

CHAPTER-7
TRIANGLES

1 Exercise 7.1

Q2. $ABCD$ is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$ as shown in figure 1. Prove that

1. $\triangle ABD \cong \triangle BAC$
2. $BD = AC$
3. $\angle ABD = \angle BAC$

Construction

The input parameters for construction are shown in 1:

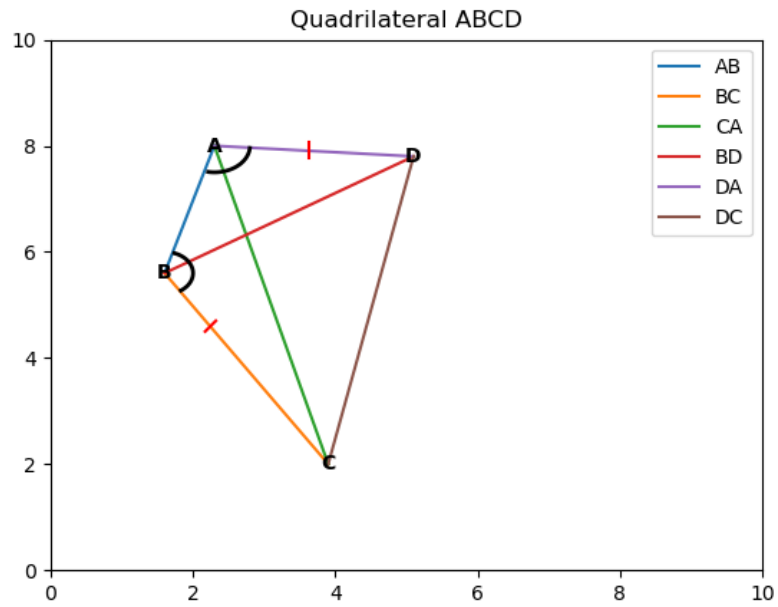


Figure 1: Quadrilateral ABCD

Symbol	Values	Description
θ	120°	$\angle BAD = \angle ABC$
a	9	AB
c	5	BC
\mathbf{e}_1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	basis vector

Table 1: Parameters

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c \cos \theta \\ c \sin \theta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -c \cos \theta \\ c \sin \theta \end{pmatrix} \quad (1)$$

Solution:

$$\mathbf{A} - \mathbf{D} = \mathbf{B} - \mathbf{C} \quad (2)$$

$$\angle DAB = \angle CBA \quad (3)$$

To Prove:

$$\triangle ACB \cong \triangle ADB \quad (4)$$

$$BD = AC \quad (5)$$

$$\angle ABD = \angle BAC \quad (6)$$

Proof:

In $\triangle ABD$ and $\triangle BAC$

Let equation of AB be $y = 0$, which can be written as:

$$\mathbf{n}^\top \mathbf{X} = 0, \quad (7)$$

$$(8)$$

where

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (9)$$

$$(10)$$

From the above assumptions, we get the coordinates of C and D as

$$\mathbf{C} = \begin{pmatrix} 4.3 \\ -2.5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -4.3 \\ -2.5 \end{pmatrix} \quad (11)$$

$$(12)$$

Finding the angles(according to assumptions):

$$\text{Let } \theta_1 = \angle ADB \quad (13)$$

$$\mathbf{m}_1 = \mathbf{D} - \mathbf{A} = \begin{pmatrix} -4.7 \\ -2.5 \end{pmatrix}, \mathbf{m}_2 = \mathbf{D} - \mathbf{B} = \begin{pmatrix} -13.7 \\ -2.5 \end{pmatrix} \quad (14)$$

$$\theta_1 = \cos^{-1} \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (15)$$

$$\Rightarrow \theta_1 = \cos^{-1} \frac{\begin{pmatrix} -4.7 & -2.5 \end{pmatrix} \begin{pmatrix} -13.7 \\ -2.5 \end{pmatrix}}{(9.2)(15.8)} = 61^\circ \quad (16)$$

$$\text{Let } \theta_2 = \angle ACB \quad (17)$$

$$\mathbf{n}_1 = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4.7 \\ -2.5 \end{pmatrix}, \mathbf{n}_2 = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 13.7 \\ -2.5 \end{pmatrix} \quad (18)$$

$$\theta_2 = \cos^{-1} \frac{\mathbf{n}_1^\top \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (19)$$

$$\Rightarrow \theta_2 = \cos^{-1} \frac{\begin{pmatrix} 4.7 & -2.5 \end{pmatrix} \begin{pmatrix} 13.7 \\ -2.5 \end{pmatrix}}{(9.2)(15.8)} = 61^\circ \quad (20)$$

from (16) and (20)

$$\angle ABD = \angle CAB \quad (\text{Sum of the angles in a triangle is } 180^\circ)$$

Since all the angles and sides of triangles CAB and CAD are equal , from the definition of congruency both the triangles are said to be congruent to each other.

$$\triangle ACB \cong \triangle ADB \quad (21)$$

$$BD = AC \quad (22)$$

$$\angle ABD = \angle BAC \quad (23)$$