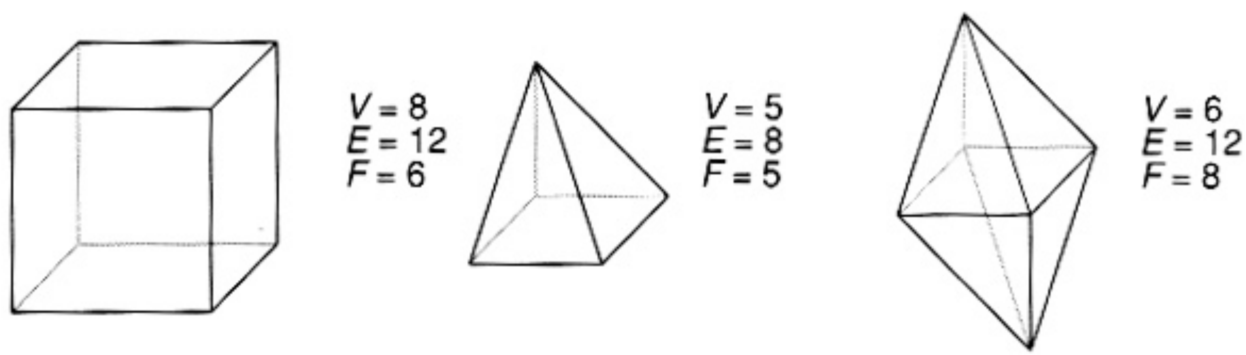
Triangle meshes

CS 4620 Lecture 11

Notation

- n_T = #tris; n_V = #verts; n_E = #edges
- Euler: $n_V - n_E + n_T = 2$ for a simple closed surface
  - and in general sums to small integer
  - argument for implication that n_T:n_E:n_V is about 2:3:1



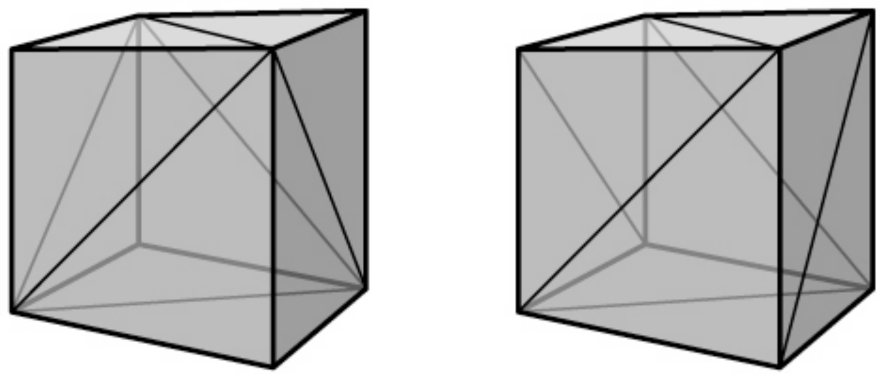
[Foley et al.]

Validity of triangle meshes

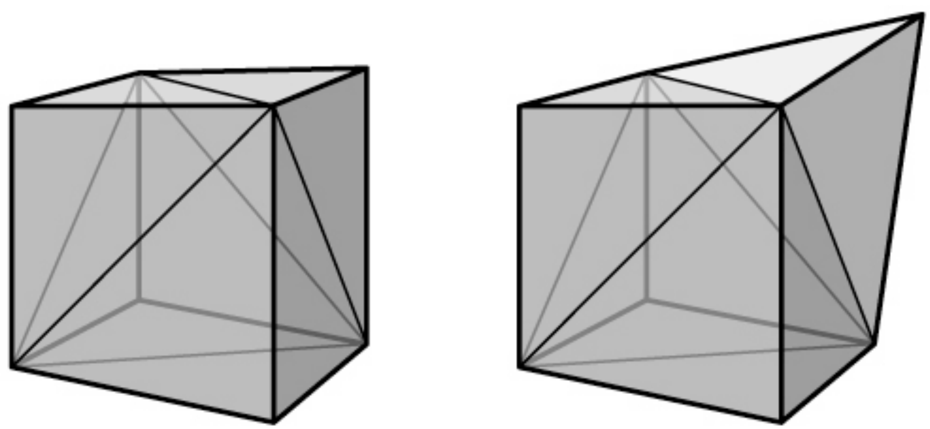
- in many cases we care about the mesh being able to bound a region of space nicely
- in other cases we want triangle meshes to fulfill assumptions of algorithms that will operate on them (and may fail on malformed input)
- two completely separate issues:
  - topology: how the triangles are connected (ignoring the positions entirely)
  - geometry: where the triangles are in 3D space

Topology/geometry examples

- same geometry, different mesh topology:

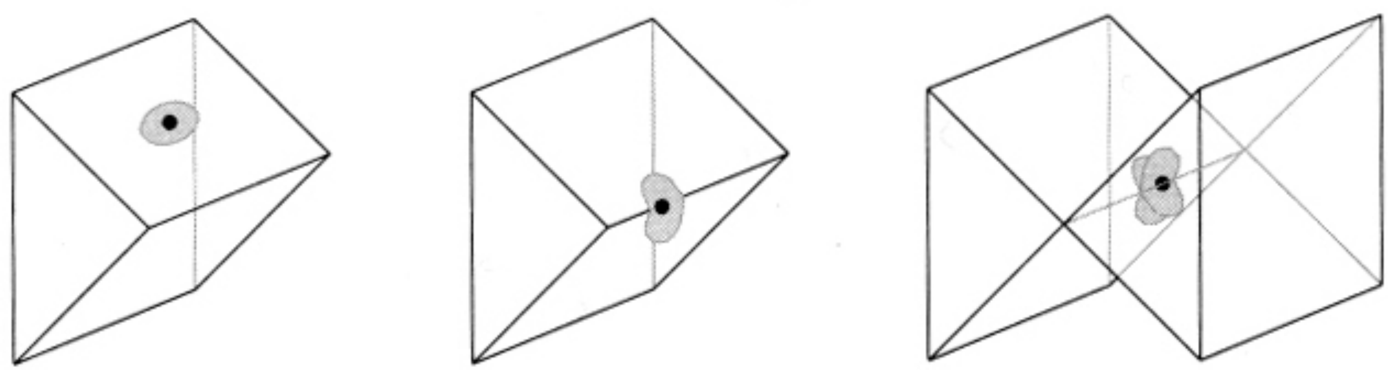


- same mesh topology, different geometry:



Topological validity

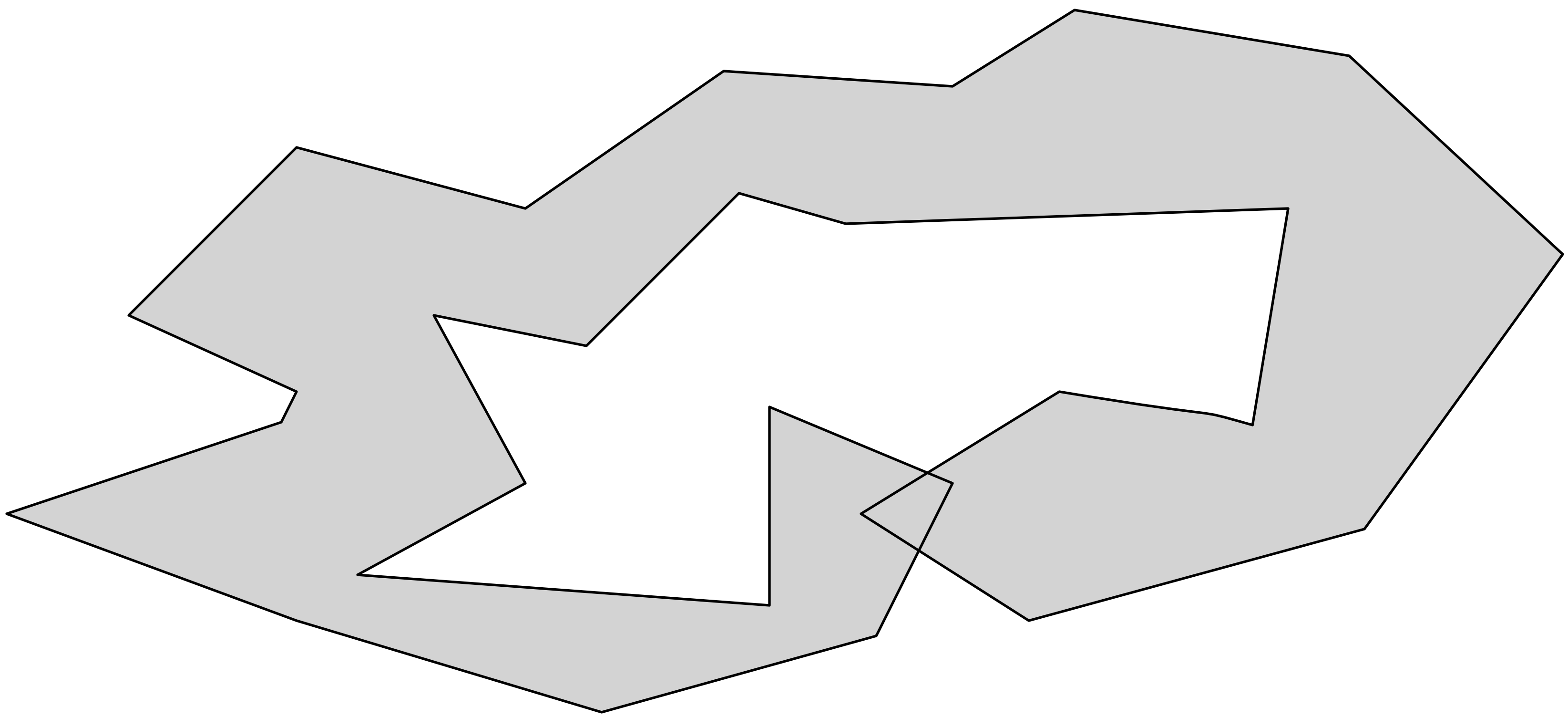
- strongest property, and most simple: be a manifold
  - this means that no points should be "special"
  - interior points are fine
  - edge points: each edge should have exactly 2 triangles
  - vertex points: each vertex should have one loop of triangles
    - not too hard to weaken this to allow boundaries



[Foley et al.]

Geometric validity

- generally want non-self-intersecting surface
- hard to guarantee in general
  - because far-apart parts of mesh might intersect



Representation of triangle meshes

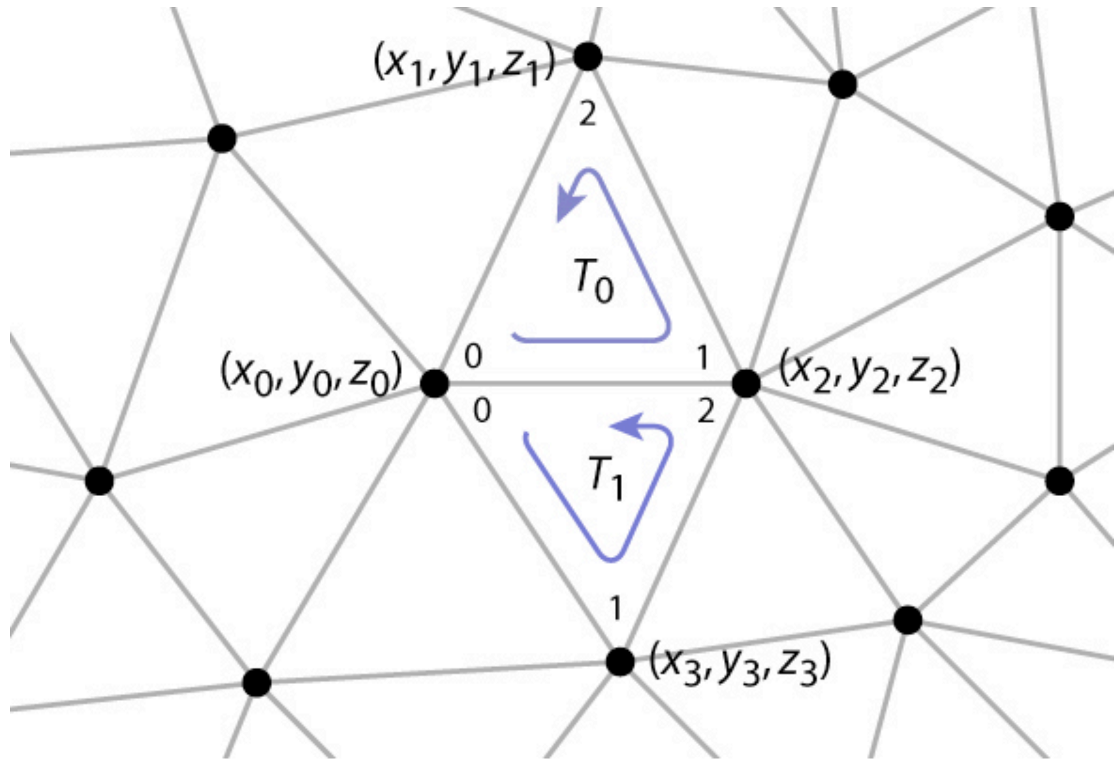
- Compactness
- Efficiency for rendering
  - enumerate all triangles as triples of 3D points
- Efficiency of queries
  - all vertices of a triangle
  - all triangles around a vertex
  - neighboring triangles of a triangle
  - (need depends on application)
    - finding triangle strips
    - computing subdivision surfaces
    - mesh editing

Representations for triangle meshes

- Separate triangles
- Indexed triangle set
  - shared vertices
- Triangle strips and triangle fans
  - compression schemes for transmission to hardware
- Triangle-neighbor data structure
  - supports adjacency queries
- Winged-edge data structure
  - supports general polygon meshes

Separate triangles

	[0]	[1]	[2]
tris[0]	x_0, y_0, z_0	x_2, y_2, z_2	x_1, y_1, z_1
tris[1]	x_0, y_0, z_0	x_3, y_3, z_3	x_2, y_2, z_2
	⋮	⋮	⋮



Separate triangles

- array of triples of points
  - float[n_T][3][3]: about 72 bytes per vertex
    - 2 triangles per vertex (on average)
    - 3 vertices per triangle
    - 3 coordinates per vertex
    - 4 bytes per coordinate (float)
- various problems
  - wastes space (each vertex stored 6 times)
  - cracks due to roundoff
  - difficulty of finding neighbors at all

Indexed triangle set

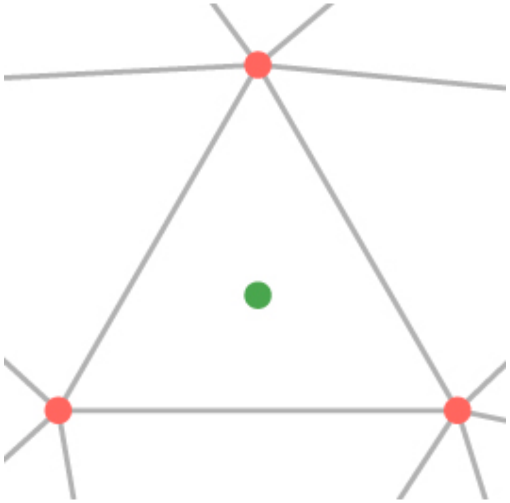
- Store each vertex once
- Each triangle points to its three vertices

```
Triangle {
    Vertex vertex[3];
    }

Vertex {
    float position[3];  // or other data
    }

// ... or ...

Mesh {
    float verts[nv][3];  // vertex positions (or other data)
    int tInd[nt][3];  // vertex indices
    }
```



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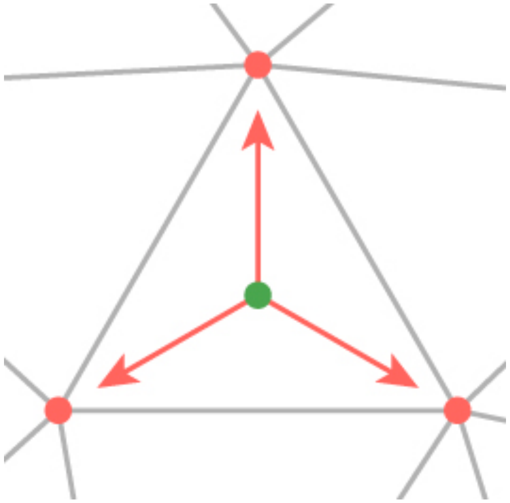
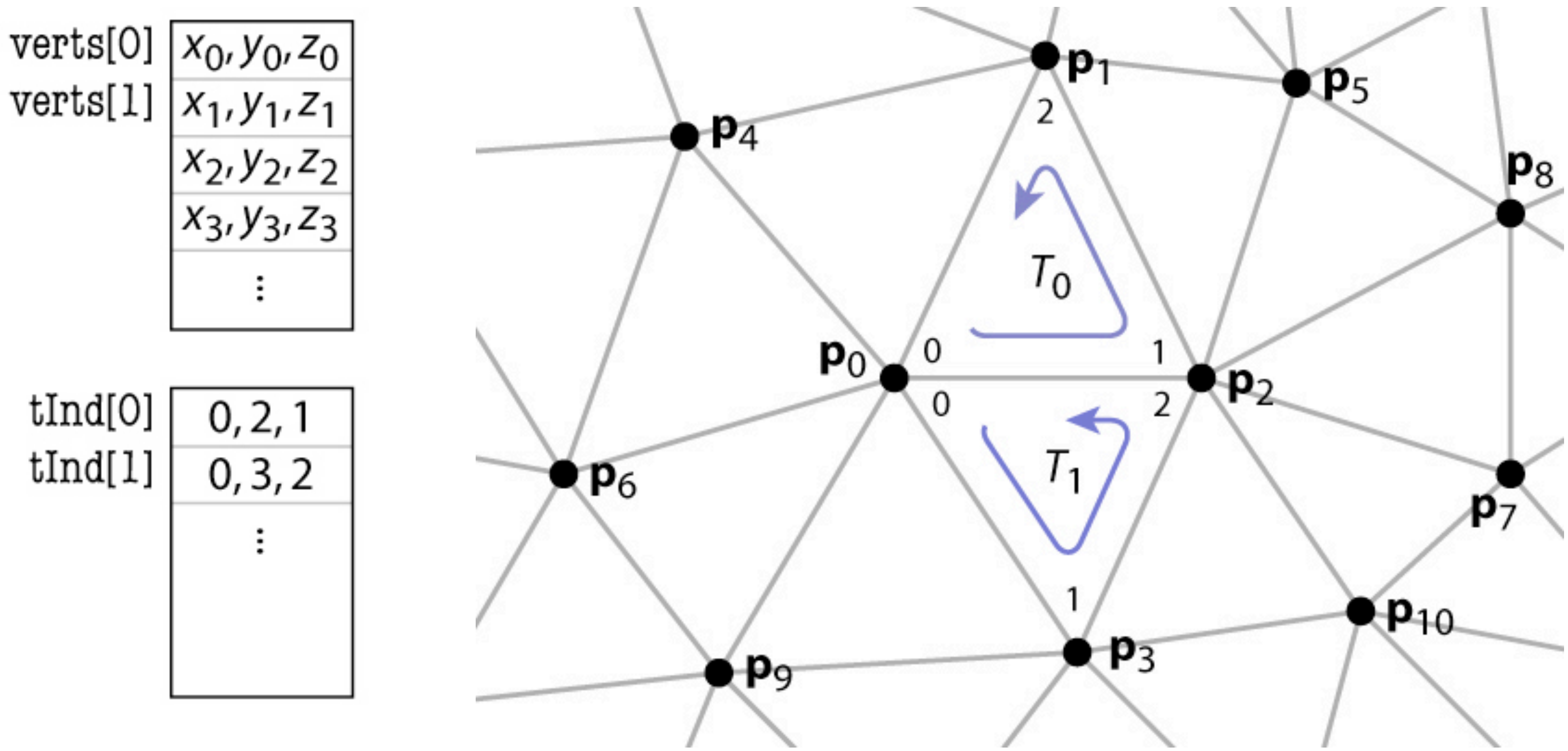


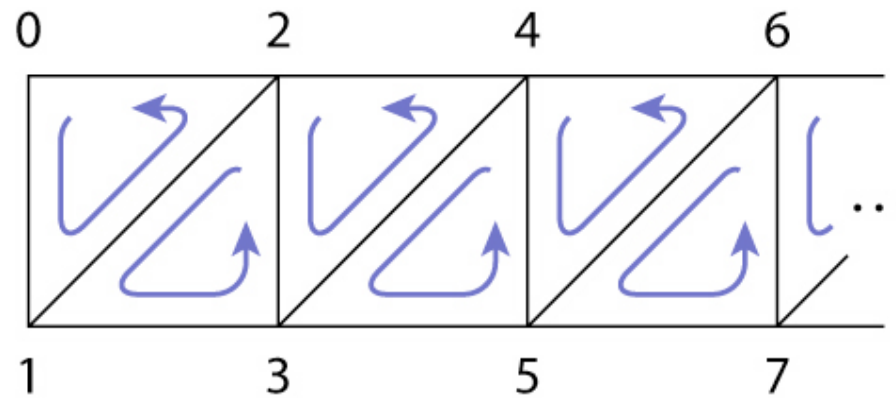
Indexed triangle set



Indexed triangle set

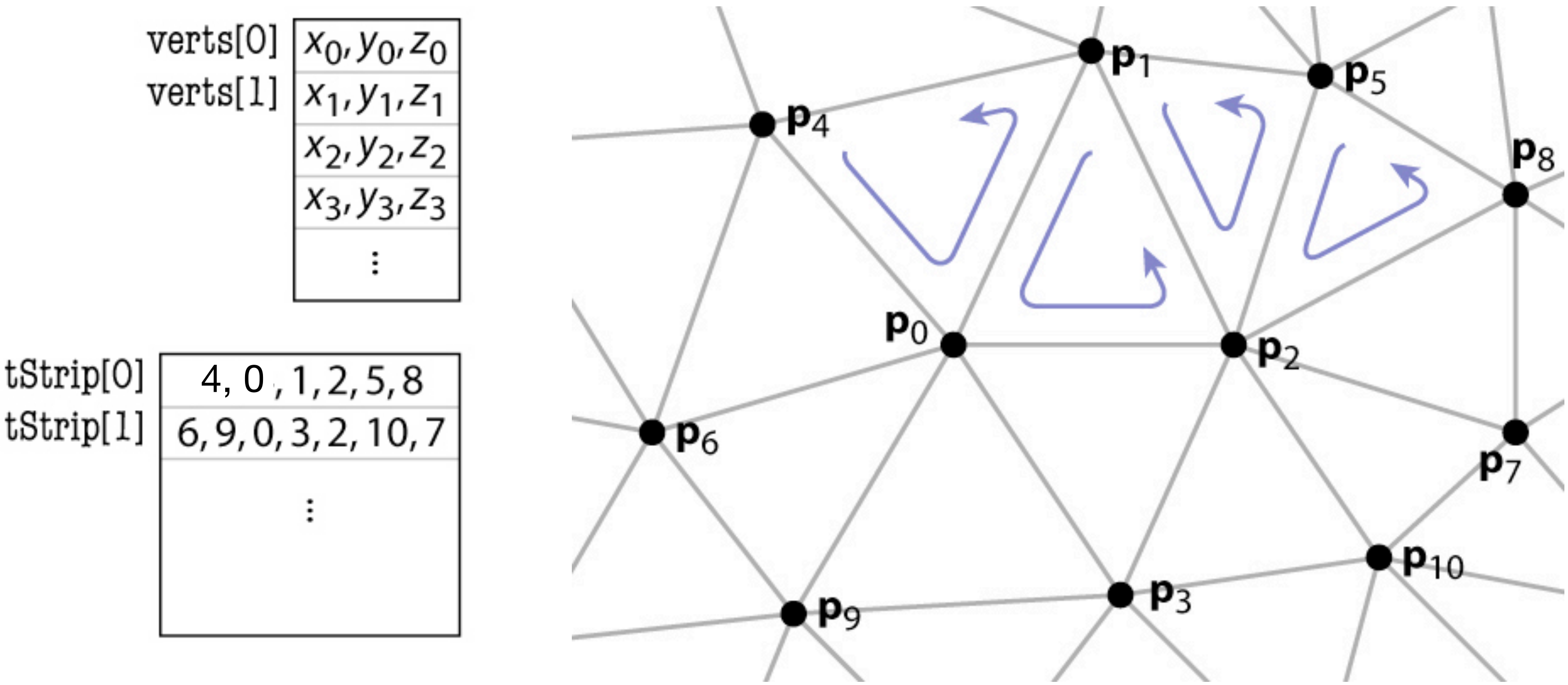
- array of vertex positions
  - float[n_V][3]: 12 bytes per vertex
    - (3 coordinates x 4 bytes) per vertex
- array of triples of indices (per triangle)
  - int[n_T][3]: about 24 bytes per vertex
    - 2 triangles per vertex (on average)
    - (3 indices x 4 bytes) per triangle
- total storage: 36 bytes per vertex (factor of 2 savings)
- represents topology and geometry separately
- finding neighbors is at least well defined

Triangle strips



- Take advantage of the mesh property
  - each triangle is usually adjacent to the previous
  - let every vertex create a triangle by reusing the second and third vertices of the previous triangle
  - every sequence of three vertices produces a triangle (but not in the same order)
  - e. g., 0, 1, 2, 3, 4, 5, 6, 7, … leads to
    (0 1 2), (2 1 3), (2 3 4), (4 3 5), (4 5 6), (6 5 7), …
  - for long strips, this requires about one index per triangle

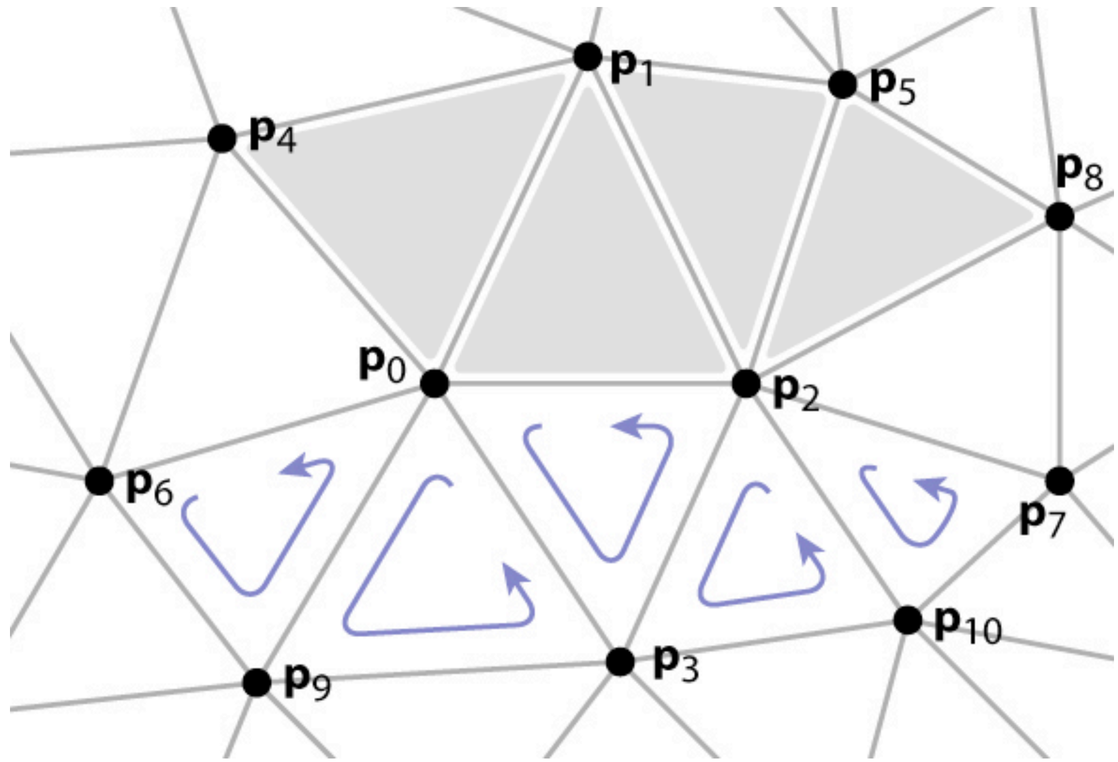
Triangle strips



Triangle strips

verts[0]	x_0, y_0, z_0
verts[1]	x_1, y_1, z_1
	x_2, y_2, z_2
	x_3, y_3, z_3
	⋮

tStrip[0]	4, 0 , 1, 2, 5, 8
tStrip[1]	6, 9, 0, 3, 2, 10, 7
	⋮

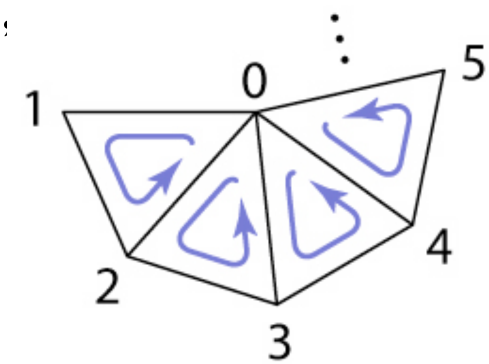


Triangle strips

- array of vertex positions
  - float[n_V][3]: 12 bytes per vertex
    - (3 coordinates x 4 bytes) per vertex
- array of index lists
  - int[n_S][*variable*]: 2 + n indices per strip
  - on average, (1 + ε) indices per triangle (assuming long strips)
    - 2 triangles per vertex (on average)
    - about 4 bytes per triangle (on average)
- total is 20 bytes per vertex (limiting best case)
  - factor of 3.6 over separate triangles; 1.8 over indexed mesh

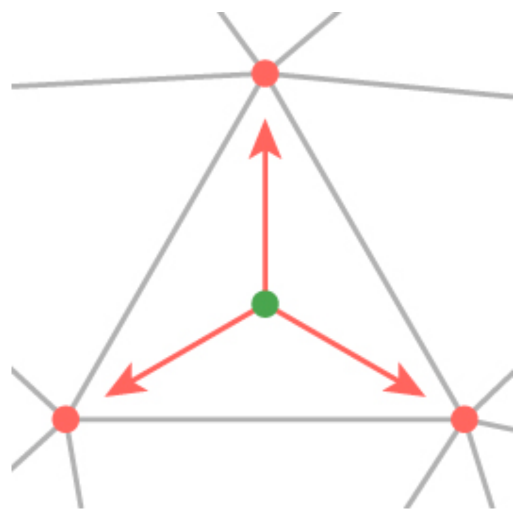
Triangle fans

- Same idea as triangle strips, but keep oldest rather than newest
  - every sequence of three vertices produces a triangle
  - e. g., 0, 1, 2, 3, 4, 5, … leads to
    (0 1 2), (0 2 3), (0 3 4), (0 3 5), …
  - for long fans, this requires about one index per triangle
- Memory considerations exactly the same as triangle strip



Triangle neighbor structure

- Extension to indexed triangle set
- Triangle points to its three neighboring triangles
- Vertex points to a single neighboring triangle
- Can now enumerate triangles around a vertex



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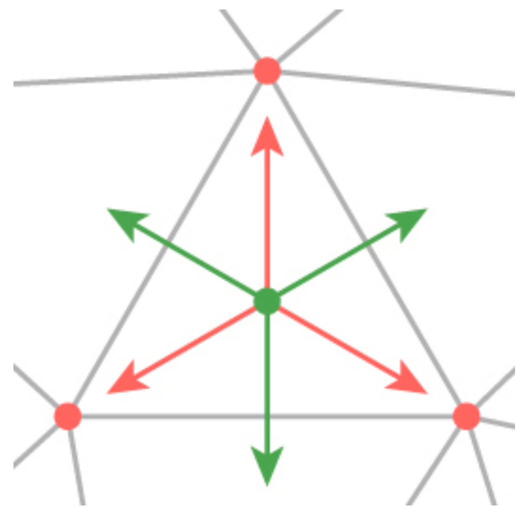

Triangle neighbor structure

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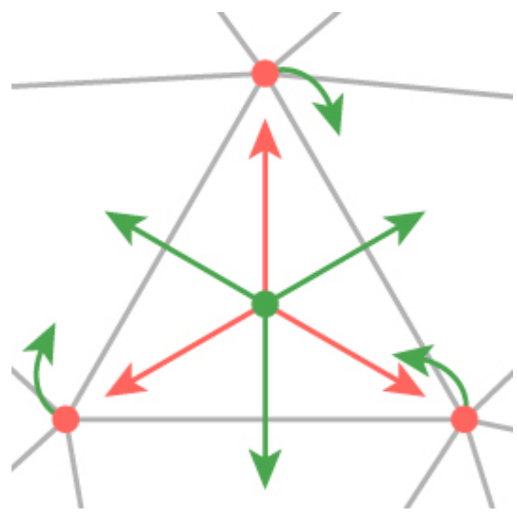


Triangle neighbor structure

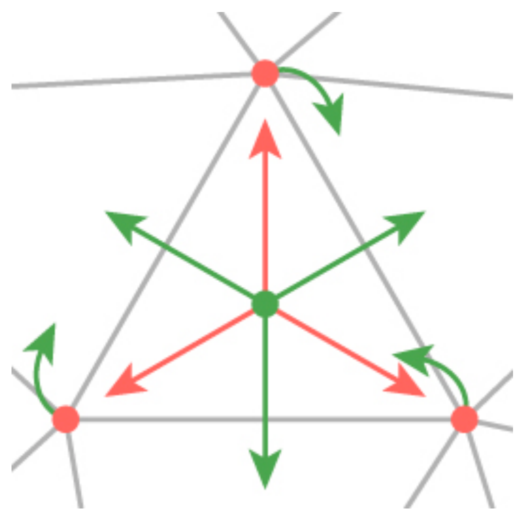
```
Triangle {
    Triangle nbr[3];
    Vertex vertex[3];
    }

// t.neighbor[i] is adjacent
// across the edge from i to i+1

Vertex {
    // ... per-vertex data ...
    Triangle t;  // any adjacent tri
    }

// ... or ...

Mesh {
    // ... per-vertex data ...
    int tInd[nt][3];  // vertex indices
    int tNbr[nt][3]; // indices of neighbor triangles
    int vTri[nv]; // index of any adjacent triangle
    }
```

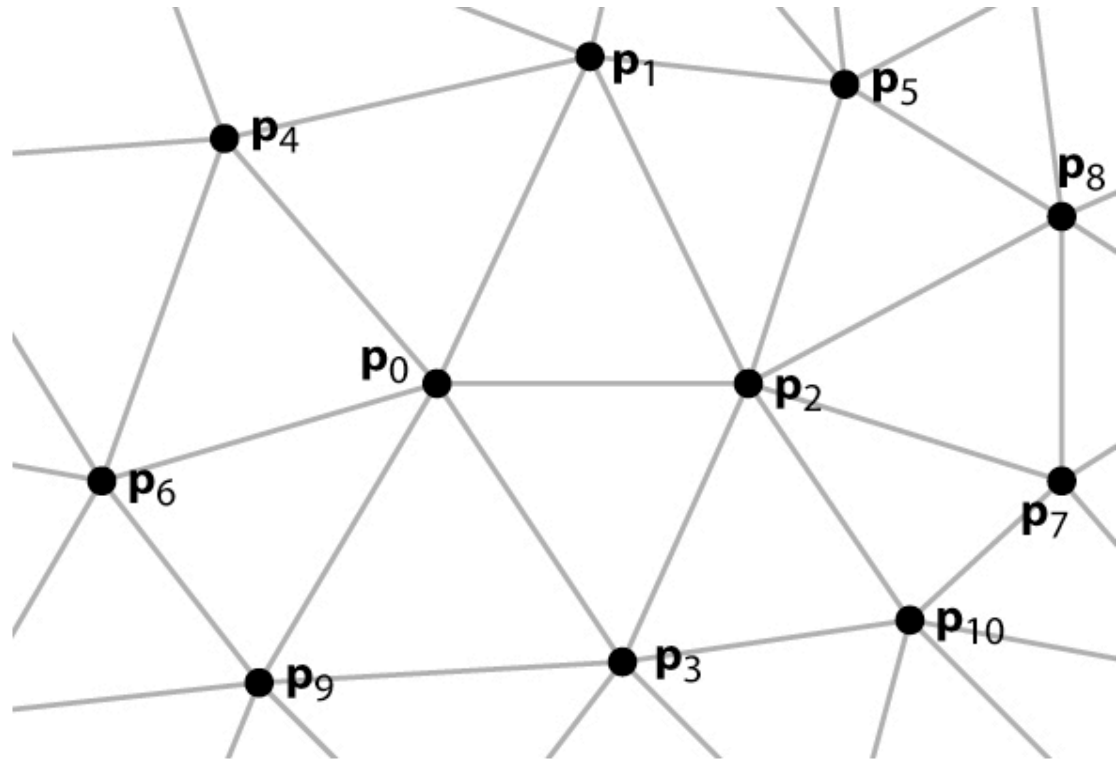


Triangle neighbor structure

vTri[0]	0
vTri[1]	6
vTri[2]	1
vTri[3]	1
	⋮

tNbr[0]	1, 6, 7
tNbr[1]	10, 2, 0
tNbr[2]	3, 1, 12
tNbr[3]	2, 13, 4
	⋮

tInd[0]	0, 2, 1
tInd[1]	0, 3, 2
tInd[2]	10, 2, 3
tInd[3]	2,10,7
	⋮

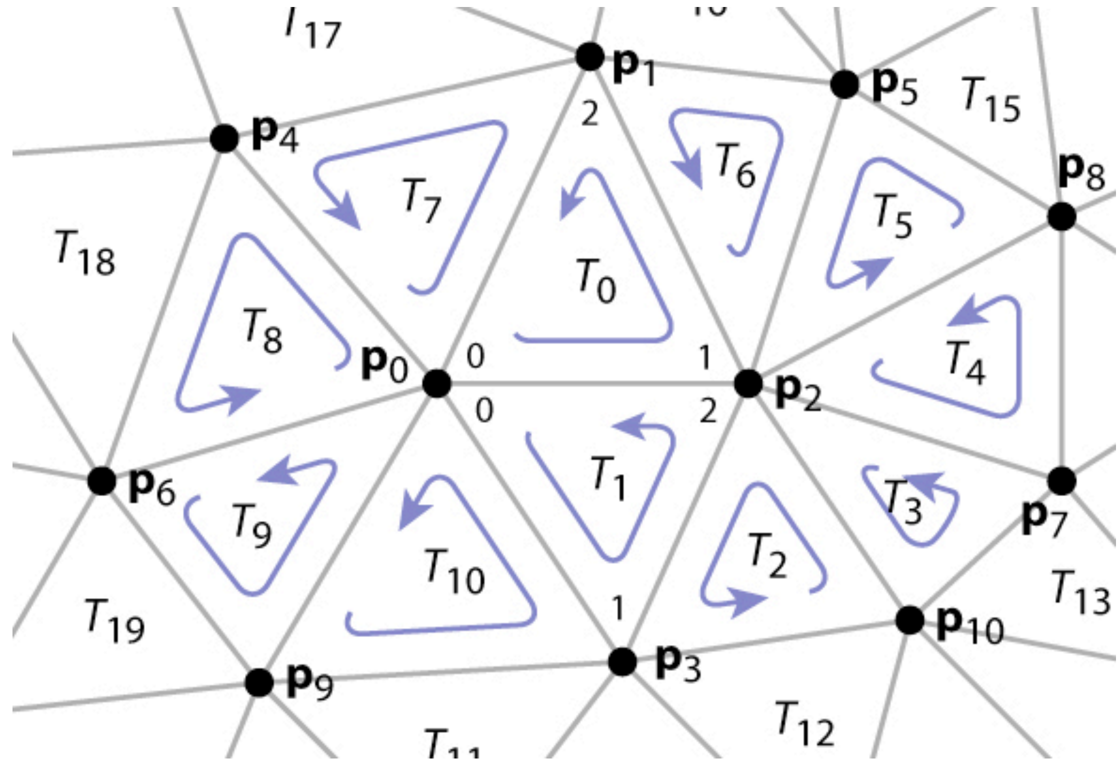


Triangle neighbor structure

vTri[0]	0
vTri[1]	6
vTri[2]	1
vTri[3]	1
	⋮

tNbr[0]	1,6,7
tNbr[1]	10,2,0
tNbr[2]	3,1,12
tNbr[3]	2,13,4
	⋮

tInd[0]	0,2,1
tInd[1]	0,3,2
tInd[2]	10,2,3
tInd[3]	2,10,7
	⋮

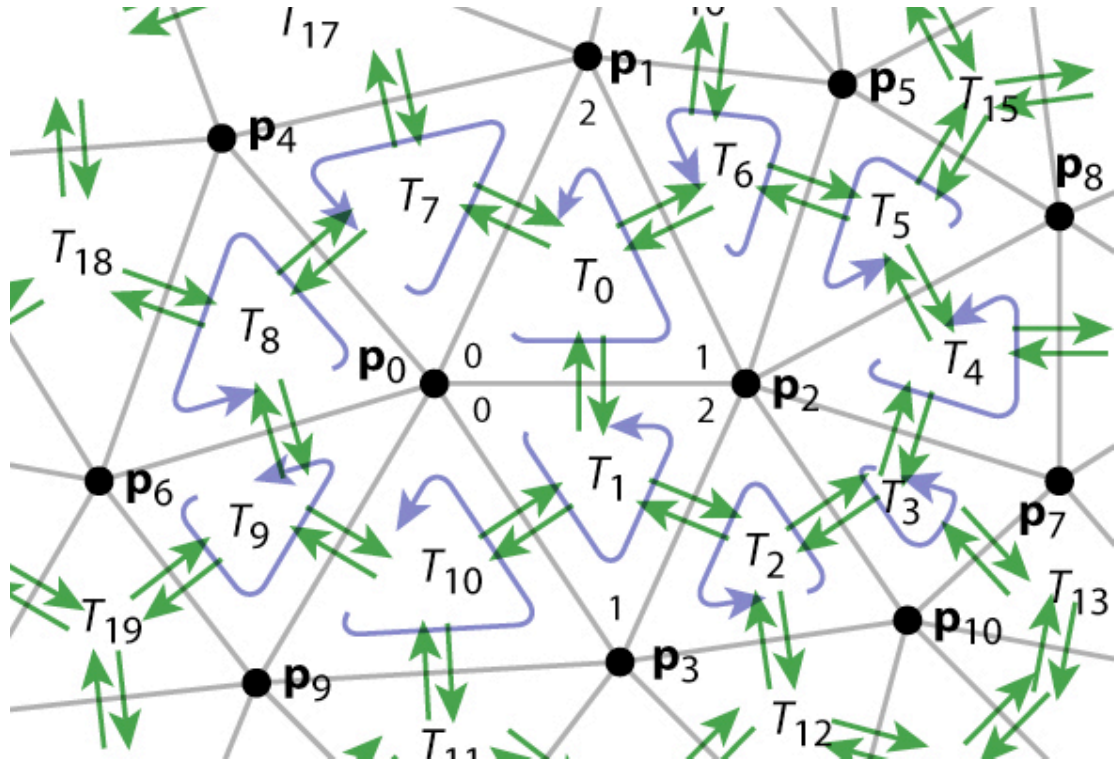


Triangle neighbor structure

vTri[0]	0
vTri[1]	6
vTri[2]	1
vTri[3]	1
	⋮

tNbr[0]	1, 6, 7
tNbr[1]	10, 2, 0
tNbr[2]	3, 1, 12
tNbr[3]	2, 13, 4
	⋮

tInd[0]	0, 2, 1
tInd[1]	0, 3, 2
tInd[2]	10, 2, 3
tInd[3]	2, 10, 7
	⋮



Triangle neighbor structure

vTri[0]	0
vTri[1]	6
vTri[2]	1
vTri[3]	1
	⋮

tNbr[0]	1, 6, 7
tNbr[1]	10, 2, 0
tNbr[2]	3, 1, 12
tNbr[3]	2, 13, 4
	⋮

tInd[0]	0, 2, 1
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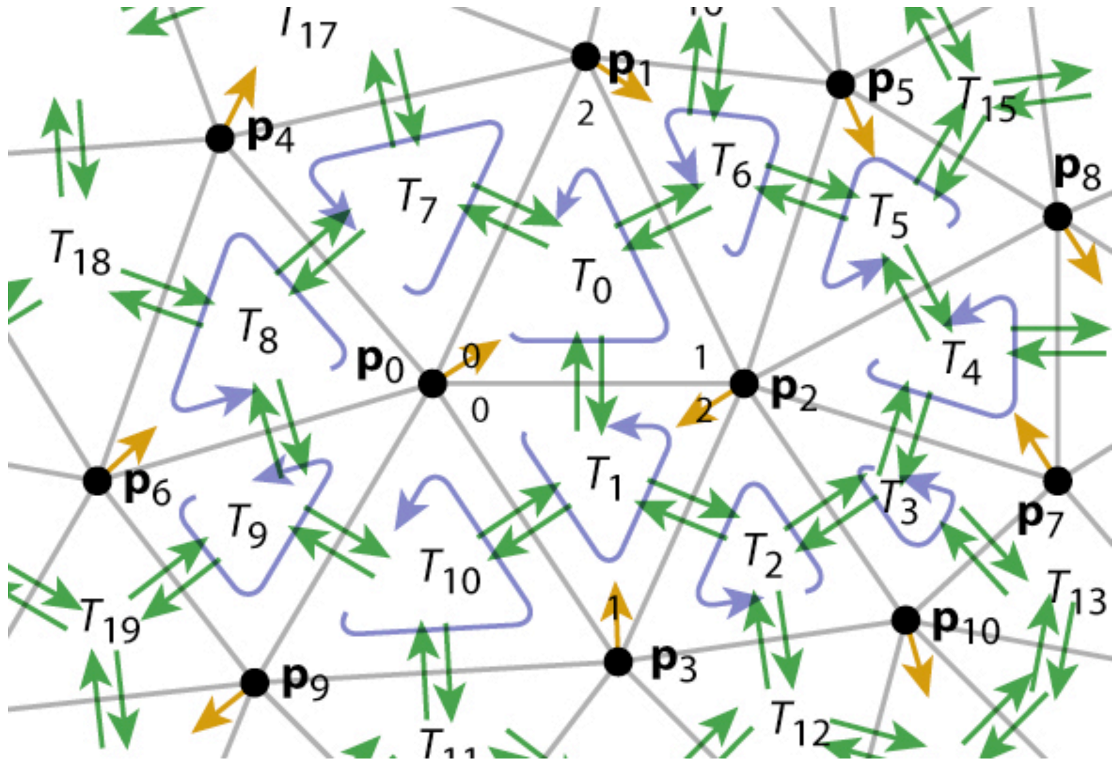
	⋮

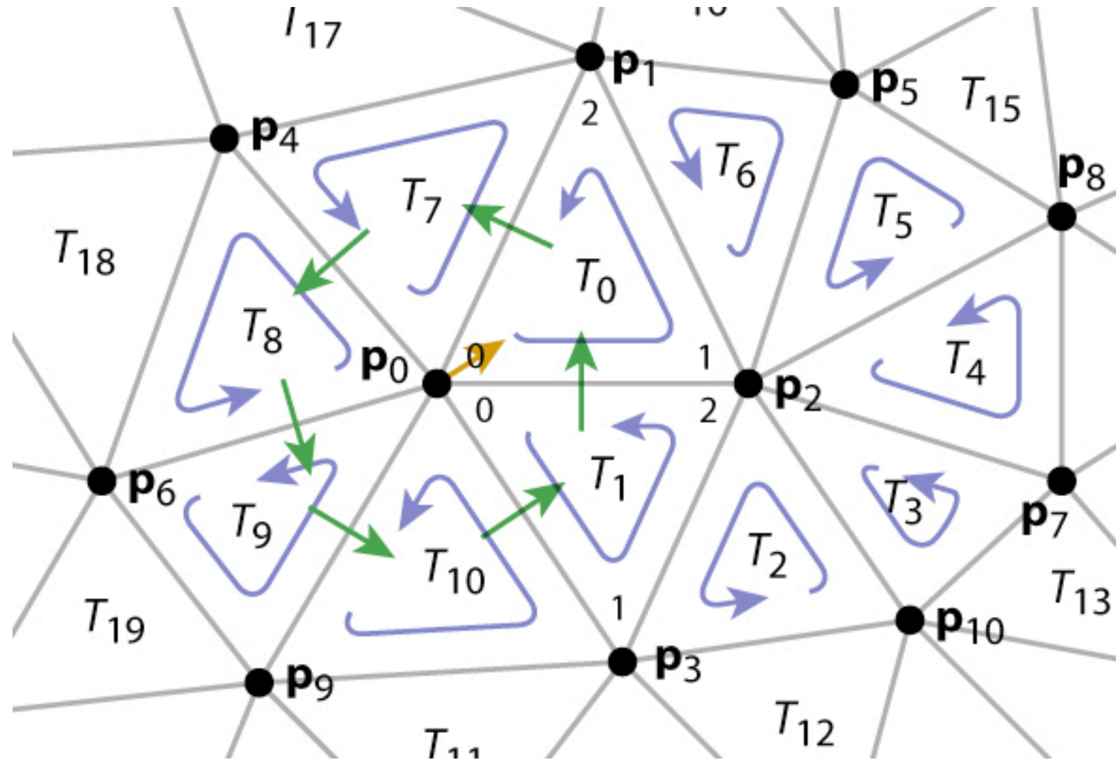


Triangle neighbor structure

vTri[0]	0
vTri[1]	6
vTri[2]	1
vTri[3]	1
	⋮

tNbr[0]	1,6,7
tNbr[1]	10,2,0
tNbr[2]	3,1,12
tNbr[3]	2,13,4
	⋮

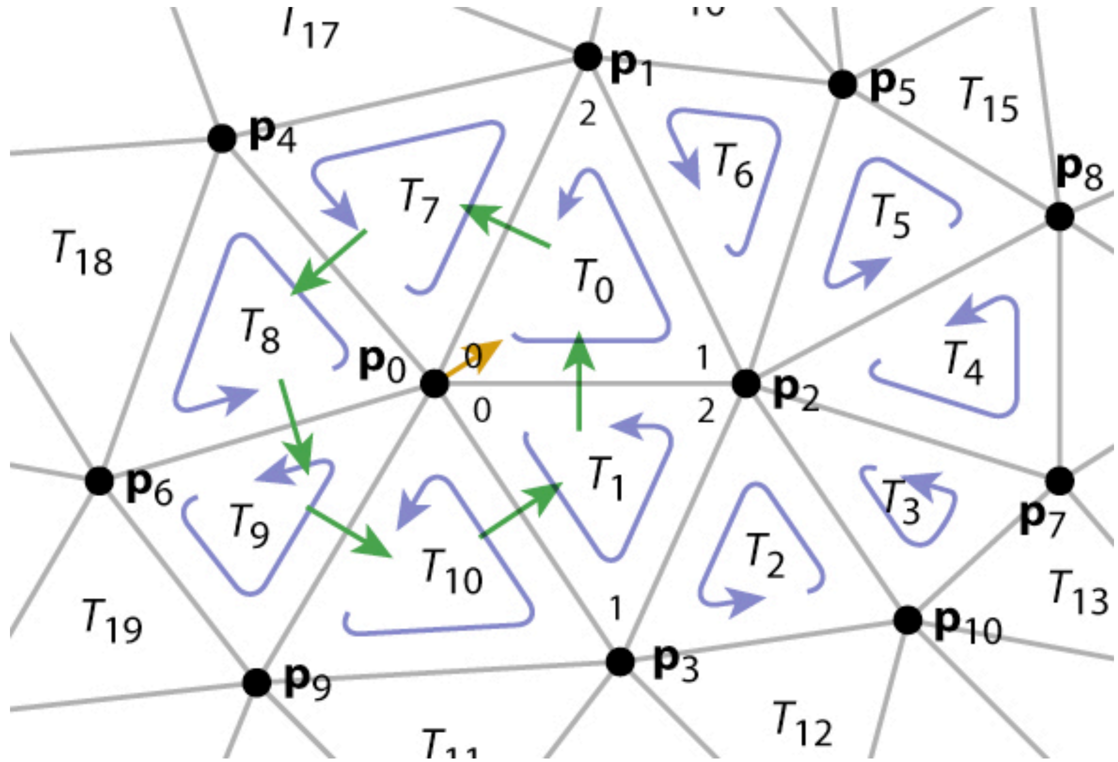
tInd[0]	0,2,1
tInd[1]	0,3,2
tInd[2]	10,2,3
tInd[3]	2,10,7
	⋮



Triangle neighbor structure

```
TrianglesOfVertex(v) {
    t = v.t;
    do {
        find t.vertex[i] == v;
        t = t.nbr[pred(i)];
        } while (t != v.t);
    }

pred(i) = (i+2) % 3;
succ(i) = (i+1) % 3;
```



Triangle neighbor structure

- indexed mesh was 36 bytes per vertex
- add an array of triples of indices (per triangle)
  - int[n_T][3]: about 24 bytes per vertex
    - 2 triangles per vertex (on average)
    - (3 indices x 4 bytes) per triangle
- add an array of representative triangle per vertex
  - int[n_V]: 4 bytes per vertex
- total storage: 64 bytes per vertex
  - still not as much as separate triangles

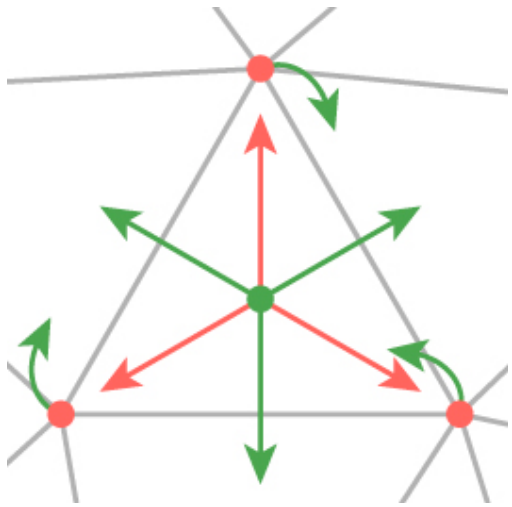
Triangle neighbor structure—refined

```
Triangle {
    Edge nbr[3];
    Vertex vertex[3];
    }

// if t.nbr[i].i == j
// then t.nbr[i].t.nbr[j] == t

Edge {
    // the i-th edge of triangle t
    Triangle t;
    int i;  // in {0,1,2}
    // in practice t and i share 32 bits
    }

Vertex {
    // ... per-vertex data ...
    Edge e;  // any edge leaving vertex
    }
```



T_0.nbr[0] = { T_1, 2 }
T_1.nbr[2] = { T_0, 0 }
V_0.e = { T_1, 0 }

Triangle neighbor structure—refined

```
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    Edge nbr[3];
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    }

// if t.nbr[i].i == j
// then t.nbr[i].t.nbr[j] == t

Edge {
    // the i-th edge of triangle t
    Triangle t;
    int i;  // in {0,1,2}
    // in practice t and i share 32 bits
    }

Vertex {
    // ... per-vertex data ...
    Edge e;  // any edge leaving vertex
    }
```

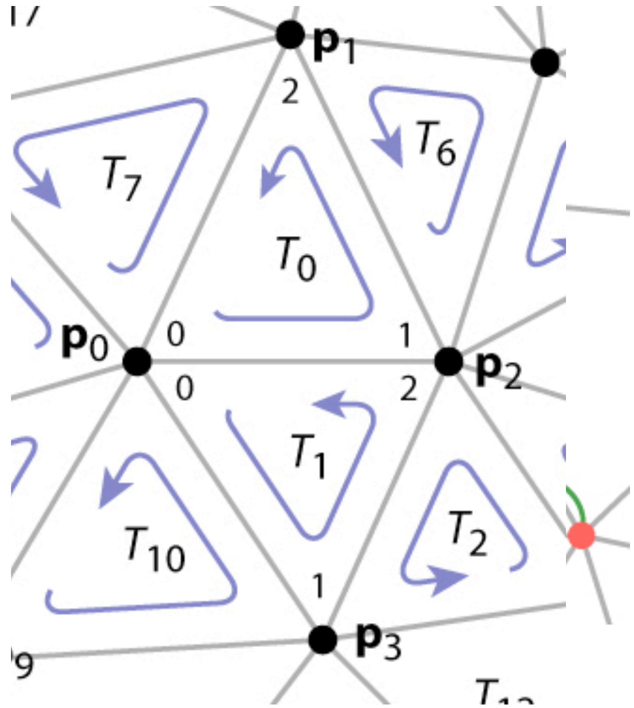

T_0.nbr[0] = { T_1, 2 }
T_1.nbr[2] = { T_0, 0 }
V_0.e = { T_1, 0 }

Triangle neighbor structure

```
TrianglesOfVertex(v) {
    {t, i} = v.e;
    do {
        {t, i} = t.nbr[pred(i)];
        } while (t != v.t);
    }

pred(i) = (i+2) % 3;
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```
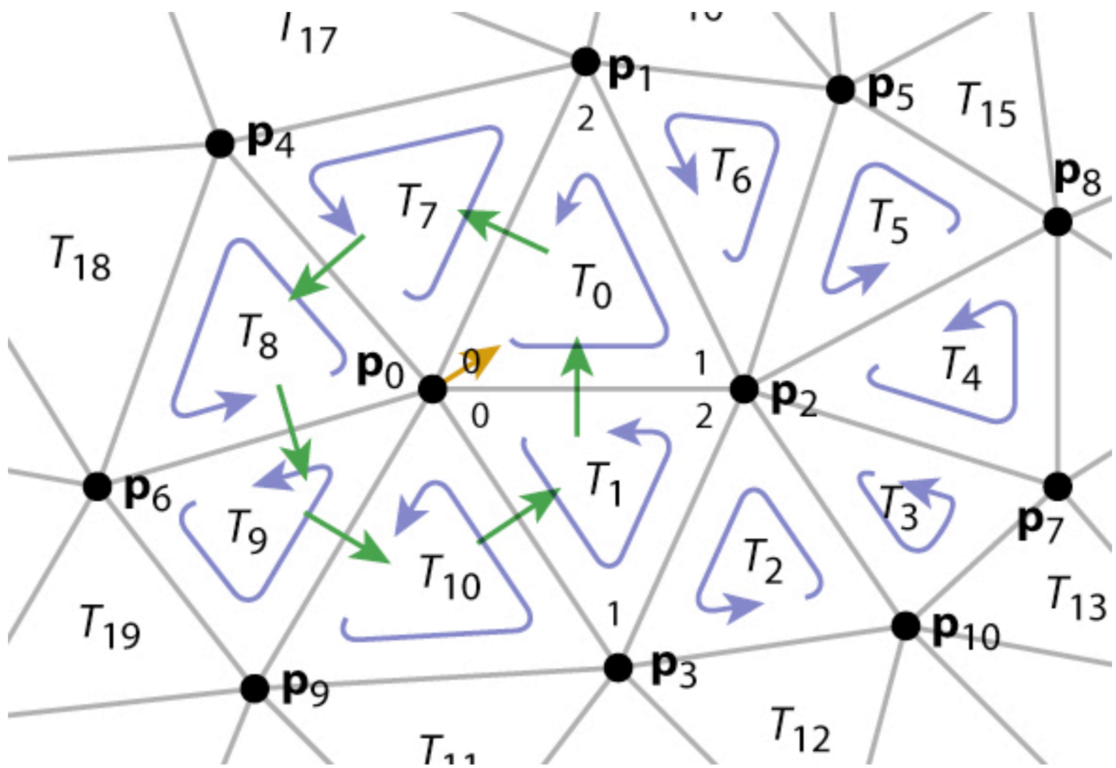



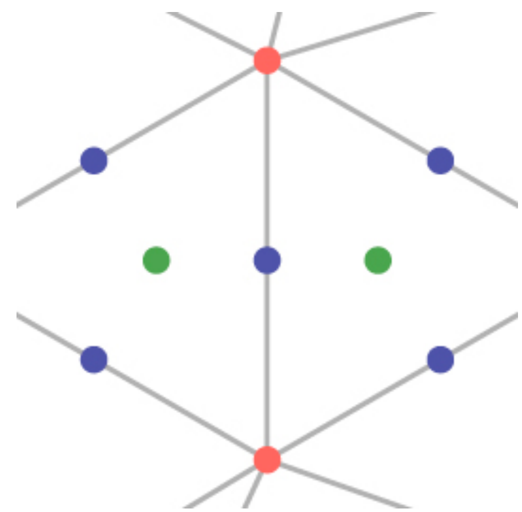
$T_0.nbr[0] = \{ T_1, 2 \}$

$T_1.nbr[2] = \{ T_0, 0 \}$

$V_0.e = \{ T_1, 0 \}$

Winged-edge mesh

- Edge-centric rather than face-centric
  – therefore also works for polygon meshes
- Each (oriented) edge points to:
  – left and right forward edges
  – left and right backward edges
  – front and back vertices
  – left and right faces
- Each face or vertex points to one edge



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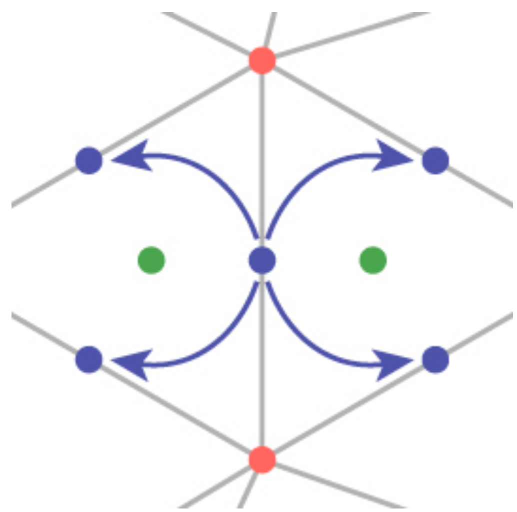

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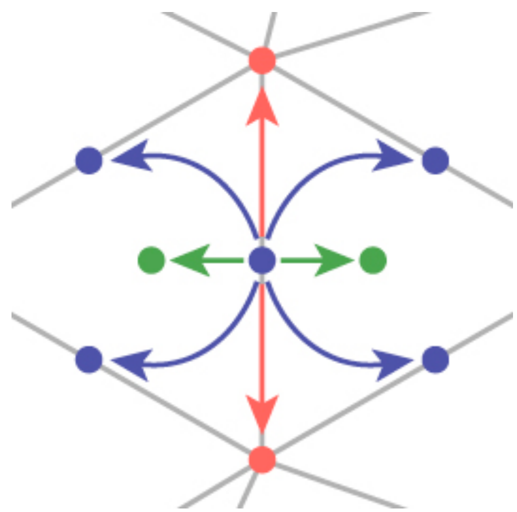
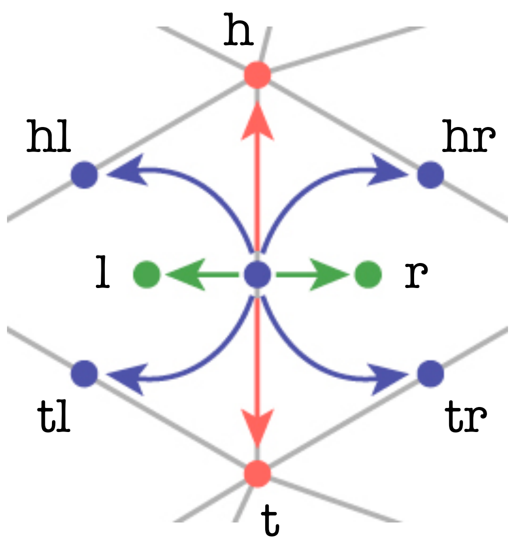


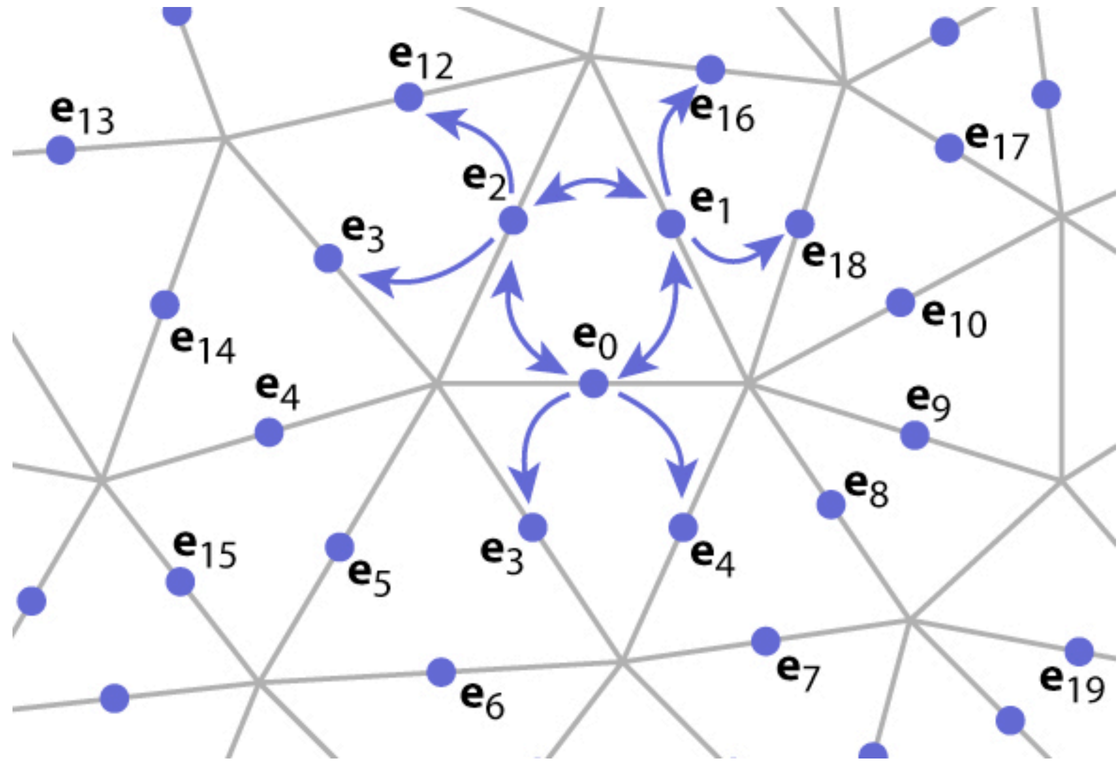
Winged-edge mesh

```
Edge {
    Edge hl, hr, tl, tr;
    Vertex h, t;
    Face l, r;
    }

Face {
    // per-face data
    Edge e;  // any adjacent edge
    }

Vertex {
    // per-vertex data
    Edge e;  // any incident edge
    }
```



Winged-edge structure

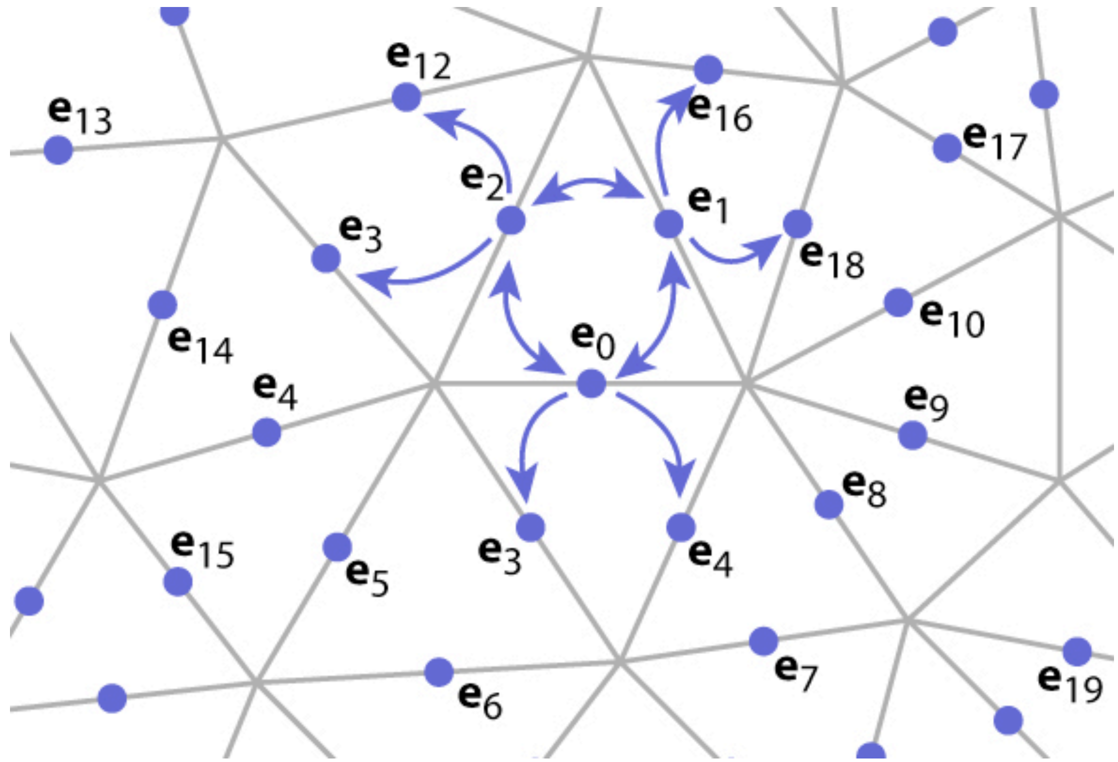
	hl	hr	tl	tr
edge[0]	1	4	2	3
edge[1]	18	0	16	2
edge[2]	12	1	3	0
		⋮		



Winged-edge structure

```
EdgesOfFace(f) {
    e = f.e;
    do {
        if (e.l == f)
            e = e.hl;
        else
            e = e.tr;
    } while (e != f.e);
}
```

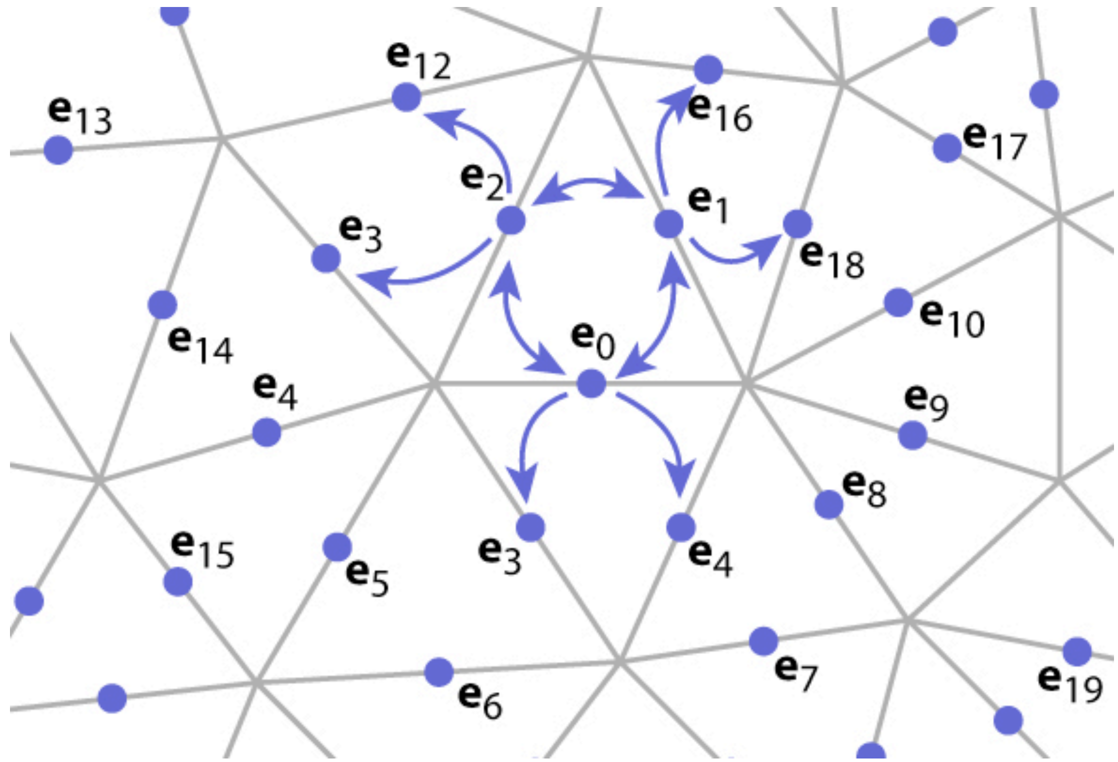
	hl	hr	tl	tr
edge[0]	1	4	2	3
edge[1]	18	0	16	2
edge[2]	12	1	3	0
		⋮		



Winged-edge structure

```
EdgesOfVertex(v) {
    e = v.e;
    do {
        if (e.t == v)
            e = e.tl;
        else
            e = e.hr;
    } while (e != v.e);
}
```

	hl	hr	tl	tr
edge[0]	1	4	2	3
edge[1]	18	0	16	2
edge[2]	12	1	3	0
		⋮		

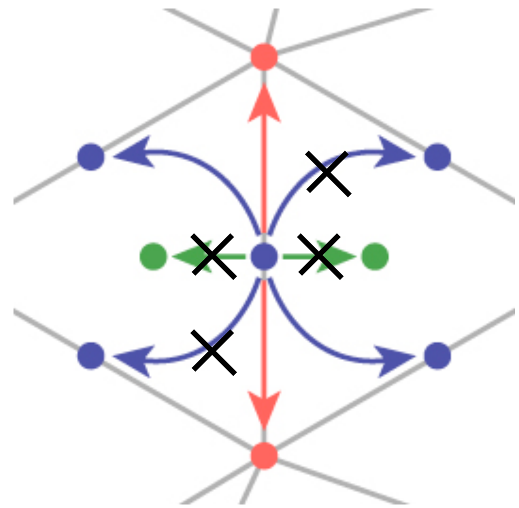


Winged-edge structure

- array of vertex positions: 12 bytes/vert
- array of 8-tuples of indices (per edge)
  - head/tail left/right edges + head/tail verts + left/right tris
  - int[n_E][8]: about 96 bytes per vertex
    - 3 edges per vertex (on average)
    - (8 indices x 4 bytes) per edge
- add a representative edge per vertex
  - int[n_V]: 4 bytes per vertex
- total storage: 112 bytes per vertex
  - but it is cleaner and generalizes to polygon meshes

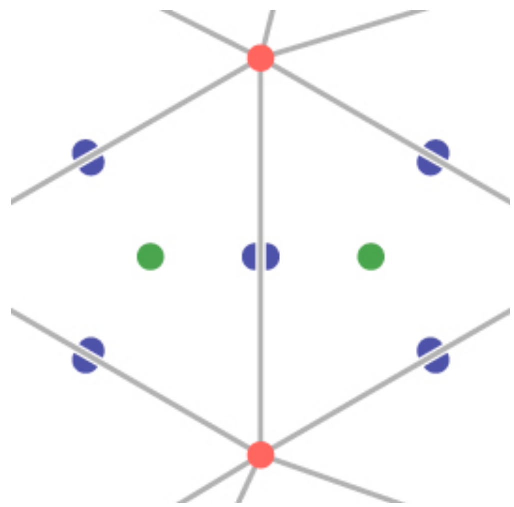
Winged-edge optimizations

- Omit faces if not needed
- Omit one edge pointer on each side
  - results in one-way traversal



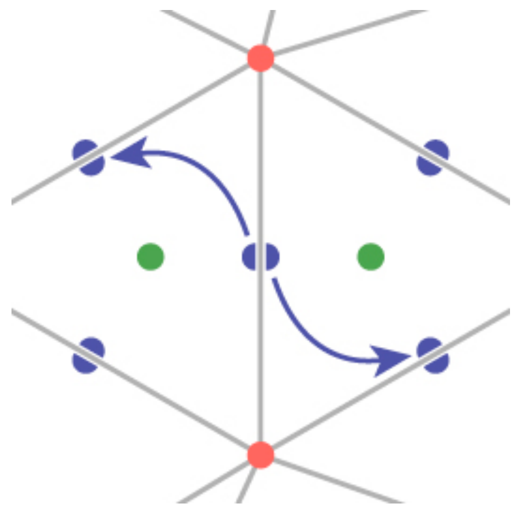
Half-edge structure

- Simplifies, cleans up winged edge
  - still works for polygon meshes
- Each half-edge points to:
  - next edge (left forward)
  - next vertex (front)
  - the face (left)
  - the opposite half-edge
- Each face or vertex points to one half-edge



Half-edge structure

- Simplifies, cleans up winged edge
  - still works for polygon meshes
- Each half-edge points to:
  - next edge (left forward)
  - next vertex (front)
  - the face (left)
  - the opposite half-edge
- Each face or vertex points to one half-edge



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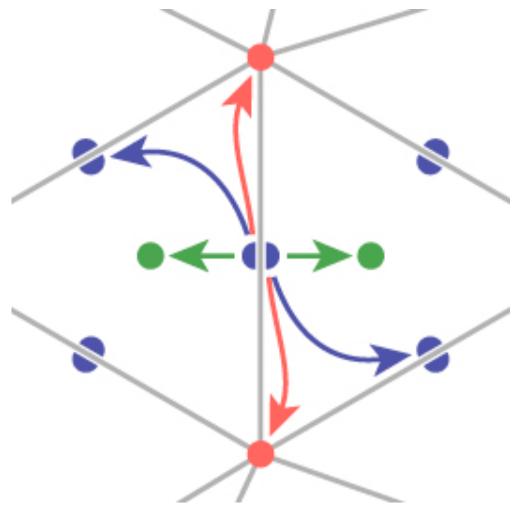
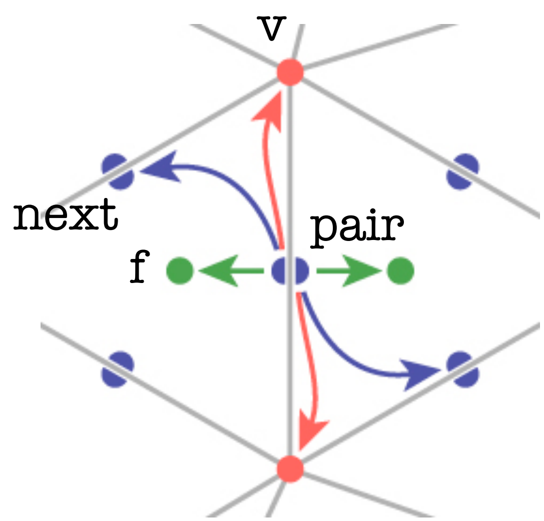


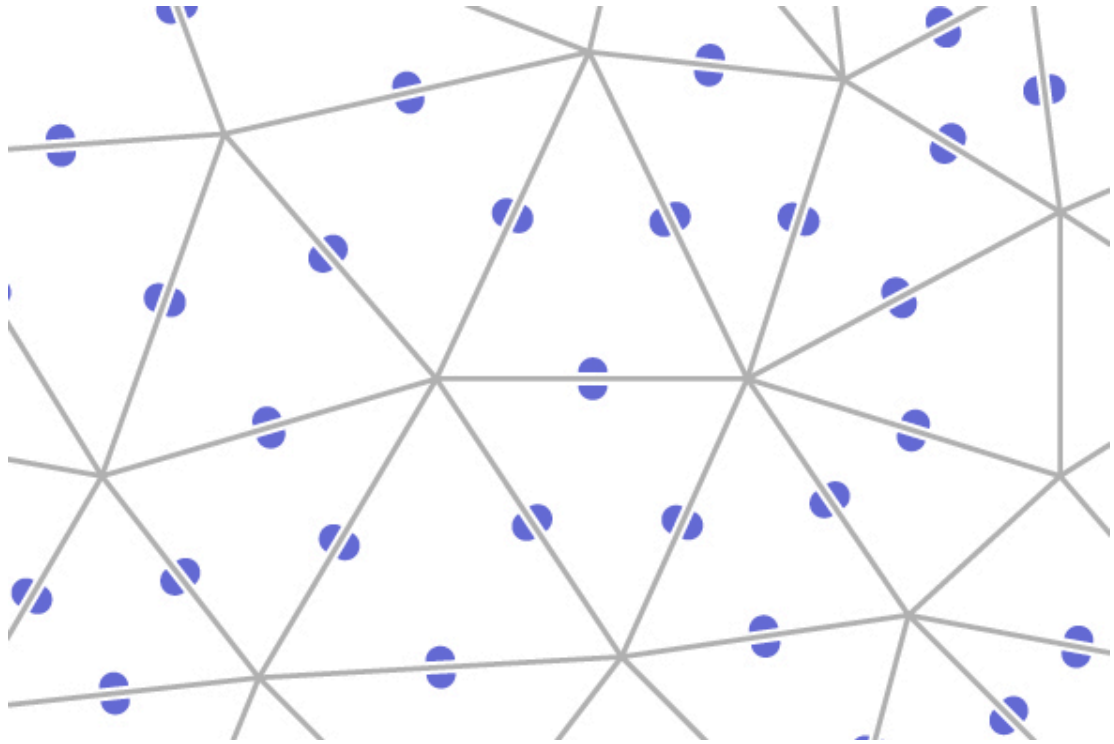
Half-edge structure

```
HEdge {
    HEdge pair, next;
    Vertex v;
    Face f;
    }

Face {
    // per-face data
    HEdge h;  // any adjacent h-edge
    }

Vertex {
    // per-vertex data
    HEdge h;  // any incident h-edge
    }
```



Half-edge structure

	pair	next
hedge[0]	1	2
hedge[1]	0	10
hedge[2]	3	4
hedge[3]	2	9
hedge[4]	5	0
hedge[5]	4	6
	⋮	



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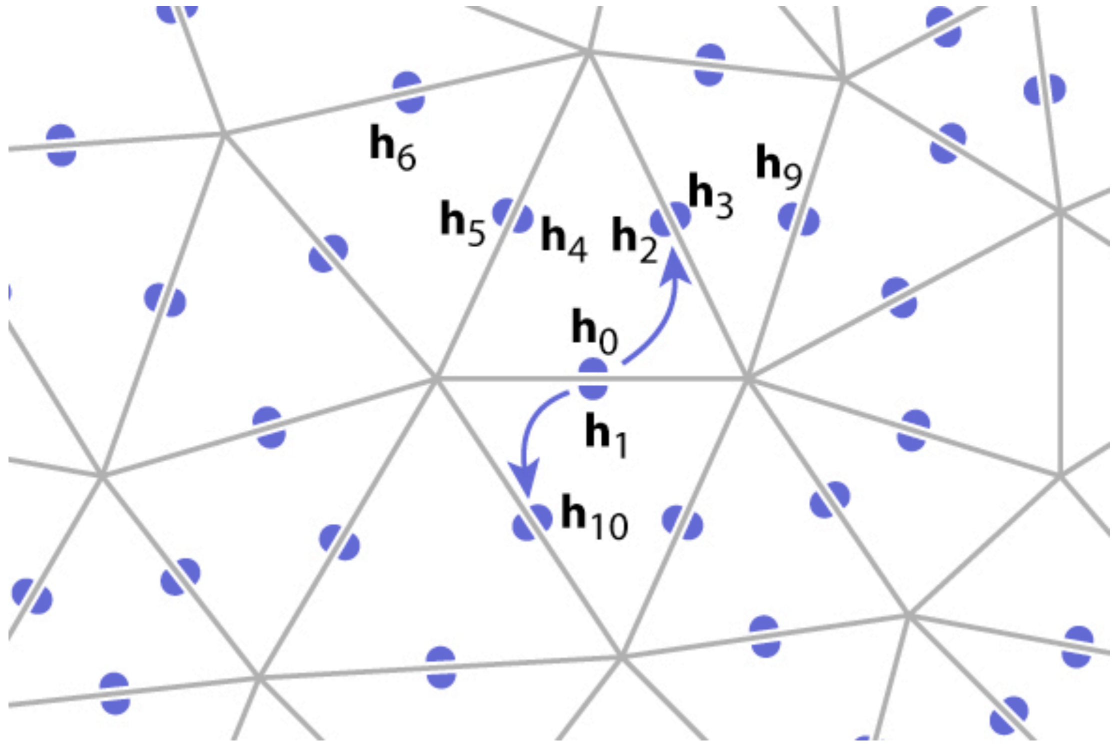

Half-edge structure

	pair	next
hedge[0]	1	2
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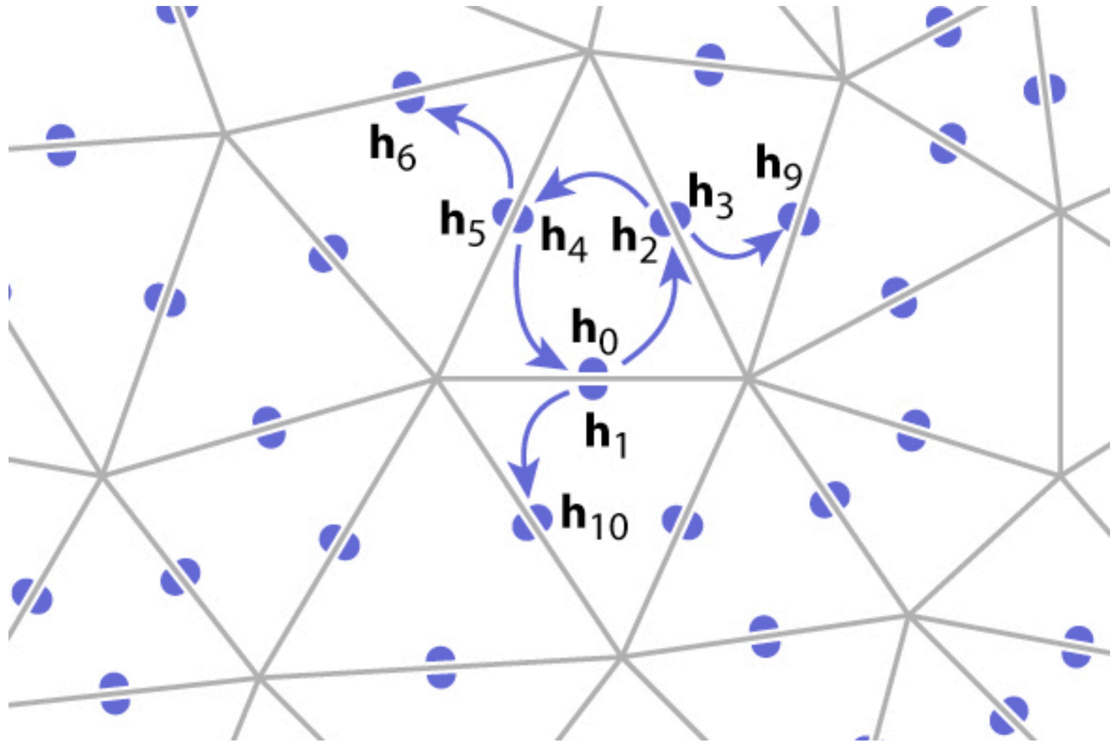
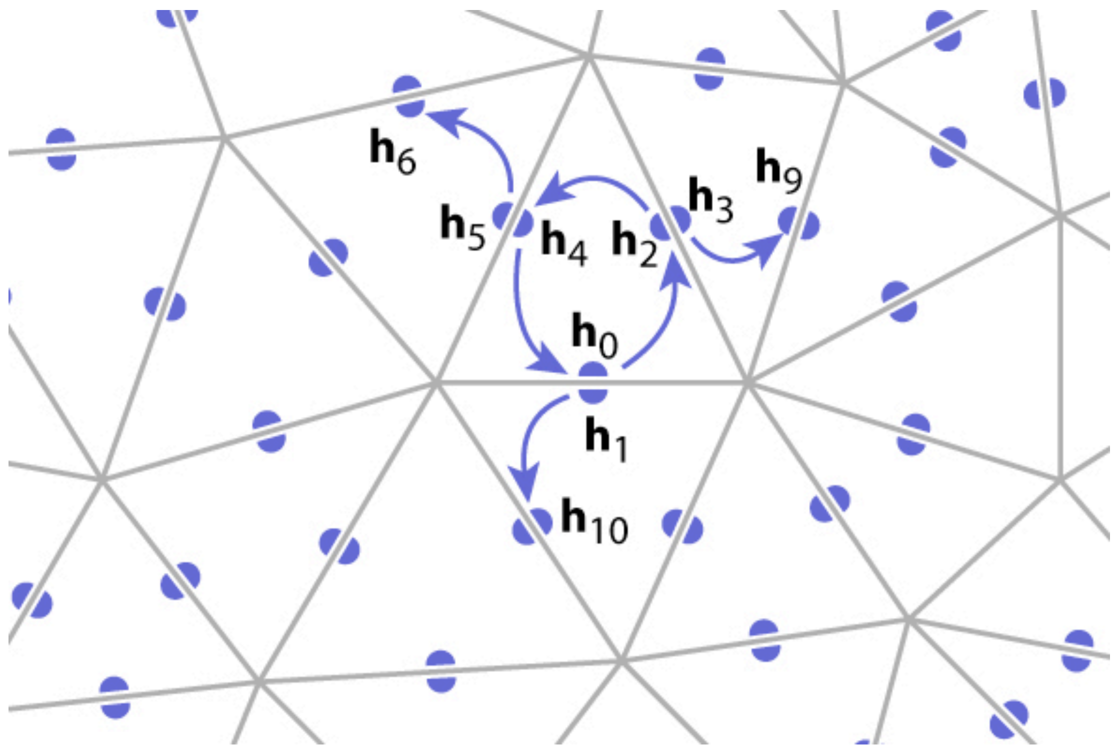


Half-edge structure

```
EdgesOfFace(f) {
    h = f.h;
    do {
            h = h.next;
        } while (h != f.h);
    }
```

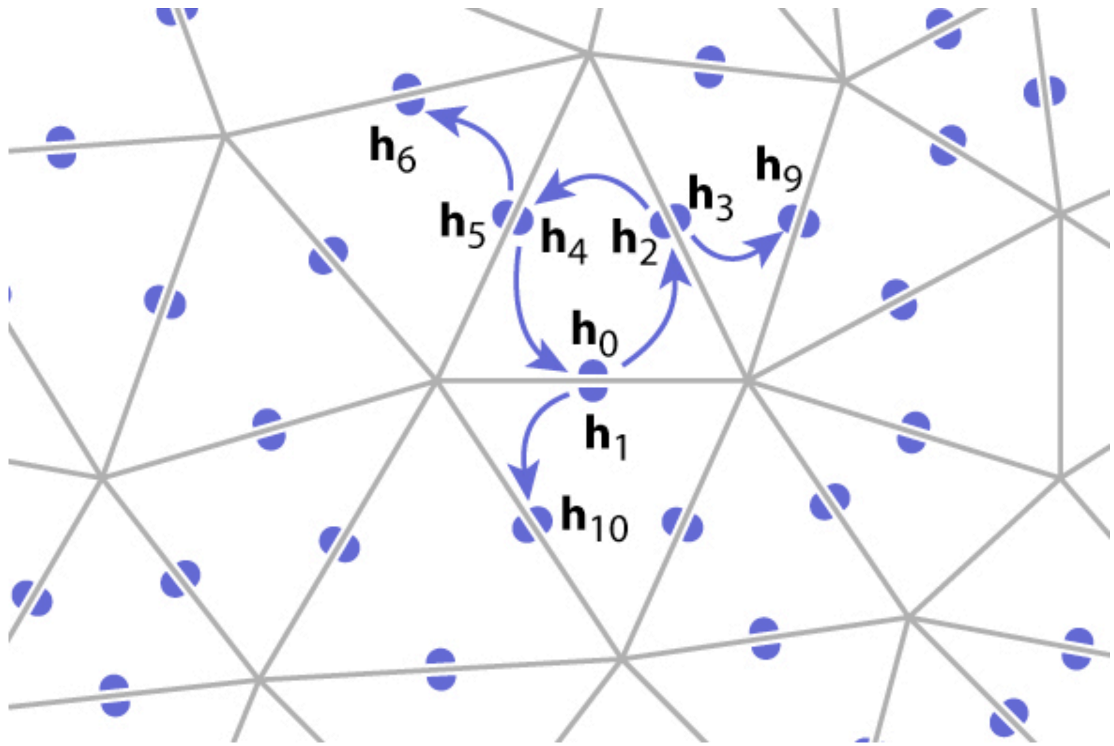
	pair	next
hedge[0]	1	2
hedge[1]	0	10
hedge[2]	3	4
hedge[3]	2	9
hedge[4]	5	0
hedge[5]	4	6
	⋮	



Half-edge structure

```
EdgesOfVertex(v) {
    h = v.h;
    do {
            h = h.pair.next;
        } while (h != v.h);
    }
```

	pair	next
hedge[0]	1	2
hedge[1]	0	10
hedge[2]	3	4
hedge[3]	2	9
hedge[4]	5	0
hedge[5]	4	6
	⋮	



Half-edge structure

- array of vertex positions: 12 bytes/vert
- array of 4-tuples of indices (per h-edge)
  - next, pair h-edges + head vert + left tri
  - int[2n_E][4]: about 96 bytes per vertex
    - 6 h-edges per vertex (on average)
    - (4 indices x 4 bytes) per h-edge
- add a representative h-edge per vertex
  - int[n_V]: 4 bytes per vertex
- total storage: 112 bytes per vertex

Half-edge optimizations

- Omit faces if not needed
- Use implicit pair pointers
  - they are allocated in pairs
  - they are even and odd in an array