Macroeconomic Stress Testing: A Household Survey Data Simulation

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Abstract

Macroeconomic stress test is a risk management technique used by financial market regulator to evaluate the potential effects on an institution's financial condition, of a set of specified changes in risk factors, corresponding to severe but plausible economic conditions. It plays an essential role in financial market risk management and supervision. This paper extends this technique to the perspective of households in the economy. It develops a stress testing framework of agent-based micro-founded simulation bounded with macro-consistency to evaluate household's financial security. This framework is then applied as to investigate Canadian households based on the recent 2023 Macroeconomic Stress Testing (MST) scenario conducted by the national regulator, the Office of the Superintendent of Financial Institutions (OSFI). The stress testing result is consistent with the statement about Canadian households in Bank of Canada's 2023 Financial System Review. Moreover, we discover household's income distribution has significant impact on stress testing results.

1 Introduction

Macroeconomic stress test (MST) is a risk management technique used by financial market regulator. It is used to evaluate the potential effects on an institution's financial condition at scenario that corresponds to severe but plausible economic conditions. The exercise result is among essential inputs for regulator to decide financial market policy and regulations. In Canada, the national regulator, the Office of the Superintendent of Financial Institutions (OSFI), conducts MST once every two years, with the most recent exercise conducted by the Canadian Domestic Systemically Important Banks (D-SIBs) at 2023.

On one hand, at the opening in Bank of Canada's 2023 Financial System Review [4], it states what our economy stands in a post-pandemic world:

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†Disclaimer: All content on this paper is my own and does not necessarily reflect the views of Canadian Imperial Bank of Commerce (CIBC) or its affiliates. This work is based on Statistics Canada's Survey of Financial Security Public Use Microdata, 2019, which contains anonymous data collected in the Survey of Financial Security. All computations based on these microdata are prepared by the author. The responsibility for the use and interpretation of these data is entirely that of the author.

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”Over the past year, financial conditions have tightened globally in response to monetary policy actions aimed at reducing inflation. Recent stresses in the banking sector in the United States and Switzerland further tightened financial conditions. ... These events have exposed vulnerabilities — notably, business models that rely excessively on an environment of low interest rates and low volatility — and serve as a reminder that risks can emerge and spread quickly. As the financial sector adjusts to higher interest rates, participants, regulators and central banks must be more vigilant about vulnerabilities and risks.”

In other words, our economy is experiencing systemic structural break. In particular, Figure 1 shows real GDP and unemployment rate are shocked by the pandemic, while Figure 2 shows the high inflation along with spiking mortgage loan rate in post-pandemic periods.[1] It is not surprising that in recent years the national regulator pays much attention to the scenario of economic downturn connecting with spiking interest rate and persistent high inflation.

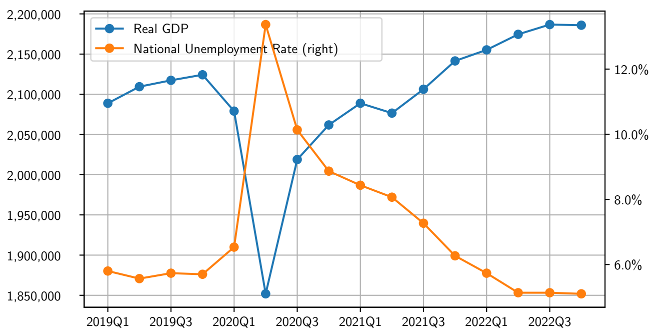


Figure 1: Canada real GDP (at market prices $M) and unemployment rate.

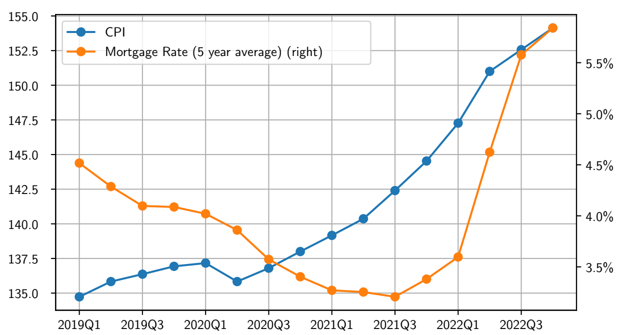


Figure 2: Canada CPI and mortgage loan rate.

[1] If without further clarification, the economic data are sourced from Statistics Canada. We recognize that Statistics Canada updates historical data indefinitely. The data used in this work is the version downloaded at July 2023.

On the other hand, the current practice in financial market risk management and supervision relies on the concept of financial institution's risk-based capital ratio:

$$\text{Risk-Based Capital Ratio} = \frac{\text{Capital}}{\text{Risk Weighted Assets}},$$

where financial institution invests substantial effort on how to effectively model the denominator, risk weighted assets (RWA).

We can breakdown RWA into three major components: credit risk, operational risk, and market risk, where credit risk is the dominant component of RWA.

Credit risk modelling starts from portfolio segmentation where we categorize by the nature of products. For example, we consider by retail portfolios such as mortgages and credit cards, while by non-retail portfolios such as corporate lending and commercial real-estate. Then we develop for each portfolio risk models that map systemic inputs such as real GDP, unemployment rate, and idiosyncratic inputs such as client's credit score, risk rating, into the risk components of Probability of Default (PD %), Loss Given Default (LGD %), Exposure At Default (EAD $).

Implicitly, we presume:

1. Credit risk is segregated in the universe of portfolios: The risk from one portfolio has no cascade effects to others.

2. Historical performance is predictive of future: Past relationships and sensitivities among risk variables will continue.

These presumptions seem to work properly when the economy is tranquil. However, when the economy is volatile, these presumptions should not be taken for granted. What our economy is experiencing in systemic structural break inevitably triggers a challenge on these presumptions and thus to current practice of credit risk modelling and regulation.

This challenge is especially vital for retail portfolios, as they are inter-connected at the level of clients/households. It would be problematic to presume that the default of one client's credit card would not affect the likelihood of default to the same client's mortgage debt. In the scenario of severe economic downturn with spiking interest rate and persistent high inflation, this inter-connection would potentially result in cascade effect of defaults among retail portfolios. Arguably, this is one of the reasons why in recent years the national regulator has been requesting D-SIBs to establish credit risk retail challenger models as alternative to the existing risk framework.

In response to this request, one can consult with two potential options. The first option is to model the inter-connection mechanically. For example, one can quantify the cross-correlation matrix among retail portfolios. This can be done by using internal and/or external risk data. This option has the advantage of relatively easy implementation and deployment to the existing risk framework. But since this option relies on historical data fitting, in the scenario with systemic structural break, historical relationships might not continue mechanically. This would substantially undermine the effectiveness of this option, which is worth noting when applying to risk supervision. For this reason, we do not consider this option in the first place.

The second option is to model from 'bottom-up'. For example, one can model at the level of individual account/client/household, explicitly specifying its state and dynamics. Two branches along this direction have emerged in existing literature.

The first branch emphasizes on the perspective of stress testing. A typical work is the Household Risk Assessment Model (HRAM) [5] developed by Bank of Canada.[2] It is a stress test framework of macro-consistent micro-simulation. It models the heterogeneous micro-level households with explicit balance sheets. The dynamics of household is by the evolution of asset and debt items in its balance sheet. Household is bounded by macro-level constraints. For example, household's income or cash flow is bounded by the constraint carried from macroeconomic employment status such as unemployment rate and unemployment duration.

Notice that the existing work of HRAM is still steps away from readiness in deployment as alternative retail challenger model, with the following reasons. First, it calibrates the model parameters by utilizing proprietary data source such as Canadian Financial Monitor survey data compiled by Ipsos Reid that is not publicly available. This non-transparency makes it difficult to adapt. Second, the granularity on household's debt items needs to improve, as it considers mortgage and consumer debt these two types only. Last but not the least, dynamics of household's cash flow is lack of microeconomic foundation. In particular, the model considers household's consumption as a residual term, whereas the decision-making in consumption is among the top priority in household's spending. Moreover, note that inflation affects household's affordability in consumption, which impacts the likelihood of household's financial security and survival. In the scenario with persistent high inflation, this impact should not be ignored.

The second branch goes to agent-based modeling (ABM). ABM simulates heterogeneous economic agents from 'bottom-up', by explicitly specifying behavioral rules of economic agents. The macro-level dynamics that captures the emergent properties of the economic system is produced by the micro-level interactions among agents. It has a variety of applications in economic research and practice, from European simulation platform EURACE [1] to Bank of Canada's alternative economic model CANVAS [3].

ABM provides flexibility by explicitly quantifying diversified economic agents' behavior and their interactions among each other, which is essential for building micro-founded model. But this degree of freedom comes with a price in model calibration. Heterogeneity and diversification in behavioral models tend to result in over-parameterization that increases the demand of data source employed for model calibration, which in turn increases the computational difficulty and expense. Another point of notice in ABM simulation is that once we initialize the model, it autonomously evolves along the dynamics with emergent properties. The modeller gives up the control on what simulation results would come up. This leads to the difficulty in harnessing model results.

In this work we aim at developing a macroeconomic stress testing framework for retail portfolio in credit risk. We adopt the second option to build our framework from 'bottom-up'. As aforementioned, we note that ABM has comparative advantages of providing economic sounded micro-foundation for modeling household's states and behavioral rules, whereas HRAM has comparative advantages of binding macro-level

[2] Our national regulator considers HRAM as one of the main references for the retail challenger models.

dynamics with a series of macroeconomic constrains preset by scenario. Therefore, it is plausible to integrate these comparative advantages from HRAM and ABM respectively under the same umbrella. In particular, we propose to develop a stress testing framework of agent-based micro-founded simulation, with boundedly macro-consistency imposed by given scenario.

Different from the existing work in HRAM, our work sets the starting point from the publicly available Survey of Financial Security (SFS) data by Statistics Canada, under the category of Financial Security Public Use Microdata. Our work presents as follows: Section 2 describes the SFS data. Section 3 presents the model specification. Section 4 explains the calibration strategy and provides an overview of utilized data source. Section 5 evaluates the stress testing results by scenarios, and Section 6 concludes.

2 The Household Data

Statistics Canada publishes a collection of data on the incomes, assets, debts of Canadian households under the umbrella of Financial Security Public Use Microdata, in the format of Public Use Microdata File (PUMF). We find from there the most recent data from Survey of Financial Security (SFS) conducted at 2019 provides a comprehensive picture of the financial status of nationwide households, see [2].

2019 SFS data contains information collected from more than $10,000$ households residing in Canada. It stores information of geographical location by province, assets, debts, net worth, income, employment status, and mortgage payment characteristics. It snapshots the financial status of Canadian households, see Figure 3 for aggregated balance sheet and 4 for debt composition.

Household records are attached with idiosyncratic weights for their representativeness of the Canadian population. For 2019 data, it has the weight range of [35.49, 9462.59]. To synchronize the weights among households, we consider the minimum weight 35.49 as universal for all households. For household with weight w, we replicate by $n = round(w/35.49)$ rows of data. For example, for household record with weight 9462.59, we replicate $n = 267$ rows of data.

This application of universal weights with replication by rounding has the effect of data adjustment to the original dataset. For a sanitary check, we show in Table 1 that the aggregate balance sheet and debt composition after adjustment are within the 95% confidence intervals of PUMF estimate as well as production estimate provided in Table 10-1 in its user guide [2]. Therefore, we consider this adjusted household dataset as our starting point, see Figure 5 and 6 for visualization of the corresponding aggregated balance sheet and debt composition, respectively.

	Data after Adjustment	PUMF Estimate Lower Bound	PUMF Estimate Upper Bound	Production Estimate Lower Bound	Production Estimate Upper Bound
Total Assets	13,704,403	13,346,660	14,080,194	13,216,053	14,013,999
Non-Property Asset	7,406,817	6,828,579	7,994,894	6,684,422	8,016,782
Property Asset	6,297,585	6,035,920	6,567,460	5,991,623	6,537,225
Total Debts	1,868,478	1,810,566	1,946,012	1,795,385	1,937,145
Net Worth	11,835,924	11,483,811	12,186,465	11,371,749	12,125,773
Mortgage Debt	1,516,094	1,436,607	1,601,925	1,419,627	1,595,216
Credit Cards	38,917	37,681	43,039	37,225	43,155
Line of Credit	139,126	127,007	152,972	125,777	155,548
Other Debt	174,340	162,761	194,586	160,628	195,355

Table 1: Data Adjustment with PUMF and Production Estimate Lower and Upper Bound (95% confidence interval).

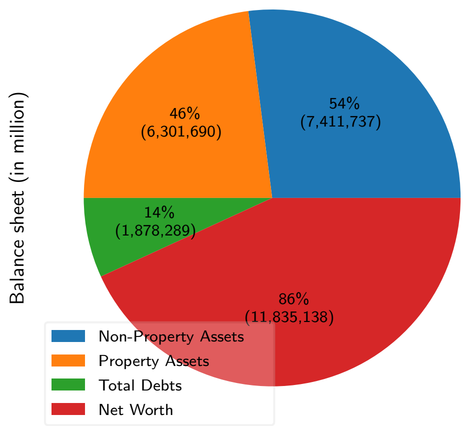


Figure 3: 2019 SFS balance sheet.

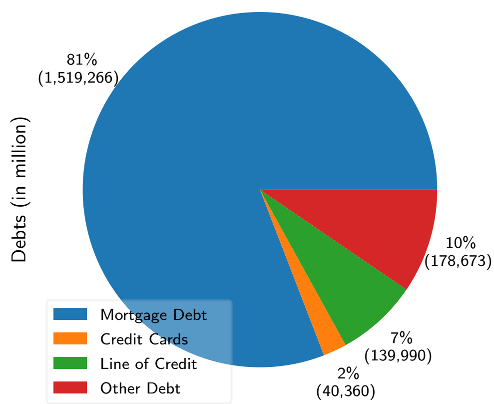


Figure 4: 2019 SFS debt composition.

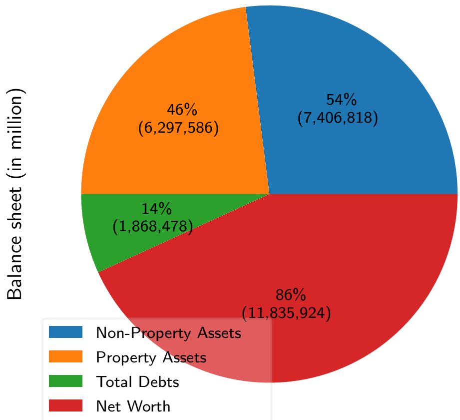


Figure 5: Initialized Balance Sheet.

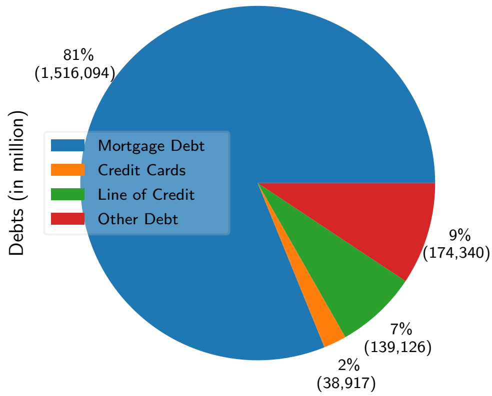


Figure 6: Initialized Debt Composition.

3 The Household Model Procedure

We consider stress testing carries at period/round $t = 0, 1, 2, \ldots, T$, with each period representing one quarter. $t = 0$ is the jump-off period or the qualifying round. Those household accounts that are non-default/performing at the end of this period are qualified to proceed with stress testing in the subsequent periods. Each stress testing period $t = 1, 2, \ldots, T$ might generate default accounts, they are replaced randomly by performing accounts at the end of each period.

Suppose household account, or household equivalently, is indexed by $h \in \mathcal{H}_t \equiv \{1, \ldots, H_t\}$ for each period $t = 0, 1, 2, \ldots, T$. Obviously, $H_0 \geq H_1 = \ldots H_T$. To simplify notation, we denote household account $h \in \mathcal{H} = \{1, \ldots, H\}$ when the period in discussion is evident.

3.1 Initialization

Household contains heterogeneous characteristics such as age and province. In particular, Table 2 shows characteristics relevant to stress testing.

Characteristics	Values
Residence Ownership	Own without mortgage, Own with mortgage, Not own.
Employment Status	full time, part time, N (for unemployed).
Retired	Yes, No.

Table 2: Household Characteristics to Stress Testing.

For those households with mortgage, the characteristics linked with mortgage, shown in Table 3, are also involved in stress testing.

Mortgage Characteristics	**Values**
Amortization	1 to 30 years
Remaining Term	1 to 6 years
Interest Rate	$(0\%, 10\%]$
Interest Rate Type	Fixed, Variable

Table 3: Household Characteristics with Mortgage to Stress Testing.

The state and dynamics of household are captured by its income statement and balance sheet. For simplification, we assume that each household has the same structures for its income statement and balance sheet, shown in Table 4 and 5, respectively. [3]

Disposable Income Y^{di}
Consumption C^c
Interest Expense C^{ir}
Mortgage Payment C^{mtg}
Net Cash Flow CF^{net}

Table 4: Household Income Statement.

Assets	**Debts and Net Worth**
Non-property Asset A^{np}	Mortgage Debt B^{mtg}
Property Asset A^p	Credit Cards B^{card}
	Line of Credit B^{loc}
	Other Debt B^{other}
	Net Worth E
Total Assets A^{total}	Total Debts and Net Worth $B^{total} + E$

Table 5: Household Balance Sheet.

We assume household in each period realizes its income and expense, which results in the equivalence of accrual and cash basis in financial reporting. By accounting principle, household in each period holds the accounting equality for its income statement:

$$CF^{net} = Y^{di} - C^c - C^{ir} - C^{mtg}. \tag{1}$$

At the end of period, household's net cash flow CF^{net} is updated to its balance sheet under net worth E. It holds the accounting equality for its balance sheet:

$$A^{total} = B^{total} + E. \tag{2}$$

[3] From the perspective of financial reporting, here we admit an abuse of terminology: the term "Net Cash Flow" in Table 4 refers to "Net Income" normally used in financial reporting, while the terms "Debts" and "Net Worth" in Table 5 refer to "Liabilities" and "Equity", respectively.

We assume that a collection of macroeconomic variables (MEVs) as drivers is preset by stress testing scenario such that the dynamics of household for each round t is under these constraints. In particular, as shown in Table 6, our work considers the following MEVs capturing 5 major categories for our economy: business cycle, job market, inflation, interest rate, and housing market.

Category	Macroeconomic Driver
Business Cycle	National Disposable Income Growth Rate $Z^{DI_{gr}}[t]$
Job Market	National Unemployment Rate $Z^{UR}[t]$
	National Unemployment Duration $Z^{UD}[t]$
Inflation	National CPI Growth Rate $Z^{CPI_{gr}}[t]$
Interest Rate	Borrower Loan Rate $Z^{IR_{loan}}[t]$
	Prime Rate $Z^{IR_{prime}}[t]$
Housing Market	National House Price Index Growth Rate $Z^{HPI_{gr}}[t]$

Table 6: MEVs in Stress Testing.

3.2 Employment

At the beginning of period t, household updates its employment statues constrained by MEV unemployment rate $Z^{UR}[t]$ and unemployment duration $Z^{UD}[t]$.

Denote the subset of households in labor force by $\mathcal{H}^{labor}[t] \subseteq \mathcal{H}$. Unemployed household in $\mathcal{H}^{un}[t]$ is randomly selected by:

$$\mathcal{H}^{un}[t] = rnc(\mathcal{H}^{labor}[t], Z^{UR}[t] \times count(\mathcal{H}^{labor}[t]), p^{labor}[t]), \tag{3}$$

where $rnc(X, n, p)$ is *random choice function* that selects n sample from the set X with p specifying the probabilities associated with each entry in X.

Equation (3) means that we randomly select from the population of labor force $\mathcal{H}^{labor}[t]$ the unemployed households with the size of unemployment rate times the labor force head count $Z^{UR}[t] \times count(\mathcal{H}^{labor}[t])$, by the associated probability assigned to each household in labor force $p^{labor}[t]$. The assigned probability is calculated by:

$$p^{labor}[t] = normalize(\max(Z^{UD}[t] - \mathcal{H}^{labor}[unempperiod,\ t], 0) + 1), \tag{4}$$

with the $normalize()$ function scales the vector or array to a unit norm.

$\mathcal{H}^{labor}[unempperiod, t]$ in Equation (4) records the count of consecutive unemployed periods each household in labor force has up-to-date. The term $\max(Z^{UD}[t] - \mathcal{H}^{labor}[unempperiod,\ t], 0)$ calculates the positive gap of the given unemployment duration over the number of consecutive unemployed periods.

For example, consider 10 households in labor force are employed, except for the first household having up-to-date 2 unemployed periods and the second household unemployed in previous period, then $\mathcal{H}^{labor}[unempperiod,\ t] = [2, 1, 0, \ldots, 0]$. If the given unemployment duration is $Z^{UD}[t] = 1.5$, $\max(Z^{UD}[t] - \mathcal{H}^{labor}[unempperiod,\ t], 0) = [0, 0.5, 1.5, 1.5, \ldots, 1.5]$. In this case, the corresponding assigned probability is thus:

$$\begin{aligned} p^{labor}[t] &= normalize([0, 0.5, 1.5, 1.5, \ldots, 1.5] + 1) \\ &= [4.4\%, 6.7\%, 11.1\%, 11.1\%, \ldots, 11.1\%]. \end{aligned}$$

This implies that household with consecutive unemployed periods more than the given unemployment duration is least likely to be selected as unemployed for the next period. Household with consecutive unemployed periods less than the given unemployment duration is more likely to be unemployed when its unemployed period is less. Intuitively speaking, we assume household is unemployment averse with which the longer time span it stays unemployed, the less likelihood it remains unemployed for the coming period.

Once unemployed households in $\mathcal{H}^{un}[t]$ are determined, so are the employed households in $\mathcal{H}^{em}[t]$ by

$$\mathcal{H}^{em}[t] = \mathcal{H}^{labor}[t] - \mathcal{H}^{un}[t]. \tag{5}$$

Similarly, from employed households in $\mathcal{H}^{em}[t]$, full-time employed household in $\mathcal{H}^{em,f}[t]$ is randomly selected from $\mathcal{H}^{em}[t]$ by

$$\mathcal{H}^{em,f}[t] = rnc(\mathcal{H}^{em}[t], ratio^{f}_{em}[t-1] \times count(\mathcal{H}^{em}[t]), p^{em,f}[t]). \tag{6}$$

Equation (6) shows that the selected sample size is consistent with full-time employment ratio in previous period [4]

$$ratio^{f}_{em}[t-1] := \frac{count(\mathcal{H}^{em,f}[t-1])}{count(\mathcal{H}^{em}[t-1])}, \tag{7}$$

with probabilities normalized by

$$p^{em,f}[t] = normalize(ind(\mathcal{H}^{em}[emp\,status, t] == full\,time) + 1). \tag{8}$$

Here $ind()$ is the indicator function that returns the value of 1 if household has employment status "$emp\,status$" as "$full\,time$" at the beginning of period t, and 0 otherwise. Equation (8) implies that the household previously with "$full\,time$" is more likely to be selected as "$full\,time$".

Once full-time employed households in $\mathcal{H}^{em,f}[t]$ are determined, part-time employed households in $\mathcal{H}^{em,p}[t]$ are set by:

$$\mathcal{H}^{em,p}[t] = \mathcal{H}^{em}[t] - \mathcal{H}^{em,f}[t]. \tag{9}$$

Households are then assigned with wage ratio depending on their updated employment status:

$$\mathcal{H}^{labor}[wage\,ratio,\, t] = \begin{cases} w_f, & if\,emp\,status == full\,time; \\ w_p, & if\,emp\,status == part\,time; \\ w_{un}, & if\,emp\,status == unemployed; \end{cases} \tag{10}$$

where w_f, w_p, and w_{un} are given parameters by model calibration at Section 4.

[4] When period $t = 0$, the full-time employment ratio in previous period is calculated from the adjusted household dataset discussed in Section 2.

3.3 Disposable Income

Household's disposable income Y^{di} at period t is under constraint by the given MEV disposable income growth rate $Z^{DI_{gr}}[t]$. Denote the average income for all households in previous period by $AvgInc^{\mathcal{H}}[t-1]$. [5]

We update the average income by:

$$AvgInc^{\mathcal{H}}[t] = AvgInc^{\mathcal{H}}[t-1] \times (1 + Z^{DI_{gr}}[t]). \tag{11}$$

Then, we calculate household's disposable income Y^{di} by:

$$\mathcal{H}[Y^{di}, t] = \mathcal{H}[wage\, ratio, t] \times AvgInc^{\mathcal{H}}[t] \times \mathcal{H}[\epsilon^{Y^{di}}, t]. \tag{12}$$

Note that Equation (12) is in vector/array format with the emphasis on households realizing disposable income simultaneously. In particular, $\mathcal{H}[Y^{di}, t]$ in Equation (12) refers to the array of disposable income for all households, $\mathcal{H}[wage\, ratio, t]$ is the array of wage ratio for all households updated by Equation (10) from Section 3.2, $AvgInc^{\mathcal{H}}[t]$ is a scalar value calculated by Equation (11) broadcasting to all households, and $\mathcal{H}[\epsilon^{Y^{di}}, t] \sim N(1, 1)$ is the array of idiosyncratic random shock applied to each household.

Due to the introduction of idiosyncratic random shock, household's disposable income by Equation (12) might violate the MEV constraint on its growth rate $Z^{DI_{gr}}[t]$ in aggregate-level. We thus apply the adjustment by:

$$\mathcal{H}[Y^{di}, t] = \mathcal{H}[Y^{di}, t] \times \frac{sum(\mathcal{H}[Y^{di}, t-1]) \times (1 + Z^{DI_{gr}}[t])}{sum(\mathcal{H}[Y^{di}, t])}. \tag{13}$$

3.4 Consumption

Given MEV CPI growth rate $Z^{CPI_{gr}}[t]$, household calculates its consumption C^c for period t. Denote the average household consumption in previous period as $AvgCon^{\mathcal{H}}[t-1]$. [6] Assuming average household consumption growth is driven by CPI, we update the average consumption by:

$$AvgCon^{\mathcal{H}}[t] = AvgCon^{\mathcal{H}}[t-1] \times (1 + Z^{CPI_{gr}}[t]). \tag{14}$$

Then, we calculate average household consumption ratio $AvgCon^{\mathcal{H}}_{ratio}[t]$ by the updated average income [7] and consumption:

$$AvgCon^{\mathcal{H}}_{ratio}[t] = AvgCon^{\mathcal{H}}[t] / AvgInc^{\mathcal{H}}[t]. \tag{15}$$

The calculation of household consumption in vector/array form is thus:

$$\mathcal{H}[C^c, t] = mean(\mathcal{H}[Y^{di}, t] \times AvgCon^{\mathcal{H}}_{ratio}[t], AvgCon^{\mathcal{H}}[t]). \tag{16}$$

[5] At period $t = 0$, the average income for all households in previous period $AvgInc^{\mathcal{H}}[-1]$ is set by model calibration at Section 4.

[6] Analogous to average income $AvgInc^{\mathcal{H}}[t-1]$, when period $t = 0$, $AvgCon^{\mathcal{H}}[-1]$ is set by model calibration at Section 4.

[7] Note that the adjustment in Equation (13) from Section 3.3 affects the average income $AvgInc^{\mathcal{H}}[t]$, thus we need to re-calculate $AvgInc^{\mathcal{H}}[t]$ before we apply the calculation in Equation (15).

Here $mean(X, Y)$ calculates the average of array X and Y element-wise. In particular, Equation (16) shows household takes the average between the inferred consumption by household income Y^{di} multiplying the average consumption ratio $AvgCon^{\mathcal{H}}_{ratio}[t]$ and the average household consumption applied to all households $AvgCon^{\mathcal{H}}[t]$. Intuitively, we ask household to have heterogeneous consumption while to some extent dragging back the heterogeneity to average level in the population of all households.

To be in sync with the MEV constraint $Z^{CPI_{gr}}[t]$, we apply the adjustment by:

$$\mathcal{H}[C^c, t] = \mathcal{H}[C^c, t] \times \frac{sum(\mathcal{H}[C^c, t-1]) \times (1 + Z^{CPI_{gr}}[t])}{sum(\mathcal{H}[C^c, t])}. \tag{17}$$

3.5 Interest Expense

For simplicity, we assume household has to pay with loan rate for its line of credit B^{loc} and other debt B^{other} at each period t. The loan rate is given by MEV borrower loan rate $Z^{IR_{loan}}[t]$. Thus, household's interest expense C^{ir} is calculated in vector/array form by:

$$\mathcal{H}[C^{ir}, t] = (\mathcal{H}[B^{loc}, t] + \mathcal{H}[B^{other}, t]) \times Z^{IR_{loan}}[t]. \tag{18}$$

3.6 Mortgage Payment

Households with mortgage contracts in $\mathcal{H}^{mtg}$ need to calculate mortgage payment C^{mtg} at each period t. For simplicity, we assume household has two types of mortgage contracts: with fixed interest rate that mortgage rate is fixed by the contract, and with variable interest rate that mortgage rate updates in each period according to market condition. We further assume that household with variable mortgage contract in $\mathcal{H}^{mtg_V}$ updates its mortgage rate r^{mtg} by the given MEV prime rate $Z^{IR_{prime}}[t]$ at each period t:

$$\mathcal{H}^{mtg_V}[r^{mtg}, t] = Z^{IR_{prime}}[t]. \tag{19}$$

Household with mortgage contracts in $\mathcal{H}^{mtg}$ then calculates mortgage payment pay^{mtg} according to its mortgage amount p^{mtg}, its mortgage rate r^{mtg}, its mortgage amortization period $amort^{mtg}$. The calculation is by the vector/array form:

$$\mathcal{H}^{mtg}[pay^{mtg}, t] = mtgCal(\mathcal{H}^{mtg}[p^{mtg}, t], \mathcal{H}^{mtg}[r^{mtg}, t], \mathcal{H}^{mtg}[amort^{mtg}, t]). \tag{20}$$

Here $mtgCal(P, r, N)$ is the mortgage calculator function that calculates quarterly mortgage payment for mortgage amount P, interest rate for the mortgage r, and amortization period N.

Mortgage payment pay^{mtg} can be broken down into two components: principal paid pay^{mtg_p}, and interest expense paid pay^{mtg_r}:

$$\mathcal{H}^{mtg}[pay^{mtg}, t] = \mathcal{H}^{mtg}[pay^{mtg_p}, t] + \mathcal{H}^{mtg}[pay^{mtg_r}, t]. \tag{21}$$

3.7 Credit Stage

To settle household's income statement at the end of period t, it calculates net cash flow CF^{net} by:

$$\mathcal{H}[CF^{net}, t] = \mathcal{H}[Y^{di}, t] - \mathcal{H}[C^{c}, t] - \mathcal{H}[C^{ir}, t] - \mathcal{H}[pay^{mtg}, t]. \tag{22}$$

Net cash flow by Equation (22) is equivalent to the net income in terms of financial reporting, it will be used to determine household's credit stage at the end of each period t.

Assume before determining credit stage, household is able to update its net worth by property value appreciation/depreciation, given MEV house price index growth rate $Z^{HPI_{gr}}[t]$. The update on household's property asset A^{p} is by:

$$\mathcal{H}[A^{p}, t] = \mathcal{H}[A^{p}, t-1] \times (1 + Z^{HPI_{gr}}[t]). \tag{23}$$

Correspondingly, household updates its net worth by property value change:

$$\mathcal{H}[E, t] = \mathcal{H}[E, t-1] + (\mathcal{H}[A^{p}, t] - \mathcal{H}[A^{p}, t-1]). \tag{24}$$

Another key determinant for household's credit stage is how much credit it is allowed to borrow, denoted as effective credit space, denoted as CS^{eff}. Assume the household can borrow from the bank at maximum its credit limit:

$$Credit\,Limit := \max(Y^{di} \times Limit_{LTI}, A^{p} \times Cap_{LTV}),$$

which is its disposable income Y^{di} times a loan-to-income limit $Limit_{LTI}$ or the market value of its property A^{p} times a loan-to-value cap Cap_{LTV}, whichever is larger.[8]

Since the household might already bear with existing loans, the credit space that the bank would allow for the household is thus its credit limit minus its existing loans. Specifically, we assume the credit space is the credit limit deducted by mortgage debt and line of credit, which is:

$$Credit\,Limit - (B^{mtg} + B^{loc}).$$

On the other hand, the household would not bear the debt more than what it can afford. Specifically, it would not borrow more than its net worth. This makes up the household's effective credit space CS^{eff} by:

$$CS^{eff} = \{\min\left(E, Credit\,Limit - (B^{mtg} + B^{loc})\right)\}^{+}, \tag{25}$$

where the superscript $^{+}$ denotes for only the non-negative value, for example $\{-1.0\}^{+} = 0.0$.

With household's net cash flow CF^{net}, updated net worth E, and effective credit space CS^{eff}, now we are ready to determine household's credit stage at the end of period t. In our work, we assume 3 types of credit stages as follows:

[8] In practice, the loan-to-income limit is around four to five, while the loan-to-value ratio ranges from 30% to 80%. The specific values of $Limit_{LTI}$ and Cap_{LTV} used in our work are set by model calibration at Section 4.

- Credit stage 1: net cash flow $CF^{net} \geq 0$. Household in credit stage 1 has healthy credit condition and is unlikely to deteriorate in subsequent period.
- Credit stage 2: net cash flow $CF^{net} < 0$, effective credit space CS^{eff} enough to cover net cash flow deficit CF^{net} as $CS^{eff} + CF^{net} \geq 0$. Household in credit stage 2 has significantly deteriorated credit condition in subsequent period. In other words, household in credit stage 2 is likely to default in the upcoming period.
- Credit stage 3: net cash flow $CF^{net} < 0$, effective credit space CS^{eff} *not* enough to cover net cash flow deficit as $CS^{eff} + CF^{net} < 0$. Household in credit stage 3 is credit-impaired, or in other words, default.

The decision tree of credit stage determination is shown in Figure 7.

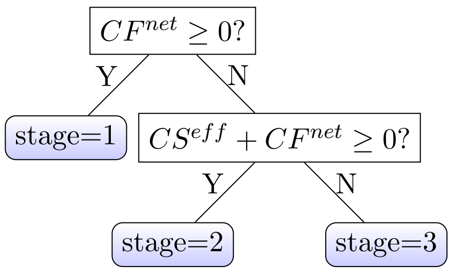


Figure 7: Decision Tree of Credit Stage Determination.

3.8 Balance Sheet Settlement

Household settles its balance sheet according to its credit stage determined at Section 3.7.

For credit stage 1 household, it has positive net cash flow $CF^{net} \geq 0$. We assume it uses CF^{net} to pay back line of credit if there is any. This leads to an increase of its net worth by CF^{net}. Another increase in net worth is from the pay back of mortgage debt principal C^{mtg_p}. The net effect on its balance sheet settlement is summarized in Table 7.

	Debit	**Credit**
Mortgage Debt (-)	C^{mtg_p}	
Line of Credit (-)	CF^{net}	
Net Worth (+)		$CF^{net} + C^{mtg_p}$

Table 7: Balance Sheet Settlement: Credit Stage 1 Household.

The first row in Table 7 shows the liability item "Mortgage Debt" has "Debit" of C^{mtg_p}, which means a decrease "(-)" of existing mortgage loan by household's mortgage payment. The second row shows the liability item "Line of Credit" has "Debit" of CF^{net}, which means a decrease "(-)" in line of credit loan when household pays back the loan by its net cash flow. The last row shows the "Credit" or the increase "(+)" of "Net Worth" by $CF^{net} + C^{mtg_p}$ in total, which balances the decrease of liability items. Note that the total debit amount is equal to the total credit amount, by which we maintain the accounting equality in Equation (2).

For credit stage 2 household, it has negative net cash flow $CF^{net} < 0$, while its effective credit space is enough to cover this net cash flow deficit as $CS^{eff} + CF^{net} \geq 0$. Assume the household is able to expand "Line of Credit" up to the limit of its effective credit space to fill the net cash flow deficit. the net effect on its balance sheet settlement is summarized in Table 8.

	Debit	**Credit**
Mortgage Debt (-)	C^{mtg_p}	
Line of Credit (+)		$\lvert CF^{net}\rvert$
Net Worth (+)		$-\lvert CF^{net}\rvert + C^{mtg_p}$

Table 8: Balance Sheet Settlement: Credit Stage 2 Household.

The first row in Table 8 shows the liability item "Mortgage Debt" has "Debit" of C^{mtg_p}, which means a decrease "(-)" of mortgage loan by household's mortgage payment. The second row shows the liability item "Line of Credit" has "Credit" or increase "(+)" of amount $|CF^{net}|$, which means household expands its line of credit loan up to cover its net cash flow deficit $|CF^{net}|$. The last row shows the net effect of $-|CF^{net}| + C^{mtg_p}$ is "Credit" or increase "(+)" to its "Net Worth" account.[9] Again, the total debit amount is equal to the credit amount, by which household maintains the accounting equality in Equation 2.

Credit stage 3 household is with negative net cash flow $CF^{net} < 0$, while its effective credit space is *not* enough to cover its cash flow deficit as $CS^{eff} + CF^{net} < 0$. Household is default and it goes through the liquidation process. Assume there is no fire sale cost that household can fully liquidate its assets at market value. Liabilities are liquidated by the following order:

1. Credit cards;
2. Mortgage debt;
3. Line of credit;
4. Other debt.

After liquidation, default household is replaced randomly by performing household in exit-entry mechanism as shown in Section 3.10, to keep the total account number constant during stress testing periods $t = 1, 2, \ldots, T$.

3.9 Mortgage Contract Renewal

For performing household with credit stage 1 or 2, if its mortgage contract expires at the end of period t, it requests a new mortgage contract. We assume household can choose between fixed-rate or variable-rate contract, with mortgage term of 2 years or remaining amortization, whichever is shorter.

[9]Here we demonstrate the case when $-|CF^{net}| + C^{mtg_p} \geq 0$. When $-|CF^{net}| + C^{mtg_p} < 0$, the balance sheet settlement is "Debit" or decrease "(-)" to its "Net Worth" amount by $|-|CF^{net}| + C^{mtg_p}|$.

Given the MEV prime rate $Z^{IR_{prime}}[t]$, we construct the fixed two year MTG rate $Z^{IR_{fixed2}}[t]$ as perfect-foresight of the average prime rate in forward-looking 2-year span by

$$Z^{IR_{fixed2}}[t] = mean(Z^{IR_{prime}}[t : t + 7]), \tag{26}$$

where $Z^{IR_{prime}}[t : t + 7]$ refers to the array/vector of prime rate foreword-looking from period t to $t + 7$.

Household can choose between fixed-rate contract with a fixed mortgage rate of $Z^{IR_{fixed2}}[t]$ and variable-rate contract with a variable mortgage rate equal to prime rate $Z^{IR_{prime}}[t]$ at each period.

Probability of household choosing fixed rate mortgage contract is given by:

$$\frac{Z^{IR_{prime}}[t]}{Z^{IR_{prime}}[t] + Z^{IR_{fixed2}}[t]}. \tag{27}$$

Equation (27) implies that if the current prime rate is higher than the 2-year forward-looking average with $Z^{IR_{prime}}[t] > Z^{IR_{fixed2}}[t]$ then more household would choose fixed rate contract with mortgage rate of $Z^{IR_{fixed2}}[t]$, and vice versa.

3.10 Exit-Entry Mechanism

At the end of period $t = 0$, denote all performing accounts after qualifying round as $\mathcal{H}^{perform}$. Obviously, $\mathcal{H}^{perform}$ contains households with and without mortgage contract.

At the end of each stress testing period $t = 1, \ldots, T$, default accounts are replaced by randomly selected accounts from $\mathcal{H}^{perform}$. To maintain the account numbers with mortgage contract in the population of households during stress testing, default accounts with mortgage contract are randomly replaced by those with mortgage contract in $\mathcal{H}^{perform}$, and the same applies to the replacement for default accounts without mortgage contract.

4 Model Calibration

Assume $T = 12$ for a 3-year stress testing. We initialize/calibrate the wage ratios used in Equation (10), the initial average income $AvgInc^{\mathcal{H}}[-1]$ used in Equation (11) at period $t = 0$, the initial average consumption $AvgCon^{\mathcal{H}}[-1]$ used in Equation (14) at period $t = 0$, and loan-to-income limit $Limit_{LTI}$ with loan-to-value cap Cap_{LTV} used in Equation 25 in Table 9 respectively.

In particular, unemployed wage ratio is regarded as the basic rate for Employment Insurance (EI) benefit. [10] With full-time wage ratio is set to 1, part-time wage ratio is set to be in the middle of unemployment wage ratio and full-time wage ratio. Average income and consumption are taken from Statistics Canada Table 36-10-0587-01 by the 2019 annual data.[11]

[10] For more information, see `https://www.canada.ca/en/services/benefits/ei/ei-regular-benefit/benefit-amount.html`.

[11] See `https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=3610058701`.

Loan-to-income limit works as a proxy for the regulation that OSFI proposes recently a loan-to-income (LTI) limit on the portfolios of federally regulated financial institutions for new uninsured mortgage loans,[12] whereas loan-to-value cap works as an estimate by current practice.

Name	**Variable**	**Data Source**
Stress testing periods	$T = 12$	
Full-time wage ratio	$W_f = 1.0$	
Unemployed wage ratio (EI benefit basic rate)	$W_{un} = 0.55$	By regulation
Part-time wage ratio	$W_p = (W_f + W_{un})/2$	
Average Income	$AvgInc^{\mathcal{H}}[-1] = 104,112$	Table 36-10-0587-01
Average Consumption	$AvgCon^{\mathcal{H}}[-1] = 83,076$	Table 36-10-0587-01
Loan-to-income limit	$Limit_{LTI} = 4.5$	Proxy by regulation
Loan-to-value cap	$Cap_{LTV} = 0.8$	Estimate by current practice

Table 9: Model Calibration: Variables and Values.

5 Simulations

We evaluate our model by Monte-Carlo simulations for each given scenario. The evaluation results are contingent to the scenario setup. We are interested in two scenarios for stress testing: HIST and MST.

5.1 Scenarios

HIST scenario mimics the pandemic periods at 2020 to 2022, where the economy shows unemployment spiking up along with disposable income suddenly dropping, followed by CPI rising to a high level and as a lagged response interest rates rising up at the end. The MEVs time series in this scenario is taken from the corresponding economic series published by Statistics Canada.[13] Table 10 shows the data source and the conducted data pre-process. This scenario works as benchmark in our stress testing exercise.

[12]See `https://www.osfi-bsif.gc.ca/en/news/loan-income-limit`.

[13]One exception is on disposable income: The historical data of disposable income has distortion by government's pandemic relief program, which leads to a spiking up at disposable income during COVID-19 periods. This breaks down of the connection between high unemployment rate and low disposable income. Thus, we use real GDP data which is more in sync with high unemployment rate to cover disposable income in HIST scenario.

MEV	Vector Identifier	Data Source Frequency	Data Preprocess
Disposable Income Growth Rate, National	v62305752	Q (quarterly)	
Unemployment Rate, National	v2062815	M (monthly)	re-frequency from M to Q
Unemployment Duration, National	v1078668391	M (monthly)	re-frequency from M to Q
CPI Growth Rate, National	v41690914	M (monthly)	re-frequency from M to Q
Borrower Loan Rate	v80691312	W (weekly)	re-frequency from W to Q
Prime Rate	v80691311	W (weekly)	re-frequency from W to Q
House Price Index Growth Rate, National	v111955442	M (monthly)	re-frequency from M to Q, re-index by 2022Q3 as 100

Table 10: HIST scenario data sources from Stat Canada.

On the other hand, MST mimics the scenario considered at OSFI MST 2023 exercise, where the economy shows a gradually rising unemployment, persistently high interest rate and high inflation, along with housing price cutting down.

Appendix A shows the visualized comparison on MEVs between these two scenarios.

5.2 Results

We conduct 20 Monte-Carlo simulations for each scenario. [14] Taking average from these simulations gives the stress testing results for HIST and MST.

5.2.1 Default Rates

Figure 8 and Table 11 show household's default rate and mortgage (MTG) default rate for HIST and MST, respectively. MST has in general higher default rate and mortgage default rate than HIST.

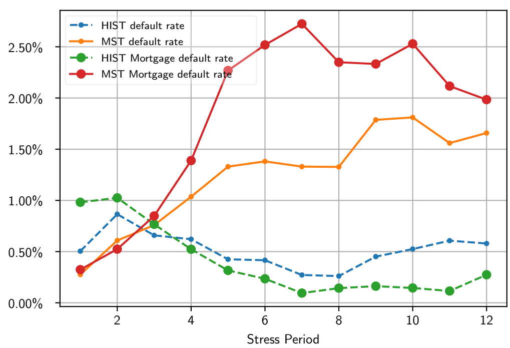


Figure 8: Default rate and MTG default rate.

In particular, HIST default rate averages at 0.52%, with its peak at period 2 for 0.87% and then gradually dropping down to the level of 0.5%. This mainly reflects the same pattern shown in HIST unemployment rate, see Figure 20a. HIST mortgage default rate averages at 0.40%, with its peak at the first two periods at the level of 1.0%

[14] Appendix C provides a series of visualized justifications on the distributions of default rates by each Monte-Carlo simulation, which gives us confidence on the robustness of our average results by 20 Monte-Carlo simulations.

and then gradually dropping down to the level of 0.2%. This implies that households do not have sufficient financial buffer when facing the unexpected pandemic shock if without government's Canada Emergency Response Benefit (CERB) program.

On the other hand, MST has default rate and mortgage default rate average at 1.24% and 1.83%, respectively. The MST scenario of gradual rising in unemployment rate with persistently high inflation and interest rate, together with housing price cutting down, undermines household's financial security, which results in persistently high default rate in MST. Moreover, the mortgage default rate first rises along with the default rate, then quickly surpasses household's default rate and remains at very high level above 2%. This indicates the existence of cascade effect in the universe of retail portfolios in terms of credit risk.

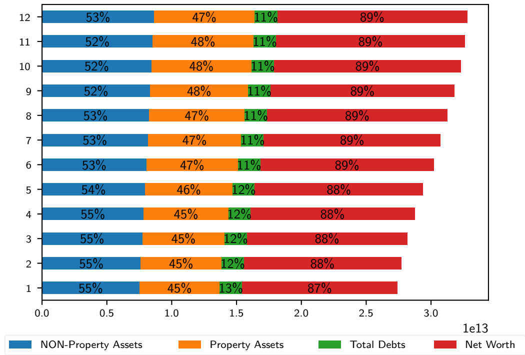
Stress Period	**HIST Default Rate**	**MST Default Rate**	**HIST Mortgage Default Rate**	**MST Mortgage Default Rate**
1	0.50%	0.28%	0.98%	0.32%
2	0.87%	0.61%	1.02%	0.52%
3	0.66%	0.76%	0.77%	0.85%
4	0.62%	1.04%	0.52%	1.39%
5	0.42%	1.33%	0.32%	2.27%
6	0.42%	1.38%	0.23%	2.52%
7	0.27%	1.33%	0.10%	2.72%
8	0.26%	1.33%	0.14%	2.35%
9	0.45%	1.79%	0.16%	2.33%
10	0.52%	1.81%	0.14%	2.53%
11	0.61%	1.56%	0.12%	2.12%
12	0.58%	1.66%	0.27%	1.98%

Table 11: Default rate and MTG default rate.

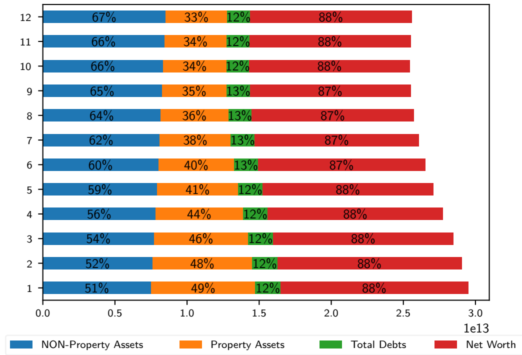
5.2.2 Household Balance Sheet

Figure 9 shows the composition of households balance sheet in aggregate-level. Figure 9a shows that HIST has an expanding balance sheet with relatively stable proportions of non-property assets, property assets, total debts, and net worth. This provides the implication that most households in HIST can bear with the transitory pandemic shock with no significant structural change in the balance sheet composition.

As a comparison, Figure 9b shows a contracting balance sheet in MST, with a shrinking proportion of property assets that results from the housing price cutting down. The shrinking size of the aggregate balance sheet indicates households in MST are more financial stressed than HIST.



(a) HIST.



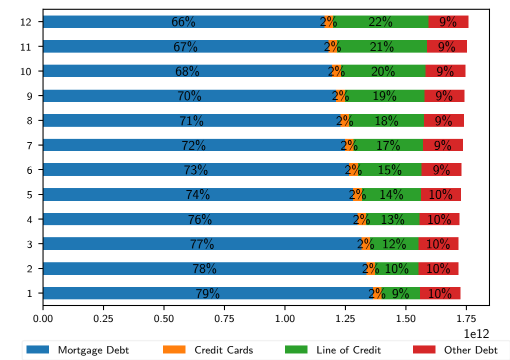
(b) MST.

Figure 9: Households Balance Sheets.

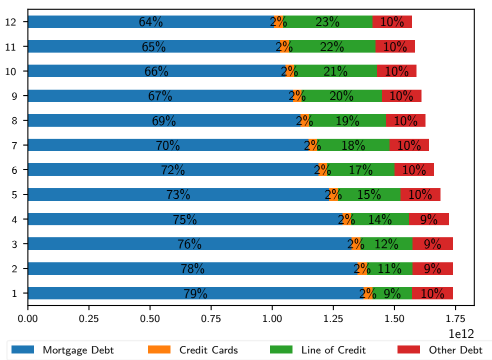
5.2.3 Household Liabilities

A further break down in the proportion of debt composition is shown at Figure 10, with Figure 10a for HIST and Figure 10b for MST, respectively. Both scenarios have shrinking proportion in mortgage debt while expanding proportion in line of credit. This reflects in economic downturn household has the tendency of maintaining its mortgage debt to be non-default with the cost of expanding line of credit whenever it is possible, even though line of credit in general bears with higher interest rate.

Notice that in MST the amount of total debt is decreasing whereas that in HIST is slightly increasing. This seems to suggest households in MST have reduced financial flexibility.



(a) HIST.

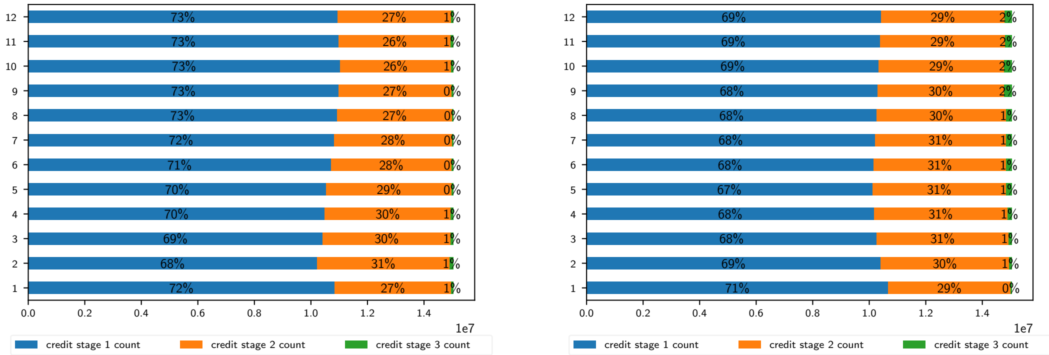


(b) MST.

Figure 10: Households Liabilities.

5.2.4 Credit Stage

Not surprisingly, MST households have slightly more proportion in credit stage 2 than HIST, as shown in Figure 11. This indicates that households are more financially fragile in MST than in HIST scenario.



(a) HIST. (b) MST.

Figure 11: Households Credit Stages.

5.3 With Income Distribution

Before drawing a conclusion, there is one more thing we need to pay attention to. We use in Section 4 from Statistics Canada Table 36-10-0587-01 the average income $AvgInc^{\mathcal{H}}[-1] = 104,112$ and average consumption $AvgCon^{\mathcal{H}}[-1] = 83,076$ applied to all households. Implicitly we assume that households have homogeneous income and consumption. More importantly, we assume that their income is enough to cover consumption. We may regard this as universal average income and consumption. On the other hand, we discover from the same table that income and consumption are heterogeneously distributed, see Table 12 and Figure 12 for the distributions by income quintile at the year of 2019.

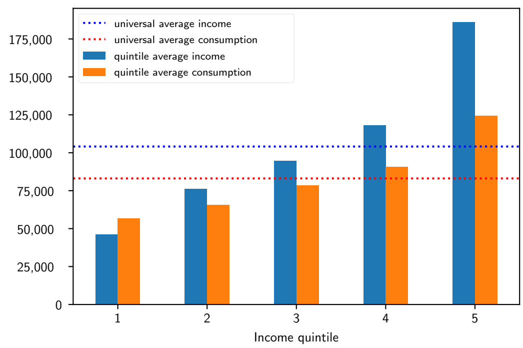


Figure 12: Income and Consumption Distribution: at the Year of 2019.

In particular, we see from Table 12 that households at the first quintile on average have income not sufficient to cover consumption. This triggers a question whether the stress testing results so far with homogeneous income and consumption have underestimation on the financial severity among households. To answer this question, we integrate the quintile incomes and consumptions into our stress testing framework.

Quintile	Average Income	Average Consumption
1	46,100	56,504
2	76,145	65,462
3	94,545	78,585
4	117,900	90,573
5	185,868	124,252

Table 12: Quintile Average Income and Consumption: at the Year of 2019.

5.3.1 Quintile Disposable Income

Given the quintile thresholds calculated from the adjusted household dataset discussed in Section 2, households are partitioned into quintiles $\mathcal{H}_1$, ..., $\mathcal{H}_5$ according to disposable income Y^{di}, as shown in Table 13:

Households in Quintile	Condition
$\mathcal{H}_1$	$Y^{di} < 51,750$
$\mathcal{H}_2$	$51,750 \leq Y^{di} < 77,249$
$\mathcal{H}_3$	$77,249 \leq Y^{di} < 107,472$
$\mathcal{H}_4$	$107,472 \leq Y^{di} < 152,236$
$\mathcal{H}_5$	$Y^{di} \geq 152,236$

Table 13: Quintile Income Partition.

At the beginning of period t, households inherit from previous period $t-1$ their positions in these quintiles. [15] For each quintile $k = 1, 2, \ldots, 5$, denote the average income for households at this quintile in previous period by $AvgInc^{\mathcal{H}_k}[t-1]$. [16] Given MEV disposable income growth rate $Z^{DI_{gr}}[t]$, households at quintile k update the average income by:

$$AvgInc^{\mathcal{H}_k}[t] = AvgInc^{\mathcal{H}_k}[t-1] \times (1 + Z^{DI_{gr}}[t]), \quad k = 1, 2, \ldots, 5. \tag{28}$$

Then, we calculate household's disposable income Y^{di} at quintile k by:

$$\mathcal{H}_k[Y^{di}, t] = \mathcal{H}_k[wage\,ratio, t] \times AvgInc^{\mathcal{H}_k}[t] \times \mathcal{H}_k[\epsilon^{Y^{di}}, t], \quad k = 1, 2, \ldots, 5. \tag{29}$$

Analogous to Equation (12), Equation (29) is in vector format with $\mathcal{H}_k[\epsilon^{Y^{di}}, t] \sim N(1, 1)$ as idiosyncratic random shock.

After households in all quintiles updating disposable income by Equation (29), we apply the same adjustment as by Equation (13) in Section 3.3.

$$\mathcal{H}[Y^{di}, t] = \mathcal{H}[Y^{di}, t] \times \frac{sum(\mathcal{H}[Y^{di}, t-1]) \times (1 + Z^{DI_{gr}}[t])}{sum(\mathcal{H}[Y^{di}, t])}. \tag{13}$$

[15] At the beginning of period $t = 0$, the income partitions are carried from the adjusted household dataset discussed in Section 2, by applying the conditions described at Table 13.

[16] At period $t = 0$, the average income for households at quintile k in previous period $AvgInc^{\mathcal{H}_k}[-1]$ is set by values from Table 12. For example, the first quintile household has average income $AvgInc^{\mathcal{H}_1}[-1] = 46,100$.

5.3.2 Quintile Consumption

We assume households update their positions in quintiles after updating their disposable income Y^{di} at Section 5.3.1, then they update their consumption C^c for period t given MEV CPI growth rate $Z^{CPI_{gr}}[t]$.

For each quintile $k = 1, 2, \ldots, 5$, denote the average household consumption in previous period as $AvgCon^{\mathcal{H}_k}[t-1]$. [17] Assuming average household consumption growth is driven by CPI, we update the average consumption at quintile k by:

$$AvgCon^{\mathcal{H}_k}[t] = AvgCon^{\mathcal{H}_k}[t-1] \times (1 + Z^{CPI_{gr}}[t]), \quad k = 1, 2, \ldots, 5. \tag{30}$$

Then, we calculate average household consumption ratio $AvgCon^{\mathcal{H}_k}_{ratio}[t]$ at quintile k by the updated average income [18] and consumption:

$$AvgCon^{\mathcal{H}_k}_{ratio}[t] = AvgCon^{\mathcal{H}_k}[t] / AvgInc^{\mathcal{H}_k}[t], \quad k = 1, 2, \ldots, 5. \tag{31}$$

The calculation of household consumption at quintile k in vector/array form is thus:

$$\mathcal{H}_k[C^c, t] = mean(\mathcal{H}_k[Y^{di}, t] \times AvgCon^{\mathcal{H}_k}_{ratio}[t], AvgCon^{\mathcal{H}_k}[t]), \quad k = 1, 2, \ldots, 5. \tag{32}$$

To be in sync with the MEV constraint $Z^{CPI_{gr}}[t]$, after all households finish the calculation for consumptions by Equation (32), we apply the same adjustment as by Equation (17) in Section 3.4:

$$\mathcal{H}[C^c, t] = \mathcal{H}[C^c, t] \times \frac{sum(\mathcal{H}[C^c, t-1]) \times (1 + Z^{CPI_{gr}}[t])}{sum(\mathcal{H}[C^c, t])}. \tag{17}$$

With these new implementations in quintile income and consumptions, we conduct the same rounds of Monte-Carlo simulations for each scenario as in Section 5.2. Taking average from these simulations gives the stress testing results for HIST and MST with quintile distribution for income and consumption.

5.3.3 Default Rates

When taking income distribution into consideration in stress testing, household are under severer financial stress than with universal income distribution, which reflects in a substantial increasing in household's default rate and mortgage default rate, shown in Figure 13 and Table 14. Notice that HIST average default rate increases from 0.52% with universal income distribution to 1.02% with quintile income distribution, HIST average mortgage default rate from 0.40% to 0.71%, MST average default rate from 1.24% to 2.24%, and MST average mortgage default rate from 1.83% to 3.22%.

In particular, with quintile income distribution, HIST default rate has the peak at period 2 for 1.59%, then gradually drops down to the level of 0.8%, with tilting up at the last two periods to the level higher than 1.0%. HIST mortgage default rate shares

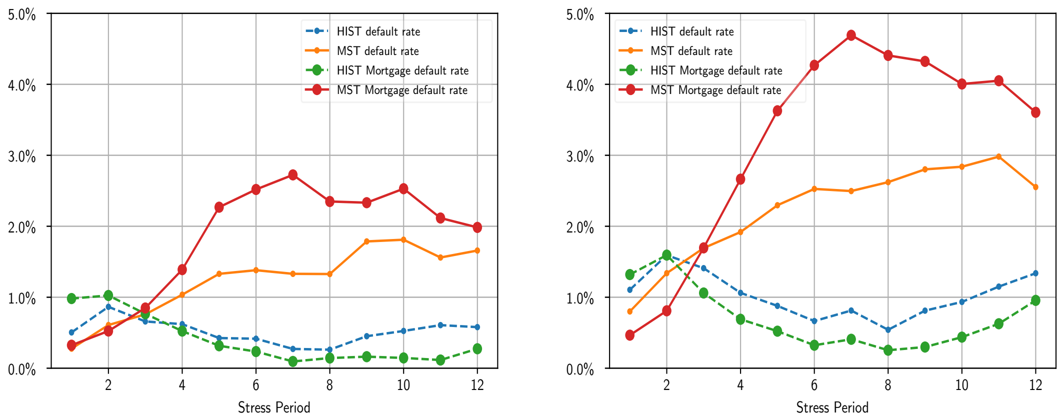
[17] Analogous to average income at quintile k, when period $t = 0$, $AvgCon^{\mathcal{H}_k}[-1]$ is set by values from Table 12. For example, the first quintile household has average consumption $AvgCon^{\mathcal{H}_1}[-1] = 56,504$.

[18] Note that the update in households quintiles $\mathcal{H}_k$ and the adjustment in Equation (13) from Section 3.3 affect the average income $AvgInc^{\mathcal{H}_k}[t]$, thus we need to re-calculate $AvgInc^{\mathcal{H}_k}[t]$ before we apply the calculation in Equation (31).

similar pattern with its peak at period 2 for 1.59%, then gradually dropping down to the level of 0.4%, with tilting up at the last period close to the level of 1.0%.

On the other hand, MST default rate gradually rises up to the peak at period 11 for 2.98%, with a drop at the last period to 2.55%. MST mortgage default rate rises up to the peak at period 7 for 4.69%, then remains at the level of 4% with a drop at the last period to 3.61%.

For a straightforward visual comparison, Figure 13a and 13b show in the same scale the default rates for the stress testing with universal and quintile income distribution. It shows that the introduction of quintile income distribution magnifies the financial stress that households have to face.



(a) With Universal Income Distribution. (b) With Quintile Income Distribution.

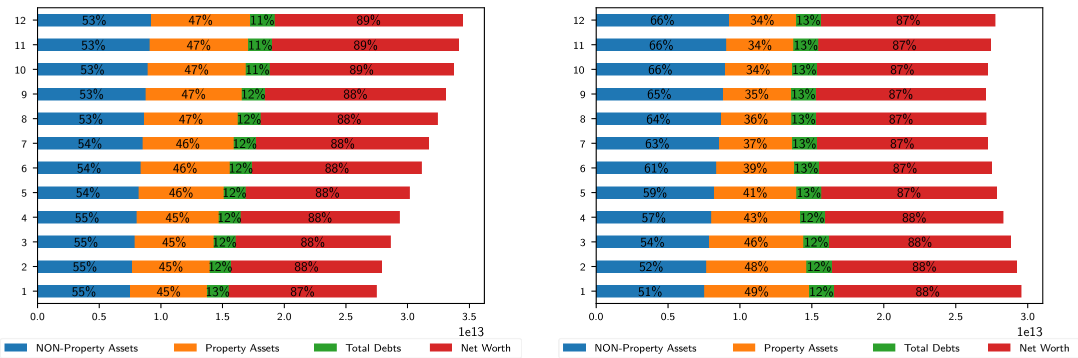
Figure 13: Default rate and MTG default rate.

Stress Period	HIST Default Rate	MST Default Rate	HIST Mortgage Default Rate	MST Mortgage Default Rate
1	1.10%	0.80%	1.32%	0.47%
2	1.59%	1.34%	1.59%	0.81%
3	1.41%	1.69%	1.06%	1.70%
4	1.06%	1.92%	0.69%	2.66%
5	0.88%	2.30%	0.52%	3.63%
6	0.66%	2.53%	0.32%	4.27%
7	0.81%	2.50%	0.41%	4.69%
8	0.54%	2.62%	0.25%	4.41%
9	0.81%	2.80%	0.30%	4.32%
10	0.93%	2.84%	0.44%	4.00%
11	1.15%	2.98%	0.63%	4.05%
12	1.34%	2.55%	0.96%	3.61%

Table 14: Default rate and MTG default rate.

5.3.4 Household Balance Sheet

Figure 14 shows the composition of households balance sheet in aggregate-level. Similar to Figure 9 with universal income distribution, we observe that HIST has an expanding balance sheet while MST a shrinking balance sheet. MST has the proportion of property assets shrinking along with a slightly rising in the proportion of total debts.



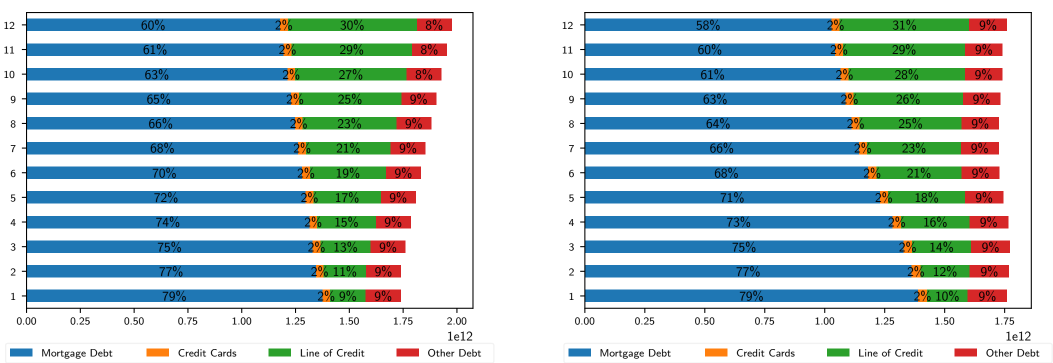
(a) HIST. (b) MST.

Figure 14: Households Balance Sheets: with quintile income partition.

5.3.5 Household Liabilities

Figure 15 shows the break down in the proportion of debt composition. Comparing with Figure 10 with universal income distribution, we notice substantial decrease in the proportion of mortgage debt with significant expanding in the proportion of line of credit in both HIST and MST.

With quintile income distribution, households are more financially stressed, and thus have to put more effort into maintaining mortgage debt to be non-default with the cost of expanding line of credit whenever it is possible. This can be seen from the proportion of line of credit rises at HIST from with universal income distribution for 22% to 30% with quintile income distribution, at MST from 23% to 31%.

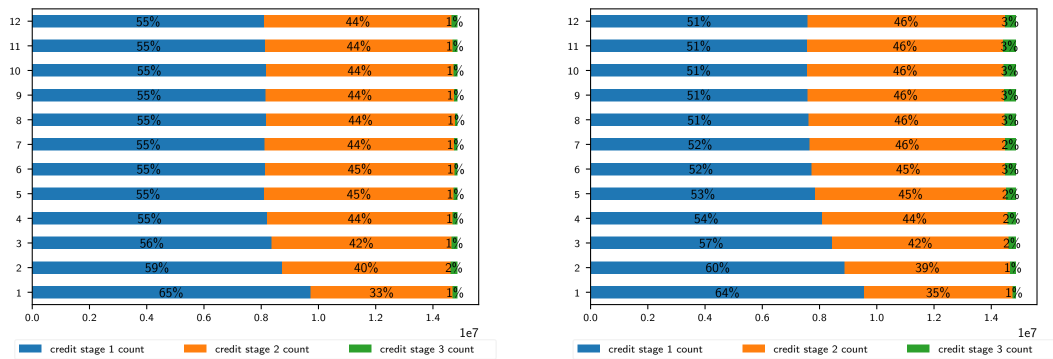


(a) HIST. (b) MST.

Figure 15: Households Liabilities: with quintile income partition.

5.3.6 Credit Stage

More households stand at credit stage 2 under quintile income distribution than those under universal income distribution, with HIST 44% and MST 46% with quintile income distribution against HIST 27% and MST 29% with universal income distribution, as shown in Figure 16 and 11, respectively. This indicates the income distribution accelerates the financial fragility of households in our economy for both HIST and MST scenarios.



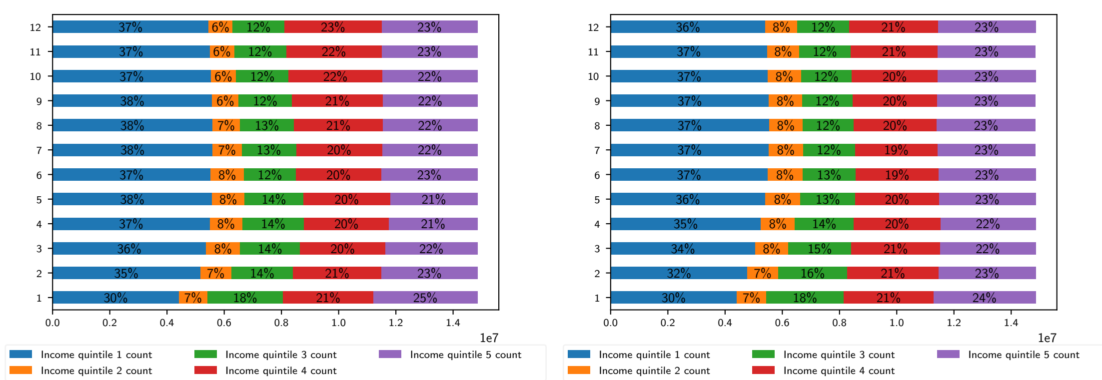
(a) HIST. (b) MST.

Figure 16: Households Credit Stages: with quintile income partition.

5.3.7 Income Distribution

Figure 17 shows the composition of income quintiles, where Figure 17a for HIST and Figure 17b for MST respectively.

Both scenarios show significant increase in the proportion of 1st quintile with the lowest income level, the 3rd quintile is shrinking, and the 5th quintile with the highest income level does not hurt significantly. This middle-class squeezing effect shows up in both HIST and MST, which seems to suggest middle class households endure most damage in either type of economic downturn.



(a) HIST. (b) MST.

Figure 17: Households Income Partition.

6 Concluding Remark

We have developed from "bottom-up" a macroeconomic stress testing framework for retail portfolio in credit risk. This framework of agent-based micro-founded simulation integrates those comparative advantages from methodologies of HRAM and ABM, with its data structure rooted at publicly available household's survey data.

As an application, we conduct a stress testing exercise with the HIST and MST scenarios. The stress testing results indicate transitional pandemic shock has temporary effect against household's financial health while the persistently high interest rate and inflation along with cutting down housing price undermine the financial soundness of our economy. Moreover, we show that income distribution has significant impact on our stress testing results, which also demonstrates the middle-class squeezing effect that households confront in economic downturn.

In our work, we present two options of stress testing, one with universal income distribution and the other with quintile income distribution. To see which option works as better proxy to our economy, recall that HIST scenario by our configuration mimics the pandemic period of 2020Q1 to 2022Q4. Thus, it is plausible to compare HIST default rates with actual default rates for the same period of 2020Q1 to 2022Q4.

Notice that Canada Mortgage and Housing Corporation (CHMC) publishes at "Mortgage and Consumer Credit Trends Data, 2023-Q4" the delinquency rates of retail products, including credit card and mortgage. [19] We consider these CHMC delinquency rates of credit card and mortgage as the benchmark for historical default rate and mortgage default rate, respectively. Figure 18 and Table 15 compare default rates with universal and quintile income distribution against these benchmarking delinquency rates.

HIST with universal income distribution has its default rate much lower than the benchmark, while its mortgage default rate is close to or lower than the benchmark for the period of 2021Q2 to 2022Q3. For the other option, HIST with quintile income distribution has its default rate higher than that with universal income distribution. It hits the benchmark at its peak for the period of 2020Q2, then it stays below for the rest periods and catches up the benchmark again at the last period of 2024Q4. The mortgage default rate with quintile income distribution is higher than the benchmark as well as that with universal income distribution.

As stated in Section 5.1, HIST configuration does not include the distortion in disposable income by government's pandemic relief program that came into effect during pandemic period in our economy. Therefore, we would expect HIST default rates in our stress testing exercise at a higher level than historical benchmarks. In this regard, we consider the option with quintile income distribution roughly a better proxy for presenting our stress testing results.

[19] See `https://www.cmhc-schl.gc.ca/professionals/housing-markets-data-and-research/housing-data/data-tables/mortgage-and-debt/mortgage-consumer-credit-trends-cmas`.

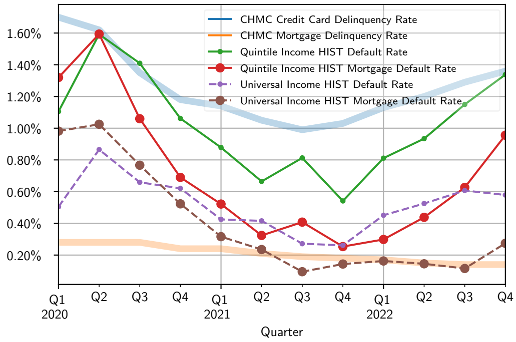


Figure 18: Compare to CHMC benchmark default rates.

Quarter	CHMC CARDS Delinquency Rate	CHMC MTG Delinquency Rate	Quintile Income HIST Default Rate	Quintile Income HIST MTG Default Rate	Universal Income HIST Default Rate	Universal Income HIST MTG Default Rate
2020Q1	1.70%	0.28%	1.10%	1.32%	0.50%	0.98%
2020Q2	1.62%	0.28%	1.59%	1.59%	0.87%	1.02%
2020Q3	1.35%	0.28%	1.41%	1.06%	0.66%	0.77%
2020Q4	1.18%	0.24%	1.06%	0.69%	0.62%	0.52%
2021Q1	1.14%	0.24%	0.88%	0.52%	0.42%	0.32%
2021Q2	1.05%	0.21%	0.66%	0.32%	0.42%	0.23%
2021Q3	0.99%	0.19%	0.81%	0.41%	0.27%	0.10%
2021Q4	1.03%	0.18%	0.54%	0.25%	0.26%	0.14%
2022Q1	1.13%	0.17%	0.81%	0.30%	0.45%	0.16%
2022Q2	1.20%	0.15%	0.93%	0.44%	0.52%	0.14%
2022Q3	1.29%	0.14%	1.15%	0.63%	0.61%	0.12%
2022Q4	1.36%	0.14%	1.34%	0.96%	0.58%	0.27%

Table 15: Compare to CHMC benchmark default rates.

In our MST results, households have gradually increasing default rate and mortgage default rate, with expanding proportion of line of credit. This implies that elevated interest rate and declining housing price reduce the financial flexibility of households. If the high interest rate with declining housing price is persistent with high inflation, financial crisis with cascade effect hitting mortgage market is underway, which is shown in MST with continuously rising in default rate above the level of 2.5% followed by 5-quarter of skyrocketing mortgage default rate above the level of 4%. These findings are consistent with the statement in Bank of Canada's 2023 Financial System Review:

> "Elevated interest rates and declining house prices have reduced the financial flexibility of many households. While most households are proving resilient to increases in debt-servicing costs, early signs of financial stress are emerging. The share of households affected by higher interest rates will continue to rise over the next few years as homeowners renew their mortgages."

Several limitations should be kept in mind. The first limitation is on the household's survey data that we employ as the starting point. We assume that this survey dataset is representative to households in our economy. Note that it is not unlikely those households standing at either extreme in income distribution are less inclined to participate in the survey. Thus, the representativeness of our stress testing results should be interpreted with caution. Another limitation is the missing of explicit modeling other components in our economy such as firms, banks, government, import, export. Tax scheme is not considered in our work either. These missing pieces interacting with households produce more comprehensive picture of how our economy dynamics look like. Hence, in the future we would like to extend along this direction. Last but not the least, behavioral rules employed in our work for households are mainly driven by economic heuristics combined with inference from the survey data. It leaves much space for further improvement and refinement with calibration at more granular scale in terms of modeling economic agent's behavioral and interaction rules, which shall be considered as another direction in our future work.

Our framework employs financial accounting for formulating states and dynamics of micro-level economic agents as well as aggregate-level household in our economy. This presentation is rooted directly from public use microdata from Statistics Canada, which can be shared with other data source from the same vein. It then utilizes quantitative tools from statistics and computer programming with heuristics from economics to solve economic problems in practice. This invokes a mission of conducting economic research and modeling in line with financial engineering. In this regard, we would like to invoke this mission as economic engineering.

References

[1] Christophe Deissenberg, Sander van der Hoog, and Herbert Dawid. Eurace: A massively parallel agent-based model of the european economy. *Applied Mathematics and Computation*, 204(2):541–552, 2008.

[2] Centre for Income and Socio economic Well-being Statistics. 2019 survey of financial security: Public use microdata file user guide. Archived, Statistics Canada, 2021. `STATCAN.income-revenu.STATCAN@canada.ca`.

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[4] Tiff Macklem, Carolyn Rogers, Paul Beaudry, Toni Gravelle, Sharon Kozicki, and Nicolas Vincent. Financial system review—2023. Publications, Bank of Canada, 2023.

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A Scenario Visualization

Here we show visualized comparison between HIST and MST scenario MEVs in Figure 19 to Figure 23 for disposable income growth rate, unemployment rate, unemployment duration, CPI growth rate, borrower loan rate, prime rate, and house price index (HPI), respectively.

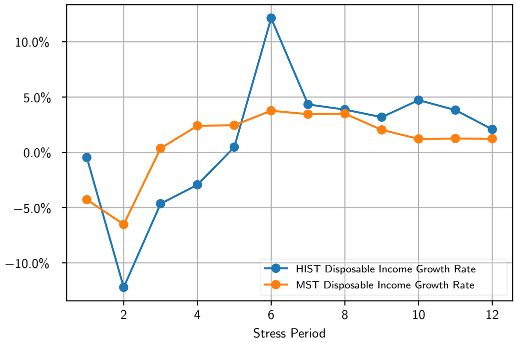
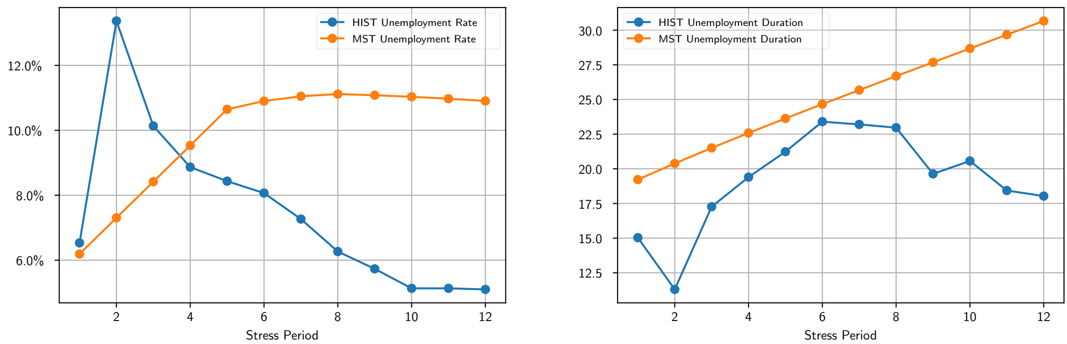


Figure 19: Scenarios: Disposable income.



(a) Unemployment Rate.

(b) Unemployment Duration.

Figure 20: Scenarios: Job Market.

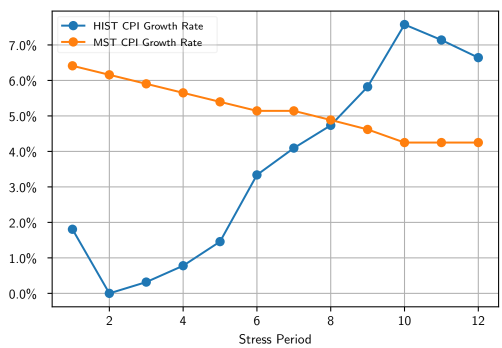
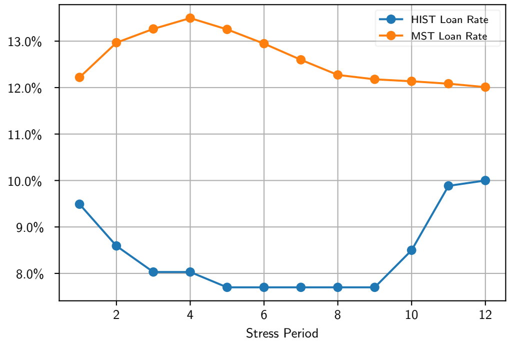
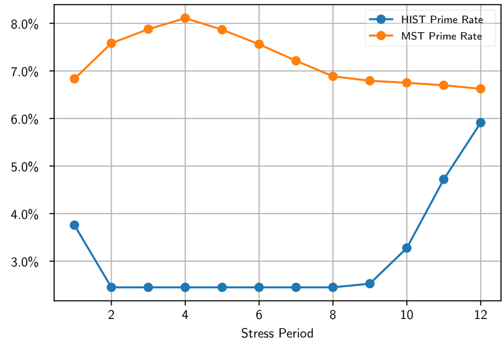


Figure 21: Scenarios: CPI Growth Rate.



(a) Borrower Loan Rate.



(b) Prime Rate.

Figure 22: Scenarios: Interest Rate.

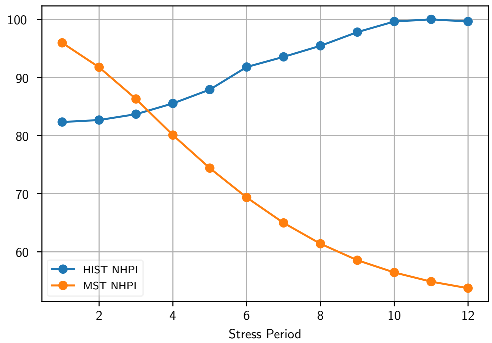


Figure 23: Scenarios: House Price Index.

B Mortgage Deferral Program

Here we investigate the effect of mortgage deferral program that was employed by the government during pandemic. Basically, this deferral program allows households to extend their mortgage obligation terms as long as they are able to cover their interest expense generated from the mortgage contract. This implies households can still be regarded as non-default even when their effective credit space is not enough to cover its net cash flow deficit. In particular, we extend the determination of household's credit stage with mortgage deferral program by:

- Credit stage 1: net cash flow $CF^{net} \geq 0$;
- Credit stage 2: net cash flow $CF^{net} < 0$, effective credit space CS^{eff} enough to cover net survival cash flow SCF^{net} with $CS^{eff} + SCF^{net} \geq 0$, where net survival cash flow SCF^{net} is by

$$\mathcal{H}[SCF^{net}, t] = \mathcal{H}[Y^{di}, t] - \mathcal{H}[C^{c}, t] - \mathcal{H}[C^{ir}, t] - \mathcal{H}[pay^{mtg_r}, t].$$

- Credit stage 3: effective credit space CS^{eff} not enough to cover survival net cash flow SCF^{net} with $CS^{eff} + SCF^{net} < 0$.

Household's credit stage decision tree with mortgage deferral program is depicted in Figure 24:

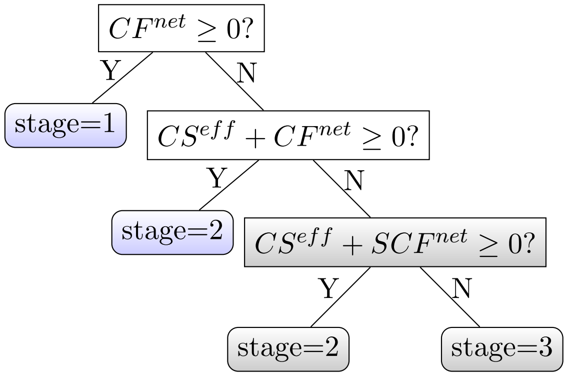


Figure 24: Decision Tree of Credit Stage Determination.

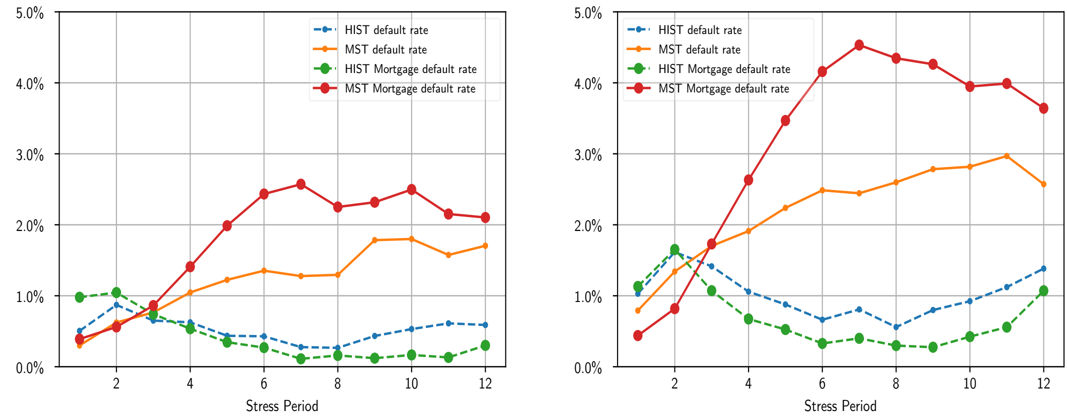
For the new type of credit stage 2 household with mortgage deferral, it has net cash flow $CF^{net} < 0$, $CS^{eff} + CF^{net} < 0$, and $CS^{eff} + SCF^{net} \geq 0$.

Assume the household is able to expand "Line of Credit" up to the limit of its effective credit space to fill the survival net cash flow deficit. the net effect on its balance sheet settlement is summarized in Table 16.

	Debit	**Credit**
Mortgage Debt (-)	$CS^{eff} + SCF^{net}$	
Line of Credit (+)		CS^{eff}
Net Worth (+/-)		SCF^{net}

Table 16: Balance Sheet Settlement, Credit Stage 2 Household with Mortgage Deferral Program.

With mortgage deferral program, default rate and mortgage default rate for HIST and MST scenarios under universal income distribution and quintile income distribution are shown in Figure 25. Comparing to the results with no mortgage deferral program, we notice that mortgage deferral program has indeed lowered a bit mortgage default rates. This indicates that mortgage deferral program helps in a limited sense to provide support for household's financial robustness.

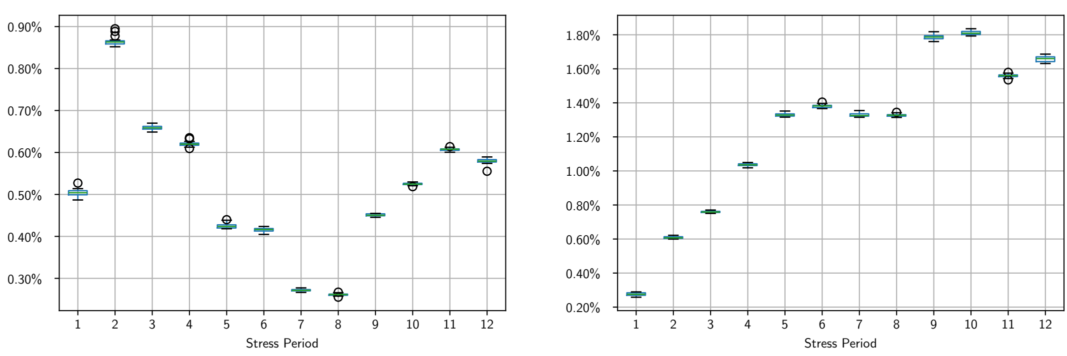


(a) With Universal Income Distribution. (b) With Quintile Income Distribution.
Figure 25: Default rate and MTG default rate, with Mortgage Deferral Program.

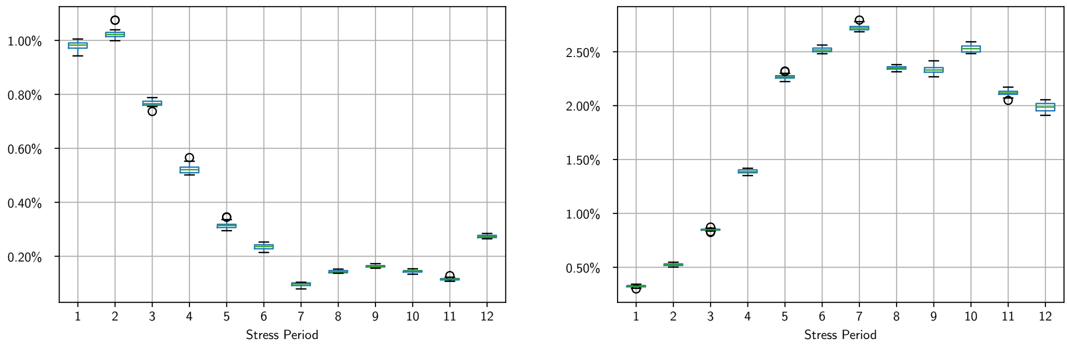
C Distributions of Simulated Default Rates

Here we visualize by Box plots the distribution of default rates and mortgage default rates for each scenario with universal income distribution and with quintile income distribution by each Monte-Carlo simulation. We also report these Box plots for simulations with mortgage deferral program.

As seen from these visualizations, default rates and mortgage default rates produced from each simulation are close to each other. This thus gives us confidence to report results for 20 Monte-Carlo simulations by averaging.

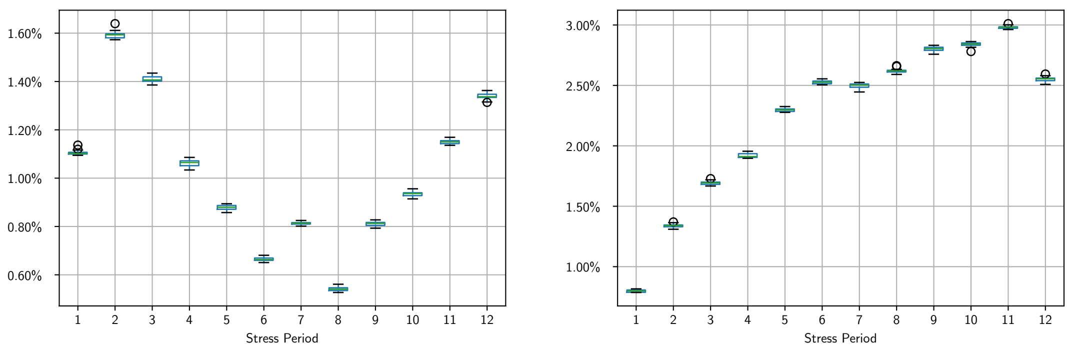


(a) HIST. (b) MST.
Figure 26: Distribution by Simulation: Default Rate, with Universal Income Distribution.



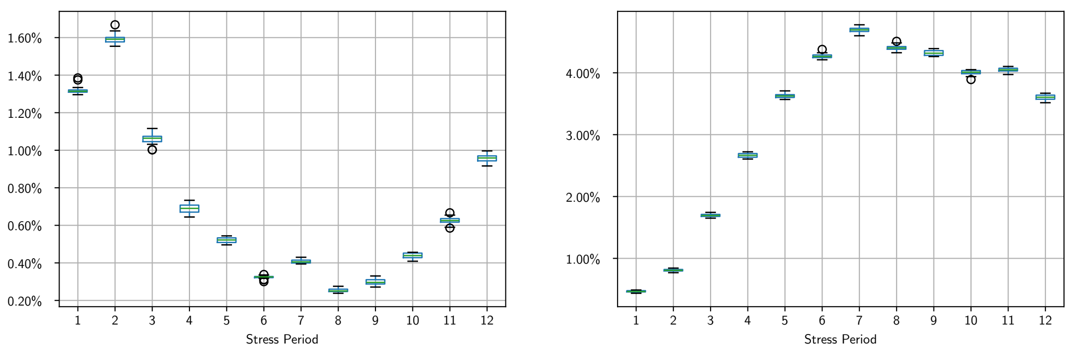
(a) HIST. (b) MST.

Figure 27: Distribution by Simulation: Mortgage Default Rate, with Universal Income Distribution.



(a) HIST. (b) MST.

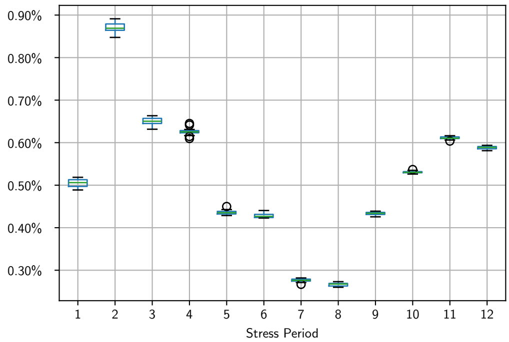
Figure 28: Distribution by Simulation: Default Rate, with Quintile Income Distribution.



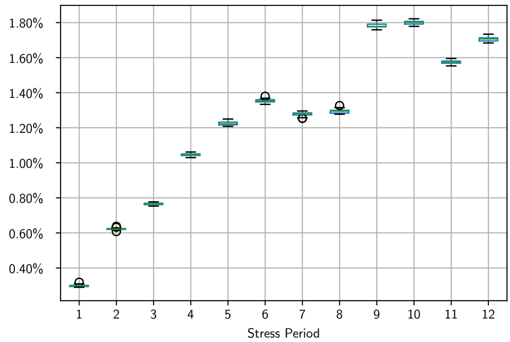
(a) HIST. (b) MST.

Figure 29: Distribution by Simulation: Mortgage Default Rate, with Quintile Income Distribution.

C.1 Extra Plots: with Mortgage Deferral Program

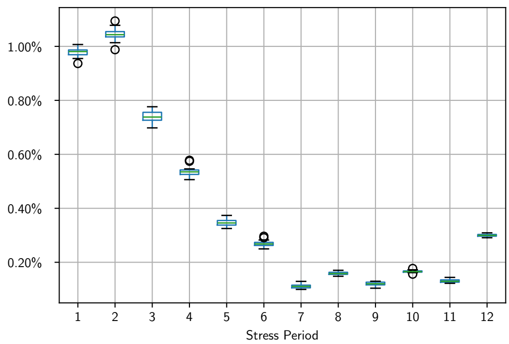


(a) HIST.

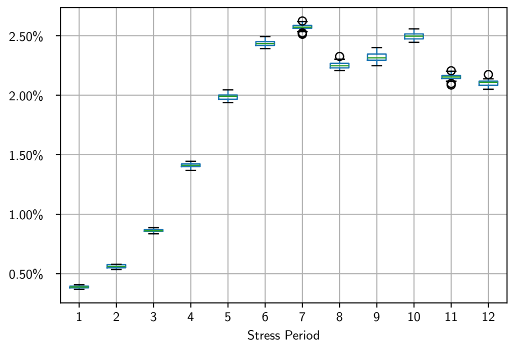


(b) MST.

Figure 30: Distribution by Simulation: Default Rate, with Universal Income Distribution.

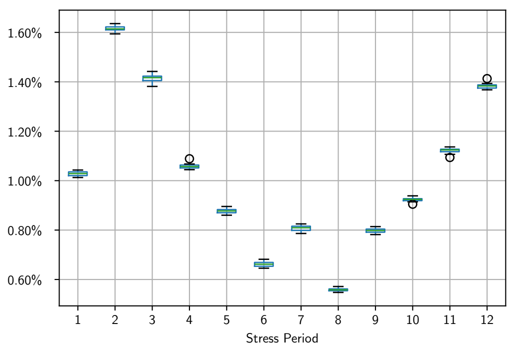


(a) HIST.

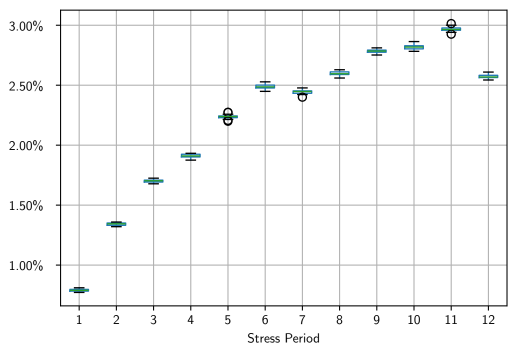


(b) MST.

Figure 31: Distribution by Simulation: Mortgage Default Rate, with Universal Income Distribution.

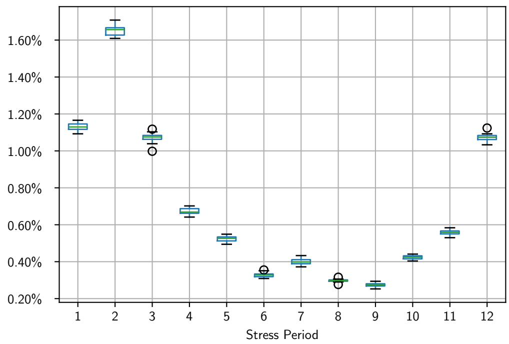


(a) HIST.

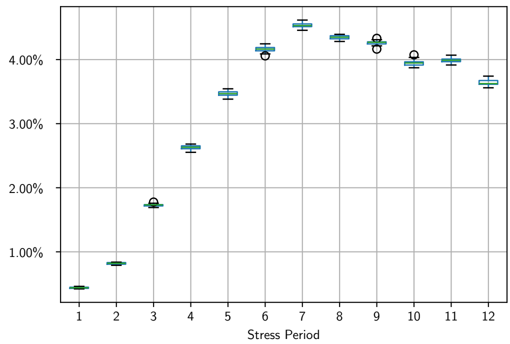


(b) MST.

Figure 32: Distribution by Simulation: Default Rate, with Quintile Income Distribution.



(a) HIST.



(b) MST.

Figure 33: Distribution by Simulation: Mortgage Default Rate, with Quintile Income Distribution.