(b) ||x+y|| = ||x-y||. (c) $||x+y||^2 = ||x||^2 + ||y||^2$. (a) $\langle x, y \rangle = 0$. || x+y||2 = (x+y). (x+y) = x.x + 2xy+ y.y = (x24)=0 =) Assume = < x, x> + 2<x,y> + <y,y> = = ||x||² + 2.0 + ||y||²= - 11x112 + 11x112 (1) 1x-y112= (x-y)(x-y) - x-x - 2xy + y.y= = <x,x> - 2 <x,y> + (4,4> = = 11x11 - 2.0 + 114112= = 11x112+114112 (2) from (1) and (2) => || X-41| = || X-41| (b) Assume that ||x+y||= ||x-y|| =) $\|x-y\|^2 = \|x-y\|^2 = (x-y)(x-y) = (x-y)(x-y)$ 1X1) 2+2 <xxy>+114112= 11X112-2<xxy>+114112 => 2<x,y) =- 2<x,y> => <x,y> -0 =) || x||² + ||y||² = ||x+y||² (c) Assume that 11 x 4112 = 11 x 12-111412 -> (x2y)(x2y) = ||x||2 + ||y||2 (1x12+2<x,4>+11,412=11x12+11412 => (x,4)=0 (a) Since (b) =) (c) (2) (0) (b), (c) are equivalent
(c) => (a)

2. \bigstar For $x, y \in \mathbb{R}^n$ prove that the following statements are equivalent: