Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză

Curs: Dynamical Systems

Primăvara 2024

Lecture 9 - List of problems

1. How many solutions has the following problem: a) $x'' + t^2 x = 0$, $x(0) = 0$? b) $x''+t^2x = 0$, $x(0) = 0$, $x'(0) = 0$? c) $x''+t^2x = 0$; $x(0) = 0$, $x'(0) = 0$, $x''(0) = 1$? $\diamond$

2. We consider the following nonlinear planar systems

$\dot{x} = -x + xy, \quad \dot{y} = -2y + 3y^2.$

a) Find its equilibria and study their stability.

b) Find $\varphi(t, 0, 2/3)$, $\varphi(t, 4, 0)$ and $\varphi(t, 1, 2/3)$.

c) Represent in the phase plane the orbits corresponding to the initial values $(0, 2/3)$, $(4, 0)$ and $(1, 2/3)$. $\diamond$

3. Let $\omega > 0$ be a fixed parameter. The second order linear equation $\theta''+\omega^2\theta = 0$ describes the *small* oscillations of a simple idealized pendulum. Here $\theta(t)$ is the angle (measured in radians) between the rod and the vertical and t is the time variable measured in minutes. Find the solution of the IVP

$$\theta'' + \omega^2\theta = 0, \quad \theta(0) = \frac{\pi}{6}, \quad \theta'(0) = 0.$$

Describe the oscillations of the pendulum. After how much time the pendulum will return to the initial state? Represent the phase portrait of the associated planar dynamical system. Specify its type. Justify that the equilibrium solution $\theta(t) = 0$ for all $t \in \mathbb{R}$ is stable, but it is not an attractor. $\diamond$

4. Let $\omega > 0$ be a fixed parameter. The second order nonlinear equation $\theta'' + \omega^2 \sin\theta = 0$ describes the oscillations of a simple idealized pendulum.

(a) Find the equilibria. Use the linearization method to study their stability.

(b) Find a global first integral. Justify that the equilibrium solution $\theta(t) = 0$ for all $t \in \mathbb{R}$ is stable, but it is not an attractor. $\diamond$