Math 111-003: Intro. to Contemporary Math.
Exam 1 (Fall 16)
Wednesday, September 21, 2016

Name: _____

Exam Instructions

- DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO
- Do not remove this first page - you will turn in this entire exam.
- Complete this exam using only a pencil and a simple calculator (not a cellphone).
- The exam consists of multiple choice and short answer questions. Record your answers on this page in the following manner:

  (A) (B) (C) (D) (E)

- For the multiple choice, only this front page will be graded and no partial credit will be awarded.
- The exam is out of 100 points. There are 16 multiple choice questions each worth 4 points. There are 2 short answer problems each worth 18 points. These short answer problems have multiple parts.
- Throughout the exam, MA = Majority Criterion, CO = Condorcet Criterion, MO = Monotonicity Criterion, and IIA = Independence-of-Irrelevant-Alternatives Criterion.

Multiple Choice Answers

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)
16. (A) (B) (C) (D) (E)

Scoring		
#1-16:	/64	
#17:	/18	
#18:	/18	
Total:	**/100**	=

Multiple Choice Section

- There are 16 multiple choice questions, each worth 4 points.
- Circle and/or write the letter corresponding to the **BEST** answer, then **transfer your answer to the front page.**

Problems 1-5 concern the following blank preference schedule. The columns are numbered 1 to 8 from left to right. The numbers at the bottom of each column are there to help denote the column number.

?	?	?	?	?	?	?	?
A	A	B	B	C	C	D	D
B	D	C	A	D	B	A	C
C	C	D	D	A	A	B	B
D	B	A	C	B	D	C	A
1	2	3	4	5	6	7	8

1. Which column represents voters who like Candidate **C** the most and Candidate **B** the least?
   A. Column 1
   B. Column 2
   C. Column 3
   D. Column 4
   E. Column 5

2. Candidate **B** will get 3 Borda points per vote from which pair of columns?
   A. Column 1 and Column 6
   B. Column 4 and Column 7
   C. Column 5 and Column 6
   D. Column 3 and Column 8
   E. No column gives 3 points per vote

3. During an election using **Plurality with Elimination**, Candidate **D** wins after Candidates **A** and **C** are eliminated. From which pair of columns did Candidate **D** receive more first place votes **after A** and **C** are eliminated?
   A. Column 1 and Column 6
   B. Column 3 and Column 4
   C. Column 2 and Column 4
   D. Column 1 and Column 4
   E. Column 2 and Column 5

4. For an election using **Pairwise Comparison**, which of the following columns will help **C** beat **D**, **AND B** beat **A**, in the individual pairwise comparisons?
   A. Column 5 and Column 6
   B. Column 3 and Column 4
   C. Column 3 and Column 6
   D. Column 4 and Column 5
   E. Column 3 and Column 5

?	?	?	?	?	?	?	?
A	A	B	B	C	C	D	D
B	D	C	A	D	B	A	C
C	C	D	D	A	A	B	B
D	B	A	C	B	D	C	A
1	2	3	4	5	6	7	8

5. Suppose Candidate **C** wins the election using an unknown method. Suppose we want to test if the voting method can violate the Monotonicity criterion by changing ballots from the **eighth** column. Which of these ballot orderings is a valid change (note: the ballot orderings are listed as 1st-2nd-3rd-4th)?

   A. A, C, B, D

   B. D, C, B, A

   C. C, B, D, C

   D. C, D, A, B

   E. C, D, B, A

6. An unknown voting method was used in the following three elections:

Election 1

7	3	5
B	C	A
A	A	C
C	B	B

Winner: B

Election 2

7	2	4
B	C	A
A	A	C
C	B	B

Winner: B

Election 3

3	2	6
B	C	A
A	A	C
C	B	B

Winner: B

   Which election(s) show that the voting method **violates MA**?

   A. Election 1

   B. Election 2

   C. Election 3

   D. Elections 1 and 2

   E. Elections 1 and 3

7. Which election(s) from problem 6 show that the voting method **always satisfies MA**?

   A. Election 1

   B. Election 2

   C. Election 3

   D. Elections 1 and 2

   E. None of the elections show that the voting method always satisfies the majority criterion.

8. Plurality with Elimination always satisfies which fairness criterion?

   A. MA

   B. CO

   C. MO

   D. IIA

   E. None of the above

9. Plurality can violate which fairness criterion?
   A. MA
   B. CO
   C. MO
   D. All of the above
   E. None of the above

10. Which voting method always satisfies CO?
   A. Plurality
   B. Borda Count
   C. Plurality with Elimination
   D. Pairwise Comparison
   E. None of the above

11. Which voting method can violate MO (monotonicity)?
   A. Plurality
   B. Borda Count
   C. Plurality with Elimination
   D. Pairwise Comparison
   E. None of the above

12. **Arrow's Theorem** says that _____ always satisfies all four fairness criteria.
   A. Plurality Method
   B. Borda
   C. Plurality with Elimination
   D. Pairwise Comparison
   E. No voting method

13. The following election below uses the *plurality method*. We will use this election to test the plurality method against one of the fairness criterion. Notice that **A is the original winner with plurality.**

10	7	11	9
D	B	A	C
C	A	D	B
B	C	B	A
A	D	C	D

We will remove candidates from the election to see if **A** will win in each modified election.

Which fairness criterion are we testing for?
   A. MA
   B. MO
   C. IIA
   D. CO
   E. None of the above.

14. Remove Candidate **B** from the election. Will **A** win or lose the modified election (you may want to draw the new preference schedule)? Is the test **valid or not**? Does this test show a **violation**, **satisfaction**, or an **inconclusive result**?

    A. A wins, and the test is inconclusive

    B. A loses, and the test shows a violation

    C. A wins, and the test shows satisfaction

    D. A loses, but the test is invalid

    E. A loses, and the test is inconclusive

15. Put **B** back and remove Candidate **A** from the election. Will **A** win or lose the modified election (you may want to draw the new preference schedule)? Is the test **valid or not**? Does this test show a **violation**, **satisfaction**, or an **inconclusive result**?

    A. A wins, and the test is inconclusive

    B. A loses, and the test shows a violation

    C. A wins, and the test shows satisfaction

    D. A loses, but the test is invalid

    E. A loses, and the test is inconclusive

16. Put **A** back and remove Candidate **C** from the election. Will **A** win or lose the modified election (you may want to draw the new preference schedule)? Is the test **valid or not**? Does this test show a **violation**, **satisfaction**, or an **inconclusive result**?

    A. A wins, and the test is inconclusive

    B. A loses, and the test shows a violation

    C. A wins, and the test shows satisfaction

    D. A loses, but the test is invalid

    E. A loses, and the test is inconclusive

Short answer

- There are 2 short answer questions. each short answer question may have multiple parts.
- Write your answer in the space provided next to each question.
- Partial credit may be given depending on the quality of work shown.

17. The following election uses an *unknown voting method*. We will use this **single** election to test the unknown method against the **CO criterion**.

26	28	3
C	A	D
D	D	B
A	B	C
B	C	A

(a) Fill in results for the **Pairwise Comparisons**. For each of the individual pairwise comparisons, determine:
how many voters were in each camp, and then
write the winner or "TIE" in the event of a tie.

A vs B: _____ to _____, _____ B vs C: _____ to _____, _____

A vs C: _____ to _____, _____ B vs D: _____ to _____, _____

A vs D: _____ to _____, _____ C vs D: _____ to _____, _____

(b) How many pairwise comparison points did each candidate get?

A's points: _____ B's points: _____ C's points : _____ D's points : _____

(c) Which candidate (if any) **won all** of their comparisons?

(d) If the unknown method declares **D** as the winner, does the election show a **violation**, **satisfaction**, or an **inconclusive result**?

(e) If the unknown method declares **B** as the winner, does the election show a **violation**, **satisfaction**, or an **inconclusive result**?

18. Use these clues to fill in the preference schedule. Each clue tells you what to put above a column. **Each clue refers to a different column.**

    (1) **10** voters turned in ballots that help **A** win with the **Plurality method** and during comparisons, these voters **never** went into **B's** camp.

    (2) **8** voters turned in ballots that give **A** last place votes. If an unknown voting method picks **A** as the winner and these ballots were **changed** to test with the **MO criterion**, they would read **A, B, C**.
    Note: Your answer goes above the original ballots, not the changed ones.

    (3) **6** voters turned in ballots that give **A** two Borda points per vote, and during testing with the **IIA** criterion, if **A** was **removed,** these ballots would give **B** two Borda points per vote instead.

    (4) The remaining three columns have **zeroes** at the top of them.

A	**A**	**B**	**B**	**C**	**C**
B	**C**	**C**	**A**	**A**	**B**
C	**B**	**A**	**C**	**B**	**A**

This is the end of the exam. Take any extra time you may have to go back and carefully check your work/answers.