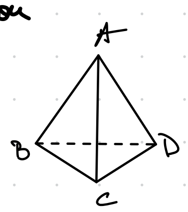
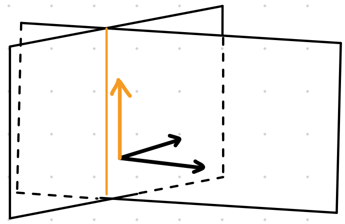
4.18. ABCD tetrahedron

A(2,-1,1)
B(5,5,4)
C(3,2,-1)
D(4,1,3)





Determine the common perpendicular on AB and CD

$$\vec{d_0} \times \vec{d_1} = \begin{vmatrix} i & j & k \\ p_0 & n_0 & q_0 \\ p_1 & n_1 & q_1 \end{vmatrix}$$

$\vec{AB}(3,6,3)$
$\vec{CB}(1,-1,4)$

$$\Rightarrow \vec{AB} \times \vec{CB} = \begin{vmatrix} i & j & k \\ 3 & 6 & 3 \\ 1 & -1 & 4 \end{vmatrix} = 24i - 3k + 3j - 6k + 3i - 12j =$$
$$= 27i - 9j - 9k$$

A(2,-1,1)

$$\begin{vmatrix} x-2 & y+1 & z-1 \\ 3 & 6 & 3 \\ 27 & -9 & -9 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} x-2 & y+1 & z-1 \\ 1 & 2 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 0$$

$$-2x+4-z+1+3y+3-6z+6+x-2+y+1=0$$
$$-x+4y-7z+13=0$$

C(3,2,-1)

$$\begin{vmatrix} x-3 & y-2 & z+1 \\ 3 & -1 & -1 \\ 1 & -1 & 4 \end{vmatrix} = 0 \quad = -4x+12-3z-\cancel{3}-y+\cancel{2}+z+\cancel{1}-x+3-12y+24=$$
$$= -5x-13y-2z-39=0$$

$$\begin{cases} -x+4y-7z+13=0 \\ -5x-13y-2z-39=0 \end{cases}$$

* 2.1, 2.5, 2.10, 2.11, 2.14, 2.18, 2.19, 2.26, 2.27, 2.30

4.17

$\vec{a}(8,4,1)$ $\quad$ $\vec{c}(1,1,1)$
$\vec{b}(2,2,1)$ $\quad$ $\vec{d}(x,y,z)$

a) $\measuredangle(\vec{d},\vec{a}) = \measuredangle(\vec{d},\vec{b})$

$|\vec{a}| = \sqrt{64+16+1} = \sqrt{81} = 9$

$|\vec{b}| = \sqrt{4+4+1} = \sqrt{9} = 3$

$\vec{d} \cdot \vec{a} = |\vec{d}| \cdot |\vec{a}| \cdot \cos(\measuredangle(\vec{d},\vec{a}))$

$\vec{d} \cdot \vec{b} = |\vec{d}| \cdot |\vec{b}| \cdot \cos(\measuredangle(\vec{d},\vec{b}))$

$$\Rightarrow \quad 8x+4y+z = 9\cdot\sqrt{x^2+y^2+z^2}\cdot\cos(\widehat{d,a})$$

$$2x+2y+z = 3\sqrt{x^2+y^2+z^2}\cdot\cos(\widehat{d,b})$$

$$\Rightarrow \quad \frac{8x+4y+z}{9} = {}^{3/}\frac{2x+2y+z}{3}$$

$$8x+4y+z = 6x+6y+3z$$

$$2x-2y-2z=0$$

$$x-y-z=0$$

b) $\vec{d}\perp\vec{c} \Rightarrow \vec{d}\,\vec{c}=0 \Rightarrow x+y+z=0$

c) (a,b,c) and (a,b,d) have the same orientation

$$[\vec{a},\vec{b},\vec{c}] = \begin{vmatrix} 8 & 4 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \cancel{16}+2+4-2-\cancel{8}-\cancel{8} = 4>0 \Rightarrow \text{right oriented}$$

$$[\vec{a},\vec{b},\vec{d}] = \begin{vmatrix} 8 & 4 & 1 \\ 2 & 2 & 1 \\ x & y & z \end{vmatrix} = 16z+2y+4x-2x-8y-8z =$$

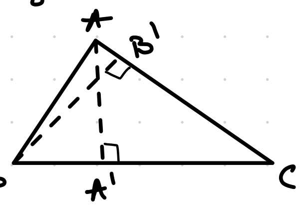
$$= 2x-6y+8z>0$$

$$\begin{cases} x-y-z=0 \\ x+y+z=0 \\ x-3y+4z>0 \end{cases} \quad + \Rightarrow x=0$$

$$y=-z \Rightarrow 3z+4z>0$$

$$z>0 \Rightarrow y<0 \qquad \Rightarrow \text{one vector is } \vec{d}(0,-1,1)$$

3.18 $A(1,2)$, $B(3,-2)$, $C(5,6)$ orthocenter? circumcenter?



$\vec{BC}=(2,8)$

$\vec{AB}=(2,-4)$

orthocenter

$$BC: \begin{cases} x=3+2\lambda \quad |\cdot(-4) \\ y=-2+8\lambda \end{cases}$$

$$BC: y-4x=-14 \Rightarrow 4x-y-14=0$$

$$n_{BC}=\langle(4,-1)\rangle$$

$$AA'\perp BC \Rightarrow AA': \begin{cases} x=1+4\lambda \\ y=2-\lambda \quad |\cdot(4) \end{cases}$$

$$AA' = x+4y=9$$

$$AC: \begin{cases} x=1+4\lambda \\ y=2+4\lambda \end{cases}$$

$$AC: x-y=-1$$

$\vec{n}_{AC} = \langle (1, -1) \rangle$

$BB' \perp AC \Rightarrow \quad BB' = \begin{cases} x = 3 + \lambda \\ y = -2 - \lambda \end{cases}$ $(+)$

$BB' = x + y = 1$

$AA' : \quad x + 4y = 9$

$BB' : \quad x + y = 1$ $(-)$

$3y = 8$

$y = \frac{8}{3} \; ; \quad x = \frac{-5}{3} \Rightarrow H\left(\frac{-5}{3}, \frac{8}{3}\right)$ orthocenter

circumcenter – mediatoare!

$B''(3,4) \quad A''(4,2)$

$\left.\begin{matrix} \vec{n}_{AC} = \langle (1,-1) \rangle \\ BB'' \perp AC \end{matrix}\right\} \Rightarrow \begin{cases} x = 3 + \lambda \\ y = 4 - \lambda \end{cases}$

$BB'' = x + y = 7$

$\left.\begin{matrix} \vec{n}_{BC} = \langle (4,-1) \rangle \\ AA'' \perp BC \end{matrix}\right\} \Rightarrow \begin{cases} x = 4 + 4\lambda \\ y = 2 - \lambda \quad (\cdot 4) \end{cases}$ $(+)$

$AA'' : x + 4y = 12$

$\begin{cases} AA'' : & x + 4y = 12 \\ BB'' : & x + y = 7 \end{cases}$ $(-)$

$3y = 5 \Rightarrow y = \frac{5}{3}$

$x = {}^{3/}7 - \frac{5}{3} = \frac{21-5}{3} = \frac{16}{3} \Rightarrow AA'' \cap BB'' = \left\{ O\left(\frac{16}{3}, \frac{5}{3}\right) \right\}$