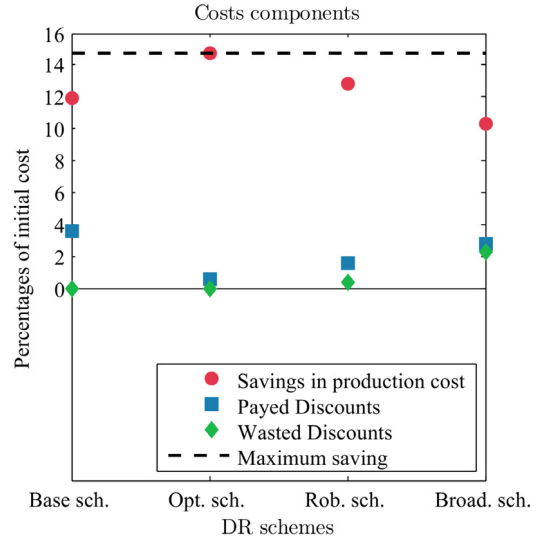**Fig. 4.** Cost savings normalized to the initial cost, for various flexibility parameters μ.

Fig. 5. Analysis of the components of the cost savings. All the quantities are normalized to the initial cost.

Let U be the total number of users. Under the base mechanism, if $q_{j \to i} U$ users (or better $\lceil q_{j \to i} U \rceil$) are offered the discount R_i to move their consumption from i to j, Eq. (9) assumes that $q_{j \to i} U P_{j \to i}(R_i)$ of them accept. In reality the number of those who accept is a Binomial random variable $U^a_{i \to j} \triangleq \text{Bin}(\lceil q_{j \to i} U \rceil, P_{j \to i}(R_i))$. Similar considerations hold for all the mechanisms.

About the second aspect, we consider that the energy utility solves any of the specific optimization problems introduced above, starting from the forecast E^0_i of the aggregate baseline consumption for $i = 1, \ldots, N$. Let $e^0_i = E^0_i / U$ be the forecast of the individual baseline consumption. We assume that the actual baseline consumptions $(\tilde{e}^0_i)$ are i.i.d. random variables with expected value $\text{E}[\tilde{e}^u_i] = e^0_i$ (i.e. the energy utility forecasts are unbiased) and

coefficient of variation δ. The actual aggregate baseline consumption is then a random variable $\tilde{E}^0_i = \sum_{u=1}^{U} \tilde{e}^0_i$ with expected value the forecast E^0_i and coefficient of variation $\delta / \sqrt{U}$. This coefficient captures the uncertainty of aggregate consumption forecasts.

We have then evaluated how the performance metrics change if these two sources of randomness are present, but are ignored by the energy utility in the optimization phase. For the base and optimized mechanisms we maintain that the energy utility can estimate the exact amount of energy shifted between two slots, even if this value is different from what predicted the day ahead.

We considered $U = 13.6$ millions users (equal to the current estimate of Ontario's population). Simulating the behavior of each user would have been very time-demanding because we would
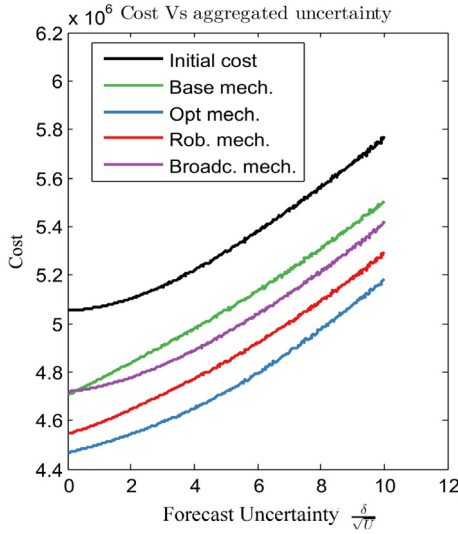
Fig. 6. Global cost versus relative uncertainty $\frac{\delta}{\sqrt{U}}$ (%).

Fig. 7. Savings versus relative uncertainty $\frac{\delta}{\sqrt{U}}$ (%). Savings are normalized by the initial cost.

have had to generate NU random consumptions and roughly NU random choices for the offers. Instead we have approximated by matching the first two moments: (i) the Binomial random variables $U_{j\to i}$ as gaussian ones and (ii) the aggregate consumption of a group of U' users as a lognormal random variable as suggested in [8]. We need then to generate only $O(N^2)$ random variables.

We report in Figs. 6 and 7 respectively the final total costs and the cost savings achieved by the DR mechanisms for $\mu = 1/3$ and different values of the aggregate relative forecast uncertainty $\frac{\delta}{\sqrt{U}}$. Usually the consumption forecast errors over large populations are evaluated to be of a few percents. Results are averaged over 10^5 realizations of the set of random variables. Fig. 6 indicates that costs increase with uncertainty for both the DR mechanism and the idealized schemes. This feature is explained by the Jensen's inequality as the cost functions (16), (3), (12) and (23) are convex in the aggregated forecast $\mathbf{E^0}$. The figures confirm the relative ranking of the DR mechanisms shown in Fig. 4, but for the base mechanism that appears to be the most sensitive to the effect of noise and becomes worse than the broadcast mechanism already for values of $\delta/\sqrt{U}$ as small as 0.2%.

6. Discussion

The models in this paper are based on the microeconomic theory of utility maximization and consumer rationality: users weigh expected costs and benefits and choose the most beneficial actions. This assumption is quite common in the field (see for example the literature overview in [9,10]). In this framework, the possibility to draw quantitative conclusions depends on the availability of accurate aggregate sensitivity functions characterizing how users react to given economic incentives. Experiments have been carried on to this purpose in many countries, like in U.S.A. (Carolina and California), Germany, Switzerland, India [11]. In particular, some of these studies measure the *elasticity of substitution*, that has indeed been introduced to quantify load shifting. It is defined as the relative change of the ratio of the peak to off-peak demand, divided by the relative change of the peak to off-peak price. Elasticity varies significantly from one region to another and from one season to another, but in most of the cases the measured values suggest the possibility of major cost reductions, achievable through demand response mechanisms. These empirical studies can be used to start tuning more accurate models like those we consider in this paper.
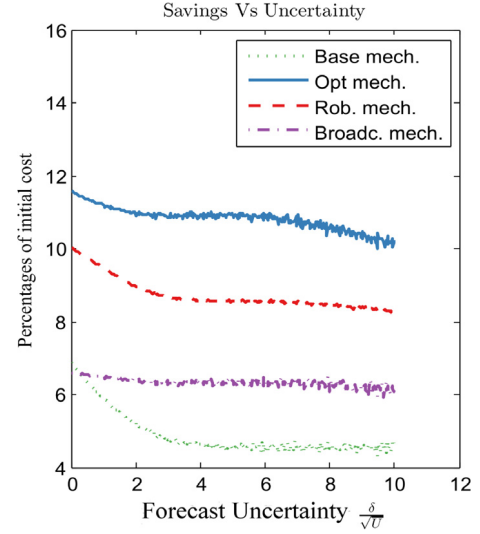
Moreover, once similar pricing schemes are deployed, the system could learn over time better users' models.

At the same time, experimental work has shown that individuals often do not act as the rational users envisaged by microeconomic theory. In particular many cognitive biases affect human decisions and many choices are not consistent with utility maximization forecasts [12]. These observation lead to the rise of the field of behavioral economics. As regards energy use, the authors of [13] show how users' choices exhibit time inconsistency, framing and reference dependence, bounded rationality and the (unconscious) adoption of different decision heuristics. In general the influence of psychological factors should be considered in the design of new energy policies and pricing schemes. For example, [14] advocates that decentralized energy generation and control can transform the users from simple energy consumers to energy citizens and simultaneously make them more responsive to demand response mechanisms. These conclusions are based on data from focus groups in Great Britain, and they are also supported by a similar investigation in Croatia [15]. Another survey in New Zealand [16] confirms that users are not only price driven: their response is equally sensitive to supply security considerations (power grid congestion can increase the black-out risk) and slightly less to CO_2 emissions (from diesel generators activated to face peak demand). Overall, we see that this line of works suggests solutions that are complementary to ours: once users are sensitized to safety and environmental issues and are made aware of the large picture and involved as much as possible in all the phases of the energy cycle—as suggested by [14–16], economic incentives have still an important role and our work can contribute to correctly design them.

7. Conclusions

In this paper, we have shown that macroscopic descriptions of DR mechanisms can hide important assumptions that can jeopardize the mechanisms' implementation. For this reason, our proposal moved from a microscopic description that explicitly models each user's decision. We have then introduced four DR mechanisms with various assumptions on the provider's capabilities. Interestingly, contrarily to previous studies, we find that the optimization problems that result from our mechanisms are complex and can be solved numerically only through a heuristic. Moreover,

our results show that the performance of DR mechanisms under reasonable assumptions on the provider's capabilities are significantly lower than those suggested by previous studies, but that the gap reduces when the population's flexibility increases.

Acknowledgments

Appendix A. Problem (3)–(4) is not convex

We prove the problem is not convex by providing the following counterexample:

- $N = 3$ slots.
- Initial consumption in each slot: $E_1^0 = 1, E_2^0 = 1, E_3^0 = 2$.
- Sensitivity functions to rewards: $S_2(R_i, |2 - i|) = \frac{1 - e^{-R_i/\alpha_2}}{|2-i|}$, $S_3(R_i, t) = \frac{1 - e^{-R_i/\alpha_3}}{|3-i|}$, concave in R_i and decreasing in $|j - i|$. Consider $\alpha_2 = 1, \alpha_3 = 1/10$.
- Cost functions: $c_i(x) = \beta_i x$. Consider $\beta_1 = 1, \beta_2 = 11, \beta_3 = 1/2$.

The derivative versus R_1 of the cost in $\mathbf{R} = \mathbf{0}$ is

$$
\begin{aligned}
\left.\frac{\partial \text{cost}_{\text{base}}}{\partial R_1}\right|_{\mathbf{R}=\mathbf{0}} &= R_1 E_2^0 S_2'(R_1, t) + E_2^0 S_2(R_1, t) + R_1 E_3^0 S_3'(R_1, t) \\
&\quad + E_2^0 S_3(R_1, t) \\
&\quad + \beta_1 (E_2^0 S_2'(R_1, t) + E_3^0 S_3'(R_1, t)) \\
&\quad - \beta_2 E_2^0 S_2'(R_1, t) - \beta_3 E_3^0 S_3'(R_1, t) = \\
&= \beta_1 E_2^0 \frac{e^{-R_1/\alpha_2}}{\alpha_2 |2 - 1|} + \beta_1 E_3^0 \frac{e^{-R_1/\alpha_3}}{\alpha_3 |3 - 1|} \\
&\quad - \beta_2 E_2^0 \frac{e^{-R_1/\alpha_2}}{\alpha_2 |2 - 1|} - \beta_3 E_3^0 \frac{e^{-R_1/\alpha_3}}{\alpha_3 |3 - 1|} = \\
&= \frac{\beta_1}{\alpha_2} + \frac{\beta_1}{\alpha_3} - \frac{\beta_2}{\alpha_2} - \frac{\beta_3}{\alpha_3} = -5 < 0,
\end{aligned}
$$

then it is convenient for the energy provide to increase the reward R_1.

The second derivative (in $\mathbf{R} = \mathbf{0}$) is

$$
\begin{aligned}
\left.\frac{\partial^2 \text{cost}_{\text{base}}}{\partial R_1^2}\right|_{\mathbf{R}=\mathbf{0}} &= E_2^0 S_2'(R_1, t) + R_1 E_2^0 S_2''(R_1, t) + E_2^0 S_2'(R_1, t) \\
&\quad + E_3^0 S_3'(R_1, t) + R_1 E_3^0 S_3'(R_1, t) + E_3^0 S_3'(R_1, t) \\
&\quad + \beta_1 E_2^0 S_2''(R_1, t) + \beta_1 E_3^0 S_3''(R_1, t) \\
&\quad - \beta_2 E_2^0 S_2''(R_1, t) - \beta_3 E_3^0 S_3'(R_1, t) = \\
&= 2 \frac{e^{-R_1/\alpha_2}}{\alpha_2} + 2 \frac{e^{-R_1/\alpha_3}}{\alpha_2} - \beta_1 \frac{e^{-R_1/\alpha_2}}{\alpha_2^2} \\
&\quad - \beta_1 \frac{e^{-R_1/\alpha_3}}{\alpha_3^2} + \beta_2 \frac{e^{-R_1/\alpha_2}}{\alpha_2^2} + \beta_3 \frac{e^{-R_1/\alpha_3}}{\alpha_3^2} = \\
&= \frac{2}{\alpha_2} + \frac{2}{\alpha_3} - \frac{\beta_1}{\alpha_2^2} - \frac{\beta_1}{\alpha_3^2} + \frac{\beta_2}{\alpha_2^2} + \frac{\beta_3}{\alpha_3^2} = -18 < 0,
\end{aligned}
$$

then the function is not convex.

Moreover, this non-convexity is at a point where the derivative of the total cost is negative.

Appendix B. Proof of discontinuity of broadcast mechanism cost function

Suppose the provider broadcasts discount rates $\mathbf{R}$ such that

$$
R_z = \begin{cases} R > 0 & \text{if } z = j - k, j + k \\ 0 & \text{otherwise.} \end{cases} \tag{B.1}
$$

The consumption shifted out of j will then be equally divided between the slots $j + k$ and $j - k$. In formulas, from (22) follows:

$$
P_{j \rightarrow (j-k)}(\mathbf{R}) = P_{j \rightarrow (j+k)}(\mathbf{R}) = \Pr\left(R - \beta_j k \geq 0\right) = \frac{1}{2} F_j\left(\frac{R}{|k|^{t_j}}\right) \tag{B.2}
$$

Consider now another discount $\mathbf{R}^*$ such that

$$
R_z = \begin{cases} R + \epsilon & \text{if } z = j + k \\ R & \text{if } z = j - k \\ 0 & \text{otherwise.} \end{cases} \tag{B.3}
$$

where ϵ is a positive.

Then all the consumption shifted out of j will be moved to $j + k$. From (22) follows

$$
\begin{aligned}
P_{j \rightarrow (j+k)}(\mathbf{R}) &= F_j\left(\frac{R}{|k|^{t_j}}\right) \\
P_{j \rightarrow (j-k)}(\mathbf{R}) &= 0.
\end{aligned} \tag{B.4}
$$

A positive arbitrary small ϵ generate a finite change in probabilities $P_{j \rightarrow i}$. Hence $P_{j \rightarrow i}(\mathbf{R})$ are discontinuous in $\mathbf{R}$ which is why the final cost (23) is discontinuous.

Appendix C. Mechanisms ranking

We start proving Proposition 4.1, i.e. that the optimized mechanism achieves a lower cost than the robust and the base mechanisms.

We first observe that the base mechanism solves the same problem solved by the optimized mechanism, but with the additional constraint that fractions $q_{j \rightarrow i}$ have to be constant as indicated in Eq. (10). The feasibility set of the base mechanism's optimization problem is then a subset of the feasibility set of the optimized mechanism's optimization problem. It follows that the optimized mechanism achieves a lower cost.

We now consider the robust mechanism, and assume that the pair $(\mathbf{R}_{\text{rob.}}, \mathbf{q}_{\text{rob.}})$ is an optimizer of ((16)–(19)). Choose the pair $(\mathbf{R}_{\text{opt}}, \mathbf{q}_{\text{opt}})$ such that:

$$
\forall(j, i) \text{ s.t. } j \neq i \begin{cases} R_{j \rightarrow i} = R_i \\ q_{j \rightarrow i} = q_i \end{cases}
$$

which implies that

$$
\forall(j, i) \text{ s.t. } j \neq i, \ P_{j \rightarrow i}(R_i) q_i = P_{j \rightarrow i}(R_{j \rightarrow i}) q_{j \rightarrow i}
$$

and set

$$
(R_{j \rightarrow j}, q_{j \rightarrow j}) = (0, 0) \ \forall j = 1, \ldots, N
$$

The two configurations $(\mathbf{R}_{\text{rob.}}, \mathbf{q}_{\text{rob.}})$ and $(\mathbf{R}_{\text{opt}}, \mathbf{q}_{\text{opt}})$ generate the same final distribution E_1.

However while the robust mechanism awards with a discount R_i a fraction q_i of the consumption E_i^0 originally scheduled in i, the optimized scheme does not. So the optimized scheme can achieve the same distribution provided by the robust mechanism paying less rewards. It follows that the total cost achieved by the optimized mechanism is never larger than that of the robust mechanism. This last point completes the proof.

The ranking cannot be extended further as we show through the following examples.
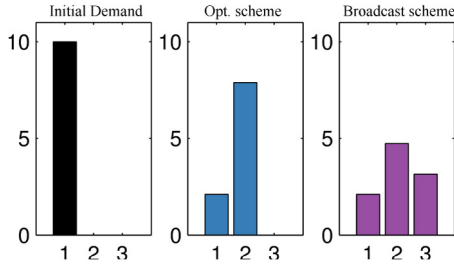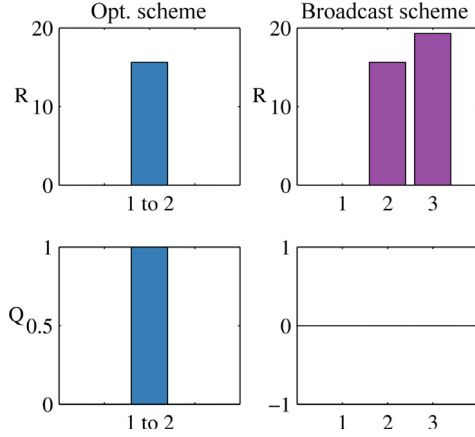
Fig. C.8. Initial and final consumptions in Example C.1.

Fig. C.10. Initial and final consumptions in Example C.2.

Fig. C.9. Optimized and broadcast schemes minimizers in Example C.1.
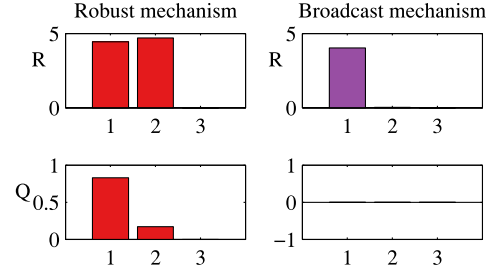
Fig. C.11. Robust mechanism and broadcast scheme minimizers in Example C.2.

Example C.1. We show first a case where the broadcast scheme (23)–(24) provides larger savings than the optimized one (12)–(15). Consider a three timeslots scenario with initial consumption

$$\mathbf{E_0} = \begin{cases} 10 & j = 1 \\ 0 & j = 2, 3 \end{cases}$$

timeslot-dependent costs $c_j(\cdot)$

$$c_j(E) = \begin{cases} 100E & j = 1 \\ 10E & j = 2 \\ 1E & j = 3 \end{cases}$$

and a maximum discount rate $B = 20$; finally assume discomforts are formulated as in (27) with

$$Pr(\beta_j \leq R) = 1 - e^{-\frac{R}{6}} \tag{C.1}$$

Electricity provider can reduce the cost it carries by shifting consumption in timeslots 2 and 3 where there are zero demands and lower marginal production costs. We report in Figs. C.8 and C.9 respectively the rearranged consumptions and the minimizers of the optimized and broadcast schemes. We observe that the optimized mechanism offer to all the users to move in to slot 2, while the broadcast mechanism is able to have some of them moving to the more convenient slot 3. The final cost of the optimized scheme (12)–(15) is 311 and is larger than the final cost of the broadcast one (23)–(24), 286.

The broadcast scheme is able to outperform the optimized scheme, because users can choose between two different offers and then the more flexible ones move to slot 3 and the less flexible ones to slot 2. On the contrary, in order to move some consumption to slot 3, the optimized mechanism should offer to a fraction $q_{1\to3}$ to move, but those who do not accept the offer would stay in 1 with
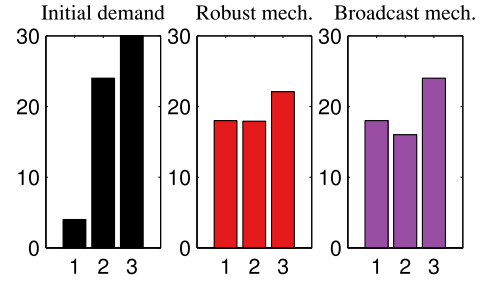
corresponding elevated costs. Observe that this example has been built in such a way that the broadcast mechanism does not "waste" rewards paying for consumption that does not move.

Note that, due to Proposition 4.1, in this scenario the broadcast scheme allows larger savings than the robust mechanism and the base scheme.

Example C.2. We show here that the robust mechanism (16)–(19) can perform better than the broadcast scheme (23)–(24). Consider the following three timeslots scenario:

$$\mathbf{E_0} = \begin{cases} 6 & j = 1; \\ 24 & j = 2; \\ 30 & j = 3; \end{cases}$$

a timeslot independent cost $c(\cdot)$, such that

$$c'(E) = \begin{cases} 1 & E \leq 9 \\ 9 & E \in [9, 18] \\ 36 & E \in [18, 27] \\ 78 & E \geq 27 \end{cases}$$

and a maximum discount $B = 10$. Discomforts as formulated in (27) with

$$Pr(\beta_j \leq R) = \frac{R}{B}$$

We report in Figs. C.10 and C.11 respectively the rearranged consumptions and the minimizers of the robust and broadcast schemes. The final cost of the robust mechanism (16)–(19), 580.75 is smaller than the final cost of the broadcast scheme (23)–(24), 594.

The broadcast scheme assigns only a reward R_1 to the first timeslot. Flexible enough consumption will be moved from timeslots 2 and 3. Notably, the fraction of consumption shifted from timeslot 2 is twice larger than the fraction of consumption shifted from 3 because the discomfort is linear in the distance (see Eq. (27)). The broadcast scheme cannot alter this proportion, though it could be beneficial as in timeslot 3 there is a larger demand. It then achieves an unbalanced consumption in slot 1 and 2.
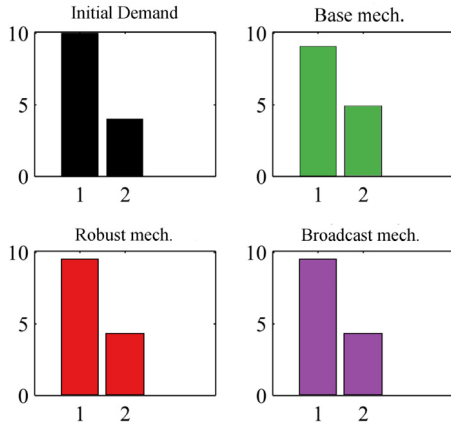
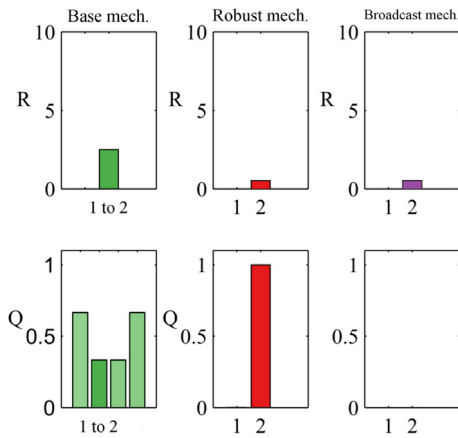**Fig. C.12.** Initial and final consumptions in Example C.3.

Fig. C.13. Robust mechanism, optimized and broadcast schemes minimizers in Example C.3.

On the contrary the robust mechanism can equalize consumption in slots 1 and 2 by offering to a small fraction of the population to move to slot 2.

Observe that, because of Proposition 4.1, in this scenario also the optimized mechanism returns larger savings than the broadcast one.

Example C.3. In this final example, we show that the base scheme can perform better than the robust mechanism and broadcast scheme. Consider a two timeslots scenario with initial consumption

$$\mathbf{E_0} = \begin{cases} 10 & j = 1 \\ 4 & j = 2 \end{cases}$$

a timeslot independent cost $c(\cdot)$, such that

$$c(E) = \begin{cases} 10E & E \leq 7 \\ 15E - 35 & E \geq 7 \end{cases}$$

a maximum discount rate $B = 10$ and discomforts formulated as in (27) with

$$Pr(\beta_j \leq R) = \frac{R}{B}$$

In this scenario the robust mechanism and the broadcast one achieve a cost equal to 154.75, larger than the cost of the base mechanism, 152.92. We report in Figs. C.12 and C.13 respectively the rearranged consumptions and the minimizers of the three mechanisms.

In this scenario all the schemes the robust and broadcast mechanisms are equivalent: they both shift the same amount of consumption to slot 2 and pay the same amount of rewards. The base scheme performs better than the other two since it does not waste any discount, i.e. it does not award discount to the demand E_2^0 originally scheduled in timeslot 2. The base and broadcast schemes do waste some discount and therefore offer smaller discounts than the base scheme, rearrange smaller amount of demand and return a larger cost.

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