Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză

Curs: Dynamical Systems

Primăvara 2024

Lecture 14 - List of problems

1. Find a range of values for $h > 0$ such that the attractor equilibrium point of $x' = x^2 + 5x + 6$ is also an attractor fixed point of the discrete dynamical system associated to the Euler's numerical formula with stepsize $h > 0$ for the given differential equation. $\diamond$

2. We consider the pray-predator system

$\dot{x} = x(1 - y), \quad \dot{y} = -y(2 - x).$

(a) Find the expression of a first integral in $(0, \infty) \times (0, \infty)$. Check it using the corresponding first order partial differential equation.

(b) If $(2, 1)$ is an equilibrium point, is it hyperbolic? Is it stable? $\diamond$

3. (a) Let $A \in \mathcal{M}_n(\mathbb{R})$ and $\eta \in \mathbb{R}^n$. Write a representation formula for the solution of the IVP

$$X' = AX, \quad X(0) = \eta.$$

(b) Let $t \in \mathbb{R}$. Using the definition of the matrix exponential, compute

$$e^{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}, \quad e^{t\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}, \quad e^{t\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}, \quad e^{t\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}.$$

(c) Let $A, J, P \in \mathcal{M}_n(\mathbb{R})$ and assume that P is invertible and $A = PJP^{-1}$. Prove that $e^A = Pe^J P^{-1}$. $\diamond$

4. Let $a_{11}, a_{12}, a_{21}, a_{22} \in \mathbb{R}$ be such that $a_{12} \neq 0$. Show that the roots of the characteristic equation corresponding to the second-order difference equation obtained by reducing the linear system

$$x_{k+1} = a_{11}x_k + a_{12}y_k, \quad y_{k+1} = a_{21}x_k + a_{22}y_k$$

are the eigenvalues of the matrix associated to this system. $\diamond$

5. We consider the IVP $\quad y' = -200y, \quad y(0) = 1,$
where the unknown is the function $y(t)$.

a) Find the solution and its limit as $t \to \infty$.

b) Write the Euler's numerical formula with constant step-size h.

c) For $h = 0.001$, and, respectively, $h = 0.01$ find the solution $(y_k)_{k \geq 0}$ of the difference equation found at b) and decide if it satisfies $\lim\limits_{k \to \infty} y_k = 0$.

d) Find a range of values for the step-size h such that the solution $(y_k)_{k \geq 0}$ of the difference equation found at b) satisfies $\lim\limits_{k \to \infty} y_k = 0$. $\diamond$