Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică
Secţia: Informatică engleză
Curs: Dynamical Systems
Primăvara 2024

Seminar 6

1. Find the general solution of each of the following equations, looking first for some solutions of the form $x = t^r$, with $r \in \mathbb{R}$.

a) $t^2x'' - 8tx' + 20x = 0$, $t \in (0, \infty)$; b) $t^2x'' - 6x = 0$, $t \in (0, \infty)$;

c) $t^2x'' + tx' + x = 0$, $t \in (0; \infty)$. $\diamond$

2. Find the general solution of the following linear planar system using the reduction method $x' = 2x - 5y$, $y' = x - 2y$. $\diamond$

3. Consider the following planar system $x' = -y(x^2 + y^2)$, $y' = x(x^2 + y^2)$.

a) Does this system have other equilibria besides $(0, 0)$? Justify.

b) Verify that $\varphi(t, 1, 0) = (\cos t, \sin t)$, $\varphi(t, 2, 0) = (2\cos 4t; 2\sin 4t)$ for all $t \in \mathbb{R}$. Find $\varphi(t, \eta, 0)$ for each $\eta > 0$. Represent the corresponding orbits.

c) Decide the validity of the statement: "Any solution is periodic." $\diamond$