CHAPTER-7
TRIANGLES

1 Exercise 7.1

Q2. $ABCD$ is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$ as shown in figure 1. Prove that

1. $\triangle ABD \cong \triangle BAC$
2. $BD = AC$
3. $\angle ABD = \angle BAC$

Construction
The input parameters for construction are shown in 1:

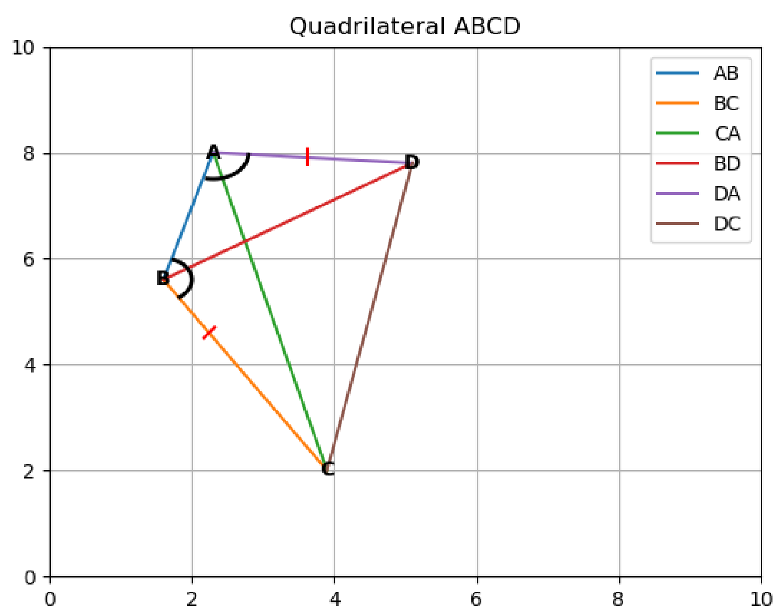

Figure 1: Quadrilateral ABCD

Symbol	Values	Description
θ	$120°$	$\angle BAD = \angle ABC$
a	9	AB
c	5	BC
$\mathbf{e}_1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	basis vector

Table 1: Parameters

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c\cos\theta \\ c\sin\theta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -c\cos\theta \\ c\sin\theta \end{pmatrix} \tag{1}$$

Solution:

$$\mathbf{A} - \mathbf{D} = \mathbf{B} - \mathbf{C} \tag{2}$$
$$\angle DAB = \angle CBA \tag{3}$$

To Prove:

$$\triangle ACB \cong \triangle ADB \tag{4}$$
$$BD = AC \tag{5}$$
$$\angle ABD = \angle BAC \tag{6}$$

Proof:
In $\triangle ABD$ and $\triangle BAC$
Let equation of AB be $y = 0$, which can be written as:

$$\mathbf{n}^\top \mathbf{x} = 0, \tag{7}$$
$$\tag{8}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{9}$$
$$\tag{10}$$

From the above assumptions, we get the coordinates of C and D as

$$\mathbf{C} = \begin{pmatrix} 4.3 \\ -2.5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -4.3 \\ -2.5 \end{pmatrix} \tag{11}$$
$$\tag{12}$$

Finding the angles(according to assumptions):

$$\text{Let } \theta_1 = \angle ADB \tag{13}$$

$$\mathbf{m_1} = \mathbf{D} - \mathbf{A} = \begin{pmatrix} -4.7 \\ -2.5 \end{pmatrix}, \mathbf{m_2} = \mathbf{D} - \mathbf{B} = \begin{pmatrix} -13.7 \\ -2.5 \end{pmatrix} \tag{14}$$

$$\theta_1 = \cos^{-1} \frac{\mathbf{m_1}^\top \mathbf{m_2}}{\|\mathbf{m_1}\| \, \|\mathbf{m_2}\|} \tag{15}$$

$$\implies \theta_1 = \cos^{-1} \frac{\begin{pmatrix} -4.7 & -2.5 \end{pmatrix} \begin{pmatrix} -13.7 \\ -2.5 \end{pmatrix}}{(9.2)(15.8)} = 61^\circ \tag{16}$$

$$\text{Let } \theta_2 = \angle ACB \tag{17}$$

$$\mathbf{n_1} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4.7 \\ -2.5 \end{pmatrix}, \mathbf{n_2} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 13.7 \\ -2.5 \end{pmatrix} \tag{18}$$

$$\theta_2 = \cos^{-1} \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\| \, \|\mathbf{n_2}\|} \tag{19}$$

$$\implies \theta_2 = \cos^{-1} \frac{\begin{pmatrix} 4.7 & -2.5 \end{pmatrix} \begin{pmatrix} 13.7 \\ -2.5 \end{pmatrix}}{(9.2)(15.8)} = 61^\circ \tag{20}$$

from (16) and (20)

$$\angle \text{ ABD} = \angle \text{ CAB} \quad \text{(Sum of the angles in a triangle is } 180^\circ\text{)}$$

Since all the angles and sides of triangles CAB and CAD are equal , from the definition of congruency both the triangles are said to be congruent to each other.

$$\triangle ACB \cong \triangle ADB \tag{21}$$

$$BD = AC \tag{22}$$

$$\angle ABD = \angle BAC \tag{23}$$