

# Chapter 5

1, 2, 5, 11, 12, 13, 14, 16

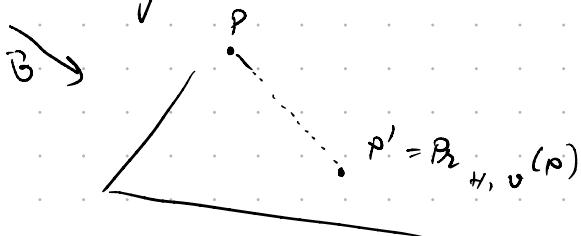
$f: \mathbb{A}^m \rightarrow \mathbb{A}^m$  - affine morphism

$\forall A, B \in \mathbb{A}^m$

$$f(\vec{AB}) = \overrightarrow{f(A) f(B)}$$

$$f(P) = A \cdot P + b, A \in \mathbb{M}_{n \times m}(\mathbb{R}), b \in \mathbb{M}_n(\mathbb{R})$$

Projection on hyperplane along a dir  $\vec{v}$



$$H: a_1x_1 + \dots + a_nx_n + a_{n+1} = 0$$

$$a = (a_1, \dots, a_n) = \vec{m}_H$$

$$P_{H, \vec{v}}(P) = \left( J_m - \frac{v \otimes a}{\langle v, a \rangle} \right) \cdot P - \frac{a_{n+1}}{\langle v, a \rangle} \cdot a$$

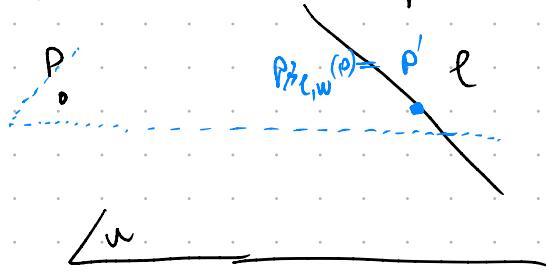
$$v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{pmatrix} \quad w = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix}$$

$$v \otimes w = v \cdot w^T = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \cdot [y_1 \dots y_m] = \begin{bmatrix} x_1 y_1 & x_1 y_2 & \dots & x_1 y_m \\ \vdots & \vdots & \ddots & \vdots \\ x_m y_1 & \dots & x_m y_m \end{bmatrix}$$

$$\langle v, w \rangle = v^T \cdot w = x_1 y_1 + x_2 y_2 + \dots + x_m y_m$$

$$P_{H, \vec{v}}^\perp = P_{H, a} \Rightarrow P_{H, \vec{v}}(P) = J_m - \left( \frac{a + a}{\langle a, a \rangle} \right) P - \frac{a_{n+1}}{\langle a, a \rangle} \cdot a$$

Projection on a line  $\ell$  parallel to a hyperplane  $w$ .



$Q \in \ell, v \in \Delta(\ell)$

$$\rightarrow P_{\ell, w}(P) = \frac{v \otimes a}{\langle v, a \rangle} \cdot P - \left( I_n - \frac{v \otimes a}{\langle v, a \rangle} \right) \cdot a$$

$$P_{\ell, \perp}(P) = \frac{a \otimes a}{\langle a, a \rangle} \cdot P - \left( I_n - \frac{a \otimes a}{\langle a, a \rangle} \right) \cdot Q$$

J.1. - coord planes  
- coord axis.

(xoy):  $z=0$

(yoz):  $x=0$

(zox):  $y=0$

$$Ox: \begin{cases} y=0 \\ z=0 \end{cases}, oy: \begin{cases} z=0 \\ x=0 \end{cases}, oz: \begin{cases} x=0 \\ y=0 \end{cases}$$

$$P_{n, \text{xoza}}(P) = \left( I_n - \frac{a \otimes a}{\langle a, a \rangle} \right) \cdot P - \frac{a_{n+1}}{\langle a, a \rangle} \cdot a$$

$$a \otimes a = (0, 0, 1) \cdot \begin{pmatrix} 0 & & & \\ 0 & 0 & & \\ 0 & & 1 & \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow P_{n, \text{xoza}}(P) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot P$$

$$P_{n, \text{yoza}}(P) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot P$$

$$P_{y=2, a}(\mathbf{P}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \mathbf{P}$$

We choose  $\mathbf{Q} = \mathbf{0} = (0, 0, 0)$

$$\begin{aligned} P_{\text{ox}}^{-1}(\mathbf{P}) &= \frac{(1, 0, 0) \otimes (1, 0, 0)}{\langle (1, 0, 0), (1, 0, 0) \rangle} \cdot \mathbf{P} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \mathbf{P} \end{aligned}$$

$$P_{\text{oy}}^{-1}(\mathbf{P}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \mathbf{P}$$

$$P_{\text{o2}}^{-1}(\mathbf{P}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \mathbf{P}$$

$$\text{Ref}_{H, \mathbf{v}}(\mathbf{P}) = \left( I_3 - \frac{2(\mathbf{v} \otimes \mathbf{v})}{\langle \mathbf{v}, \mathbf{v} \rangle} \right) \cdot \mathbf{P} - \frac{2\mathbf{v}\mathbf{v}^T}{\langle \mathbf{v}, \mathbf{v} \rangle} \cdot \mathbf{v}$$

T.4. Orthogonal reflection of  $P(6, -5, 1)$  in  $H: 2x - 3y + 2z - 4 = 0$  by dot the matrix form  
 $(2, 0, 0) \in H$

$$\vec{\alpha} = \vec{\omega} = (2, -3, 1)$$

$$P_{H, \vec{\alpha}}^{-1}(\mathbf{P}) = \frac{(2, -3, 1) \otimes (2, -3, 1)}{\langle (2, -3, 1), (2, -3, 1) \rangle} \cdot \mathbf{P} - \frac{-4}{\langle (2, -3, 1), (2, -3, 1) \rangle} \cdot \mathbf{P}$$

$$= \frac{1}{7} \begin{bmatrix} 4 & -6 & 2 \\ -6 & 9 & -3 \\ 2 & -3 & 1 \end{bmatrix} \cdot \mathbf{P} + \frac{4}{7} (2, -3, 1)$$

$$= \begin{bmatrix} 10 & 6 & -2 \\ 6 & 5 & 3 \\ -2 & 3 & 13 \end{bmatrix} \cdot \mathbf{P} + \frac{1}{7} (4, -6, 2)$$

$$= \begin{bmatrix} 10 & 6 & -2 \\ 6 & 5 & 3 \\ -2 & 3 & 13 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 5 \end{bmatrix} \begin{bmatrix} 20 \\ 60 - 20 - 10 \\ 26 - 25 + 5 - 10 \\ -2 = 15 + 65 \\ 80 \\ 32 \end{bmatrix} = \begin{bmatrix} 20 \\ 26 \\ 38 \end{bmatrix} + \frac{1}{7} (4, -6, 2) / 2$$

$$= \begin{bmatrix} 10 \\ 13 \\ 19 \end{bmatrix} + \frac{1}{7} (2, -3, 1)$$

$$\text{Reflection} \Rightarrow 2 \cdot (2, 1, 3) - \begin{pmatrix} 6 \\ -5 \\ 7 \end{pmatrix} \begin{pmatrix} -2 \\ 7 \end{pmatrix} = \begin{pmatrix} \frac{88}{7} \\ -\frac{11}{7} \\ \frac{15}{7} \end{pmatrix}$$

$$f(p) = Ap + b$$

$$f = \begin{pmatrix} A & | & b \\ 0 & | & 1 \end{pmatrix}$$

$$f(\bar{p}) = \begin{pmatrix} A & | & b \\ 0 & | & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

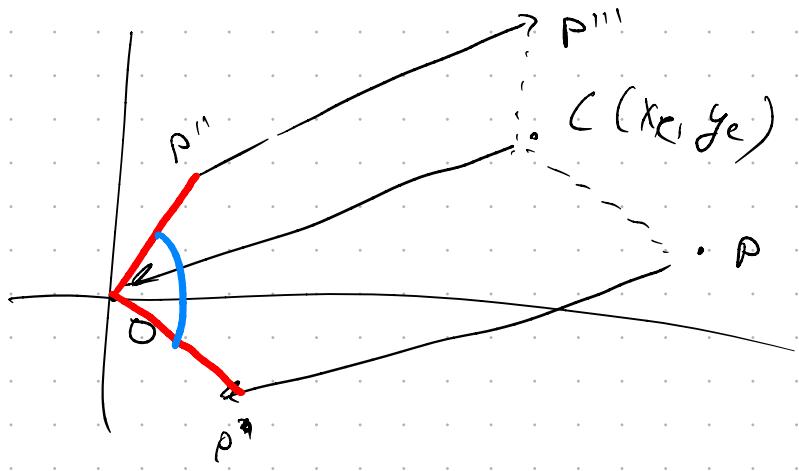
$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} & | & b_1 \\ \vdots & & & & | & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} & | & b_m \\ 0 & 0 & \dots & 0 & | & 1 \end{pmatrix} \xrightarrow{x_1 \quad \dots \quad x_m}$$

$$\begin{pmatrix} a_{11}x_1 + \dots + a_{1m}x_m + b_1 \\ \vdots \\ a_{m1}x_1 + \dots + a_{mm}x_m + b_m \end{pmatrix}$$

$$\begin{pmatrix} A & P + b \\ | & \end{pmatrix}$$

$$\text{Rot}_\theta(p) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \cdot p$$

$$\overline{\text{Rot}}_\theta = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$\text{Rot}_{C,\theta}(P) = ?$$

$$P' = T_{(-x_C, -y_C)}(P)$$

$$P'' = \text{Rot}_O(P)$$

$$P''' = T_{(x_C, y_C)}(P')$$

$$\Rightarrow \text{not}_{C,\theta}(P) = T_{(x_C, y_C)} \circ \text{Rot}_O \circ T_{(-x_C, -y_C)}(P)$$

$$\overline{\text{Rot}_{C,\theta}(P)} = \overline{T_{(x_C, y_C)}} \circ \overline{\text{Rot}_O} \circ \overline{T_{(-x_C, -y_C)}(P)}$$

$$= \begin{pmatrix} 1 & 0 & x_C \\ 0 & 1 & y_C \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -x_C \\ 0 & 1 & -y_C \\ 0 & 0 & 1 \end{pmatrix}$$

=

$$14. \quad A(1,1), B(4,1), C(2,3)$$

Find

$$\text{Ref}_{AB} \circ \text{Rot}_{C,\frac{\pi}{2}}(ABC)$$

$$\text{Rot}_{c, \frac{\pi}{2}}(\bar{A}) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\text{Ref}_{\vec{AB}, \vec{AB}}^{\perp}(P) = \frac{\vec{AB} \otimes \vec{AB}}{\langle \vec{AB}, \vec{AB} \rangle} \cdot P - \left( g_2 \cdot \frac{\vec{AB} \otimes \vec{AB}}{\langle \vec{AB}, \vec{AB} \rangle} \right) A$$

$$\text{Ref}_{\vec{AB}, \vec{AB}}^{\perp}(P) = \left( 2 \cdot \frac{\vec{AB} \otimes \vec{AB}}{\langle \vec{AB}, \vec{AB} \rangle} - g_2 \right) P - \left( I_2 - \frac{\vec{AB} \otimes \vec{AB}}{\langle \vec{AB}, \vec{AB} \rangle} \right) A$$

$$\vec{AB} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$\vec{AB} \otimes \vec{AB} = \begin{pmatrix} 9 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\langle \vec{AB}, \vec{AB} \rangle = 9$$

$$\Rightarrow \text{Ref}_{\vec{AB}, \vec{AB}}^{\perp}(P) = \left( \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) P - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} -$$

$$- \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} P - \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} P + \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{aligned}
 \text{Rd}_{A_B}^T &= \left( \begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right) \Rightarrow \text{Rd}_{A_0}^T \in \text{Rec}_{\mathbb{D}_2}(A) \\
 &= \left( \begin{array}{ccc} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \right) \\
 &= \left( \begin{array}{c} 1 \\ -1 \\ 1 \end{array} \right) \Rightarrow A^T \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}
 \end{aligned}$$