TTK4190 Guidance and Control of Vehicles

Draft Solution of Assignment 2

Info

Guidelines

- This presentation is only intended as a draft of a possible way to solve the assignment.
- There may exist better ways to solve the problems, so don't (necessarily) be alarmed if your solution deviates from this.

Outline

1. Open-Loop Analysis

2. Autopilot for Course Hold using Aileron and Successive Loop Closure

Problem 1 - Open-loop Analysis

Problem 1a)

What is the ground speed of the aircraft (numerical value) in the absence of wind?

What we know

- The relationship between ground speed (V_g), air speed (V_a), and wind (V_w) is given by the wind triangle
- The wind speed is zero

Solution

$$V_g = V_a + V_w = 580km/h + 0km/h = 580km/h \approx 161.11m/s$$

Problem 1 - Open-loop Analysis

Problem 1b)

Write down two expressions for the sideslip angle β in the absence of wind. One expression should depend on aircraft velocity and the other on aircraft heading.

Notes

- The book is somewhat messy when it comes to the relationship between sideslip, wind and air velocity.
- The sideslip and crab angle are equal when there is no wind (and not necessarily zero).

Solution

The first expression (Eq. (28) p.20 in Beard & McLain) is based on the air velocity:

$$\beta = \sin^{-1}\left(\frac{v_r}{\sqrt{u_r^2 + v_r^2 + w_r^2}}\right) = \sin^{-1}\left(\frac{v}{\sqrt{u^2 + v^2 + w^2}}\right) = \sin^{-1}\left(\frac{v}{V_g}\right)$$

The second expression is based on the course and heading angle (crab equal to sideslip without wind):

$$\beta = \chi - \psi$$

Problem 1 - Open-loop Analysis

Problem 1c)

Compute the Dutch-roll natural frequency and relative damping ratio for the aircraft.

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \\ \dot{\delta}_a \end{bmatrix} = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & 0 & 1 & -0.001 & 0 \\ -10.6 & 0 & -2.87 & 0.46 & -0.65 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 0 & 0 & -7.5 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \\ \delta_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 7.5 \end{bmatrix} \delta_a^c$$

Solution - First Part

We are only looking at the equations for the yaw rate and the sideslip (neglect other terms):

$$\dot{\beta} = -0.322\beta - 1.12r$$
$$\dot{r} = 6.87\beta - 0.32r$$

- Using the damp function on the reduced system yields $\zeta = 0.115$ and $\omega_n = 2.79$.
- Using the damp function directly on $\mathbf{A}$ yields $\zeta = 0.111$ and $\omega_n = 2.85$.

Problem 1 - Open-loop Analysis

Problem 1c)

Describe how the Dutch roll mode affect the yaw and roll motion? How would the motion change with increased relative damping ratio?

Solution - Second Part

- ▸ Dutch Roll Animation
- Since the damping ratio is low and under-damped, the tail will be wagging from side-to-side.
- Increased damping ratio would reduce the oscillations.
- A yaw damper can be used to limit the effect of the dutch-roll mode.

Problem 1 - Open-loop Analysis

Problem 1d)

Compute the spiral-divergence mode. Is the mode unstable?

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \\ \dot{\delta}_a \end{bmatrix} = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & 0 & 1 & -0.001 & 0 \\ -10.6 & 0 & -2.87 & 0.46 & -0.65 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 0 & 0 & -7.5 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \\ \delta_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 7.5 \end{bmatrix} \delta_a^c$$

Assumptions

- $\dot{p} = p = 0$
- Rudder command is negligible

Solution

We are only looking at the equations for roll rate and yaw rate:

$$\dot{p} = -10.6\beta + 0.46r - 0.65\delta_a = 0$$
$$\dot{r} = 6.87\beta - 0.32r - 0.02\delta_a$$

Problem 1 - Open-loop Analysis

Problem 1d)

Solution

By solving the first equation for β and inserting it into the second equation:

$$\dot{r} = -0.0218r - 0.442\delta_a$$

- The pole (mode) is -0.0218, which means that the mode is very slow and stable.
- "damp(A)" gives a pole of -0.00056.

Problem 1 - Open-loop Analysis

Problem 1e)

Compute the roll mode. Is the roll mode faster or slower than the spiral-divergence mode?

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \\ \dot{\delta}_a \end{bmatrix} = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & 0 & 1 & -0.001 & 0 \\ -10.6 & 0 & -2.87 & 0.46 & -0.65 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 0 & 0 & -7.5 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \\ \delta_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 7.5 \end{bmatrix} \delta_a^c$$

Assumptions

- We are only concerned with the equation for the roll rate p.
- $\beta = r = \delta_r = 0$

Solution

The equation for the roll rate can (with the assumptions above) be written as

$$\dot{p} = -2.87p - 0.65\delta_a$$

- The pole (mode) is -2.87 and it is much faster than the spiral-divergence mode.
- "damp(A)" yields -2.88.

Outline

1. Open-Loop Analysis

2. Autopilot for Course Hold using Aileron and Successive Loop Closure

Problem 2 - Autopilot Design

Problem 2 a)

Explain why the smallest signed angle function `ssa(.)` *should be applied to the angle errors* e_χ *and* e_ϕ

Smallest Signed Angle (SSA)

The operator ssa : $\mathbb{R} \to [-\pi, \pi)$ maps the unconstrained angle $\tilde{x} = x - x_0 \in \mathbb{R}$ representing the difference between the two angles x and x_0 to the smallest difference between the angles

$$\tilde{x}_s = \mathrm{ssa}(\tilde{x})$$

where $\tilde{x}_s \in \mathbb{S}^1$.
Since the angle errors e_χ and e_ϕ can lie outside $[-\pi, \pi)$ it is necessary to confide them within this interval using the ssa function.

Problem 2 - Autopilot Design

Problem 2b)

Find numerical values for a_{ϕ_1} and a_{ϕ_2} based on the state-space model.

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \\ \dot{\delta}_a \end{bmatrix} = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & 0 & 1 & -0.001 & 0 \\ -10.6 & 0 & -2.87 & 0.46 & -0.65 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 0 & 0 & -7.5 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \\ \delta_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 7.5 \end{bmatrix} \delta_a^c$$

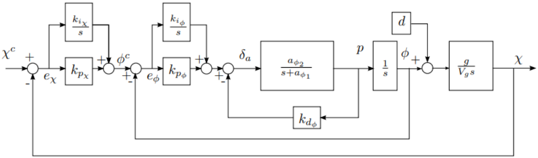

Figure: Block diagram of simplified system

Assumptions

- We are considering a simplified system between roll, roll rate, and aileron
- Ignore sideslip and yaw rate, i.e., $\beta = r = 0$.

Solution

With the assumptions outlined above, the simplified roll rate equation becomes:

$$\dot{p} = -2.87p - 0.65\delta_a$$

Laplace transform:

$$H(s) = \frac{p(s)}{\delta_a(s)} = \frac{-0.65}{s + 2.87} = \frac{a_{\phi_2}}{s + a_{\phi_1}}$$

Therefore,

$$a_{\phi_1} = 2.87$$
$$a_{\phi_2} = -0.65$$

- The answer can be extracted directly from the roll mode.

Problem 2c) - Successive Loop Closure

Main Considerations

I will not go through the exact solution in detail. Some of the main concerns are:

- The inner loop must have a much higher bandwidth than the outer loop.
- It is necessary to start with the inner most-loop $\rightarrow$ Gains in roll loop identified first.
- Gains in the course loop are identified in the end.
- Using the Evans form is necessary in order to find the interval for k_{i_ϕ}

Roll Loop (inner loop)

By following the procedure outlined in Sec. 6.3.1 in Beard & McLain, we get:

$$k_{p_\phi} = \frac{\delta_a^{max}}{e_\phi^{max}} sign(a_{\phi_2}) = \frac{30^o}{15^o}(-1) = -2$$

$$\omega_{n_\phi} = \sqrt{|a_{\phi_2}| \frac{\delta_a^{max}}{e_\phi^{max}}} \approx 1.14$$

$$k_{d_\phi} = \frac{2\zeta_\phi \omega_{n_\phi} - a_{\phi_1}}{a_{\phi_2}} \approx 1.94$$

where the damping ratio (design parameter) is chosen as $\zeta_\phi = 0.707$.

Problem 2c) - Successive Loop Closure

Integral Gain in Roll loop

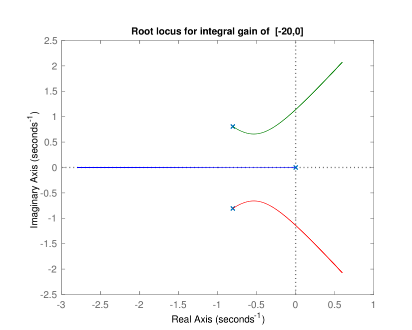

Figure: Root-Locus For Negative Gain

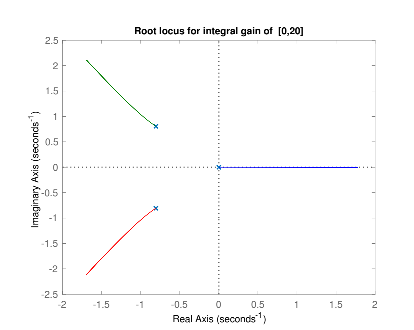

Figure: Root-Locus For Positive Gain

- The feasible interval for the gain is $k_{i_\phi} \in [-3.21, 0]$.

Problem 2c) - Successive Loop Closure

Course Loop (outer loop)

- The inner loop is now approximated as a DC-gain of 1.
- The bandwidth should be at least 5 times lower than that of the inner loop.
- Choosing a bandwidth ten times slower than in the roll loop and a damping ratio of 1 yields:

$$\omega_{n_\chi} = \frac{1}{W_\chi}\omega_{n_\phi} = \frac{1}{10} \cdot 1.14 = 0.114$$

$$k_{p_\chi} = 2\zeta_\chi \omega_{n_\chi} \frac{V_g}{g} = 2 \cdot 1 \cdot \frac{1.14}{10} \cdot \frac{161.11}{9.81} \approx 3.74$$

$$k_{i_\chi} = \frac{\omega_{n_\chi}^2 V_g}{g} = \left(\frac{1.14}{10}\right)^2 \cdot \frac{161.11}{9.81} \approx 0.21$$

Problem 2d) - Integral gain in the roll loop

Can you come up with a reason for why it in some cases may be beneficial to remove the integral action from the roll loop?

- The first thing you should do is ask yourself if integral action is necessary or not.
- In general, you don't want to add integral action unless you need it - can lead to delay and instability in your system.

Integrator in roll loop

The transfer function from ϕ^c to ϕ without integral action can be shown to be:

$$\frac{\phi}{\phi^c}(s) = \frac{k_{p_\phi} a_{\phi_2}}{s^2 + (a_{\phi_1} + k_{d_\phi} a_{\phi_2})s + k_{p_\phi} a_{\phi_2}}$$

For a constant set point $\phi^c(s) = \frac{1}{s}\bar{\phi}$, it can be shown using the final value theorem that

$$\lim_{t \to \infty} \phi(t) = \lim_{s \to 0} s\phi(s) = \bar{\phi}$$

$\Rightarrow$ Integral action is not necessary as we don't have any disturbance in the inner roll loop.

- Also worth mentioning that we don't really "care" in this case if the roll angle deviates from the setpoint as long as the course angle converges.

Problem 2e) - Simulation of Simplified System

Present simulation results for the system in Figure 1 with course changing maneuvers: choose a series of steps (you may choose yourself the desired values).

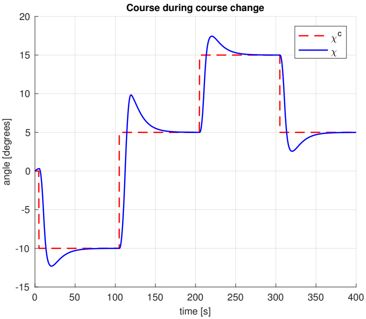

Figure: Course during course changing maneuvers.

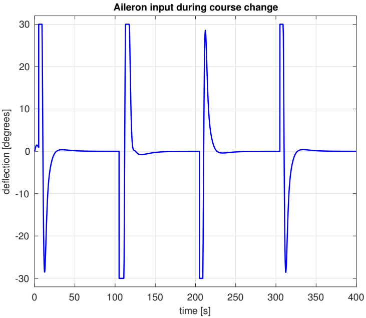

Figure: Aileron input during course changing maneuvers.

Problem 2f) - Simulation of the State-Space Model

Present simulation results for course changing maneuvers (choose the same desired values as in Problem 2d) with the complete state-space model.

- We replace the simplified roll dynamics with the full state-space model.
- The real coordinated-turn equation should be used instead of the simplified version.
- The lowpass filter is already a part of the state-space model (no need to add this in Simulink).

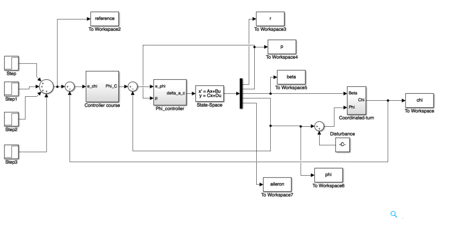

Figure: Overall system

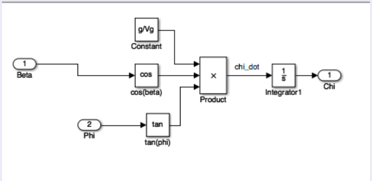

Figure: Coordinated-turn

Problem 2f) - Simulation of State Space Model and Real Coordinated-Turn Equation

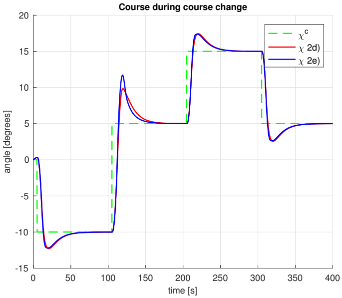

Figure: Course during course changing maneuvers.

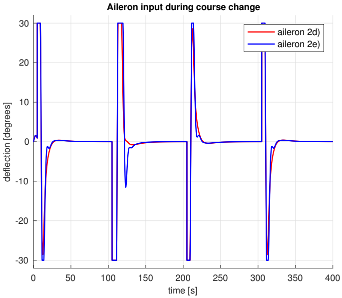

Figure: Aileron during course changing maneuvers.

- The simplified model is, in this case, an accurate approximation of the "real" dynamics.
- Remember that the relationship between heading and roll actually is nonlinear.

Problem 2g) - Integrator windup

Would you claim that integrator windup is a problem in your simulations? If that is the case, can you propose a solution that would limit the problem?

- When the input is saturated, the integrator will continue to accumulate error which can lead to large overshoots and consequently large delay
- There exists a number of different anti-windup schemes
- The easiest one to implement is to turn off the integrator by setting $k_{i_\chi} = 0$ when the input goes into saturation. This can be achieved with a simple switching mechanism in Simulink

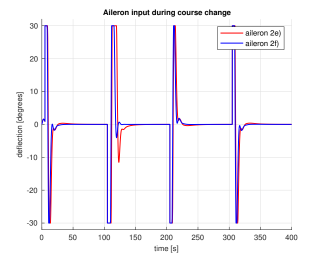

Figure: Aileron during course changing maneuvers.

Problem 2g) - Integrator windup

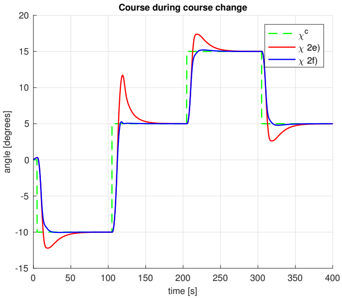

Figure: Course during course changing maneuvers.

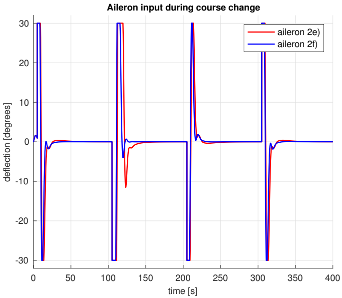

Figure: Aileron during course changing maneuvers.

- How well you are able to control windup will also depend on the size of the step error.
- By using a simple switching mechanism it is possible to avoid overshoot.
- If you have overshoot and the input is saturated, you should have some sort of proposal/action to handle windup.