$K = (O, B)$ - reference system (ref frame / coord. system)

$O \in \mathcal{E}^n$

B - basis of V^n

<u>Notations</u>: V, V' - vec. spaces

$\varphi : V \to V'$ - lin. map

B, B' - basis of V, V'

Instead of $[f]_{BB'}$, we use $M_{B'B}(f) = \left([f(v_1)]_{B'}, [f(v_2)]_{B'}, \dots [f(v_n)]_{B'} \right)$

, where $B = (v_1, \dots, v_n)$

V - vector space

B, B' - basis of V

Base change matrix from B to B' is $M_{B'B} := M_{B'B}(id) = [id]_{BB'}$

We use it like this: $\forall\ v \in V$:

$[v]_{B'} = M_{B'B} [v]_B$

$K = (O, B)$

$K' = (O', B')$

$\forall\ P \in \mathcal{E}^n$: $[P]_K := [\underbrace{\overrightarrow{OP}}_{\vec{r}_P}]_B$

We want $[P]_{K'}$ in terms of $[P]_K$

$\Rightarrow [P]_{K'} = [\overrightarrow{O'P}]_{B'} = M_{B'B} [\overrightarrow{O'P}]_B = M_{B'B} [\overrightarrow{OP} - \overrightarrow{OO'}]_B = M_{B'B}\left([\overrightarrow{OP}]_B - [\overrightarrow{OO'}]_B\right) =$

$= M_{B'B}\left([P]_K - [O']_K\right) = M_{B'B}[P]_K - M_{B'B}[\overrightarrow{OO'}]_B = M_{B'B}[P]_K - [\overrightarrow{OO'}]_{B'} =$

$= M_{B'B}[P]_K + [\overrightarrow{O'O}]_{B'} = M_{B'B}[P]_K + [O]_{K'}$

From now on, we will write $[\vec{v}]_B =: [\vec{v}]_K$ &

$$M_{B'B} = M_{K'K}$$

$\boxed{1.16.}$ $K = (O, \vec{i}, \vec{j})$

$K' = (O', \vec{i}', \vec{j}')$ - coord. sys. for $\mathcal{E}^2$

$[O']_K = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$, $\vec{i}' = -2\vec{i} + \vec{j}$ $\quad$ 1 0

$\vec{j}' = \vec{i} + 2\vec{j}$ $\quad$ 0 1

A, B, C - points in $\mathcal{E}^2$ w/ $[A]_K = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $[B]_K = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $[C]_K = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

- det. the base change m. from K to K' & find coords.
- find the —"— —"— K' to K & check the results.

$$M_{KK'} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow M_{K'K} = \frac{1}{5}\begin{bmatrix} 2 & -1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

$$[A]_{K'} = M_{K'K}[\vec{O'A}]_K = M_{K'K}[A]_K - M_{K'K}[O']_K$$

$$= \frac{1}{5}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\left[\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 7 \\ -1 \end{bmatrix}\right]$$

$$= \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}\begin{bmatrix} 6 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$[B]_{K'} = M_{K'K}[\vec{O'B}]_K = M_{K'K}[B]_K - M_{K'K}[O']_K$$

$$= \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}\left[\begin{bmatrix} 2 \\ 4 \end{bmatrix} - \begin{bmatrix} 7 \\ -1 \end{bmatrix}\right] =$$

$$= \frac{1}{5}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$[C]_{K'} = M_{K'K}[\vec{O'C}]_K = M_{K'K}[\vec{OC} - \vec{OO'}]_K$$

$$= \frac{1}{5}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ -1 \end{bmatrix}\right)$$

$$= \frac{1}{5}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$= \frac{2}{5}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \frac{2}{5}\begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$M_{KK'} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$[A]_K = [\vec{OA}]_K = M_{KK'}[\vec{O'A} - \vec{O'O}]_{K'}$$

$$= M_{KK'}[A]_{K'} - M_{KK'}[O]_{K'}$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$[\overrightarrow{OO'}]_K = M_{K'K} \cdot (\overrightarrow{O'O})_K = \frac{1}{5}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -1 \end{bmatrix} =$$

$$= \frac{1}{5}\begin{bmatrix} -15 \\ 5 \end{bmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$[B]_K = [\overrightarrow{OB}]_K = M_{KK'}\left([\overrightarrow{O'B}]_{K} - [\overrightarrow{O'O}]_K\right)$$

$$= M_{KK'}\left([B]_{K'} - [O]_{K'}\right)$$

$$= M_{KK'}\left([B]_{K'} - [O'O]_{K'}\right)$$

$$= M_{KK'}\left([B]_{K'} + [O']_{K'}\right)$$

$$= M_{KK'} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ 5 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$[C]_K = [OC]_K = M_{KK'}[OC]_{K'}$$

$$= M_{KK'}\left([O'C]_{K'} - [OO']_{K'}\right) =$$

$$= M_{KK'}[C]_{K'} + [O']_K =$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix} =$$

$$= \begin{bmatrix} -4 \\ 2 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\boxed{1.17}$ ABCD - tetrahedron & $K_A = (A, \vec{AB}, \vec{AC}, \vec{AD})$

M - mid. point of [BC] $\quad K_A' = (A, \vec{AB}, \vec{AD}, \vec{AC})$

$K_B = (B, \vec{BA}, \vec{BC}, \vec{BD})$

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a) Find the coords. of ABCD, M in the coord. sys

b) the change matrix from K_A to K_A'

c) — " — K_B to K_A.

a) K_A.

$$A = \vec{r_A} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$B = \vec{r_B} = \vec{AB} = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$C = \vec{r_C} = \vec{AC} = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$D = \vec{r_D} = \vec{AD} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

M - mid of BC $\Rightarrow$ $M = \vec{r_m} = \frac{1}{2}(\vec{AB} + \vec{AC}) = \begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\\ 0\end{bmatrix}$

K_A'

$$A = \begin{bmatrix}0\\0\\0\end{bmatrix} \quad C = \begin{bmatrix}0\\0\\1\end{bmatrix} \quad D = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$B = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad M = \begin{bmatrix}\frac{1}{2}\\0\\ \frac{1}{2}\end{bmatrix}$$

K_B.

$\frac{1}{2}\, \vec{AB} + \vec{AC}$

$$A = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad C = \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \frac{1}{2}(-\cancel{BA} + CB + \cancel{BA}) = \frac{1}{2} - \vec{BC}$$

$$B = \begin{bmatrix}0\\0\\0\end{bmatrix} \quad D = \begin{bmatrix}0\\0\\1\end{bmatrix} \quad M = \begin{bmatrix}0\\ -\frac{1}{2}\\ 0\end{bmatrix}$$