

Algebra - recap

algebraic structures set + operation(s) -5 set, 4: s x s -> 5

(x, y) -> x \* y operation (internal law) semigroup monoid
semi-group group

- associativity: & x,4,2 & 5 (x\*y)=2 = x + (4+3)

- mentral element: ] e ES: xFx ES: x \* e = x

invertability . \* x es : ] x' as : x \* x' = e

- commutativity : X x, y e 5 : x + y = y + x

group + commutativity = abelian (commutative) group

 $G_{K}: qroups: (Z, +), (R, +), (R, +), (C, +), (M_{n}(R), +), (Z_{2}, +), (Z_{2}, +), (Z_{2}, +)$ 

(5<sub>n</sub>, 0), (Z<sup>2</sup>, +) mornoids that are not groups:  $(\mathcal{M}_{h}(R), \cdot)$ ,  $(A^{\frac{1}{2}}, \circ)$  with mention element  $id_{A}: A \rightarrow A$   $(\mathcal{U}_{4}, \cdot)$ 

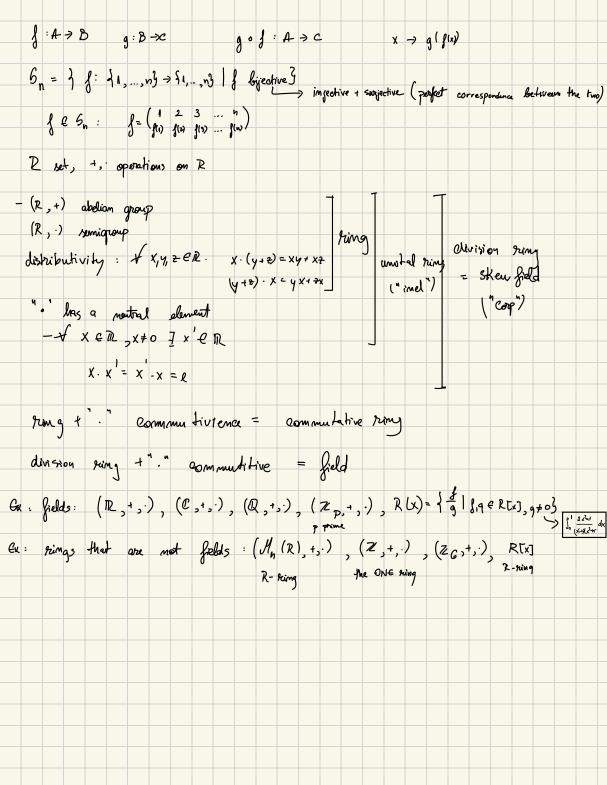
GLn(R) = A & M(R) | dot A + of

How does one invert a matrix?

- A\*
- A-1 = 1 dut A

(Zn,+,) field (≥) m prime

B= { g: A > B} + + = { g: A > A}



Yoly nomials R commutative unital laing (e.g. a field) A polynomial over R is a formal sum of the form f= ax + a x + a . x + a; er the coefficients of x m = deg of (degree of g), an= leading coefficient a = free torm Gx: dug (x2-5x4)=2 deg ( +x - 9) =1 deg (5) =0 deg (0) = - 00 deg (f.g)= deg f + deg q (if R is field) Counter example: R=Z4 (2x+3) (2x2+2)=6x2+2x+3 ference A- set when the operations from 2 make sense we com define  $\tilde{j}:A \rightarrow A$ X -> a, x 1 ... + a, x 19, x 19. The polymormial function of I on A Thu Fundamental theorem of algebra Any ge C[x] has roots in C Corellary If the resols of fare x1, x2..., xn EC, then f = an(x-x1)(x-x2)....(x-xn)

Ex) 
$$\int_{-\infty}^{\infty} x^2 \cdot 1 \in \mathbb{R}(x)$$
, invaring the sin  $\mathbb{R}$ 
 $\int_{-\infty}^{\infty} (x-i)(x+i)$  reducible in  $\mathbb{R}$ 
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Then Carlideau division

 $K = \{uable = 0 \}$ 
 $\int_{-\infty}^{\infty} 2 \in K(x)$  with dig  $x < dg$  so that  $\int_{-\infty}^{\infty} 2 \int_{-\infty}^{\infty} 1 \int_{-\infty}$