Lecture 6 – Backtracking

1. Backtracking

Backtracking method:

- generally applicable to problems that have several solutions.
- allows the generation of all solutions to a problem
- depth limited search in the space of solutions to the problem
- exponential as execution time
- general method for solving problems in the class of Constraint Satisfaction Problems (CSP)
- Prolog is suitable for solving CSP class problems
    - the **control structure** used by the Prolog interpreter is based on backtracking

Formalization

- the problem solution is a list vector (*x1*, ..., *xn*), with *xi* ∈ *Di*
- the solution vector is incrementally generated
- we denote by *col* the list that collects a solution to the problem
- we denote by **final** the predicate verifiying whether the *col* list is a solution to the problem
- we denote by **constraints** the predicate verifying whether the *col* list may lead to a solution of the problem

To determine a solution to the problem, the following cases are possible:

i. if *col* satisfies **final**, then there is a solution to the problem;
ii. otherwise, we extend *col* with an element *e* (which we will call **candidate**) such that col∪*e* satisfies **constraints**

Remarks

- if the generation of the element *e* the *col* collector at point ii will be extended with is non-deterministic, then all solutions to the problem will be generated
- there are problems with no final conditions imposed

The general recursive model for generating solutions is:

$solution(col) =$
1. col $\quad\quad\quad\quad$ $if\ col\ verifies\ the\ final\ conditions$
2. $solution(col \cup e)$ $\quad$ $if\ e\ is\ a\ possible\ candidate\ and$
$\quad\quad\quad\quad$ $col \cup e\ verifies\ the\ continuation\ conditions$

The simplified Prolog code structure follows:

```
% E – a candidate value for the current variable

candidate(E, …) :-
        …

% X – the candidate solution

final(X, …) :-
        …

% E – the candidate value for the current variable
% X – the past variables with assigned values
```

verify X+E → valid

```
constraints(E, X, …) :-
        …
```

```
% S – the solution array – the solution to the problem

problem(…, S) :-
        solution([], S, …).

% X – the candidate solution – the complete array

solution(X, X, …) :-
        final(X, …).

% X – the past variables with assigned values
% S – the solution array
% E – the candidate value for the current variable

solution(X, S, …) :-
        candidate(E, …),
        constraints(E, X, …),
        solution([E | X], S, …).
```

EXAMPLE 1.1 Write a non-deterministic predicate that generates combinations with k ≠ 1 elements of a non-empty set whose elements are non-zero natural numbers, so that the sum of the elements in the combination is a given value S.

```
? combSum ([3, 2, 7, 5, 1, 6], 3, 9, C).
/* flow model (i, i, i, o) – non-deterministic */
/* k = 3, S = 9 */
C = [2, 1, 6];
C = [3, 5, 1].

? allCombSum ([3, 2, 7, 5, 1, 6], 3, 9, LC).

LC = [[2, 1, 6], [3, 5, 1]].
```

For solving this problem, we will give three solutions, the last one based on the backtracking method described above.

Solution 1 We generate the solutions of the problem using direct recursion.

To determine the combinations of a list [E | L] (which has the head E and the tail L) taken K, of given sum S the following cases are possible:

i. if K = 1 and E is equal to S, then [E] is a combination
ii. determine a combination with K elements of the list L, having the sum S;
iii. place the element E on the first position in the combinations with K-1 elements of the list L, of sum S-E (if K> 1 and SE> 0).

The recursive model for generation is:

$combSum(l_1\ l_2 \ldots l_n, k, S) =$
1. (l_1) $\quad\quad\quad\quad$ $if\ k = 1\ and\ l_1 = S$
2. $combSum(l_2 \ldots l_n, k, S)$
3. $l_1 \oplus combSum(l_2 \ldots l_n, k-1, S-l_1)$ $\quad$ $if\ k > 1\ and\ S - l_1 > 0$

We will use the non-deterministic predicate combSum which will generate all combinations. If you want to collect combinations in a list, you can use the findall predicate.

The SWI-Prolog program is as follows:

```
% combSum(L: list, K: integer, S: integer, C: list)
% (i, i, I, o) – non-deterministic

                L    k S  C
combSum([H | _], 1, H, [H]).
combSum([_ | T], K, S, C) :-
        combSum(T, K, S, C).
combSum([H | T], K, S, [H | C]) :-
        K > 1,
        S1 is S-H,
        S1 > 0,
        K1 is K-1,
        combSum(T, K1, S1, C).
```

work **Solution 2** The combinations with k elements are first generated and then verified whether the sum of elements is S. This solution is not efficient, considering that combinations are generated that may not have sum S.

The following predicates will be used:

- the comb predicate for generating a combination with k elements from a list, predicate described in **lecture 5**
- the predicate sum (L: list of numbers, S: integer), flow model (i, i) that checks if the sum of the elements of the list L is equal to S.

```
% combSum(L: list, K: integer, S: integer, C: list)
% (i, i, i, o) – non-deterministic
combSum(L, K, S, C) :-
        comb(L, K, C),
        sum(C, S).
```

```
comb([H | _], 1, [H]).
comb([_ | T], K, C) :-
        comb(T, K, C). k combination of T
comb([H | T], K, [H | C]) :-
        K > 1,
        K1 is K-1,
        comb(T, K1, C). k-1 comb. of T
```

<u>Solution 3</u> We use a non-deterministic predicate **candidate**(E: element, L: list), flow model (o, i), which generates all the elements of a list.

```
? candidate (E, [1, 2, 3]).
E = 1;
E = 2;
E = 3.
```

$candidate(l_1 l_2 \dots l_n) =$

1. l_1 $\quad\quad\quad$ *if l is not empty*
2. $candidate(l_2 \dots l_n)$

```
% candidate(E: item, L: list)
% (o, i) – non-deterministic
candidate(E, [E | _]).          member funct.
candidate(E, [_ | T]) :-
    candidate(E, T).
```

The main predicate will generate a candidate E and will start generating the solution with this element.

```
% combSum(L: list, K: integer, S: integer, C: list)
% (i, i, i, o) – non-deterministic
combSum(L, K, S, C) :-
    candidate(E, L),
    combAux(L, K, S, C, 1, E, [E]).
```

The non-deterministic auxiliary predicate combAux will generate combinations of a given sum.

```
% combAux (L: list, K: integer, S: integer, C: list, Lg: integer,
Sum: integer, Col: list)
% (i, i, i, o, i, i, i) – non-deterministic
combAux(_, K, S, C, K, S, C) :- !.
combAux(L, K, S, C, Lg, Sum, [H | T]) :-
    Lg < K,
    candidate(E, L),
  ? E < H,
    Sum1 is Sum + E,
    Sum1 =< S,
    Lg1 is Lg + 1,
    combAux(L, K, S, C, Lg1, Sum1, [E, H | T]).

% alternative for the second clause
% refactoring – an auxiliary predicate to verify the condition

% constraints(L:list, K:integer, S:integer, Lg:integer,
    Sum:integer, H:integer, E:integer)
% (i, i, i, i, i, i, o) – non-deterministic
% generates an E from L that verifies the constraints with the
   past variables

constraints(K, S, LenRes, SumRes, HRes, E) :-
   LenRes < K,
   E < HRes,
   NewSum is SumRes + E,
   NewSum =< S.

% return the solution
combAux(_, K, S, C, K, S, C) :- !.

% get a legal candidate and add it to the partial solution found
so far
combAux(L, K, S, C, LenRes, SumRes, [HRes|TRes]):-
   candidate(E, L),
   constraints(L, K, S, LenRes, SumRes, HRes, E),
   NewLen is LenRes+1,
   NewSum is SumRes + E,
   combAux(L, K, S, C, NewLen, NewSum, [E, HRes|T]).
```

```
           i ,  o
candidate([], []).
candidate([H|T], [H|S]) :-
          H mod 2 =:= 1,
          candidate(T, S).
 candidate([_|T], S) :-
           candidate(T, S).


 constraint(S) :-
      suma_list(S, sum),
       Sum mod 2 =:= 0.

  sumSP(L, S) :-
       candidate(L, S),
        constraint(S).
```

Hw.

EXAMPLE 1.2. Given a set represented as list, the subsets of even sum formed by odd numbers only should be generated.

```
? submSP([1, 2, 3, 4, 5], S).
/* flow model (i, o) – non-deterministic */
S = [1, 3];
S = [1, 5] ;
S = [3, 5];
```

EXAMPLE 1.3 Given two natural values n (n> 2) and v (v non-zero), a Prolog predicate is required which returns the permutations of the elements 1, 2 ...., n with the property that any two consecutive elements have the difference in absolute value greater than or equal to v.

```
        ? permutations (4, 2, P)          (n = 4, v = 2)
        P = [2, 4, 1, 3];
        P = [3, 1, 4, 2];
        ....

        ? candidate (4, I).
        I = 4;
        I = 3;
        I = 2;
        I = 1.
```

Recursive models

candidate (n) =

1. n

2. candidate (n-1) if n > 1

The following predicates will be used

- the non-deterministic predicate **candidate** (N, I) (i, o) generates a solution candidate (a value between 1 and N);
- the non-deterministic predicate **permutations_aux** (N, V, LResult, Length, LCollectors) (i, i, o, i, i) collects the elements of a permutation in LCollectors of length Length. Collection will stop when the number of collected items (Length) is n. In this case, LCollectors will contain a solution permutation, and LResult will be bound to LCollectors.
- the non-deterministic predicate **permutations** (N, V, L) (i, i, o) generates a solution permutation;
- the predicate **member** (E, L) which tests the membership of an element in a list (to collect in solution only the distinct elements).

```
% candidate (N: integer, I: integer)
% (i, o) – non-deterministic
candidate (N, N).
candidate (N, I) :-
        N> 1,
        N1 is N-1,
        candidate (N1, I).

% permutations (N: integer, V: integer, L: list)
% (i, i, o) – non-deterministic
permutations (N, V, L) :-
        candidate (N, I),
        permutations_aux (N, V, L, 1, [I]).
```

```
% permutations_aux (N: integer, V: integer, L: list, Lg: integer,
Col: list)
% (i, i, o, i, i) – non-deterministic
permutations_aux (N, _, Col, N, Col) :- !.
permutations_aux(N, V, L, Lg, [H | T]) :-
        candidate (N, I),
        abs (H-I) >= V,
        \+ member (I, [H | T]), % (i, i)   I haven't added it yet
        Lg1 is Lg + 1,
        permutations_aux (N, V, L, Lg1, [I, H | T]).
```

EXAMPLE 1.4 Consider a set of non-null natural numbers represented as a list. Determine all the possibilities to write a number N as a sum of elements from this list.

```
% list=integer*
% candidate(list, integer) (i, o) – non-deterministic
% an element possibly to be added in the solution list
candidate([E|_],E).      member function
candidate([_|T],E) :-
        candidate(T,E).

% solution(list,integer,list) (i,i,o) – non-deterministic
solution(L, N, Rez) :-
        candidate(L,E),
        E =< N, → the sum cannot be computed
        solution_aux(L, N, Rez, [E], E).

% solution_aux(list,integer,list,list,integer) (i,i,o,i,i) – non-
deterministic
% the fourth parameter collects the solution
% the fifth parameter represents the sum of all elements in the
collector
solution_aux(_, N, Rez, Rez, N) :- !.
solution_aux(L, N, Rez, [H | Col], S) :-
        candidate(L, E),
   ?    E < H,
        S1 is S+E,
        S1 =< N, we can still add
        solution_aux(L, N, Rez, [E, H | Col], S1).
```

<u>EXAMPLE 1.5</u> THE PROBLEM OF THE THREE HOUSES.

1. The Englishman lives in the first house on the left.
2. In the house immediately to the right of the one where the wolf is, he smokesLucky Strike.
3. The Spaniard smokesKent.
4. The Russian has a horse.

Who smokes LM? Whose dog is it?

We remark that the problem has two solutions:

I.

English	dog	LM
Spanish	wolf	Kent
Russian	horse	LS

II.

English	wolf	LM
Russian	horse	LS
Spanish	dog	Kent

This is a typical constraint satisfaction problem.

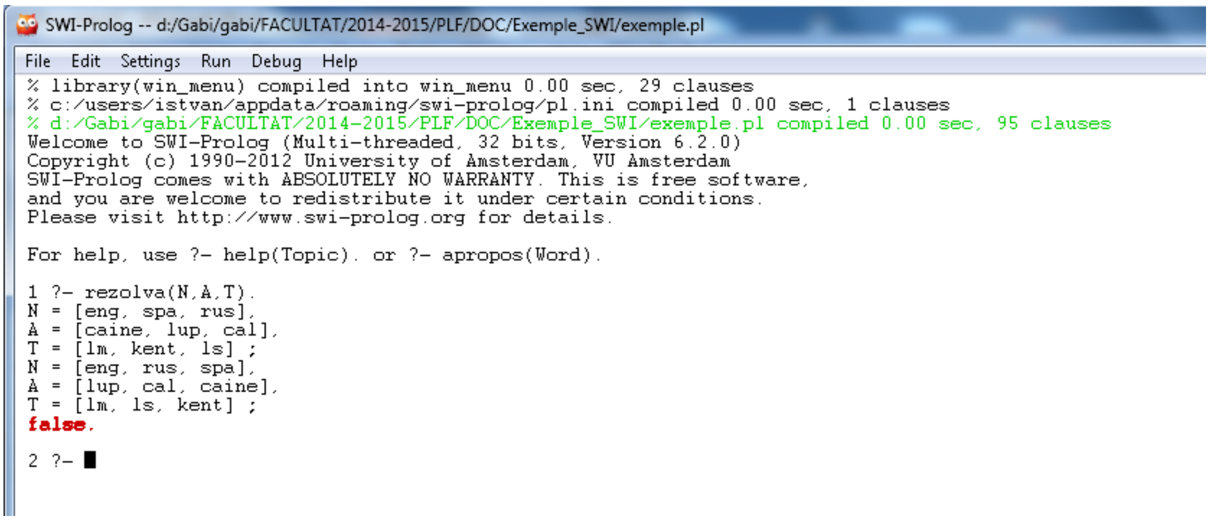
We encode the data of the problem and remark that a solution consists of triplets of the form (N, A, T) where:

N belongs to the set **[eng, spa, rus]**
A belongs to the set **[dog, wolf, horse]**
T belongs to the set **[lm, ls, ken]**

We will use the following predicates:

- non-deterministic predicate **solve** (N, A, T) (o, o, o) which generates a solution of the problem

- non-deterministic predicate **candidates** (N, A, T) (o, o, o) that generates all candidates for solution
- deterministic predicate **constraints** (N, A, T) (i, i, i) which verifies whether a candidate for solution satisfies the constraints imposed by the problem
- non-deterministic predicate **perm** (L, L1) (i, o) which generates the permutations of the list L



SWI-Prolog -- d:/Gabi/gabi/FACULTAT/2014-2015/PLF/DOC/Exemple_SWI/exemple.pl
File Edit Settings Run Debug Help

```
% library(win_menu) compiled into win_menu 0.00 sec, 29 clauses
% c:/users/istvan/appdata/roaming/swi-prolog/pl.ini compiled 0.00 sec, 1 clauses
% d:/Gabi/gabi/FACULTAT/2014-2015/PLF/DOC/Exemple_SWI/exemple.pl compiled 0.00 sec, 95 clauses
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For help, use ?- help(Topic). or ?- apropos(Word).

1 ?- rezolva(N,A,T).
N = [eng, spa, rus],
A = [caine, lup, cal],
T = [lm, kent, ls] ;
N = [eng, rus, spa],
A = [lup, cal, caine],
T = [lm, ls, kent] ;
false.

2 ?-
```

```
% solve - (o, o, o)
solve (N, A, T) :-
        candidates(N, A, T),
        constraints(N, A, T).

% candidates - (o, o, o)
candidates(N, A, T) :-
        perm([eng, spa, rus], N),
        perm([dog, wolf, horse], A),
        perm([lm, kent, ls], T).
```

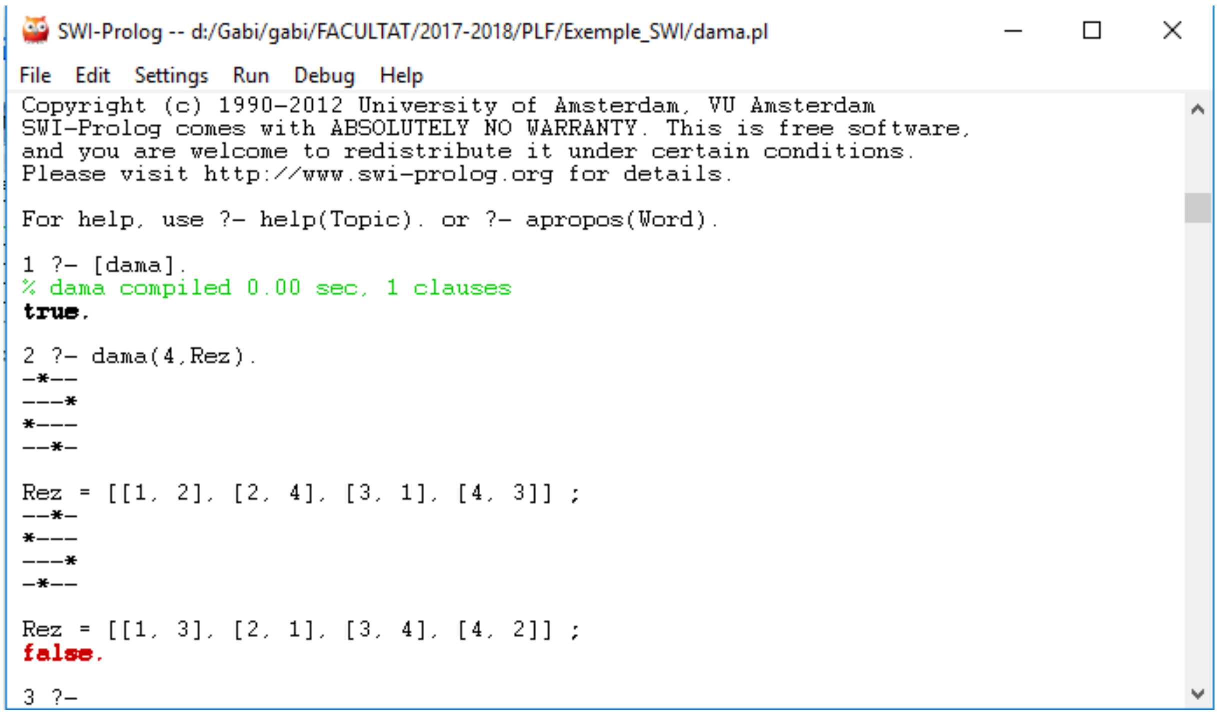
```
% constraints - (i, i, i)
constraints(N, A, T) :-
        aux (N, A, T, eng, _, _, 1),
        aux (N, A, T, _, wolf, _, Nr1),
        right (Nr1, Nr2),
        aux (N, A, T, _, _, ls, Nr2),
        aux (N, A, T, spa, _, kent, _),
        aux (N, A, T, rus, horse, _, _).

% right - (i, o)
right (I, J) :-
        J is I + 1.

% aux (i, i, i, o, o, o, o)
aux ([N1, _, _], [A1, _, _], [T1, _, _], N1, A1, T1,1).
aux ([_, N2, _], [_, A2, _], [_, T2, _], N2, A2, T2,2).
aux ([_, _, N3], [_, _, A3], [_, _, T3], N3, A3, T3,3).

% insert (s)
insert (E, L, [E | L]).
insert (E, [H | L], [H | T]) :-
        insert (E, L, T).

% perm (i, o)
perm ([], []).
perm ([H | T], L) :-
        perm (T, P),
        insert (H, P, L).
```

<u>EXAMPLE 1.6</u> THE PROBLEM OF THE FIVE HOUSES.

There are 15 facts and 2 questions:
Who has a zebra and who drinks water?

1) There are 5 colored houses in a row, each with an owner, an animal, a cigarette, a drink.
2) The English lives in the red house.
3) The Spanish has a dog.
4) They drink coffee in the green house.
5) The Ukrainian drinks tea.
6) The green house is next to the white house.
7) The Winston smoker has a serpent.
8) In the yellow house they smoke Kool.
9) In the middle house they drink milk.
10) The Norwegian lives in the first house from the left.
11) The Chesterfield smoker lives near the man with the fox.
12) In the house near the house with the horse they smoke Kool.
13) The Lucky Strike smoker drinks juice.
14) The Japanese smokes Kent.
15) The Norwegian lives near the blue house.

<u>EXAMPLE 1.7</u> Arrange N queens on a NxN chessboard so that they do not attack each other.


SWI-Prolog -- d:/Gabi/gabi/FACULTAT/2017-2018/PLF/Exemple_SWI/dama.pl
File Edit Settings Run Debug Help

```
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For help, use ?- help(Topic). or ?- apropos(Word).

1 ?- [dama].
% dama compiled 0.00 sec, 1 clauses
true.

2 ?- dama(4,Rez).
-*--
---*
*---
--*-

Rez = [[1, 2], [2, 4], [3, 1], [4, 3]] ;
--*-
*---
---*
-*--

Rez = [[1, 3], [2, 1], [3, 4], [4, 2]] ;
false.

3 ?-
```
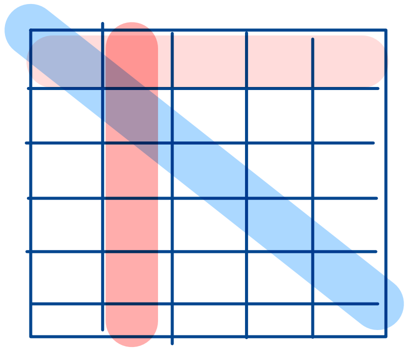

```
% (integer, list *) - (i, o) non-deterministic
queen (N, Rez) :-
        candidate (E, N),
        queen_aux (N, Rez, [[N, E]], N),
        printSolution (N, Rez).

% (integer, integer) - (o, i) non-deterministic
candidate (N, N).
candidate (E, I) :-
        I > 1,
        I1 is I-1,
        candidate (E, I1).

% (integer, list *, list *, integer) - (i, o, i, i) non-deterministic
queen_aux (_, Rez, Rez, 1) :- !.
```



```
queen_aux (N, Rez, C, Lin) :-
        candidate (Col1, N),
        Lin1 is Lin-1,
        valid (Lin1, Col1, C),
        queen_aux (N, Rez, [[Lin1, Col1] | C], Lin1).

% (integer, integer, list *) - (i, i, i) deterministic
valid (_, _, []).
valid (Lin, Col, [[Lin1, Col1] | T]) :-
        Col =\= Col1,
        DLin is Col-Col1,              }
        DCol is Lin-Lin1,              } diagonal
        abs (DLin) =\= abs (DCol),     }
        valid (Lin, Col, T). → check next

% (integer, list *) - (i, i) deterministic
printSolution (_, []) :- nl.
printSolution (N, [[_, Col] | T]) :-
        printLine (N, Col),
        printSolution (N, T).

% (integer, char) - (i, o) deterministic
character (1, '*') :- !.
character(_,'-').

% (integer, list *) - (i, i) determin.
printLine (0, _) :- nl, !.
printLine (N, Col) :-
        character (Col, C),
        write (C),
        N1 is N-1, Col1 is Col-1,
        printLine (N1, Col1).
```

HOMEWORK

1. The **chess knight path** problem: Find the path a knight has to follow on a chess board assuming: (1) legal chess knight moves, (2) the knight touches each cell exactly once, and (3) the knight touches all cells on the chess board.

2. A list of distinct integer elements is given. Generate all subsets with **elements in strictly ascending order**.

3. A list of distinct integer elements is given. Generate all subsets with **k elements in arithmetic progression**.

2. Database manipulation

There are four database manipulation predicates: **assert**, **retract**, **asserta**, **assertz**.

listing	displays the contents of the internal database
assert(*fact*)	adds *fact* to the internal database
assert((*rule*))	adds *rule* to the internal database
retract(*fact*)	removes the first occurrence of *fact* from the internal database
assertz(…)	places asserted material at the *end* of the database
asserta(…)	places asserted material at the *beginning* of the database
retract(*fact*(_)), fail.	removes all occurrences of *fact*.

Example:

Consider the predicate definition:

```
additiontable(A) :-
        member(B, A),
        member(C, A),
        D is B+C,
        assert(sum(B, C, D)),
        fail.
```

The following goal fails, but there is a side effect on the database.

```
?- additiontable([0, 1, 2, 3, 4, 5, 6, 7, 8, 9]).
```

The database now contains:

```
sum(0,0,0).
sum(0,1,1).
sum(0,2,2).
sum(0,3,3).
sum(0,4,4).
sum(0,5,5).
sum(0,6,6).
…
```

The following goal removes all these facts from the database:

```
?- retract(sum(_, _, _)), fail.
```