SUBJECT 1

1. Transform the formula U=$(p \to q \wedge r) \to (p \to q) \vee (p \to r)$ into its equivalent DNF and CNF. Decide the type of the formula (inconsistent, valid, consistent, contingent). Theory: relations: normal forms --- the properties: validity, inconsistency.

2. Check the distributive property of the existential quantifier over conjunction using a syntactic proof method. The theorem of soundness and completeness of the method.

3. Draw a logic circuit having 3 inputs and containing all basic and derived gates. Write the corresponding Boolean function and simplify it. Draw the simplified circuit.

SUBJECT 2

1. Using lock resolution (2 different indexings) check whether this deduction holds or not. $p \to q, \neg(q \to r) \to \neg p \vdash p \to r$.

2. Using a semantic proof method check if the formula: $(\forall x)A(x) \vee (\forall x)B(x)$ is a logical consequence of the formula: $(\forall x)(A(x) \wedge B(x))$. Theory.

3. Simplify the following Boolean function using Veitch diagram: $f(x_1, x_2, x_3) = x_1x_3 \vee x_1x_2x_3 \vee \bar{x}_1x_3 \vee \bar{x}_1\bar{x}_2x_3$. Implement the logic circuits corresponding to the initial form of *f* and to all the simplified forms of *f*.

SUBJECT 3

1. Using a refutation proof method prove that the separation of the premises law is a theorem.

2. Evaluate the formula U=$((\forall x)p(x) \to (\forall x)q(x)) \to (\forall x)(p(x) \wedge q(x))$ under two interpretations: one with a finite domain (2 elements) and the other with an infinite domain. Are they models or anti-models of the formula?

3. Using Quine's method simplify the Boolean function: $f(x_1, x_2, x_3, x_4) = x_1\bar{x}_2x_3 \vee x_1x_2x_3\bar{x}_4 \vee x_1x_2\bar{x}3x_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4$. Implement the logic circuit associated to a simplified form of *f*.

SUBJECT 4

1. Write all the models and the anti-models of the formula: V=$((p \wedge \neg r) \to q) \to \neg p \wedge \neg q \wedge r$. Theory.

2. Using linear (input/unit) resolution check whether the following set of formulas is inconsistent. $S = \{p(x) \wedge q(x) \vee r(x), \neg q(y) \vee r(y), r(a) \wedge \neg p(a)\}$. Theory.

3. For the Boolean function $f(x, y, z) = x(y \oplus z) \vee y(x \oplus z) \vee x(\bar{y} \downarrow \bar{z}) \vee (x \downarrow y)\bar{z}$; draw the corresponding logic circuit using basic and derived gates. Simplify the function and draw the logic circuits associated to all simplified forms of the initial function using only basic gates.