

8. Consider family of ellipses $\mathcal{E}_a: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 a=? st. \mathcal{E}_a is tangent to the line $\ell: x-y+5=0$

$$\mathcal{E}: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\ell: y = kx + m$$

ℓ is tangent to \mathcal{E} if $m^2 = a^2k^2 + b^2$

$$\Rightarrow 25 = a^2 + b^2$$

$$25 = a^2 + b^2 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$$

$$\sqrt{5}x + 6 = y.$$

9. Consider a family of lines $\mathcal{L}_c: \sqrt{5}x - y + c = 0$

for what values of c is ℓ_c tangent to $\mathcal{E}: \frac{x^2}{9} + \frac{y^2}{25} = 1$?

From the quadratic \Rightarrow

$$c^2 = 5 + 1 \Rightarrow c^2 = 9 \Rightarrow c = \pm 3$$

10. Det. the common tangents of the ellipses:

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \quad \text{assume } y = kx + m.$$

$$\frac{x^2}{9} + \frac{y^2}{18} = 1 \quad \Rightarrow m^2 = 45k^2 + 9 \\ m^2 = 9k^2 + 18$$

$$\Rightarrow 45k^2 + 9 = 54k^2 + 18$$

$$36k^2 = 9$$

$$k^2 = \frac{1}{36}$$

$$k = \pm \frac{1}{6}$$

$$\Rightarrow m^2 = \frac{81}{4} \Rightarrow m = \pm \frac{9}{2}$$

$$\Rightarrow \ell: \pm \frac{1}{6}x \pm \frac{9}{2}$$

18. tangents to the hyperbola $H: x^2 - y^2 = 16$

which contain $M(-1, 7)$

$$x^2 - y^2 = 16 \quad | : 16$$

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

$$l_k: y = kx \pm \sqrt{a^2k^2 - b^2}$$

$$7 = -k \pm \sqrt{k^2 \cdot 16 - 16}$$

$$7 + k = \pm \sqrt{(k^2 \cdot 16 - 16)} \quad (1)$$

$$49 + 14k - 16 = k^2 \cdot 16 - 16$$

$$65 + 14k - 15k^2 = 0$$

$$15k^2 - 14k - 65 = 0$$

$$\Delta = 196 + 60 \cdot 65$$

$$= 196 + 3900$$

$$\therefore \log 6 = 2^{12}$$

$$b_{1,2} = \frac{14 \pm \sqrt{64}}{30} = \frac{28}{30} = \frac{39}{15} = \frac{13}{5}$$

$$= -\frac{14}{30} = -\frac{7}{15}$$

Exercise 20.

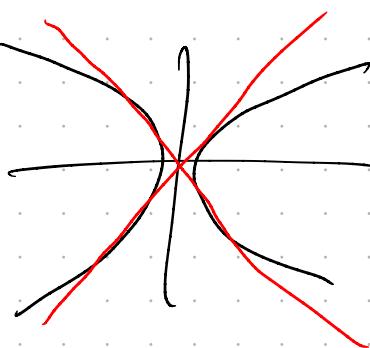
Find the area of the triangle determined by the asymptotes of the hyperbola $H: \frac{x^2}{5} - \frac{y^2}{9} = 1$ and the line $l: 9x + 2y - 24 = 0$.

$$\Rightarrow y = -\frac{9}{2}x + 12$$

asymptotes:

$$y = \pm \frac{3}{\sqrt{5}}x$$

$$\Rightarrow y = \pm \frac{3}{\sqrt{5}}x$$



$$\frac{3}{2}x = -\frac{9}{2}y + 12$$

$$\Rightarrow 6x - 12 \Rightarrow \boxed{x=2} \Rightarrow y = -3 \Leftrightarrow k = 3 \\ \Rightarrow A(2, 3)$$

$$-\frac{3}{2}x = -\frac{9}{2}y + n$$

$$3x - 12 \Rightarrow \boxed{x=4} \Rightarrow y = -18 + n = -6 \Rightarrow B(4, -6)$$

$$A_D = \frac{1}{2} \begin{vmatrix} 2 & 3 & 1 \\ 4 & -6 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2} [-12 + 0 + 0 - 0 - 0 - 12] = \boxed{-12}$$

zu a)

Find an eq for the tangent lines to $H: \frac{x^2}{20} - \frac{y^2}{5} = 1$

which are orthogonal to the line $l: 4x + 3y - 4 = 0$

$$y = \pm \frac{5}{a}x = \pm \frac{\sqrt{5}}{2\sqrt{5}}x = \pm \frac{1}{2}x \Rightarrow$$

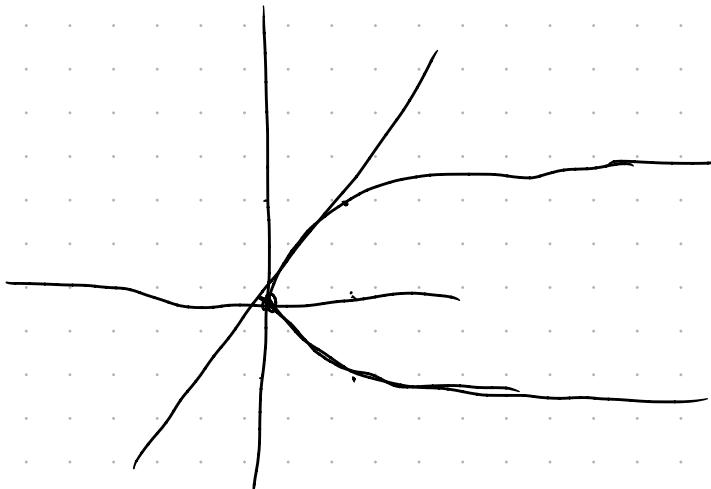
$$y = -\frac{5}{3}x + \frac{4}{3} \Rightarrow k_l = -\frac{5}{3} \Rightarrow k = \frac{5}{3}$$

$$l: y = kx \pm \sqrt{20k^2 - 5}$$

$$y = \frac{3}{5}x \pm \sqrt{\frac{20}{5} \cdot \frac{9}{5} - 5}$$

$$y = \frac{3}{5}x \pm \frac{1}{2}$$

26. For which value of k is the line $y = kx + 2$ tangent to the parabola $P: y^2 = 4x$?



$$P_p: y^2 = 2px$$

$$\text{tangent line: } y = kx + \frac{p}{2k}$$

$$y^2 = 4x = 2px \Rightarrow p = 2.$$

$$y = kx + \frac{1}{k}$$

$$\Rightarrow p = \frac{1}{2}$$

27: $P: y^2 = 16x$ - Determine the tangents to P .

$$\curvearrowleft P = 8$$

- s.t. a) parallel to $\ell: 3x - 2y + 30 = 0$
 b) \perp to $3x + 2y + 7 = 0$

$$-3x + 2y - 30 = 0$$

$$y = \frac{3}{2}x + 15 \Rightarrow k\ell = \frac{3}{2}, \Rightarrow k$$

$$\boxed{y = \frac{3}{2}x + \frac{15}{2}}$$

$$y = -2x - \frac{7}{2} \Rightarrow k\ell = -2 \Rightarrow k = \frac{1}{2}$$

$$\boxed{y = \frac{1}{2}x - 8.5}$$

28. Determine the tangents to the parabola P:

$y^2 = 16x$, which contain the point $P(-2, 2)$

$p=8$

$$y = kx + \frac{p}{2k}$$

$$2 = -2k + \frac{8}{2k} \quad | \cdot dR.$$

$$4k = -4k^2 + 8 \quad | :4$$

$$k^2 + k - 2 = 0 \quad b=1$$
$$(k-1)(k+2) = \begin{cases} k=1 \\ k=-2 \end{cases}$$