Determining the Thermal Conductivity of Borosilicate Glass to Design a Better Aquarium Heater

Ashwan Kadam & Patrick Trunfio

A common sight in any home aquarium is a water heater. These devices are used to maintain the temperature of an aquarium constant for the health of the fish. Fish are cold blooded animals and cannot regulate their own body temperature, relying on the temperature of the surrounding water [1]. For most aquarium fish, their tank must be kept within a range of 15 °F. or they risk disease, infection, and even death [1]. To ensure that an aquarium is kept at the proper temperature, fish owners use water heaters that maintain a constant temperature or periodically turn on when the temperature falls below a set value.

These heaters are necessary to the survival of the fish, but can also pose a danger if poorly designed. The warmth given off by a heater often attracts the fish in a tank to it; the warmth encourages the growth of plants and algae. However, if the surface temperature of the heater is too high, this can cause burns to the fish's scales, potentially leading to life threatening infections. In this experiment, we propose a way to understand the heat transfer in an aquarium heater and extract the thermal conductivity to design a heater that will not burn a fish.

To do this, we must first understand the design of a typical water heater; Figure 1 shows the design of the Ebo-Jager 300 W Heater [2]. In this heater, as in most aquarium heaters, there is an internal heating element that supplies a constant temperature to the glass surrounding it. This in turn supplies heat to the water surrounding it through convection at its outer surface. The use of glass as the outer surface is preferred because it is inert and does not react with any of the chemicals commonly found in tanks, while metal heaters pose the risk of rusting or releasing poisonous materials into the water [1]. The specific material preferred is borosilicate glass made by adding silicon and boron oxides during the typical glass manufacturing process [3]. This results in a material with a very low coefficient of thermal expansion which prevents damage or cracking from heat shock occurring.

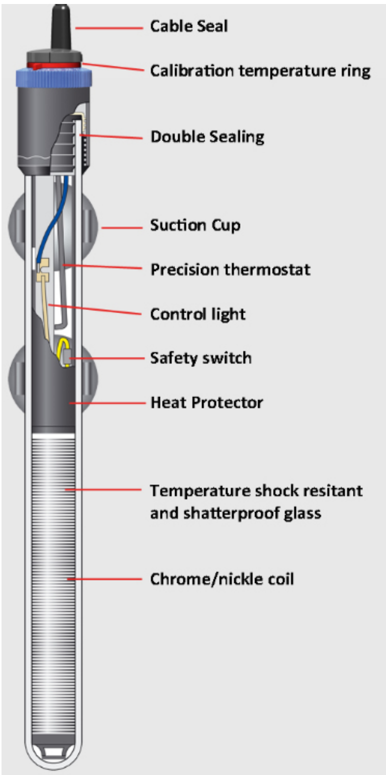

Figure 1: Design of an aquarium water heater [2]

Previous work has been done to determine the values of thermal properties of this material before [4], [5], however they suggest different properties. Bauccio [4] describes the behavior of the material as consistent for the temperature range 0-300 °C with a linear coefficient of thermal expansion, while [5] found nonlinear behavior over this same range. Due to these inconsistencies, we are looking to define the thermal properties of the material through our own experiment, focusing on a transient situation to see if the thermal properties noticeably change over the temperature range that aquarium heaters operate at. Determining these characteristics will also allow us to design an aquarium heater that can maintain a safe surface temperature for fish.

In order to evaluate this system, we must first develop a model to relate the different system parameters so that thermal characteristics can be extracted. To begin, we are considering the heater to be large in size compared to the thickness of glass, allowing the glass surface to be considered as a plane wall. We can then represent the system as follows; where $T_{\infty 1}$ and $T_{\infty 2}$ are the ambient temperature at each wall surface with T_1 and T_2 as the temperature at each surface.

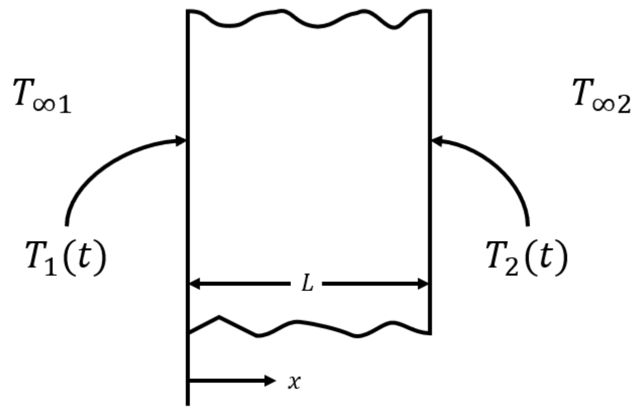

Figure 2: Physical model of heater system under analysis

Conduction is the only significant mechanism of heat transfer across a plane wall, and we assume that the temperature gradient across it is linear [6] giving the following relation for temperature over the length of the wall:

$$\frac{\partial^2 T}{\partial x^2} = 0 \qquad \text{Equation 1}$$

Integrating Equation 1 twice with respect to x and substituting the boundary conditions $T(0) = T_1$ and $T(L) = T_2$ gives:

$$T(x) = \frac{T_2 - T_1}{L} x + T_1 \qquad \text{Equation 2}$$

The energy balance of the wall is then:

$$E(t) = mc_p \frac{\int_0^L T(x)dx}{L} = \frac{mc_p}{2}(T_2(t) - T_1(t)) \qquad \text{Equation 3}$$

Where m is the mass of the wall and c_p is the specific heat of the material. For the situation being modeled, the initial temperature of the tank is equal to $T_{\infty 2}$. When the system is suddenly exposed to heat at surface 1 from the heating element, we assume the environment in the heater reaches a constant temperature $T_{\infty 1}$. Using the no slip condition [6], we can define: for $t > 0, T_1(t) = T_{\infty 1}$ making T_1 a constant. Taking the derivative of Equation 3 with respect to time then gives:

$$\frac{\partial E}{\partial t} = \frac{mc_p}{2}\frac{\partial T_2}{\partial t} = \frac{\rho A_s L c_p}{2}\frac{\partial T_2}{\partial t} \qquad \text{Equation 4}$$

Where ρ is the density of the glass and A_s is the surface area of the wall normal to the x-axis. In the scenario being considered, there is assumed to be no energy generation or transfer to or from the wall except for conduction through the wall, and convection at surface 2. Fourier's law defines heat transfer through conduction as:

$$\dot{Q}_{cond} = kA_s\frac{\Delta T}{L} = kA_s(T_2(t) - T_{\infty 1}) \qquad \text{Equation 5}$$

Where k is the thermal conductivity of the wall material. For the convection at surface 2, the heat transfer is defined by Newton's law of cooling.

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = hA_s(T_2(t) - T_{\infty 2}) \qquad \text{Equation 6}$$

Equation 5 can then be combined with Equation 6 and substituted into Equation 4 as the $\frac{\partial E}{\partial t}$ term giving:

$$-(kA_s(T_2(t) - T_{\infty 1}) \ + hA_s(T_2(t) - T_{\infty 2})) = \frac{\rho A_s L c_p}{2}\frac{\partial T_2}{\partial t} \qquad \text{Equation 7}$$

Equation 7 can be arranged into the following first order model:

$$\frac{kT_{\infty 1} + hLT_{\infty 2}}{k + hL} = \frac{\rho c_p L^2}{2(k + hL)}\frac{\partial T_2}{\partial t} + T_2 \qquad \text{Equation 8}$$

The coefficient to the differential term in Equation 8 can be replaced by a time constant $\boldsymbol{\tau}$, that describes the time response of the system regardless of input.

$$\tau = \frac{\rho c_p L^2}{2(k + hL)} \qquad \text{Equation 9}$$

The solution to the model in Equation 8 is as follows:

$$T_2(t) = \frac{kT_{\infty 1} + k(T_{\infty 2} - T_{\infty 1})e^{-t/\tau} + hLT_{\infty 2}}{k + hL} \qquad \text{Equation 10}$$

The validity of this solution and the model it was derived from will be evaluated by comparing it to the actual response of a system similar to the one shown in Figure 2.

Research Objective

The objective of this experiment is to determine the minimum thickness of borosilicate glass necessary for safe heat transfer in an aquarium heater. A first order model of the transient heat conduction for borosilicate glass will be tested to validate its accuracy and to determine the thermal conductivity and surface temperature of the material at a specific thickness. Those will be used to calculate the surface temperature of glass for different thicknesses to determine the length necessary for optimal safety.

Research Plan

To experimentally determine the thermal conductivity of the glass, it will be subjected to a constant temperature at one surface using a steam chamber while the change in temperature of the opposite surface is measured. The following equipment will be necessary to complete the experiment:

- Borosilicate Glass Plate, 1 cm thick
- Surface thermocouple, Type K
- Arduino board and code to process T/C data
- Pot of water
- Heat source: stove or hot plate
- Steam chamber with removable lid
- Rubber hose
- Condensation reservoir such as a beaker

The setup of these components is shown below in Figure 3.

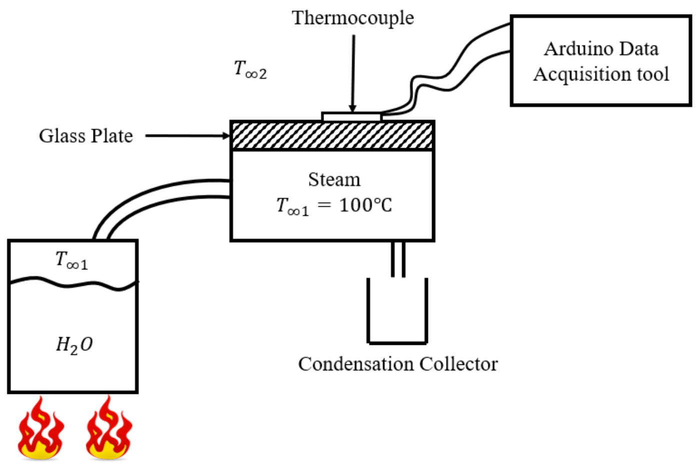

Figure 3: Experimental setup to be used

The heat source that will be used is the steam generated from the boiling water connected to the steam chamber below the glass plate. To conduct the experiment, the glass plate will originally be replaced with a metal plate as the steam chamber heats up until it is at a sufficiently steady state. Before being placed over the steam chamber, the glass plate will be at room temperature, $T_{\infty 2}$. The temperature of this will be measured by the thermocouple, then with data recording still on, the glass plate will quickly be substituted for the metal plate. Both surfaces of the plate are assumed to be isothermal so the thermocouple can be placed anywhere on the top surface.

With regards to the data acquisition, the thermocouple will connect to an Arduino Uno, which will be utilizing Parallax to continuously record temperature data until the user stops the program. The temperature data collected will be plotted against time to demonstrate the change in the free surface temperature as a result of conduction. From this data the value of τ, the system time constant can be determined and subsequently the value of k. From Equation 10, it can be seen that when $t = \tau$, $T_2(\tau) = 0.63\Delta T$. From the initial value and steady state temperatures, ΔT, the total change in temperature of the free surface, and then $T_2(\tau)$ can be determined. The time at which this temperature occurs will be the experimental value of τ for the thermal system.

Using Equation 9, the thermal conductivity, k, can be determined. The remaining variables in the equation will substituted in using known system properties. The specific heat of borosilicate glass, c_p, and heat transfer coefficient, h, for air at 1 atm and $T_{\infty 2}$ can be found in reference books such as [6], and [4]. The remaining quantities ρ and L will be determined by measuring the dimensions of the plate to determine its volume and length, along with its mass found using a scale. After all these values are found, k, can be found. The uncertainty of variables will be determined in 2 ways. For non-experimental values, the uncertainty will be either the last significant figure of the given value or the standard deviation. Experimental values will have their uncertainty as the resolution or standard deviation of the measurements. To estimate the response of the system, the values shown in Table 1 will be used with the model in Equation 8.

Table 1: Values used in simulation of results retrieved from [6]

Quantity	**Value**	**Projected Uncertainty**
L [cm]	1	± 0.01
h [W/m^2-K]	26	± 0.5
ρ [kg/m^3]	2225	± 0.5
c_p [J/kg]	835	± 0.5
k [W/m-K]	1.1	± 0.05
$T_{\infty 2}$[°C]	25	± 0.25

These values were input into a simulation of the system to show its response. That plot is in Figure 4 below.

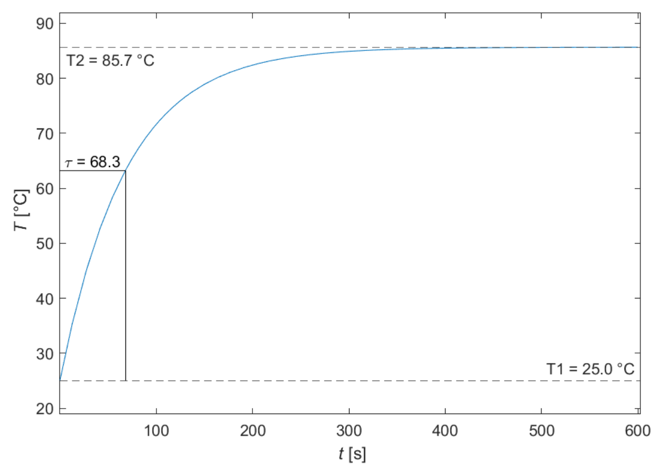

Figure 4: Expected experimental results with τ identified

For the experimental situation, τ will be the known value, and the system will be solved for k. That value will be compared to values determined in other experiments such as the example in Table 1. If the experiment also demonstrates that the model of system behavior is accurate, the experimental τ and k can be used to determine the necessary glass thickness for an aquarium water heater using Equation 10 where $T_2(t)$ will be the desired surface temperature to avoid harming fish, $T_{\infty 1}$ will be the temperature of the internal heating element, $T_{\infty 2}$ will be the water temperature, and h will be the heat transfer coefficient for water at $T_{\infty 2}$. With these known quantities we will be able to predict the behavior of the glass surface for numerous temperatures, fluid conditions, and thicknesses without requiring additional experimental data.

Bibliography

[1] C. W. Emmens, Keeping and Breeding Aquarium Fishes, Academic Press, 1953.

[2] Fish Tanks Direct, "Ebo Jager 300 Watt Aquarium Heater," 2020. [Online]. Available: https://www.fishtanksdirect.com/ebo-jager-300-watt-aquarium-heater-3619090.aspx. [Accessed 8 April 2021].

[3] L. Pedesseau, S. Ispas and W. Kob, "First-principles study of a sodium borosilicate glass-former. I. The liquid state," *Phys. Rev. B,* vol. 91, no. 13, 14 April 2015.

[4] M. Bauccio, Ed., ASM Engineered Materials Reference Book, 2 ed., ASM International, 1994.

[5] N. Bouras, M. A. Madjoubi, M. Kolli, S. Benterki and M. Hamidouche, "Thermal and Mechanical Characterization of Borosilicate Glass," *Physics Procedia,* vol. 2, no. 3, pp. 1135-1140, 2009.

[6] Y. A. Cengel and A. J. Ghajar, Heat and Mass Transfer, Fundamentals & Applications, 5 ed., McGraw-Hill Education, 2015.

Appendix

```
%% Define System Parameters
Ts = 100;
Ta = 25;
k = 1.1;
h = 26;
Lvec = 0.01;
dens = 2225 ;
cp = 835;
tau = dens*cp*Lvec^2/(2*(k+h*Lvec));
L = Lvec(1);
%% Define system model and solve
syms T2(t)
ode = Ts*k/L + h*Ta == dens*cp*L/2*diff(T2,t) + (k/L + h)*T2 ;
cond = T2(0) == Ta;
sol = dsolve(ode,cond);
%% Plot system and annotate points of interest
fplot(sol, [0,600])
hold on
plot([0,tau],[0.63*(85.66-Ta)+Ta,Ta + 0.63*(85.66-Ta)],'k')
plot([tau,tau],[0.63*(85.66-Ta)+Ta,Ta],'k')
xlabel('{\it t} [s]')
ylabel('{\it T} [°C]')
text(5,65,sprintf('\\tau = %.1f',tau))
yline(Ta,'--',sprintf('T1 = %.1f °C', Ta),'LabelVerticalAlignment','top')
yline(85.66,'--',sprintf('T2 = %0.1f °C',85.66),'LabelHorizontalAlignment'
,'left','LabelVerticalAlignment','bottom')
```