2 a) Find solutions of the form $x_k = a3^k$ of the diff eq. $x_{k+1} = 2x_k + 3^k$, $k \geq 0$

$x_k = a\,3^k$

$x_{k+1} = 2_k + 3^k$, $k \geq 0$

$a \in \mathbb{R}$

$\left.\begin{matrix} x_k = a3^k \\ x_{k+1} = 2x_k + 3^k \end{matrix}\right\} \Rightarrow a3^{k+1} = 2a3^k + 3^k$

$3^k(3a) = 3^k(2a+1) \Rightarrow a = 1$

$\Rightarrow x_k = 3^k$ is a solution of the diff. eq.

5. Find the expression of the Fibbonacci sequence

$x_{k+2} = x_{k+1} + x_k \qquad x_0 = 0 \quad x_1 = 1$

$\Rightarrow x_k - x_{k-1} - x_{k-2} = 0$

$\Rightarrow t^2 - t - 1 = 0$

$\delta = 1 + 4 = 5 \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

$\Rightarrow x_k = a\left(\frac{1+\sqrt{5}}{2}\right)^k + b\left(\frac{1-\sqrt{5}}{2}\right)^k$

IVP: $x_0(\text{IVP}) = 0$

$0 = a\left(\frac{1+\sqrt{5}}{2}\right)^0 + b\left(\frac{1-\sqrt{5}}{2}\right)^0$

$0 = a + b \Rightarrow a = -b$

IVP: $x_1(\text{IVP}) = 1$

$1 = a \cdot \left(\frac{1+\sqrt{5}}{2}\right)^1 + b\left(\frac{1-\sqrt{5}}{2}\right)^1$

$\left.\begin{matrix} 1 = \dfrac{a + a\sqrt{5} + b - b\sqrt{5}}{2} \\ a = -b \end{matrix}\right\} \Rightarrow 1 = \frac{-2b\sqrt{5}}{2} = -b\sqrt{5} \Rightarrow b = \frac{-1}{\sqrt{5}} \Rightarrow a = \frac{1}{\sqrt{5}}$

$x_k(\text{IVP}) = \dfrac{\left(\frac{1+\sqrt{5}}{2}\right)^k + \left(\frac{1-\sqrt{5}}{2}\right)^k}{\sqrt{5}}$

3. Find solutions of the form $x_k = ak + b$ of the diff. eq.

a) + b) $\qquad x_{k+1} = -5x_k - k$, $k \geq 0$

I $\quad x_{k\,hom}$: $x_{k+1} = -5x_k$

$\Rightarrow x_{k\,hom} = c \cdot (-5)^k$

II $\quad x_{k\,part}$: $x_k = ak + b$

$a(k+1) + b = -5(ak+b) - k$

$ak + a + b = (-5a - 1)k - 5b$

$$\begin{cases} a = -5a - 1 \\ a + b = -5b \end{cases} \Rightarrow \quad a = \frac{-1}{6} \quad b = \frac{a}{-6} = \frac{1}{6^2}$$

$$x_{k\,part} = \frac{-1}{6}k - \frac{1}{6^2}$$

$$\Rightarrow x_k = c(-5)^k - \frac{1}{6}k - \frac{1}{6^2}$$

c) IVP $x_{k+1} = -5x_k - k \quad x_0 = -1$

$$x_0 = -1$$

$$x_0 = c \cdot (-5)^0 - \frac{1}{6} \cdot 0 - \frac{1}{6^2}$$

$$-1 = c - \frac{1}{36} \Rightarrow c = \frac{-36-1}{36} = \frac{-37}{36}$$

$$x(ivp) = \frac{-37}{36}(-5)^k - \frac{1}{6}k + \frac{1}{36}$$