2's complement. Discution and examples

Mathematically, the two's complement REPRESENTATION of a NEGATIVE number is the value $2^n - V$, where V is the absolute value of the represented number

Needed only with signed interpretation on negative numbers

„We interpret representation and represent interpretation"

$-\overline{abc}$

↳ $\overline{abc}$ = binary representation

↳ $-\overline{abc}$ = ? = $-$(2's complement of $\overline{abc}$)

↓

binary representation of $-\overline{abc}$

$$\boxed{2^n - V = \text{2's complement}}$$

Methods:

1) $2^n - V$

1001 0011 = 93h = 147

which is the value in signed interpretation ?

= $-$(2's complement of 1001 0011)

```
 1 0000 0000 -
   1001 0011
 ___________
  |0110 1101| ───→ 2's complement of 1001 0011
    6    D
```

0110 1101 b = 6D h = 109

the absolute value interpreted with sign has the value of -109

2) invert all the bits and ADD 1

```
  0 1 1 0 1 1 0 0 +
                1
 ________________
  0 1 1 0 1 1 0 1
```

3) preserve starting from right to left all 0's until the first 1, including it and invert all the others

1001 0011 ⟶ 0110 1101

! These are the only methods to get the 2's complement in binary configuration

Derived from the 2's complement we can say that the sum of the absolute value of the 2 complementary numbers is always the cardinal of the set of values representable on that size

4) $256 - 147 = 109 \Rightarrow -\overline{abc} = -109$

↑ the fastest if you're asked about the interpretation of that value

---

→ On 8 bits we have the possibility to represent 2^8 values

unsigned byte → $[0, 255]$ → 2^8 values

signed byte → $[-128, 127]$ → 2^8 values

→ On 16 bits we can represent 2^{16} values → $[0, 65535]$
$[-32768, 32767]$

a) Which is the signed interpretation (also: value representing the signed interp.) of 1001 0011? ii)

b) ———— || ———— of 93h? ii) 93 h is a zipped representation of 1001 0011

c) ———— || ———— of 147 in base 10? → stupid question
147 is already an interpretation, not a representation

i) 0110 1101

ii) -109

iii) 6D h

iv) $+147$

147 and -109 are NOT complementary values, their base two representations are

Complementary code only makes sense when you connect it to a negative value.

unsigned ∩ signed $= [0, 255] \cap [-128, 127] = [0, 127]$
(only absolute values)

every time you take a number from here ⇒ their interpretation is the SAME

any nr. that starts in binary with a 0 has the same interpretation for both signed and unsigned

When to use the 2's complement?

a) $\overline{0xx\ldots x} = \overline{abc}$ (unsigned) - which will be the value of this REPRESENTATION in the SIGNED repr?

the same associated value in base 10, no matter if you talk about signed or unsigned representation

⇒ you DON'T need 2's complement

b) $\overline{0xx\ldots x} = \overline{abc}$ (unsigned) - which will be the binary representation of "$-\overline{abc}$"?

* 2's complement → since we talk about NEGATIVE numbers

0 1 1 0 1 1 0 1 → 1 0 0 1 0 0 1 1

c) $\overline{1xx\ldots x} = \overline{abc}$ (unsigned) - which will be the value of this represent. in the signed representation

$-$(2's complement of $\overline{1xx\ldots x}$)

1 0 0 1 0 0 1 1

0 1 1 0 1 1 0 1

$-(01101101) = \underline{-109}$

d) $\overline{1xx\ldots x} = \overline{abc}$ (unsigned) → which will be the binary representation of "$-\overline{abc}$"?

the 2's complement of the initial configuration

1 0 0 1 0 0 1 1 = 147

?

$\boxed{-147 = ?}$

2's complement: 0 1 1 0 1 1 0 1

positive, WRONG

!!!

$-147 \notin [0, 255]$

$-147 \notin [-128, 127]$ → it CANNOT be represented in memory on a BYTE

cbw : 0000 0000 1001 0011

1111 1111 0110 1101 → the correct 2's complement of $\underline{-147}$

* vezi tabel din lecture notes

* this does not apply to $\boxed{128}$. it's complement is exactly the same

As a result, we may conclude that the involvement of the “2’s complement” is manifest in 3 cases :

Binary format	Interpretation	Value	In what way is involved “2’s complement”	Answer
0xxx	Unsigned	+abc	-	-
	Signed	+abc	How do we represent -abc ? (b)	2’s complement of 0xxx
1xxx	Unsigned	+def	-	-
	Signed		Which value will have 1xxx in the SIGNED interpretation ? (c)	-(2’s complement of 1xxx)
	Signed		How do we represent -def ? (d)	2’s complement of 1xxx’s UNSIGNED extension on 2 * sizeof (1xxx)

(with the exception of the representations of the form $\overline{100}$…. (-128, $+128, -32768, +32768$ *etc*).

Let’s notice that the “2’s complement” is NOT involved in any way when we approach only unsigned representations ! Example:

Binary nr	Interpretation	Value	“2’s complement” involvement	Answer
01101101	Unsigned	+109	-	-
	Signed	+109	How do we represent -109 ? (b)	10010011
10010011	Unsigned	+147	-	-
	Signed		Which value will have 10010011 in the SIGNED interpretation ? (c)	- (01101101) = -109
	Signed		How do we represent -147 ? (d)	The 2’s complement of 00000000 10010011 which is 11111111 01101101

Columns 3 and 4 from above can be summarized as follows:

Number X in binary representation begins with	-X begins with	-X is represented on	Examples:
0	1	Same sizeof as X	109 = 01101101 ; -109 = 10010011
1	1	2 * sizeof(X)	147 = 10010011; -147 = 11111111 01101101

(exceptions are the representations of the form $\overline{100}$…. (-128, $+128, -32768, +32768$ *etc*).

Applications '26 "40 EXAM QUESTIONS

1) Which is the minimum numbers of bits on which we can represent -147?

'28 '35 – how to answer

On n bits in the general case we can represent 2^{n-1} values:

↳ either the UNSIGNED values $[0, 2^n - 1]$

SIGNED values $[-2^{n-1}, 2^{n-1} - 1]$

start from the reasoning

On 9 bits : 2^9 values $= 512$

$[0, 511]$

$[-256, 255]$

$\Rightarrow -147 \in [-256, 255]$

$\Rightarrow$ the minimum number of bits needed for representing -147 is $\boxed{9}$

2)

$101101101 = 16Dh = 256 + 6 \cdot 16 + 13 = 365$ in UNSIGNED representation

$101101101 = -(\text{2's comp}) = -010010011 = -093h = -147$ in SIGNED repr.

* vezi sfârșitul înregistrării 38'