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cs3252-01

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7.1.3

a. S, A, B, c are all nullable

S → 0A0 | 00 | 1B1 | 11 | BB | B

A → c

B → S | A

c → S

b. Basis: (S, S)
(A, A)
(B, B)
(c, c)

Induction: (A, A) then (A, c) then (A, S)

pair	production
(A, A)	none
(A, c)	none
(A, S)	0A0	00	1B1	11	BB
(B, B)	none
(B, A)	none
(B, S)	0A0	00	1B1	11	BB
(B, c)	none
(c, c)	none
(c, S)	0A0	00	1B1	11	BB
(A, B)	none
(c, B)	none
(c, A)	none
(S, S)	0A0	00	1B1	11	BB
(S, B)	none
(S, A)	none
(S, c)	none

S → 0A0 | 00 | 1B1 | 11 | BB

A → 0A0 | 00 | 1B1 | 11 | BB

B → 0A0 | 00 | 1B1 | 11 | BB

c → 0A0 | 00 | 1B1 | 11 | BB

c. $S \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

c is unreachable

S, A, B are identical so A and B are useless

$S \rightarrow 0S0 \mid 00 \mid 1S1 \mid 11 \mid SS$

d. $S \rightarrow AB \mid AA \mid CD \mid CC \mid SS$

$A \rightarrow 0$

$B \rightarrow SA$

$C \rightarrow 1$

$D \rightarrow SC$

 # 7.1.4

a. $S \rightarrow AAA \mid AA \mid A$

$A \rightarrow aA \mid a$

b. Basis: (S, S)

(A, A)

Induction:

$(S, S) \rightarrow AAA \mid AA$

$(S, A) \rightarrow aA \mid a$

$(A, A) \rightarrow aA \mid a$

$S \rightarrow AAA \mid AA \mid aA \mid a$

$A \rightarrow aA \mid a$

c. S is not useless

neither is A   since $S \rightarrow a$

$A \rightarrow a$

$S \rightarrow AAA \mid AA \mid aA \mid a$

$A \rightarrow aA \mid a$

d. $A \rightarrow a$

$B \rightarrow AB \mid a$

$S \rightarrow BB \mid a$

a. Nullable: S, A, B, C, D

$S \rightarrow aAa \mid aa \mid bBb \mid bb$

$A \rightarrow c \mid a$

$B \rightarrow c \mid b$

$C \rightarrow CDE \mid DE \mid CE \mid E$

$D \rightarrow A \mid B \mid ab$

b. unit productions

$(S, S) \rightarrow aAa \mid aa \mid bBb \mid bb$

$(A, A) \rightarrow a$

$(A, C) \rightarrow CDE \mid DE \mid CE$

$(A, E) \rightarrow$ none

$(B, B) \rightarrow b$

$(B, C) \rightarrow CDE \mid DE \mid CE$

$(B, E) \rightarrow$ none

$(C, C) \rightarrow CDE \mid DE \mid CE$

$(C, E) \rightarrow$ none

$(D, D) \rightarrow ab$

$(D, A) \rightarrow a$

$(D, C) \rightarrow CDE \mid DE \mid CE$

$(D, E) \rightarrow$ none

$(D, B) \rightarrow b$

$S \rightarrow aAa \mid aa \mid bBb \mid bb$

$A \rightarrow a \mid CDE \mid DE \mid CE$

$B \rightarrow b \mid CDE \mid DE \mid CE$

$C \rightarrow CDE \mid DE \mid CE$

$D \rightarrow ab \mid a \mid b \mid CDE \mid DE \mid CE$

c. useless symbols

1. nongenerating : E

$S \rightarrow aAa \mid aa \mid bBb \mid bb$

$A \rightarrow a$

$B \rightarrow b$

$\underline{C \rightarrow \text{none}}$

$D \rightarrow ab \mid a \mid b$

D is not reachable

so,

$S \rightarrow aAa \mid aa \mid bBb \mid bb$

$A \rightarrow a$

$B \rightarrow b$

d. chomsley

$S \rightarrow CA \mid AA \mid DB \mid BB$

$A \rightarrow a$

$B \rightarrow b$

$C \rightarrow AA$

$D \rightarrow BB$

* 7.2.1

For parts a, b, and c n is the integer of the pumping lemma

for all $z \in L$, and $|z| \geq n$, $z = uvwxy$

1. $|vwx| \leq n$

2. $vx \neq \varepsilon$

3. for all $i \geq 0$, $uv^i w x^i y \in L$

a. $a^i b^j c^k \mid i < j < k$

It is clear that any string of the form $a^n b^{(n+1)} c^{(n+2)}$ is in L.

Case 1: vwx consists only of a's

   Thus, it can easily be seen that pumping upward for any $i > 1$ gives a string with $a^m b^n$ where $m > n$.

, case 2: vwx has both a's and b's

similarly pumping v and x upward will clearly quickly violate i.e $j \leq k$.

case 3: vwx has only b's

pumping will obviously result in more b's than c's for pumping where $i > 1$

case 4: vwx has b's and c's

If pumped down for $i = 0$, the number of a's $\geq$ number of b's

case 5: only c's

pumped down for $i = 0$, number of b's $\geq$ number of c's

Thus, the language is not context-free.

b. $\{ a^n b^n c^i \mid i \leq n \}$

clearly, $a^n b^n c^n \in L$.

case 1: vwx is only a's, then pump up will have $a^m b^n$ $m > n$

case 2: a's and b's, pump more a's than b's

case 3: b's: pump down, $a^n b^{<n} c^n \notin L$

case 4: b's and c's pump down, then $a^m b^n$ where $m > n$

case 5: c's: pump upward, $a^n b^n c^m$ $m > n$

Thus, the language is not context free.

c. $\{ 0^p \mid p \text{ is prime} \}$

p is a prime $\geq n + 2$ so $z = 0^p$

If $|vwz| = m$, $|wy| = p - m$

$u (vwx)^{p-m} y = |uy| + (p-m)|vwx| =$

$= p - m + (p - m) m$

$= (m+1)(p-m)$

for $m+1$ because $m \geq 2$ since v and $x \neq \varepsilon$

for $p-m$ $m \leq n$ and $p \geq n+2$ so $p - m > 2$

thus, the language is not context free.