Lecture 3 - Deterministic and non-deterministic predicates

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Bibliography

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findall → capture all solutions of a deterministic predicate

1. Deterministic and non-deterministic predicates

Types of predicates, **ideal implementation:**

- **deterministic**
  - a deterministic predicate has only one solution
- **non-deterministic** alternate clauses to find more solutions
  - an non-deterministic predicate has several solutions

⇒ always end with a red false

SWI Prolog implementation:

- A **deterministic predicate** always succeeds exactly once and does not leave a choicepoint.
- A **semi-deterministic predicate** succeeds at most once. If it succeeds it does not leave a choicepoint.
- A **non-deterministic predicate** is the most general case and no claims are made on the number of solutions and whether or not the predicate leaves a choicepoint on the last solution.

Remark. A predicate can be deterministic in one flow model, and non-deterministic in other flow models.

1.1 Predefined predicates

var (X)	= true if X is free, false if bound
number (X)	= true if X is bound to a number
integer (X)	= true if X is bound to an integer
float (X)	= true if X is bound to a real number
atom (X)	= true if X is bound to an atom
atomic (X)	= atom (X) or number (X)

* difference in semantics ?

1.2 Collect all solutions

Prolog provides ways to collect in a list all the solutions of a predicate.

findall(X, goal_on_X, L)
setof(X, goal_on_X, L)
bagof(X, goal_on_X, L)

They have the following parameters:

- the first parameter specifies the predicate parameter to be collected in the list;
- the second parameter specifies the predicate to be solved;
- the third parameter specifies the list where all the solutions will be collected.

The difference between these three predicates follows:

- findall
  - collects all the solutions of X in L
  - any variable in goal not in the first argument is not instantiated
- setof
  - removes from L all duplicate outputs and answers are sorted
  - any variable in goal not in the first argument results in separate solutions
- bagof
  - does not remove from L duplicates and the answers may not be sorted
  - any variable in goal not in the first argument results in separate solutions

EXAMPLE 1:

```
p (a, b).
p (b, c).
p (a, c).
p (a, d).
all (X, L) :- findall (Y, p (X, Y), L).

?- all (a, L).
L = [b, c, d]
```

EXAMPLE 2:

```
age(peter, 7).
age(ann, 5).
age(pat, 8).
age(tom, 5).
age(ann, 5).
```

?- setof(Child, age(Child, Age), Results).

```
Age = 5
Results = [ann, tom] ;

Age = 7
Results = [peter] ;

Age = 8
Results = [pat] .
```

?- bagof(Child, age(Child, Age), Results).

Age = 5
Results = [ann, tom, ann] ;

Age = 7
Results = [peter] ;

Age = 8
Results = [pat] .

?- findall(Child, age(Child, Age), Results).

Results = [peter, ann, pat, tom, ann] .

A way to group parameters in pairs uses the notation **A/B** or **A-B**:

?- setof(Age/Child, age(Child, Age), Results).

Results = [5/ann, 5/tom, 7/peter, 8/pat].

?- setof(Age-Child, age(Child, Age), Results).

Results = [5-ann, 5-tom, 7-peter, 8-pat].

?- bagof(Age/Child, age(Child, Age), Results).

Results = [7/peter, 5/ann, 8/pat, 5/tom, 5/ann].

?- findall(Age/Child, age(Child, Age), Results).

Results = [7/peter, 5/ann, 8/pat, 5/tom, 5/ann].

1.3 Negation - “not”, “\+”

Negation is true if the subgoal fails (can't be proven to be true).

not(subgoal (Arg1, ..., ArgN))
\+ subgoal (Arg1, ..., ArgN)

?- \+ (2 = 4).	?- not (2 = 4).
true.	**true.**

1.4 Lists and recursion

In Prolog, a list is an object that contains an arbitrary number of other objects. The lists in SWI-Prolog are heterogeneous (the component elements can have different types). Lists are built using square brackets. Their elements are separated by a comma.

Here are some examples:

[1, 2, 3]
[dog, cat, canary]
[“valerie ann”,“jennifer caitlin”,“benjamin thomas”]

If we were to declare the type of a (homogeneous) list with integer elements, we would use a domain declaration of the following type

element = integer
list = element *

1.4.1 Head & tail of a list

A list is a recursive object. It consists of two parts: the head, which is the first element of the list, and the tail, which is the rest of the list. The head of the list is an element, and the tail is a list:

The head of the list [a, b, c] is
The head of the list [c] is c

The tail of [a, b, c] is [b, c]
The tail of [c] is []

The empty list [] cannot be split into head and tail.

1.4.2 List processing

Prologue provides a way to make the head and tail of a list explicit. Instead of separating the elements of a comma list, we will separate the head of the queue with the character '|'.

For example, the following lists are equivalent:

after it there's always a list

[a b c] [a | [b, c]] [a | [b | [c]]] [a | [b | [c | []]]]

Also, before the '|' sign several elements may be written, not just the first. For example, the list [a, b, c] above is equivalent to

[a | [b, c]] [a, b | [c]] [a, b, c | []]

- Following the unification of list [a, b, c] with list [H | T] (H, T being free variables)
  - H binds to a; T binds to [b, c]
- Following the unification of list [a, b, c] with list [H | [H1 | T]] (H, H1, T being free variables)
  - H binds to a; H1 binds to b; T binds to [c]

1.4.3 Using lists

Because a list is a recursive data structure, recursive algorithms are needed to process it. The basic way to process the list is to work with it, performing certain operations with each element of it, until the end is reached.

Such an algorithm generally needs two clauses. One of them says what to do with an empty list. The other says what to do with a non-empty list, which can be broken down in its head and its tail.

2. Examples

EXAMPLE 2.1 Given a non null natural number N, compute F = N!. Simulate the iterative computation process.

$I \leftarrow 1$
$P \leftarrow 1$
While $I < N$ **do**
    $I \leftarrow I + 1$
    $P \leftarrow P * I$
End While
$F \leftarrow P$

$$fact(n) = fact\_aux(n, 1, 1)$$

$$fact\_aux(n, i, p) = \begin{cases} p & if\ i = n \\ fact\_aux(n, i+1, p*(i+1)) & otherwise \end{cases}$$

The description is not directly recursive, collector variables (i, P) are used

```
% fact (N: integer, F: integer)
% (i, i), (i, o) - deterministic
fact (N, F) :- fact_aux (N, F, 1, 1).

% fact_aux (N: integer, F: integer, I: integer, P: integer)
% (i, i, i, i), (i, o, i, i) - deterministic
fact_aux (N, F, N, F) :- !. % fact_aux (N, F, I, P) :- I is N, F is P,!.
fact_aux (N, F, I, P) :- NewI is I + 1,
                         NewP is P * NewI,
                         fact_aux (N, F, NewI, NewP).
```

exit clause

F is untouched and will be bound back on the exit clause

The result is a tail recursion. All cycling variables were introduced as arguments to the fact_aux predicate.

HOMEWORK:

Write a factorial predicate (N, F) that works in all three flow models (i, i), (i, o) and (o, i).

EXAMPLE 2.2 Verify the membership of an item in a list.

% (i, i)
To describe the membership in a list we will construct the member predicate (element, list) which will investigate whether a certain element is a member of the list. The algorithm to be implemented would (declaratively) be the following:

1. E is a member of the list L if it is its head.
2. Otherwise, E is a member of the list L if it is a member of the tail of L.

From a procedural point of view:

1. To find a member of the list L, find his head;
2. Otherwise, find a member of the tail of L.

$$member(E, l_1 l_2 \dots l_n) = \begin{cases} false & if\ is\ empty \\ true & if \quad l_1 = E \\ member(E, l_2 \dots l_n) & otherwise \end{cases}$$

Version 1

```
% member1(e: element, L: list)
% (i, i) - (non) deterministic
% (o, i) - non-deterministic

member1(E, [E | _]).
member1(E, [_ | L]) :-
        member1(E, L).
```

Version 2

```
% member2 (e: element, L: list)
% (i, i) - deterministic
%(o, i) - deterministic

member2(E, [E | _]) :- !.
member2(E, [_ | L]) :-
        member2(E, L).
```

→ stops when it finds the first occ.

Version 3

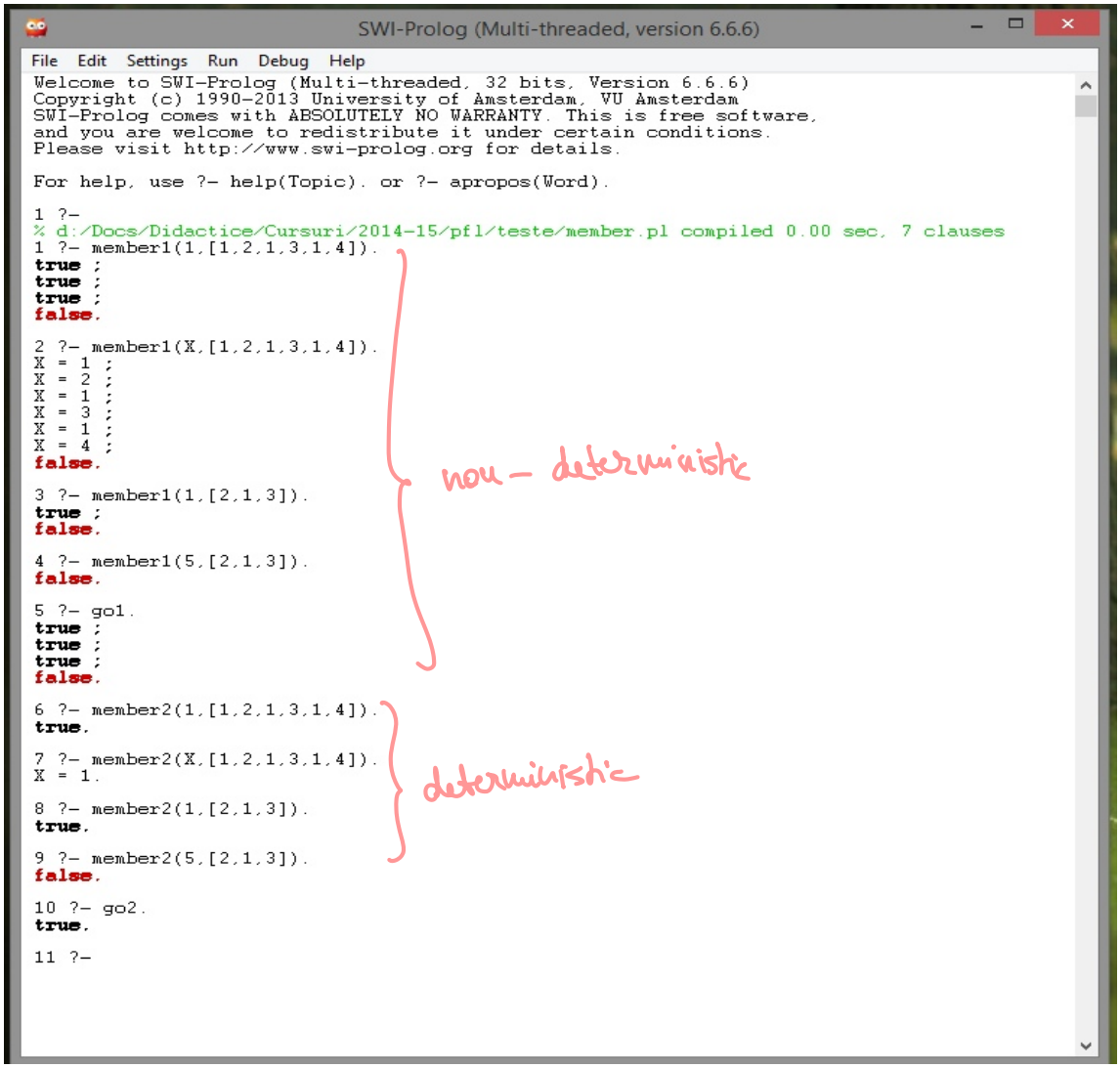
```
% member3 (e: element, L: list)
% (i, i) - (non) deterministic
% (o, i) - non-deterministic

member3(E, [_ | L]) :-
        member3(E, L).
member3(E, [E | _]).

?- member3(E, [1,2,3]).
E = 3;
E = 2;
E = 1.

?-member3 (4, [1,2,3]).
false.

?- member3 (2, [1,2,3]).
true;
false.
```



As it can be seen, the predicate **member** it also works in the flow model (o, i), in which it is **non-deterministic**.

<u>EXAMPLE 2.3</u> Add an item at the end of a list.

? add (3, [1, 2], L).
L = [1, 2, 3]

Recursive formula:

$$add(e, l_1 l_2 \dots l_n) = \begin{cases} (e) & daca \quad l\ e\ vida \\ l_1 \oplus add(e, l_2 \dots l_n) & altfel \end{cases}$$

Version 1

```
% add (e: element, L: list, LRez: list)
% (i, i, o) – deterministic

add (E, [], [E]).
add (E, [H | T], [H | Rez]) :-
        add (E, T, Rez).
% add (E, [H | T], Rez) :-
%       add (E, T, L), Rez = [H | L].
```

→ bound to the result of the construct

Version 2

```
% add (L: list, e: element, LRez: list)
% (i, i, o) – deterministic

add ([], E, [E]).
add ([H | T], E, [H | Rez]) :-
        add (T, E, Rez).
% add ([H | T], E, Rez) :-
%       add (T, E, L), Rez = [H | L].
```

The time complexity of the operation of adding an item to the end of the list of a list of n items is $\theta(n)$.

EXAMPLE 2.4 Determine the inverse of a list.

Version A (direct recursion)

$$invers(l_1 l_2 \dots l_n) = \begin{cases} \varnothing & daca\ l\ e\ vida \\ \\ invers(l_2 \dots l_n) \oplus l_1 & altfel \end{cases}$$

```
% inverse (L: list, LRez: list)
% (i, o) – deterministic

inverse ([], []).
inverse ([H | T], Rez) :-
        inverse (T, L),
        add (H, L, Rez).
```

add the head at the end of the tail

The time complexity of the operation of reversing a list of n elements (using the addition at the end) is $\theta(n^2)$.

Version B (with collector variable)

$$invers\_aux(l_1 l_2 \dots l_n, Col) = \begin{cases} Col & daca\ l\ e\ vida \\ \\ invers\_aux(l_2 \dots l_n, l_1 \oplus Col) & altfel \end{cases}$$

$$invers(l_1 l_2 \dots l_n) = invers\_aux(l_1 l_2 \dots l_n, \varnothing)$$

```
% inverse (L: list, LRez: list)
% (i, o) – deterministic

inverse (L, Rez) :-
        invers_aux ([], L, Rez).
```

```
% invers_aux (Col: list, L: list, LRez: list)
% Col is the collector
% (i, i, o) - deterministic

invers_aux (Col, [], Col).
invers_aux (Col, [H | T], Rez) :-
        invers_aux ([H | Col], T, Rez).
```

→ collector in reverse order

The time complexity of the operation of reversing a list of n elements (using a collector variable) is θ(n).

Remark. Using a collector variable does not lead to a decrease in complexity in all cases. There are situations in which the use of a collector variable increases the complexity (ex: the addition in collectors is done at the end of it, not at the beginning).

EXAMPLE 2.5 Determine the list of even items in a list (keep the order of the items in the original list).

Version A (direct recursion)

$$even(l_1 l_2 \dots l_n) = \begin{cases} \varnothing & daca\ l\ e\ vida \\ l_1 \oplus even(l_2 \dots l_n) & daca\ \ l_1\ par \\ even(l_2 \dots l_n) & altfel \end{cases}$$

```
% even (L: list, LRez: list)
% (i, o) – deterministic

even([], []).
even([H | T], [H | Rez]) :-
        H mod 2 =: = 0,
        !,
        even(T, Rez).
even([_ | T], Rez) :-
        even(T, Rez).
```

* clauses are not split in if – then – else
* they are all tried

! → if the condition holds, don't go to try the other one

The time complexity of the operation is θ(n), n being the number of items in the list.

$$T(n) = \begin{cases} 1 & if\ n = 0 \\ T(n-1) + 1 & otherwise \end{cases}$$

Version B (with collector variable)

$$even\_aux(l_1 l_2 \dots l_n, Col) = \begin{cases} Col & daca\ l\ e\ vida \\ even\_aux(l_2 \dots l_n, Col \oplus l_1) & daca\ \ l_1\ par \\ even\_aux(l_2 \dots l_n, Col) & altfel \end{cases}$$

$$even(l_1 l_2 \dots l_n) = even\_aux(l_1 l_2 \dots l_n, \varnothing)$$

```
% even (L: list, LRez: list)
% (i, o) – deterministic

even(L, Rez) :-
        even_aux (L, Rez, []).

% even_aux(L: list, LRez: list, Col: list)
% (i, o, i) – deterministic

even_aux ([], Rez, Rez).
even_aux ([H | T], Rez, Col) :-
        H mod 2 =: = 0,
        !,
        add (H, Col, ColN), % add at the end
        even_aux (T, Rez, ColN).
even_aux ([_ | T], Rez, Col) :-
        even_aux (T, Rez, Col).
```

The time complexity of the operation is θ(n2), n being the number of items in the list.

not good
we parse the list twice instead

<u>EXAMPLE 2.6</u> Given a numerical list, determine the list of strictly ascending pairs of elements in the list.

?- pairs ([2, 1, 3, 4], L)
L = [[2, 3], [2, 4], [1, 3], [1, 4], [3, 4]]

?- pairs ([5, 4, 2], L)
false

We will use the following predicates:

- the nondeterministic predicate pair (element, list, list) (flow model (i, i, o)), which will produce pairs in ascending order between the given element and elements of the argument list

?- pair (2, [1, 3, 4], L)
L = [2, 3]
L = [2, 4]

pair([H|T], Res):-
    pairs(H, T, Res).
pair([_|T], Res):-
    pair(T, Res).

$pair(e, l_1 l_2 \ldots l_n) =$
1. (e, l_1) $e < l_1$
2. $pair(e, l_2 \ldots l_n)$

```
% pair (E: element, L: list, LRez: list)
% (i, i, o) – non-deterministic

pair (A, [B | _], [A, B]) :-
        A < B.
pair (A, [_ | T], P) :-
        pair (A, T, P).
```

- the nondeterministic predicate pairs (list, list) (flow model (i, o)), which will produce pairs in ascending order between the elements of the argument list

? pairs ([2, 1, 4], L)
L = [2, 4]
L = [1, 4]

$pairs(l_1 l_2 \dots l_n) =$
1. $pair(l_{1,} l_2 \dots l_n)$
2. $pairs(l_2 \dots l_n)$

```
% pairs (L: list, LRez: list)
% (i, o) – non-deterministic

pairs ([H | T], P) :-
        pair(H, T, P).
pairs ([_ | T], P) :-
        pairs(T, P).
```

- the main predicate allPairs (list, listRez) (flow model (i, o)), which will collect all solutions of the nondeterministic predicate pairs.

```
% allPairs (L: list, LRez: list)
% (i, o) –deterministic

allPairs (L, LRes) :-
        findall (X, pairs (L, X), LRez).
```