# Voltage calc

Saturday, November 9, 2024 1:53 PM

$V_{max} = maximum\ Voltage$
$V_{min} = minimum\ Voltage$
$V_{nom} = nominal\ Voltage$
$A_h = Total\ ampacity$
$\alpha = amp\ hours\ consumed$

- Constraints
  1. $V(0) = V_{max}$
     - *We start at maximum voltage when 0 amp hours have been consumed*
  2. $\frac{1}{A_h}\int_0^{A_h} V(\alpha)d\alpha = V_{nom}$
     - *Our average voltage is the nominal voltage on the interval [0, total ampacity]*
  3. $V(A_h) = V_{min}$
     - *We end at minimum voltage when total ampacity has been consumed*
  4. $\frac{d}{d\alpha}V(\alpha = 0) = \frac{V_{nom} - V_{max}}{kA_h}$
     - *Define the initial slope to be a ratio dependent on a scaling factor k*

- Function
  - $V(\alpha) = a\alpha^3 + b\alpha^2 + c\alpha + d$
    - *Constraint 1*
      - $d = V_{max}$
    - *Constraint 2*
      - $\int V(\alpha)d\alpha = \frac{a\alpha^4}{4} + \frac{b\alpha^3}{3} + \frac{c\alpha^2}{2} + d\alpha$
      - $\frac{1}{A_h}\int_0^{A_h} V(\alpha)d\alpha = \frac{aA_h^3}{4} + \frac{bA_h^2}{3} + \frac{cA_h}{2} + V_{max}$
      - $V_{nom} = \frac{aA_h^3}{4} + \frac{bA_h^2}{3} + \frac{cA_h}{2} + V_{max}$
      - $V_{nom} - V_{max} = \frac{aA_h^3}{4} + \frac{bA_h^2}{3} + \frac{cA_h}{2}$
      - $\frac{aA_h^3}{4} = V_{nom} - V_{max} - \frac{bA_h^2}{3} - \frac{cA_h}{2}$
      - $a = \frac{4}{A_h^3}(V_{nom} - V_{max}) - c\frac{2}{A_h^2} - b\frac{4}{3A_h}$
    - *Constraint 3*
      - $V(A_h) = aA_h^3 + bA_h^2 + cA_h + d$
      - $V_{min} = \left(\frac{4}{A_h^3}(V_{nom} - V_{max}) - c\frac{2}{A_h^2} - b\frac{4}{3A_h}\right)A_h^3 + bA_h^2 + cA_h + V_{max}$
      - $V_{min} - V_{max} = c(-2A_h + A_h) + b\left(-\frac{4}{3}A_h^2 + A_h^2\right) + 4V_{nom} - 4V_{max}$
      - $V_{min} - V_{max} - 4V_{nom} + 4V_{max} = -A_hc - \frac{1}{3}A_h^2b$
      - $4V_{nom} - 3V_{max} - V_{min} = A_hc + \frac{1}{3}A_h^2b$

- $\frac{1}{3}A_h^2 b = 4V_{nom} - 3V_{max} - V_{min} - A_h c$
- $b = \frac{3(4V_{nom} - 3V_{max} - V_{min})}{A_h^2} - \frac{3}{A_h}c$

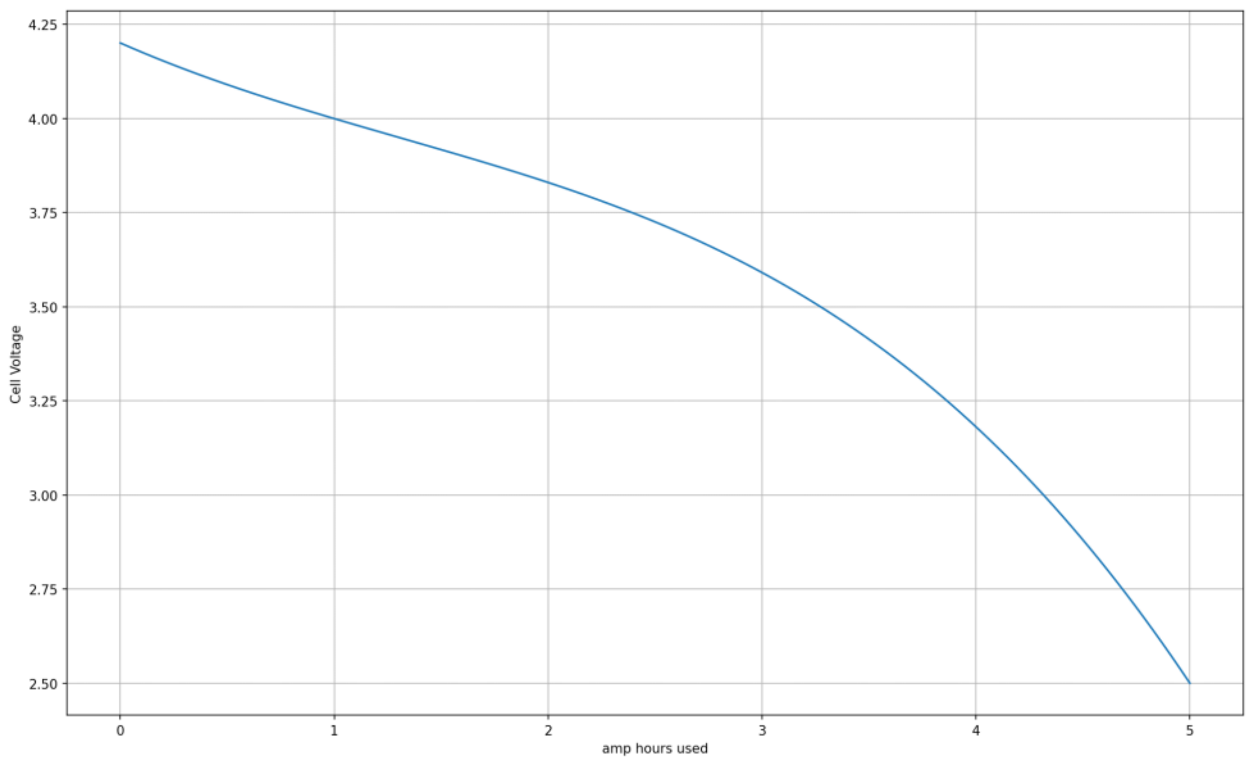
- *Constraint 4*
  - $\frac{d}{d\alpha}V(\alpha = 0) = c$
  - $c = \mu$
    - *Where $\mu$ is selected to best fit testing data*

- *Coefficients*
  - $d = V_{max}$
  - $c = \mu$
  - $b = \frac{3(4V_{nom} - 3V_{max} - V_{min})}{A_h^2} - \frac{3}{A_h}\mu$
  - $a = \frac{4}{A_h^3}(V_{nom} - V_{max}) - \mu\frac{2}{A_h^2} - \left(\frac{3(4V_{nom} - 3V_{max} - V_{min})}{A_h^2} - \frac{3}{A_h}\mu\right)\frac{4}{3A_h}$

$$V(\alpha) = \left[\frac{4}{A_h^3}(V_{nom} - V_{max}) - \mu\frac{2}{A_h^2} - \left(\frac{3(4V_{nom} - 3V_{max} - V_{min})}{A_h^2} - \frac{3}{A_h}\mu\right)\frac{4}{3A_h}\right]\alpha^3 + \left[\frac{3(4V_{nom} - 3V_{max} - V_{min})}{A_h^2} - \frac{3}{A_h}\mu\right]\alpha^2 + \mu\alpha + V_{max}$$

- *Using the Samsung 50s cell*

  $V_{max} = 4.2V$
  $V_{min} = 2.5V$
  $V_{nom} = 3.6V$
  $A_h = 5Ah$
  $\mu = -0.25$

- 


- To convert this function to be with respect to watt-hour consumed, we integrate over the curve on the interval $[0, A_h]$ to get capacity and do a similar calculation

- $\omega = watt\text{-}hours\ consumed$
- $\beta = total\ capacity$
  - $\int_0^{A_h} V(\alpha)d\alpha = \beta$
- $V(\omega) = i\omega^3 + j\omega^2 + k\omega + l$
  - *Coefficients*
    - $l = V_{max}$
    - $k = \frac{2\mu}{A_h}$
    - $j = \frac{3(4V_{nom} - 3V_{max} - V_{min})}{\beta^2} - \frac{3}{\beta}\mu$
    - $i = \frac{4}{\beta^3}(V_{nom} - V_{max}) - \frac{2}{\beta^2}\left(\frac{2\mu}{A_h}\right) - \left(\frac{3(4V_{nom} - 3V_{max} - V_{min})}{\beta^2} - \frac{3}{\beta}\mu\right)\frac{4}{3\beta}$

$$V(\omega) = \left[\frac{4}{\beta^3}(V_{nom} - V_{max}) - \frac{2}{\beta^2}\left(\frac{2\mu}{A_h}\right) - \left(\frac{3(4V_{nom} - 3V_{max} - V_{min})}{\beta^2} - \frac{3}{\beta}\mu\right)\frac{4}{3\beta}\right]\omega^3 + \left[\frac{3(4V_{nom} - 3V_{max} - V_{min})}{\beta^2} - \frac{3}{\beta}\mu\right]\omega^2 + \frac{2\mu}{A_h}\omega + V_{max}$$

- *Using the same Samsung 50s cell*
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