2. Write a right-oriented basis of V^3

$\vec a(3,-1,2)$, $\vec b(1,2,-1)$

? $\vec a \times \vec b$, $(2\vec a+\vec b)\times \vec b$, $(2\vec a+\vec b)\times(2\vec a-\vec b)$

$$\vec a \times \vec b = \begin{vmatrix} \vec i & \vec j & \vec k \\ 3 & -1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = \vec i + 6\vec k - 2\vec j + \vec k - 4\vec i + 3\vec j = 5\vec i + \vec j + 7\vec k$$

$$(2\vec a+\vec b)\times \vec b = 2\vec a\times\vec b + \underbrace{\vec b\times\vec b}_{\vec 0} = 2\vec a\times\vec b = 10\vec i + 2\vec j + 14\vec k$$

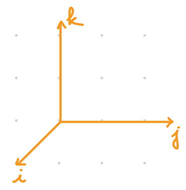
$$(2\vec a+\vec b)\times(2\vec a-\vec b) = -2\vec a\times\vec b + \vec b\times 2\vec a = 2(\vec b\times 2\vec a) = -2(2\vec a\times\vec b) = -4\vec a\times\vec b = -4(5\vec i+\vec j+7\vec k) = -20\vec i - 4\vec j - 28\vec k$$

$\vec u(u_1, u_2, u_3)$

$\vec v(v_1, v_2, v_3)$

$\vec u = u_1\vec i + u_2\vec j + u_3\vec k$

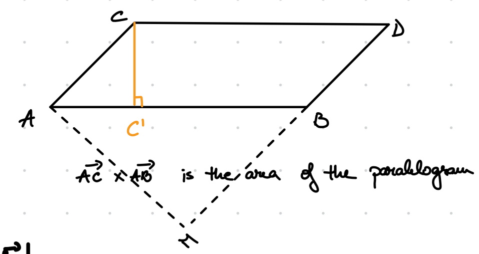
$$\vec u \times \vec v = \begin{vmatrix} \vec i & \vec j & \vec k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$



* 2, 3, 4, 10a, 11a, 13, 16, 17

3. $\vec{AB}(6,0,1)$

$\vec{AC}(1.5, 2, 1)$



$$CC' = \frac{|\vec{AC}\times\vec{AB}|}{|\vec{AB}|}$$

$$\vec{AC}\times\vec{AB} = \begin{vmatrix} \vec i & \vec j & \vec k \\ \frac{3}{2} & 2 & 1 \\ 6 & 0 & 1 \end{vmatrix} = 2\vec i + 6\vec j - 12\vec k - \frac{3}{2}\vec j = 2\vec i + \frac{9}{2}\vec j - 12\vec k$$

$$|\vec{AC}\times\vec{AB}| = \sqrt{2^2 + \frac{81}{4} + 12^2} = \sqrt{4 + \frac{81}{4} + 144} = \sqrt{\frac{673}{4}}$$

$$|\vec{AB}| = \sqrt{36+1} = \sqrt{37}$$

$$CC' = \frac{\sqrt{673}}{2\sqrt{37}}$$

$$|\vec{AC}| = \sqrt{\frac{9}{4}+4+1} = \sqrt{\frac{9+16+4}{4}} = \frac{\sqrt{29}}{2}$$

$$AH = \frac{|\vec{CA}\times\vec{AB}|}{|\vec{AC}|} = \frac{\sqrt{673}}{\sqrt{29}}$$

4. $$\vec a\times\vec b = \begin{cases} \vec 0 & \text{if } \vec a, \vec b \text{ are lin. dependent} \\ & \vec a, \vec b \text{ are lin. independent} \end{cases}$$

$\vec a(2,3,-1)$

$\vec b(1,-1,3)$

a) $$\vec a \times \vec b = \begin{matrix} \vec i & \vec j & \vec k \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{matrix}$$ *table

10 a) (i, j, k) orthogonal basis

$\phi : \mathbb{V}^3 \to \mathbb{V}^3$

$\phi(v) = w \times v$

$w = -i + 3j + k$

$\phi(v) = w \times v = (-i+3j+k) \times (v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k})$

$$\phi(v) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 3 & 1 \\ v_1 & v_2 & v_3 \end{vmatrix} = 3v_3 \vec{i} - v_2 \vec{k} + v_1 \vec{j} - 3v_1 \vec{k} - v_2 \vec{i} + v_3 \vec{j}$$

$$= (3v_3 - v_2)\vec{i} + (v_1 + v_3)\vec{j} - (v_2 + 3v_1)\vec{k}$$

$v_1 = 1$, $v_2 = v_3 = 0 \Rightarrow \phi(i) = \vec{j} - 3\vec{k}$

$v_2 = 1$, $v_1 = v_3 = 0 \Rightarrow \phi(j) = -\vec{i} - \vec{k}$

$v_3 = 1$ $v_1 = v_2 = 0 \Rightarrow \phi(k) = 3\vec{i} + \vec{j}$

$$[\phi]_B = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$$

11 a / L4 a) Prove the Grassmann identity

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Let $\vec{a}(a_1, a_2, a_3)$
$\vec{b}(b_1, b_2, b_3)$
$\vec{c}(c_1, c_2, c_3)$

$\vec{a} \times (\vec{b} \times \vec{c}) =$ takes too long

I $\vec{b}, \vec{c}$ are lin. dependent

$\vec{b} = \lambda \vec{c}$

$\vec{b} \times \vec{c} = \vec{0} \Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = 0$

$(\vec{a} \cdot \lambda \vec{b})\vec{b} - (\vec{a} \cdot \vec{b})\lambda b = \lambda(\vec{a} \cdot \vec{b})\vec{b} - \lambda(\vec{a} \cdot \vec{b}) b = \vec{0}$ | true

II $\vec{b}, \vec{c}$ are lin. independent

$\vec{a} \times (\vec{b} \times \vec{c})$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (b_2 c_3 - b_3 c_2)\vec{i} - (b_1 c_3 - b_3 c_1)\vec{j} + (b_1 c_2 - b_2 c_1)\vec{k}$$

$\vec{a} \times (\vec{b} \times \vec{c}) = a_2 b_1 c_2 \vec{i} - a_1 b_1 c_2 \vec{j}$

13. $A(1, 2, -1)$ $B(0, 1, 5)$ $C(-1, 2, 1)$ $D(2, 1, 3)$

$\vec{AB}(-1, -1, 6)$
$\vec{AC}(-2, 0, 2)$
$\vec{AD}(1, -1, 4)$

$$\begin{vmatrix} -1 & -1 & 6 \\ -2 & 0 & 2 \\ 1 & -1 & 4 \end{vmatrix} = 0 + 12 - 2 - 2 - 8 = 0$$

16. $V(ABCD) = 5$

$A(2,1,-1) \quad B(3,0,1) \quad C(2,-1,3)$

det $D(x_D, y_D, z_D) = ?$

$$V(ABCD) = \frac{1}{6}\,[\vec{AB}, \vec{AC}, \vec{AD}]$$

$\vec{AB} = (1,-1,2)$

$\vec{AC} = (0,-2,4)$

$\vec{AD} = (x_D-2, y_D-1, z_D+1)$

$$[\vec{AB}, \vec{AC}, \vec{AD}] = \begin{vmatrix} 1 & -1 & 2 \\ 0 & -2 & 4 \\ -2 & y_D-1 & 1 \end{vmatrix} = -2 + \cancel{8} - \cancel{8} - 4(y_D-1) = -4y_D + 2 = 30$$

$$y_D = \frac{28}{-4} = -7 \Rightarrow D(0,-7,0)$$

* parțial:
- ecuațiile dreptelor
- ecuațiile planurilor
- \+ să se construiască vectori / planuri $\parallel$ $\perp$ cu ceva dat
- produs scalar / produs vectorial

* pozițiile relative? planuri / drepte

* teorie 11/L4?

* intersecție de drepte

↳ scriem tot în funcție de vect. de poziție

↳ vedem să nu fie o comb. liniară de vectori

! o dreaptă în 3D e descrisă de 2 ecuații

dimensiunea e intrinsecă obiectului, nu spațiului de care aparține

* vectorul normal ne spune tot ce trebuie să știm despre plan

coord. vectorului sunt coeficienți pt. plan

$OA \perp \pi \qquad OA = (1,-1,3)$

$\Rightarrow \pi: 1\cdot x + (-1)\cdot y + 3z + D = 0$

$A \in \pi \Rightarrow 1\cdot 1 + -1\cdot -1 + 3\cdot 3 + D = 0$

$1 + 1 + 9 + D = 0 \Rightarrow D = -11$

$\pi: x - y + 3z - 11 = 0$