Voltage calc

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V_{max} = maximum Voltage
V_{min} = minimum Voltage
V_{nom} = nominal Voltage
A_h = Total ampacity
α = amp hours consumed

- Constraints
  1. $V(\alpha = 0) = V_{max}$
     - *We start at maximum voltage when 0 amp hours have been consumed*
  2. $\frac{1}{A_h}\int_0^{A_h} V(\alpha)d\alpha = V_{nom}$
     - *Our average voltage is the nominal voltage on the interval [0, total ampacity]*
  3. $V(A_h) = V_{min}$
     - *We end at minimum voltage when total ampacity has been consumed*
  4. $\frac{d}{d\alpha}V(\alpha = 0) = -\frac{V_{max} - V_{min}}{A_h}$
     - *Define the initial slope to be that of a linear line from the point* $(0, V_{max}) \to (A_h, V_{min})$

- Function
  - $V(\alpha) = a\alpha^3 + b\alpha^2 + c\alpha + d$
    - *Constraint 1*
      - $d = V_{max}$
    - *Constraint 2*
      - $\int V(\alpha)d\alpha = \frac{a\alpha^4}{4} + \frac{b\alpha^3}{3} + \frac{c\alpha^2}{2} + d\alpha$
      - $\frac{1}{A_h}\int_0^{A_h} V(\alpha)d\alpha = \frac{aA_h^3}{4} + \frac{bA_h^2}{3} + \frac{cA_h}{2} + V_{max}$
      - $V_{nom} = \frac{aA_h^3}{4} + \frac{bA_h^2}{3} + \frac{cA_h}{2} + V_{max}$
      - $V_{nom} - V_{max} = \frac{aA_h^3}{4} + \frac{bA_h^2}{3} + \frac{cA_h}{2}$
      - $\frac{aA_h^3}{4} = V_{nom} - V_{max} - \frac{bA_h^2}{3} - \frac{cA_h}{2}$
      - $a = \frac{4}{A_h^3}(V_{nom} - V_{max}) - c\frac{2}{A_h^2} - b\frac{4}{3A_h}$
    - *Constraint 3*
      - $V(A_h) = aA_h^3 + bA_h^2 + cA_h + d$
      - $V_{min} = \left(\frac{4}{A_h^3}(V_{nom} - V_{max}) - c\frac{2}{A_h^2} - b\frac{4}{3A_h}\right)A_h^3 + bA_h^2 + cA_h + V_{max}$
      - $V_{min} - V_{max} = c(-2A_h + A_h) + b\left(-\frac{4}{3}A_h^2 + A_h^2\right) + 4V_{nom} - 4V_{max}$
      - $V_{min} - V_{max} - 4V_{nom} + 4V_{max} = -A_h c - \frac{1}{3}A_h^2 b$
      - $4V_{nom} - 3V_{max} - V_{min} = A_h c + \frac{1}{3}A_h^2 b$
      - $\frac{1}{3}A_h^2 b = 4V_{nom} - 3V_{max} - V_{min} - A_h c$
      - $b = \frac{3(4V_{nom} - 3V_{max} - V_{min})}{A_h^2} - \frac{3}{A_h}c$

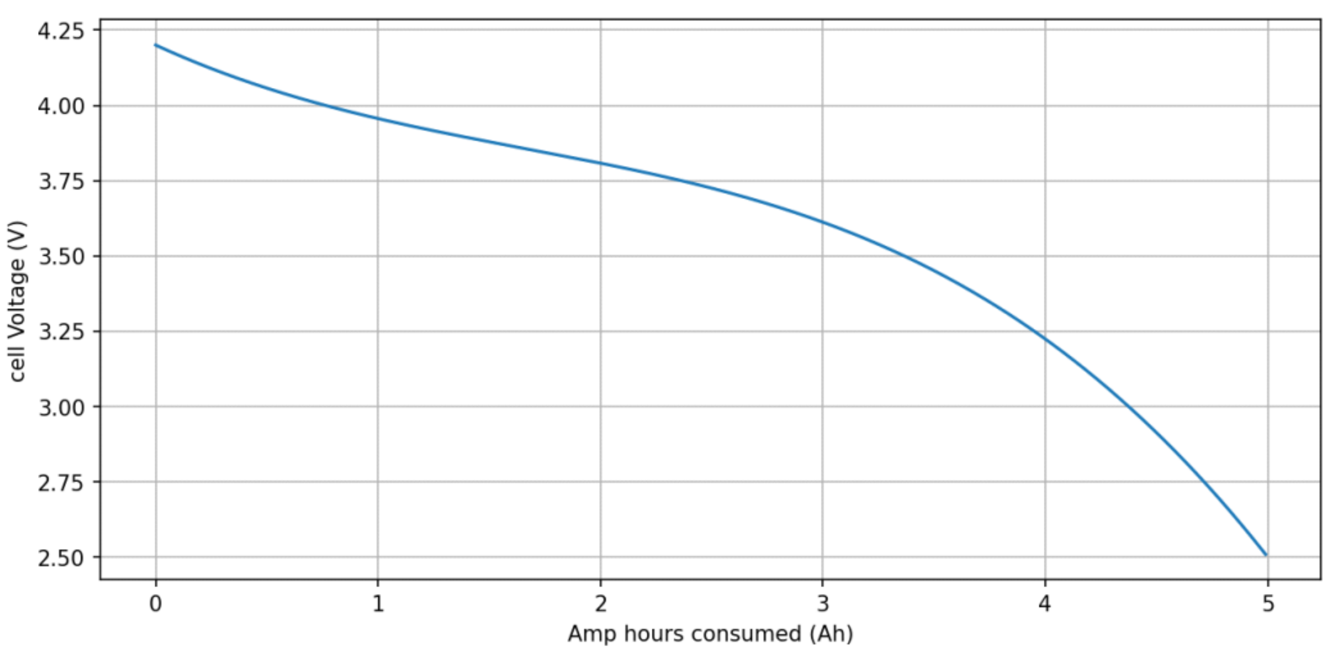
- *Constraint 4*
    - $\frac{d}{d\alpha}V(\alpha = 0) = c$
    - $c = -\frac{V_{max} - V_{min}}{A_h} = \frac{V_{min} - V_{max}}{A_h}$

- *Coefficients*
    - $d = V_{max}$
    - $c = \frac{V_{min} - V_{max}}{A_h}$
    - $b = \frac{3(4V_{nom} - 3V_{max} - V_{min})}{A_h^2} - \frac{3}{A_h}\left(\frac{V_{min} - V_{max}}{A_h}\right)$
        - $b = \frac{3(4V_{nom} - 2V_{max} - 2V_{min})}{A_h^2}$
        - $b = \frac{6}{A_h^2}(2V_{nom} - V_{max} - V_{min})$
    - $a = \frac{4}{A_h^3}(V_{nom} - V_{max}) - \left(\frac{V_{min} - V_{max}}{A_h}\right)\frac{2}{A_h^2} - \left(\frac{6}{A_h^2}(2V_{nom} - V_{max} - V_{min})\right)\frac{4}{3A_h}$
        - $a = \frac{4}{A_h^3}(V_{nom} - V_{max}) - \frac{2}{A_h^3}(V_{min} - V_{max}) - \frac{8}{A_h^3}(2V_{nom} - V_{max} - V_{min})$
        - $a = \frac{2}{A_h^3}(2V_{nom} - 2V_{max} - V_{min} + V_{max} - 8V_{nom} + 4V_{max} + 4V_{min})$
        - $a = \frac{2}{A_h^3}(-6V_{nom} + 3V_{max} + 3V_{min})$
        - $a = -\frac{6}{A_h^3}(2V_{nom} - V_{max} - V_{min})$
            - $a = -\frac{b}{A_h}$

$$V(\alpha) = \left[-\frac{6}{A_h^3}(2V_{nom} - V_{max} - V_{min})\right]\alpha^3 + \left[\frac{6}{A_h^2}(2V_{nom} - V_{max} - V_{min})\right]\alpha^2 + \left[\frac{V_{min} - V_{max}}{A_h}\right]\alpha + V_{max}$$

- *Using the Samsung 50s cell*

  $V_{max} = 4.2V$
  $V_{min} = 2.5V$
  $V_{nom} = 3.6V$
  $A_h = 5Ah$

- 

- To convert this function to be with respect to watt-hour consumed, we integrate over the curve on the interval $[0, A_h]$ to get capacity and do a similar calculation
    - $\omega = \textit{watt-hours consumed}$
    - $\beta = \textit{total capacity}$
        - $\int_0^{A_h} V(\alpha)d\alpha = \beta$
    - $V(\omega) = i\omega^3 + j\omega^2 + k\omega + l$
        - *Coefficients*
            - $l = V_{max}$
            - $k = \frac{V_{min} - V_{max}}{\beta}$
            - $j = \frac{6}{\beta^2}\left(2V_{nom} - V_{max} - V_{min}\right)$
            - $i = -\frac{6}{\beta^3}\left(2V_{nom} - V_{max} - V_{min}\right)$

$$V(\omega) = \left[-\frac{6}{\beta^3}\left(2V_{nom} - V_{max} - V_{min}\right)\right]\omega^3 + \left[\frac{6}{\beta^2}\left(2V_{nom} - V_{max} - V_{min}\right)\right]\omega^2 + \left[\frac{V_{min} - V_{max}}{\beta}\right]\omega + V_{max}$$

- *Using the same Samsung 50s cell*
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