# Household Liquidity Policy*

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**Abstract**

We assess 'household liquidity policy', a novel approach to stimulating aggregate demand that relies on relaxed regulation instead of conventional fiscal tools. We analyse the effectiveness of these liquidity policies, focusing on a form that was widely used during the Covid–19 pandemic: early access to retirement savings accounts. In a heterogeneous agent model with retirement and present–biased households we find both liquidity and conventional fiscal policies can achieve similar boosts to aggregate consumption but have different distributional implications. Relative to fiscal policy, liquidity policy benefits wealthier workers, retirees, and future generations, due to its lower tax burden and added flexibility, but it is also highly regressive. Liquidity policy shifts the future financial burden of present–day stimulus onto poorer and more present–biased workers, who only feel the impact when it is too late to adjust.

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# 1 Introduction

Suppose the government wants to stimulate aggregate demand. The conventional way to do this is with government transfers or spending, funded by deficits. Such 'fiscal stimulus' boosts demand in part because people who are liquidity constrained tend to have high marginal propensities to consume (Kaplan and Violante, 2014; Fagereng et al., 2021; Carroll et al., 2021; Aguiar et al., 2024) and so they spend more when they receive extra income. The key to this mechanism is that policy relaxes household liquidity constraints, not necessarily that they receive more income. In principle, another policy that relaxes constraints will also stimulate aggregate demand, even if it leaves total household resources unchanged. One example of this alternative approach is to give households early access to their otherwise illiquid retirement savings during periods of aggregate distress.[1] These policies were used sporadically for many years, but became common during Covid–19, with more than thirty countries granting some form of relief from retirement saving regulations (OECD, 2021). We call stimulus policy that uses regulation, rather than the government budget, 'household liquidity policy'.[2]

In this paper, we compare household liquidity policy with conventional fiscal stimulus. Our paper makes two main contributions. First, theoretically, we build a model that captures the tradeoffs inherent to each approach to stimulus, allowing the first direct comparison of liquidity policy with fiscal stimulus. These tradeoffs stem from the difference in funding—fiscal policy is funded with future taxes that cause distortions and redistribution, whereas liquidity policy is self–funded, thereby causing no distortions nor redistribution, but reducing the ability to commit resources to the future for those who need it.

Our second contribution is to show that, whilst both approaches are capable of the same short–term stimulus, liquidity policy is highly regressive. Comparing the two approaches, liquidity policy is more beneficial to wealthy workers, retirees and future generations, but for reasons unrelated to the stimulus itself—they value the option to rebalance their portfolios that liquidity policy grants, and wish to avoid the future taxes implied by fiscal policy. Pursuing stimulus via liquidity policy effectively privatises the cost of aggregate demand management, and this cost is paid by the people who actually spend the money (poorer, and more present–biased workers) when they retire and find they have less to live on. Choosing liquidity policy therefore places the burden of stimulus on the shoulders of precisely the people retirement policy seeks to protect.

We begin our analysis by building a model that allows us to compare the two policies. The start-

[1] By no means the only example, others of which include student debt, mortgage repayment, or rent deferrals; changes in regulations limiting collateralised loans; or changes to mandatory pre–payment of income taxes.

[2] Throughout this paper, we will use the term 'liquidity' or 'liquid resources' to refer to resources that can be used close–to immediately for consumption. This would include cash, bank and saving deposits, available consumer credit, and investments in securities that can be sold at will. It excludes real property and other durable investments, holdings of private companies, and contingent assets like insurance policies and most retirement accounts.

ing point is a two–asset heterogeneous–household model featuring idiosyncratic risk and incomplete markets. This ensures that there is demand for liquidity due to high MPCs for households that are close to their constraints. As well as this, a fiscal authority sets taxes respecting a fiscal rule. This kind of environment has been used to analyse the impact of fiscal policy in various papers (e.g. Kaplan and Violante, 2014; Auclert et al., 2024; Bayer et al., 2023), and allows us to do the same.

We add three novel features to capture the tradeoffs presented by liquidity policy. First, an overlapping–generations life–cycle with working and retirement phases (Blanchard, 1985; Yaari, 1965). This creates a need to save for retirement in addition to the standard precautionary motive. Second, a portion of the population is subject to naive present–bias (Laibson, 1997; Maxted et al., 2024; Maxted, 2024), leading to over–consumption (and under–saving) in the present. Left to their own devices, households' myopia will leave them far too poor in retirement. This creates a role for government intervention, the third model addition, in the form of retirement policy. Retirement policy consists of two pillars: first, a tax–funded unconditional state pension, and second, regulations on a retirement account designed to mirror mandatory defined contribution pension systems that are increasingly common across the world (OECD, 2023). These are a limit on withdrawals prior to retirement, a mandatory contribution rate out of labour earnings, and a tax concession on mandatory contributions and earnings within the account. This retirement account replaces the illiquid asset that is standard in two–asset heterogeneous–agent models; here, illiquidity is due to regulations to address myopia, and differences in return are due to tax concessions.

The retirement regulations are set in an optimal retirement policy exercise that sets mandatory contribution rates and tax concessions to maximise the well–being of prospective newborns into the society. This exercise is able to rationalise the contribution rates and concessions we observe. At the same time, all but the poorest workers' portfolios are over–weight in the retirement account relative to what they would choose for themselves. This is the price of helping the present–biased save for retirement: they will only save if the money is illiquid, and the government cannot distinguish types so everyone must be required to save at the same rate into the same illiquid environment.

To perform the stimulus experiment, we shock the model with unanticipated transfers (fiscal policy) and early access to retirement savings (liquidity policy) designed to boost household consumption by the same amount over one quarter. Matching the short–term stimulus from the two approaches allows us to compare their long–run implications. There are three main differences (i) the tax changes driven by the fiscal rule distort inter–temporal consumption smoothing. This distortion is much greater under fiscal than liquidity policy; (ii) these taxes also cause redistribution, with lower future consumption by retirees and future generations subsidising the transfers received by workers; (iii) liquidity policy undermines retirement adequacy for present–biased workers, reducing their consumption upon retirement, and more so the less wealthy they were to begin with.

To weight these qualitative differences in the impacts of the two policies, we estimate the compensating variation that would make each household indifferent between liquidity policy and its fiscal alternative. Liquidity policy is better for retirees, future generations and wealthy workers—the groups that (a) do not benefit from the stimulus so much, but (b) will be liable for higher taxes under fiscal policy, and (c) are more likely to be overweight in their retirement accounts relative to total assets. By contrast, fiscal policy is preferred by working households with low wealth, a group disproportionately comprised of present–biased households. In aggregate, the group that benefit more from liquidity policy constitute a majority (70% of the population in our baseline calibration), liquidity policy may be politically popular despite its regressivity, concentrating the payment for stimulus on the vulnerable group of present–biased and poorer workers.

The degree to which one approach dominates the other depends on how the government sets retirement policy, and their fiscal rule. Aggregating the compensating variations, liquidity policy is marginally better for society the stricter is retirement policy, and the more aggressively the fiscal rule retires government debt. For example, liquidity policy is welfare *improving* for all but the poor present–biased types in our baseline calibration. This is because it allows some portfolio re–balancing, particularly valuable to wealthier workers. In an alternative calibration with lower mandatory contribution rates, wealthy workers are less over–invested in their retirement accounts, and there is much lower from benefit liquidity policy as a result. Similarly, the main results are based on an exponential fiscal rule that sets a half–life for debt gaps of 14 years (as in Galí, 2020). This rule front–loads the cost of stimulus under fiscal policy more than liquidity policies do because debt is 'retired' at a faster rate than workers retire. If the fiscal rule is instead relaxed to match the rate of retirement, or be even looser, then the implied taxes are less onerous and the relative benefit of liquidity policy is reduced.

**Related literature.** This paper brings together two large strands of literature. On one hand, the influential heterogeneous agent macro literature explains fiscal and monetary policy transmission based on liquidity constraints and the distinction between liquid and illiquid assets (see e.g. Kaplan and Violante, 2014; Kaplan et al., 2018; Bayer et al., 2019; Auclert et al., 2024). While this literature generally assumes that retirement accounts play an important role in household illiquidity, retirement policy is assumed exogenous, and none of these papers consider policies that alter the illiquidity of these accounts. On the other hand, there is a growing public economics literature that evaluates retirement policy and the optimal degree of illiquidity in retirement saving systems (see e.g. Amador et al., 2006; Moser and Olea de Souza e Silva, 2019; Beshears et al., 2020a; Andersen et al., 2024; Beshears et al., 2020b). While these papers assume that the level of illiquidity in retirement systems is a societal choice, they are largely silent on macroeconomic considerations related to fiscal stimulus. To the best of our knowledge, our paper is the first to offer a positive and normative evaluation of household liquidity policy relative to traditional fiscal stimulus.

A growing empirical literature analyses past episodes of household liquidity policies (Argento et al., 2015; Kreiner et al., 2019; Andersen, 2020; Hamilton et al., 2023; Preston, 2022; Schneider and Moran, 2024; Shapiro and Slemrod, 1995). We bring these stimulus packages together under the banner of household liquidity policy and analyse them theoretically in a modeling environment that allows for direct comparison with conventional fiscal policy. This allows for positive and normative comparisons of the two approaches, which may help to design future stimulus packages.

Our work complements Hamilton et al. (2023), which analyses Australia's early withdrawal program during Covid–19, using detailed micro data to identify who withdraws and what they do with the money. The authors show empirically that around one in six working age people withdrew, the modal withdrawal was all of the $20,000 allowed, and these households on average spent 40% of the funds within eight weeks. They argue that this is evidence of present–bias, which they estimate in a structural model. Our paper makes a very different but complementary contribution. While the above authors identify the MPC and the strength of present-bias, we take present-bias as given, and instead develop a model that captures the key trade-offs between household liquidity policy and traditional fiscal policy. This allows us to perform the first positive, normative, and distributional comparison of these two very different approaches to stimulus.

Our model is also informed by empirical work by Schneider and Moran (2024), who use a survey-elicited measure of psychological self-control, combined with the early release of retirement wealth in Australia, to estimate the relative importance of behavioral biases versus situational factors in accounting for early withdrawal from retirement accounts. The authors find that self-control heterogeneity plays an important role in predicting early withdrawal, and is a stronger predictor than other behavioural factors such as financial literacy, planning horizons, or personality traits. Overall, individuals in the top quintile of self-control issues are 60% more likely to withdraw than those in the bottom quintile. Informed by these empirical results, we also incorporate heterogeneity in present-bias into our quantitative model, evaluate how the two policies affect long-term retirement adequacy, and examine how the welfare implications of liquidity policy differ for individuals with versus without present-bias.

Some papers use quantitative models to explore the role of retirement accounts in stimulating the economy, but none compare liquidity policy to traditional fiscal policy. Love (2017) proposes a policy to stimulate the economy using counter–cyclical matching to retirement contributions, which he evaluates in a life-cycle model. Graves (2023) develops a HANK model to analyze the flight–to–liquidity that occurs following an increase in unemployment. He conducts one counterfactual exercise showing the effect of lower withdrawal penalties on aggregate consumption during Covid–19. Finally, Kaplan et al. (2020b) explore the tradeoff between health outcomes and economic impacts of policy choices during Covid–19 in the USA. They combine a HANK model with an SIR module of disease transmission, and use it to assess the impact of the various economic and health policies used in the USA. A part of the CARES Act that they model is the USA's removal of the withdrawal penalties from individual retirement accounts, as in Graves (2023), but this is not the focus of their analysis. Our paper (1) characterises the different channels through which household liquidity policy differs from traditional fiscal policy, (2) evaluates the distributional implications of the two policies, and (3) conducts a welfare analysis of the two policies, something that no previous paper has attempted.

Our modeling approach builds upon Attanasio et al. (2024) and Maxted et al. (2024) who show the importance of present-bias for hand-to-mouth behavior and fiscal policy, but do not consider household liquidity policy. We contribute to the broader heterogeneous–agent macro literature by providing a new micro foundation for the illiquid accounts commonly featured in two-asset macro models. This illiquidity is generally modeled as an exogenous feature of the world, when in reality it is usually a result of government policy. Empirically, household budgets are made up of only two types of genuinely illiquid asset: housing and retirement savings (Fagereng et al., 2019). In both cases, much of the illiquidity is due to regulation, e.g. restrictions or penalties on withdrawals from retirement accounts, and limits to home equity withdrawal. Modelling it as such opens the option for liquidity policy in our environment.

Finally, we contribute to the literature about retirement system design. A common thread in this literature is that imposed illiquidity is justified to help households overcome biases in their decision–making. The government has a role in mandating some form of retirement saving, and faces a problem of how to optimally balance the long–run commitment that households need against the short–run need for flexibility to insure working–life idiosyncratic risk, and also to balance the welfare of the behaviourally biased against those who are not (Beshears et al., 2020a; Moser and Olea de Souza e Silva, 2019; Amador et al., 2006). Our government faces the same type of problem, but is constrained to consider mandatory DC systems, as implemented in many countries (OECD, 2023). We show that the optimal DC system involves mandatory contributions close to those actually observed, but does not involve any tax concessions, in stark contrast to the systems in place. Instead, we rationalise these tax concessions as necessary to encourage people to opt into the system, at least at the beginning of their careers.

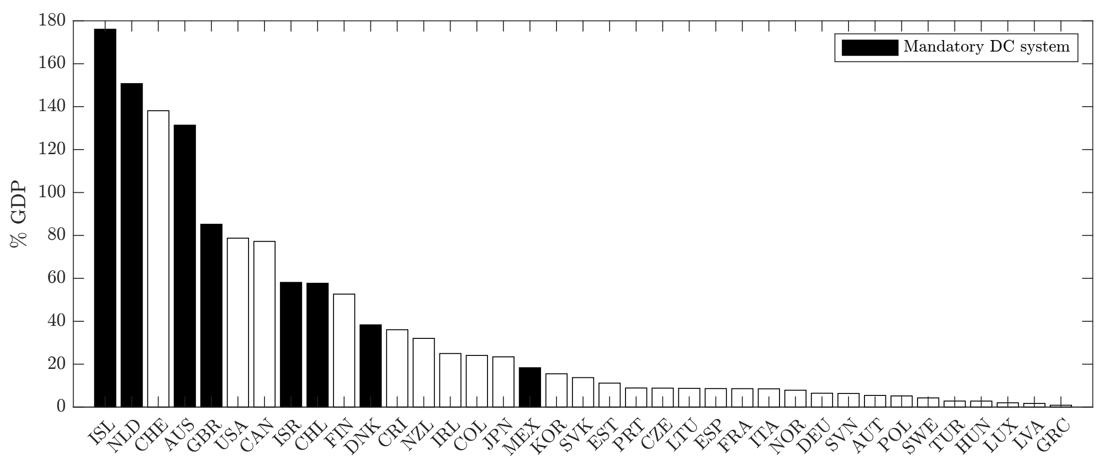
One major feature of the public literature on retirement system design and reform is the impact of retirement systems on the decision to retire. These papers (e.g. Blundell et al., 2016; Kolsrud et al., 2024) emphasise the distorting effects retirement policy can have on labour supply and estimate the optimal design subject to these distortions and the fiscal externalities they impose. We abstract entirely from the retirement decision in this paper. Rather we take retirement to be a fact, which creates a need for extra savings, but we let the event itself arrive randomly. This simplifies the problem by removing a household decision without undermining the point of our analysis, which is to explore the impact of different approaches to stimulus for a given retirement system.

**Road map** The paper proceeds as follows. In Section 2 we describe defined contribution retirement schemes, the relevant institutional setting for the paper, and identify two key parameters that we will map to our model. In Section 3 we detail the model, and calibrate its standard parameters in Section 4. In Section 5 we set up and solve the government's optimal retirement policy problem and show that this approach rationalises the key parameters identified in Section 2. Section 5.6 validates the macro–moments in the stationary solution of our model calibrated to the optimal retirement policy. We then turn to the stimulus policy experiments in Section 6, showing the two approaches are similar in aggregate, but have very different distributional implications. We evaluate their differences with a welfare analysis in Section 7, finishing with robustness checks. Section 8 concludes.

## 2 Defined contribution retirement accounts

The need to provide for retirement is universal and government intervention to meet some of this need is also very common. The government's involvement is justified for a variety of reasons. These include a redistributive motive, to help avoid poverty in retirement, a protective motive, to short–circuit the moral hazard created by the redistributive motive (i.e. households neglect to save for retirement, anticipating the government will bail them out), and a paternalistic one, correcting for biases that reduce working–life saving (Feldstein, 1985; Beshears et al., 2015). The means with which countries address these needs vary a lot, but usually involve some mix of state and private provisions, with the latter becoming increasingly important as governments grapple with the strain of unfunded state pension provisions coupled with ageing populations (OECD, 2018).

Figure 1: Retirement system assets




Our focus in this paper is on countries with private, mandatory defined contribution (DC) schemes.[3] In DC pension systems, working–age people make contributions into regulated investment accounts, and they have a claim on the balance and accumulated returns upon retirement; these systems can build up substantial resources, as illustrated in Figure 1 which plots total retirement assets across OECD countries. The exact design features of DC accounts differ across countries, but they can be broadly understood in terms of rules defining (a) liquidity during working life, (b) contributions, (c) tax treatment, and (d) the state pension they are combined with.

**Liquidity during working life** Regulations affecting access to DC accounts differ across countries. In some settings, like the USA and UK, participants are allowed to withdraw prior to retirement, but they pay a penalty (10% in the USA, 55% in the UK). In others, like Australia, withdrawals are not allowed except in very dire personal circumstances (e.g. terminal illness or extreme financial hardship) effectively making the accounts completely illiquid. In either case, the illiquidity is created by regulation, rather than because the assets are difficult to transact in.

**Contributions** Regulations affecting contributions generally govern (a) whether any contributions are mandatory and (b) limits on voluntary contributions. Mandatory contributions, when they exist, are usually set as a proportion of pre–tax employment income, commonly in the range of 10–20% (see Table 1 for some examples from OECD countries). Voluntary contributions to these schemes are often allowed as well, but are usually limited to maximum nominal amounts per year because they attract tax concessions.

Table 1: Mandatory contribution rates in OECD DC systems

| | AUS | CHL | DNK | ICE | ISR | MEX | NLD | UK* |
|---|---|---|---|---|---|---|---|---|
| $\xi$ (p.p.) | 12 | 10 | 12 | 15.5 | 13 | 15 | 18.6 | 8 |

Source: OECD (2023) Table 3.4, p. 141. For all OECD countries with privately funded DC schemes, and no other mandatory private system. *The UK's is a default, rather than mandatory.

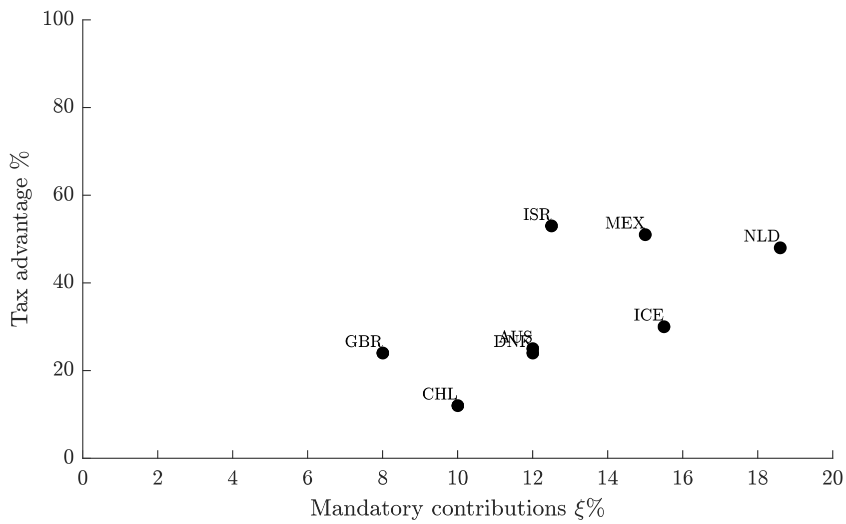
**Tax treatment** There are three potentially taxable flows in DC systems—contributions, investment returns, and withdrawals—and systems differ in whether each of these is taxed (potentially at concessional rates) or exempt, leading to a three-letter code describing them. In a system like USA 401(k)s, for example, contributions are made from pre–tax income, and returns are exempt as well, but withdrawals are taxed at the personal marginal tax rate, so it is coded EET. This is

[3] Such schemes are common, and increasingly being adopted as countries attempt to reduce the fiscal burden of state–only systems facing ageing populations (OECD, 2018). Among OECD countries, for example, 20 have some form of regulated, private retirement savings vehicle—Australia, Belgium, Canada, Chile, Costa Rica, Denmark, Estonia, Germany, Iceland, Ireland, Israel, Lithuania, Mexico, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom, United States (OECD, 2023). Of these, 8 mandate contributions into funded DC schemes—Australia, Chile, Denmark, Iceland, Israel, Mexico, Netherlands, United Kingdom (OECD, 2023), where the UK's contributions are a default, rather than mandatory.

the most common approach. By contrast, Australia's Superannuation contributions from pre–tax income are taxed at a concessional 15% rate, as are returns, and withdrawals are tax free, so it is coded TT.E[4]

Figure 2 shows the combinations of mandatory contribution rates ($\xi$) and tax advantages across the collection of OECD countries with mandatory DC systems.[5] The tax advantage variable comes from OECD (2018), and represents the tax savings from a given flow of contributions over a typical working life, relative to if they were saved in a regular investment account. Tax advantages are ubiquitous in these accounts, though they range from quite small (worth a discount of around 10% in Chile) to substantial (around 50% in Israel and Mexico).

Figure 2: DC system design in OECD countries




In the model introduced in Section 3, the retirement system features parameters encoding mandatory contributions and tax concessions, and we rationalise the features seen here as the optimal policy when society features a portion of the population that is present–biased, and the government needs to ensure workers to opt–in to the system at the start of their careers. The tax concessions, then, serve the *political* purpose of building buy–in into the system.

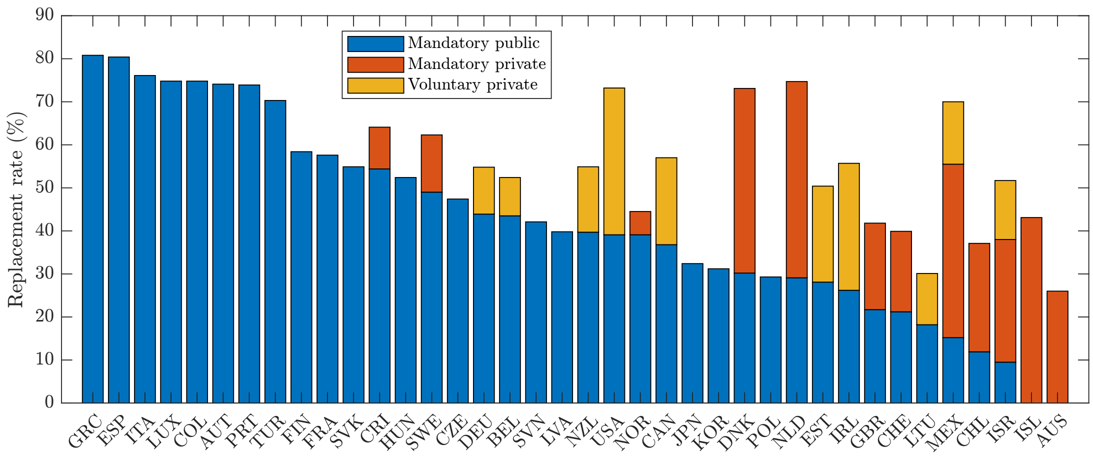
**State pension** DC systems are usually introduced to reduce the burden of retirement provision on current taxation by shifting the responsibility onto households themselves. In mandatory DC systems, government retirement provision is usually still present, but less generous. We can see

[4]In the model introduced in Section 3, we use a TTE system so that we can control the tax concession granted for contributions and returns inside the account. EET systems, whilst more common, typically apply taxes at full marginal rates, and so offer fewer degrees of freedom.

[5]Note this excludes countries that mix these with other mandatory private options.

this by looking at the state pension replacement rate for an average income earner across countries with and without mandatory private systems, plotted across OECD countries in Figure 3. This replacement rate for an average earner is 31% on average for OECD countries with mandatory systems, compared with 56% on average for OECD countries with only mandatory public systems, a substantial difference.[6]

Figure 3: Retirement system replacement rates for average earners, OECD countries




**Early access as stimulus** In response to economic crises, many countries have used access to these pools of resources to stimulate demand. Denmark was one of the first to introduce such measures during the Global Financial Crisis, allowing early withdrawals and temporary suspensions of contributions (Kreiner et al., 2019). This approach became more widespread during the Covid-19 pandemic, in which three approaches were used across at least 31 countries: allowing limited withdrawals when they were previously banned, as seen in Australia, Chile, and Peru (Hamilton et al., 2023; Madeira, 2022; OECD, 2021); removing withdrawal penalties, as in the United States (Graves, 2023); and reducing or deferring mandatory contributions in countries like Singapore, Malaysia, and Vietnam.

These policies resulted in significant outflows from retirement systems, with the most dramatic cases observed in Latin America. In Peru, a staggering 18.3% of assets in retirement savings plans were withdrawn, followed closely by Chile at 14.6%. Iceland and Australia saw smaller, but still substantial, withdrawals during Covid–19 of 3% and 1.4% of assets, respectively (OECD, 2021, p. 27).

[6]These are population weighted averages of the mandatory public gross replacement rates for an average earner across OECD countries with and without a mandatory private system in place (OECD, 2023, Table 4.2)

In the remainder of the paper we will analyse traditional fiscal and liquidity policy in an economy with government intervention into retirement policy motivated by paternalistic concerns. The model detailed in the next section features present–biased households who unwittingly save too little, and the government responds by forcing them to save in a defined contribution scheme with the features described above—limits on working–life withdrawal, mandatory contributions from labour income, and tax concessions. The liquidity policy we explore in Sections 6 to 7 is early withdrawal opportunities, like those used in Australia, Peru, and Chile. An alternative would be to reduce mandatory contribution rates. This works as stimulus as well, but it is more regressive because it grants relatively more liquidity to higher earners. We focus on withdrawal opportunities because it is the most directly comparable to fiscal transfers.

# 3 A model of households with illiquid retirement accounts

Our environment is a continuous–time, infinite horizon model featuring a measure of households, and a fiscal authority. Prices (the interest rate and wage) are fixed, and the equilibrium between households and fiscal authority is reached with a tax rate that balances the government budget.

## 3.1 Households

Households are differentiated by their stage of life (working or retired) and four time–varying state variables: their idiosyncratic productivity $z$ and employment state, the balance in their liquid account $b$, their illiquid account balance $a$, and their present–bias $\beta$. We collect these states into the tuple $x = (b, a, z, \beta)$, where $z$ captures both life–stage, workforce status and employed productivity.

### 3.1.1 Life–cycle transitions

Households live through working–life and then retirement (Yaari, 1965; Blanchard, 1985), transitioning out of each phase with fixed Poisson intensities $(\delta_R, \delta)$. When young, they work, and make consumption and asset allocation decisions. In retirement, they make the same decisions but can no longer receive the market wage. This creates a need for provision that is met personally by any assets they retire with, and collectively by the distribution of a fixed state–pension $w_R$. With these resources, retirees solve a cake–eating problem until they die, and are replaced by workers with no assets.

This lifecycle structure is a substantial simplification of how our careers typically progress. The most important departure from reality is that the transition into retirement is random, rather than a choice. This abstracts from an important part of the public economics literature that looks at the implications for retirement systems on incentives to retire, and the optimal pension reform to achieve affordability in the face of ageing populations (e.g. Kolsrud et al., 2024). Simplifying the transitions to be random gains us tractability, by reducing the households' choice set, without sacrificing structure that's important for our central question; transition rates are calibrated to match the average spans of working life and retirement.

#### 3.1.2 Idiosyncratic risk

Working–age households are subject to two types of idiosyncratic risk to their income. First, they jump in and out of employment with Poisson finding and separation intensities $(\lambda_f, \lambda_s)$. Second, whilst employed, their log–labour productivity is a diffusion that follows an Ornstein–Uhleneck process

$$d \ln z_t = -\theta_z(\ln z_t - \ln \bar{z})dt + \sigma_z dW_t \tag{1}$$

Where $\theta_z$ captures its persistence, $\bar{z}$ is the stationary mean, normalised to 1, $W_t$ is a Wiener process, and $\sigma_z$ is the weight on this noise. Newborn workers are employed, and draw their productivity from the stationary distribution of $z$. Retired workers are not subject to any idiosyncratic risk beyond the chance of dying.

#### 3.1.3 Budget constraints

All households have access to two accounts for storing wealth—a liquid and an illiquid account. At any given point in time, households have two choice variables—consumption $c > 0$, funded from their liquid account, and voluntary transfers between the two accounts $d \in \mathbb{R}$.

**Liquid account** The law of motion for the liquid account is

$$\dot{b} = r_b(b)b + (1 - \xi(x))y(x) - d - \chi(d, a) - (1 + \tau_c)c + T(x) - \tau_b(x) \tag{2}$$

Drift in liquid assets comes from various sources. First, asset returns, where $r_b(b)$ is the balance–dependent rate of return on the liquid account. We assume positive balances attract a return of $r_b$, and that borrowing, whilst allowed, comes with an extortionate penalty $\omega >> 0$ so that $r_b(b < 0) = r_b + \omega$.[7] This assumption creates a soft–borrowing constraint, which will be important later. $y(x)$ is idiosyncratic income, assumed to be $wz$ when working, $w_U$ when unemployed, and $w_R$ when retired. $T(x)$ captures fiscal transfers, $c$ is consumption, which attracts a tax $\tau_c$, and $\tau_b(x)$ is a state–contingent income tax function, discussed in full in Section 3.2.

[7] This setup reflects the empirical reality that very few households are actually borrowing constrained (Lee and Maxted, 2023), as in the wealthy hand–to–mouth literature (Kaplan and Violante, 2014). Instead, many hover close to zero liquid assets, rotate credit card balances (but not at their limit), and rarely exhaust *all* avenues for borrowing, which come with ever more onerous terms (pay–day loans, pawn shops, loan sharks etc).

Working households are required to contribute a proportion of their income $\xi(x)$ into their illiquid account (equal to zero in unemployed and retired states). As we will discuss in Section 3.2, this is one of the levers of regulation the government uses in retirement policy. As well as this, households may make voluntary transfers into (and out of) the illiquid account ($d$). These transfers are subject to a constraint $d \geq \gamma$ which is another lever of the government retirement policy, discussed in Section 3.2. Any voluntary transfers are subject to adjustment costs $\chi(d, a)$ which, following Kaplan et al. (2018), have the structure

$$\chi(d, a) = \chi_0(d)|d| + \frac{\chi_1}{2}\frac{d^2}{a} \tag{3}$$

The convex nature of this function puts a handbrake on the voluntary transfer policy, necessary to avoid jumps in continuous time, and its structure leads to analytical solutions for $d$. The linear cost $\chi_0(d)$ may differ for withdrawals and deposits.[8]

**Illiquid account** The law of motion for the illiquid account is

$$\dot{a} = r_a a + \xi(x)wz + d - \tau_a(x) \tag{4}$$

Where $r_a$ is the rate of return on the illiquid asset, and $\tau_a(x)$ is a state–contingent tax function, detailed in Section 3.2. Borrowing is not allowed in the illquid account, $a \geq 0$. As we will discuss in Section 3.2, and in contrast to the standard treatment in two–asset heterogeneous agent models (e.g. Kaplan et al., 2018), this illiquid account represents the households' regulated retirement accounts.

#### 3.1.4 Present–biased Preferences

A portion of the population $\mu \in [0, 1]$ is subject to present–bias, the rest are standard exponential discounters. The present–biased households have 'instantaneous gratification' (IG) preferences, the continuous–time analogue to quasi–hyperbolic discounting (Harris and Laibson, 2013; Laibson and Maxted, 2023; Maxted, 2024).

In working life, unbiased households' recursive preferences are as follows[9]

$$v(x_t) = \lim_{\Delta \to 0} \max_{c_t, d_t} u(c_t)\Delta + e^{-\rho\Delta}\,\mathbb{E}\left[v(x_{t+\Delta})\right] \tag{5}$$

[8] This allows us to impose a withdrawal penalty on the illiquid account to replicate e.g. the USA's 10% tax penalty.

[9] This is derived by separating the present $\Delta$ from the future in the integral

$$v(x_t) = \max_{c_s, d_s} \mathbb{E} \int_t^\infty D(s-t)u(c_s)ds$$

where the discount function is $D(s-t) = e^{-\rho(s-t)}$

Where $v(\cdot)$ is the value the household places on states $x$ in time $t$, which comes from a combination of the utility they gain from optimal consumption for the present moment $\Delta$, and the expected, discounted value placed on the state variables they're left with in the next moment ($\mathbb{E}[\cdot]$ captures all idiosyncratic risk transitions).

By contrast, the equivalent expression for present–biased households is[10]

$$v^{\beta}(x_t) = \lim_{\Delta \to 0} \max_{c_t, d_t} u(c_t)\Delta + \beta \cdot e^{-\rho\Delta}\, \mathbb{E}\left[v^E(x_{t+\Delta})\right] \tag{6}$$

The present–biased value in Equation (6) differs from (5) in two important ways. First, the continuation value is discounted by an extra $\beta \leq 1$ on top of the exponential discount $e^{-\rho\Delta}$. This is the source of present–biased behaviour—the IG agent values the future less than their exponential counterpart. Second, the continuation value $v^E(x_{t+\Delta})$ represents the value the household *believes* they will place on expected states in the future, which may not be how they actually value them. A 'sophisticated' agent holds correct beliefs: they will have the same value function in future as in the present (i.e. $v^E(x) = v(x)$) whereas 'naivete' wedges them apart.

We assume complete naivete i.e. $v^E(x) = v(x)$ where $v(x)$ is an exponential discounter's value function.[11] This assumption simplifies the analysis because the solution can be reached in two steps: (1) solve the exponential discounter's problem to find $v(x)$ and use the solution as the IG consumer's continuation value to (2) solve the IG consumer's problem for each type.

**Household problem and solution** During each stage of life, households choose consumption and voluntary transfers to maximise utility. Appendix A.1 shows that the exponential household's problem results in the working and retired value functions $v(x)$ and $v_R(x)$, and the policy functions $(c(x), d(x), c_R(x), d_R(x))$, and that the biased households' perceived value and policies are recovered directly from the exponential household results.

**Lemma 3.1** (Present–biased solution (Maxted, 2024))**.** Assuming (1) CRRA utility with risk–aversion $\sigma$, and (2) never–binding soft–borrowing constraint, the naive IG consumer's value and policies are scale transformations of the exponential discounter's equivalents

$$c^{\beta}(x) = \beta^{-\frac{1}{\sigma}} \cdot c(x) \quad \text{and} \quad v^{\beta}(x) = \beta \cdot v(x) \quad \text{and} \quad d^{\beta}(x) = d(x) \tag{7}$$

*Proof.* Derived in Appendix A.2. □

[10] As above, but where the discount is now the step function

$$D(s-t) = \begin{cases} 1 & \text{if } s - t = 0 \\ \beta \cdot e^{-\rho(s-t)} & \text{if } s - t > 0 \end{cases}$$

[11] This is an innocuous assumption. Maxted (2024) shows that under two assumptions (1) CRRA utility and (2) soft–borrowing constraint, a problem with any degree of sophistication is isomorphic to a fully naive agent with a lesser degree of present–bias.

The intuition behind this result, from Maxted (2024), is that an exponential discounter sets consumption so that marginal utility equals the marginal continuation value of liquid resources in future. The present–biased household does the same, but they perceive their marginal continuation value to be lower by $\beta$, and so consume more. Note that the present–bias only comes into play in decisions that trade between the present and future. The voluntary transfer choice is about balancing marginal values in the future, and as such is unaffected by present–bias.

These policies describe the optimal drift in the two accounts. Combined with the exogenous transitions in employment status, productivity, and life–cycle, they induce a stationary joint distribution $h(x)$ over all the households' state variables. We describe the Kolmogorov forward equation that characterises this distribution in Appendix A.3.

Present–biased households over–consume relative to the exponential discounters they believe themselves to be by a factor of $\beta^{-\frac{1}{\sigma}} > 1$. As a result, left to their own devices, these households are left with less savings in retirement. Crucially, they regret ending up in this position: it is not the result of rational planning, but rather a series of mistakes that they would not have made if they could commit in advance to a state–contingent consumption plan. This regret leaves room for the government to intervene to help resolve this commitment problem with retirement policy.

## 3.2 Retirement policy

The government sets retirement policy, which consists of the unconditional state pension $w_R$ and a variety of parameters governing the households' illiquid accounts. In other macro papers, the illiquid account is usually taken to represent housing, or some other difficult–to–transact–but–attractive asset. In our model, it is an individual retirement account (IRA).[12] Our treatment of this account differs from the usual in some important ways.

The underlying asset is the same as in the liquid account, so the gross rates of return are equal.[13]

$$r_b = r_a = r$$

Second, the account's illiquidity stems from four regulatory parameters. The government can penalise withdrawals during working life ($\chi_0(d < 0) = \chi_0$), or limit them directly by imposing the constraint $d \geq \gamma$. The source of the illiquidity in the retirement account is therefore regulatory; we set $\chi_0(d > 0) = 0$ and the convex adjustment cost $\chi_1$ to be trivially low.

[12] Superannuation in Australia, IRAs and 401(k)s in the USA, SIPPs in the UK etc

[13] In developed countries at least, participation in DC plans doesn't change the span of assets available too much. Retail consumers may not be able to access alternatives like hedge funds themselves, but these tend to make up a small portion of DC retirement funds, which tend to be mainly invested in market securities, or to hold real assets like infrastructure directly that can be accessed through market securities as well.

In line with our discussion of DC schemes in Section 2, the government also mandates that a certain proportion $\xi \in [0,1)$ of labour income be deposited into the retirement account, and offers a concession $\varphi \in [0,1]$ so that contributions into and returns within the retirement account are subject to less tax. We discuss how the government government chooses these policy parameters in Section 5.

The mandatory contribution rate and tax concession affect the tax functions introduced in Equations 2 and 4 as follows:

$$\tau_b(x) = \tau\left[rb \cdot (b > 0) + (1 - \varphi\xi)wz\right] \tag{8}$$

$$\tau_a(x) = \tau(1 - \varphi)ra \tag{9}$$

The variable $\varphi$ reduces the tax liability for mandatory contributions as well as asset returns within the account, implementing a concessional TTE system discussed in Section 2. If $\varphi = 1$, then contributions are made from pre–tax income, and returns are tax–free. We assume that retirees are free to withdraw from their retirement account, continues to receive tax concessions, but cannot deposit. At all stages in life, voluntary contributions are made from post–tax income, and withdrawals are not taxed.

Retirement policy gives life to the illiquid asset—it is illiquid due to regulations designed to discourage withdrawal during working life, and any difference in asset returns comes from preferential tax treatment. This setup closely mirrors the defined contribution systems discussed in Section 2. Retirement systems across the world are complex and vary from country to country (Beshears et al., 2015), but the setup in our model allows us to approximate many of their key features. In particular, we can span three of the first first four pillars in the World Bank's Conceptual Framework for classifying pension schemes (Holzmann et al., 2008): the state pension $w_R$ protects households from poverty and affects redistribution (Pillar 0), mandatory contributions to the retirement account can be used to force personal saving in line with income, correcting for myopia and other errors (Pillars 2), and tax concessions in the retirement account can support voluntary savings as well (Pillar 3).[14]

Implementing a mandatory defined contribution system, like the Australian Superannuation system, for example, is done by banning withdrawals and mandating contributions from labour income, as well as granting a tax discount of about 37.5% to an average earner;[15] under current legislation, the settings would be $(\chi_0, \gamma, \xi, \varphi) = (0, 0, 0.11, 0.375)$; and the average state pension per pensioner is around 20% average employed income.[16]

[14] What's missing is Pillar 1: a state–organised defined–benefit pillar like the USA's social security system, but this is beside the point for the purposes of this paper.

[15] Concessional contributions and returns within Superannuation are taxed at 15%, compared to a 23% income tax bill for a person on average income with no dependents.

[16] Australia's state pension is asset and income means tested so this average hides a lot of variation.

### 3.3 Fiscal authority

The fiscal authority pays benefits to the retired ($w_R$) and unemployed ($w_U$), services debt ($rB$), makes government purchases of the consumption good $G$, and makes discretionary transfers to households $T(x)$. It takes in taxes on consumption, capital and labour income. Deficits are funded by changes in government debt $\dot{B}$, defined below.

$$\begin{aligned}\dot{B} = &\underbrace{G + w_U\pi_U + w_R\pi_R + rB + \int T(x)h(x)dx}_{\text{Spending}} \\ &- \underbrace{\int \left(\tau_c \cdot c(x) + \tau_b(x) + \tau_a(x)\right) h(x)dx}_{\text{Tax revenues}}\end{aligned} \tag{10}$$

Where $\pi_U$ and $\pi_R$ represent the measures of unemployed and retired people, and $z_U$ and $z_R$ represent those states (e.g. $\pi_U = \int h(b, a, z = z_U, \beta)dx$). Borrowing may be restricted by an exponential fiscal rule (as in Auclert et al., 2023; Galí, 2020; Angeletos et al., 2023):

$$\dot{B} = -\mu\left(B - \bar{B}\right) \tag{11}$$

This rule sets an exponential decay of the government debt gap around some ideal level $\bar{B}$, where $\mu \geq 0$ (if zero, always hold debt constant; if $\infty$ return to $\bar{B}$ immediately).[17] We assume that government debt is fixed at the target in the stationary solution, and that the fiscal authority uses the consumption tax rate as the marginal tool to meet the fiscal rule in both the stationary and dynamic solutions. We also assume that this rule is suspended whilst any stimulus is active (whether fiscal or liquidity policy), and reactivated immediately after.

The fiscal authority has a balance sheet to manage and a rule to meet, but no objectives beyond this. Control of the discretionary policy levers and benefit levels are left to the government, whose choices the fiscal authority takes as given.

### 3.4 Stationary equilibrium

For fixed prices $(w, r)$ and given retirement policy settings $(w_R, \chi_0, \gamma, \xi, \varphi)$, a stationary equilibrium is the set of working–life and retirement value functions $(v(x), v_R(x))$ and policy functions $(c(x), c_R(x), d(x), d_R(x))$, the measure over household states $h(x)$, and the consumption tax rate $\tau_c$ such that

[17] Under this fiscal rule, debt must follow the path $B_{t+s} - \bar{B} = \left(B_t - \bar{B}\right)e^{-\mu s}$ yielding a half–life of $\ln 2/\mu$ in units of model frequency.

– The values and policies solve the household problems (Appendices A.1 and A.2)

– $h(x)$ is stationary (Appendix A.3)

– $\tau_c$ balance the budget (Equation 10)

In equilibrium, the household and fiscal authority constitute a combined model–block that takes prices and policy settings as given, and produces aggregate demand for a consumption good (the sum of household and government demand), net asset supply (total household wealth, net of government debt), and aggregate labour supply, which is an endowment process.

**Stimulus policy** This model allows us to analyse the two approaches to stimulus policy we're interested in. Stimulus is taken to mean policy interventions that increase aggregate consumption above its stationary level in the short–run. And the two methods available to the policy–maker are (a) fiscal transfers $\Delta T_t$, funded by deficits and repaid via future tax changes that meet the fiscal rule, or (b) household liquidity policies i.e. regulatory changes to either restrictions on retirement account withdrawals $\Delta\gamma_t$, or mandatory retirement contribution rates $\Delta\xi_t$; we focus on the former in this paper.

### 3.5 Computational solution

We solve the model using a finite–difference scheme (Achdou et al., 2022), with discrete, non–linear grids for the two endogenous assets, and three states for the idiosyncratic productivity process. The value–function updates are affected using the nested–drift algorithm introduced by Sabet and Schneider (2024), which is monotone and consistent, and is very stable to parameter choices as a result. And the stationary solution makes further use of the tentative-guess algorithm detailed in (Sabet and Schneider, 2024), to ensure the starting guess is appropriately close to the solution for the local convergence of this finite–difference scheme (Barles and Souganidis, 1991) to be guaranteed.

## 4 Calibration

We calibrate the model to match an advanced economy with a mandatory DC retirement system. To do this, most parameters are set to standard values in the literature. The remaining parameters govern retirement policy, which are set in the optimal policy exercise detailed in Section 5.

Our calibration achieves two goals that set the scene for our policy experiments. First, the retirement policy settings reflect and rationalise what we observe in countries with mandatory DC systems: mandatory contributions, no withdrawals allowed, and tax concessions. This is necessary to ensure a fair comparison between fiscal and liquidity policy. If retirement policy were too restrictive, liquidity policy will be both very effective and unambiguously welfare improving; if too loose, then there isn't enough firepower for liquidity policy to be a viable substitute for fiscal. Second, the aggregate MPC among workers is empirically realistic. This is necessary to ensure the consumption boost from fiscal (the desired policy outcome) is matched by an appropriate increase in government debt (which sets the longer–run policy impact). This goal is achieved by estimating the present–biased share $\eta$ that matches the model's aggregate MPC to empirical estimates, discussed below.

The calibrated parameters are outlined in Table 2, and detailed below[18].

Table 2: Calibrated parameters

| **Parameter** | **Symbol** | **Baseline calibration** | **Source** |
|---|---|---|---|
| Retirement intensity | $\delta_R$ | 1/160 | 40 year av. work–life |
| Death intensity | $\delta$ | 1/80 | OECD (2023) |
| Sep. & find. intensity | $(\lambda_s, \lambda_f)$ | (0.0587,1.2) | Shimer (2005) & BLS |
| Log–income process | $(\rho_z, \sigma_z^2)$ | (0.9136,0.0426) | Floden and Lindé (2001) |
| Risk aversion | $\sigma$ | 2 | Standard |
| Discount rate | $\rho$ | 0.0025 | Carroll et al. (2017) |
| Present–bias | $(\beta_L, \beta_H)$ | (0.5,1) | Ganong and Noel (2020) |
| Present–bias share | $\eta$ | 0.5 | Target MPC $\in [0.15, 0.25]$ |
| Risk–free real rate | $r$ | 0.0051 | 2% p.a. |
| Borrowing penalty | $\omega$ | 0.4024 | 500% p.a. |
| Wage | $w$ | 0.25 | Numeraire |
| Unemployment benefit | $w_U$ | 0.1 | Shimer (2005) |
| Income tax | $\tau$ | 25% | OECD average |
| Consumption tax | $\tau_c$ | 12% | Budget balance |
| Government spending | $G$ | 0.0238 | G/GDP = 15% |
| Steady state debt | $\bar{B}$ | 0.1589 | Debt/GDP = 25% |
| Fiscal rule | $\mu$ | 0.0128 | Galí (2020) |
| Adjustment costs | $(\chi_0, \chi_1)$ | $(0, 0.001)$ | Trivial |

**Life–cycle** Working lives and retirement last an average of 40 and 20 years, respectively. This assumes a working life spanning 25–65, and matches the OECD average expected life–years after retirement (OECD, 2023, p. 192). After death, retirees are replaced by employed workers with zero assets and productivity drawn from its stationary distribution.[19]

[18] Time is continuous, with a base frequency of one quarter

[19] We also impose two extra rules on the transition from working life to retirement that households do not anticipate. The first is a 'forced–retirement' level in working households' illiquid account which does what it says on the tin. This limit is necessary to ensure the state–space is compact and that retirement balances don't get out of hand, but is set to $a_{max} = 15$, sufficiently high that it only affects a small measure of workers and keeps the retired population at realistic levels. And second, households that retire with negative total assets (liquid plus illiquid) are bankrupted. In the model, this means their gross positions in both accounts are returned to zero (the same as if they were born into retirement). Without bankruptcy, a small measure of households retire with the maximum debt, and they are stuck there because it is an absorbing state for present–biased households. This is a disastrous position to be in for these households—their consumption is close to zero and so their value is orders of magnitude lower than at other points in the state space. If the risk of destitution is not addressed, then the government's retirement policy is primarily focused on managing it, rather than the more prosaic concern of general retirement adequacy. The bankruptcy rule avoids the issue.

**Working–life idiosyncratic risk** Working households face idiosyncratic risk from jumps into, and out of, unemployment, and diffusion in their employed labour productivity. The jump transitions are governed by finding and separation rates of 0.0587 and 1.2 respectively; the former matches the quarterly separation rate in Shimer (2005), and the latter matches the mean 2.5 months spent in unemployment from the US Bureau of Labor Statistics.[20] The labour productivity process is calibrated to match the estimated AR(1) process in log–income residuals after individual characteristics effects are stripped out, from Floden and Lindé (2001).[21].

$$\ln z_t = 0.9136 \ln z_{t-1} + u_t,\ u_t \sim N(0, 0.0426)$$

We cast this in continuous time, following Achdou et al. (2022), so it becomes a monthly Ornstein–Uhlenbeck process, which we discretise over $k = 3$ points with reflecting barriers at $\ln z \in [-\sigma, \sigma]$, and normalise so the stationary distribution of $z$, in levels, has a mean of one.[22]

**Preferences** Households have CRRA preferences over consumption

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

With a coefficient of relative risk aversion equal to $\sigma = 2$, as is standard. Households discount the future at $\rho = 0.0025$, which corresponds to an annual discount rate of 0.99. Recall that households also face the risk of retirement and death. This boosts the effective annual discount rate to 0.97 and 0.94 for employed and retired households, within the range of standard estimates (Carroll et al., 2017).

**Prices & fiscal** The base rate of return is $r = 0.0051$ per quarter (2% p.a.) and the borrowing penalty is $\omega = 0.4024$ (500% p.a.), set to be extortionate to impose a soft–borrowing constraint. The annual wage for a unit of effective labour is the numeraire (quarterly wage $w = 0.25$) so all other monetary values are relative to this. The government pays an unemployed benefit $w_u = 0.1$ to match the standard replacement rate of 0.4 from Shimer (2005). The baseline income tax is set to $\tau = 25\%$, the OECD average personal income tax rate for a single person with no children on

[20] Table A.12 'Unemployed persons by duration of unemployment: Monthly, Seasonally Adjusted'; recent average outside of recessions.

[21] This calibration is used in Maxted et al. (2024) and Guerrieri and Lorenzoni (2017) The estimates are from PSID data covering 1988–1991. More recent estimates of the same AR(1) process all get numbers around this i.e. with auto–regressive parameter at least 0.9, and standard–deviation at least 0.2 (e.g. Kaplan et al., 2020a; Chang et al., 2013; Guvenen et al., 2023).

[22] This yields a stationary distribution defined by

$$\{z_L, z_M, z_H\} = \{0.681, 0.9516, 1.4652\} \text{ and } \{\pi_L, \pi_M, \pi_H\} = \{0.2686, 0.4628, 0.2686\}$$

Where $\pi_i$ represent the stationary probability of being in state $i$.

the average wage. In both the stationary solution and the dynamic exercises later, the consumption tax $\tau_c$ is set internally to meet the fiscal rule.

Steady state government spending is $G = 0.0238$, targeting 15% of GDP, and steady state debt levels are $\bar{B} = 0.1589$, or 25% annual GDP. In both cases, GDP here is taken to be the aggregate income of the steady state employed population multiplied by 3/2 to adjust for the capital share. The exponential parameter on the fiscal rule is set to $\mu = 0.0128$, such that debt gaps are closed with a half–life of 13.5 years, following Galí (2020).[23] This parameter has no influence on the stationary solution, in which the rule dictates budget balance.

**Adjustment costs** The adjustment cost function serves no purpose other than to deliver analytical solutions for the voluntary contribution policy. It is possible to use the function to impose an early withdrawal penalty during working life, but this is discussed in the next section. The baseline parameters that set common adjustment costs between working and retired households are set so the costs are trivial $(\chi_0, \chi_1) = (0, 0.001)$.

**Present–bias** The population is split into present–biased and exponential households. The present–biased have $\beta = 0.5$, similar to Ganong and Noel (2020) and Laibson et al. (2024), and the exponential have $\beta = 1$. The size of the present–biased population is set to $\eta = 0.5$. This leads to an average present–bias in the population of 0.75, which is within the range estimated in papers with only one type (Laibson et al., 2024; Hamilton et al., 2023), but a greater biased proportion than estimated by Ganong and Noel (2020), who find only 25% of their sample (unemployed workers) exhibit this degree of bias. This parameter has crucial importance for the model because it sets the aggregate marginal propensity to consume. As we show in Section 5.6, setting $\eta = 0.5$ leads to an equilibrium MPC in the centre of the range of empirical estimates, and we explore the sensitivity of our results to this parameter in Section 5.5.

**Retirement policy framework** We restrict attention to mandatory DC schemes backed by a fixed state pension because this is the relevant policy framework within which liquidity policy has been used. Withdrawals are not allowed $\gamma = 0$ and the withdrawal penalty is set to zero $\chi_0(d < 0) = 0$. The state pension is set to $w_R = 0.075$, a replacement rate of 30% average worker income, which matches the average replacement rate across OECD countries with a mix of state and private pension schemes.[24] The remaining retirement policy parameters are the mandatory contribution rate $\xi$ and the tax concession $\varphi$. We saw in Section 2 that these parameters vary across countries

[23] This matches the European Union's fiscal compact, which includes a provision that excess debt should be reduced by $1/20^{th}$ each year (Galí, 2020).

[24] This is the replacement rate of average income provided by the state pension ('Mandatory Public'), weighted average across OECD states that combine state and private components in their pension schemes (OECD, 2023, Table 4.2, p. 153). Where the state is the sole pension provider, e.g. Austria, Spain, or Colombia, this weighted average is 56% in OECD countries.

but that contributions cluster in the range $8 - 20\%$ and tax concessions are universally granted to these illiquid savings environments. In the next section, we set these remaining parameters optimally.

# 5 Optimal retirement policy

In this section the retirement policy parameters are set to implement the optimal mandatory DC scheme. Within the constraints of the policy framework, the government chooses the optimal mandatory contribution rate ($\xi$) and tax concession ($\varphi$).

## 5.1 The government's problem

The government's problem is similar to that in the literature on paternalistic savings policies in that they must balance two tradeoffs. First, following Amador et al. (2006), there is both a need to provide households with commitment (to overcome present bias) and flexibility (to insure against idiosyncratic risk). This prompts government intervention to help the present–biased save, but puts a limit on how much mandatory saving is appropriate. Second, present–bias is heterogeneous but unobserved by the government. This introduces a need for the government to balance the interests of the biased against the unbiased, potentially prompting screening or compensation (Moser and Olea de Souza e Silva, 2019; Beshears et al., 2020a). Our setting differs from the extant literature by limiting the government's options to a DC system with uniform settings across the working–age population.

We evaluate welfare of these policy settings as follows.

**Definition 5.1** (Social welfare criterion)**.** Social welfare in the stationary solution is defined as the expected long–run value for a prospective newborn.

$$W(\xi, \varphi) = \mathbb{E}_{(z,\beta)}[\hat{v}(0, 0, z, \beta; \xi, \varphi)] \tag{12}$$

$$\hat{v}(x; \xi, \varphi) = \mathbb{E} \int e^{-\rho s} u\left(c(x_s; \xi, \varphi)\right) ds \tag{13}$$

This is found in two steps. First we find the value of actual (rather than anticipated) behaviour[25] in Equation 13, referred to as the 'long–run value' (O'Donoghue and Rabin, 2006; Bernheim and Taubinsky, 2018; Naik and Reck, 2024). Using this value means the government anticipates but does not adopt the bias of its subjects when evaluating policy choices. Second we restrict attention to newborns (zero assets) and find their expected long–run value, based on the stationary distribution over $(z, \beta)$ in Equation 12. Defining the criterion like this means comparisons of welfare

[25]Both $w(x)$ and $v(x)$ are based on the incorrect assumption that the policy rules are $\check{c}(x)$.

under different policy regimes pose the question: 'which society would you prefer to be born into, anticipating potential present–bias?'.

Having defined a welfare criterion, we consider two approaches to the problem that differ in how powerful the government is. These lead to two distinct equilibria, defined below.[26]

**Definition 5.2** (Social equilibrium)**.** The social equilibrium is a stationary equilibrium with retirement policy settings $(\tilde{\xi}, \tilde{\varphi})$ that maximise steady state social welfare.

$$(\tilde{\xi}, \tilde{\varphi}) = \arg\max_{\xi,\varphi} W(\xi, \varphi)$$

The social equilibrium is the outcome of an all–powerful government's problem, in the sense that they can guarantee compliance with the policy settings they choose. As we will show in the next section, the optimal tax concession is zero in the social equilibrium. To rationalise the ubiquity of these tax concessions, we suppose the government is subject to an extra participation constraint, optimisation under which leads to the 'buy–in' equilibrium.

**Definition 5.3** (Buy–in equilibrium)**.** The buy–in equilibrium is a stationary equilibrium with retirement policy settings $(\xi^*, \varphi^*)$ that maximise social welfare subject to a participation constraint: no newborn chooses to opt out of the system by setting their own personal retirement parameters to $(\xi_i, \varphi_i) = (0, 0)$.

$$(\xi^*, \varphi^*) = \arg\max_{\xi,\varphi} W(\xi, \varphi) \;\text{ s.t. }\; \hat{v}(0, 0, z, \beta; \xi^*, \varphi^*, \tau_c) \geq \hat{v}(0, 0, z, \beta; 0, 0, \tau_c) \;\forall\, z, \beta$$

The buy–in equilibrium is the outcome when we imagine each household has the option at the start of their careers to opt out of the system[27]. In this case the government must cajole compliance for the system to be stable. The government wants to implement a pooling equilibrium where no–one opts out because (i) their policy tools don't allow for screening as in Moser and Olea de Souza e Silva (2019), and (ii) a separating equilibrium where only the present–biased opt in is not possible anyway, due to naivete. Note that this amounts to ensuring the rational agents opt in: assuming naivete, everyone thinks themselves rational.

## 5.2 The social equilibrium

Figure 4 plots the social welfare under different combinations of retirement policy settings, with warmer colours representing greater welfare. The social equilibrium is labelled, and picks the highest welfare point in the area plotted.

[26] We don't mean that there are multiple equilibria, but that the government will select different equilibria under these different constraints.

[27] Call this the 'Dubai option'.

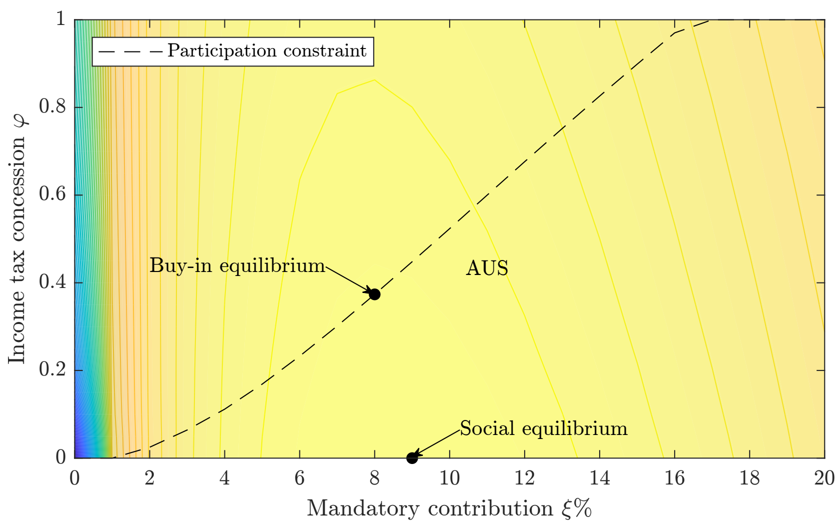
**Result 5.4** (Social equilibrium)**.** The social equilibrium is $(\tilde{\xi}, \tilde{\varphi}) = (0.09, 0)$

This contribution rate is toward the lower end of the mandatory contribution rates in countries with compulsory DC schemes, with OECD examples outlined in Table 1.

Note that the social equilibrium involves no tax concession. The plotted results are for a population with $\eta = 0.5$, but the optimality of zero tax concession in social equilibrium holds no matter how small or large a share of the population are biased—it is never necessary to compensate people for a policy that helps them, and tensions between biased and unbiased workers are better resolved by adjusting the contribution rate than with tax concessions.

This result reflects a finding in the empirical public literature that a majority of people are not responsive to incentives to save for retirement, and those that are tend to save more in retirement accounts by reducing savings elsewhere (Chetty et al., 2014; Choukhmane and Palmer, 2024). The use of tax concessions to encourage retirement savings is therefore likely to incur a fiscal cost with little welfare benefit. In our setting, greater tax concessions necessitate higher tax rates on consumption to make up for the eroded base. The result here tells us the marginal welfare benefit from the concessions is less than the welfare cost of greater taxes.

Figure 4: Social welfare under policy settings




## 5.3 The buy–in equilibrium

Tax concessions are ubiquitous in actually–existing DC schemes, as discussed in Section 2, and so to rationalise them we explore a political dimension to the government's problem—the need to implement a pooling equilibrium where agents opt–in to the retirement system at the beginning of their careers. The black dashed line in Figure 4 traces out the limited menu facing the government who has to deal with a participation constraint. In this setting, the tax concession ensures participation. Each contribution rate $\xi$ has a minimum concession $\varphi(\xi)$ necessary to implement a pooling equilibrium, and this minimum is increasing in the contribution rate $\varphi'(\xi) > 0$. The government selects among these $(\xi, \varphi(\xi))$ to maximise welfare. The result is the buy–in equilibrium, with an optimal contribution rate lower than in the social equilibrium, and more tax concessions.

**Result 5.5** (Buy–in equilibrium)**.** The Buy–in equilibrium is $(\xi^*, \varphi^*) = (0.08, 0.37)$

This result is in line with what's observed in the real world—substantial tax concessions coupled with mandatory contributions close to the OECD examples in Table 1—and this is our preferred calibration as a result. The tax treatment in the model is a TTE system, with concessions for the taxes on entry and returns. Direct comparison to other countries is difficult because most countries use an EET system. The one country that is comparable is Australia, identified on Figure 4, which taxes mandatory contributions from pre–tax income and Superannuation returns at 15%, a $\varphi = 0.4$ discount on the standard 25% rate for an average worker.[28] With a mandatory contribution rate currently at 11%, this is remarkably close to the buy–in euqilibrium in our model.

## 5.4 The impact of retirement policy for working–age people

Retirement policy substantially raises working households' saving rates and retirement adequacy, and this effect is stronger for the present–biased households. Table 3 shows the impact of retirement policy for workers, by type. To see behaviour without mandatory savings first we set $\xi = 0$ and re–solve the model. In this scenario, working households' saving rate is 10% of earnings, with a large difference between present–biased and unbiased households (4% and 16% respectively). With mandatory savings, the average saving rate is higher (23%), with hardly any difference between the two types of household.

Table 3: Effect of mandatory saving on workers, by type

| | **Average saving rate (% earnings)** | | **Median %$\Delta C$ at retirement** | |
|---|---|---|---|---|
| | Present–biased | Unbiased | Present–biased | Unbiased |
| Without mandatory saving | 4 | 16 | −27 | −13 |
| With mandatory saving | 22 | 24 | 16 | −12 |

Similarly retirement adequacy, which we measure as the median expected change in consumption from working to the first year of retirement, is much improved by the policy (see Table 3).[29] Without

[28]Specifically this is the average personal income tax rate on labour income for a single person with no children on the average wage, from the OECD's 'Labour taxation - average and marginal tax wedge decompositions' series in 2023.

[29]Specifically, we find the difference between the workers' consumption policy and the average expected consumption rate over the first year of retirement, using the employed worker's state distribution to identify moments.

mandatory saving, unbiased people's consumption drops by a median of 13% upon retirement, and more than double this for present–biased households. With mandatory saving, the unbiased household's consumption drop barely changes, whereas biased households now see an increase in their consumption of 16%, as they move from being working–poor to having resources to spend.

## 5.5 Sensitivity to present–biased share

Table 4 shows how the government's solutions change with the present–biased population share.[30] The optimal mandatory contribution rate is increasing in the present–biased population share, reflecting the increasing level of need in society. The tax concession necessary in the buy–in equilibrium is increasing as well, to ensure early–career opt–in to the system.[31] The aggregate worker MPC in the buy–in equilibrium is also increasing in $\eta$, both due to the combination of a greater share of present–biased households, and the greater mandatory contribution rate.

Table 4: government solutions with different population mixes

| Present–biased share $\eta$ | 10% | 20% | 30% | 40% | **50%** | 60% | 70% | 80% | 90% |
|---|---|---|---|---|---|---|---|---|---|
| *Buy–in Equilibrium* | | | | | | | | | |
| $\xi^*$ | 0.04 | 0.06 | 0.06 | 0.07 | **0.08** | 0.08 | 0.08 | 0.09 | 0.09 |
| $\varphi^*$ | 0.11 | 0.23 | 0.23 | 0.30 | **0.37** | 0.37 | 0.37 | 0.45 | 0.45 |
| Worker MPC | 0.05 | 0.10 | 0.14 | 0.18 | **0.22** | 0.26 | 0.30 | 0.34 | 0.38 |
| *Social Equilibrium* | | | | | | | | | |
| $\tilde{\xi}$ | 0.05 | 0.06 | 0.07 | 0.08 | **0.09** | 0.09 | 0.09 | 0.10 | 0.10 |

## 5.6 Model validation

Table 5 compares aggregate moments among workers in the stationary solution to the model to equivalents in the USA. The model produces an average quarterly MPC of 0.22 among workers (0.24 for the whole population), within the range of recent empirical estimates for non–durable consumption (Ganong et al., 2023; Jappelli and Pistaferri, 2014; Sahm et al., 2010, 2012; Fagereng et al., 2021; Kueng, 2018; Kaplan and Violante, 2022). The MPC is this high because 40% of workers are classed as hand–to–mouth, in line with standard estimates from the literature (e.g. Aguiar et al., 2024; Kaplan et al., 2020a; Kaplan and Violante, 2022).[32]

The model comes close to aggregate wealth–to–income ratios: recording total financial wealth of 6.4 times labour income, compared to 4.1 in the data; and liquid wealth measuring one year's worth of labour income, compared to 0.6 in the data. Having greater total wealth is natural, given our assumption of a mandatory DC retirement system. The actual USA features a voluntary DC

[30] The grid–search is restricted to whole percentage–points in $\xi$.

[31] The optimal tax concession is zero in the social equilibrium across all levels of the present–biased population.

[32] Hand–to–mouth status is defined by having liquid assets less than half of monthly labour income (Kaplan and Violante, 2022)

Table 5: Aggregate moments for workers

| Moment | Model | Data | Source |
|---|---|---|---|
| Quarterly MPC | 0.22 | $[0.15, 0.25]$ | Kaplan and Violante (2022) |
| % HTM | 40 | 41 | ” |
| Liquid wealth / labour income | 1.0 | 0.6* | ” |
| Fin. wealth / labour income | 6.4 | 4.1* | ” |
| Personal saving rate | 7% | $[0, 10]$% | OECD range |
| Median $\Delta c$ on retirement | $-1$% | $-3.5$% | Aguila et al. (2011) |

*From the bottom 95% of the empirical wealth distribution.

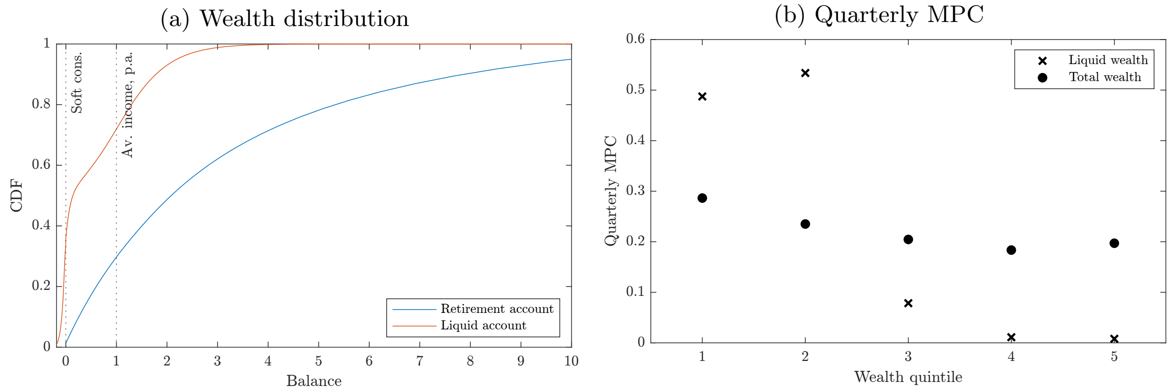
system, combined with government–backed Social Security that does not count in wealth statistics. The average household saving rate is 7%, higher than in the USA, but in the range of OECD rates.

The median drop in consumption upon retirement is $-1\%$, which is well within estimates from the existing literature; for example Aguila et al. (2011) estimate a median change of $-3.5\%$ with substantial variation around this. The size and sign of this change has been the subject of a lot of empirical work. Initial estimates, focused on food, found a substantial drop in consumption on retirement (Banks et al., 1998; Bernheim et al., 2001; Aguiar and Hurst, 2005). Later work explained this as substitution to home–production, meaning that such a measured drop doesn't reflect a loss in welfare (Aguiar and Hurst, 2005, 2013). Later work, expanding the definition to include other categories of spending, showed that there is no average drop among individuals who retire voluntarily, but that there is a lot of heterogeneity around this average across households. The change in spending at retirement is driven by both wealth (Aguila et al., 2011) and unobservable characteristics (Moran et al., 2021). Our median result matches the literature in that it is close to zero, and the average change is increasing in total wealth as well.

**Across the distribution** Figure 5a plots the distributions of liquid and retirement assets. Very few households have any liquid assets at all, a result of their present–bias leading them to under–save. The distribution of retirement assets is approximately exponential, because it reflects the age distribution.

Figure 5b plots the household MPC across the quintiles of liquid and total wealth. The MPC is high in the first two quintiles of the liquid account distribution, which are closest to the soft–borrowing constraint, and declining rapidly after that. The MPC is roughly flat with total wealth. This mirrors reality well, as various studies have shown that the MPC is sharply declining in liquid wealth, but much less so with total wealth (e.g. Fagereng et al., 2021; Ganong et al., 2023).

This model is well situated for the stimulus policy experiments in the next section. The MPC matches empirical estimates, and so we can be confident that the sizing of fiscal policy will be appropriate. Similarly, because retirement policy is set optimally, the cost to society of liquidity policy is appropriate. One implication of the lack of voluntary contributions is that everyone will




take advantage of opportunities to withdraw. Implemented like this liquidity policy will be identical to a fiscal transfer funded by a simultaneous lump sum tax on retirement accounts. We will discuss this further in the next section.

# 6 Stimulus policy

In the remaining sections of the paper we explore the effects of household liquidity policy in comparison to conventional fiscal stimulus. We first establish that liquidity policy indeed works similarly to fiscal stimulus in aggregate: opening a window during which people can withdraw from their retirement accounts leads to such withdrawals, and the greater liquidity boosts consumption in much the same way as a fiscal transfer. From the perspective of the economy, the two approaches are very similar—they increase liquidity in the present by incurring debt, but they differ in where this debt sits (on the government's books, representing an implicit liability for households in the form of future taxes, or against workers' retirement accounts), and the process through which it is repaid.

Having established that both approaches 'work' as stimulus in aggregate, we then explore the other implications that each stimulus approach has. These include distortions to inter–temporal choices from changing tax rates, inter–generational distributional effects, and reduced retirement adequacy for workers. To weigh these distinct implications appropriately, we use a welfare analysis to quantify the degree of households' preference for liquidity policy over fiscal stimulus. We show that liquidity policy is better by the retired, richer and unbiased workers, and future generations i.e. those who will (a) not gain much from the stimulus benefits of either policy, but will (b) pay more in taxes under fiscal, and (c) are more likely to be over–weighted in their retirement accounts. We conclude by exploring the sensitivity of the welfare analysis to various dimensions of the problem—the nature of the fiscal rule, size of the intervention, and targeting of the fiscal stimulus.

## 6.1 Defining the stimulus policy experiments

We assume that the planner wishes to induce a specific stimulus to aggregate household consumption over a set period of time, and it is deciding which intervention to use. In the baseline experiment we set the desired stimulus to be 5% of stationary average household consumption over a period of one quarter (i.e. $\Delta = 1$). To frame the results it is useful to define a policy–dependent aggregate that accumulates average household consumption over a period $\Delta$

$$C(T, \gamma) = \int_0^{\Delta} \left( \int c_{t+s}(x; T, \gamma) \cdot h_{t+s}(x) dx \right) ds$$

The stationary aggregate, for example, is $\bar{C} = C(0, 0)$; and the target for a given policy mix $(T, \gamma)$ is therefore

$$C(T, \gamma) = 1.05 \times \bar{C}$$

Both interventions are modeled as MIT shocks to the policy instrument that last for a duration of one–quarter.[33] The baseline fiscal intervention is a shock to $T$, paid to workers, that solves: $C(T^*, 0) = 1.05 \times \bar{C}$.[34] And the baseline liquidity policy intervention is similarly a shock to $\gamma$ that lasts for one quarter such that $C(0, \gamma^*) = 1.05 \times \bar{C}$. In both cases, taxes adjust endogenously to meet the fiscal rule defined in Section 3.3. All other variables, like wages and the interest rate, are held fixed to isolate the direct effects of the policies.

## 6.2 Common aggregate stimulus

The first thing to establish is that liquidity policy works as stimulus in the model. The amounts needed to achieve a 5% consumption boost are below.

**Result 6.1** (Stimulus equivalence)**.** The baseline interventions are very close to the same magnitude at $T^* = 0.0706$ and $\gamma^* = -0.0723$.

The calibrated stimulus policies transfer the equivalent of 7.06% average annual income to working households under fiscal policy, or allow them to withdraw an amount equivalent to 7.23% of average annual income over a quarter under liquidity policy. In the US, this is equivalent to nearly \$5,500 in each case. The calibrated policy counterfactual is therefore large for a fiscal stimulus (e.g. the US CARES Act transferred \$1,200 per person and an extra \$500 per child), but small for a

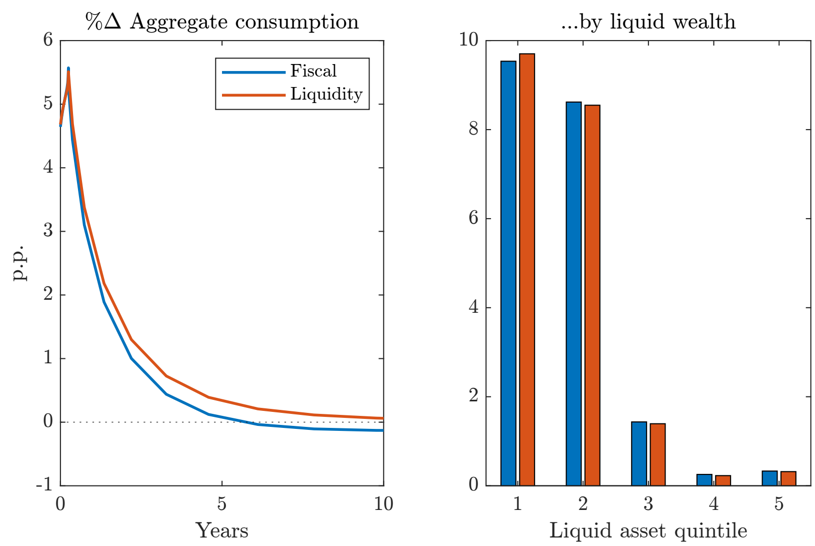
[33] Unanticipated before time–0 but known thereafter.

[34] We choose to target workers in the baseline fiscal exercise so its magnitude can be compared to the alternative liquidity policy, which is only effective for the working–age. We explore an alternative where the whole population receives the transfer in Section 7.2.

liquidity policy (e.g. in Australia people were allowed to withdraw up to $AU20,000 from their superannuation accounts). The fact that the calibrated numbers are so close suggests policymakers should see them as equivalents, at least in terms of their aggregate impact on aggregate demand.[35]

Figure 6 shows the impulse responses of aggregate consumption under the two different stimulus policies. Liquidity policy clearly has a stimulative effect on consumption, just as fiscal does, prompting a 5% increase in the aggregate consumption rate over the stationary equivalent ($\bar{C}$), as they were calibrated to do. After the immediate stimulus, liquidity policy produce less drag on consumption in the medium term (left panel). The two paths for consumption diverge within a few years, with the one under fiscal policy going negative as the stimulative effects wear off and the repayment plan kicks in. Under both policies this stimulus is mainly driven by workers who started off with less liquidity (right panel), as expected because these people tend to have higher MPCs (Figure 5b).

Figure 6: Consumption response to different policies

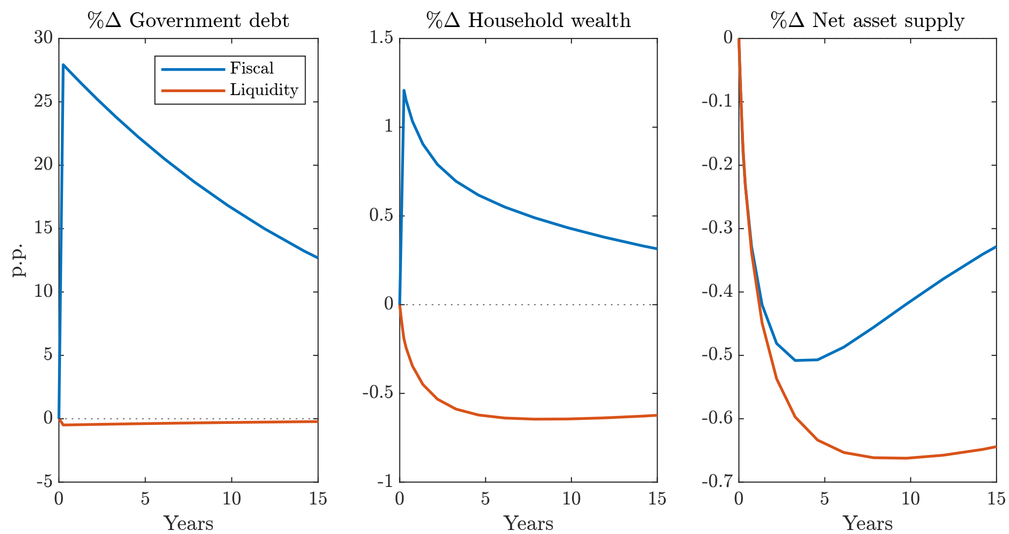



[35]This aggregate equivalence would be ameliorated by a participation decision, which we do not model here. We explore the participation channel in Schneider and Moran (2024), showing that 1/6 people withdrew when allowed in the Australian Covid–19 program, and we establishing that self–control issues were an important driver of this decision, alongside more standard drivers of demand for liquidity like assets and income shocks. Furthermore Hamilton et al. (2023) show that, conditional on participation, the MPC was 40%—higher than aggregate estimates. The high MPC can be explained by a combination of present–bias, and the self–selection of the most needy. A back of the envelope exercise then suggests an aggregate MPC out of liquidity policy of $0.4 \times 1/6 = 7\%$, relative to the 20% or so out of fiscal policy, meaning liquidity policy is only half as potent as fiscal stimulus, at least in the Australian context.

## 6.3 Difference in funding

The difference in funding mechanism is apparent by comparing the responses of government debt and household wealth under the two policies. In Figure 7, government debt jumps under fiscal policy as the transfers drive deficits, and then glides down as the debt is repaid with higher taxes. This increased government debt funds a concurrent increase in household wealth (visible in the middle panel) under fiscal policy, which reduces as households spend the extra liquidity on consumption and higher taxes. By contrast, there is no immediate impact on household wealth from liquidity policy (which only alters asset allocation), but a similar gradual decline as households (a) consume the extra liquid resources, and (b) retire with fewer illiquid resources.

Figure 7: Funding channels




The government debt accumulated under fiscal policy represents an implicit liability for households: the present value of their increased future tax bill, not recognised in their balance sheets. To show the consolidated household financial position, the right panel plots their net asset supply—the difference between households' wealth (the sum of liquid and illiquid assets) and government debt. The path of this aggregate is quite similar under the two policies for the first few years, and only starts to deviate in later years. The effects of both policies last a very long time. The duration of fiscal policy's impact clearly depends on the fiscal rule, but liquidity policy's effects necessarily span generations. They only wash out of the system after all affected workers retire, and die, which will be many decades after the stimulus occurred.

From the perspective of the aggregate economy, both policies do the same thing in qualitative terms. They increase liquidity for workers, driving short–term consumption, and they fund this with decreased illiquid wealth, whether in retirement accounts or greater future taxes. The difference between the two is where decreased illiquid wealth is situated, who repays it, and when. Under fiscal policy it sits as a debt on the government's balance sheet, and it is repaid with the (very broad–based) consumption tax in a process determined by the fiscal rule. Under liquidity policy it sits like a debt on households' balance sheets (as a negative entry in their retirement accounts), and it is 'repaid' in lump sums by workers as they retire, in a process determined by the retirement rate. The primary reason the aggregate asset supply curves diverge in the right panel of Figure 6 is that the fiscal rule 'retires' debt at a faster rate than workers retire.[36]

Although the two approaches are qualitatively similar at the aggregate level, they have potentially different distributional and normative implications because of the difference in the size of the repayment tax base, and the redistribution and distortions caused by the repayment mechanism. The taxes used to repay increased government debt have a very broad base, applying to workers and retirees regardless of their status when the fiscal policy was implemented. By contrast liquidity policy is repaid from a narrower base—only the workers that extracted liquidity are forced to 'repay' by having lower future–value retirement savings—and it is not at all redistributive. One other difference between these repayment mechanisms at the aggregate level is whether they cause inter–temporal distortions.

## 6.4 Inter–temporal distortion

One aggregate implication comes from the inter–temporal distortions caused by changing tax rates. Knowing the tax rate is growing (declining), households will shift consumption into the present (future)—a distortion away from the optimal smoothing path.

**Lemma 6.2** (Euler distortions)**.** Assuming the borrowing constraint is non–binding, the changing consumption tax rate distorts the Euler equation

$$\mathbb{E}\left[\frac{\dot{c}(x)}{c(x)}\right] = \frac{1}{\sigma}[r_b(b)b + r(b)) - \rho] - \underbrace{(1+\tau_t)(1-\beta^{1/\sigma})\partial_b \hat{c}_t(x)}_{\text{Bias distortion}} - \underbrace{\frac{1}{\sigma}\left[\frac{\dot{\tau}}{1+\tau_t}\right]}_{\text{Tax distortion}}$$

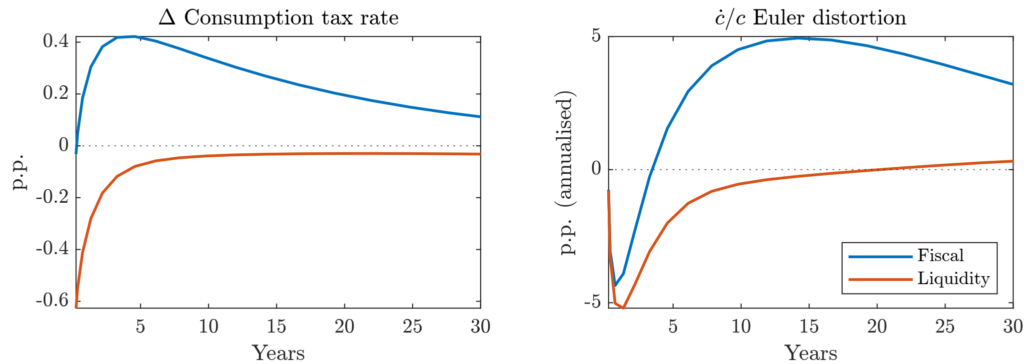
*Proof.* Derived in Appendix A.4.2 □

Figure 8 plots the consumption tax over time under the two policies (left panel), as well as the distortions to the optimal expected consumption growth path they imply (right panel). Under fiscal policy, consumption taxes climb steadily, peaking 0.4 p.p. higher than the stationary level about five years after the stimulus, and they glide down slowly over following decades. By contrast under liquidity policy the consumption tax drops with the stimulus. This is because the stimulus

[36] We explore an alternative fiscal rule that equalises these rates in Section 7.2.

increases the tax base and the fiscal authority needs to keep the budget balanced.[37] It climbs back again slowly over following decades as aggregate consumption returns to normal.

Figure 8: Euler distortions




Both policies imply short–term distortions to consumption growth because they both imply climbing tax rates over the first few years. These distortions effectively disappear under liquidity policy after the first few years, as the consumption tax rate stabilises close to the stationary level. But the distortions persist under fiscal policy for decades, with a maximal impact of causing expected annual consumption growth to be more than 4 percentage points higher than in the stationary solution.

We have established that both policies stimulate consumption in much the same way, but they differ in their funding mechanisms, and the common inter–temporal distortions these imply. These results are all at the aggregate level, but the different approaches to stimulus distribute their impacts quite differently across households states, and we explore this in the next section.

## 6.5 Distributional implications

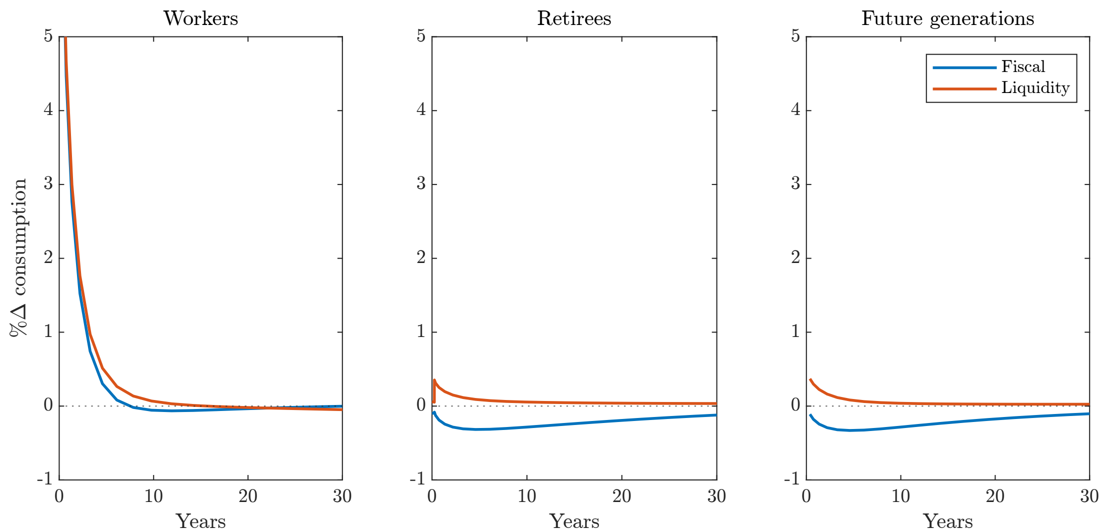
### 6.5.1 Conflicting generational interests

Questions about the long–run implications of government policies are often framed as generational conflicts. It's useful to define three distinct groups within the population because they are affected by the policies differently. These are workers and retirees, those who are working whilst the stimulus occurs, and retired at time–0,respectively, and the future generations, those whose working lives begin after the stimulus ends.

Figure 9 shows the average consumption path, relative to the stationary solution, for these distinct groups under each stimulus policy. Workers' consumption is boosted the most, by design,

[37] We explore an alternative scenario with an asymmetric fiscal rule in Section 7.2

Figure 9: Generational consumption




and there is very little drag in the later years as the repayment plans for each policy kicks in. By contrast, retirees and future generations both experience a drag on their consumption under fiscal policy—they must pay higher taxes and didn't receive any stimulus—and a boost under liquidity policy—caused by the tax distortions discussed in Section 6.4.

#### 6.5.2 Retirement (in)adequacy

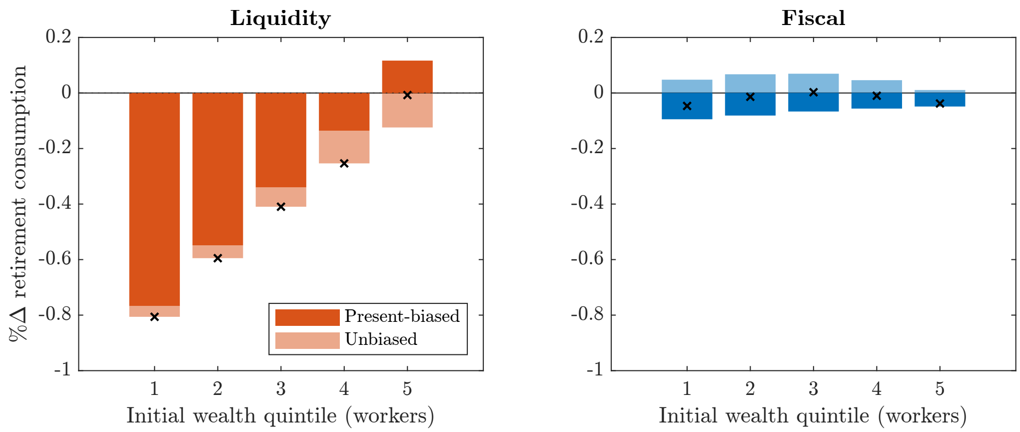
Reduced retirement adequacy—fewer resources on retirement for affected households—is the main implication of liquidity policy, and it only affects the workers who withdrew from their accounts and spent the money. To measure retirement adequacy, we compare the consumption rate that the cohort of workers at time–0 can expect the moment they retire under the different stimulus policies, relative to the equivalent in the stationary solution.

Figure 10 plots this change for the section of this cohort that retires 20 years after the stimulus[38], and breaks down contributions by wealth quintile and present–bias types. The figure shows that liquidity policy has a much more severe impact on retirement adequacy than fiscal, which has almost no effect on adequacy. This impact is stronger the less wealthy workers are, and mainly driven by the behaviour of biased households at all points in the wealth distribution.

There are two forms of non–Ricardian behaviour at play in our model. First is the standard inability to smooth due to incomplete markets, and the second is the over–consumption brought on by present–bias. All stimulus relies on there being some non–Ricardian households. These results for retirement adequacy show that liquidity policy concentrates the duty of payment for stimulus

[38]Results are qualitatively the same for any retirement date.

Figure 10: Relative outcomes at retirement, by wealth and present–bias type




mainly on the shoulders of the biased ones. That is, exactly the group for whom the illiquidity in the retirement system is designed.

# 7 Welfare

We have established that both approaches work to stimulate household consumption, but that they come with various contrasting implications in different dimensions. To bring this together and compare how their respective implications weigh against each other, and for whom, we compare welfare under each policy using a compensating variation (CV). At time–0, for each point in the state–space, we find the change in liquid assets that would be required to make the household indifferent to liquidity policy instead of the baseline fiscal intervention

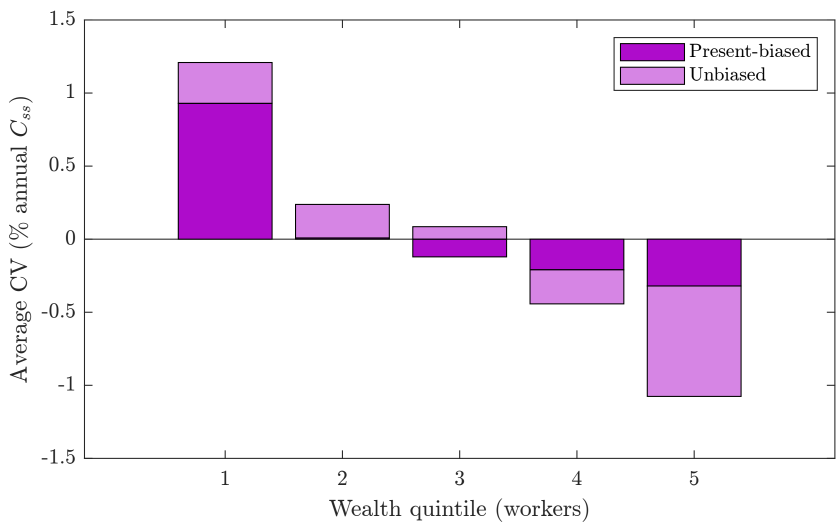
$$\hat{v}(b + CV(x), a, z, \beta; 0, \gamma) = \hat{v}(x; T, 0)$$

We find this compensating variation from two perspectives: (a) the near–sighted evaluation finds the CV based on households' perceived value functions i.e. accepting any naivete they have, and (b) the long–run evaluation finds the CV based on a correct anticipation of how they will behave (O'Donoghue and Rabin, 2006; Bernheim and Taubinsky, 2018; Naik and Reck, 2024), defined in Equation 13. This compensating variation shows the intensity of a preference for fiscal policy—positive numbers indicate the need to compensate for using liquidity policy instead, and negative numbers indicate a willingness to *pay* for liquidity policy to be used.

## 7.1 Winners and losers from liquidity vs fiscal policy

Who benefits from liquidity policy over fiscal? Wealthy workers and retirees. Retirees don't stand to gain anything from the stimulus, but they must subsidise the stimulus to workers, and their consumption is further distorted by the changing taxes, as we saw in Figure 9. To highlight the importance of wealth for workers' preferences, Figure 11 plots the average compensating variation across wealth quintiles showing that enthusiasm for fiscal policy is declining in wealth. Wealthy workers are more likely to be Ricardian, more likely over–weighted in their retirement account, and (like retirees) also expect to pay greater taxes under fiscal. Hence their preference for liquidity policy. Figure 11 also disaggregates by type, showing that the majority of the benefit of fiscal over liquidity policy is accruing to biased households in the bottom quintile of the wealth distribution. Biased households are over–respresented in this quintile because their bias leads them into debt, and because of this they stand the most to lose from having lower retirement assets. This result confirms the intuition that many opponents to liquidity policy voiced—that it places the cost of support on the shoulders of the least well off, and the most in need of help to save for retirement.

Figure 11: Worker preference for fiscal policy, by wealth & type




In aggregate, the best approach to stimulus depends on the welfare criterion. Liquidity policy is very popular—it is preferred by an overwhelming majority, whether evaluated by households' biased or long–run preferences (69% and 72%). And the average compensating variation across the whole population is negative—for the long-run evaluation the average CV is $-1.6\%$ of average annual stationary consumption—meaning there is more than enough money to fund compensation because the average person would be willing to pay to switch from fiscal to liquidity policy. By contrast, a Utilitarian social planner would choose fiscal policy, under which average welfare is greater than under liquidity policy.

Why the difference? Liquidity policy concentrates repayment for the stimulus on a small and relatively vulnerable group. It's popular because this small group doesn't constitute a majority. But it's a disaster for welfare because their marginal utility of consumption is *much* greater than for others in the population. Liquidity policy is preferred in average compensating variation terms for the same reason—their marginal utility is extraordinarily responsive to resources i.e. they can be bought off cheaply.

Bringing this together, the apparently optimal approach would be to use liquidity policy to do the stimulus, and to combine this with lump–sum transfers from wealthy workers, and the retired, to poorer and present–biased workers. Unfortunately, this begs the question. If these transfers were possible, then neither fiscal nor liquidity policy would be the optimal way to stimulate consumption. Instead, a program of lump–sum transfers moving liquidity from low– to high–MPC households would be probably be better (Oh and Reis, 2012). Lacking such an instrument, we're left with the political question of how to weigh the needs of the many against those of the few.

## 7.2 Sensitivity

In this section we see how sensitive the baseline results are to different fiscal rules, targeting and magnitudes of the stimulus, and ex–ante retirement policy. In aggregate, liquidity policy is preferred to fiscal across all of our robustness checks around the baseline calibration for the retirement policy. It wins a majority vote in each case, and also costs a *negative* average compensating variation. This popularity is not due to naivete—the results are all based on long–run evaluations, in which the evaluator correctly anticipates naive present–biased behaviour. In fact naive evaluations would be even more strongly in favour of liquidity policy. So the liquidity policy's widespread appeal is robust to these different assumptions. It is, however, quite regressive, with present–biased and less wealthy workers benefiting much more from fiscal stimulus. By contrast, if retirement policy were looser, then it is less binding on working–age people and this leads to reduced benefits from liquidity policy, tipping the balance back in favour of fiscal as the better approach to stimulus, both in aggregate and out of a concern for distributional impacts.

**Fiscal rule symmetry** The baseline results came from a symmetric fiscal rule—governments set tax rates to get debt back to target whether it is below or above this target. One of the implications of such a rule is that liquidity policy is accompanied by a persistent tax cut, to balance the government's books as the consumption tax's base is expanded. The populations we found to prefer liquidity policy may feel this way mainly because they like the tax cut. Although temporary tax cuts are often part of stimulus packages, we may also expect governments to apply fiscal rules asymmetrically, with the actual allowable debt level anywhere between zero and $\bar{B}$. In this case the fiscal rule would only activate when debt was above the upper bound, and otherwise taxes would stay fixed and deficits or surpluses allowed.

Table 6 shows that using an asymmetric fiscal rule reverses the average worker preference for liquidity policy. Workers on average now all prefer fiscal policy; the previous preference for liquidity policy among the unbiased workers was apparently driven by the tax cuts attached. Similarly, the retirees are still in favour of liquidity policy, but their enthusiasm is dampened (though not by enough to tip the scales to fiscal policy in aggregate).

Table 6: CV to prefer liquidity policy (% annual $C_{ss}$)

| | Baseline | Asymmetric | Consolidating | Accommodating |
|---|---|---|---|---|
| Total | -1.6 | -0.1 | -2.1 | -0.8 |
| Workers | -0.1 | 1.6 | -0.5 | 0.8 |
| *Biased* | *0.6* | *1.9* | *0.2* | *1.3* |
| *Unbiased* | *-0.8* | *1.3* | *-1.1* | *0.2* |
| Retired | -4.2 | -3 | -4.8 | -3.5 |

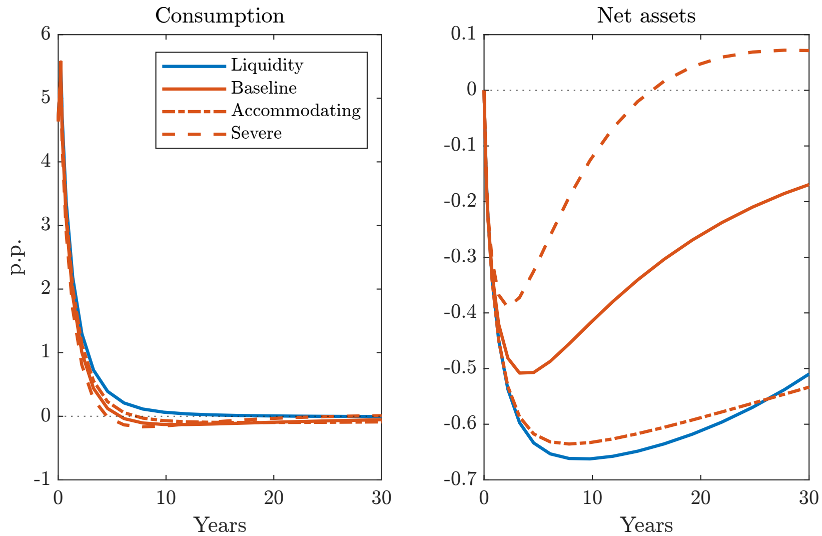
**Fiscal rule severity** The timing of the fiscal rule may also matter. In the baseline results, the fiscal rule is set to mirror the EU's fiscal compact, following Galí (2020), which sets a target half–life for debt gaps of 13.5 years ($\mu = 0.0128$). Here we consider two alternative rules—the 'consolidating' one sets a stricter repayment schedule, with $\mu = 0.0256$ so the half–life is halved, and the 'accommodating' one sets a more lax schedule, with $\mu = 0.0063$. The accommodating rule is set so that government debt is 'retired' at the same rate as the working population. This is an interesting special case because it means the timing under liquidity and fiscal policy are essentially identical.

Figure 12 shows the aggregate consumption and net wealth plots under each alternative fiscal rule. They show what you'd expect—the drag on consumption is greater the stricter the rule, as net assets return to their stationary level quicker. Table 6 shows that the preference for liquidity policy diminishes with the intensity of the fiscal rule—it is greatest for the consolidating rule, followed by the baseline, and then the accommodating rule. As the fiscal rule relaxes, the taxes required to repay it are pushed further into the future and so they feel less onerous to the present generations, increasing the relative appeal of fiscal policy.

**Targeting** The baseline policy counterfactuals pitted liquidity policy against a fiscal transfer targeted at workers. This was designed to make the recipient group match, so they were more comparable. Here we explore how the main results differ when everyone receives the transfer.

If everyone receives the transfer then the total transfer per person needs to be smaller to achieve the same consumption stimulus. Specifically, to match the liquidity intervention we now need a transfer of $T^* = 0.0403$ for one quarter (versus 0.0706 when targeting workers), a total outlay for

Figure 12: Aggregate impact of fiscal rule severity




the government of around 10% less than when just targeting workers (retirees have a greater MPC than the average worker).

Table 7 shows that in this scenario, the overall long–run preference for liquidity policy is stronger. This is driven by a reversal of preferences for retirees, who now prefer to receive the fiscal transfer, and biased workers, who prefer to withdraw from their retirement accounts than receive a smaller fiscal transfer. Unbiased workers are also much more in favour of the liquidity policy in this scenario. This result indicates how much the preference for, or against, redistribution was driving attitudes to fiscal policy.

**Magnitude** Table 7 shows long–run CVs under different magnitudes for the stimulus—50% and 150% the size of the baseline exercise—and their equivalent liquidity interventions. It shows a close–to linear relationship: liquidity is preferred overall in each case, with everyone's preference scaling up, or down, with the impulse magnitude.

Table 7: CV to prefer liquidity policy (% annual $C_{ss}$)

| | Baseline | Un–targeted | 2.5% impulse | 7.5% impulse | Lower ex–ante $\xi$ |
|---|---|---|---|---|---|
| Total | -1.6 | -2.1 | -0.8 | -2.6 | 1.4 |
| Workers | -0.1 | -4.9 | -0.1 | -0.1 | 4.1 |
| *Biased* | *0.6* | *-3.9* | *0.2* | *0.8* | *5.9* |
| *Unbiased* | *-0.8* | *-5.9* | *-0.4* | *-1.0* | *2.3* |
| Retired | -4.2 | 2.8 | -2.0 | -7.0 | -3.4 |

**Looser retirement policy** The baseline results all work in an environment with retirement policy set optimally in the exercise in Section 5. This policy is designed with the needs of newborns in mind, and one of its side–effects is to leave older workers generally over–weighted in their retirement accounts, particularly if they are not present–biased. If we suppose instead that the government set retirement policy with the whole population in mind, but with the same participation constraint as the buy–in equilibrium, then the optimal policy sets contribution rates to about half what they are in the baseline, and much smaller tax concessions as well

$$(\hat{\xi}, \hat{\varphi}) = (0.04, 0.12)$$

We redo the stimulus policy experiments with this calibration of the model's stationary solution. Fiscal and liquidity policy both now need to release more money to achieve the same stimulus $\hat{T} = 0.0816$ and $-\hat{\gamma} = 0.0875$ because the aggregate worker MPC is reduced to 0.2.

Table 7 shows that, with this starting retirement policy, fiscal is the better stimulus option for the average person. The average worker prefers it, with biased workers having the stronger (long–run) preference, whilst the average retiree still prefers liquidity policy because it means they avoid future taxes. Two forces drive the result for workers. With looser ex–ante retirement policy, there are both fewer wealthy workers who are over–weighted in their retirement accounts, and more poorer workers who will suffer from the reduced retirement adequacy. Liquidity policy is thus an even more regressive stimulus option in settings where retirement contributions are low, or even not mandatory, as in the USA.

# 8 Conclusion

Household liquidity policy is becoming an increasingly popular tool used by governments seeking to stimulate the economy, growing from a few scattered uses during the global financial crisis, to a popular response used by over 30 countries during the Covid–19 recession. While a growing literature evaluates who withdrew and what they did with the money, our paper makes a very different contribution. In short, we provide the first analysis of the distributional and welfare implications of household liquidity policy compared to traditional fiscal stimulus.

In our analysis, we capture many of the key trade-offs faced by policy makers thinking about the efficacy and equity concerns related to these two different approaches to stimulus. While this represents an important first step in understanding household liquidity policy, there are still a number of directions in which we could meaningfully extend our analysis, which we plan to pursue in future work.

At the most general level, our analysis could be expanded to capture the indirect, general equilibrium, effects of the stimulus policies in question. First, the stimulus approaches may have different implications for production. The model presented here works as a combined household–government block that can embed into a richer general equilibrium environment. Insofar as the aggregate effects coming out of this block are the same across the two approaches to stimulus, there will be no difference in their indirect effects on prices and production. However, if we extend the model to include labour supply, then inter–temporal distortions arising from taxes will drive a wedge between the two approaches that will bear on production—it will be more distorted under fiscal stimulus, than liquidity policy. Our future work will seek to quantify how important these production impacts are, and whether they have flow–on distributional implications.

Second, the source of the shock that precedes the stimulus may influence how we evaluate the two approaches. In our stimulus policy experiments we take it as given that the government wishes to achieve a 5% boost to household consumption. This allowed us to focus on the question of interest and to characterise the tradeoffs inherent in the two approaches. In practice, stimulus programs are implemented in response to shocks, and it may be that the nature of the original shock alters the relative desirability of liquidity versus fiscal policy.

Third, there may be important interactions with asset prices. Liquidity policy allows sales of assets at market lows, compounding the disadvantage to households that cash–out and spend the money because they realise capital losses (or at least reduced gains) by withdrawing from their retirement accounts and not reinvesting outside of them. Furthermore, in a similar vein, liquidity policy may inadvertently exacerbate flight–to–safety dynamics we observe during times of aggregate stress, increasing funding costs for risky ventures.

Finally, we have modelled our stimulus policies as unanticipated shocks, so there is no role for anticipation. This was appropriate of the liquidity policies deployed in many countries during Covid–19 as this was the first time such measures were implemented in many places. Now that Pandora's box has been opened, however, forward–looking households may expect their retirement accounts to be somewhat more liquid than they were in the past. Such anticipation will alter households' engagement with retirement savings systems, and the repeated use of retirement resources for aggregate demand management will also alter the optimal design of retirement policy itself.

# A Household problem

## A.1 Exponential household's problem

The exponential household's HJB equation at each stage of life is below.

**Working life** Suppose we have already substituted the drift in productivity with an $N$–state discrete process, and that this process and jumps in and out of unemployment are governed by Poisson intensities $\lambda^{z \to z'}$. During working life, the HJB is

$$\begin{aligned} \rho v(x) = \max_{c,d} & \left\{ u(c) + \partial_b v(x) \cdot \dot{b}(x) + \partial_a v(x) \cdot \dot{a}(x) \right\} \\ & + \sum_{z'} \lambda^{z \to z'} \left[ v(x') - v(x) \right] + \delta_R \left[ v_R(x) - v(x) \right] \end{aligned} \tag{14}$$

Where $\dot{b}(x)$ and $\dot{a}(x)$ are defined by equations 2 and 4. This problem's FOC are

$$u'(c(x)) = (1+\tau)\partial_b v(x) \tag{15}$$

$$\partial_a v(x) = \partial_b v(x)(1 + \chi_d(d(x), a)) + \kappa(x) \tag{16}$$

Where $\kappa(x)$ is the Lagrange multiplier on the withdrawal constraint.

**Retirement** During retirement, the HJB is

$$(\rho + \delta) v_R(x) = \max_{c,d} \left\{ u(c) + \partial_b v_R(x) \cdot \dot{b}(x) + \partial_a v(x) \cdot \dot{a}(x) \right\} \tag{17}$$

Where $\dot{b}(x)$ and $\dot{a}(x)$ are defined by equations 2 and 4. This problem's FOC are

$$u'(c_R(x)) = (1+\tau)\partial_b v_R(x) \tag{18}$$

$$\partial_a v_R(x) = \partial_b v_R(x)(1 + \chi_d(d_R(x), a)) \tag{19}$$

In either stage of life, the soft–borrowing constraint ensures the exponential household will never borrow and so these FOC always hold.

## A.2 Present–biased household's problem

For a present–biased naif with bias parameter $\beta$, the HJB equation changes in two ways from the exponential equivalent—all terms except the flow utility are pre–pended by $\beta$, and the value in the HJB is the rational value, not the household's actual valuation.

**Working life**

$$\beta\rho v(x) = \max_{\hat{c},\hat{d}} \Big\{ u(\hat{c}) + \beta\partial_b v(x) \cdot \dot{b}(x) + \beta\partial_a v(x) \cdot \dot{a}(x) \Big\} \\ + \beta \sum_{z'} \lambda^{z\to z'} \left[v(x') - v(x)\right] + \beta\delta_R \left[v_R(x) - v(x)\right] \tag{20}$$

This problem's FOC are

$$u'(\hat{c}(x)) = \beta(1+\tau)\partial_b v(x) = \beta u'(c(x)) \tag{21}$$

$$\partial_a v(x) = \partial_b v(x)(1 + \chi_d(\hat{d}(x), a)) + \kappa(x) \tag{22}$$

Where the latter is the same as the exponential agent's, and so $\hat{d}(x) = d(x)$

**Retired** Similarly, the retired present–biased naif's problem is to solve the following.

$$\beta(\rho+\delta)v_R(x) = \max_{\hat{c}_R,\hat{d}_R} \Big\{ u(\hat{c}_R) + \beta\partial_b v_R(x) \cdot \dot{b}(x) + \beta\partial_a v(x) \cdot \dot{a}(x) \Big\} \tag{23}$$

Where $\dot{b}(x)$ and $\dot{a}(x)$ are defined by equations 2 and 4. This problem's FOC are

$$u'(\hat{c}_R(x)) = \beta(1+\tau)\partial_b v_R(x) = \beta u'(c_R(x)) \tag{24}$$

$$\partial_a v_R(x) = \partial_b v_R(x)(1 + \chi_d(\hat{d}_R(x), a)) \tag{25}$$

## A.3 Discretised system and Kolmogorov forward equation

The policy functions described above describe the optimal drift in the endogenous state variables—the balances in the liquid and retirement accounts. All other state transitions—between productivity and employment status within working life, and transitions between life stages—are exogenous. Together, these state–transition rules define how the distribution of households across the state–space moves around over time, and therefore define the Kolmogorov Forward Equation (Achdou et al., 2022). We can solve the stationary distribution of each type separately because, by assumption, the types are permanent.

**Exponential agents** The above HJB equation can be expressed as a linear system, discretised over the state space, as follows

$$\rho\mathbf{V} = u(\mathbf{c}) + \mathbf{A}\mathbf{V}$$

Where $\mathbf{A}$ captures the finite–difference transition rates between all the states, from the perspective of the households (i.e. they anticipate retirement and death but not rebirth), and $\mathbf{V} = \begin{bmatrix}\mathbf{v}' & \mathbf{v}'_R\end{bmatrix}'$ stacks the discretised working and retired values together. The solution to this system linear is

$$\mathbf{V} = [\rho\mathbf{I} - \mathbf{A}]^{-1} u(\mathbf{c})$$

Where updates are implemented using the semi–implicit scheme with update control step–size $\Delta$

$$\mathbf{V}^{n+1} = [(1/\Delta + \rho)\mathbf{I} - \mathbf{A}^n]^{-1} [u(\mathbf{c}^n) + \mathbf{V}^n/\Delta]$$

We use the nested–drift algorithm in Sabet and Schneider (2024) to find the policy functions that define $\mathbf{A}^n$ and $\mathbf{c}^n$ for a given value guess $\mathbf{V}^n$. In practice, we solve the retired value first, and then use this as an input into the working–life value solution; doing so reduces the computational burden of inverting the matrix in the semi–implicit update step. Following Achdou et al. (2022), the stationary distribution discretised over the same state grids ($\mathbf{g}$) is the solution to the linear system

$$0 = \tilde{\mathbf{A}}'\mathbf{g}$$

Where $\tilde{\mathbf{A}}$ adjusts $\mathbf{A}$ for the state transitions households do not anticipate, which are (1) rebirth as a zero–asset worker after dying in retirement, (2) forced retirement if illiquid assets reach the threshold, and (3) bankruptcy upon retirement if total assets are negative.

**Present–biased agents** The solution process and formulae are the same, with the exception that their state transition matrix $\mathbf{A}^\beta$ does not solve their value function, it is only used to solve their Kolmogorov Forward equation.

## A.4 Euler equation

### A.4.1 Present–biased naif, stationary

Following Maxted (2024), Appendix A.4, but with an added consumption tax. The following solves the Euler for the working household. First, find the derivative of the expected value Equation 14 with respect to the liquid asset $b$.

$$\begin{aligned}\rho\partial_b v(x) &= u'(c(x))\partial_b c(x) + \partial_{bb} v(x) \cdot \dot{b}(x) + \partial_b v(x)\partial_b \dot{b}(x) \\ &\quad + \sum_{z'} \lambda^{z\to z'} [\partial_b v(x') - \partial_b v(x)] + \delta_R [\partial_b v_R(x) - \partial_b v(x)]\end{aligned}$$

Apply the realised FOC i.e. that $u'(\hat{c}(x)) = \beta(1+\tau)\partial_b v(x)$ so $\partial_b v(x) = \frac{u'(\hat{c}(x))}{\beta(1+\tau)}$

$$\begin{aligned}(\rho - \partial_b r(b)b - r(b))\, u'(\hat{c}(x)) = (1+\tau)\left(\beta u'(c(x)) - u'(\hat{c}(x))\right)\partial_b c(x) + \partial_b \hat{c}(x) u''(\hat{c}(x)) \cdot \dot{b}(x) \\ + \sum_{z'} \lambda^{z \to z'} \left[u'(\hat{c}(x')) - u'(\hat{c}(x))\right] + \delta_R \left[u'(\hat{c}_R(x)) - u'(\hat{c}(x))\right]\end{aligned}$$

Optimisation implies the relationship $u'(\hat{c}(x)) = \beta u'(c(x))$, which we can use to eliminate the first term on the RHS so the equation simplifies to

$$\begin{aligned}(\rho - \partial_b r(b)b - r(b))\, u'(\hat{c}(x)) = \partial_b \hat{c}(x) u''(\hat{c}(x)) \cdot (r(b)b - (1+\tau_c)c(x) + other) \\ + \sum_{z'} \lambda^{z \to z'} \left[u'(\hat{c}(x')) - u'(\hat{c}(x))\right] + \delta_R \left[u'(\hat{c}_R(x)) - u'(\hat{c}(x))\right]\end{aligned}$$

Note that we can collect many of these terms into the time–derivative of expected marginal utility ($\mathbb{E}[du'(c(x))]/dt$), by Ito's Lemma, after we add and subtract $u''(\hat{c}(x))\partial_b \hat{c}(x)\hat{c}(x)(1+\tau)$

$$(\rho - \partial_b r(b)b - r(b))\, u'(\hat{c}(x)) = (1+\tau)u''(\hat{c}(x))\partial_b \hat{c}(x)(\hat{c}(x) - c(x)) + \mathbb{E}[du'(c(x))]/dt$$

And using CRRA utility we know $c(x) = \beta^{1/\sigma}\hat{c}(x)$

$$\begin{aligned}(\rho - \partial_b r(b)b - r(b)) &= (1+\tau)\frac{u''(\hat{c}(x))\hat{c}(x)}{u'(\hat{c}(x))}\partial_b \hat{c}(x)(1-\beta^{1/\sigma}) + \frac{\mathbb{E}[du'(c(x))]/dt}{u'(c(x))} \\ \mathbb{E}\left[\frac{\dot{c}}{c}\right] &= \frac{1}{\sigma}\left[\partial_b r(b)b - r(b) - \rho - \underbrace{\sigma(1+\tau)(1-\beta^{1/\sigma})\partial_b \hat{c}(x)}_{\text{Naive bias distortion}}\right]\end{aligned} \tag{26}$$

Hence the naif's Euler equation is a distorted version of the standard one, where the distortion scales with the bias, marginal propensity to consumer, and consumption tax.

### A.4.2 Present–biased naif, dynamic

Using the same steps as above, but with an added term $\partial_t v_t(x)$ on the HJB equation, which eventually introduces an influence from drift in the tax rate.

$$\mathbb{E}\left[\frac{\dot{c}}{c}\right] = \frac{1}{\sigma}\left[(r_b(b)b + r(b)) - \rho - \sigma(1+\tau_t)(1-\beta^{1/\sigma})\partial_b \hat{c}_t(x) - \frac{\dot{\tau}}{1+\tau_t}\right] \tag{27}$$

The dynamic tax adds two distortions, relative to the stationary Euler. First, its level alters the bias distortion we found in the previous section, relative to its stationary level. Second, expected drift in the tax rate introduces inter–temporal smoothing distortions common to all agents. The analysis in the main text focuses on the latter.

## B Conditional expectations in continuous–time

**Goal** Find the expected path of a variable (a policy or state variable), given a starting point in the state space ($x_0$), from the perspective[39] of an agent at that point in the state–space.

$$y^e_t(x_0) = \mathbb{E}[y(x_t)|x_0 = x]$$

Say we have solved our problem, with both the state–space and time discretised, and we have a sequence of policy vectors, transition matrices and (starting) distributions vectorised over the state–space

$$\{\mathbf{y}_s, \mathcal{A}_s, \mathbf{g}_s\}_{s=1}^t$$

where $\partial_t \mathbf{g}_t = \mathcal{A}'_t g_t$. If we wanted to use these to build the path of the aggregate policy, we use

$$Y_t = \mathbf{y}'_t \mathbf{g}_t$$

So the expected aggregate in time–$t$ is the policy in that period integrated over the *future* distribution at the start of that period.

Note that the future distribution $\mathbf{g}_t$ is constructed from the starting distribution and the implicit transition matrices

$$\mathbf{g}_t = \mathcal{L}_{t-\Delta}\mathbf{g}_{t-\Delta} = \begin{cases} \mathbf{g}_1 & \text{if } t = 1 \\ \prod_{s=1}^{t-1} \mathcal{L}_s \mathbf{g}_1 & \text{if } t > 1 \end{cases}$$

Where the implicit transition matrix is defined by $\mathcal{L} = (\mathcal{I} - \Delta\mathcal{A}'_t)^{-1}$. Substituting this into the definition of the aggregate, we have

$$Y_t = \begin{cases} \mathbf{y}'_1 \mathbf{g}_1 & \text{if } t = 1 \\ \mathbf{y}'_t \prod_{s=1}^{t-1} \mathcal{L}_s \mathbf{g}_1 & \text{if } t > 1 \end{cases}$$

Focusing on the future cases ($t > 1$), we can see that the aggregate is made up of three terms—the future policy: $\mathbf{y}_t$, the starting distribution: $\mathbf{g}_1$, and a matrix that affects the expected movement of measure around the state–space between the start and period $t$: $\prod_{s=1}^{t-1} \mathcal{L}_s$. We can collect these terms in different ways to get the same aggregate.

[39] Important if (a) expectations are not fully rational, or (b) agents don't internalise some transitions like reincarnation.

**Future distribution** In the standard definition, the matrix is applied to the starting distribution, and so the expected aggregate in time–$t$ is the policy in that period integrated over the *future* distribution, as discussed.

**Expected policy** If we instead apply the matrix to the future policy, then the future aggregate in time–$t$ is the expected policy in that period from the POV of a point in the state space in period–$t$, integrated over the *starting* distribution.

$$Y_t = \mathbf{y}_t^e \mathbf{g}_1 \text{ where } \mathbf{y}_t^e = \mathbf{y}_t' \prod_{s=1}^{t-1} \mathcal{L}_s$$

The two approaches measure the same aggregate, but they create very different sub–aggregate objects—the first a future distribution, and the second an expected policy. The latter $\mathbf{y}_t^e$ is exactly the conditional expectation[40] we are seeking—it tells us the *expected* value of the policy $y$ at some future point in time $t$ for each *starting–point* in the state–space.

**Goal** Say we now want to describe expected policies by quantile of some distribution. This is really easy with above object. Only two steps required:

1. Calculate the quantile for each point in the state space
2. Find the average of the conditional policy, conditional on the quantile $x_0$ is in

$$y_t^e(q) = \mathbb{E}[y_t^e(x_0)|x_0 \in q] = \frac{\int_{x\in q} y_t^e(x) dG(x)}{\int_{x \in q} dG(x)}$$

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[40]This is a discretised implementation of the Feynman–Kac formula.

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