

Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secția: Informatică engleză

Curs: Dynamical Systems

Primăvara 2024

## Seminar 7

**1.** Let  $\lambda \in \mathbb{R}^*$  and  $\eta \in \mathbb{R}$  be fixed parameters. Find the unique solution  $(x_k)_{k \geq 0}$  of the initial value problem  $x_{k+1} = \lambda x_k$ ,  $x_0 = \eta$ .

Note that the solution is a geometric progression. What is the long term behavior of this sequence? Discuss with respect to  $\lambda$  and  $\eta$ .  $\diamond$

**2.** (a) Find solutions of the form  $x_k = a 3^k$  of the difference equation  $x_{k+1} = 2x_k + 3^k$ ,  $k \geq 0$ . Here we look for  $a \in \mathbb{R}$ .

(b) Find the general solution of  $x_{k+1} = 2x_k + 3^k$ .

(c) Find the solution of the IVP  $x_{k+1} = 2x_k + 3^k$ ,  $x_0 = 0$ .  $\diamond$

**3.** (a) Find solutions of the form  $x_k = ak + b$  of the difference equation  $x_{k+1} = -5x_k - k$ ,  $k \geq 0$ . Here we look for  $a, b \in \mathbb{R}$ .

(b) Find the general solution of  $x_{k+1} = -5x_k - k$ .

(c) Find the solution of the IVP  $x_{k+1} = -5x_k - k$ ,  $x_0 = -1$ .  $\diamond$

**4.** Find the general solution of

(a)  $x_{k+2} - 6x_{k+1} + 9x_k = 0$ .

(b)  $x_{k+2} - 2x_{k+1} + x_k = 0$ .

(c)  $x_{k+2} + x_{k+1} + x_k = 0$ .  $\diamond$

**5.** Find the expression of the Fibonacci sequence

$$x_{k+2} = x_{k+1} + x_k, \quad x_0 = 0, \quad x_1 = 1.$$

**6.** Find the linear homogeneous difference equation of minimal order that has the solution  $(x_k)_{k \geq 0}$  such that

(a)  $x_k = \frac{7}{2^k} - \frac{2}{3^k}$ ,  $k \geq 0$ .

(b)  $x_k = 7\operatorname{Re}(i^k) - 2\operatorname{Im}(i^k)$ ,  $k \geq 0$ .