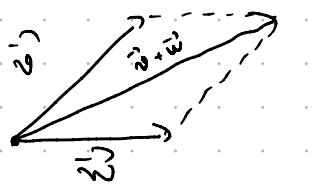
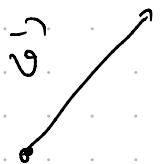
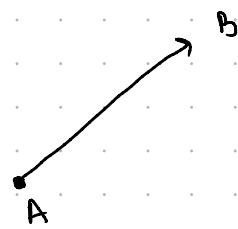


\mathbb{E}^n - n -dimensional Euclidean space

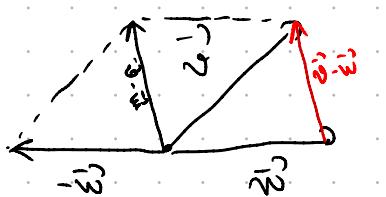
(usually $n=2$ or $n=3$)

V^n - space of vectors in \mathbb{E}^n

(IR-Vector space)



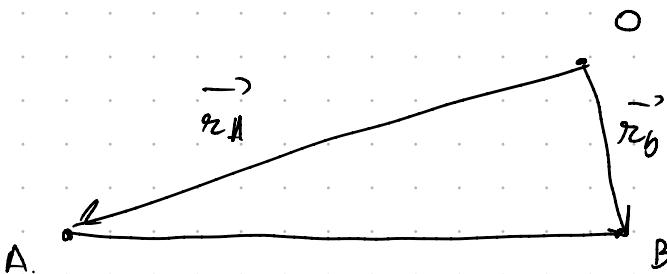
ADDITION
parallelogram rule



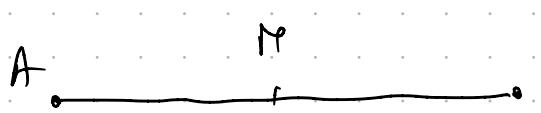
SUBTRACTION
triangle rule

$$A_1, A_2, \dots, A_k \in \mathbb{E}^n \Rightarrow \overrightarrow{A_1 A_2} + \overrightarrow{A_2 A_3} + \dots + \overrightarrow{A_{n-1} A_n} = \overrightarrow{0}$$

If we fix $0 \in \mathbb{E}^n \Rightarrow \forall A \in \mathbb{E}^n$ we can define $\vec{r}_A = \overrightarrow{OA}$
= the position vector of A (w.r.t. 0)



$$\vec{AB} = \vec{r}_B - \vec{r}_A$$

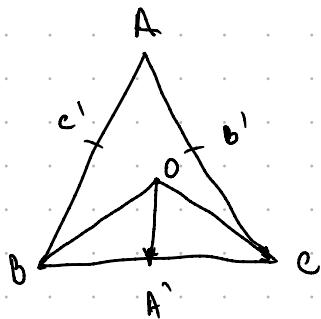


$A, M, B \in JE^m$

$$M - \text{midpoint of } [AB] \Leftrightarrow \vec{r}_M = \frac{1}{2} (\vec{r}_A + \vec{r}_B)$$

6. A', B', C' mid points of sides of a triag. Show that $H \circ e JE^m$ We have

$$\vec{OA'} + \vec{OB'} + \vec{OC'} = \vec{OA} + \vec{OB} + \vec{OC}$$

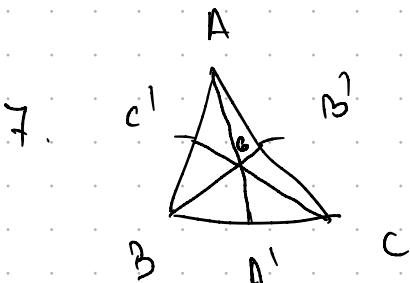


$$\vec{OA'} = \frac{1}{2} (\vec{OB} + \vec{OC})$$

$$\vec{OB'} = \frac{1}{2} (\vec{OA} + \vec{OC})$$

$$\vec{OC'} = \frac{1}{2} (\vec{OA} + \vec{OB})$$

$$\begin{aligned} \vec{OA'} + \vec{OB'} + \vec{OC'} &= \frac{1}{2} (2\vec{OA} + 2\vec{OB} + 2\vec{OC}) \\ &= (\vec{OA} + \vec{OB} + \vec{OC}) \end{aligned}$$



$A' - \text{mid } BC$

$B' - \text{mid } AC$

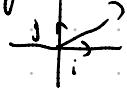
$C' - \text{mid } AB$

A, B, C collinear ($\Rightarrow \exists l \in JE^m$), $A, B, C \in l \Leftrightarrow \vec{AB}, \vec{BC}$ linearly dependent ($\Rightarrow \vec{AB} = \alpha \vec{BC}$)

$K = (O, B)$ - reference system

$O \in JE^m$

$B - \text{basis of } V^m$



$i_j^2, \text{ basis of } V^2$

$$\Rightarrow [A]_K = [\vec{r}_A]_B$$

$$A, B, C \text{ collinear} \Leftrightarrow [\vec{AB}]_B = \alpha [\vec{AC}]_B$$

8. a) A(3, -5), B(-1, 2), C(-5, 9)

b) A(11, 2), B(1, -3), C(31, 13)

c) A(1, 0, -1), B(0, -1, 2), C(-1, 2, 5)

d) A(-1, -1, -4), B(1, 1, 0), C(2, 2, 2)

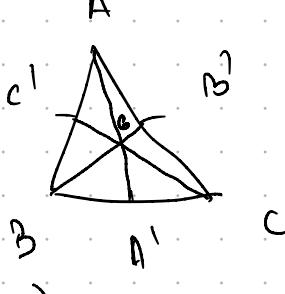
a) $\vec{AB} = \vec{r}_B - \vec{r}_A = (-1, 2) - (3, -5) = \underline{(-4, 7)}$ $\Rightarrow \vec{AB} = 1 \cdot \vec{BC}$
 $\vec{BC} = \vec{r}_C - \vec{r}_B = (-5, 9) - (-1, 2) = \underline{(-4, 7)}$ \Rightarrow collinear.

b) $\vec{AB} = \vec{r}_B - \vec{r}_A = (1, -3) - (11, 2) = (-10, -5)$
 $\vec{BC} = \vec{r}_C - \vec{r}_B = (31, 13) - (1, -3) = (30, 16)$ \Rightarrow they are not collinear.

c) $\vec{AB} = (0, -1, 2) - (1, 0, -1) = (-1, -1, 3)$
 $\vec{BC} = (1, 2, 5) - (0, -1, 2) = (-1, 3, 3)$ \Rightarrow lin. dependent
 \Rightarrow linear combination.

d) $\vec{AB} = (1, 1, 0) - (-1, -1, -4) = (2, 2, 4)$
 $\vec{BC} = (2, 2, 2) - (1, 1, 0) = (1, 1, 2)$ \Rightarrow lin. dep.
 \Rightarrow collinear.

7



$$\vec{AA'} = \vec{r}_{A'} - \vec{r}_A = \frac{1}{2} (\vec{r}_B + \vec{r}_C) - \vec{r}_A$$

$$\vec{AG} = \vec{r}_G - \vec{r}_A = \frac{1}{3} (\vec{r}_A + \vec{r}_B + \vec{r}_C) - \vec{r}_A$$

$$\vec{AG} = \alpha \cdot \vec{AA'}$$

$$\frac{1}{3} \vec{r}_A + \frac{1}{3} \vec{r}_B + \frac{1}{3} \vec{r}_C - \vec{r}_A = \frac{\alpha}{2} \vec{r}_B + \frac{\alpha}{2} \vec{r}_C - \alpha \vec{r}_A$$

$$\Rightarrow -\frac{2}{3}\vec{r}_A + \frac{1}{3}\vec{r}_B + \frac{1}{3}\vec{r}_C = \frac{\alpha}{2}\vec{r}_B + \frac{\alpha}{2}\vec{r}_C - \alpha\vec{r}_A$$

$$\Rightarrow \alpha = \frac{2}{3} \Rightarrow \text{collinear.}$$

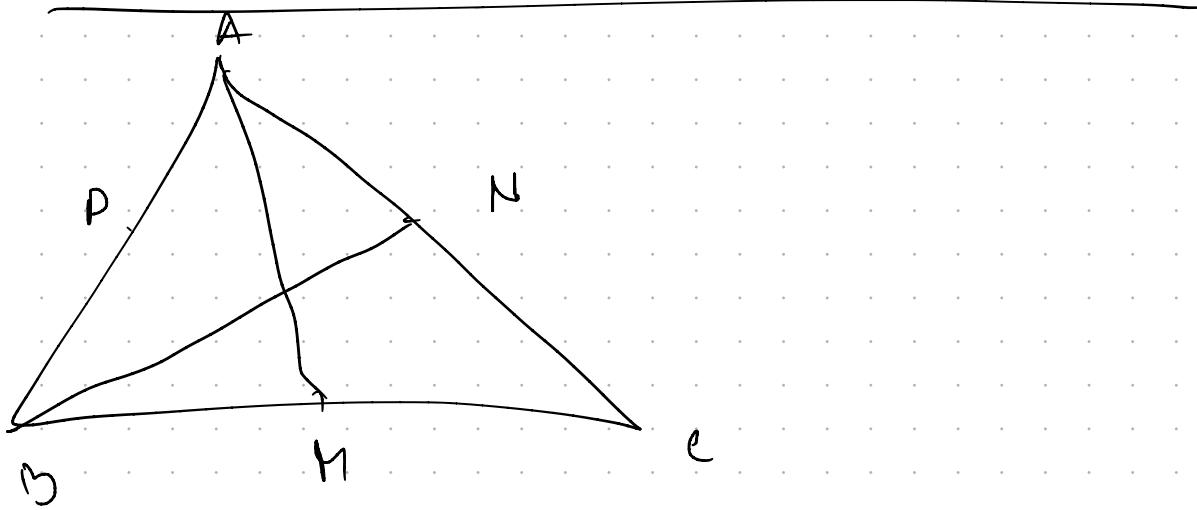
A 

$A, B \in Je^m$ distinct

$$T \in AB \Rightarrow$$

$\exists \lambda \in \mathbb{R}$ s.t.

$$\vec{r}_T = \lambda \vec{r}_A + (1-\lambda) \vec{r}_B \quad (\text{the vector equation of the line})$$



$$AM \cap BN = \{G\}$$

Show that $G \in CP$

$G \in AM \Rightarrow \exists \lambda \in \mathbb{R}$:

$$\vec{r}_G = \lambda \vec{r}_A + (1-\lambda) \vec{r}_M =$$

$$= \lambda \vec{r}_A + \frac{1-\lambda}{2} \vec{r}_B + \frac{1-\lambda}{2} \vec{r}_C$$

$$G \in BN \Rightarrow \exists \mu \in \mathbb{R} \text{ s.t. } \vec{r}_G = \mu \vec{r}_B + \frac{1-\mu}{2} \vec{r}_A + \frac{1-\mu}{2} \vec{r}_C$$

$$\Rightarrow \left(x - \frac{1-\mu}{2} \right) \vec{r}_A + \left(\frac{1-x}{2} - \mu \right) \vec{r}_B + \left(\frac{1-x}{2} - \frac{1-\mu}{2} \right) \vec{r}_C$$

$$= \vec{0}$$

ΔABC ~ non-degenerate $\Rightarrow \vec{v} = \vec{AB}$ and $\vec{w} = \vec{AC}$ are lin. indep.

$$\vec{r}_B = \vec{r}_A + \vec{v}$$

$$\vec{r}_C = \vec{r}_A + \vec{w}$$

$$\Rightarrow \left(x - \frac{1-\mu}{2} \right) \vec{r}_A + \left(\frac{1-x}{2} - \mu \right) (\vec{r}_A + \vec{v}) + \left(\frac{1-x}{2} - \frac{1-\mu}{2} \right) (\vec{r}_A + \vec{w}) = \vec{0}$$

$$\vec{r}_A \left(x - \frac{1-\mu}{2} + \frac{1-x}{2} - \mu + \frac{1-x}{2} - \frac{1-\mu}{2} \right) + \left(\frac{1-\lambda}{2} - \frac{1-\mu}{2} \right) \vec{v} + \left(\frac{1-\lambda}{2} - \frac{1-\mu}{2} \right) \vec{w} = \vec{0}$$

$$= \vec{0} \Rightarrow \left(\frac{1-\lambda}{2} - \frac{1-\mu}{2} \right) \vec{v} + \left(\frac{1-\lambda}{2} - \mu \right) \vec{w} = 0$$

v, w - lin. indep.

$$\begin{cases} \frac{1-\lambda}{2} - \frac{1-\mu}{2} = 0 \Rightarrow 1-\lambda = 1-\mu \Rightarrow \lambda = \mu \\ \frac{1-\lambda}{2} - \mu = 0 \Rightarrow 1-\lambda = 2\mu \Rightarrow 3\lambda = 1 \Rightarrow \lambda = \mu = \frac{1}{3} \end{cases}$$

$$\Rightarrow \vec{r}_G = \frac{1}{3} (\vec{r}_A + \vec{r}_B + \vec{r}_C)$$

$$\vec{G_P} = \frac{1}{2} (\vec{r}_A + \vec{r}_B) - \frac{1}{3} (\vec{r}_A + \vec{r}_B + \vec{r}_C)$$

$$= \frac{1}{6} \vec{r}_A + \frac{1}{6} \vec{r}_B - \frac{1}{3} \vec{r}_C$$

$$\vec{CP} = \frac{1}{2} (\vec{r}_A + \vec{r}_B) - \vec{r}_C$$

$$= 3 \vec{G_P} \Rightarrow G \in CP \Rightarrow AN, BN, CP \sim \text{congruent in } P \Rightarrow$$

$$\frac{P(G)}{C_P} = \frac{1}{3}$$