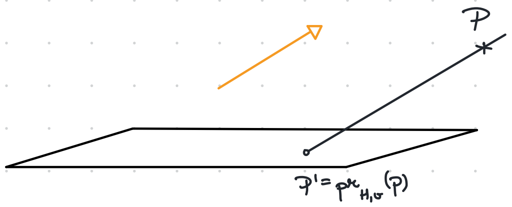
5.1. Consider an orthonormal coordinate system K of $\mathcal{E}^n$ where $n=2$ or 3. Starting from the matrix form of the projections and reflections described in this chapter, deduce the matrices of:

a) orthogonal projections on the coordinate axes and the coordinate hyperplanes of K.

$$Pr_{H,v} P = \left(\mathcal{I}_n - \frac{a \otimes v}{\langle a, v \rangle} \right) \cdot P - \frac{a_{n+1}}{\langle a, v \rangle} \cdot a$$



$$H: a_1x_1 + a_2x_2 + \dots + a_nx_n + a_{n+1} = 0$$

pr

$$a \otimes v = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} (x_1 \dots x_n) = \begin{pmatrix} a_1x_1 & \dots & a_nx_1 \\ \vdots & & \vdots \\ a_nx_1 & & a_nx_n \end{pmatrix}$$

$(xOy): z=0$
$(yOz): x=0$
$(xOz): y=0$

$Ox: \begin{cases} y=0 \\ z=0 \end{cases}$

$Oy: \begin{cases} x=0 \\ z=0 \end{cases}$

$Oz: \begin{cases} x=0 \\ y=0 \end{cases}$

$a = (0,0,1)$

$$a \otimes a = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (0\ 0\ 1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$a_{n+1} = 0$

$$pr_{xOy,a} P = \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \cdot P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot P$$

$$pr_{\ell,w} P = \frac{w \otimes a}{\langle w, a \rangle} \cdot P - \left(\mathcal{I}_n - \frac{w \otimes a}{\langle w, a \rangle} \right) \cdot Q$$

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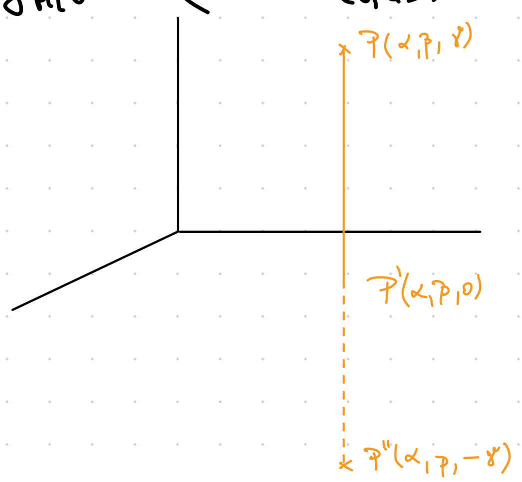
$$Ref_{\ell,w}(P) = \left(-\mathcal{I}_n + 2 \cdot \frac{v \otimes a}{\langle v, a \rangle} \right) \cdot P - \left(\mathcal{I}_n - \frac{v \otimes a}{\langle v, a \rangle} \right) Q$$

$$Ref_{H,v}(P) = \left(\mathcal{I}_n - 2 \cdot \frac{v \otimes a}{\langle v, a \rangle} \right) \cdot P - 2 \cdot \frac{a_{n+1}}{\langle v, a \rangle} v$$



$$Ref^{\perp}_{xOy}(P) = \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \right) P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} P$$

2. $\pi : xOy \Rightarrow u = (0,0,1)$

$$Pr_{H,v} P = \left(I_n - \frac{a \otimes v}{\langle a, v \rangle} \right) \cdot P - \frac{a_{n+1}}{\langle a, v \rangle} \cdot \vec{a}$$

+ Tabla

4. $P(6,-5,5)$

$\pi : 2x - 3y + z - 4 = 0$

$$Ref_{\pi} P = \left(I_n - 2 \cdot \frac{v \otimes a}{\langle v, a \rangle} \right) P - 2 \cdot \frac{a_{n+1}}{\langle v, a \rangle}$$

$\vec{v} = (2,-3,1)$

$$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \begin{pmatrix} 2 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -6 & 2 \\ -6 & 9 & -3 \\ 2 & -3 & 1 \end{pmatrix}$$

$\langle v, v \rangle = 14$

$$\Rightarrow Ref_{\pi} P = \begin{pmatrix} \frac{3}{7} & \frac{6}{7} & \frac{-2}{7} \\ \frac{6}{7} & \frac{-2}{7} & \frac{3}{7} \\ -\frac{2}{7} & \frac{3}{7} & \frac{6}{7} \end{pmatrix} \cdot P + \begin{pmatrix} \frac{8}{7} \\ -\frac{12}{7} \\ \frac{4}{7} \end{pmatrix}$$

5.

$$Pr_{\vec{a}}^{\perp} (\vec{b}) = \frac{\langle \vec{a}, \vec{b} \rangle}{\langle \vec{a}, \vec{a} \rangle} \cdot \vec{a}$$

$$Pr_H^{\perp} (P) = \left(I_n - \frac{\vec{a} \otimes \vec{a}}{|a|^2} \right) \cdot P - \frac{a_{n+1}}{|a|^2} \cdot \vec{a}$$

Let $\vec{a}(a_1, a_2)$ $\quad a^{\perp}(a_2, -a_1)$

$H: a_2 x - a_1 y = 0$

$$a^{\perp} \otimes a^{\perp} = \begin{pmatrix} a_2 \\ -a_1 \end{pmatrix} \begin{pmatrix} a_2 & -a_1 \end{pmatrix} = \begin{pmatrix} a_2^2 & -a_1 a_2 \\ -a_1 a_2 & a_1^2 \end{pmatrix}$$

$$Pr_H^{\perp} \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{1}{a_1^2 + a_2^2} \begin{pmatrix} a_2^2 & -a_1 a_2 \\ -a_1 a_2 & a_1^2 \end{pmatrix} \right.$$