

Ch 7: 9, 10 a) b)

Ch 8: 3, 4, 5, 10, 14

$$Q: a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{10}x + 2a_{01}y + a_{00} = 0$$

$$M_Q = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$$

$$\hat{M}_Q = \begin{pmatrix} a_{11} & a_{12} & a_{10} \\ a_{12} & a_{22} & a_{01} \\ a_{10} & a_{01} & a_{00} \end{pmatrix}$$

$$Q: \begin{pmatrix} x & y & 1 \end{pmatrix} \cdot \hat{M}_Q \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

isometric invariants:

$$\hat{\Delta} = \det(\hat{M}_Q)$$

$$\Delta = \det(M_Q)$$

$$T = \text{Tr}(M_Q)$$

$\hat{\Delta}$	Δ	T	Curve A
0	>0		a point
	=0		two lines on the <u>empty set</u>
	<0		two lines
$\neq 0$	>0	<0	ellipse (positivity)
	>0	>0	empty set
	=0		parabola (equals 0)
	<0		hyperbola (negativity)

7.9. Discuss the type of the curve $x^2 + 2xy + y^2 - 6x - 16 = 0$

how: solve this using the techniques from last time

$$\hat{\Delta} = \begin{pmatrix} 1 & \frac{x}{2} & -6 \\ \frac{x}{2} & 1 & 0 \\ -6 & 0 & -16 \end{pmatrix} = -16 + 0 + 0 - 36 - 0 + 16 \cdot \frac{x^2}{4} = 4x^2 - 48$$

$$\text{I } \hat{\Delta} = 0 \Rightarrow 4x^2 - 48 = 0 \Rightarrow x^2 = 12 \Rightarrow x = \pm 2\sqrt{3}$$

$$D = 1 - \frac{x^2}{4} < 0 \text{ due to } \text{Ges.}$$

$$\exists D > 0 \Rightarrow 4x^2 > 16 \Rightarrow x^2 > 4 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$$

$\hookrightarrow D < 0$

7.16. Decide what surfaces are described by the following eq:

a) $x^2 + 2y^2 + z^2 + xy + yz + zx = 1$

b) $xy + yz + zx = 1$

$$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx + y^2 - xy - yz - zx = 1$$

$$= (x+y+z)^2 + y^2 - xy - yz - zx = 1.$$

$$-(x+y+z)^2 + y^2 - x \cdot \frac{1}{2}y + \frac{x^2}{4} - \left(\left(\frac{x}{2}\right)^2 + 2 \cdot \frac{x}{2} \cdot z + z^2 \right) + z^2 - 2 \cdot z \cdot \frac{y}{2} + \frac{y^2}{4} - \frac{yz}{2}$$

$$-(x+y+z)^2 + \left(y - \frac{x}{2}\right)^2 - \left(\frac{x}{2} + z\right)^2 + \left(z - \frac{y}{2}\right)^2 - \frac{yz}{2} = 1$$

$$= \underbrace{\left(x - \frac{y}{2} - \frac{z}{2}\right)^2}_{x'} - \frac{y^2}{4} - \frac{z^2}{4} - \frac{yz}{2} = 1$$

$$= x'^2 - \frac{y^2}{4} - \frac{z^2}{4} + \frac{yz}{2} = 1$$

$$= x'^2 - \left(\left(\frac{y}{2}\right)^2 - 2 \cdot \frac{y}{2} \cdot \frac{z}{2} + \frac{z^2}{4} \right) = 1$$

$$= x'^2 - \left(\frac{y}{2} - \frac{z}{2} \right)^2$$

$$= x'^2 + y^2 + \frac{z^2}{4} = 1 \quad \text{--- elliptic}$$

b) $xy + yz + zx = 1$

$$x = y + x'$$

$$(y+x') \cdot y + yz + 2 \cdot (y+x') \cdot z = 1$$

$$y^2 + yx' + 2yz + zx' = 1$$

$$y^2 + yx' + 2yz - \left(\left(\frac{1}{2}x' \right)^2 + z^2 + 2 \cdot \frac{1}{2}x'z \right) - \frac{1}{2}x'^2 - z^2 = 1$$

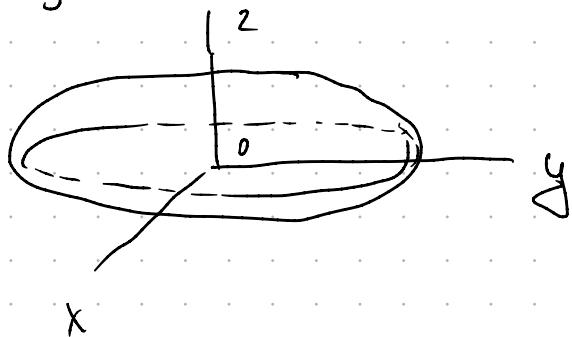
$$\underbrace{\left(y + \frac{1}{2}x' + z \right)^2}_{y'} - \left(\frac{1}{2}x' \right)^2 - z^2 = 1$$

y'

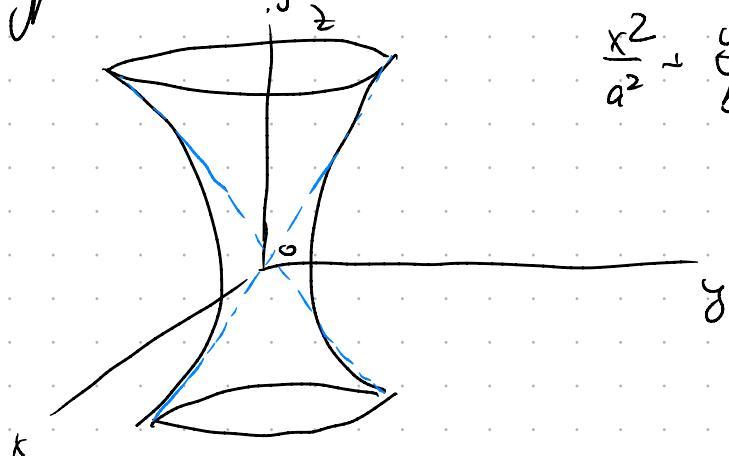
$$\Rightarrow -\frac{x'^2}{4} + y'^2 - z^2 = 1 \Rightarrow \text{two sheet hyperboloid}$$

ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



hyperboloid of one sheet:

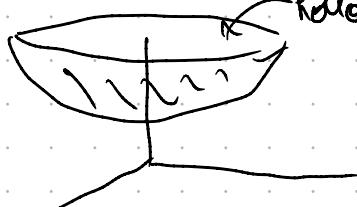


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

hyperboloid of two sheets

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

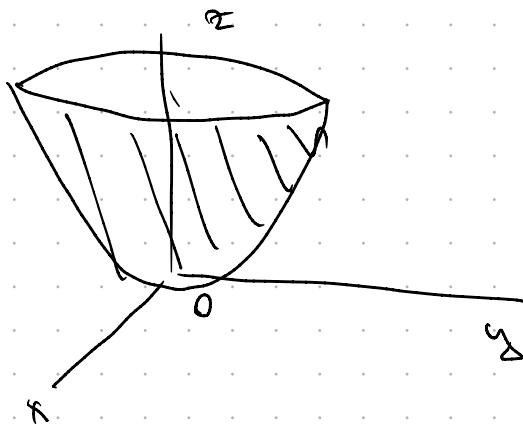
follow



elliptic paraboloid

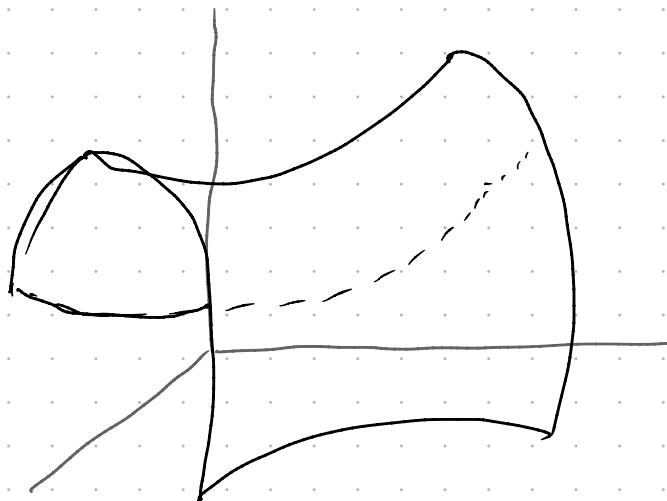
$$\frac{x^2}{P} + \frac{y^2}{Q} = z$$

$$P, Q > 0$$



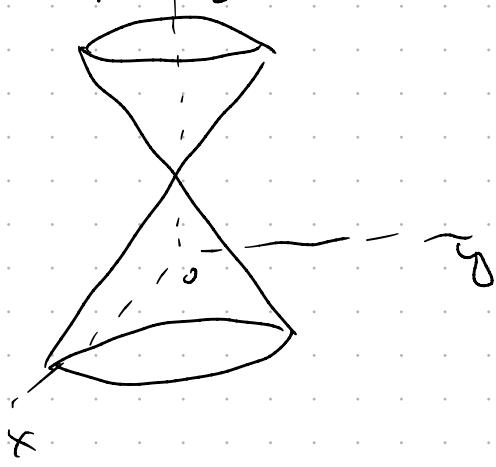
hyperbolic paraboloid

$$\frac{x^2}{P} - \frac{y^2}{Q} = z$$



Elliptic cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$



Tangent line:

$$\frac{\partial f}{\partial x}(x_0, y_0, z_0) + \frac{\partial f}{\partial y}(y_0, z_0) + \frac{\partial f}{\partial z}(z_0, y_0, z_0)$$

8.4. Dd. the tangent planes to the ellipsoid

$$E_{2,3,2\sqrt{2}}: \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{8} = 1$$

which are parallel to the plane $\pi: 3x - 2y + 2z = 0$

$$\frac{x_0}{4} + \frac{y_0}{9} + \frac{z_0}{8} = 1$$

$$\vec{m}_E = \left(\frac{x_0}{4}, \frac{y_0}{9}, \frac{z_0}{8} \right)$$

$$\vec{m}_{\pi} = (3, -2, 5)$$

$$T_E \parallel \pi \Rightarrow \vec{m}_{\pi} \parallel \vec{m}_{T_E} \Rightarrow \frac{10}{12} = \frac{60}{-18} = \frac{20}{40} = t$$

$$\frac{x_0}{6} = \frac{y_0}{-9} = \frac{z_0}{20} = t$$

$$\Rightarrow x_0 = 6t$$

$$y_0 = -9t$$

$$z_0 = 40t$$

$$\Rightarrow \frac{(12t)^2}{11} + \frac{(-18t)^2}{9} + \frac{(40t)^2}{8} = 1$$

$$56t^2 + 36t^2 + 200t^2 = 1$$

$$272t^2 = 1 \Rightarrow t = \pm \sqrt{\frac{1}{272}} = \pm \frac{1}{2\sqrt{68}} = \pm \frac{1}{4\sqrt{17}}$$

$$\Rightarrow P_1\left(\frac{3}{\sqrt{17}}, -\frac{9}{2\sqrt{17}}, \frac{10}{\sqrt{17}}\right), P_2\left(-\frac{3}{\sqrt{17}}, \frac{9}{2\sqrt{17}}, -\frac{10}{\sqrt{17}}\right)$$