AME 408 Final Project
Centrifugal Governor

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Group #11
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Design

This design study looks at the optimization of a highly repeatable centrifugal governor. The governor is attached to a solid shaft and rotates at very high velocities within a cylindrical housing. Figure 1 below shows the overall shape of the part.

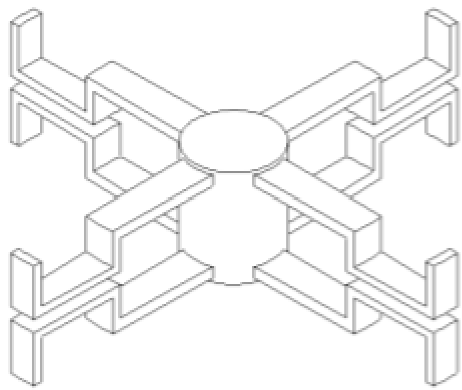


Figure 1: General overview of the governor's design

For this study, we attempted to optimize the design of the governor such that it met the following requirements: the governor remained within its housing at speed up to 6000 rad/s, the maximum radial displacement of the governor is less than 2.3 mm, the maximum allowable stress of the material is not exceeded at any point on the part. The governor is manufactured from MIRAGE 250 CVM, a hardened steel alloy with the material properties shown in the table below.

Table 1: MIRAGE 250 CVM Properties

Property	**Value**
Modulus of Elasticity, E	205 [GPa]
Poisson's Ratio, v	.3
Mass Density, ρ	7850 [kg/m^3]
Maximum Allowable Stress, σ_a	1750 [MPa]

To optimize the part, 5 parameters (L_1, L_2, W, t, & d) were varied until the optimal performance was identified. This included making the legs as long, wide, and thin as possible while still following the requirements. An additional stipulation was placed on the values for L_1 and L_2 by the design criteria of the part. The value for L_1 must fall between 1.4*L_2 and 1.8*L_2. These parameters and their corresponding dimension are shown in Figure 2 below.

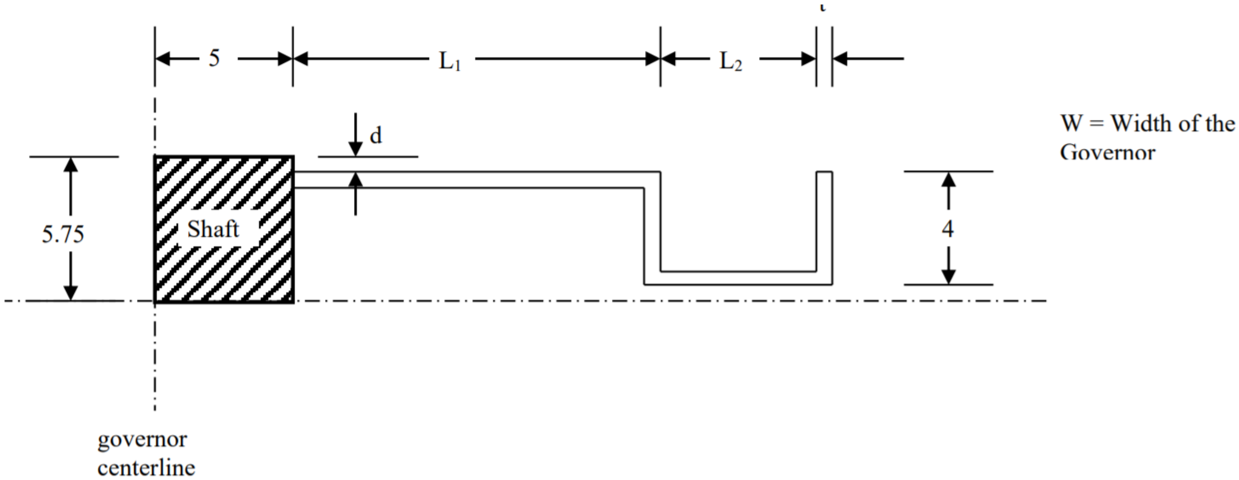
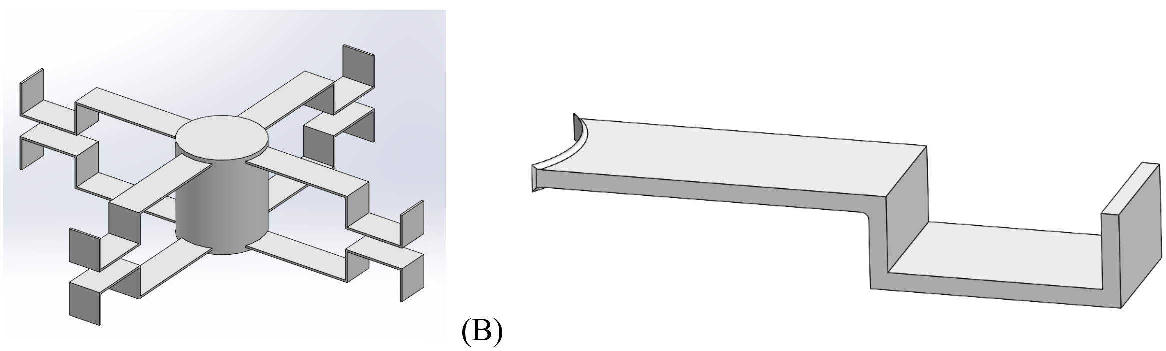


Figure 2: Parameters to be varied as shown on a single leg of the governor. *W* as described is the width of each leg coming out from the drawing.

To prepare for simulations, the governor was first fully modelled in SolidWorks. Following this, different configurations of the part were developed to prepare for simulation (Figure 3A). To evaluate the radial displacement and stress in the governor, CosmoWorks 3-D Solid Static simulations were used. Due to the symmetry of the governor we were able to reduce the computational strain of the simulations by reducing the part to only a single leg (Figure 3B). In addition, we were able to remove the shaft from the part for simulations as well due to the assumed rigidity of the shaft.



(A) (B)

Figure 3: The modelling of the part for simulation. (A) shows the fully modelled governor, while (B) shows the simplified model used for static simulations

For the static simulations of the governor, several constraints and loads were applied to accurately simulate the behavior of the part. With the simplified leg model, the inside face connected to the shaft was fully constrained given that the shaft was assumed to be rigid. For loading, a single centrifugal load was applied with a velocity of 6000 rad/s and no acceleration. These constraints and loads can be seen in Figure 4 below.

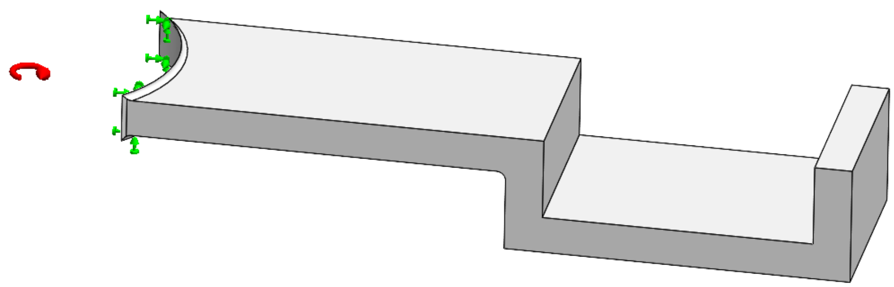


Figure 4: Fixtures and Loading applied to the part for static simulations. The fixtures can be seen in green, while the centrifugal loading can be seen as the red arrow.

With these constraints and loads in place the axial and vertical displacements were observed to ensure that the governor did not expand beyond its housing. The maximum principle stress was also noted each trial to confirm that it did not exceed the maximum allowable stress of the material. In addition, as a requirement, the design of the governor was such that the maximum stress did not occur at the connection point between the leg and the governor.

For frequency simulations, the entire part had to be simulated, and simplification could not be used as an incorrect result would occur. To set up the simulations constraints and loadings, the part was constrained radially as the shaft is treated as a rigid body. Furthermore, the top and bottom faces of the shaft were constrained in the normal direction for no vertical movement. The centrifugal force was applied as well, with in-plane loading effect neglected. Convergence for the frequency values were performed and discussed later in the report. These constraints are shown in Figure 5 below along with an example mesh size of 1 mm.

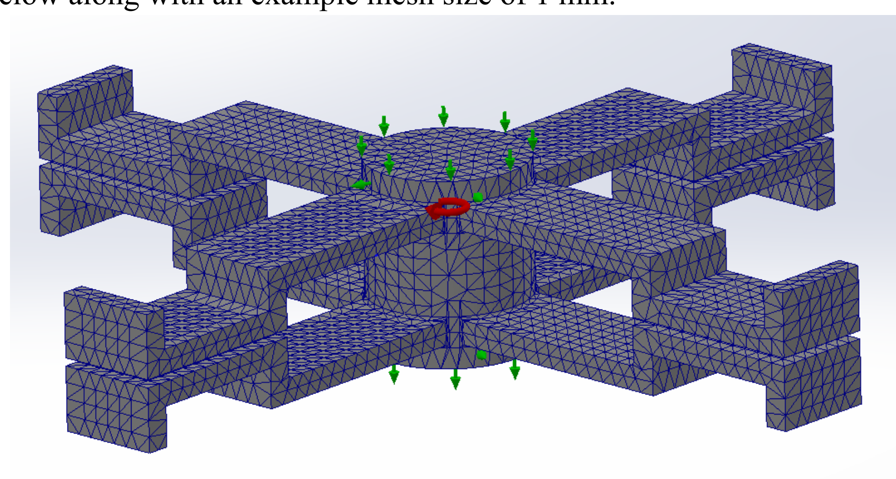


Figure 5: Fixtures and loading applied to the governor for frequency simulations.

Results

The first step in optimizing the part was identifying a global mesh size that would provide reliable simulation results without placing undue strain on the computational resources available to complete simulations. To do this, initial simulations on the part, tracking stress and radial displacement were done until convergence was observed. The initial part used for these simulations had the following values for each parameter.

Table 2: Part parameters used in initial simulations

Parameter	**Value [mm]**
L_1	15.75
L_2	9
t	0.25
W	4
d	0.5

The convergence simulations were first attempted without any fillets on the component and produced the following results:

Table 3: Initial Mesh Sizing (No Fillets)

Study #	**Global Mesh [mm]**	**Radial Displacement of Edge [mm]**		**Max V-M Stress [MPa]**	
1	0.25	20.76	N/A	9.952E+03	N/A
2	0.125	20.82	0.29%	1.212E+04	17.89%
3	0.0625	20.84	0.10%	1.246E+04	2.73%

From Table 3 and visual inspection of the stress in the part, it was apparent stress concentrations were occurring at the point the legs meet the shaft as well as the first bend in the leg at the end of L_1. To prevent this from occurring, the convergence table was repeated, but with the maximum allowable fillets placed at these two vertices. This entailed a 0.2 mm fillet at each edge between the shaft and leg, as well as a 0.3 mm fillet on the first bend of the leg. With these two alterations made, mesh sizing was repeated and the results can be seen in Table 4 below.

Table 4: Mesh Sizing with Fillets

Study #	**Global Mesh [mm]**	**Radial Displacement of Edge [mm]**		**Max V-M Stress [MPa]**	
1	0.25	20.25	N/A	9.622E+03	N/A
2	0.125	20.32	0.34%	1.094E+04	12.05%
3	0.0625	20.32	0.00%	1.168E+04	6.34%

With the alterations made to the part, a single stress concentration was still observed on the governor, however the convergence in radial displacement was more pronounced, with no difference at all between mesh sizes of 0.125 and 0.0625 mm. The single stress concentration would not converge even with further mesh refinement unless geometric changes were made to the part, so the lack of convergence was temporarily ignored. Due to the identical values in displacement across the different mesh sizes, it was decided to utilize a larger mesh size for faster simulations. As the thickness for this part was 0.5 mm, it was decided for all future simulations to use a mesh size of $t/2$.

To approach the optimization of the governor's parameters, the following approach was used. Since the criteria of the part required

a. The centrifugal governor to fit within the annular space at all times
    i. The total radial length to be kept under 30mm
    ii. d must always be greater than the total vertical displacement.
b. The maximum von Mises stress to be kept under 1750 MPa
    i. And does not occur at the shaft
c. The fundamental frequency to be kept under 2000 Hz
d. Making the part as long, wide, and thin as possible

The process involved starting with an arbitrary thickness of 0.5mm and finding a set of lengths which kept the total deformed axial length under the housing size of 30mm. When attempts to increase the length to achieve the longest leg possible failed, the thickness was increased to allow for larger L_1 and L_2 values. With L_2 increased, L_1 had a larger range in which it could be altered. The thickness had to be further increased in order to reduce vertical displacement of the leg as well as to reduce the maximum overall stress on the part. This increase in thickness allowed for larger L_1 and L_2 values while still meeting requirements. The position of the leg on the shaft, d, was increased in order to allow the leg more room for vertical displacement in order for it to remain within the housing. Lastly, the width of the leg, which was found to have marginal effect on displacement was increased to a maximum to satisfy criteria a.

Using the previously described approach, the following results were obtained. Trial 24 produced the optimal governor as identified by our simulations.

Table 5: Optimization Trials

Trial	L_1 [mm]	L_2 [mm]	t [mm]	W [mm]	d [mm]	Radial Disp [mm]	Total Length [mm]	Y Disp [mm]	Height Outside Housing [mm]	Max Stress [MPa]
1	9	5	0.5	5	0.5	1.16	20.66			
2	12.5	7	0.5	5	0.5	2.554	27.554			
3	10	7	0.5	5	0.5	1.8	24.3			
4	15.25	9	0.5	5	0.5	4.335	34.085			
5	12.75	9	0.5	5	0.5	3.267	30.517			
6	12.75	9	0.75	5	0.5	1.301	28.801			
7	12.75	9	0.75	5	1	1.301	28.801			
8	14	9	0.75	5	0.5	1.508	30.258			
9	13.75	9	0.75	5	0.5	1.465	29.965			
10	13.75	9	0.75	6	0.5	1.461	29.961			
11	13.75	9	0.5	6	0.5	3.667	31.917			
12	12.75	9	0.5	6	0.5	3.257	30.507			
13	13.75	9	0.5	6	0.5	3.667	31.917			
14	13.75	9	0.75	6	0.5	1.461	29.961			
15	13.75	9	0.75	6	0.5	1.463	29.963			
16	13.75	9	0.75	6	1	1.456	29.956	6.488	5.488	
17	13.75	9	0.75	6	1		28.5			2397
18	13.75	9	1	6	1	0.715	29.465	3.326	2.326	
19	13.75	9	1.25	6	1	0.4028	29.4028	1.96	0.96	1941
20	12.75	9	1.25	6.5	1.5	0.325	28.325	1.485	-0.015	
21	13	9	1.25	6.5	1.5	0.367	28.617	1.745	0.245	
22	14.25	9	1.5	6.5	1.5	0.2589	30.0089	1.337	-0.163	1438
23	13.25	9	1.25	6.5	1.65	0.3786	28.8786	1.814	0.164	1674
24	14	9	1.5	6.5	1.5	0.2517	29.7517	1.289	-0.211	1530

To confirm the accuracy of the final simulation results, the iterative mesh sizes were used using the final dimensions to confirm convergence and demonstrate the validity of our results. The results of this can be seen in Table 6 below:

Table 6: Convergence at Final Dimensions for Trial 20

Study #	Global Mesh [mm]	Radial Displacement of Edge [mm]		Vertical Displacement [mm]		Max V-M Stress [MPa]	
1	0.625	0.3550	N/A	1.674	N/A	1,555.00	N/A
2	0.3	0.3559	0.25%	1.676	0.12%	1,862.00	16.49%
3	0.15	0.3562	0.08%	1.678	0.12%	1,970.00	5.48%

After finer mesh simulations the max von Mises stress converged to a value greater than the max allowable stress, thus the part had to be further configured to reduce stress. This was done by increasing the thickness by 0.25mm, allowing for an increase in L_1 due to a reduction in the radial displacement.

Thus, the next part that satisfied the requirements during initial simulations was Trial 24. Once again, to confirm the accuracy for the final results, the mesh sizes were reduced for several simulations to confirm the convergence of these values in Table 7 below. For this analysis, the fundamental frequency of the part was also determined to ensure convergence to a value below 2000 Hz. The results can be seen in Table 8 below:

Table 7: Convergence of Final Dimensions for Trial 24

Study #	Global Mesh [mm]	Radial Disp of Edge [mm]		Vertical Disp [mm]		Max V-M Stress [MPa]	
1	0.5	0.2513	N/A	1.288	N/A	1,337.00	N/A
2	0.25	0.2516	0.12%	1.289	0.08%	1,617.00	17.32%
3	0.15	0.2517	0.04%	1.289	0.00%	1,661.00	2.65%

Table 8: Convergence of the fundamental frequency for Trial 24

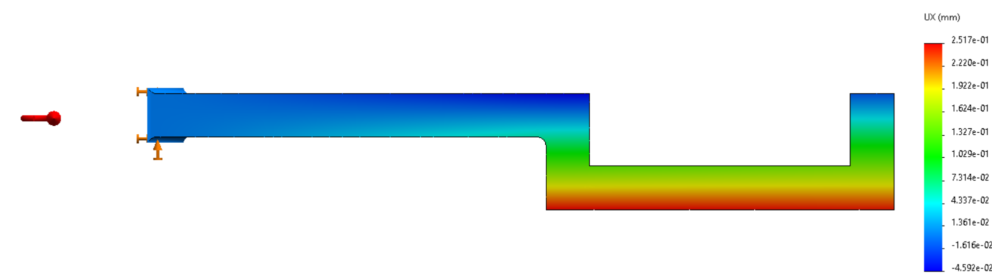
Mesh Size (mm)	Natural Frequency (Hz)	
1	2001.6	N/A
0.75	1997.7	0.20%
0.5	1994.2	0.18%

It was determined that the maximum radial displacement calculated during all simulations (in Table 5) was the distance from the center of the shaft to the midpoint of the furthest displaced edge of the leg. However, since the governor leg is in an annular space and the shape of the leg is rectangular the first points on the leg to "hit" the outer bounds would be the corners of the leg (rather than the midpoint). Therefore the distance from the center of the shaft to the corner of the furthest displaced edge of the leg would also be required to remain under 30mm, this was calculated using total displaced length and width with Pythagorean's Theorem.

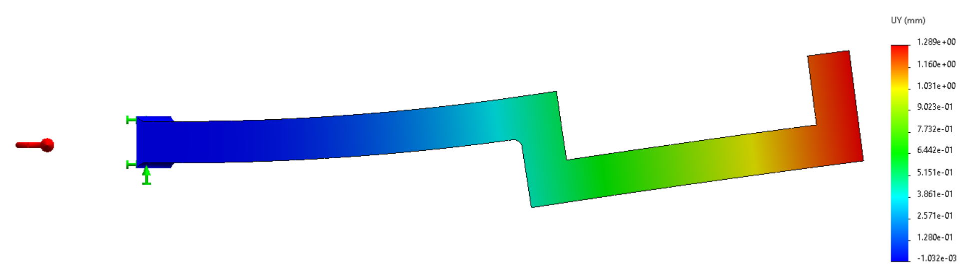
Therefore, the tabular values were then compared to each of the analysis criteria in which

a. The governor fit in the housing at all times
    i. The maximum radial length 29.929 ± 0.001 mm is less than 30mm
    ii. The maximum vertical displacement 1.289 ± 0.001 mm is less than $d = 1.5$ mm
b. The maximum von Mises stress 1661 ± 4 MPa is less than the allowable 1750 MPa
c. The fundamental frequency 1994.2 ± 0.4 Hz is less than 2000 Hz.
d. All while keeping the part as long, wide, and thin as possible
    i. Any longer would violate criteria a.
    ii. Any thinner would violate criteria b.

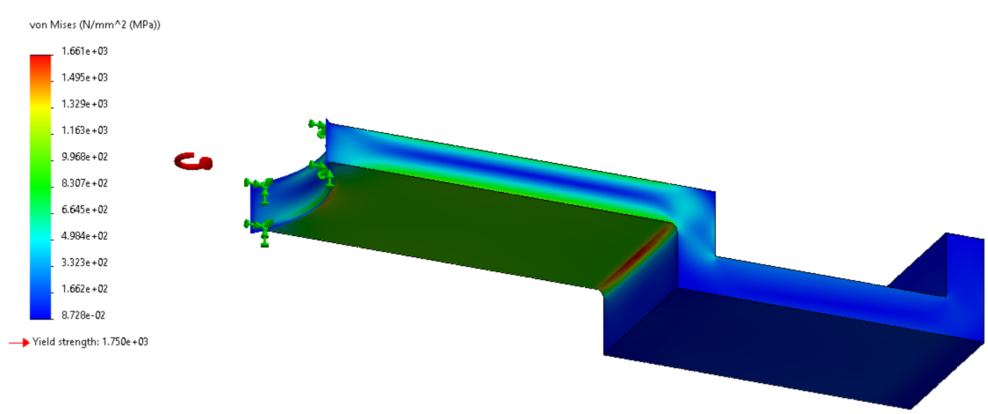
The results of the final simulation for the parameters selected can be seen in the figures below.



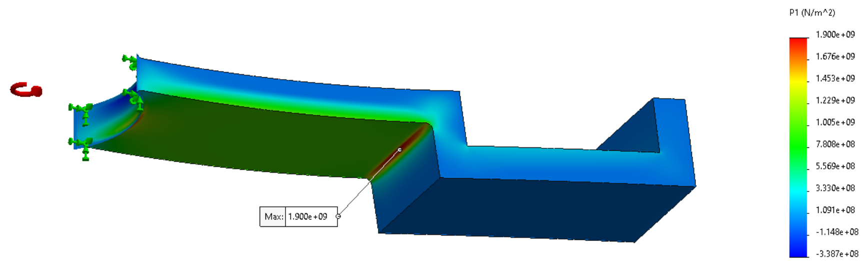
(A)



(B)



(C)



(D)

Figure 6: Results of Static Simulations. (A) shows the radial displacement. (B) shows residual displacement. (C) shows the Von Mises Stress. (D) shows the first principal stress

For trial 24, the first non-rigid body fundamental frequency result found was in the second amplitude plot, as the animation displayed bending in the arms. The first amplitude plot did not have any part of the solid body deform. Therefore it was not considered. The result of the latest frequency simulation is shown below in Figure 7

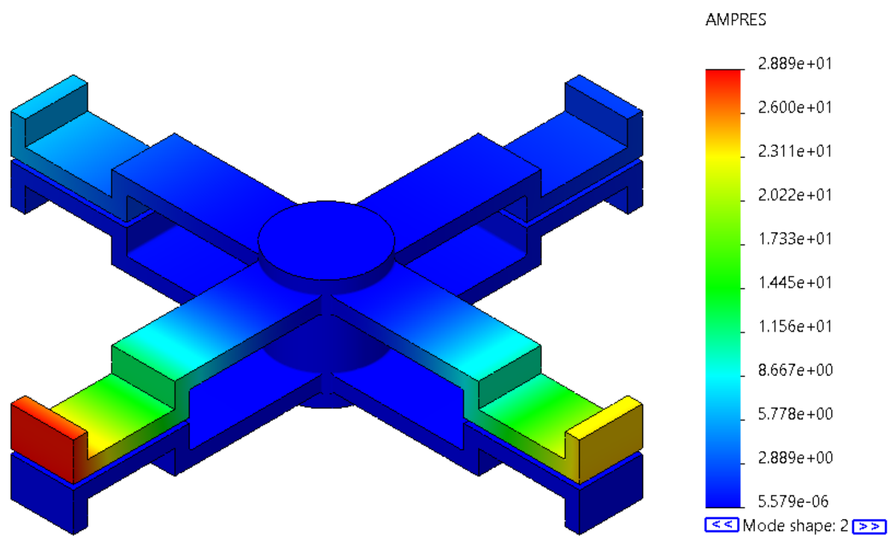


Figure 7: Results of Frequency Simulations

From the static simulations completed we were also able to generate a plot of the radial displacement of the governor as a function of angular velocity. This plot can be seen in Figure 8.

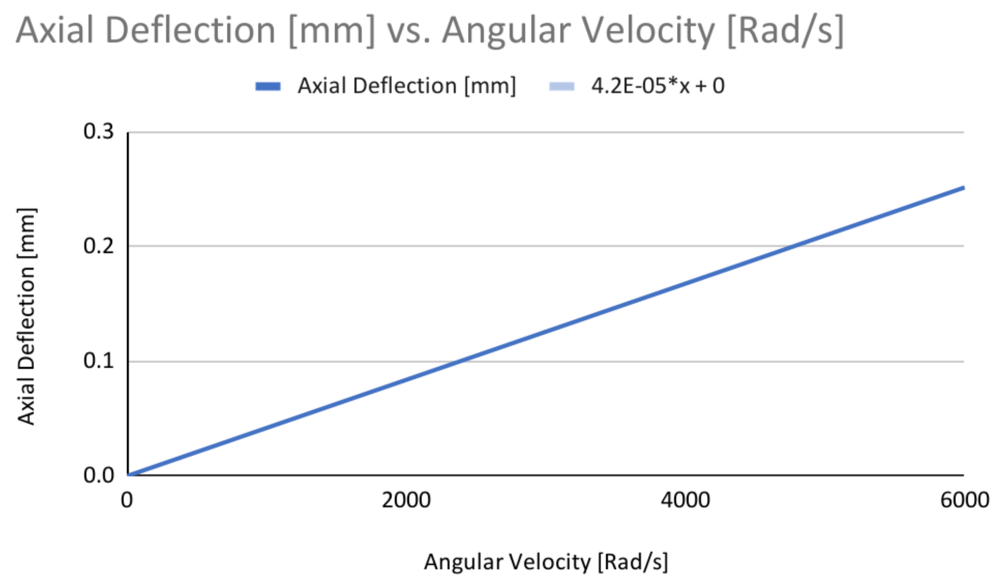


Figure 8: Radial Displacement vs Angular Velocity

Final Dimensions

The table below shows the final dimensions and qualities of the governor and its simulation results.

Table 9: Final dimensions of the Governor

Quantity	Value
L_1	14 [mm]
L_2	9 [mm]
t	1.5 [mm]
W	6.5 [mm]
d	1.5 [mm]
Mass, m	24.5 [grams]
Fundamental Frequency	1997 [hz]
Maximum Von Mises Stress, σ_{VM}	1661 [MPa]
Maximum Radial Displacement	0.2517 [mm]
Maximum Vertical Displacement	1.289 [mm]
Final Static Mesh Size	.15 [mm]
Final Frequency Mesh Size	0.5 [mm]

These values meet all of the design criteria and were found to be the longest, thinnest, and most wide the leg could be in doing so.

Conclusion

While there were many different ways to attack this problem and various correct solutions, it came down to which of the design criteria were prioritized over others. A leg which prioritized length, while its thickness would be the smallest to meet the design requirements, would still be larger than a leg design which prioritized being as thin as possible; which would in turn have to be shorter than the first design. Ultimately, a good balance was achieved which pushed the governor to the limits of its boundary while still keeping the stress under the maximum allowable as well as keeping it away from the joint between the leg and the shaft.