

CHAPTER-7  
TRIANGLES

## 1 Exercise 7.1

Q2.  $ABCD$  is a quadrilateral in which  $AD = BC$  and  $\angle DAB = \angle CBA$  as shown in figure ??. Prove that

1.  $\triangle ABD \cong \triangle BAC$
2.  $BD = AC$
3.  $\angle ABD = \angle BAC$

### Construction

The input parameters for construction are shown in ??:

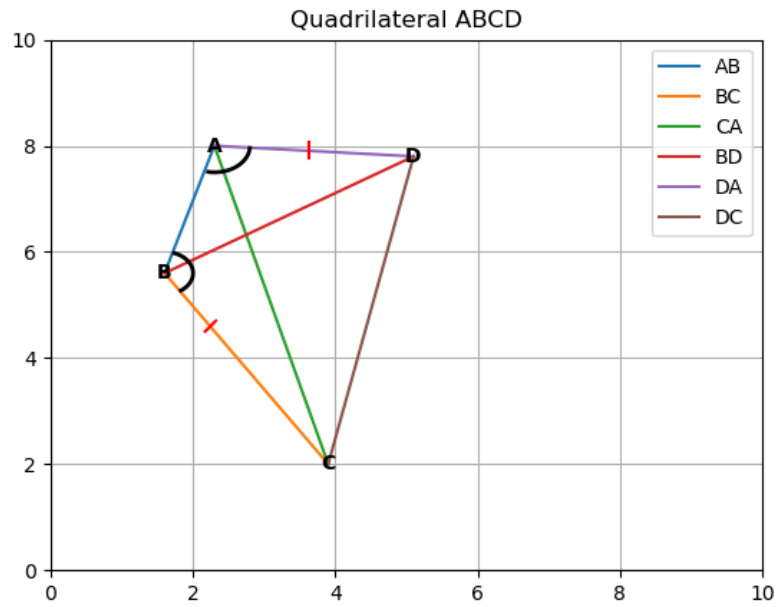


Figure 1: Quadrilateral ABCD

Symbol	Values	Description
$\theta$	$120^\circ$	$\angle BAD = \angle ABC$
$a$	9	$AB$
$c$	5	$BC$
$\mathbf{e}_1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	basis vector

Table 1: Parameters

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c \cos \theta \\ c \sin \theta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -c \cos \theta \\ c \sin \theta \end{pmatrix} \quad (1)$$

**Solution:**

$$\mathbf{A} - \mathbf{D} = \mathbf{B} - \mathbf{C} \quad (2)$$

$$\angle DAB = \angle CBA \quad (3)$$

**To Prove:**

$$\triangle ACB \cong \triangle ADB \quad (4)$$

$$BD = AC \quad (5)$$

$$\angle ABD = \angle BAC \quad (6)$$

**Proof:**

In  $\triangle ABD$  and  $\triangle BAC$

Let equation of  $AB$  be  $y = 0$ , which can be written as:

$$\mathbf{n}^\top \mathbf{X} = 0, \quad (7)$$

$$(8)$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (9)$$

$$(10)$$

From the above assumptions, we get the coordinates of  $C$  and  $D$  as

$$\mathbf{C} = \begin{pmatrix} 4.3 \\ -2.5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -4.3 \\ -2.5 \end{pmatrix} \quad (11)$$

$$(12)$$

Finding the angles(according to assumptions):

$$\text{Let } \theta_1 = \angle ADB \quad (13)$$

$$\mathbf{m}_1 = \mathbf{D} - \mathbf{A} = \begin{pmatrix} -4.7 \\ -2.5 \end{pmatrix}, \mathbf{m}_2 = \mathbf{D} - \mathbf{B} = \begin{pmatrix} -13.7 \\ -2.5 \end{pmatrix} \quad (14)$$

$$\theta_1 = \cos^{-1} \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (15)$$

$$\Rightarrow \theta_1 = \cos^{-1} \frac{\begin{pmatrix} -4.7 & -2.5 \end{pmatrix} \begin{pmatrix} -13.7 \\ -2.5 \end{pmatrix}}{(9.2)(15.8)} = 61^\circ \quad (16)$$

$$\text{Let } \theta_2 = \angle ACB \quad (17)$$

$$\mathbf{n}_1 = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4.7 \\ -2.5 \end{pmatrix}, \mathbf{n}_2 = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 13.7 \\ -2.5 \end{pmatrix} \quad (18)$$

$$\theta_2 = \cos^{-1} \frac{\mathbf{n}_1^\top \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (19)$$

$$\Rightarrow \theta_2 = \cos^{-1} \frac{\begin{pmatrix} 4.7 & -2.5 \end{pmatrix} \begin{pmatrix} 13.7 \\ -2.5 \end{pmatrix}}{(9.2)(15.8)} = 61^\circ \quad (20)$$

from (16) and (20)

$$\angle ABD = \angle CAB \quad (\text{Sum of the angles in a triangle is } 180^\circ)$$

Since all the angles and sides of triangles  $CAB$  and  $CAD$  are equal , from the definition of congruency both the triangles are said to be congruent to each other.

$$\triangle ACB \cong \triangle ADB \quad (21)$$

$$BD = AC \quad (22)$$

$$\angle ABD = \angle BAC \quad (23)$$