TTK4190 Guidance and Control of Vehicles

Assignment 1

Kinematics and Kinetics

Fall 2021

Deadline Monday the 13th of September at 23:59

Objective

You will learn how to implement and simulate the attitude kinematics and kinetics of a rigid body in space. The theory is also applicable to underwater vehicles, robotic systems and unmanned aerial vehicles.

Grading

This assignment must be passed to get access to the final exam. The overall impression of how well you have understood the problems will be the basis for the evaluation. You need at least 60 % of the assignment correct to pass. **You are encouraged/supposed to work in groups of 2-4 people, but are allowed to do the assignment individually if that is preferred**. Note that the grading will be equally severe if you do the work individually. The participants in the group will receive the same feedback.

Deadline and Delivery Details

The assignment must be handed in by 23:59 on Monday the 13th of September. You must deliver a report and produce your own Matlab files for each simulation problem. Simulink should not be used. Matlab m-files should not be included in the delivery. Make sure all plots clearly show the required data (title, legend, label and tag the axes, set the grid on and so forth). An example of how to simulate the attitude dynamics of a rigid-body (using unit quaternions) is demonstrated in the file attitude.m, which is posted on Blackboard and displayed in Appendix A.1. It is recommended to use this file as a template and change the necessary parts of the code. You are encouraged to use and look into the MSS toolbox (wwww.marinecontrol.org), which includes several useful functions for this assignment and you need the toolbox to run the template. The report has to be handed in via Blackboard. You are encouraged to write the report on your PC using your favorite editor (LaTeX, Word, Pages...). It is also possible to hand in a scanned version, but in the end it has to be in PDF format. Paper versions are not accepted.

Problem 1 - Attitude Control of Satellite

Consider a satellite with inertia matrix $\mathbf{I}_g = mR_{33}^2 \mathbf{I}_3$, $m = 180$ kg, $R_{33} = 2.0$ m. The equations of motion are:

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{T}_q(\mathbf{q})\omega \\ \mathbf{I}_g\dot{\omega} - \mathbf{S}(\mathbf{I}_g\omega)\omega &= \tau \end{aligned} \tag{1}$$

Problem 1.1 What is the equilibrium point $\mathbf{x}_0$ of the open-loop system $\mathbf{x} = [\varepsilon^\top, \omega^\top]^\top$ corresponding to $\mathbf{q} = [\eta, \varepsilon_1, \varepsilon_2, \varepsilon_3]^\top = [1, 0, 0, 0]^\top$ and $\tau = \mathbf{0}$? It is not necessary to include the state η in the analysis since it is a function of ε, that is $\eta = \sqrt{1 - \varepsilon^\top\varepsilon}$. Linearize the spacecraft model about $\mathbf{x} = \mathbf{x}_0$ and write down the expressions for the $\mathbf{A}$ and $\mathbf{B}$ matrices.

Hint: use (2.77) in Fossen (2021)

Problem 1.2 Consider the attitude control law:

$$\tau = -\mathbf{K}_d\omega - k_p\varepsilon \tag{2}$$

where $k_p > 0$ is a scalar control gain, $\mathbf{K}_d = k_d\,\mathbf{I}_3$ is a controller gain matrix with $k_d > 0$ and ε is the imaginary part of the unit quaternion. Let $k_p = 2$ and $k_d = 40$ and verify that the linearized closed-loop system is stable. Would you prefer real or complex poles in this particular application? Explain why/why not?

Problem 1.3 Let $k_p = 2$ and $k_d = 40$. Simulate the attitude dynamics (1) of the closed-loop system with the control law given by (2) for initial conditions $\phi(0) = -5^o$, $\theta(0) = 10^o$ and $\psi(0) = -20^o$ by modifying attitude.m. The initial angular velocities are zero. Plot the results (convert the resulting $\mathbf{q}(t)$ to Euler angles for easier visualization). Does the behavior of the system match your expectations? Explain why/why not. How would you modify the control law to follow nonzero constant reference signals? Include figures of the Euler angles, angular velocities and the control inputs in the report.

Problem 1.4 Consider the modified attitude control law:

$$\tau = -\mathbf{K}_d\omega - k_p\tilde{\varepsilon} \tag{3}$$

where $\tilde{\varepsilon}$ is the error in the imaginary part of the quaternion (between the setpoint and true state). The quaternion error is defined as:

$$\tilde{\mathbf{q}} := \begin{bmatrix} \tilde{\eta} \\ \tilde{\varepsilon} \end{bmatrix} = \bar{\mathbf{q}}_d \otimes \mathbf{q} \tag{4}$$

where $\mathbf{q}_d$ is the desired quaternion, $\mathbf{q}$ is the current state and $\bar{\mathbf{q}} = [\eta, -\varepsilon^\top]^\top$ denotes the conjugate (sometimes called the inverse) of a quaternion $\mathbf{q}$. The quaternion product is defined as:

$$\mathbf{q}_1 \otimes \mathbf{q}_2 = \begin{bmatrix} \eta_1\eta_2 - \varepsilon_1^\top\varepsilon_2 \\ \eta_1\varepsilon_2 + \eta_2\varepsilon_1 + \mathbf{S}(\varepsilon_1)\varepsilon_2 \end{bmatrix} \tag{5}$$

where $\mathbf{S}(\varepsilon_1)$ is the skew-symmetric matrix.

Write down the matrix expression for the quaternion error $\tilde{\mathbf{q}}$ on component form. What is $\tilde{\mathbf{q}}$ after convergence, that is $\mathbf{q} = \mathbf{q}_d$?

Problem 1.5 Let $k_p = 20$ and $k_d = 400$. Simulate the attitude dynamics of the closed-loop system with the control law given by (3). The desired attitude is given by the time-varying reference signal $\mathbf{q}_d(t)$ corresponding to $\phi(t) = 0, \theta(t) = 15\cos(0.1t)$ and $\psi(t) = 10\sin(0.05t)$ (all in degrees) and the initial values are equal to the ones in Problem 1.3. Does the behavior of the system match your expectations? Explain why/why not. Include the same set of figures as in Problem 1.3 and the tracking error in the report.

Problem 1.6 Consider the modified attitude control law:

$$\tau = -\mathbf{K}_d\tilde{\omega} - k_p\tilde{\varepsilon} \tag{6}$$

where $\tilde{\omega} = \omega - \omega_d$ is the difference between the desired and current angular velocity. Let the desired attitude be given by the reference signals from Problem 1.5 and calculate the desired angular velocity using (see equation (2.39) in Fossen (2021)).

$$\omega_d = \mathbf{T}_{\Theta}^{-1}(\Theta_d)\dot{\Theta}_d$$

where Θ_d is the desired Euler angles. Let $k_p = 20$ and $k_d = 400$ and simulate the controller (6) in Matlab with the same parameters, initial conditions and reference signals as in Problem 1.5. Deliver the same set of figures and compare the results. Does the behavior of the system match your expectations? Explain why/why not. Can you propose a way to further improve the control law?

Problem 1.7 It can be shown that:

$$\dot{\tilde{\eta}} = -\frac{1}{2}\tilde{\varepsilon}^{\top}\tilde{\omega} \tag{7}$$

Assume setpoint regulation, that is $\omega_d = \mathbf{0}$, $\varepsilon_d =$ constant and η_d = constant, and the control law given by (3). Consider the Lyapunov function candidate (Fjellstad and Fossen, 1994):

$$V = \frac{1}{2}\tilde{\omega}^{\top}\mathbf{I}_g\tilde{\omega} + 2k_p(1-\tilde{\eta}) \tag{8}$$

Explain why V is positive and radially unbounded. Show that:

$$\dot{V} = -k_d\omega^{\top}\omega \tag{9}$$

Use a suitable Lyapunov theorem/method to prove that the equilibrium of the closed-loop system is asymptotically stable. You need to use the nonlinear system dynamics. Is the system globally or locally asymptotically stable?

1 Appendix

1.1 attitude.m

```
% M-script for numerical integration of the attitude dynamics of a rigid
% body represented by unit quaternions. The MSS m-files must be on your
% Matlab path in order to run the script.
%
% System:                        .
%                                q = T(q)w
%                                .
%                              I w - S(Iw)w = tau
% Control law:
%                              tau = constant
%
% Definitions:
%                              I = inertia matrix (3x3)
%                              S(w) = skew-symmetric matrix (3x3)
%                              T(q) = transformation matrix (4x3)
%                              tau = control input (3x1)
%                              w = angular velocity vector (3x1)
%                              q = unit quaternion vector (4x1)
%
% Author:                     2018-08-15 Thor I. Fossen and Hakon H. Helgesen

%% USER INPUTS
h = 0.1;                       % sample time (s)
N  = 400;                      % number of samples. Should be adjusted

% model parameters
m = 180;
r = 2;
I = m*r^2*eye(3)               % inertia matrix
I_inv = inv(I);

% constants
deg2rad = pi/180;
rad2deg = 180/pi;

phi = -10*deg2rad;              % initial Euler angles
theta = 10*deg2rad;
psi = 5*deg2rad;

q = euler2q(phi,theta,psi);     % transform initial Euler angles to q

w = [0 0 0]';                   % initial angular rates

table = zeros(N+1,14);          % memory allocation

%% FOR-END LOOP
for i = 1:N+1,
t = (i-1)*h;                    % time
tau = [0.5 1 -1]';                 % control law

[phi,theta,psi] = q2euler(q); % transform q to Euler angles
[J,J1,J2] = quatern(q);         % kinematic transformation matrices

q_dot = J2*w;                          % quaternion kinematics
w_dot = I_inv*(Smtrx(I*w)*w + tau);    % rigid-body kinetics

table(i,:) = [t q' phi theta psi w' tau'];  % store data in table

q = q + h*q_dot;                   % Euler integration
w = w + h*w_dot;

q  = q/norm(q);                 % unit quaternion normalization
end
```

```
%% PLOT FIGURES
t       = table(:,1);
q       = table(:,2:5);
phi     = rad2deg*table(:,6);
theta   = rad2deg*table(:,7);
psi     = rad2deg*table(:,8);
w       = rad2deg*table(:,9:11);
tau     = table(:,12:14);


figure (1); clf;
hold on;
plot(t, phi, 'b');
plot(t, theta, 'r');
plot(t, psi, 'g');
hold off;
grid on;
legend('\phi', '\theta', '\psi');
title('Euler angles');
xlabel('time [s]');
ylabel('angle [deg]');

figure (2); clf;
hold on;
plot(t, w(:,1), 'b');
plot(t, w(:,2), 'r');
plot(t, w(:,3), 'g');
hold off;
grid on;
legend('x', 'y', 'z');
title('Angular velocities');
xlabel('time [s]');
ylabel('angular rate [deg/s]');

figure (3); clf;
hold on;
plot(t, tau(:,1), 'b');
plot(t, tau(:,2), 'r');
plot(t, tau(:,3), 'g');
hold off;
grid on;
legend('x', 'y', 'z');
title('Control input');
xlabel('time [s]');
ylabel('input [Nm]');
```

References

R. W. Beard and T. W. McLain. Small Unmanned Aircraft. Theory and Practice. Princeton University Press, 2012.

T. Fossen, Handbook of Marine Craft Hydrodynamics and Motion Control. John Wiley & Sons, 2021.

O.-E. Fjellstad and T. I. Fossen, “Quaternion feedback regulation of underwater vehicles,” vol. 2, 1994, pp. 857–862.

N. A. Chaturvedi, A. K. Sanyal, and N. H. McClamroch, “Rigid-body attitude control,” IEEE Control Systems, vol. 31, no. 3, pp. 30–51, June 2011.