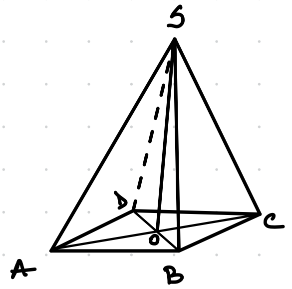
11. Show that $\vec{SA}+\vec{SB}+\vec{SC}+\vec{SD}=4\vec{SO}$



$\vec{SA}=\vec{SO}+\vec{OA}$ $\quad \vec{OA}=\frac{\vec{CB}}{2}+\frac{\vec{BA}}{2}$

$\vec{SB}=\vec{SO}+\vec{OB}$ $\quad \vec{OB}=\frac{\vec{DC}}{2}+\frac{\vec{CB}}{2}$

$\vec{SC}=\vec{SO}+\vec{OC}$ $\quad \vec{OC}=\frac{\vec{BD}}{2}+\frac{\vec{DC}}{2}$

$\vec{SD}=\vec{SO}+\vec{OD}$ $\quad \vec{OD}=\frac{\vec{BC}}{2}+\frac{\vec{BA}}{2}$

$\vec{SA}+\vec{SB}+\vec{SC}+\vec{SD}=4\vec{SO}$

2.9. Determine parametric equations for the plane π in the following cases:

a) π contains the point $M(1,0,2)$ and is parallel to the vectors $\mathbf{a}_1(3,-1,1)$ and $\mathbf{a}_2(0,3,1)$,

b) π contains the points $A(-2,1,1)$, $B(0,2,3)$ and $C(1,0,-1)$,

c) π contains the point $A(1,2,1)$ and is parallel to $\mathbf{i}$ and $\mathbf{j}$,

d) π contains the point $M(1,7,1)$ and is parallel coordinate plane Oyz,

e) π contains the points $M_1(5,3,4)$ and $M_2(1,0,1)$, and is parallel to the vector $\mathbf{a}(1,3,-3)$,

f) π contains the point $A(1,5,7)$ and the coordinate axis Ox.

a) $\pi \ni M(1,0,2)$ $\qquad \pi \parallel a_1, a_2$

$$\Rightarrow \pi = \begin{cases} x = x_A + \lambda x_{\vec{v}} + \mu x_{\vec{w}} \\ y = y_A + \lambda y_{\vec{v}} + \mu y_{\vec{w}} \\ z = z_A + \lambda z_v + \mu z_{\vec{w}} \end{cases}$$

$$\Rightarrow \begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_{\vec{v}} & y_{\vec{v}} & z_{\vec{v}} \\ x_{\vec{w}} & y_{\vec{w}} & z_{\vec{w}} \end{vmatrix} = 0$$

$$\Rightarrow \pi = \begin{cases} x = 1 + 3\lambda + 0\mu \\ y = 0 - 1\cdot\lambda + 3\mu \\ z = 2 + \lambda + \mu \end{cases}$$

$$\Rightarrow \begin{vmatrix} x-1 & y & z-2 \\ 3 & -1 & 1 \\ 0 & 3 & 1 \end{vmatrix} = 0 \Rightarrow D(\pi) = \langle (3,-1,1),(0,3,1) \rangle$$

b) $A(-2,1,1)$ $\qquad B(0,2,3)$ $\qquad C(1,0,-1)$

$\vec{AB} = (0-(-2), 2-1, 3-1) = (2,1,2)$

$\vec{BC} = (1,-2,-4)$

$$\Rightarrow \pi = \begin{cases} x = 0 + \lambda\cdot 2 + \mu\cdot 1 \\ y = 2 + \lambda\cdot 1 + \mu\cdot(-2) \\ z = 3 + \lambda\cdot 2 + \mu\cdot(-4) \end{cases}$$

$$\Rightarrow \begin{vmatrix} x & y-2 & z-3 \\ 2 & 1 & 2 \\ 1 & -2 & -4 \end{vmatrix} = 0 \Rightarrow D(\pi) = \langle (2,1,2),(1,-2,-4) \rangle$$

c) $A(1,2,1)$ $\qquad \pi \parallel (1,0,0),(0,1,0) \Rightarrow \vec{v}=(1,0,0)$, $\vec{w}=(0,1,0)$

$$\pi = \begin{cases} x = 1 + \lambda\cdot 1 + \mu\cdot 0 \\ y = 2 + \lambda\cdot 0 + \mu\cdot 1 \\ z = 1 + \lambda\cdot 0 + \mu\cdot 0 \end{cases}$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0 \Rightarrow D(\pi) = \langle (1,0,0), (0,1,0) \rangle$$

d) $M(1,7,1)$ $\quad \pi \parallel Oyz$

$$\pi = \begin{vmatrix} x-1 & y-2 & z-1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow D(\pi) = \langle (0,1,0), (0,0,1) \rangle$$

e) $M_1(5,3,4)$ $\quad M_2(1,0,1) \in \pi$ $\quad \pi \parallel a_1(1,3,-3)$

$$\overrightarrow{M_1M_2} = (-4,-3,-3)$$

$$\pi = \begin{cases} x = 5 + \lambda - 4\mu \\ y = 3 + 0 - 3\mu \\ z = 4 + \lambda - 3\mu \end{cases}$$

$$\Rightarrow D(\pi) = \langle (1,0,1), (-4,-3,-3) \rangle$$

f) $A(1,5,7) \in \pi$ $\quad \pi \parallel Ox \Rightarrow$ determined by $\begin{cases} Ox = (1,0,0) \\ OA = (1,5,7) \end{cases}$

$$\pi = \begin{cases} x = 1 + \lambda + \mu \\ y = 5 + \quad 5\mu \\ z = 7 + \quad 7\mu \end{cases} \Rightarrow D(\pi) = \langle (1,0,0), (1,5,7) \rangle$$

2.3. With the assumptions in Example 1.20, give parametric equations and Cartesian equations for the lines AB, AC, BC both in the coordinate system $\mathcal{K}$ and in the coordinate system $\mathcal{K}'$.

Example 1.20 (In dimension 2)**.** Let $\mathcal{K} = (O,(\mathbf{i},\mathbf{j}))$ and $\mathcal{K}' = (O',(\mathbf{i}',\mathbf{j}'))$ be two coordinate systems (reference frames) of $\mathbb{E}^2$. Suppose that we know O', $\mathbf{i}'$ and $\mathbf{j}'$ relative to $\mathcal{K}$:

$$[O']_{\mathcal{K}} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}_{\mathcal{K}}, \quad \mathbf{i}' = -2\mathbf{i} + \mathbf{j} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}_{\mathcal{K}}, \quad \mathbf{j}' = \mathbf{i} + 2\mathbf{j} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{\mathcal{K}}.$$

$A(x_A, y_A)$ $\qquad K(O, i, j)$ $\qquad i' = -2i + j$

$B(x_B, y_B)$ $\qquad K'(O', i', j')$ $\qquad j' = i + 2j$

$C(x_C, y_C)$ $\qquad M_{KK'} = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$

$$\begin{cases} x = i \cdot x_A + j + y_A \\ \vdots \quad \ldots \end{cases} \Rightarrow [x]_{K'} = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_A \\ y_A \end{pmatrix} = \begin{pmatrix} -2x_A + y_A \\ x_A + 2y_A \end{pmatrix}$$

$$[x]_{K'} = (-2x_A + y_A) i' + (x_A + 2y_A) j'$$