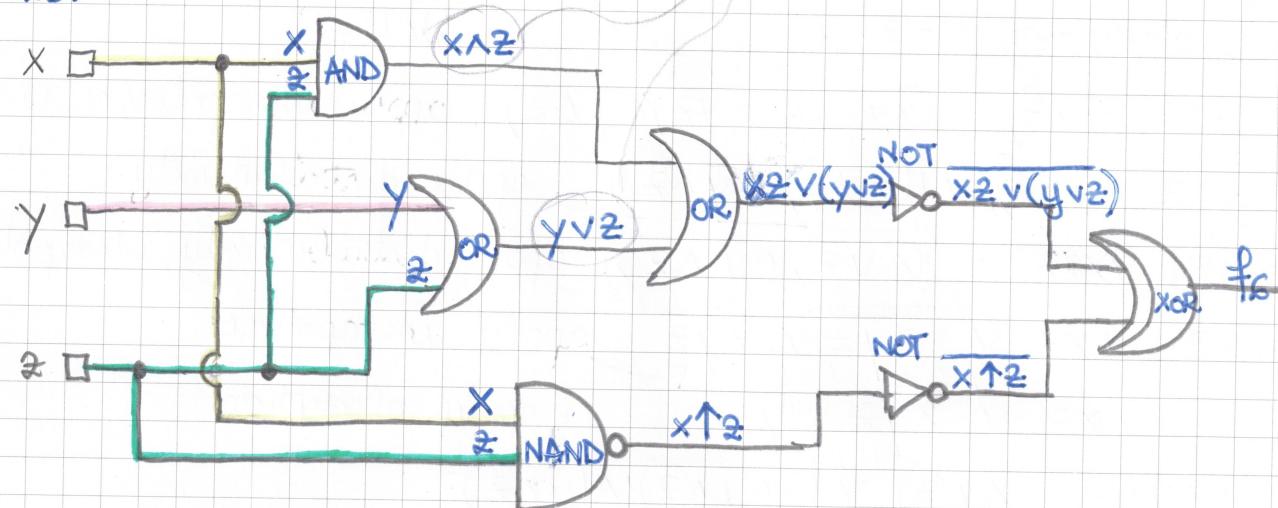


(Ex 1) Write the corresponding Boolean fm associated to the following logic circuit, then simplify it and draw simplified equivalent circuit using only basic gates.

1.6.

write output at each gate



$$f_6(x, y, z) = (\overline{xz} \vee (\overline{y} \vee z)) \oplus \overline{x \uparrow z}$$

replace \oplus

$$= (\overline{xz} \vee (\overline{y} \vee z)) (x \uparrow z) \quad \text{F}$$

$$\vee (\overline{xz} \vee (\overline{y} \vee z)) (\overline{x \uparrow z})$$

$$= (\overline{xz} \vee \overline{y} \vee z) (\overline{x \uparrow z}) \vee$$

$$\vee \overline{\overline{xz} \vee (\overline{y} \vee z)} (\overline{x \uparrow z})$$

replace \uparrow

$$= (\overline{xz} \vee \overline{y} \vee z) (\overline{x \wedge z}) \vee$$

$$\vee (\overline{xz} \vee \overline{y} \vee z) (\overline{x} \vee \overline{z})$$

$$= (\overline{xz} \vee \overline{y} \vee z) (\overline{x \wedge z}) \vee$$

$$\vee ((\overline{x} \vee \overline{z}) \wedge \overline{y} \wedge \overline{z}) (\overline{x} \vee \overline{z})$$

$$= (\overline{xz} \vee \overline{y} \vee z) \overline{xz} \vee$$

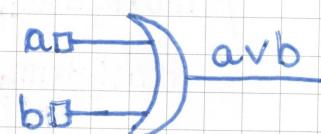
$$\vee ((\overline{x} \vee \overline{z}) \wedge \overline{y} \wedge \overline{z}) (\overline{x} \vee \overline{z})$$

identify gates given

GATES



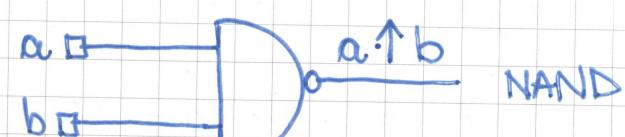
AND



OR



XOR



NAND

$$a \oplus b = \bar{a}b \vee a\bar{b}$$

$$a \uparrow b = \bar{a} \wedge \bar{b} = \bar{a} \vee \bar{b}$$

write formulas you use

$$= xz(xz \vee y \vee z) \vee ((\bar{x} \vee \bar{z}) \wedge \bar{y} \wedge \bar{z})(\bar{x} \vee \bar{z}) \quad \text{ABSORPTION}$$

$$= (x \wedge z)((x \wedge z) \vee y \vee z) \vee (\bar{x} \vee \bar{z})(\bar{x} \vee \bar{z}) \wedge \bar{y} \wedge \bar{z} \quad U \wedge (U \vee V) \equiv U$$

$$= (x \wedge z) \wedge ((x \wedge z) \vee y \vee z) \cdot v \\ \vee ((\bar{x} \vee \bar{z}) \wedge (\bar{x} \vee \bar{z}) \wedge \bar{y} \wedge \bar{z}) \quad \text{IDEMPOTENCE}$$

apply absorption $U = z \dots$

$$= (x \wedge z)(y \vee z) \vee ((\bar{x} \vee \bar{z}) \wedge (\bar{x} \vee \bar{z}) \wedge \bar{y} \wedge \bar{z}) \quad \text{apply idempotence } U = \bar{x} \vee \bar{z}$$

$$= (x \wedge z \wedge (y \vee z)) \vee ((\bar{x} \vee \bar{z}) \wedge \bar{y} \wedge \bar{z}) \quad \text{apply distributivity}$$

$$= xzy \vee \cancel{xz\bar{z}} \vee ((\bar{x} \vee \bar{z}) \wedge \bar{y} \wedge \bar{z}) \quad \text{apply distributivity, idempotence}$$

$$= xzy \vee xz \vee \cancel{\bar{x}\bar{y}\bar{z}} \vee \cancel{\bar{z}\bar{y}\bar{z}} \quad \text{apply idempotence}$$

* $\begin{array}{|c|c|c|c|} \hline & x & \bar{x} & . \\ \hline y & m_4 & & \\ \hline \bar{y} & m_2 & m_3 & m_5 \\ \hline \end{array}$ apply absorption

$$= (x \wedge y \wedge z) \vee (x \wedge z) \vee (\bar{x} \wedge \bar{y} \wedge \bar{z}) \vee (\bar{y} \wedge \bar{z})$$

$$= (x \wedge z) \vee (\bar{y} \wedge \bar{z})$$

$$= xz \vee \bar{y}\bar{z} \quad \leftarrow \text{we must check that it is the simplest form}$$

x	.	\bar{x}	.
y	m_4		
\bar{y}	m_2	m_3	m_5

$$\max_1 = m_4 \vee m_5 = \bar{y}\bar{z}$$

$$\max_2 = m_4 \vee m_5 = \bar{x}\bar{y}$$

$$\max_3 = m_5 \vee m_7 = xz$$

$$M(f) = \{ \max_1, \max_2, \max_3 \}$$

$$C(f) = \{ \max_1, \max_3 \}$$

$$* \begin{array}{l} 111 \\ xzy \\ \hline \end{array} : m_4$$

$$xz: \begin{array}{l} 111 \\ xyz \\ \hline m_7 \end{array}, \begin{array}{l} 101 \\ xy\bar{z} \\ \hline m_5 \end{array}$$

$$\begin{array}{l} 000 \\ \bar{xy}\bar{z} \\ \hline m_0 \end{array}$$

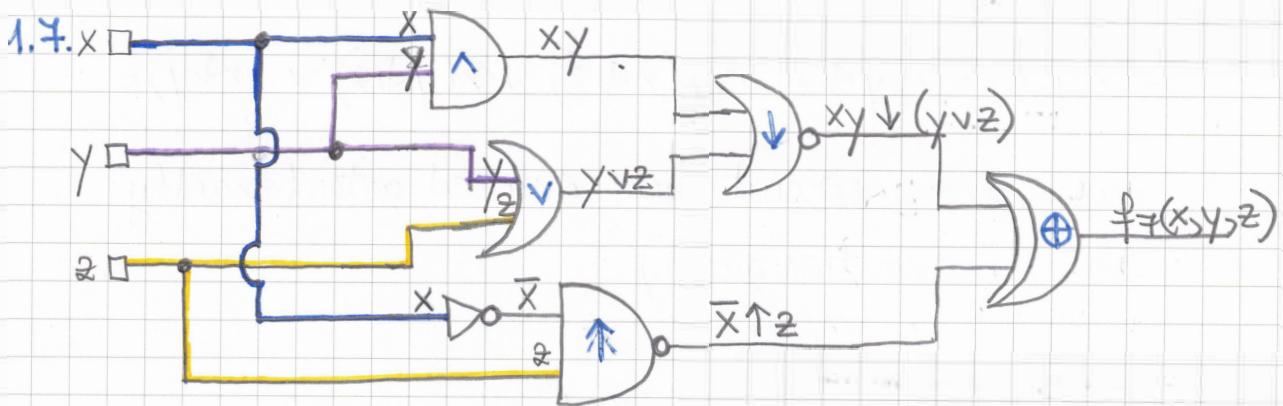
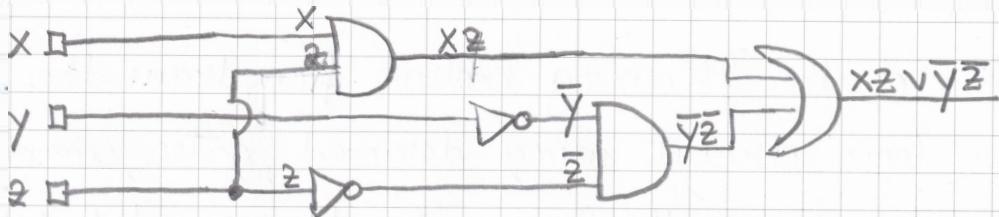
$$\begin{array}{l} 000 \\ \bar{y}\bar{z} \\ \hline \bar{xy}\bar{z} \\ \hline m_0 \end{array}, \begin{array}{l} 000 \\ \bar{xy}\bar{z} \\ \hline m_4 \end{array}$$

$$\left. \begin{array}{l} M(f) \neq C(f) \\ C(f) \neq \emptyset \end{array} \right\} \Rightarrow 2^{\text{nd}} \text{ case of simplification}$$

\Rightarrow We denote by $g = \bigvee_{\max_i(f)} \max_i$ the disjunction of central monomials belonging to every simplified form

$g = \max_1 \vee \max_3 \Rightarrow \max_1, \max_3$ cover all minterms \Rightarrow

$$\Rightarrow f_g(x, y, z) = \max_1 \vee \max_3 \\ = xz \vee \bar{y}\bar{z}$$



$$f_g(x, y, z) = (xy \downarrow (y \vee z)) \oplus (\bar{x} \uparrow z)$$

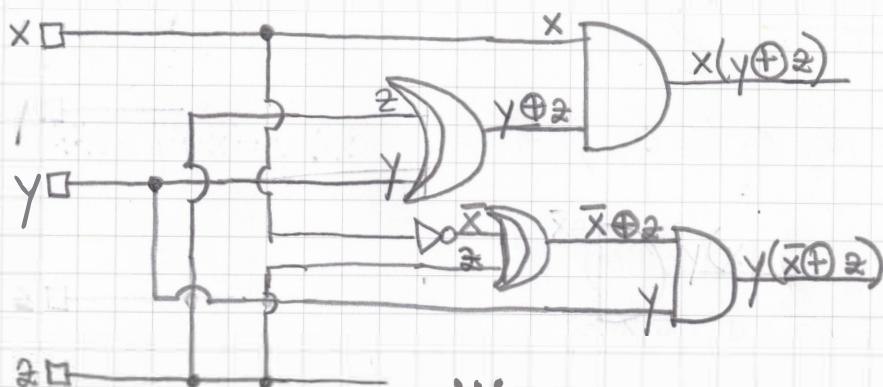
$$x \downarrow y = \bar{x} \vee \bar{y} = \bar{x} \bar{y}$$



Ex 2 For each of the following Boolean fns, draw the corresponding logic circuit using derived gates, simplify the function and draw the logic circuits associated to all simplified forms of the initial fn using basic gates.

$$2.7. f_7(x,y,z) = \underline{x(y \oplus z)} \vee \underline{y(\bar{x} \oplus z)} \vee x(y \downarrow z) \vee (x \downarrow y)\bar{z}$$

Step 1: draw logic circuit w/ derived gates (exactly like the function given)



draw circuit for $x(y \downarrow z)$
 $(x \downarrow y)z$

and then $\Rightarrow f_7$

Step 2: simplify fn

$$\begin{aligned} f(x,y,z) &= x(y\bar{z} \vee \bar{y}\bar{z}) \vee y(x\bar{z} \vee \bar{x}\bar{z}) \vee x(\bar{y} \wedge \bar{z}) \vee (\bar{x} \wedge \bar{y})\bar{z} \\ &= \underline{\underline{xy\bar{z}}} \vee \underline{\underline{x\bar{y}\bar{z}}} \vee \underline{\underline{xy\bar{z}}} \vee \underline{\underline{x\bar{y}\bar{z}}} \vee \underline{\underline{x\bar{y}z}} \vee \underline{\underline{\bar{x}\bar{y}\bar{z}}} \\ &\text{m}_6 \quad \text{m}_2 \quad \text{m}_7 \quad \text{m}_5 \quad \text{m}_4 \quad \text{m}_0 \end{aligned}$$

x \ yz	00	01	11	10
0	m_0	m_{10}	m_{11}	m_{12}
1	m_{45}	m_5	m_7	m_6
			\max_1	

x \ yz	00	01	11	10
0	m_0			m_2
1	m_4	m_5	m_7	m_6

not okay, can double factorise

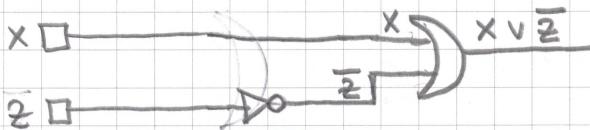
$$\max_1 = m_0 \vee m_4 \vee m_2 \vee m_6 = \overline{z}$$

$$\max_2 = m_4 \vee m_5 \vee m_7 \vee m_6 = x$$

$$M(f) = C(f) = \{\max_1, \max_2\} \Rightarrow 1^{\text{st}} \text{ case of simplification}$$

$$\Rightarrow f(x, y, z) = \max_1 \vee \max_2$$

$$= x \vee \overline{z}$$



$$f_7(x, y, z) = (xy \downarrow (y \vee z)) \oplus (\bar{x} \uparrow z)$$

$$= (\bar{y} \wedge \bar{z}) \oplus (x \vee \bar{z})$$

$$= xy \vee y\bar{z} \vee xz$$

	x	\bar{x}
y	m_7 , m_6 , m_2	m_3
\bar{y}	m_5	m_4
z	\bar{z}	\bar{z}

$$\begin{aligned} xy &: \overbrace{xy\bar{z}}^{111}, \overbrace{\bar{xy}\bar{z}}^{110} \\ y\bar{z} &: \cancel{\overbrace{xy\bar{z}}^{111}}, \overbrace{\bar{xy}\bar{z}}^{100} \\ xz &: \overbrace{xy\bar{z}}^{101}, \overbrace{\bar{xy}\bar{z}}^{111} \end{aligned}$$

$$\max_1 = m_6 \vee m_2 = y\bar{z}$$

$$\max_2 = m_6 \vee m_7 = xy$$

$$\max_3 = m_7 \vee m_5 = xz$$

$$M(f) = \{ \max_1, \max_2, \max_3 \}$$

$$C(f) = \{ \max_1, \max_3 \}$$

$$\left. \begin{array}{l} M(f) \neq C(f) \\ C(f) \neq \emptyset \end{array} \right\} \Rightarrow \begin{array}{l} 2^{\text{nd}} \text{ case of} \\ \text{simplification} \end{array}$$

$$g = \bigvee_{m \in C(f)} m = \max_1 \vee \max_3 = y\bar{z} \vee xz$$

→ all minterms covered ⇒ 1 unique simplified form

$$f_7^S(x, y, z) = y\bar{z} \vee xz$$

$$a \downarrow b = \overline{ab} = \overline{a}\overline{b}$$

$$xy \downarrow (y \vee z) =$$

$$= \overline{xy} \wedge \overline{y \vee z}$$

$$= (\overline{x} \vee \overline{y}) \wedge \overline{y} \wedge \overline{z}$$

$$= \overline{y} \wedge \overline{z}$$

$$\bar{x} \uparrow z = \overline{\bar{x} \wedge z} = x \vee z$$

$$a \uparrow b = \overline{ab} = \overline{a} \vee \overline{b}$$

$$a \oplus b = \overline{ab} \vee \overline{ab}$$

$$\bar{y} \bar{z} \oplus (x \vee z) =$$

$$= \overline{\bar{y} \bar{z}} \wedge (x \vee z) \vee$$

$$\vee \bar{y} \bar{z} \wedge \overline{x \vee z}$$

$$= ((y \vee z) \wedge (x \vee z)) \vee \underbrace{\bar{y} \bar{z} \bar{x} \bar{z}}_0$$

$$= (y \vee z) \wedge (x \vee z)$$

$$= xy \vee y\bar{z} \vee xz \vee \underbrace{z\bar{z}}_0$$

$$= xy \vee y\bar{z} \vee xz$$