1. Exercise Sheet
Advanced Automatic Speech Recognition

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Exercise 1

a)

We know that the following equation holds

$$\sum_{N=1}^{\infty} p(N) = 1 \tag{1}$$

Knowing that we have a bigram language model, we can derive:

$$\sum_{N=1}^{\infty} p(N) \sum_{w_1^N} p(w_1^N) = \sum_{N=1}^{\infty} p(N) \sum_{w_1^N} \prod_{i=1}^{N} p(w_i|w_{i-1})$$

$$(sum\ over\ all\ words\ in\ vocab = \sum_{N=1}^{\infty} p(N) \prod_{i=1}^{N} \sum_{w_i} p(w_i|w_{i-1})$$

$$= \sum_{N=1}^{\infty} p(N) \prod_{i=1}^{N} 1$$

$$= \sum_{N=1}^{\infty} p(N)$$

$$(using\ equation\ 1)\ = 1$$

b)

i)

The sentence end token on language model allows for language model normalization on word level independent of the length of the sentence. Without the sentence end token the following must always be true:

$$\prod_{i=1}^{N} w_i = (\prod_{i=1}^{N-1} w_i)w_N \leq \prod_{i=1}^{N-1} w_i$$

The above equation logically holds, but having a corpus with a finite amount of sentences, it might very well be the case, that a sentence containing a subsentence is more likely to occur as a full sentence than the subsentence as a full sentence.

Therefore, the sentence end model makes it possible to include sentence ends in the model and in doing so also ensures language model normalization on word level

$$\sum_{w \in \mathbb{V} \cup \{\$\}} p(w|v) = 1, \forall v \in \mathbb{V}$$

ii)

$$\sum_{N=1}^{\infty} \sum_{w_1^N : w_N = \$, w_n \in \mathbb{V}, \forall n=1,...,N-1} p(w_1^N) = \sum_{N=1}^{\infty} \sum_{w_1^N : w_N = \$, w_n \in \mathbb{V}, \forall n=1,...,N-1} \prod_{i=1}^{N} p(w_i|w_{i-1})$$

$$= \sum_{N=1}^{\infty} \prod_{i=1}^{N-1} (\sum_{w_i} p(w_i|w_{i-1})) p(\$|w_{i-1}))$$

$$= \sum_{N=1}^{\infty} \prod_{i=1}^{N-1} (\sum_{w_i} p(w_i|w_{i-1})) p(\$)$$

$$(using\ assumption) = \sum_{N=1}^{\infty} \prod_{i=1}^{N-1} (1 - p(\$)) p(\$)$$

$$= \sum_{N=1}^{\infty} (1 - p(\$))^{N-1} p(\$)$$

$$(using\ geometric\ series) = \frac{1}{1 - (1 - p(\$))} p(\$)$$

$$= 1$$

iii)

We can imply from ii) that

$$p(N) = \sum_{w_1^N : w_N = \$, w_n \in \mathbb{V}, \forall n=1,...,N-1} \prod_{i=1}^{N} p(w_i|w_{i-1}) = (1 - p(\$))^{N-1} p(\$)$$

Exercise 2