LECTURE 11 - Examples of list processing using MAP functions

MAP Functions - Recap

Usage:

(MAP-function function list-1 list-2 … list-n)

How-to:

MAP-function	get parameters	pack results
MAPCAR	CAR	LIST
MAPLIST	CDR	LIST
MAPCAN	CAR	NCONC
MAPCON	CDR	NCONC
MAPC	CAR	list-1
MAPL	CDR	list-1

1. Consider the following definitions:

```
(defun f (L e)
    (list e L)
)

(setq L '(1 2 3))
(setq e 4)
```

(mapcar #'*f* L e) evaluates to NIL – why?

(mapcar #'(lambda (L) (*f* L e)) L) evaluates to ((4 1) (4 2) (4 3))

2. Consider the following definition:

```
(defun f (L)
    (list L)
)
```

(mapcar #'*f* '(1 2 3)) evaluates to ((1) (2) (3))
(mapcan #'*f* '(1 2 3)) evaluates to (1 2 3)

Notice the equivalence of the following

(mapcan #'*f* L)
(apply #'append (mapcar #'*f* L))

3. Define a function that returns the length of a nonlinear list (in number of atoms at any level).

(LG '(1 (2 (a) c d) (3))) = 6

$$\lg(L) = \begin{cases} 1 & daca \quad L\ e\ atom \\ \sum_{i=1}^{n} \lg(L_i) & daca\ L\ e\ lista\ (L_1 \ldots L_n) \end{cases}$$

```
(DEFUN LG (L)
    (COND   ((null L) 0)
        ((ATOM L) 1)
        (T (APPLY #'+ (MAPCAR 'LG L)))
    )
)
```

4. Define a function that, given a nonlinear list returns the number of sublists (including the list) with even length (at the superficial level).

(NR '(1 (2 (3 (4 5) 6)) (7 (8 9)))) = 4

We will use a auxiliary function that returns T if the argument list has an even number of elements at the surface level, NIL otherwise.

$$nr(L) = \begin{cases} 0 & \textit{daca L e atom} \\ 1 + \sum_{i=1}^{n} \lg(L_i) & \textit{daca L e lista } (L_1 \ldots L_n) \textit{ si n e par} \\ \sum_{i=1}^{n} \lg(L_i) & \textit{altfel} \end{cases}$$

```
(DEFUN EVEN (L)
    (COND
        ((= 0 (MOD (LENGTH L) 2)) T)
        (T NIL)
    )
)

(DEFUN NR (L)
    (COND
        ((ATOM L) 0)
        ((EVEN L) (+ 1 (APPLY #'+ (MAPCAR #'NR L))))
        (T (APPLY #'+ (MAPCAR #'NR L)))
    )
)
```

? Other version for NR ?

5. Define a function that, given a nonlinear list, returns the list of atoms (from any level) in the list.

(ATOMI '(1 (2 (3 (4 5) 6)) (7 (8 9)))) = (1 2 3 4 5 6 7 8 9)

$$atomi(L) = \begin{cases} (L) & daca \quad L \; e \; atom \\ \bigcup_{i=1}^{n} atomi(L_i) & daca \; L \; e \; lista \; (L_1 \ldots L_n) \end{cases}$$

```
(DEFUN ATOMI (L)
	(COND
		((ATOM L) (LIST L))
		(T (MAPCAN #'ATOMI L))
	)
)
```

Remark: The same requirement could be solved using the MAPCAR function.

```
(DEFUN ATOMI (L)
	(COND
		((ATOM L) (LIST L))
		(T (APPLY #'APPEND (MAPCAR #'ATOMI L)))
	)
)
```

For the following examples, we let the reader deduce the recursive solution formulas.

6. Define a function that, given a nonlinear list returns the list with all negative numerical atoms at any level removed (keeping the list structure).

(ELIMIN '(A (1 B (-1 3 C)) 2 -3)) = (A (1 B (3 C)) 2)

Remark: An auxiliary ELIM function will be used (let the reader notice the need to use this function)

```
(DEFUN ELIM (L)
    (COND
        ((AND (REALP L) (MINUSP L)) NIL)
        ((ATOM L) (LIST L))
        (T (LIST (MAPCAN #'ELIM L)))
    )
)

(DEFUN ELIMIN (L)
    (CAR (ELIM L))
)
```

7. Define a function which, given a nonlinear list, returns T if all sublists (including the list) have even length (at the surface level), or NIL otherwise.

```
(VERIF '(1 (2 (3 (4 5))))) = T
(VERIF '(1 (2 (3 (4 5 6))))) = NIL
```

Remark: A function (EVEN L) (defined above) and an auxiliary function (MYAND L) will be used, which having as argument a list consisting only of the values T and NIL checks if all the elements in the list are T.

```
(DEFUN MYAND (L)
    (COND
        ((NULL L) T)
        ((NOT (CAR L)) NIL)
        (T (MYAND (CDR L)))
    )
)

(DEFUN VERIF (L)
    (COND
        ((ATOM L) T)
        ((NOT (EVEN L)) NIL)
        (T (FUNCALL #'MYAND
                    (MAPCAR #'VERIF L)))
    )
)
```

If L is NOT EVEN, the condition that all lists are EVEN is false, and the computation ends. Otherwise, the result is a logical AND on all the recurrences comming from MAPCAR.

8. We could represent a general tree in Lisp as a list of the form

(root subtree1 subtree2 ...)

Define a function which, given a tree, returns the number of nodes in the tree.

(NR '(1 (2) (3 (5) (6)) (4))) = 6

; (CDR L) is the list of subtrees of L

```
(DEFUN NR (L)
    (COND
        ((NULL (CDR L)) 1)
        (T (+ 1 (APPLY #’+ (MAPCAR #‘NR (CDR L))))
    )
)
```

; other option – count all atoms

; take care – avoid the empty lists

; why?

```
(defun nr (Tree)
  (cond
    ((null Tree) 0)
    ((atom (car Tree)) (+ 1 (apply #'+ (mapcar #'nr (cdr Tree)))))
  )
)

(print (nr '(1 (2) (3 (5) (6)) (4))))
```

9. Define a function which, given a tree represented as above, returns the depth of the tree (maximum level - root level is assumed 0).

```
(AD '(1 (2) (3 (5) (6)) (4))) = 2

(DEFUN AD (L)
    (COND
        ((NULL (CDR L)) 0)
        (T (+ 1 (APPLY #'MAX
                    (MAPCAR #'AD (CDR L))))
    )
)
```

10. Define a function which, given a nonlinear list, returns the list of atoms that appear on any level, but in reverse order.

```
(INVERS '(A (B C (D (E))) (F G))) = (G F E D C B A)
```

a. recursive, without MAP functions

```
(DEFUN INVERS (L)
  (COND
      ((NULL L) NIL)
      ((ATOM (CAR L)) (APPEND
                  (INVERS (CDR L)) (LIST (CAR L))))
      (T (APPEND (INVERS (CDR L)) (INVERS (CAR L))))
  )
)
```

b. using MAP functions

```
(DEFUN INVERS (L)
      (COND
            ((ATOM L) (LIST L))
            (T (MAPCAN #'INVERS (REVERSE L)))
      )
)

; why using mapcan?
; what happens if we use mapcar?
```

11. A matrix can be represented in Lisp as a list whose elements are lists representing the lines of the matrix.

((line1) (line2)….)

Define a function which, given two matrices of order n return their product (as a matrix).

(PRODUCT '((1 2) (3 4)) ‘((2 -1) (3 1))) = ((8 1) (18 1))

Remark: We will use two auxiliary functions: a function (COLUMNS L) that returns the list of columns of the parameter matrix L and a function (PR L1 L2) that returns as a matrix the result of multiplying the matrix L1 (list of rows) with the list L2 (a list of columns of a matrix).

```
(DEFUN COLS (L)
      (COND
            ((NULL (CAR L)) NIL)
            (T (CONS (MAPCAR #’CAR L)
                              (COLS (MAPCAR #’CDR L))))
))

; compute one value of the product matrix
(DEFUN PR1 (L1 L2)
      (APPLY #’+ (MAPCAR #'* L1 L2))
)

; compute one line of the product matrix
(DEFUN PR2 (L1 M2)
      (MAPCAR #’(LAMBDA (L) (PR1 L1 L)) M2)
)
```

```
; compute all lines of the product matrix
(DEFUN PR (M1 M2)
        (MAPCAR #’(LAMBDA (L) (PR2 L M2)) M1)
)

(DEFUN PRODUCT (M1 M2) (PR M1 (COLS M2)) )
```

Comments for PR:

A version of PR2 with PR1 subsumed

```
(DEFUN PR2 (L1 MAT2)
        (MAPCAR #’(LAMBDA (L)
                (APPLY #’+ (MAPCAR #'* L1 L)) )
        MAT2)
)
```

Write a version of PR with PR1 and PR2 subsumed.

Comments for COLS:

Instead of

```
        (COLS ’((2 -1) (3 1)))
```

we may try

```
        (MAPCAR #’LIST ’(2 -1) ’(3 1))
```

with the same results. But notice we unwrapped the matrix.
This leads us to

```
        (APPLY #’MAPCAR #’LIST ’((2 -1) (3 1)))
```

So we may define COLS as

```
        (DEFUN COLS (L)
                (APPLY #’MAPCAR #’LIST L)
        )
```

12. Write a function to return the number of occurrences of a certain element in a nonlinear list at any level.

(*nrap* 'a '(1 (a (3 (4 a) a)) (7 (a 9)))) = 4

$$nrap(e,l) = \begin{cases} 1 & daca\ l = e \\ 0 & dacă\ l\ e\ atom \\ \sum_{i=1}^{n} nrap(e, l_i) & altfel, l = (l_1 l_2 \dots l_n)\ e\ lista \end{cases}$$

```
(defun nrap(e L)
	(cond
		((equal L e) 1)
		((atom L) 0)
		(t (apply #'+ (mapcar #'(lambda(L) (nrap e L) ) l ) ))
	)
)
```

13. Given a nonlinear list, write a function to return the list with all negative numeric atoms removed. Use a MAP function.

Ex: (*stergere* '(a 2 (b -4 (c -6)) -1)) → (a 2 (b (c)))

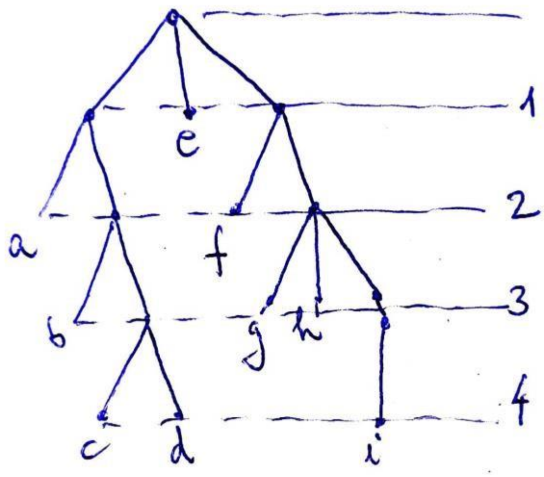
$$sterg(l) = \begin{cases} \emptyset & daca\ l\ numeric\ negativ \\ l & dacă\ l\ e\ atom \\ sterg(l_i) & altfel, l = (l_1 l_2 \ldots l_n)\ e\ lista \end{cases}$$

```
(defun sterg(L)
    (cond
        ((and (numberp L) (minusp L)) nil)
        ((atom L) (list L))
        (t (list (apply #'append (mapcar #'sterg L) ) ))
    )
)

(defun stergere (L)
    (car (sterg L))
)
```

14. Write a function to return the list of atoms at depth n from a non-linear list. The superficial level is assumed 1.

(lista '((a (b (c d))) e (f (g h (i)))) 3) returns (b g h)
(lista '((a (b (c d))) e (f (g h (i)))) 4) returns (c d i)
(lista '((a (b (c d))) e (f (g h (i)))) 5) returns NIL



<u>Recursive model</u>

$$lista(l,n) = \begin{cases} (l) & \text{dacă } n = 0 \text{ si } l \text{ atom} \\ \emptyset & \text{dacă } n = 0 \\ \emptyset & \text{dacă } l \text{ atom} \\ \bigcup_{i=2}^{k} lista\,(l_i, n-1) & altfel, l = (l_1 l_2 \ldots l_k) \text{ e } lista \end{cases}$$

```
(defun lista(L n)
        (cond
                ((and (= n 0) (atom L)) (list L))
                ((= n 0) nil)
                ((atom L) nil)
                (t (mapcan #'(lambda(L) (lista L (- n 1)) ) L))
        )
)
```

15. Generate the power set of a set. I.e., return the list of all sublists of a linear list. Use a MAP function.

Ex: (*subm* ‘(1 2)) → (nil (1) (2) (1 2))

<u>Remark</u>

(setq e 1)

(mapcar #'(lambda(L) (cons e L)) '(2 3)) returns ((1 2) (1 3))

```
(defun subm (L)
        (cond
                ((null L) (list nil))
                (t ((lambda (s)
                        (append s
                                (mapcar #'(lambda (sb) (cons (car L) sb)) s)
                        )
                    ) (subm (cdr L)))
                )
        )
)
```

16. Given a set represented as a linear list, write a function to generate the list of permutations of that set. Use a MAP function.

Ex: (*permutari* ‘(1 2 3)) → ((1 2 3) (1 3 2) (2 1 3) (2 3 1) (3 1 2) (3 2 1))

```
(defun permutari (L)
 (cond
      ((null (cdr L)) (list L))
      (t (mapcan #'(lambda (e)
                          (mapcar #'(lambda (p) (cons e p) )
                                (permutari (remove e L))
                          )
                    )
                    L
      ))
   )
)
```

17. Generate the cartesian product of two given sets. Use a MAP function.

E.g. (CARTESIAN ′(1 2 3) ′(11 22 33 44)) should return

```
( (1 11) (1 22) (1 33) (1 44)
  (2 11) (2 22) (2 33) (2 44)
  (3 11) (3 22) (3 33) (3 44)
  (4 11) (4 22) (4 33) (4 44) )
```

```
(DEFUN CARTESIAN (L1 L2)
    (MAPCAN #'(LAMBDA (X)
        (MAPCAR #'(LAMBDA (Y)
            (LIST X Y)
        ) L2)
    ) L1)
)
```

Comments:

The MAPCAR call produces the list of all pairs of a certain x (in L1) with all y (in L2).

The MAPCAN call concatenates all of the above for all x (in L1).

The LAMBDA calls are needed to hide quite a complex functionality inside single function required for the MAP functions.