Household Liquidity Policy*

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Abstract

We assess 'household liquidity policy', a novel approach to stimulating aggregate demand that relies on relaxed regulation instead of conventional fiscal tools. We analyse the effectiveness of these liquidity policies, focusing on a form that was widely used during the Covid–19 pandemic: early access to retirement savings accounts. In a heterogeneous agent model with retirement and present–biased households we find both liquidity and conventional fiscal policies can achieve similar boosts to aggregate consumption but have different distributional implications. Relative to fiscal policy, liquidity policy benefits wealthier workers, retirees, and future generations, due to its lower tax burden and added flexibility, but it is also regressive. Liquidity policy shifts the future financial burden of present–day stimulus onto poorer and more present–biased workers, who only feel the impact when it is too late to adjust.

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1 Introduction

Suppose the government wants to stimulate aggregate demand. The conventional way to do this is with government transfers or spending, funded by deficits. Such 'fiscal stimulus' boosts demand in part because people who are liquidity constrained tend to have high marginal propensities to consume (MPC) and so they spend more when they receive extra income (Kaplan and Violante, 2014; Fagereng et al., 2021; Carroll et al., 2021; Aguiar et al., 2024). The key to this mechanism is that policy relaxes household liquidity constraints, not necessarily that households receive more income. In principle, other policies that relax liquidity constraints can also stimulate aggregate demand, even if such policies leave total household resources unchanged. One example of this alternative approach is to give households early access to their otherwise illiquid retirement savings during periods of aggregate distress.[1] These policies were used sporadically for many years, but became common during Covid–19, with more than thirty countries granting some form of relief from retirement saving regulations (OECD, 2021). We call stimulus policy that uses regulation, rather than the government budget, 'household liquidity policy'.

In this paper, we compare household liquidity policy with conventional fiscal stimulus. Focusing on liquidity policies that relax retirement regulations, our paper makes two main sets of contributions. First, we develop a heterogeneous–agent model that captures the key tradeoffs to each of the different approaches, providing the first comparison of liquidity policy with fiscal stimulus. These tradeoffs stem from the difference in funding—fiscal policy is funded with future taxes that cause distortions and redistribution, whereas liquidity policy is self–funded, thereby causing no distortions nor redistribution, but also reducing the ability to commit resources to the future for those who need it.

Our second contribution is to show that, whilst both approaches are capable of the same short–term stimulus, liquidity policy is regressive. Comparing the two approaches, liquidity policy is more beneficial to wealthy workers, retirees and future generations, but for reasons unrelated to the stimulus itself—these individuals value the option to rebalance their portfolios, and wish to avoid the future taxes (and their distortions) implied by fiscal policy. Pursuing stimulus via liquidity policy effectively privatises the cost of aggregate demand management, and this cost is paid by the people who actually spend the money (the less wealthy, and more present–biased, workers) when they retire and find they have less to live on. Choosing liquidity policy therefore places the burden of stimulus on the shoulders of precisely the people retirement policy seeks to protect.

To begin, we develop a quantitative model that allows us to compare the aggregate and distributional effects of the two different policies. Our starting point is a two–asset heterogeneous–household model featuring idiosyncratic risk and incomplete markets, where households save in a liquid account, with a borrowing constraint, or illiquid account. The two-asset framework helps to

[1] By no means the only example, others of which include student debt, mortgage repayment, or rent deferrals; changes in regulations limiting collateralised loans; or changes to mandatory pre–payment of income taxes.

ensure that there is demand for liquidity, from households that are close to their constraints.[2] We combine this household with a fiscal authority constrained by a fiscal rule, yielding the standard heterogeneous–agent environment for analysing fiscal stimulus (as in Kaplan and Violante, 2014; Auclert et al., 2024; Bayer et al., 2023).

To also analyse liquidity policy, we extend this model with three important features to capture the key tradeoffs. First, we include an overlapping–generations life–cycle with work and retirement phases (Blanchard, 1985; Yaari, 1965). This creates a need to provide for retirement, in addition to the standard precautionary saving motive. Second, a portion of the population is subject to naive present–bias, leading to over–consumption (and under–saving) in the present (Laibson, 1997; Maxted et al., 2024; Maxted, 2024). As a result, our model captures one of the principal rationales for government intervention in retirement saving—the view that many individuals are myopic and lack the ability to save for retirement if left entirely to their own devices (Feldstein, 1985). In light of the above, we add realistic retirement policy, which consists of two pillars: first, a tax–funded state pension that goes to all retirees, and second, defined–contribution individual retirement accounts with realistic tax subsidies, restrictions on early withdrawal, and the possibility of mandatory contributions, depending on the calibration. This retirement account replaces the illiquid asset that is standard in two–asset heterogeneous–agent models; here, illiquidity is due to regulations to address myopia, and differences in return are due to tax concessions.

We parameterise the model with standard values, and select the degree of present bias to match empirical estimates of the aggregate MPC.[3] The retirement regulations are set in an optimal policy exercise that sets mandatory contribution rates and tax concessions to maximise the well–being of prospective newborns into the society, subject to a constraint that everyone opts in at the start of their careers. This exercise is able to rationalise the contribution rates and tax concessions we observe in mandatory DC systems around the world. An implication of this calibration is that, in the stationary solution to our model, all but the poorest workers' portfolios are over–invested in the retirement account relative to what they would choose for themselves. This is the price of helping the present–biased save for retirement: they will only save if the money is illiquid, but because the government cannot distinguish type, nor career–stage, everyone is required to save at the same rate into the same illiquid environment.

To perform the stimulus experiment, we shock the model with unanticipated transfers (fiscal policy) and early access to retirement savings (liquidity policy) designed to boost household con-

[2]Throughout this paper, we will use the term 'liquidity' or 'liquid resources' to refer to resources that can be used close–to immediately for consumption. This would include cash, bank and saving deposits, available consumer credit, and investments in securities that can be sold at will. It excludes real property and other durable investments, holdings of private companies, and contingent assets like insurance policies and most retirement accounts.

[3]We solve the model using a new algorithm developed in Sabet and Schneider (2024), which is monotone and consistent, and so robust to parameter choices. This is in contrast to existing methods for solving continuous–time HA models with multiple endogenous state variables e.g. the drift–splitting approach in Kaplan et al. (2018), which is often unstable.

sumption by the same amount over one quarter. Matching the short–term stimulus from the two approaches allows us to compare their long–run implications. There are three main differences (i) the tax changes driven by the fiscal rule distort inter–temporal consumption smoothing, and this distortion is much greater under fiscal than liquidity policy; (ii) these taxes also cause redistribution, with lower future consumption by retirees and future generations subsidising the transfers received by workers; (iii) liquidity policy undermines retirement adequacy for present–biased workers, reducing their consumption upon retirement, and more so for the individuals who were initially less wealthy.

After describing the mechanisms through which the two policies operate, we then use our model to quantify their relative importance for household well-being. We find the compensating variation that would make each household indifferent between liquidity policy and its fiscal alternative. We show that household liquidity policy is better for wealthy workers, retirees, and future generations. These individuals do not benefit much from traditional fiscal stimulus, but are still liable for higher taxes under fiscal policy. Further, wealthy workers value the ability to rebalance their portfolio more than their less wealthy counterparts. By contrast, fiscal policy is preferred by working households with low wealth, a group disproportionately comprised of present–biased households. Aggregating across this heterogeneity, we find that roughly 70 percent of households prefer liquidity policy over traditional fiscal stimulus in our baseline calibration. In short, liquidity policy may be politically popular despite its regressivity, as it concentrates the costs of stimulus on a relatively small subset of society, namely the present–biased and low–wealth workers.

The degree to which one approach dominates the other depends on how the government sets retirement policy, and its fiscal rule. Aggregating the compensating variations, liquidity policy is marginally better for society the stricter is retirement policy, and the more aggressively the fiscal rule retires government debt. For example, liquidity policy is welfare *improving* for all but the poor present–biased types in our baseline calibration. This is because it allows some portfolio re–balancing, particularly valuable to wealthier workers. In an alternative calibration with lower mandatory contribution rates, wealthy workers are less over–invested in their retirement accounts, and there is much lower benefit from liquidity policy as a result. Similarly, the baseline calibration is based on an exponential fiscal rule that sets a half–life for debt gaps of 14 years (as in Galí, 2020). This rule front–loads the cost of stimulus under fiscal policy more than liquidity policy because debt is 'retired' at a faster rate than workers retire. If the fiscal rule is instead relaxed to match the rate of retirement, or be even looser, then the implied taxes are less onerous and the relative benefit of liquidity policy is reduced.

Related literature. This paper brings together two large strands of literature. On one hand, the influential heterogeneous agent macro literature explains fiscal and monetary policy transmission based on liquidity constraints and the distinction between liquid and illiquid assets (see e.g. Kaplan and Violante, 2014; Kaplan et al., 2018; Bayer et al., 2019; Auclert et al., 2024). While this literature generally assumes that retirement accounts play an important role in household illiquidity, retirement policy is assumed exogenous, and none of these papers consider policies that alter the illiquidity of these accounts. On the other hand, there is a growing public economics literature that evaluates retirement policy and the optimal degree of illiquidity in retirement saving systems (see e.g. Amador et al., 2006; Moser and Olea de Souza e Silva, 2019; Beshears et al., 2020a; Andersen et al., 2024; Beshears et al., 2020b). While these papers assume that the level of illiquidity in retirement systems is a societal choice, they are largely silent on macroeconomic considerations related to fiscal stimulus. To the best of our knowledge, our paper is the first to offer a positive and normative evaluation of household liquidity policy relative to traditional fiscal stimulus.

A growing empirical literature analyses past episodes of household liquidity policies (Argento et al., 2015; Kreiner et al., 2019; Andersen, 2020; Hamilton et al., 2024; Preston, 2022; Schneider and Moran, 2024; Shapiro and Slemrod, 1995). We bring these stimulus packages together under the banner of household liquidity policy and analyse them theoretically in a modeling environment that allows for direct comparison with conventional fiscal policy. This allows for positive and normative comparisons of the two approaches, which may help to design future stimulus packages.

Our work complements Hamilton et al. (2024), which analyses Australia's early withdrawal program during Covid–19, using detailed micro data to identify who withdraws and what they do with the money. The authors show empirically that around one in six working age people withdrew, the modal withdrawal was all of the $20,000 allowed, and these households on average spent 40% of the funds within eight weeks. They argue that this is evidence of present–bias, which they estimate in a structural model. Our paper makes a different but complementary contribution. While the above authors identify the MPC and the strength of present-bias, we take present-bias as given, and instead develop a model that captures the key trade-offs between household liquidity policy and traditional fiscal policy. This allows us to perform the first positive, normative, and distributional comparison of these two different approaches to stimulus.

Our model is also informed by empirical work by Schneider and Moran (2024), who use a survey-elicited measure of psychological self-control, combined with the early release of retirement wealth in Australia, to estimate the relative importance of behavioral biases versus situational factors in accounting for early withdrawal from retirement accounts. The authors find that self-control heterogeneity plays an important role in predicting early withdrawal, and is a stronger predictor than other behavioural factors such as financial literacy, planning horizons, or personality traits. Overall, individuals in the top quintile of self-control issues are 60% more likely to withdraw than those in the bottom quintile. Informed by these empirical results, we also incorporate heterogeneity in present-bias into our quantitative model, evaluate how the two policies affect long-term retirement adequacy, and examine how the welfare implications of liquidity policy differ for individuals with versus without present-bias.

Some papers use quantitative models to explore the role of retirement accounts in stimulating the economy, but none compare liquidity policy to traditional fiscal policy. Love (2017) proposes a policy to stimulate the economy using counter–cyclical matching to retirement contributions, which he evaluates in a life-cycle model. Graves (2023) develops a HANK model to analyze the flight–to–liquidity that occurs following an increase in unemployment. He conducts one counterfactual exercise showing the effect of lower withdrawal penalties on aggregate consumption during Covid–19. Finally, Kaplan et al. (2020b) explore the tradeoff between health outcomes and economic impacts of policy choices during Covid–19 in the USA. They combine a HANK model with an SIR module of disease transmission, and use it to assess the impact of the various economic and health policies used in the USA. A part of the CARES Act that they model is the USA's removal of the withdrawal penalties from individual retirement accounts, as in Graves (2023), but this is not the focus of their analysis. Our paper (1) characterises the different channels through which household liquidity policy differs from traditional fiscal policy, (2) evaluates the distributional implications of the two policies, and (3) conducts a welfare analysis of the two policies, something that no previous paper has attempted.

We contribute to the broader heterogeneous–agent macro literature by providing a new micro foundation for the illiquid accounts commonly featured in two-asset macro models (Kaplan and Violante, 2014; Kaplan et al., 2018; Bayer et al., 2019; Auclert et al., 2024). This illiquidity is generally modelled as an exogenous feature of the world, when in reality it is usually a result of government policy. Empirically, household budgets are made up of only two types of genuinely illiquid asset: housing and retirement savings (Fagereng et al., 2019). In both cases, much of the illiquidity is due to regulation, e.g. restrictions or penalties on withdrawals from retirement accounts, and limits to home equity withdrawal. Modelling it as such opens the option for liquidity policy in our environment. Our modelling approach also builds upon Attanasio et al. (2024) and Maxted et al. (2024) who show the importance of present–bias for hand-to-mouth behaviour and fiscal policy, but do not consider household liquidity policy.

Finally, we contribute to the literature about retirement system design. A common thread in this literature is that imposed illiquidity is justified to help households overcome biases in their decision–making. The government has a role in mandating some form of retirement saving, and faces a problem of how to optimally balance the long–run commitment that households need against the short–run need for flexibility to insure working–life idiosyncratic risk, and also to balance the welfare of the behaviourally biased against those who are not (Beshears et al., 2020a; Moser and Olea de Souza e Silva, 2019; Amador et al., 2006). Our government faces the same type of problem, but is constrained to consider mandatory DC systems, as implemented in many countries (OECD, 2023). We show that the optimal DC system involves mandatory contributions close to those actually observed, but does not involve any tax concessions, in stark contrast to the systems in place. Instead, we rationalise these tax concessions as necessary to encourage people to opt into the system, at least at the beginning of their careers. As such, these concessions serve a political, rather than policy, purpose.

One major feature of the public literature on retirement system design and reform is the impact of retirement systems on the decision to retire. These papers (e.g. Blundell et al., 2016; Kolsrud et al., 2024) emphasise the distorting effects retirement policy can have on labour supply and estimate the optimal design subject to these distortions and the fiscal externalities they impose. We abstract entirely from the retirement decision in this paper. Rather we take retirement to be a fact, which creates a need for extra savings, but we let the event itself arrive randomly. This simplifies the problem by removing a household decision without undermining the focus of our analysis, which is to explore the impact of different approaches to stimulus for a given retirement system.

Road map The paper proceeds as follows. In Section 2 we describe defined contribution retirement schemes, the relevant institutional setting for the paper, and identify two key parameters that we will map to our model. In Section 3 we detail the model, and calibrate its standard parameters in Section 4. In Section 5 we set up and solve the government's optimal retirement policy problem and show that this approach rationalises the key parameters identified in Section 2. We then turn to the stimulus policy experiments in Section 6, showing the two approaches are similar in aggregate, but have different distributional implications. We evaluate their differences with a welfare analysis in Section 7, finishing with robustness checks. Section 8 concludes.

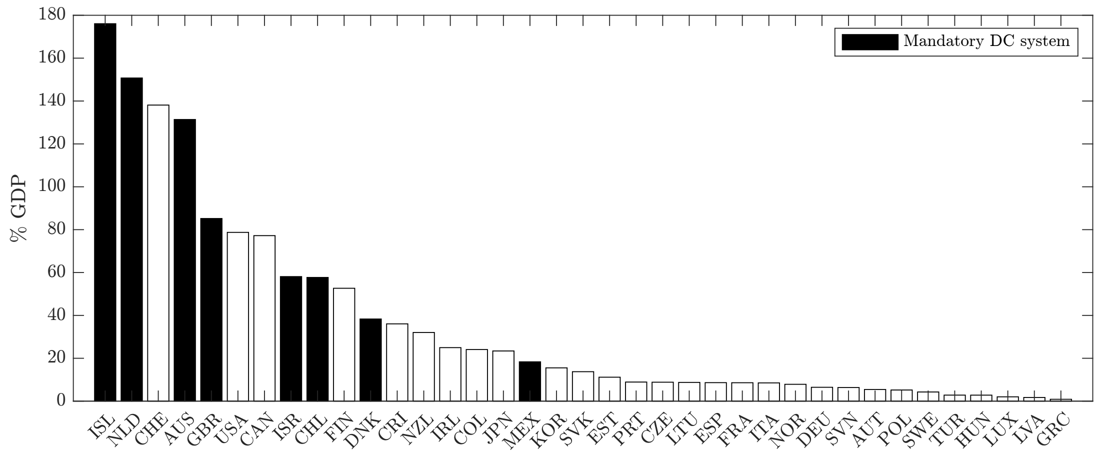
2 Defined contribution retirement accounts

The need to provide for retirement is universal and government intervention to meet some of this need is also common. The government's involvement is justified for a variety of reasons. These include a redistributive motive, to help avoid poverty in retirement, a protective motive, to short–circuit the moral hazard created by the redistributive motive (i.e. households neglect to save for retirement, anticipating the government will bail them out), and a paternalistic one, correcting for biases that reduce working–life saving (Feldstein, 1985; Beshears et al., 2015). The means with which countries address these needs vary a lot, but usually involve some mix of state and private provisions, with the latter becoming increasingly important as many governments grapple with the strain of unfunded state pension provisions coupled with ageing populations (OECD, 2018).

Our focus in this paper is on countries with private, mandatory defined contribution (DC) schemes.[4] In DC pension systems, working–age people make contributions into regulated investment

[4]Such schemes are common, and increasingly being adopted as countries attempt to reduce the fiscal burden of state–only systems facing ageing populations (OECD, 2018). Among OECD countries, for example, 20 have some form of regulated, private retirement savings vehicle—Australia, Belgium, Canada, Chile, Costa Rica, Denmark, Estonia, Germany, Iceland, Ireland, Israel, Lithuania, Mexico, Netherlands, New Zealand, Norway, Sweden,

Figure 1: Retirement system assets



Source: values from OECD 2022 Total Pension Funds' Assets, % of GDP; classification as mandatory DC system from (OECD, 2023, Table 4.2)

accounts, and they have a claim on the balance and accumulated returns upon retirement; these systems can build up substantial resources, as illustrated in Figure 1, which plots total retirement assets across OECD countries. The exact design features of DC accounts differ across countries, but they can be broadly understood in terms of rules defining (a) liquidity during working life, (b) contributions, (c) tax treatment, and (d) the state pension they are combined with.

Liquidity during working life Regulations affecting access to DC accounts differ across countries. In some settings, like the USA and UK, participants are allowed to withdraw prior to retirement, but they pay a penalty (10% in the USA, 55% in the UK). In others, like Australia, withdrawals are not allowed except in dire personal circumstances (e.g. terminal illness or extreme financial hardship) effectively making the accounts completely illiquid. In either case, the illiquidity is created by regulation, rather than because the assets are difficult to transact in.

Contributions Regulations affecting contributions generally govern (a) whether any contributions are mandatory and (b) limits on voluntary contributions. Mandatory contributions, when they exist, are usually set as a proportion of pre–tax employment income, commonly in the range of 10–20% (see Table 1 for some examples from OECD countries). Voluntary contributions to these schemes are often allowed as well, but are usually limited to maximum nominal amounts per year because they attract tax concessions.

Switzerland, United Kingdom, United States (OECD, 2023). Of these, 8 mandate contributions into funded DC schemes—Australia, Chile, Denmark, Iceland, Israel, Mexico, Netherlands, United Kingdom (OECD, 2023), where the UK's contributions are a default, rather than mandatory.

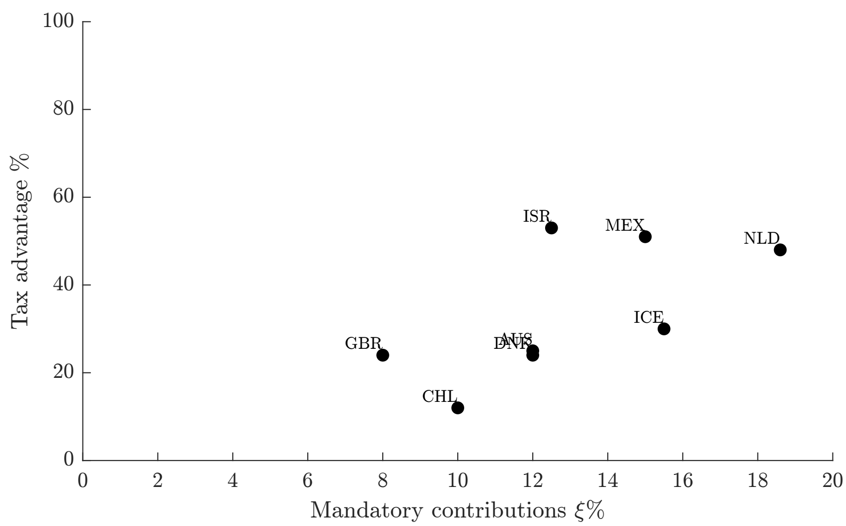
Table 1: Mandatory contribution rates in OECD DC systems

	AUS	CHL	DNK	ICE	ISR	MEX	NLD	UK*
ξ (p.p.)	12	10	12	15.5	13	15	18.6	8

Source: OECD (2023) Table 3.4, p. 141. For all OECD countries with privately funded DC schemes, and no other mandatory private system. *The UK's is a default, rather than mandatory.

Tax treatment There are three potentially taxable flows in DC systems—contributions, investment returns, and withdrawals—and systems differ in whether each of these is taxed (potentially at concessional rates) or exempt, leading to a three-letter code describing them. In a system like USA 401(k)s, for example, contributions are made from pre–tax income, and returns are exempt as well, but withdrawals are taxed at the personal marginal tax rate, so it is coded EET. This is the most common approach. By contrast, Australia's Superannuation contributions from pre–tax income are taxed at a concessional 15% rate, as are returns, and withdrawals are tax free, so it is coded TTE.[5]

Figure 2: DC system design in OECD countries



Source: mandatory contribution rates (OECD, 2023, Table 3.4) and tax advantage (OECD, 2018, Table 3.2)

Figure 2 shows the combinations of mandatory contribution rates (ξ) and tax advantages across the collection of OECD countries with mandatory DC systems.[6] The tax advantage variable comes from OECD (2018), and represents the tax savings from a given flow of contributions over a typical

[5]In the model introduced in Section 3, we use a TTE system so that we can control the tax concession granted for contributions and returns inside the account. EET systems, whilst more common, typically apply taxes at full marginal rates, and so offer fewer degrees of freedom.

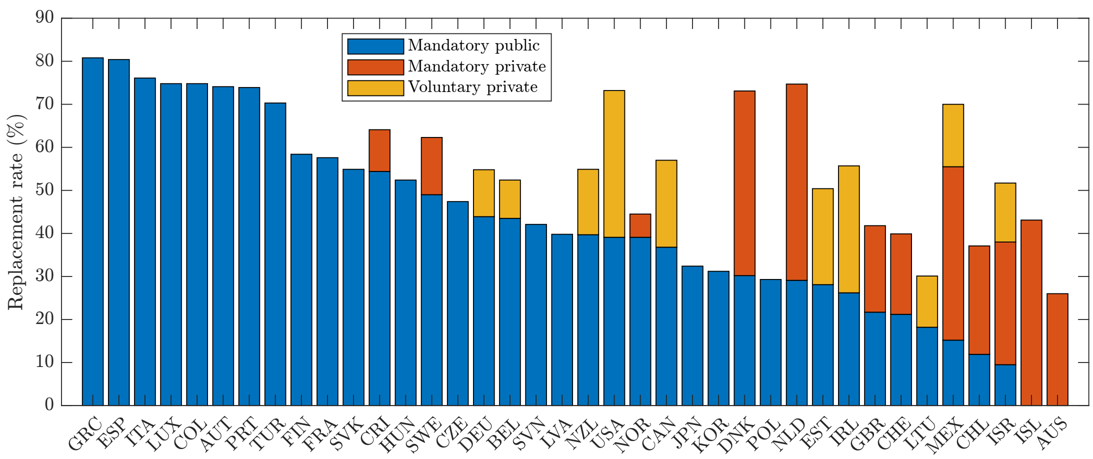
[6]Note this excludes countries that mix these with other mandatory private options.

working life, relative to if they were saved in a regular investment account. Tax advantages are ubiquitous in these accounts, though they range from quite small (worth a discount of around 10% in Chile) to substantial (around 50% in Israel and Mexico).

In the model introduced in Section 3, the retirement system features parameters encoding mandatory contributions and tax concessions, and we rationalise the features seen here as the optimal policy when society features a portion of the population that is present–biased, and the government needs to ensure workers to opt–in to the system at the start of their careers. The tax concessions, then, serve the *political* purpose of building buy–in into the system.

State pension DC systems are usually introduced to reduce the burden of retirement provision on current taxation by shifting the responsibility onto households themselves. In mandatory DC systems, government retirement provision is usually still present, but less generous. We can see this by looking at the state pension replacement rate for an average income earner across countries with and without mandatory private systems, plotted across OECD countries in Figure 3. This replacement rate for an average earner is 31% on average for OECD countries with mandatory systems, compared with 56% on average for OECD countries with only mandatory public systems, a substantial difference.[7]

Figure 3: Retirement system replacement rates for average earners, OECD countries



Source: OECD (2023) Table 4.2

Early access as stimulus In response to economic crises, many countries have used access to these pools of resources to stimulate demand. Denmark was one of the first to introduce such

[7]These are population weighted averages of the mandatory public gross replacement rates for an average earner across OECD countries with and without a mandatory private system in place (OECD, 2023, Table 4.2)

measures during the Global Financial Crisis, allowing early withdrawals and temporary suspensions of contributions (Kreiner et al., 2019). This approach became more widespread during the Covid-19 pandemic, in which three approaches were used across at least 31 countries: allowing limited withdrawals when they were previously banned, as seen in Australia, Chile, and Peru (Hamilton et al., 2024; Madeira, 2022; OECD, 2021); removing withdrawal penalties, as in the United States (Graves, 2023); and reducing or deferring mandatory contributions in countries like Singapore, Malaysia, and Vietnam.

These policies resulted in significant outflows from retirement systems, with the most dramatic cases observed in Latin America. In Peru, a staggering 18.3% of assets in retirement savings plans were withdrawn, followed closely by Chile at 14.6%. Iceland and Australia saw smaller, but still substantial, withdrawals during Covid–19 of 3% and 1.4% of assets, respectively (OECD, 2021, p. 27).

In the remainder of the paper we will analyse traditional fiscal and liquidity policy in an economy with government intervention into retirement policy motivated by paternalistic concerns. The model detailed in the next section features present–biased households who unwittingly save too little, and the government responds by forcing them to save in a defined contribution scheme with the features described above—limits on working–life withdrawal, mandatory contributions from labour income, and tax concessions. The liquidity policy we explore in Sections 6 to 7 is early withdrawal opportunities, like those used in Australia, Peru, and Chile. An alternative would be to reduce mandatory contribution rates. This works as stimulus as well, but it is more regressive because it grants relatively more liquidity to higher earners. We focus on withdrawal opportunities because it is the most directly comparable to fiscal transfers.

3 A model of households with illiquid retirement accounts

Our environment is a continuous–time, infinite horizon model featuring a measure of households, and a fiscal authority. Prices (the interest rate and wage) are fixed, and the stationary equilibrium between households and fiscal authority is reached with a tax rate that balances the government budget.

3.1 Households

Households are differentiated by their stage of life (working or retired) and four time–varying state variables: their idiosyncratic productivity z and employment state, the balance in their liquid account b, their illiquid account balance a, and their present–bias β. We collect these states into the vector $x = (b, a, z, \beta)$, where z captures life–stage, workforce status and employed productivity.

3.1.1 Life–cycle transitions

Households live through working–life and then retirement (Yaari, 1965; Blanchard, 1985), transitioning out of each phase with fixed Poisson intensities (δ_R, δ). When young, they work, and make consumption and asset allocation decisions. In retirement, they make the same decisions but can no longer receive the market wage. This creates a need for provision that is met personally by any assets they retire with, and collectively by the distribution of a fixed state–pension w_R. With these resources, retirees solve a cake–eating problem until they die, and are replaced by workers with no assets.

This lifecycle structure is a substantial simplification of how our careers typically progress. The most important departure from reality is that the transition into retirement is random, rather than a choice. This abstracts from an important part of the public economics literature that looks at the implications for retirement systems on incentives to retire, and the optimal pension reform to achieve affordability in the face of ageing populations (e.g. Kolsrud et al., 2024). Simplifying the transitions to be random gains us tractability, by reducing the households' choice set, without sacrificing structure that's important for our central question; transition rates are calibrated to match the average spans of working life and retirement.

3.1.2 Idiosyncratic risk

Working–age households are subject to two types of idiosyncratic risk to their income. First, they jump in and out of employment with Poisson finding and separation intensities (λ_f, λ_s). Second, whilst employed, their log–labour productivity is a diffusion that follows an Ornstein–Uhleneck process

$$d \ln z_t = -\theta_z(\ln z_t - \ln \bar{z})dt + \sigma_z dW_t \tag{1}$$

Where θ_z captures its persistence, $\bar{z}$ is the stationary mean, normalised to 1, W_t is a Wiener process, and σ_z is the weight on this noise. Newborn workers are employed, and draw their productivity from the stationary distribution of z. Retired workers are not subject to any idiosyncratic risk beyond the chance of dying.

3.1.3 Budget constraints

All households have access to two accounts for storing wealth—a liquid and an illiquid account. At any given point in time, households have two choice variables—consumption $c > 0$, funded from their liquid account, and voluntary transfers between the two accounts $d \in \mathbb{R}$.

Liquid account The law of motion for the liquid account is

$$\dot{b} = r_b(b)b + (1 - \xi(x))y(x) - d - \chi(d, a) - (1 + \tau_c)c + T(x) - \tau_b(x) \tag{2}$$

Drift in liquid assets comes from various sources. First, asset returns, where $r_b(b)$ is the balance–dependent rate of return on the liquid account. We assume positive balances attract a return of r_b, and that borrowing, whilst allowed, comes with an extortionate penalty $\omega >> 0$ so that $r_b(b < 0) = r_b + \omega$.[8] This assumption creates a soft–borrowing constraint, which will be important later. $y(x)$ is idiosyncratic income, assumed to be wz when working, w_U when unemployed, and w_R when retired. $T(x)$ captures fiscal transfers, c is consumption, which attracts a tax τ_c, and $\tau_b(x)$ is a state–contingent income tax function, discussed in full in Section 3.2.

Working households are required to contribute a proportion of their income $\xi(x)$ into their illiquid account (equal to zero in unemployed and retired states). As we will discuss in Section 3.2, this is one of the levers of regulation the government uses in retirement policy. As well as this, households may make voluntary transfers into (and out of) the illiquid account (d). These transfers are subject to a constraint $d \geq \gamma$ which is another lever of the government retirement policy, discussed in Section 3.2. Any voluntary transfers are subject to adjustment costs $\chi(d, a)$ which, following Kaplan et al. (2018), have the structure

$$\chi(d, a) = \chi_0(d)|d| + \frac{\chi_1}{2}\frac{d^2}{a} \tag{3}$$

The convex nature of this function puts a handbrake on the voluntary transfer policy, necessary to avoid jumps in continuous time, and its structure leads to analytical solutions for d. The linear cost $\chi_0(d)$ may differ for withdrawals and deposits.[9]

Illiquid account The law of motion for the illiquid account is

$$\dot{a} = r_a a + \xi(x)wz + d - \tau_a(x) \tag{4}$$

Where r_a is the rate of return on the illiquid asset, and $\tau_a(x)$ is a state–contingent tax function, detailed in Section 3.2. Borrowing is not allowed in the illquid account, $a \geq 0$. As we will discuss in Section 3.2, and in contrast to the standard treatment in two–asset heterogeneous agent models (e.g. Kaplan et al., 2018), this illiquid account represents the households' regulated retirement accounts.

[8]This setup reflects the empirical reality that very few households are actually borrowing constrained (Lee and Maxted, 2023), as in the wealthy hand–to–mouth literature (Kaplan and Violante, 2014). Instead, many hover close to zero liquid assets, rotate credit card balances (but not at their limit), and rarely exhaust *all* avenues for borrowing, which come with ever more onerous terms (pay–day loans, pawn shops, loan sharks etc).

[9]This allows us to impose a withdrawal penalty on the illiquid account to replicate e.g. the USA's 10% tax penalty.

3.1.4 Present–biased Preferences

A portion of the population $\mu \in [0,1]$ is subject to present–bias, the rest are standard exponential discounters. The present–biased households have 'instantaneous gratification' (IG) preferences, the continuous–time analogue to quasi–hyperbolic discounting (Harris and Laibson, 2013; Laibson and Maxted, 2023; Maxted, 2024).

In working life, unbiased households' recursive preferences are as follows[10]

$$v(x_t) = \lim_{\Delta \to 0} \max_{c,d} u(c)\Delta + e^{-\rho\Delta}\, \mathbb{E}\left[v\left(x_{t+\Delta}(c,d)\right)\right] \tag{5}$$

Where the maximisation is constrained by the state transition functions that define $x_{t+\Delta}(c,d)$. $v(\cdot)$ is the value the household places on states x in time t, which comes from a combination of the utility they gain from optimal consumption for the present moment Δ, and the expected, discounted value placed on the state variables they're left with in the next moment ($\mathbb{E}[\cdot]$ captures all idiosyncratic risk transitions).

By contrast, the equivalent expression for present–biased households is[11]

$$v^{\beta}(x_t) = \lim_{\Delta \to 0} \max_{c,d} u(c)\Delta + \beta \cdot e^{-\rho\Delta}\, \mathbb{E}\left[v^{E}\left(x_{t+\Delta}(c,d)\right)\right] \tag{6}$$

The present–biased value in Equation (6) differs from (5) in two important ways. First, the continuation value is discounted by an extra $\beta \leq 1$ on top of the exponential discount $e^{-\rho\Delta}$. This is the source of present–biased behaviour—the IG agent values the future less than their exponential counterpart. Second, the continuation value $v^{E}(x_{t+\Delta})$ represents the value the household *believes* they will place on expected states in the future, which may not be how they actually value them. A 'sophisticated' agent holds correct beliefs: they will have the same value function in future as in the present (i.e. $v^{E}(x) = v^{\beta}(x)$) whereas 'naivete' wedges them apart.

We assume complete naivete i.e. $v^{E}(x) = v(x)$ where $v(x)$ is an exponential discounter's value function, defined in Equation (5).[12] This assumption simplifies the analysis because the solution can be reached in two steps: (1) solve the exponential discounter's problem to find $v(x)$ and use

[10]This is derived by separating the present Δ from the future in the integral

$$v(x_t) = \max_{c_s,d_s} \mathbb{E} \int_t^{\infty} D(s-t)u(c_s)ds$$

where the discount function is $D(s-t) = e^{-\rho(s-t)}$

[11]As above, but where the discount is now the step function

$$D(s-t) = \begin{cases} 1 & \text{if } s-t=0 \\ \beta \cdot e^{-\rho(s-t)} & \text{if } s-t>0 \end{cases}$$

[12]This is an innocuous assumption. Maxted (2024) shows that under two assumptions—(1) CRRA utility and (2) soft–borrowing constraint—a problem with any degree of sophistication is isomorphic to a fully naive agent with a lesser degree of present–bias.

the solution as the IG consumer's continuation value to (2) solve the IG consumer's problem for each type.

3.1.5 Household problem and solution

During each stage of life, households choose consumption and voluntary transfers to maximise perceived value.

Exponential household Suppose we have substituted the drift in labour productivity with an N–state discrete process, and that this process, and jumps in and out of unemployment, are governed by Poisson intensities $\lambda^{z \to z'}$. During working life, the Hamilton–Jacobi–Bellman equation (HJB) is

$$\rho v(x) - \partial_t v(x) = \max_{c,d} \left\{ u(c) + \partial_b v(x) \cdot \dot{b}(x) + \partial_a v(x) \cdot \dot{a}(x) \right\} + \sum_{z'} \lambda^{z \to z'} \left[v(x') - v(x) \right] + \delta_R \left[v_R(x) - v(x) \right] \tag{7}$$

Where $\dot{b}(x)$ and $\dot{a}(x)$ are defined by Equations 2 and 4. This problem's FOC define the policy functions

$$u'(c(x)) = (1 + \tau) \partial_b v(x) \tag{8}$$

$$\partial_a v(x) = \partial_b v(x)(1 + \chi_d(d(x), a)) + \kappa(x) \tag{9}$$

Where $\kappa(x)$ is the Lagrange multiplier on the withdrawal constraint.

During retirement, the equivalent HJB is

$$(\rho + \delta) v_R(x) - \partial_t v_R(x) = \max_{c,d} \left\{ u(c) + \partial_b v_R(x) \cdot \dot{b}(x) + \partial_a v(x) \cdot \dot{a}(x) \right\} \tag{10}$$

Where $\dot{b}(x)$ and $\dot{a}(x)$ are defined by equations 2 and 4. This problem's FOC define the retired policy functions

$$u'(c_R(x)) = (1 + \tau) \partial_b v_R(x) \tag{11}$$

$$\partial_a v_R(x) = \partial_b v_R(x)(1 + \chi_d(d_R(x), a)) \tag{12}$$

In either stage of life, the soft–borrowing constraint ensures the exponential household will never borrow and so these FOC always hold.

Present–biased household The biased households' perceived value and policies are recovered directly from the exponential household results.

Lemma 3.1 (Present–biased solution (Maxted, 2024))**.** Assuming (1) CRRA utility with risk–aversion σ, and (2) never–binding soft–borrowing constraint, the naive IG consumer's value and policies are scale transformations of the exponential discounter's equivalents

$$c^{\beta}(x) = \beta^{-\frac{1}{\sigma}} \cdot c(x) \quad \text{and} \quad v^{\beta}(x) = \beta \cdot v(x) \quad \text{and} \quad d^{\beta}(x) = d(x) \tag{13}$$

Proof. Derived in Appendix A. □

The intuition behind this result, from Maxted (2024), is that an exponential discounter sets consumption so that marginal utility equals the marginal continuation value of liquid resources in future. The present–biased household does the same, but they perceive their marginal continuation value to be lower by β, and so consume more. Note that the present–bias only comes into play in decisions that trade between the present and future. The voluntary transfer choice is about balancing marginal values in the future, and as such is unaffected by present–bias.

These policies describe the optimal drift in the two accounts. Combined with the exogenous transitions in employment status, productivity, and life–cycle, they induce a stationary joint distribution $h(x)$ over all the households' state variables. We describe the Kolmogorov forward equation that characterises this distribution in Appendix C.

Present–biased households over–consume relative to the exponential discounters they believe themselves to be by a factor of $\beta^{-\frac{1}{\sigma}} > 1$. As a result, left to their own devices, these households are left with less savings in retirement. Crucially, they regret ending up in this position: it is not the result of rational planning, but rather a series of mistakes that they would not have made if they could commit in advance to a state–contingent consumption plan. This regret leaves room for the government to intervene to help resolve this commitment problem with retirement policy.

State distribution The policy functions described above determine the drift in the endogenous state variables i.e. the balances in the liquid and retirement accounts. All other state transitions—–between productivity and employment status within working life, and transitions between life stages—–are exogenous. Together, these state–transition rules define how the distribution of households across the state-space moves around over time and therefore define the Kolmogorov Forward Equation (KFE). As detailed in Achdou et al. (2022), these movements are captured by the infinitesimal generator $\mathcal{A}$, the continuous–time equivalent of a discrete–time transition matrix, such that the KFE is

$$\partial_t h(x) = \mathcal{A}^* \left[h(x) \right] \tag{14}$$

Where $\mathcal{A}^*$ is the adjoint of the generator.

3.2 Retirement policy

The government sets retirement policy, which consists of the unconditional state pension w_R and regulations governing the households' illiquid accounts. In other macro papers, the illiquid account is usually taken to represent housing, or some other difficult-to-transact-but-attractive asset. In our model, it is an individual retirement account (IRA).[13] Our treatment of this account differs from the usual in some important ways.

First, the underlying asset is the same as in the liquid account, so the gross rates of return are equal.[14]

$$r_b = r_a = r$$

Second, the account's illiquidity stems from four regulatory parameters. The government can penalise withdrawals during working life ($\chi_0(d < 0) = \chi_0$), or limit them directly by imposing the constraint $d \geq \gamma$. The source of the illiquidity in the retirement account is therefore regulatory; we set $\chi_0(d > 0) = 0$ and the convex adjustment cost χ_1 to be trivially low.

Third, in line with our discussion of DC schemes in Section 2, the government also mandates that a certain proportion $\xi \in [0, 1)$ of labour income be deposited into the retirement account. And , finally, the government offers a concession $\varphi \in [0, 1]$ so that contributions into and returns within the retirement account are subject to less tax. We discuss how the government government chooses these policy parameters in Section 5.

The mandatory contribution rate and tax concession affect the tax functions introduced in Equations 2 and 4 as follows:

$$\tau_b(x) = \tau \left[rb \cdot (b > 0) + (1 - \varphi\xi)wz \right] \tag{15}$$

$$\tau_a(x) = \tau(1 - \varphi)ra \tag{16}$$

The variable φ reduces the tax liability for mandatory contributions as well as asset returns within the account, implementing a concessional TTE system discussed in Section 2. If $\varphi = 1$, then contributions are made from pre–tax income, and returns are tax–free. We assume that retirees are free to withdraw from their retirement account, continue to receive tax concessions, but cannot deposit. At all stages in life, voluntary contributions are made from post–tax income, and withdrawals are not taxed.

[13] Superannuation in Australia, IRAs and 401(k)s in the USA, SIPPs in the UK etc

[14] In developed countries at least, participation in DC plans doesn't change the span of assets available too much. Retail consumers may not be able to access alternatives like hedge funds themselves, but these make up a small portion of DC retirement funds, which tend to be mainly invested in market securities, or to hold real assets like infrastructure or commercial real estate that can also be accessed through market securities.

Retirement policy gives life to the illiquid asset—it is illiquid due to regulations designed to discourage withdrawal during working life, and any difference in asset returns comes from preferential tax treatment. This setup closely mirrors the defined contribution systems discussed in Section 2. Retirement systems across the world are complex and vary from country to country (Beshears et al., 2015), but the setup in our model allows us to approximate many of their key features. In particular, we can span three of the first first four pillars in the World Bank's Conceptual Framework for classifying pension schemes (Holzmann et al., 2008): the state pension w_R protects households from poverty and affects redistribution (Pillar 0), mandatory contributions to the retirement account can be used to force personal saving in line with income, correcting for myopia and other errors (Pillar 2), and tax concessions in the retirement account can support voluntary savings as well (Pillar 3).[15]

Implementing a mandatory defined contribution system like the Australian Superannuation system, for example, is done by banning withdrawals and mandating contributions from labour income, as well as granting a tax discount of about 37.5% to an average earner.[16] Under current legislation, the settings would be $(\chi_0, \gamma, \xi, \varphi) = (0, 0, 0.11, 0.375)$; and the average state pension per pensioner is around 20% average employed income.[17]

3.3 Fiscal authority

The fiscal authority pays benefits to the retired (w_R) and unemployed (w_U), services debt (rB), makes government purchases of the consumption good G, and makes discretionary transfers to households $T(x)$. It takes in taxes on consumption, capital and labour income. Deficits are funded by changes in government debt $\dot{B}$, defined below.

$$\begin{aligned} \dot{B} = & \underbrace{G + w_U \pi_U + w_R \pi_R + rB + \int T(x) h(x) dx}_{\text{Spending}} \\ & - \underbrace{\int \left(\tau_c \cdot c(x) + \tau_b(x) + \tau_a(x)\right) h(x) dx}_{\text{Tax revenues}} \end{aligned} \tag{17}$$

Where π_U and π_R represent the measures of unemployed and retired people, and z_U and z_R represent those states (e.g. $\pi_U = \int h(b, a, z = z_U, \beta) dx$). Borrowing is restricted by an exponential fiscal rule (as in Auclert et al., 2023; Galí, 2020; Angeletos et al., 2023):

$$\dot{B} = -\mu \left(B - \bar{B}\right) \tag{18}$$

[15] What's missing is Pillar 1: a state–organised defined–benefit pillar like the USA's social security system, but this is beside the point for the purposes of this paper.

[16] Concessional contributions and returns within Superannuation are taxed at 15%, compared to a 23% income tax bill for a person on average income with no dependents.

[17] Australia's state pension is asset and income means tested so this average hides a lot of variation.

This rule sets an exponential decay of the government debt gap around some ideal level $\bar{B}$, where $\mu \geq 0$ (if zero, always hold debt constant; if ∞, return to $\bar{B}$ immediately).[18] We assume that government debt is fixed at the target in the stationary solution, and that the fiscal authority uses the consumption tax rate as the marginal tool to meet the fiscal rule in both the stationary and dynamic solutions. We also assume that this rule is suspended whilst any stimulus is active (whether fiscal or liquidity policy), and reactivated immediately after.

The fiscal authority has a balance sheet to manage and a rule to meet, but no objectives beyond this. Control of the discretionary policy levers and benefit levels are left to the government, whose choices the fiscal authority takes as given.

3.4 Stationary equilibrium

For fixed prices (w, r) and given retirement policy settings $(w_R, \chi_0, \gamma, \xi, \varphi)$, a stationary equilibrium is the set of working–life and retirement value functions $(v(x), v_R(x))$ and policy functions $(c(x), c_R(x), d(x), d_R(x))$, the measure over household states $h(x)$, and the consumption tax rate τ_c such that

- The values and policies solve the household problem (Equations 7 to 13)
- $h(x)$ is stationary (i.e. $\partial_t h(x) = 0$ in Equation 14)
- τ_c balances the budget (Equation 17)

In equilibrium, the household and fiscal authority constitute a combined model–block that takes prices and policy settings as given, and produces aggregate demand for a consumption good (the sum of household and government demand), net asset supply (total household wealth, net of government debt), and aggregate labour supply, which is an endowment process.

Stimulus policy This model allows us to analyse the two approaches to stimulus policy we're interested in. Stimulus is taken to mean policy interventions that increase aggregate consumption above its stationary level in the short–run. And the two methods available to the policy–maker are (a) fiscal transfers ΔT_t, funded by deficits and repaid via future tax changes that meet the fiscal rule, or (b) household liquidity policies i.e. regulatory changes to either restrictions on retirement account withdrawals $\Delta\gamma_t$, or mandatory retirement contribution rates $\Delta\xi_t$; we focus on the former in this paper.

[18]Under this fiscal rule, debt must follow the path $B_{t+s} - \bar{B} = \left(B_t - \bar{B}\right) e^{-\mu s}$ yielding a half–life of $\ln 2/\mu$ in units of model frequency.

3.5 Computational solution

We solve the model using a finite-–difference scheme (Achdou et al., 2022), with discrete, non–linear grids for the two endogenous assets, and three states for the idiosyncratic productivity process. The value–function updates are computed using the semi–implicit method (Achdou et al., 2022), and the policies are derived using the nested—drift algorithm[19] introduced by Sabet and Schneider (2024), which is monotone and consistent, and is stable to parameter choices as a result. This is in contrast to existing finite-difference methods for solving continuous-time heterogeneous-agent models with two or more endogenous state variables, in particular the 'splitting the drift' variant of Achdou et al. (2022) used in Kaplan et al. (2018). The stationary solution makes further use of adaptive time–steps, detailed in (Sabet and Schneider, 2024), to ensure the starting guess can get appropriately close to the solution for local convergence to be guaranteed (Barles and Souganidis, 1991).

4 Calibration

We calibrate the model to match an advanced economy with a mandatory DC retirement system. To do this, most parameters are set to standard values in the literature. The remaining parameters govern retirement policy, which are set in the optimal policy exercise detailed in Section 5.

Our calibration achieves two goals that set the scene for our policy experiments. First, the retirement policy settings reflect and rationalise what we observe in countries with mandatory DC systems: mandatory contributions, no withdrawals allowed, and tax concessions. This is necessary to ensure a fair comparison between fiscal and liquidity policy. If retirement policy were too restrictive, liquidity policy will be both effective and unambiguously welfare improving; if too loose, then there isn't enough firepower for liquidity policy to be a viable substitute for fiscal. Second, the aggregate MPC among workers is empirically realistic. This is necessary to ensure the consumption boost from fiscal (the desired policy outcome) is matched by an appropriate increase in government debt (which sets the longer–run policy impact). This goal is achieved by estimating the present–biased share η that matches the model's aggregate MPC to empirical estimates, discussed below.

The calibrated parameters are outlined in Table 2, and detailed below[20].

Life–cycle Working lives and retirement last an average of 40 and 20 years, respectively. This assumes a working life spanning 25–65, and matches the OECD average expected life–years after retirement (OECD, 2023, p. 192). After death, retirees are replaced by employed workers with zero assets and productivity drawn from its stationary distribution.[21]

[19] Detailed in Appendices C and D

[20] Time is continuous, with a base frequency of one quarter

[21] We also impose two extra rules on the transition from working life to retirement that households do not anticipate. The first is a 'forced–retirement' level in working households' illiquid account which does what it says on the tin.

Table 2: Calibrated parameters

Parameter	Symbol	Baseline calibration	Source
Retirement intensity	δ_R	1/160	40 year av. work–life
Death intensity	δ	1/80	OECD (2023)
Sep. & find. intensity	(λ_s, λ_f)	(0.0587,1.2)	Shimer (2005) & BLS
Log–income process	(ρ_z, σ_z^2)	(0.9136,0.0426)	Floden and Lindé (2001)
Risk aversion	σ	2	Standard
Discount rate	ρ	0.0025	Carroll et al. (2017)
Present–bias	(β_L, β_H)	(0.5,1)	Ganong and Noel (2020)
Present–bias share	η	0.5	Target MPC $\in [0.15, 0.25]$
Risk–free real rate	r	0.0051	2% p.a.
Borrowing penalty	ω	0.4024	500% p.a.
Wage	w	0.25	Numeraire
Unemployment benefit	w_U	0.1	Shimer (2005)
Income tax	τ	25%	OECD average
Consumption tax	τ_c	12%	Budget balance
Government spending	G	0.0238	G/GDP = 15%
Steady state debt	$\bar{B}$	0.1589	Debt/GDP = 25%
Fiscal rule	μ	0.0128	Galí (2020)
Adjustment costs	(χ_0, χ_1)	$(0, 0.001)$	Trivial

Working–life idiosyncratic risk Working households face idiosyncratic risk from jumps into, and out of, unemployment, and diffusion in their employed labour productivity. The jump transitions are governed by finding and separation rates of 0.0587 and 1.2 respectively; the former matches the quarterly separation rate in Shimer (2005), and the latter matches the mean 2.5 months spent in unemployment from the US Bureau of Labor Statistics.[22] The labour productivity process is calibrated to match the estimated AR(1) process in log–income residuals after individual characteristics effects are stripped out, from Floden and Lindé (2001).[23].

$$\ln z_t = 0.9136 \ln z_{t-1} + u_t,\ u_t \sim N(0, 0.0426)$$

This limit is necessary to ensure the state–space is compact and that retirement balances don't get out of hand, but is set to $a_{max} = 15$, sufficiently high that it only affects a small measure of workers and keeps the retired population at realistic levels. And second, households that retire with negative total assets (liquid plus illiquid) are bankrupted. In the model, this means their gross positions in both accounts are returned to zero (the same as if they were born into retirement). Without bankruptcy, a small measure of households retire with the maximum debt, and they are stuck there because it is an absorbing state for present–biased households. This is a disastrous position to be in for these households—their consumption is close to zero and so their value is orders of magnitude lower than at other points in the state space. If the risk of destitution is not addressed, then the government's retirement policy is primarily focused on managing it, rather than the more prosaic concern of general retirement adequacy. The bankruptcy rule avoids the issue.

[22]Table A.12 'Unemployed persons by duration of unemployment: Monthly, Seasonally Adjusted'; recent average outside of recessions.

[23]This calibration is used in Maxted et al. (2024) and Guerrieri and Lorenzoni (2017). The estimates are from PSID data covering 1988–1991. More recent estimates of the same AR(1) process all get numbers around this i.e. with auto–regressive parameter at least 0.9, and standard–deviation at least 0.2 (e.g. Kaplan et al., 2020a; Chang et al., 2013; Guvenen et al., 2023).

We cast this in continuous time, following Achdou et al. (2022), so it becomes a monthly Ornstein–Uhlenbeck process, which we discretise over $k = 3$ points with reflecting barriers at on standard–deviation $\ln z \in [-\sqrt{0.0426}, \sqrt{0.0426}]$, and normalise so the stationary distribution of z, in levels, has a mean of one.[24]

Preferences Households have CRRA preferences over consumption

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

With a standard coefficient of relative risk aversion equal to $\sigma = 2$. Households discount the future at $\rho = 0.0025$, which corresponds to an annual discount rate of 0.99. Recall that households also face the risk of retirement and death. This boosts the effective annual discount rate to 0.97 and 0.94 for employed and retired households, within the range of standard estimates (Carroll et al., 2017).

Prices & fiscal The base rate of return is $r = 0.0051$ per quarter (2% p.a.) and the borrowing penalty is $\omega = 0.4024$ (500% p.a.), set to be extortionate to impose a soft–borrowing constraint. The annual wage for a unit of effective labour is the numeraire (quarterly wage $w = 0.25$) so all other monetary values are relative to this. The government pays an unemployed benefit $w_u = 0.1$ to match the standard replacement rate of 0.4 from Shimer (2005). The baseline income tax is set to $\tau = 25\%$, the OECD average personal income tax rate for a single person with no children on the average wage. In both the stationary solution and the dynamic exercises later, the consumption tax τ_c is set internally to meet the fiscal rule.

Steady state government spending is $G = 0.0238$, targeting 15% of GDP, and steady state debt levels are $\bar{B} = 0.1589$, or 25% annual GDP. In both cases, GDP here is taken to be the aggregate income of the steady state employed population multiplied by 3/2 to adjust for the capital share. The exponential parameter on the fiscal rule is set to $\mu = 0.0128$, such that debt gaps are closed with a half–life of 13.5 years, following Galí (2020).[25] This parameter has no influence on the stationary solution, in which the rule dictates budget balance.

Adjustment costs The adjustment cost function serves no purpose other than to deliver analytical solutions for the voluntary contribution policy d.[26] The baseline parameters that set

[24] This yields a stationary distribution defined by

$$\{z_L, z_M, z_H\} = \{0.681, 0.9516, 1.4652\} \text{ and } \{\pi_L, \pi_M, \pi_H\} = \{0.2686, 0.4628, 0.2686\}$$

Where π_i represent the stationary probability of being in state i.

[25] This matches the European Union's fiscal compact, which includes a provision that excess debt should be reduced by $1/20^{th}$ each year (Galí, 2020).

[26] It is simple to use this function to impose an early withdrawal penalty during working life, as in the USA.

common adjustment costs between working and retired households are set so the costs are trivial $(\chi_0, \chi_1) = (0, 0.001)$.

Present–bias The population is split into present–biased and exponential households. The present–biased have $\beta = 0.5$, similar to Ganong and Noel (2020) and Laibson et al. (2024), and the exponential have $\beta = 1$. The size of the present–biased population is set to $\eta = 0.5$. This leads to an average present–bias in the population of 0.75, which is within the range of other estimates (Laibson et al., 2024; Hamilton et al., 2024), but a greater biased proportion than estimated by Ganong and Noel (2020), who find only 25% of their sample (unemployed workers) exhibit this degree of bias. This parameter has crucial importance for the model because it sets the aggregate marginal propensity to consume. As we show in Section 5.6, setting $\eta = 0.5$ leads to an equilibrium MPC in the centre of the range of empirical estimates, and we explore the sensitivity of our results to this parameter in Section 5.5.

Retirement policy framework We restrict attention to mandatory DC schemes backed by a fixed state pension because this is the relevant policy framework within which liquidity policy has been used. Withdrawals are not allowed $\gamma = 0$ and the withdrawal penalty is set to zero $\chi_0(d < 0) = 0$. The state pension is set to $w_R = 0.075$, a replacement rate of 30% average worker income, which matches the average replacement rate across OECD countries with a mix of state and private pension schemes.[27] The remaining retirement policy parameters are the mandatory contribution rate ξ and the tax concession φ. We saw in Section 2 that these parameters vary across countries but that contributions cluster in the range $8 - 20\%$ and tax concessions are universally granted to these illiquid savings environments. In the next section, we set these remaining parameters optimally.

5 Optimal retirement policy

In this section the retirement policy parameters are set to implement the optimal mandatory DC scheme. Within the constraints of the policy framework, the government chooses the optimal mandatory contribution rate (ξ) and tax concession (φ).

5.1 The government's problem

The government's problem is similar to that in the literature on paternalistic savings policies in that they must balance two tradeoffs. First, following Amador et al. (2006), there is both a need to

[27] This is the replacement rate of average income provided by the state pension ('Mandatory Public'), weighted average across OECD states that combine state and private components in their pension schemes (OECD, 2023, Table 4.2, p. 153). Where the state is the sole pension provider, e.g. Austria, Spain, or Colombia, this weighted average is 56% in OECD countries.

provide households with commitment (to overcome present bias) and flexibility (to insure against idiosyncratic risk). This prompts government intervention to help the present–biased save, but puts a limit on how much mandatory saving is appropriate. Second, present–bias is heterogeneous but unobserved by the government. This introduces a need for the government to balance the interests of the biased against the unbiased, potentially prompting screening or compensation (Moser and Olea de Souza e Silva, 2019; Beshears et al., 2020a). Our setting differs from the extant literature by limiting the government's options to a DC system with uniform settings across the working–age population.

We evaluate welfare of these policy settings as follows.

Definition 5.1 (Social welfare criterion)**.** Social welfare in the stationary solution is defined as the expected long–run value for a prospective newborn.

$$W(\xi, \varphi) = \mathbb{E}_{(z,\beta)}[\hat{v}(0, 0, z, \beta; \xi, \varphi)] \tag{19}$$

$$\hat{v}(x; \xi, \varphi) = \mathbb{E} \int e^{-\rho s} u\left(c(x_s; \xi, \varphi)\right) ds \tag{20}$$

This is found in two steps. First we find the value of actual (rather than anticipated) behaviour[28] in Equation 20, referred to as the 'long–run value' (O'Donoghue and Rabin, 2006; Bernheim and Taubinsky, 2018; Naik and Reck, 2024). Using this value means the government anticipates but does not adopt the bias of its subjects when evaluating policy choices. Second we restrict attention to newborns (zero assets) and find their expected long–run value, based on the stationary distribution over (z, β) in Equation 19. Defining the criterion like this means comparisons of welfare under different policy regimes pose the question: 'which society would you prefer to be born into, anticipating potential present–bias?'.

Having defined a welfare criterion, we consider two approaches to the problem that differ in how powerful the government is. These lead to two distinct equilibria, defined below.[29]

Definition 5.2 (Social equilibrium)**.** The social equilibrium is a stationary equilibrium with retirement policy settings $(\tilde{\xi}, \tilde{\varphi})$ that maximise steady state social welfare.

$$(\tilde{\xi}, \tilde{\varphi}) = \arg\max_{\xi, \varphi} W(\xi, \varphi)$$

The social equilibrium is the outcome of an all–powerful government's problem, in the sense that they can guarantee compliance with the policy settings they choose. As we will show in the next section, the optimal tax concession is zero in the social equilibrium. To rationalise the ubiquity of

[28] Both $w(x)$ and $v(x)$ are based on the incorrect assumption that the policy rules are $\check{c}(x)$.

[29] We don't mean that there are multiple equilibria, but that the government will select different equilibria under these different constraints.

these tax concessions, we suppose the government is subject to an extra participation constraint, optimisation under which leads to the 'buy–in' equilibrium.

Definition 5.3 (Buy–in equilibrium)**.** The buy–in equilibrium is a stationary equilibrium with retirement policy settings (ξ^*, φ^*) that maximise social welfare subject to a participation constraint: no newborn chooses to opt out of the system by setting their own personal retirement parameters to $(\xi_i, \varphi_i) = (0, 0)$.

$$(\xi^*, \varphi^*) = \arg\max_{\xi,\varphi} W(\xi, \varphi) \;\; \text{s.t.} \;\; \hat{v}(0, 0, z, \beta; \xi^*, \varphi^*, \tau_c) \geq \hat{v}(0, 0, z, \beta; 0, 0, \tau_c) \; \forall \, z, \beta$$

The buy–in equilibrium is the outcome when we imagine each household has the option at the start of their careers to opt out of the system[30]. In this case the government must cajole compliance for the system to be stable. The government wants to implement a pooling equilibrium where no–one opts out because (i) their policy tools don't allow for screening as in Moser and Olea de Souza e Silva (2019), and (ii) a separating equilibrium where only the present–biased opt in is not possible anyway, due to naivete. Note that this amounts to ensuring the rational agents opt in: assuming naivete, everyone thinks themselves rational.

5.2 The social equilibrium

Figure 4 plots the social welfare under different combinations of retirement policy settings, with warmer colours representing greater welfare. The social equilibrium is labelled, and picks the highest welfare point in the area plotted.

Result 5.4 (Social equilibrium)**.** The social equilibrium is $(\tilde{\xi}, \tilde{\varphi}) = (0.09, 0)$

This contribution rate is toward the lower end of the mandatory contribution rates in countries with compulsory DC schemes, with OECD examples outlined in Table 1.

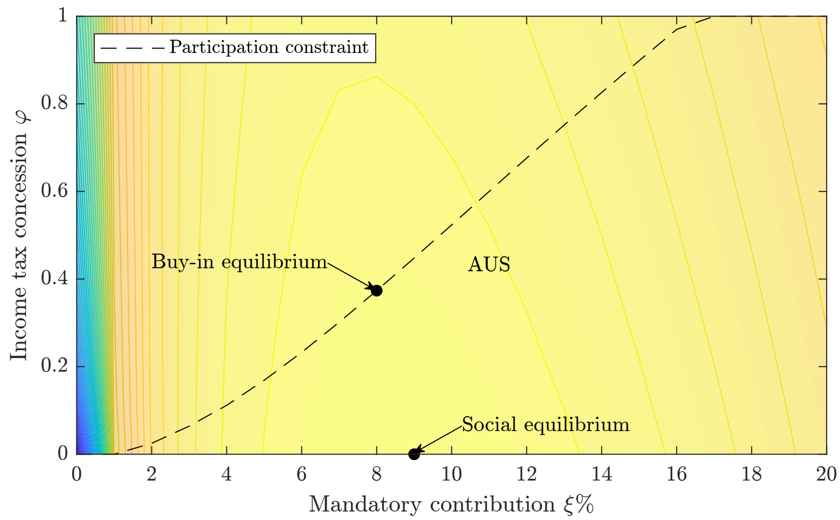
Note that the social equilibrium involves no tax concession. The plotted results are for a population with $\eta = 0.5$, but the optimality of zero tax concession in social equilibrium holds no matter how small or large a share of the population are biased—it is never necessary to compensate people for a policy that helps them, and tensions between biased and unbiased workers are better resolved by adjusting the contribution rate than with tax concessions.

This result reflects a finding in the empirical public literature that a majority of people are not responsive to incentives to save for retirement, and those that are tend to save more in retirement accounts by reducing savings elsewhere (Chetty et al., 2014; Choukhmane and Palmer, 2024). The use of tax concessions to encourage retirement savings is therefore likely to incur a fiscal cost with little welfare benefit. In our setting, greater tax concessions also necessitate higher tax rates on

[30] Call this the 'Dubai option'.

consumption to make up for the eroded base. The result here tells us the marginal welfare benefit from the concessions is less than the welfare cost of their fiscal side–effects.

Figure 4: Social welfare under policy settings



5.3 The buy–in equilibrium

Despite their apparent suboptimality, tax concessions are ubiquitous in actually–existing DC schemes, as discussed in Section 2. To rationalise them, we explore a political dimension to the government's problem—the need to implement a pooling equilibrium where agents opt–in to the retirement system at the beginning of their careers. The black dashed line in Figure 4 traces out the limited menu facing a government that has to deal with a participation constraint. In this setting, the tax concession ensures participation. Each contribution rate ξ has a minimum concession $\varphi(\xi)$ necessary to implement a pooling equilibrium, and this minimum is increasing in the contribution rate $\varphi'(\xi) > 0$. The government selects among these $(\xi, \varphi(\xi))$ to maximise welfare. The result is the buy–in equilibrium, with an optimal contribution rate lower than in the social equilibrium, and more tax concessions.

Result 5.5 (Buy–in equilibrium)**.** The Buy–in equilibrium is $(\xi^*, \varphi^*) = (0.08, 0.37)$

This result is in line with what's observed in the real world—substantial tax concessions coupled with mandatory contributions close to the OECD examples in Table 1—and this is our preferred calibration as a result. The tax treatment in the model is a TTE system, with concessions for the taxes on entry and returns. Direct comparison to other countries is difficult because most countries use an EET system. The one country that is comparable is Australia, identified on Figure 4, which taxes mandatory contributions from pre–tax income and Superannuation returns at 15%, a $\varphi = 0.4$ discount on the standard 25% rate for an average worker.[31] With a mandatory contribution rate currently at 11%, this is remarkably close to the buy–in equilibrium in our model.

5.4 The impact of retirement policy for working–age people

Retirement policy substantially raises working households' saving rates and retirement adequacy, and this effect is stronger for the present–biased households. Table 3 shows the impact of retirement policy for workers, by type. To see behaviour without mandatory savings first we set $\xi = 0$ and re–solve the model. In this scenario, working households' saving rate is 10% of earnings, with a large difference between present–biased and unbiased households (4% and 16% respectively). With mandatory savings, the average saving rate is higher (23%), with little difference between the two types of household.

Table 3: Effect of mandatory saving on workers, by type

	Average saving rate (% earnings)		**Median %ΔC at retirement**	
	Present–biased	Unbiased	Present–biased	Unbiased
Without mandatory saving	4	16	−27	−13
With mandatory saving	22	24	16	−12

Similarly, retirement adequacy, which we measure as the median expected change in consumption from working to the first year of retirement, is much improved by the policy (see Table 3).[32] Without mandatory saving, unbiased people's consumption drops by a median of 13% upon retirement, and more than double this for present–biased households. With mandatory saving, the unbiased household's consumption drop barely changes, whereas biased households now see an increase in their consumption of 16%, as they move from being working–poor to having resources to spend.

5.5 Sensitivity to present–biased share

Table 4 shows how the government's solutions change with the present–biased population share.[33] The optimal mandatory contribution rate is increasing in the present–biased population share, reflecting the increasing level of need in society. The tax concession necessary in the buy–in equilibrium is increasing as well, to ensure early–career opt–in to the system.[34] The aggregate worker

[31] Specifically this is the average personal income tax rate on labour income for a single person with no children on the average wage, from the OECD's 'Labour taxation - average and marginal tax wedge decompositions' series in 2023.

[32] Specifically, we find the difference between the workers' consumption policy and the average expected consumption rate over the first year of retirement, using the employed worker's state distribution to identify moments.

[33] The grid–search is restricted to whole percentage–points in ξ.

[34] The optimal tax concession is zero in the social equilibrium across all levels of the present–biased population.

MPC in the buy–in equilibrium is also increasing in η, both due to the combination of a greater share of present–biased households, and the greater mandatory contribution rate.

Table 4: government solutions with different population mixes

Present–biased share η	10%	20%	30%	40%	**50%**	60%	70%	80%	90%
Buy–in Equilibrium									
ξ^*	0.04	0.06	0.06	0.07	**0.08**	0.08	0.08	0.09	0.09
φ^*	0.11	0.23	0.23	0.30	**0.37**	0.37	0.37	0.45	0.45
Worker MPC	0.05	0.10	0.14	0.18	**0.22**	0.26	0.30	0.34	0.38
Social Equilibrium									
$\tilde{\xi}$	0.05	0.06	0.07	0.08	**0.09**	0.09	0.09	0.10	0.10

5.6 Model validation

Table 5 compares aggregate moments among workers in the stationary solution to the model to equivalents in the USA. The model produces an average quarterly MPC[35] of 0.22 among workers (0.24 for the whole population), within the range of recent empirical estimates for non–durable consumption (Ganong et al., 2023; Jappelli and Pistaferri, 2014; Sahm et al., 2010, 2012; Fagereng et al., 2021; Kueng, 2018; Kaplan and Violante, 2022). The MPC is this high because 40% of workers are classed as hand–to–mouth, in line with standard estimates from the literature (e.g. Aguiar et al., 2024; Kaplan et al., 2020a; Kaplan and Violante, 2022).[36]

Table 5: Aggregate moments for workers

Moment	**Model**	**Data**	**Source**
Quarterly MPC	0.22	$[0.15, 0.25]$	Kaplan and Violante (2022)
% HTM	40	41	”
Liquid wealth / labour income	1.0	0.6*	”
Fin. wealth / labour income	6.4	4.1*	”
Personal saving rate	7%	$[0, 10]$%	OECD range
Median Δc on retirement	−1%	−3.5%	Aguila et al. (2011)
*From the bottom 95% of the empirical wealth distribution.			

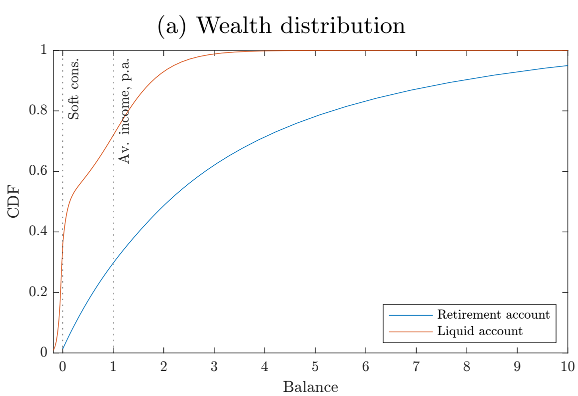
The model comes close to aggregate wealth–to–income ratios: recording total financial wealth of 6.4 times labour income, compared to 4.1 in the USA data; and liquid wealth measuring one year's worth of labour income, compared to 0.6 in the data. Having greater total wealth is natural, given our assumption of a mandatory DC retirement system. The actual USA features a voluntary DC system, combined with government–backed Social Security that does not count in wealth statistics. The average household saving rate is 7%, higher than in the USA, but in the range of OECD rates.

[35]See Appendix F for the formula used to calculate this object in the model.

[36]Hand–to–mouth status is defined by having liquid assets less than half of monthly labour income (Kaplan and Violante, 2022).

The median drop in consumption upon retirement is -1%, which is well within estimates from the existing literature; for example Aguila et al. (2011) estimate a median change of -3.5% with substantial variation around this. The size and sign of this change has been the subject of a lot of empirical work. Initial estimates, focused on food, found a substantial drop in consumption on retirement (Banks et al., 1998; Bernheim et al., 2001; Aguiar and Hurst, 2005). Later work explained this as substitution to home–production, meaning that such a measured drop doesn't reflect a loss in welfare (Aguiar and Hurst, 2005, 2013). Later work, expanding the definition to include other categories of spending, showed that there is no average drop among individuals who retire voluntarily, but that there is a lot of heterogeneity around this average across households. The change in spending at retirement is driven by both wealth (Aguila et al., 2011) and unobservable characteristics (Moran et al., 2021). Our median result matches the literature in that it is close to zero, and the average change is increasing in total wealth as well.

Across the distribution Figure 5a plots the distributions of liquid and retirement assets. Few households have any liquid assets at all, a result of their present–bias leading them to under–save. The distribution of retirement assets is approximately exponential, because it reflects the age distribution.In the baseline calibration no–one makes voluntary contributions, and so balances in the retirement account are built of 8% contributions from labour income, and asset returns, accumulated over time.



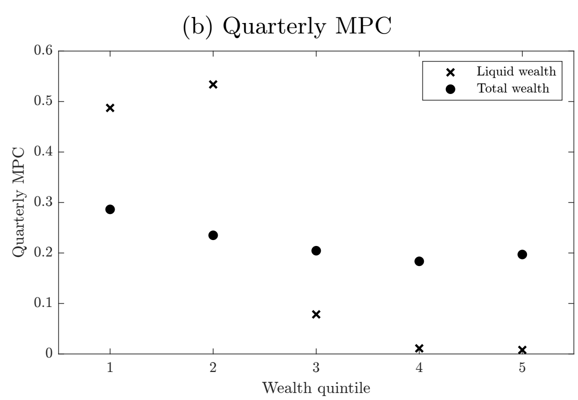


Figure 5b plots the household MPC across the quintiles of liquid and total wealth. The MPC is high in the first two quintiles of the liquid account distribution, which are closest to the soft–borrowing constraint, and declining rapidly after that. The MPC is roughly flat with total wealth. This mirrors reality well, as various studies have shown that the MPC is sharply declining in liquid wealth, but much less so with total wealth (e.g. Fagereng et al., 2021; Ganong et al., 2023).

This model is well situated for the stimulus policy experiments in the next section. The MPC matches empirical estimates, and so we can be confident that the sizing of fiscal policy will be appropriate. Similarly, because retirement policy is set optimally, the cost to society of liquidity policy is appropriate. One implication of the lack of voluntary contributions is that everyone will take advantage of opportunities to withdraw. Implemented like this liquidity policy will be identical to a fiscal transfer funded by a simultaneous lump sum tax on retirement accounts. We will discuss this further in the next section.

6 Stimulus policy

In the remaining sections of the paper we explore the effects of household liquidity policy in comparison to conventional fiscal stimulus. We first establish that liquidity policy indeed works similarly to fiscal stimulus in aggregate: opening a window during which people can withdraw from their retirement accounts leads to such withdrawals, and the greater liquidity boosts consumption in much the same way as a fiscal transfer. From the perspective of the economy, the two approaches are similar—they increase liquidity in the present by incurring debt, but they differ in where this debt sits (on the government's books, representing an implicit liability for households in the form of future taxes, or against workers' retirement accounts), and the process through which it is repaid.

Having established that both approaches 'work' as stimulus in aggregate, we then explore the other implications that each stimulus approach has. These include distortions to inter–temporal choices from changing tax rates, inter–generational distributional effects, and reduced retirement adequacy for workers. To weigh these distinct implications appropriately, we use a welfare analysis to quantify the degree of households' preference for liquidity policy over fiscal stimulus. We show in a welfare exercise that liquidity policy is better for the retired, richer and unbiased workers, and future generations i.e. those who will (a) not gain much from the stimulus benefits of either policy, but will (b) pay more in taxes under fiscal, and (c) are more likely to be over–invested in their retirement accounts. We conclude by exploring the sensitivity of the welfare analysis to various dimensions of the problem—the strictness of retirement policy, the nature of the fiscal rule, and the size and targeting of the fiscal stimulus.

6.1 Defining the stimulus policy experiments

We assume that the planner wishes to induce a specific stimulus to aggregate household consumption over a set period of time, and it is deciding which intervention to use. In the baseline experiment we set the desired stimulus to be 5% of stationary average household consumption over a period of one quarter (i.e. $\Delta = 1$). To frame the results it is useful to define a policy–dependent aggregate that accumulates average household consumption over a period Δ

$$C(T, \gamma) = \int_0^{\Delta} \left(\int c_{t+s}(x; T, \gamma) \cdot h_{t+s}(x; T, \gamma) dx \right) ds$$

The stationary aggregate, for example, is $\bar{C} = C(0,0)$; and the target for a given policy mix (T, γ) is therefore

$$C(T, \gamma) = 1.05 \times \bar{C}$$

Both interventions are modeled as MIT shocks to the policy instrument that last for a duration of one–quarter.[37] The baseline fiscal intervention is a shock to T, paid to workers, that solves: $C(T^*, 0) = 1.05 \times \bar{C}$.[38] And the baseline liquidity policy intervention is similarly a shock to γ that lasts for one quarter such that $C(0, \gamma^*) = 1.05 \times \bar{C}$. In both cases, taxes adjust endogenously to meet the fiscal rule defined in Section 3.3. All other variables, like wages and the interest rate, are held fixed to isolate the direct effects of the policies.

6.2 Common aggregate stimulus

The first thing to establish is that liquidity policy works as stimulus in the model. The amounts needed to achieve a 5% consumption boost are below.

Result 6.1 (Stimulus equivalence)**.** The baseline interventions are close to the same magnitude at $T^* = 0.0706$ and $\gamma^* = -0.0723$.

The calibrated stimulus policies transfer the equivalent of 7.06% average annual income to working households under fiscal policy, or allow them to withdraw an amount equivalent to 7.23% of average annual income over a quarter under liquidity policy. In the US, this is equivalent to nearly $5,500 in each case. The calibrated policy counterfactual is therefore large for a fiscal stimulus (e.g. the US CARES Act transferred $1,200 per person and an extra $500 per child), but small for a liquidity policy (e.g. in Australia people were allowed to withdraw up to $AU20,000 from their superannuation accounts). The fact that the calibrated numbers are so close suggests policymakers should see them as equivalents, at least in terms of their aggregate impact on aggregate demand.[39]

Figure 6 shows the impulse responses of aggregate consumption under the two different stimulus policies. Liquidity policy clearly has a stimulative effect on consumption, just as fiscal does, prompting a 5% increase in the aggregate consumption rate over the stationary equivalent ($\bar{C}$), as

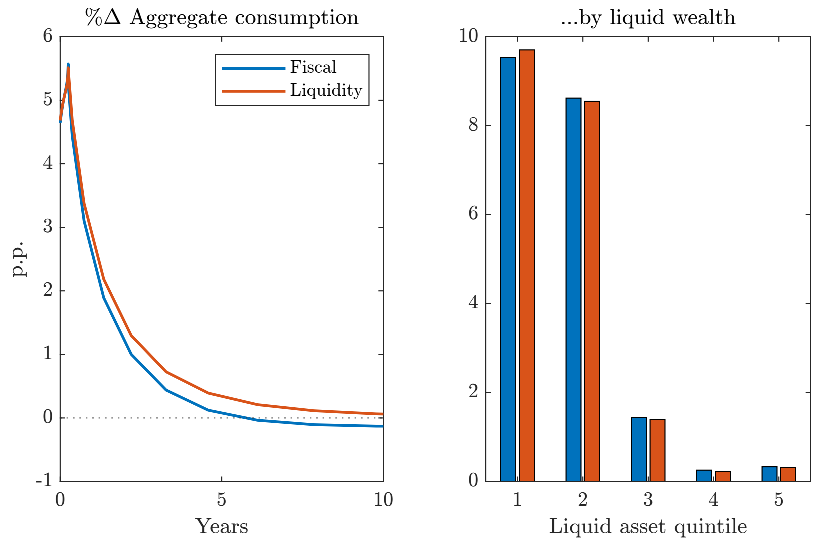
[37] Unanticipated before time–0 but known thereafter.

[38] We choose to target workers in the baseline fiscal exercise so its magnitude can be compared to the alternative liquidity policy, which is only effective for the working–age. We explore an alternative where the whole population receives the transfer in Section 7.2.

[39] This aggregate equivalence would be ameliorated by a participation decision, which we do not model here. We explore the participation channel in Schneider and Moran (2024), showing that 1/6 people withdrew when allowed in the Australian Covid–19 program, and we establishing that self–control issues were an important driver of this decision, alongside more standard drivers of demand for liquidity like assets and income shocks. Furthermore Hamilton et al. (2024) show that, conditional on participation, the MPC was 40%—higher than aggregate estimates. The high MPC can be explained by a combination of present–bias, and the self–selection of the most needy. A back of the envelope exercise then suggests an aggregate MPC out of liquidity policy of $0.4 \times 1/6 = 7\%$, relative to the 20% or so out of fiscal policy, meaning liquidity policy is only half as potent as fiscal stimulus, at least in the Australian context.

they were calibrated to do. After the immediate stimulus, liquidity policy produce less drag on consumption in the medium term (left panel). The two paths for consumption diverge within a few years, with the one under fiscal policy going negative as the stimulative effects wear off and the repayment plan kicks in. Under both policies this stimulus is mainly driven by workers who started off with less liquidity (right panel), as expected because these people tend to have higher MPCs (see Figure 5b).

Figure 6: Consumption response to different policies

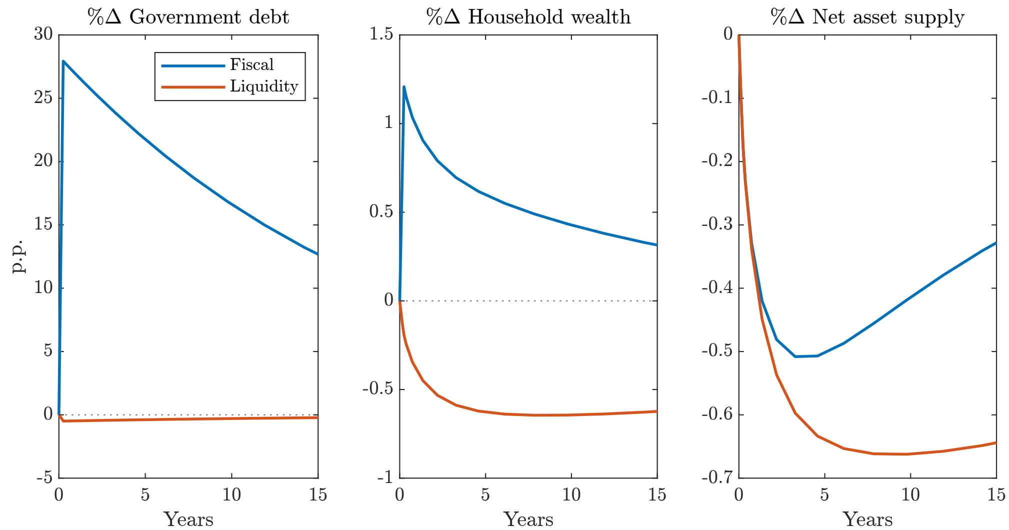


6.3 Difference in funding

The difference in funding mechanism is apparent by comparing the responses of government debt and household wealth under the two policies. In Figure 7, government debt jumps under fiscal policy as the transfers drive deficits, and then glides down as the debt is repaid with higher taxes. This increased government debt funds a concurrent increase in household wealth (visible in the middle panel) under fiscal policy, which reduces as households spend the extra liquidity on consumption and higher taxes. By contrast, there is no immediate impact on household wealth from liquidity policy (which only alters asset allocation), but a similar gradual decline as households (a) consume the extra liquid resources, and (b) retire with fewer illiquid resources.

The government debt accumulated under fiscal policy represents an implicit liability for households: the present value of their increased future tax bill, not recognised in their balance sheets. To show the consolidated household financial position, the right panel of Figure 7 plots their net asset supply—the difference between households' wealth (the sum of liquid and illiquid assets) and

Figure 7: Funding channels



government debt. This plot shows two things. First, the path of this aggregate is quite similar under the two policies for the first few years, and only starts to deviate in later years. The effects of both policies last a long time. Second, the duration of fiscal policy's impact clearly depends on the fiscal rule, but liquidity policy's effects necessarily span generations. They only wash out of the system after all affected workers retire, and die, which will be many decades after the stimulus occurred.

From the perspective of the aggregate economy, both policies do the same thing in qualitative terms. They increase liquidity for workers, driving short–term consumption, and they fund this with decreased illiquid wealth, whether in retirement accounts or greater future taxes. The difference between the two is where decreased illiquid wealth is situated, who repays it, and when. Under fiscal policy it sits as a debt on the government's balance sheet, and it is repaid with the (broad–based) consumption tax in a process determined by the fiscal rule. Under liquidity policy it sits like a debt on households' balance sheets (as a negative entry in their retirement accounts), and it is 'repaid' in lump sums by workers as they retire, in a process determined by the retirement rate. The primary reason the aggregate asset supply curves diverge in the right panel of Figure 7 is that the fiscal rule 'retires' debt at a faster rate than workers retire.[40]

Although the two approaches are qualitatively similar at the aggregate level, they have potentially different distributional and normative implications because of the difference in the size of the repayment tax base, and the redistribution and distortions caused by the repayment mecha-

[40]We explore an alternative fiscal rule that equalises these rates in Section 7.2.

nism. The taxes used to repay increased government debt have a broad base, applying to workers and retirees regardless of their status when the fiscal policy was implemented. By contrast liquidity policy is repaid from a narrower base—only the workers that extracted liquidity are forced to 'repay' by having lower future–value retirement savings—and it is not at all redistributive. One other difference between these repayment mechanisms at the aggregate level is whether they cause inter–temporal distortions.

6.4 Inter–temporal distortion

The two approaches have different implications for the path of tax rates, and the inter–temporal distortions these cause. Knowing the tax rate is growing (declining), households will shift consumption into the present (future)—a distortion away from the optimal smoothing path.

Lemma 6.2 (Euler distortions)**.** Assuming the borrowing constraint is non–binding, the changing consumption tax rate distorts the Euler equation

$$\mathbb{E}\left[\frac{\dot{c}(x)}{c(x)}\right] = \frac{1}{\sigma}[r_b(b)b + r(b)) - \rho] - \underbrace{(1+\tau_t)(1-\beta^{1/\sigma})\partial_b \hat{c}_t(x)}_{\text{Bias distortion}} - \underbrace{\frac{1}{\sigma}\left[\frac{\dot{\tau}}{1+\tau_t}\right]}_{\text{Tax distortion}}$$

Proof. Derived in Appendix B.2 □

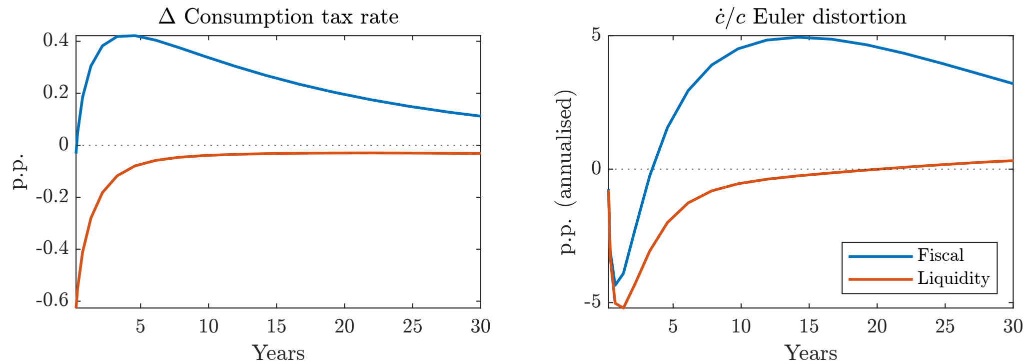
Figure 8 plots the consumption tax over time under the two policies (left panel), as well as the distortions to the optimal expected consumption growth path they imply (right panel). Under fiscal policy, consumption taxes climb steadily, peaking 0.4 p.p. higher than the stationary level about five years after the stimulus, and they glide down slowly over following decades. By contrast under liquidity policy the consumption tax drops with the stimulus. This is because the stimulus increases the tax base and the fiscal authority needs to keep the budget balanced.[41] It climbs back again slowly over following decades as aggregate consumption returns to normal.

Both policies imply short–term distortions to consumption growth because they both imply climbing tax rates over the first few years. These distortions effectively disappear under liquidity policy after the first few years, as the consumption tax rate stabilises close to the stationary level. But the distortions persist under fiscal policy for decades, with a maximal impact of causing expected annual consumption growth to be more than 4 percentage points higher than in the stationary solution.

We have established that both policies stimulate consumption in much the same way, but they differ in their funding mechanisms, and the inter–temporal distortions these imply. These results are all at the aggregate level, but the different approaches to stimulus distribute their impacts quite differently across households states, and we explore this in the next section.

[41] We explore an alternative scenario with an asymmetric fiscal rule in Section 7.2

Figure 8: Euler distortions



6.5 Distributional implications

6.5.1 Conflicting generational interests

Questions about the long–run implications of government policies are often framed as generational conflicts. It's useful to define three distinct groups within the population because they are affected by the policies differently. These are workers and retirees, those who are working whilst the stimulus occurs, and retired at time–0, respectively, and the future generations, those whose working lives begin after the stimulus ends.

Figure 9: Generational consumption

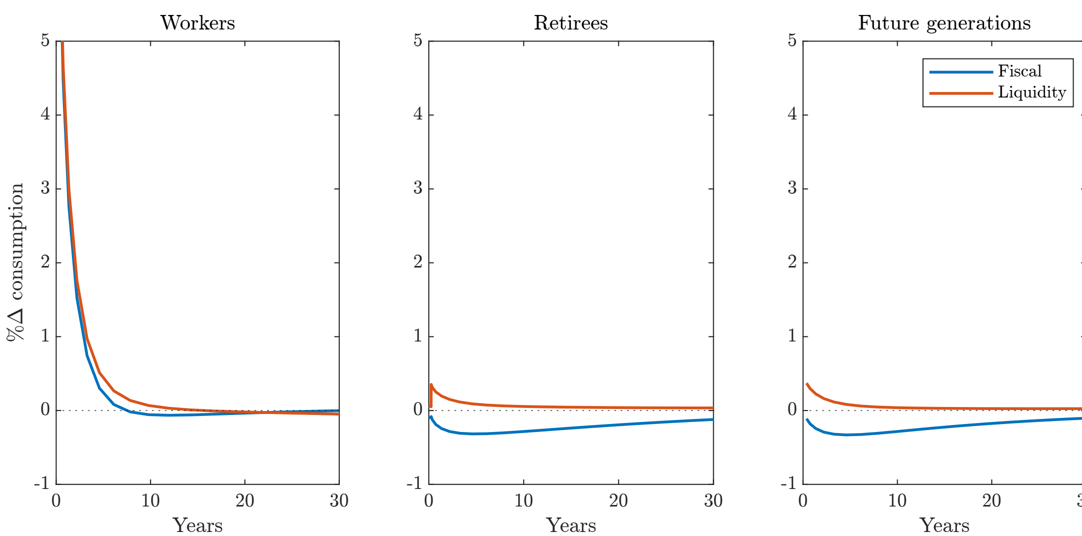


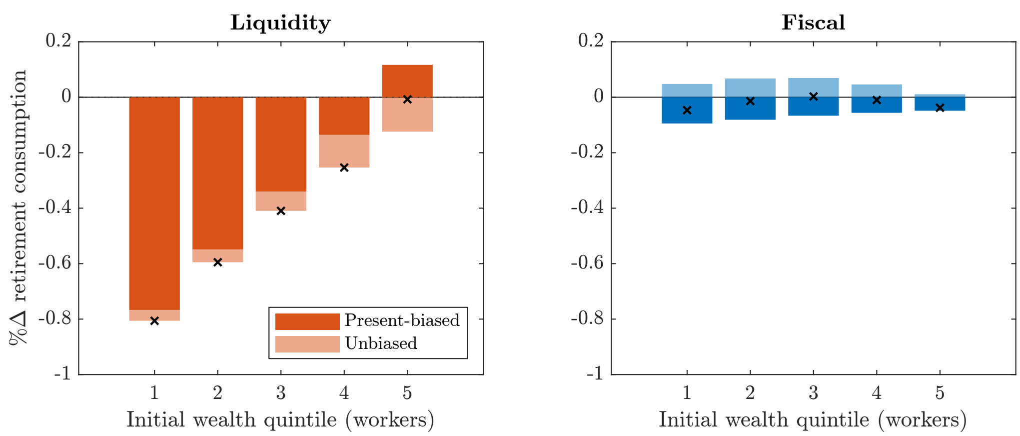
Figure 9 shows the average consumption path, relative to the stationary solution, for these distinct groups under each stimulus policy. Workers' consumption is boosted the most, by design, and there is little drag in the later years as the repayment plans for each policy kicks in. By contrast, retirees and future generations both experience a drag on their consumption under fiscal policy—they must pay higher taxes and didn't receive any stimulus—and a boost under liquidity policy—caused by the tax distortions discussed in Section 6.4.

6.5.2 Retirement (in)adequacy

Reduced retirement adequacy—fewer resources on retirement for affected households—is the main negative implication of liquidity policy, and it only affects the workers who withdrew from their accounts and spent the money. To measure retirement adequacy, we compare the consumption rate that the cohort of workers at time–0 can expect the moment they retire under the different stimulus policies, relative to the equivalent in the stationary solution.

Figure 10 plots this change for the section of this cohort that retires 20 years after the stimulus, and breaks down contributions by wealth quintile and present–bias types.[42] The figure shows that liquidity policy has a much more severe impact on retirement adequacy than fiscal, which has almost no effect on adequacy. This impact is stronger the less wealthy workers are, and mainly driven by the behaviour of biased households at all points in the wealth distribution.

Figure 10: Relative outcomes at retirement, by wealth and present–bias type



There are three forms of non–Ricardian behaviour at play in our model. First is the standard inability to smooth due to incomplete markets, second is the excess discounting caused by the OLG structure, and third is the over–consumption brought on by present–bias (Attanasio et al., 2024; Maxted et al., 2024). All stimulus relies on there being some non–Ricardian households. These results for retirement adequacy show that liquidity policy concentrates the duty of payment for

[42]Results are qualitatively the same for any retirement date.

stimulus mainly on the shoulders of the biased households. That is, exactly the group for whom the illiquidity in the retirement system is designed.

7 Welfare

We have established that both approaches work to stimulate household consumption, but that they come with various contrasting implications in different dimensions. To bring this together and compare how their respective implications weigh against each other, and for whom, we compare welfare under each policy using a compensating variation (CV). At time–0, for each point in the state–space, we find the change in liquid assets that would be required to make the household indifferent to liquidity policy instead of the baseline fiscal intervention

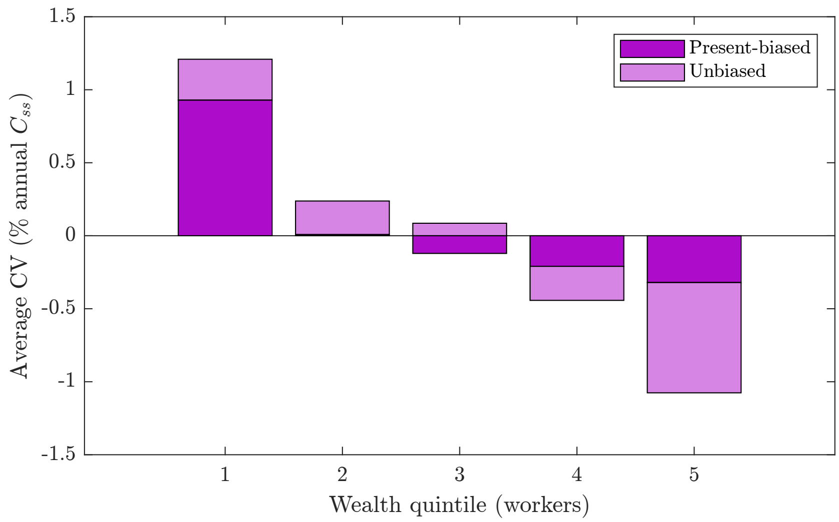
$$\hat{v}(b + CV(x), a, z, \beta; 0, \gamma) = \hat{v}(x; T, 0)$$

We find this compensating variation from two perspectives: (a) the near–sighted evaluation finds the CV based on households' perceived value functions i.e. accepting any naivete they have, and (b) the long–run evaluation finds the CV based on a correct anticipation of how they will behave (O'Donoghue and Rabin, 2006; Bernheim and Taubinsky, 2018; Naik and Reck, 2024), defined in Equation 20. This compensating variation shows the intensity of a preference for fiscal policy—positive numbers indicate the need to compensate for using liquidity policy instead, and negative numbers indicate a willingness to *pay* for liquidity policy to be used.

7.1 Winners and losers from liquidity vs fiscal policy

Who benefits from liquidity policy over fiscal? Wealthy workers and retirees. Retirees don't stand to gain anything from the stimulus, but they must subsidise the stimulus to workers, and their consumption is further distorted by the changing taxes, as we saw in Figure 9. To highlight the importance of wealth for workers' preferences, Figure 11 plots the average compensating variation across wealth quintiles showing that enthusiasm for fiscal policy is declining in wealth. Wealthy workers are more likely to be Ricardian, more likely over–invested in their retirement account, and (like retirees) also expect to pay greater taxes under fiscal. Hence their preference for liquidity policy, which allows them to escape taxes and grants more flexibility in portfolio decisions that they can ignore if they wish. Figure 11 also disaggregates by type, showing that the majority of the benefit of fiscal over liquidity policy is accruing to biased households in the bottom quintile of the wealth distribution. Biased households are over–respresented in this quintile because their bias leads them into debt, and because of this they stand to lose the most from having lower retirement assets. This result confirms the intuition that many opponents to liquidity policy voiced—that it places the cost of support on the shoulders of the least well off, and the most in need of help to save for retirement.

Figure 11: Worker preference for fiscal policy, by wealth & type



In aggregate, the best approach to stimulus depends on the welfare criterion. Liquidity policy is popular—it is preferred by an overwhelming majority, whether evaluated by households' near–sighted or long–run preferences (69% and 72%). And the average compensating variation across the whole population is negative—for the long-run evaluation the average CV is -1.6% of average annual stationary consumption—meaning there is more than enough money to fund compensation because the average person would be willing to pay to switch from fiscal to liquidity policy.

By contrast, a Utilitarian social planner would choose fiscal policy, under which average welfare is greater than under liquidity policy. Why the difference? Liquidity policy concentrates repayment for the stimulus on a small and relatively vulnerable group. It's popular because this small group doesn't constitute a majority. But it's a disaster for welfare because their marginal utility of consumption is *much* greater than for others in the population. Liquidity policy is preferred in average compensating variation terms for the same reason—their marginal utility is extraordinarily responsive to resources i.e. they can be bought off cheaply.

Bringing this together, the apparently optimal approach would be to use liquidity policy to do the stimulus, and to combine this with lump–sum transfers from wealthy workers, and the retired, to poorer and present–biased workers. Ironically, if these transfers were possible, then neither fiscal nor liquidity policy would be the optimal way to stimulate consumption. Instead, a program of lump–sum transfers moving liquidity from low– to high–MPC households would be better (Oh and Reis, 2012). Lacking such an instrument, we're left with the political question of how to weigh the needs of the many against those of the few.

7.2 Sensitivity

In this section we see how sensitive the baseline results are to different fiscal rules, targeting and magnitudes of the stimulus, and ex–ante retirement policy. In aggregate, liquidity policy is preferred to fiscal across all of our robustness checks around the baseline calibration for the retirement policy. It wins a majority vote in each case, and also costs a *negative* average compensating variation. This popularity is not due to naivete—the results in this section are all based on long–run evaluations, in which the evaluator correctly anticipates naive present–biased behaviour. In fact naive evaluations would be even more strongly in favour of liquidity policy. So the liquidity policy's widespread appeal is robust to these different assumptions. It is, however, quite regressive, with present–biased and less wealthy workers benefiting much more from fiscal stimulus. By contrast, if retirement policy were looser, then it is less binding on working–age people and this leads to reduced benefits from liquidity policy, tipping the balance back in favour of fiscal as the better approach to stimulus, both in aggregate and out of a concern for distributional impacts.

Fiscal rule symmetry The baseline results came from a symmetric fiscal rule—governments set tax rates to get debt back to target whether it is below or above this target. One of the implications of such a rule is that liquidity policy is accompanied by a persistent tax cut, to balance the government's books as the consumption tax's base is expanded. The populations we found to prefer liquidity policy may feel this way mainly because they like the tax cut. Although temporary tax cuts are often part of stimulus packages, we may also expect governments to apply fiscal rules asymmetrically, with the actual allowable debt level anywhere between zero and $\bar{B}$. In this case the fiscal rule would only activate when debt was above the upper bound, and otherwise taxes would stay fixed and deficits or surpluses allowed.

Table 6 shows that using an asymmetric fiscal rule reverses the average worker preference for liquidity policy. Workers on average now all prefer fiscal policy; the previous preference for liquidity policy among the unbiased workers was apparently driven by the tax cuts attached. Similarly, the retirees are still in favour of liquidity policy, but their enthusiasm is dampened (though not by enough to tip the scales to fiscal policy in aggregate).

Table 6: CV to prefer liquidity policy (% annual C_{ss})

	Baseline	Asymmetric	Consolidating	Accommodating
Total	-1.6	-0.1	-2.1	-0.8
Workers	-0.1	1.6	-0.5	0.8
Biased	*0.6*	*1.9*	*0.2*	*1.3*
Unbiased	*-0.8*	*1.3*	*-1.1*	*0.2*
Retired	-4.2	-3	-4.8	-3.5

Fiscal rule severity The timing of the fiscal rule may also matter. In the baseline results, the fiscal rule is set to mirror the EU's fiscal compact, following Galí (2020), which sets a target half–life for debt gaps of 13.5 years ($\mu = 0.0128$). Here we consider two alternative rules—the 'consolidating' one sets a stricter repayment schedule, with $\mu = 0.0256$ so the half–life is halved, and the 'accommodating' one sets a more lax schedule, with $\mu = 0.0063$. The accommodating rule is set so that government debt is 'retired' at the same rate as the working population. This is an interesting special case because it means the timing under liquidity and fiscal policy are essentially identical.

Figure 12: Aggregate impact of fiscal rule severity

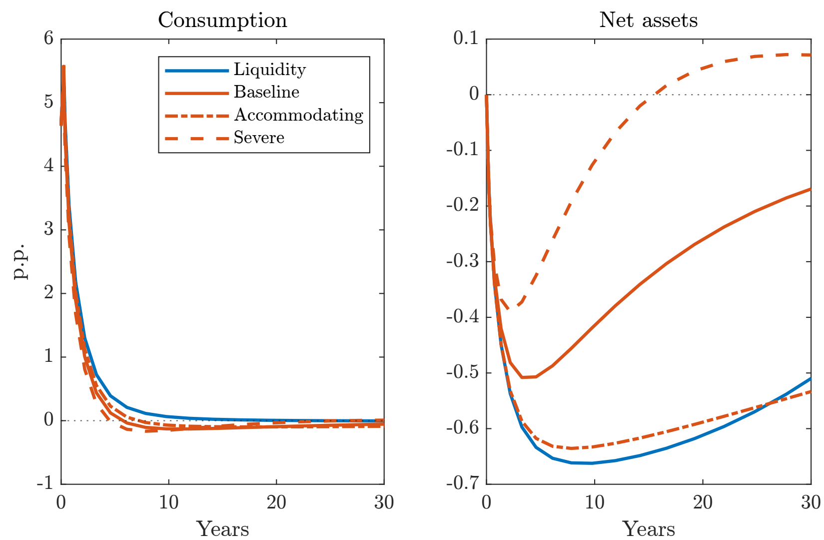


Figure 12 shows the aggregate consumption and net wealth plots under each alternative fiscal rule. They show what you'd expect—the drag on consumption is greater the stricter the rule, as net assets return to their stationary level quicker. Table 6 shows that the preference for liquidity policy diminishes with the intensity of the fiscal rule—it is greatest for the consolidating rule, followed by the baseline, and then the accommodating rule. As the fiscal rule relaxes, the taxes required to repay it are pushed further into the future and so they feel less onerous to the present generations, increasing the relative appeal of fiscal policy.

Targeting The baseline policy counterfactuals pitted liquidity policy against a fiscal transfer targeted at workers. This was designed to make the recipient group match, so they were more comparable. Here we explore how the main results differ when everyone receives the transfer.

If everyone receives the transfer then the total transfer per person needs to be smaller to achieve the same consumption stimulus. Specifically, to match the liquidity intervention we now need a transfer of $T^* = 0.0403$ for one quarter (versus 0.0706 when targeting workers), a total outlay for the government of around 10% less than when just targeting workers (retirees have a greater MPC than the average worker).

Table 7 shows that in this scenario, the overall long–run preference for liquidity policy is stronger. This is driven by a reversal of preferences for retirees, who now prefer to receive the fiscal transfer, and biased workers, who prefer to withdraw from their retirement accounts than receive a 40% smaller fiscal transfer. Unbiased workers are also much more in favour of the liquidity policy in this scenario. This result indicates how much the preference for, or against, redistribution was driving attitudes to fiscal policy.

Magnitude Table 7 shows long–run CVs under different magnitudes for the stimulus—50% and 150% the size of the baseline exercise—and their equivalent liquidity interventions. It shows a close–to linear relationship: liquidity is preferred overall in each case, with everyone's preference scaling up, or down, with the impulse magnitude.

Table 7: CV to prefer liquidity policy (% annual C_{ss})

	Baseline	Un–targeted	2.5% impulse	7.5% impulse	Lower ex–ante ξ
Total	-1.6	-2.1	-0.8	-2.6	1.4
Workers	-0.1	-4.9	-0.1	-0.1	4.1
Biased	*0.6*	*-3.9*	*0.2*	*0.8*	*5.9*
Unbiased	*-0.8*	*-5.9*	*-0.4*	*-1.0*	*2.3*
Retired	-4.2	2.8	-2.0	-7.0	-3.4

Looser retirement policy The baseline results all work in an environment with retirement policy set optimally in the exercise in Section 5. This policy is designed with the needs of newborns in mind, and one of its side–effects is to leave older workers generally over–invested in their retirement accounts, particularly if they are not present–biased. If we suppose instead that the government set retirement policy with the whole population in mind, but with the same participation constraint as the buy–in equilibrium, then the optimal policy sets contribution rates to about half what they are in the baseline, and with much smaller tax concessions as well

$$(\hat{\xi}, \hat{\varphi}) = (0.04, 0.12)$$

We redo the stimulus policy experiments with this calibration of the model's stationary solution. Fiscal and liquidity policy both now need to release more money to achieve the same stimulus $\hat{T} = 0.0816$ and $-\hat{\gamma} = 0.0875$ because the aggregate worker MPC is reduced to 0.2.

Table 7 shows that, with this starting retirement policy, fiscal is the better stimulus option for the average person. The average worker prefers it, with biased workers having the stronger (long–run) preference, whilst the average retiree still prefers liquidity policy because it means they avoid future taxes. Two forces drive the result for workers. With looser ex–ante retirement policy, there are both fewer wealthy workers who are over–invested in their retirement accounts, and more poorer workers who will suffer from the reduced retirement adequacy. Liquidity policy is thus an even more regressive stimulus option in settings where retirement contributions are low, or even not mandatory, as in the USA.

8 Conclusion

Household liquidity policy is becoming an increasingly popular tool used by governments seeking to stimulate the economy, growing from a few scattered uses during the global financial crisis, to a popular response used by over 30 countries during the COVID–19 recession. While a growing literature evaluates who withdrew and what they did with the money, our paper makes a different contribution. In short, we provide the first analysis of the distributional and welfare implications of household liquidity policy compared to traditional fiscal stimulus.

In our analysis, we capture many of the key trade-offs faced by policy makers thinking about the efficacy and equity concerns related to these two different approaches to stimulus. While this represents an important first step in understanding household liquidity policy, there are still a number of directions in which we could meaningfully extend our analysis, which we plan to pursue in future work.

At the most general level, our analysis could be expanded to capture the indirect, general equilibrium, effects of the stimulus policies in question. First, the stimulus approaches may have different implications for production. The model presented here works as a combined household–government block that can embed into a richer general equilibrium environment. Insofar as the aggregate effects coming out of this block are the same across the two approaches to stimulus, there will be no difference in their indirect effects on prices and production. However, if we extend the model to include labour supply, then inter–temporal distortions arising from taxes will drive a wedge between the two approaches that will bear on production—it will be more distorted under fiscal stimulus, than liquidity policy. Our future work will seek to quantify how important these production impacts are, and whether they have flow–on distributional implications.

Second, the source of the shock that precedes the stimulus may influence how we evaluate the two approaches. In our stimulus policy experiments we take it as given that the government wishes to achieve a 5% boost to household consumption. This allowed us to focus on the question of interest and to characterise the tradeoffs inherent in the two approaches. In practice, stimulus programs are implemented in response to shocks, and it may be that the nature of the original shock alters the relative desirability of liquidity versus fiscal policy.

Third, there may be important interactions with asset prices. Liquidity policy allows sales of assets at market lows, compounding the disadvantage to households that cash–out and spend the money because they realise capital losses (or at least reduced gains) by withdrawing from their retirement accounts and not reinvesting outside of them. Furthermore, in a similar vein, liquidity policy may exacerbate flight–to–safety dynamics we observe during times of aggregate stress, increasing funding costs for risky ventures.

Finally, we have modelled our stimulus policies as unanticipated shocks, so there is no role for anticipation. This was appropriate of the liquidity policies deployed in many countries during COVID–19 as this was the first time such measures were implemented in many places. Now that Pandora's box has been opened, however, forward–looking households may expect their retirement accounts to be somewhat more liquid than they were in the past. Such anticipation will alter households' engagement with retirement savings systems, and the repeated use of retirement resources for aggregate demand management will also alter the optimal design of retirement policy itself.

A Present–biased household's problem

For a present–biased naif with bias parameter β, the HJB equation changes in two ways from the exponential equivalent—all terms except the flow utility are pre–pended by β, and the value in the HJB is the rational value, not the household's actual valuation.

Working life

$$\begin{aligned}\beta\rho v(x) = \max_{\hat{c},\hat{d}} &\Big\{u(\hat{c}) + \beta\partial_b v(x)\cdot\dot{b}(x) + \beta\partial_a v(x)\cdot\dot{a}(x)\Big\}\\ &+ \beta\sum_{z'}\lambda^{z\to z'}\left[v(x') - v(x)\right] + \beta\delta_R\left[v_R(x) - v(x)\right]\end{aligned} \tag{21}$$

This problem's FOC are

$$u'(\hat{c}(x)) = \beta(1+\tau)\partial_b v(x) = \beta u'(c(x)) \tag{22}$$

$$\partial_a v(x) = \partial_b v(x)(1 + \chi_d(\hat{d}(x), a)) + \kappa(x) \tag{23}$$

Where the latter is the same as the exponential agent's, and so $\hat{d}(x) = d(x)$

Retired Similarly, the retired present–biased naif's problem is to solve the following.

$$\beta(\rho+\delta)v_R(x) = \max_{\hat{c}_R,\hat{d}_R}\Big\{u(\hat{c}_R) + \beta\partial_b v_R(x)\cdot\dot{b}(x) + \beta\partial_a v(x)\cdot\dot{a}(x)\Big\} \tag{24}$$

Where $\dot{b}(x)$ and $\dot{a}(x)$ are defined by Equations 2 and 4. This problem's FOC are

$$u'(\hat{c}_R(x)) = \beta(1+\tau)\partial_b v_R(x) = \beta u'(c_R(x)) \tag{25}$$

$$\partial_a v_R(x) = \partial_b v_R(x)(1 + \chi_d(\hat{d}_R(x), a)) \tag{26}$$

B Euler equation

B.1 Present–biased naif, stationary

Following Maxted (2024), Appendix A.4, but with an added consumption tax. The following solves the Euler for the working household. First, find the derivative of the expected value Equation 7 with respect to the liquid asset b.

$$\rho \partial_b v(x) = u'(c(x))\partial_b c(x) + \partial_{bb} v(x) \cdot \dot{b}(x) + \partial_b v(x) \partial_b \dot{b}(x) \\ + \sum_{z'} \lambda^{z \to z'} \left[\partial_b v(x') - \partial_b v(x)\right] + \delta_R \left[\partial_b v_R(x) - \partial_b v(x)\right]$$

Apply the realised FOC i.e. that $u'(\hat{c}(x)) = \beta(1+\tau)\partial_b v(x)$ so $\partial_b v(x) = \frac{u'(\hat{c}(x))}{\beta(1+\tau)}$

$$\left(\rho - \partial_b r(b) b - r(b)\right) u'(\hat{c}(x)) = (1+\tau)\left(\beta u'(c(x)) - u'(\hat{c}(x))\right) \partial_b c(x) + \partial_b \hat{c}(x) u''(\hat{c}(x)) \cdot \dot{b}(x) \\ + \sum_{z'} \lambda^{z \to z'} \left[u'(\hat{c}(x')) - u'(\hat{c}(x))\right] + \delta_R \left[u'(\hat{c}_R(x)) - u'(\hat{c}(x))\right]$$

Optimisation implies the relationship $u'(\hat{c}(x)) = \beta u'(c(x))$, which we can use to eliminate the first term on the RHS so the equation simplifies to

$$\left(\rho - \partial_b r(b) b - r(b)\right) u'(\hat{c}(x)) = \partial_b \hat{c}(x) u''(\hat{c}(x)) \cdot \left(r(b) b - (1+\tau_c) c(x) + other\right) \\ + \sum_{z'} \lambda^{z \to z'} \left[u'(\hat{c}(x')) - u'(\hat{c}(x))\right] + \delta_R \left[u'(\hat{c}_R(x)) - u'(\hat{c}(x))\right]$$

Note that we can collect many of these terms into the time–derivative of expected marginal utility ($\mathbb{E}[du'(c(x))]/dt$), by Ito's Lemma, after we add and subtract $u''(\hat{c}(x))\partial_b \hat{c}(x)\hat{c}(x)(1+\tau)$

$$\left(\rho - \partial_b r(b) b - r(b)\right) u'(\hat{c}(x)) = (1+\tau) u''(\hat{c}(x)) \partial_b \hat{c}(x)(\hat{c}(x) - c(x)) + \mathbb{E}[du'(c(x))]/dt$$

And using CRRA utility we know $c(x) = \beta^{1/\sigma}\hat{c}(x)$

$$\left(\rho - \partial_b r(b) b - r(b)\right) = (1+\tau)\frac{u''(\hat{c}(x))\hat{c}(x)}{u'(\hat{c}(x))}\partial_b \hat{c}(x)(1-\beta^{1/\sigma}) + \frac{\mathbb{E}[du'(c(x))]/dt}{u'(c(x))}$$

$$\mathbb{E}\left[\frac{\dot{c}}{c}\right] = \frac{1}{\sigma}\left[\partial_b r(b) b - r(b) - \rho - \underbrace{\sigma(1+\tau)(1-\beta^{1/\sigma})\partial_b \hat{c}(x)}_{\text{Naive bias distortion}}\right] \tag{27}$$

Hence the naif's Euler equation is a distorted version of the standard one, where the distortion scales with the bias, marginal propensity to consumer, and consumption tax.

B.2 Present–biased naif, dynamic

Using the same steps as above, but with an added term $\partial_t v_t(x)$ on the HJB equation, which eventually introduces an influence from drift in the tax rate.

$$\mathbb{E}\left[\frac{\dot{c}}{c}\right] = \frac{1}{\sigma}\left[(r_b(b)b + r(b)) - \rho - \sigma(1+\tau_t)(1-\beta^{1/\sigma})\partial_b \hat{c}_t(x) - \frac{\dot{\tau}}{1+\tau_t}\right] \tag{28}$$

The dynamic tax adds two distortions, relative to the stationary Euler. First, its level alters the bias distortion we found in the previous section, relative to its stationary level. Second, expected drift in the tax rate introduces inter–temporal smoothing distortions common to all agents. The analysis in the main text focuses on the latter.

C Discretised value function updates and Kolmogorov forward equation

Suppose we know the household's optimal policy functions. The stationary solution to the household's problem can be expressed as a linear system, discretised over the state space, as follows

$$\rho\mathbf{V} = u(\mathbf{c}) + \mathbf{A}\mathbf{V}$$

Where $\mathbf{A}$ captures the finite–difference transition rates between all the states, from the perspective of the households (i.e. they anticipate retirement and death but not rebirth), taking into account their optimal policies, and $\mathbf{V} = \begin{bmatrix}\mathbf{v}' & \mathbf{v}_R'\end{bmatrix}'$ stacks the discretised working and retired values together. The solution to this linear system is

$$\mathbf{V} = [\rho\mathbf{I} - \mathbf{A}]^{-1}\, u(\mathbf{c})$$

Where updates are implemented using the semi–implicit scheme with update control step–size Δ

$$\mathbf{V}^{n+1} = [(1/\Delta + \rho)\mathbf{I} - \mathbf{A}^n]^{-1}\, [u(\mathbf{c}^n) + \mathbf{V}^n/\Delta]$$

We use the nested–drift algorithm in Sabet and Schneider (2024) to find the policy functions that define $\mathbf{A}^n$ and $\mathbf{c}^n$ for a given value guess $\mathbf{V}^n$, described in Appendix D. In practice, we solve the retired value first, and then use this as an input into the working–life value solution; doing so reduces the computational burden of inverting the matrix in the semi–implicit update step. Following Achdou et al. (2022), the stationary measure[43] discretised over the same state grids ($\mathbf{g}$) is the solution to the linear system

$$0 = \tilde{\mathbf{A}}'\mathbf{g}$$

Where $\tilde{\mathbf{A}}$ adjusts $\mathbf{A}$ for the state transitions households do not anticipate, which are (1) rebirth as a zero–asset worker after dying in retirement, (2) forced retirement if illiquid assets reach the threshold, and (3) bankruptcy upon retirement if total assets are negative.

Present–biased agents For known policy functions, the solution process and formulae are the same, with the exception that their state transition matrix $\mathbf{A}^\beta$ solve for their *long–run* value function, not their perceived value.

D The nested–drift algorithm

To find the optimal policy functions for a given value, this paper uses a version of the nested–drift algorithm introduced in Sabet and Schneider (2024). Combined with implicit updates of the value, described in Appendix C, this is an approach to solving discretised systems of partial differential equations that is consistent and monotone, and so admits a solution that is stable to parameter choices. The below gives a sketch of the context for the algorithm, and its steps; see the original paper for more depth.

The problem Continuous time problems with multiple endogenous state variables, such as the one in this paper, typically have a solution defined by an HJB equation and first order conditions.

For the sake of communicating the algorithm, consider a simplified problem where households choose consumption and deposits between two accounts, where the latter are subject to adjustment costs. Their income is a random process y which switches between discrete states according to some transition intensities. The solution is thus defined by the HJB (and state transition equations) and first order conditions.

$$\begin{aligned}
\rho v(x) &= u(c) + \partial_b v(x) \cdot (r_b b + y - d - \chi(d,a) - c) + \partial_a v(x) \cdot (r_a a + d) \\
&\quad + \sum_{y' \in \mathcal{Y}} \lambda^{y \to y'} \left[v(x') - v(x) \right] \\
u'(c) &= \partial_b v(x) \\
\partial_a v(x) &= \partial_b v(x)(1 + \chi_d(d,a))
\end{aligned}$$

The above equations define the solution, but to implement it on a computer in a finite–difference scheme, we discretise the state–space x. Doing so introduces some imprecision that can be managed by using non–linear grids, placing a higher density of grid–points at areas where the value and policy functions are more likely to have curvature or kinks.

One form of imprecision is that the partial derivatives cannot be known exactly—we can find them with the forward and backward derivatives of the value at any point in the discretised state–space, or anywhere in between. So, which to choose?

Upwinding The standard approach to choosing between these options is to use 'upwinding'. That is, selecting the derivative that, when used, leads to drift in the state that goes in the direction assumed. In a single asset problem (e.g. if we fix $d = 0$ in the above), this consists of adding an additional condition to the solution. For a point in the state–space i the derivative is defined by

$$\partial_b v(x_i) = \begin{cases} \frac{v_{i+1} - v_i}{b_{i+1} - b_i} & \text{if } \dot{b}(x_i, c_i) > 0 \\ \frac{v_i - v_{i-1}}{b_i - b_{i-1}} & \text{if } \dot{b}(x_i, c_i) < 0 \\ u'(r_b b_i + y_i) & \text{if } \dot{b}(x_i, c_i) = 0 \end{cases}$$

Where the consumption policy is found from the FOC in the first two cases. In the third case, where there is no drift in the asset, the policy is identified by the drift equation and the derivative can be recovered from the first order condition.

In a two asset problem, things are more complex because one must consider the permutations of directions that each asset could be going in—both forward, both back, one still and the other drifting up, and so on. And also because multiple policies enter the one of the drift equations, and so the policies are jointly determined and can't necessarily be identified from the drift equations. In this case, they can be recovered using a root–finding algorithm based on the FOCs.

The nested–drift algorithm The algorithm we use here, originally from Sabet and Schneider (2024), solves this problem by exploring all possible cases (9 in a two–asset problem) of drift directions, using a root–finding algorithm to solve for policies in cases where necessary. The algorithm is monotone and consistent, and so satisfies the conditions necessary for local convergence (Barles and Souganidis, 1991). And it is also efficient because it places the most expensive root–finding steps last in the process, so they will only be reached if other alternatives have already failed.

The logic is as follows for each point in the state space x_{ijk}[44]:

1. Calculate key objects
   - The deposit policy that causes zero illiquid drift $\tilde{d} = -r_a a$
   - The directional derivatives of the value

$$V_b^F = \frac{v_{i+1,j,k} - v_{i,j,k}}{b_{i+1} - b_i} \text{ and } V_b^B = \frac{v_{i,j,k} - v_{i-1,j,k}}{b_i - b_{i-1}}$$
$$V_a^F = \frac{v_{i,j+1,k} - v_{i,j,k}}{a_{j+1} - a_j} \text{ and } V_a^B = \frac{v_{i,j,k} - v_{i,j-1,k}}{a_j - a_{j-1}}$$

2. Assume forward liquid drift, setting $\partial_b v = V_b^F$.
   - Calculate the consumption policy from the FOC

$$c^F = u^{-1}\left(V_b^F\right)$$

   - Calculate the deposit policy from the FOC under each assumed illiquid direction, using the FOC for forward and backward drifts, and the state transition equation for zero drift. Check the resulting deposit policies against the drift in the illiquid asset they're based on

$$d^F = \begin{cases} d^{FF} & \text{if } d^{FF} > \tilde{d} \\ d^{FB} & \text{if } d^{FB} < \tilde{d} \\ \tilde{d} & \text{otherwise} \end{cases}$$

     This step nests the upwinding for the illiquid asset within the process for finding the liquid asset's drift.
   - Find the liquid drift implied by (c^F, d^F)

$$\dot{b}^F = r_b b + y - c^F - d^F - \chi(d^F, a)$$

     and if it is positive move to the next point in the state–space. Otherwise continue.

3. Assume backward liquid drift, setting $\partial_b v = V_b^B$, and follow the equivalent steps to above to

[44] Practically many of these operations are done simultaneously for all points in the state–space, but it's simpler to show one point at a time.

find the backward policies. This time, check whether the resulting liquid drift is backward

$$\dot{b}^B = r_b b + y - c^B - d^B - \chi(d^B, a) < 0$$

If it is, move to the next point in the state–space. Otherwise continue.

4. Assume zero liquid drift, setting $\partial_b v = u'(c^0)$. Given this assumption, we know the consumption policy from the state transition equation

$$c^0 = r_b b + y - d^0 - \chi(d^0, a)$$

and the deposit policy will be defined implicitly by the FOC

$$\partial_a v(x) = u'\left(r_b b + y - d^0 - \chi(d^0, a)\right)(1 + \chi_d(d^0, a))$$

And we can solve for d^0 using this equation, and searching through different regions.

(a) Check if $d^0 > \tilde{d}$ using $\partial_a v = V_a^F$. If not, continue

(b) Check if $d^0 < \tilde{d}$ using $\partial_a v = V_a^B$. If not, continue

(c) Set $d^0 = \tilde{d}$

This algorithm results in the household's optimal policy functions $(\mathbf{c}, \mathbf{d})$ for a given value $\mathbf{V}$. It nests the up–winding of the illiquid asset within that of the liquid one. This is the efficient order to use because the illiquid asset's state transitions depend on only one policy function, and so the costly route–finding in the final steps is only necessary if all alternatives have been exhausted.

E Conditional expectations in continuous–time

The goal in this section is to find the expected path of a variable (a policy or state variable), given a starting point in the state space (x_0), from the perspective[45] of an agent at that point in the state–space.

$$y^e(x_0, t_n) = \mathbb{E}[y(x, t_n)|x_0 = x]$$

Suppose we have solved our problem, with time discretised over a sequence of N segments of a span from $t \in [0, T]$, where t_n denotes the end–point of the n–the segment such that $t_1 > 0$ and $t_N = T$. The solution is a sequence of policy vectors, transition matrices, and distributions (the measure,

[45] Important if (a) expectations are not fully rational, or (b) agents don't internalise some transitions like reincarnation.

not the density) stacked over the state–space

$$\{\mathbf{y}(t_n), \mathcal{A}(t_n), \mathbf{g}(t_n)\}_{n=1}^{N}$$

Where $\mathbf{y}(t_n)$ is the policy that applies during the span $\Delta(n) = t_n - t_{n-1}$ and $\mathbf{g}(t_n)$ is the state–distribution at the point in time t_n.

Finding the distribution updates The sequence of distributions is found using the implicit, discretised Kolmogorov Forward equation, following Achdou et al. (2022)

$$\frac{\mathbf{g}(t_n) - \mathbf{g}(t_{n-1})}{\Delta(n)} = \mathcal{A}(t_n)'\mathbf{g}(t_n)$$

Where the starting distribution is the stationary solution $\mathbf{g}(t_0) = \mathbf{g}_{ss}$ The discretised KF equation rearranges into the implicit updating equation

$$\mathbf{g}(t_n) = \mathcal{L}(t_n)\mathbf{g}(t_{n-1}) = \prod_{j=1}^{n} \mathcal{L}(t_j)\mathbf{g}_{ss}$$

With the implicit transition matrix defined by $\mathcal{L}(t_n) = [\mathcal{I} - \Delta(n)\mathcal{A}(t_n)']^{-1}$.

Aggregation To build a sequence of aggregate policies, we use

$$Y(t_n) = \mathbf{y}(t_n)'\mathbf{g}(t_{n-1})$$

So the expected aggregate at the n–th point in the time grid is the policy from that point, integrated over the distribution at that point. If we substitute the definition of the distribution into this expression we have

$$Y(t_n) = \begin{cases} \mathbf{y}(t_1)'\mathbf{g}_{ss} & \text{if } n = 1 \\ \mathbf{y}(t_n)' \prod_{j=1}^{n} \mathcal{L}(t_j)\mathbf{g}_{ss} & \text{if } n > 1 \end{cases}$$

Focusing on the cases where $n > 1$, we can see that the aggregate is made up of three terms—the future policy: $\mathbf{y}(t_n)$, the starting distribution: $\mathbf{g}_{ss}$, and a matrix that affects the expected movement of measure around the state–space between the starting point and t_n: $\prod_{j=1}^{n} \mathcal{L}(t_n)$.

We can collect these terms in two different ways to get the same aggregate. In the standard aggregation, the matrix is applied to the starting distribution, and so the expected aggregate in t_n is the policy in that period integrated over the distribution at the beginning of that period. If we instead apply the matrix to the future policy, then it produces the expected policy in t_n from the point of view of a point in the state space at t_0: $\mathbf{y}^e(t_n)$. Aggregation is then achieved by

$$Y(t_n) = \mathbf{y}^e(t_n)\mathbf{g}_{ss} \text{ where } \mathbf{y}^e(t_n) = \mathbf{y}(t_n)' \prod_{j=1}^{n} \mathcal{L}(t_j)$$

The two approaches measure the same aggregate, but they create very different sub–aggregate objects—the first a future distribution, and the second an expected policy. The latter $\mathbf{y}^e(t_n)$ is exactly the conditional expectation we are seeking—it tells us the *expected* value of the policy y at some future point in time t for each *starting–point* in the state–space.[46]

With the conditionally expected policy in hand, we can use it for other purposes than aggregation. Say we wish to describe expected policies by some quantile of the distribution over states. This is very easy with above object, with only two steps required:

1. Calculate the relevant quantile for each point in the starting state space
2. Find the average of the conditional policy, conditional on the quantile x_0 is in

$$y^e(t_n, q) = \mathbb{E}[y^e(t_n, x_0)|x_0 \in q] = \frac{\int_{x\in q} y^e(t_n, x)dG(x)}{\int_{x\in q} dG(x)}$$

F Marginal propensity to consume

Following Achdou et al. (2022) we calculate marginal propensity to consume over a discrete time–interval τ (always one–quarter in this paper) using the following formula

$$MPC_\tau(x_0) = \partial_b C_\tau(x_0) \text{ where } C_\tau(x_0) = \int_0^\tau c^e(x_0, t)dt$$

Where the conditional expected consumption policy $c^e(t, x)$ is defined in Section E.

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