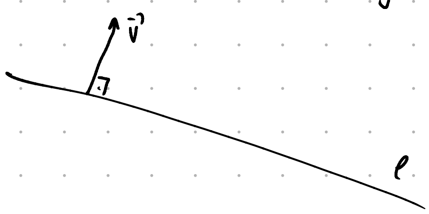
26, 29, 30, 31, 32, 33, 35, 37, 38, 39, 42

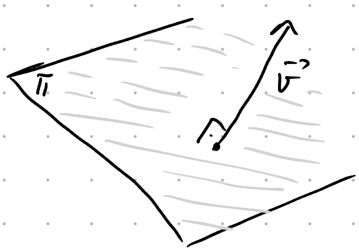
- coords in orthonormal system

l: $ax + by + c = 0$
line in $\mathbb{E}^2$

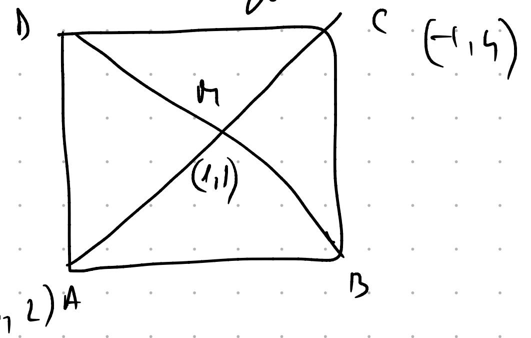
$\vec{v}(a,b)$ is a normal vector of l



π: $ax + by + cz + d = 0$ plane in $\mathbb{E}^3$
$\Rightarrow \vec{v}(a,b,c)$ - normal vector of π



3.26. $A(3,-2)$ is the vertex of a square ABCD, $\underset{(1,1)}{M}$ - int. point of the diags.
find the cartesian eq for the sides of the square



$$m_A = \frac{y_M - y_A}{x_M - x_A} = \frac{1+2}{2} = \frac{3}{2}$$

$$x_M = \frac{x_A + x_C}{2} = 1 = \frac{3 + x_C}{2} \Rightarrow x_C = -1$$

$$y_M = \frac{y_A + y_C}{2} = 1 \Rightarrow -2 + y_C = 2 \Rightarrow y_C = 4$$

$$AC \perp BD \Rightarrow m_{AC} \cdot m_{BD} = -1 \Rightarrow m_{BD} = \frac{2}{3}$$

$$y - y_M = \frac{2}{3}(x - 1)$$
$$\Rightarrow BD: \ y = \frac{2}{3}x + \frac{1}{3}$$

$AM = \sqrt{(1-3)^2 + (1+2)^2} = \sqrt{4+9} = \sqrt{13}$

$\Rightarrow AC = 2\sqrt{13}$

$B, D : \begin{cases} y = \frac{2}{3}x + \frac{1}{3} \\ (y - y_M)^2 + (x - x_M)^2 = AM^2 \end{cases}$

$\Rightarrow (y-1)^2 + (x-1)^2 = 13 \Rightarrow y^2 - 2y + x^2 - 2x = 11$

$\cancel{\frac{4}{9}x^2 + \frac{4}{9}x + \frac{1}{9} - \frac{4}{3}x - \frac{2}{3} + x^2 - 2x = 11 \quad \Big| \cdot 9}$

$\cancel{4x^2 + 4x + 1 - 12x - 6 + 9x^2 - 18x = 99}$

$\cancel{11x^2 - 26x - 94 = 0}$

$\frac{4}{9}(x-1)^2 + (x-1)^2 = 13$

$\Rightarrow \frac{13}{9}(x-1)^2 = 13 \Rightarrow (x-1)^2 = 9$

$\Rightarrow x - 1 = \pm 3$

$\Rightarrow x_1 = 4 \Rightarrow y = \frac{8}{3} + \frac{1}{3} = 3$

$x_2 = -2 \Rightarrow y = -\frac{4}{3} + \frac{1}{3} = -1$

$\Rightarrow (4,3) \ , \ (-2,-1)$

If $B(-2,-1) \Rightarrow D = (4,3)$

$\overrightarrow{AB} = (-2,-1) - (3,-2)$

$= (-5, 1)$

$\overrightarrow{DC} = (4,3) + (-1,4)$

$= (-5, +1)$

AD : $\cancel{y - y_D} = \frac{\cancel{y_D - y_A}}{x_D - x_A}(x - x_A)$

: $y - 3 = \frac{3+2}{1}(x-3)$

$y - 3 = 5(x-3)$

$y = 5x - 15 + 3$

$y = 5x - 12.$

BC: $y - y_C = \frac{y_C - y_B}{x_C - x_B}(x - x_C)$

$y - 4 = \frac{4+2}{-1-1}(x+1)$

$y - 4 = 3x + 3$

$y = 3x + 7$

3.30. $A(2,1,0)$

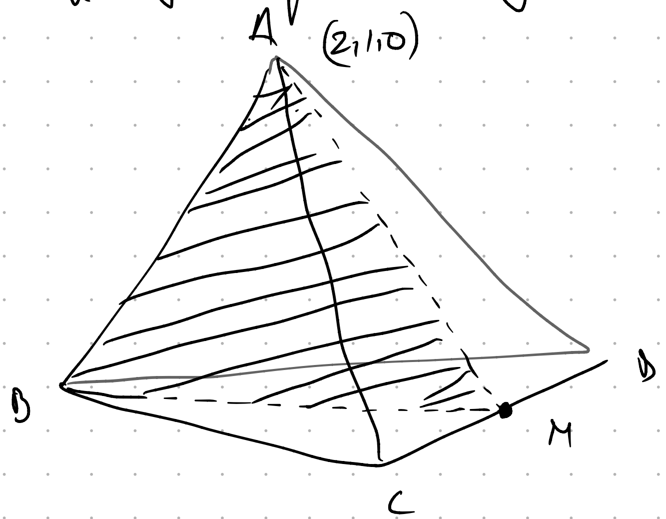
$B(1,3,5)$

$C(6,3,4)$

$D(0,-7,8)$

Tetrahedron

Cart. eq. of the plane containing [AB] and the midpoint M of [CD]



- 3 points
- 1 point & 2 vectors

$Ax + By + Cd + Z = 0$ implicit form.

$$\Pi: \begin{cases} x = x_A + \lambda x_{\vec{v}} + \mu x_{\vec{w}} \\ y = y_A + \lambda y_{\vec{v}} + \mu y_{\vec{w}} \\ z = z_A + \lambda z_{\vec{v}} + \mu z_{\vec{w}} \end{cases}$$

The cartesian (symmetric form):

$$\begin{vmatrix} x - x_A & y - y_A & z - z_A \\ x_{\vec{v}} & y_{\vec{v}} & z_{\vec{v}} \\ x_{\vec{w}} & y_{\vec{w}} & z_{\vec{w}} \end{vmatrix} = 0$$

$$\vec{BM} = \frac{1}{2}\left(\vec{BC} + \vec{BD}\right)$$

$$= \frac{1}{2}\left(\vec{BA} + \vec{AC} + \vec{BA} + \vec{AD}\right)$$

$$= \vec{BA} + \frac{1}{2}\left(\vec{AC} + \vec{AD}\right)$$

$$\vec{AB} + \vec{BM} = \vec{AM} = \frac{1}{2}\left(\vec{AC} + \vec{AD}\right)$$

$$= \frac{1}{2}\left((4, 2, 4) + (-2, -8, 8)\right)$$

$$= \frac{1}{2}(2, -6, 12)$$

$$= (1, -3, 6) = \vec{AM}$$

$\vec{AB} = (-1, 2, 5)$

$\vec{A} = (2, 1, 0)$

$$\rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & 6 \\ -1 & 2 & 5 \end{vmatrix}$$

$x = 2 + 2 - 3$

$$-15(x-2) + 2z - 6y + 6 - 3z - 12x + 24 - 5y + 5$$

$$= -15x + 30 + 2z - 6y + 6 - 3z - 12x + 24 - 5y + 5$$

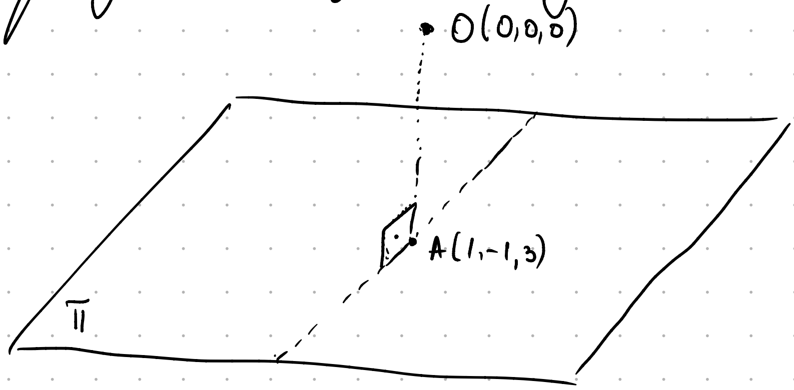
$$= -27x - 11y - z + 65 = 0$$

SAU

exam

$$\begin{vmatrix} x & y & z & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 3 & 5 & 1 \\ 3 & -2 & 6 & 1 \end{vmatrix} = 0$$

32. Det. a cartesian eq. of the plane π if $A(1,-1,+3)$ is the orthogonal projection of the origin on π



$\vec{OA}$ - normal vector of π

$\Rightarrow \vec{OA}(1,-1,3)$

$\Rightarrow \pi: x - y + 3z + d = 0$

$1 + 1 + 9 = -d$

$\Rightarrow d = -11$

$\Rightarrow \pi: x - y + 3z - 11 = 0$

33. Det. the dist. bet the planes.

$\pi_1: x - 2y - 2z + 7 = 0$

$\pi_2: 2x - 4y - 4z + 17 = 0.$

1. Check if they are $\parallel$ or not using the normal vector.

$\left.\begin{array}{l} \vec{v_1} = (1,-2,-2) \\ \vec{v_2} = (2,-4,-4) = 2\vec{v_2} \end{array}\right\} \Rightarrow$ they are $\parallel$

$\Rightarrow \text{dist}(\pi_1, \pi_2) = \text{dist}(P, \pi_2) \ \forall P \in \pi_1$

$P \in \pi_1 \Rightarrow$ let $P(-7,0,0)$

$$\Rightarrow d(P, \pi_2) = \frac{|-14+17|}{\sqrt{4+16+16}} = \frac{3}{6} = \frac{1}{2}$$

37. Det. values a and c for which the line ℓ given by the eq $\begin{cases} 3x - 2y + z + 3 = 0 \\ 4x - 3y + 4z + 1 = 0 \end{cases}$ is perpendicular.

to the plane π given by $ax + 8y + cz + 2 = 0$

$x - y + 3z - 2$

$\cancel{3x} - 2\cancel{y} + \cancel{z} + \cancel{3} = \cancel{4x} - \cancel{3y} + \cancel{4z} + \cancel{1}$

$\Rightarrow x - y + 3z - 2 = 0$

$$\left|\begin{array}{ccc|c} 3 & -2 & 1 & -3 \\ 4 & -3 & 4 & -1 \\ 1 & -1 & 3 & 2 \end{array}\right|$$

$$-\cancel{27} - \cancel{4} - \cancel{8} + \cancel{3} + \cancel{12} + \cancel{24}\ 3$$
$$-2 \quad +6 \quad -6 \quad +6 \neq 0.$$

$$\Rightarrow \begin{aligned} 3x - 2y &= -3 - z \quad | \cdot 4 \\ 4x - 3y &= -1 - 4z \quad | \cdot 3 \end{aligned}$$

$$\begin{aligned} 12x - 8y &= -12 - 4z \\ 12x - 9y &= -3 - 12z \end{aligned} \quad \ominus$$

$$-y = 9 - 8z \Rightarrow y = 8z - 9$$

$$3x - 16z - 18 = -3 - z.$$

$$3x = -3 - z + 16z + 18$$

$$x = \frac{1}{3}(15z + 15) \Rightarrow x = 5z + 5.$$

$$\Rightarrow S = \{(5\alpha + 5,\ 8\alpha - 9,\ \alpha)\}$$

Noral vedor $\Rightarrow (a, 8, c)$

$$\Rightarrow 5\alpha + 5 = a \Rightarrow 5 \cdot \frac{17}{8} + 5 = a$$

$$8\alpha - 9 = 8 \Rightarrow 8\alpha = 17 \Rightarrow \alpha = \frac{17}{8}$$

$$\alpha = c \Rightarrow \boxed{c = \frac{17}{8}}$$

$$\Rightarrow \frac{85 + 40}{8} = a \Rightarrow \boxed{\frac{125}{8} = a}$$

$$-8x + 5y - 11 = 0$$

$$y = \frac{11 + 8x}{5}$$

$$\Rightarrow z = -3x + \frac{22 + 16x}{5} - 3$$

$$z = \frac{\cancel{x+7}}{5}$$

$$\Rightarrow \ell = \left\{ \alpha,\ \frac{11 + 8\alpha}{5},\ \frac{\alpha + 7}{5} \right\}$$

$D(\ell) = \langle (1, \frac{8}{5}, \frac{1}{5}) \rangle = \langle (5, 8, 1) \rangle$

$\ell \perp \pi \iff (5, 8, 1) \parallel (a, 8, c) \Rightarrow a = 5$
$c = 1$

5

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