Databases

Lecture 5

Functional Dependencies. Normal Forms (II)

Obs. The following simple properties for functional dependencies can be easily demonstrated:

1. If K is a key of $R[A_1, A_2, \dots, A_n]$, then $K \rightarrow \beta$, $\forall \beta$ a subset of $\{A_1, A_2, \dots, A_n\}$.

- such a dependency is always true, hence it will not be eliminated through decompositions

2. If $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ - *trivial functional dependency* (*reflexivity*).

$$\Pi_\alpha(r_1) = \Pi_\alpha(r_2) \underset{\beta \subseteq \alpha}{\Rightarrow} \Pi_\beta(r_1) = \Pi_\beta(r_2) \Rightarrow \alpha \rightarrow \beta$$

↳ vertical slice of the relation

3. If $\alpha \rightarrow \beta$, then $\gamma \rightarrow \beta$, $\forall \gamma$ with $\alpha \subset \gamma$.

$$\Pi_\gamma(r_1) = \Pi_\gamma(r_2) \underset{\alpha \subset \gamma, prop.2}{\Rightarrow} \Pi_\alpha(r_1) = \Pi_\alpha(r_2) \underset{\alpha \rightarrow \beta}{\Rightarrow} \Pi_\beta(r_1) = \Pi_\beta(r_2) \Rightarrow \gamma \rightarrow \beta$$

Name	**FavLine**	**DOB**
Rachel Green	we were not on a break!	1.1.1970
Ross Geller	we were on a break!	1.1.1969
Joey Tribbiani	how you doin'?	1.1.1968
Monica Geller	I know!	1.1.1967

$$\{Name\} \rightarrow \{FavLine\}$$
$$\{Name\} \rightarrow \{DOB\}$$
$$\{Name\} \rightarrow \{FavLine, DOB\}$$
$$\{Name\} \rightarrow \{Name, FavLine, DOB\}$$

$ABC \rightarrow DE$

* If $\alpha \to \beta$, then $\gamma \to \beta$, $\forall \gamma$ with $\alpha \subset \gamma$.

$$\Pi_{\gamma}(r_1) = \Pi_{\gamma}(r_2) \underset{\alpha \subset \gamma, prop. 2}{\Rightarrow} \Pi_{\alpha}(r_1) = \Pi_{\alpha}(r_2) \underset{\alpha \to \beta}{\Rightarrow} \Pi_{\beta}(r_1) = \Pi_{\beta}(r_2) \Rightarrow \gamma \to \beta$$

Name	FavLine	Episode	...
Rachel Green	we were not on a break!	1x21	
Ross Geller	we were on a break!	4x3	
Joey Tribbiani	how you doin'?	1x21	
Joey Tribbiani	how you doin'?	9x7	
Rachel Green	we were not on a break!	2x27	
Monica Geller	I know!	3x17	
Rachel Green	we were not on a break!	2x27	

$\{Name\} \to \{FavLine\}$

$\{Name\} \subset \{Name, Episode\} \Rightarrow \{Name, Episode\} \to \{FavLine\}$

Obs. The following simple properties for functional dependencies can be easily demonstrated:

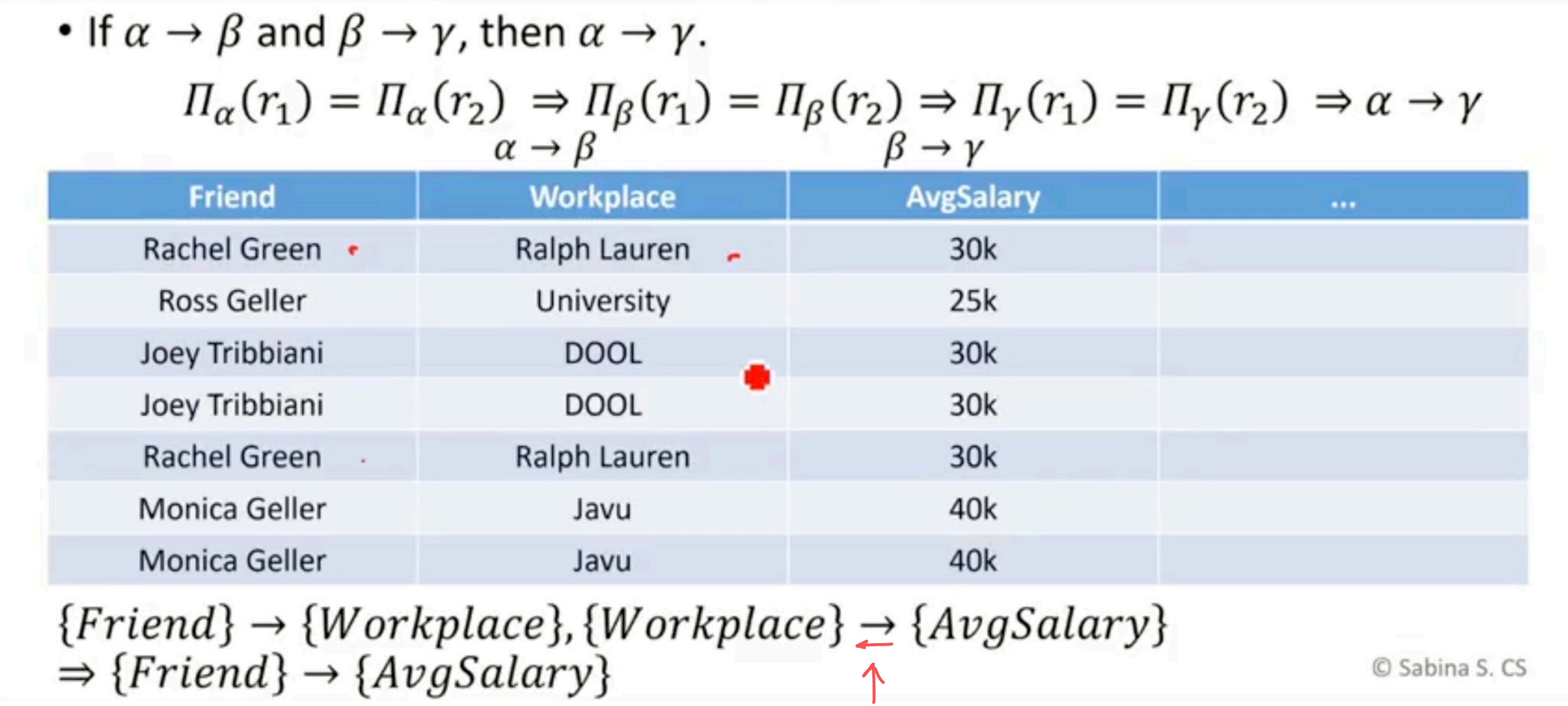
4. If $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$ - *transitivity*.

$$\Pi_\alpha(r_1) = \Pi_\alpha(r_2) \underset{\alpha \to \beta}{\Rightarrow} \Pi_\beta(r_1) = \Pi_\beta(r_2) \underset{\beta \to \gamma}{\Rightarrow} \Pi_\gamma(r_1) = \Pi_\gamma(r_2) \Rightarrow \alpha \to \gamma$$

5. If $\alpha \to \beta$ and γ a subset of $\{A_1, \dots, A_n\}$, then $\alpha\gamma \to \beta\gamma$, where $\boxed{\alpha\gamma = \alpha \cup \gamma.}$

$$\Pi_{\alpha\gamma}(r_1) = \Pi_{\alpha\gamma}(r_2) \Rightarrow \left| \begin{array}{l} \Pi_\alpha(r_1) = \Pi_\alpha(r_2) \Rightarrow \Pi_\beta(r_1) = \Pi_\beta(r_2) \\ \Pi_\gamma(r_1) = \Pi_\gamma(r_2) \end{array} \right| \Rightarrow \Pi_{\beta\gamma}(r_1) = \Pi_{\beta\gamma}(r_2)$$

$\gamma \subset \alpha$



- If $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$.

$$\Pi_\alpha(r_1) = \Pi_\alpha(r_2) \underset{\alpha \to \beta}{\Rightarrow} \Pi_\beta(r_1) = \Pi_\beta(r_2) \underset{\beta \to \gamma}{\Rightarrow} \Pi_\gamma(r_1) = \Pi_\gamma(r_2) \Rightarrow \alpha \to \gamma$$

Friend	Workplace	AvgSalary	...
Rachel Green	Ralph Lauren	30k	
Ross Geller	University	25k	
Joey Tribbiani	DOOL	30k	
Joey Tribbiani	DOOL	30k	
Rachel Green	Ralph Lauren	30k	
Monica Geller	Javu	40k	
Monica Geller	Javu	40k	

$$\{Friend\} \to \{Workplace\}, \{Workplace\} \underset{\leftarrow}{\to} \{AvgSalary\}$$
$$\Rightarrow \{Friend\} \to \{AvgSalary\}$$

↑

this does NOT go both ways

included in a key

Definition. An attribute A (simple or composite) is said to be *prime* if there is a key K and A ⊆ K (K can be a composite key; A can itself be a key). If an attribute isn't included in any key, it is said to be *non-prime*.

Definition. Let $R[A_1, A_2, \dots, A_n]$ be a relation, and let α, β be two subsets of attributes of R. Attribute β is *fully functionally dependent on α* if:

- β is functionally dependent on α (i.e., $\alpha \to \beta$) and
- β is not functionally dependent on any proper subset of α, i.e., $\forall \gamma \subset \alpha$, $\gamma \to \beta$ is not true.

$ABC \to D$

~~$BC \to D$~~

~~$A \to D$~~

Definition. A relation is in the second normal form (2NF) if:

1. it is in the first normal form and (no repeating atributes)
2. every (simple or composite) non-prime attribute is fully functionally dependent on every key of the relation.

- obs. Let R be a 1NF relation that is not 2NF. Then R has a composite key (and a functional dependency $\alpha \rightarrow \beta$, where α (simple or composite) is a proper subset of a key and β is a non-prime attribute).
- decomposition
  - relation $R[A]$ (A - the set of attributes), K - a key
  - β non-prime, β functionally dependent on α, $\alpha \subset K$ (β is functionally dependent on a proper subset of attributes from a key)
  - the $\alpha \rightarrow \beta$ dependency can be eliminated if R is decomposed into the following 2 relations:

$$R'[\alpha \cup \beta] = \Pi_{\alpha \cup \beta}(R)$$
$$R''[A - \beta] = \Pi_{A-\beta}(R)$$

- we'll analyze the relation from Example 6

EXAM[StudentName, Course, Grade, FacultyMember]

- key: {StudentName, Course}
- the functional dependency *{Course}* → *{FacultyMember}* holds => attribute *FacultyMember* is not fully functionally dependent on a key, hence the EXAM relation is not in 2NF
- this dependency can be eliminated if EXAM is decomposed into the following 2 relations:

FK

RESULTS[StudentName, Course, Grade]

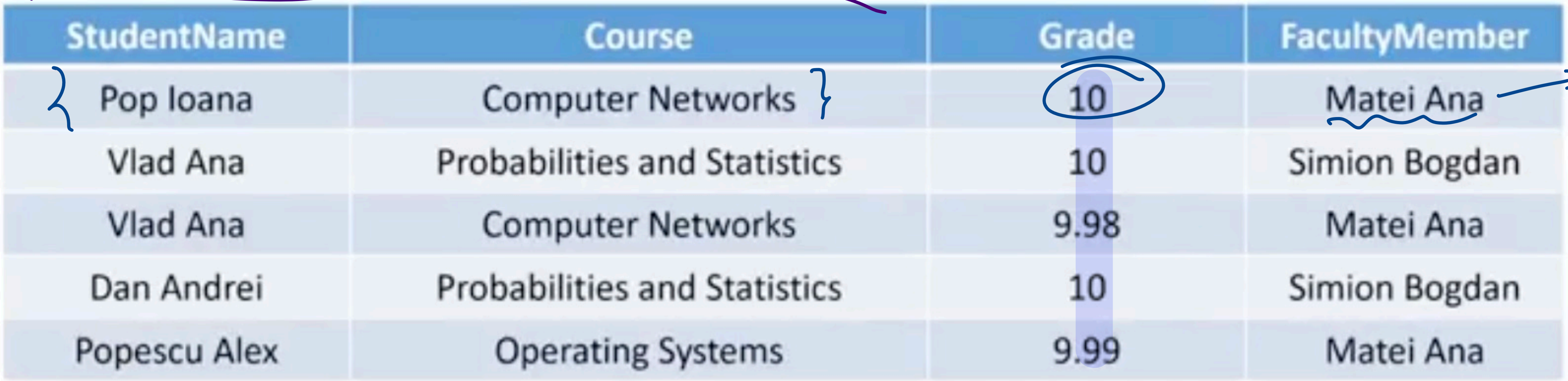
COURSES[Course, FacultyMember]

remove the redundancy

prime

non-prime

⇒ 1NF



StudentName	Course	Grade	FacultyMember
Pop Ioana	Computer Networks	10	Matei Ana
Vlad Ana	Probabilities and Statistics	10	Simion Bogdan
Vlad Ana	Computer Networks	9.98	Matei Ana
Dan Andrei	Probabilities and Statistics	10	Simion Bogdan
Popescu Alex	Operating Systems	9.99	Matei Ana

doesn't describe the exam, but rather the course

fully functionally dependent

$C \rightarrow FM$

$S, C \rightarrow FM$

$S \not\rightarrow FM$

⇒ not fully functionally depented

Example 7. Consider the following relation, storing students' learning contracts:

CONTRACTS[LastName, FirstName, CNP, CourseId, CourseName]

- key: {CNP, CourseId}
- functional dependencies: *{CNP}* → *{LastName, FirstName}*, *{CourseId}* → *{CourseName}*

Not 2NF because C ↛ LastName

→ not fully func. dep.

- to eliminate these dependencies, the relation is decomposed into the following three relations:

STUDENTS[CNP, LastName, FirstName]

COURSES[CourseId, CourseName]

LEARNING_CONTRACTS[CNP, CourseId]

many to many

all in 2NF

Pirate

* it IS 2NF because we have 1 PK

[PID, Name, Rep, Salary]

PID → Salary

PID → Name

PID → Rep

Rep → Salary

- the notion of *transitive dependency* is required for the third normal form

Definition. An attribute Z is transitively dependent on an attribute X if $\exists Y$ such that $X \rightarrow Y, Y \rightarrow Z, Y \rightarrow X$ does not hold (and Z is not in X or Y).

Definition. A relation is in the third normal form (3NF) if it is in the second normal form and no non-prime attribute is transitively dependent on any key in the relation.

Another definition: A relation R is in the third normal form (3NF) if, for every non-trivial functional dependency $X \rightarrow A$ that holds over R:

- X is a superkey, or
- A is a prime attribute.

Example 8. The BSc examination results are stored in the relation:

BSC_EXAM [StudentName, Grade, Supervisor, Department]

- the relation stores the supervisor and the department in which she works
- since the relation contains data about students (i.e., one row per student), *StudentName* can be chosen as the key

no repeating atribute ⇒ 1NF
all func. dep
} ⇒ 2NF

- the following functional dependency holds: *{Supervisor} → {Department}* ⇒ the relation is not in 3NF

StudName → Supervisor , Supervisor $\nrightarrow$ StudentName

- to eliminate this dependency, the relation is decomposed into the following 2 relations:

FK

RESULTS [StudentName, Grade, Supervisor]

SUPERVISORS [Supervisor, Department]

Department is transitively dependent on StudentName

Example 9. The following relation stores addresses for a group of people:

ADDRESSES [CNP, LastName, FirstName, ZipCode, City, Street, No]

- key: {CNP}

CNP → ZipCode   ZipCode ↛ CNP ⇒ transitively dep.

- identified dependency: *{ZipCode}* → *{City}* (can you identify another dependency in this relation?)
- since this dependency holds, relation ADDRESSES is not in 3NF, therefore it must be decomposed:

FK

ADDRESSES'[CNP, LastName, FirstName, ZipCode, Street, No]

ZIPCODES[ZipCode, City]

Example 10. The following relation stores the exam session schedule:

EX_SCHEDULE[Date, Hour, Faculty_member, Room, Group]

- the following restrictions are expressed via key definitions and functional dependencies:

* there are NO non-prime atributes

⇒ 2NF, 3NF

1. a group of students has at most one exam per day

=> *{Date, Group}* is a key

2. on a certain date and time, a faculty member has at most one exam

=> *{Faculty_member, Date, Hour}* is a key

3. on a certain date and time, there is at most one exam in a room

=> *{Room, Date, Hour}* is a key

4. a faculty member doesn't change the room in a day

=> the following dependency holds: *{Faculty_member, Date}* → *{Room}*

- all attributes appear in at least one key, i.e., there are no non-prime attributes
- given the normal forms' definitions specified thus far, the relation is in 3NF
- objective: eliminate the *{Faculty_member, Date}* → *{Room}* functional dependency

Definition. A relation is in the Boyce-Codd (BCNF) normal form if every determinant (for a functional dependency) is a key (informal definition - simplifying assumption: determinants are not too big; only non-trivial functional dependencies are considered).

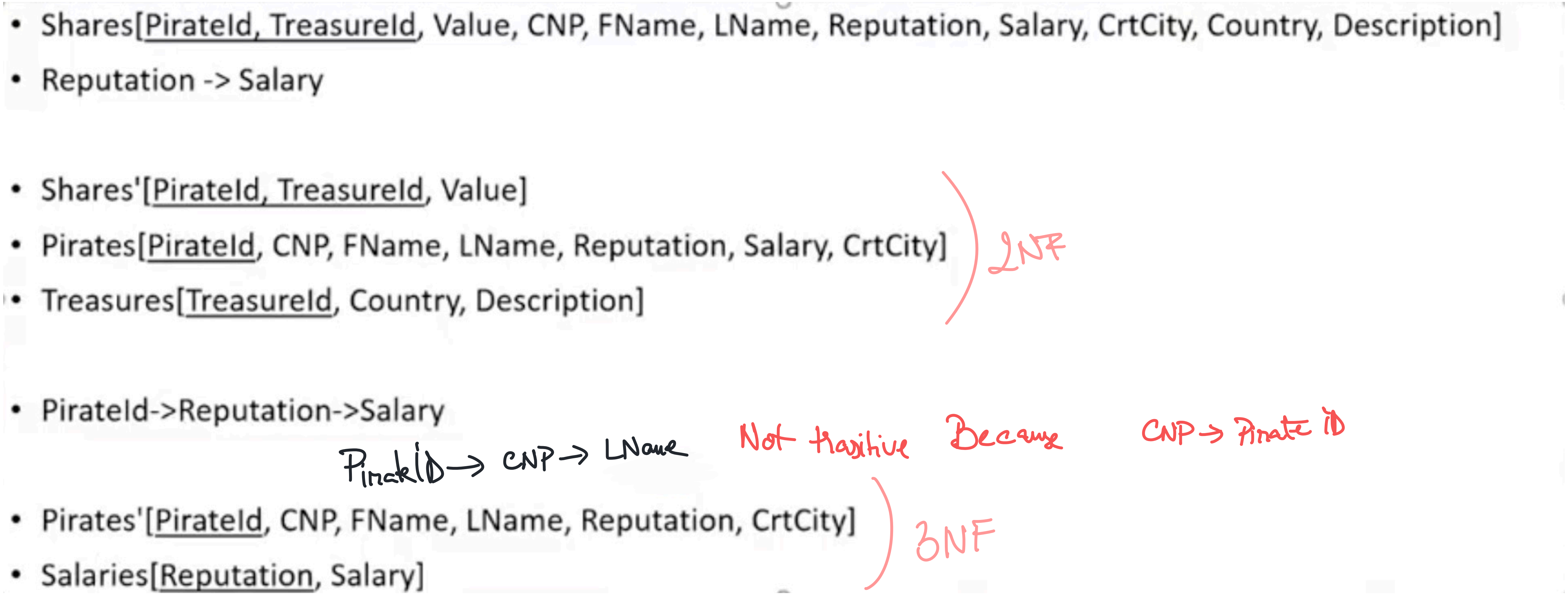
- to eliminate the functional dependency, the original relation must be decomposed into:

EX_SCHEDULE'[Date, Hour, Faculty_member, Group],

ROOM_ALLOCATION[Faculty_member, Date, Room]

- these relations don't contain other functional dependencies, i.e., they are in BCNF
- however, the key associated with the 3rd constraint, *{Room, Date, Hour}*, does not exist anymore
- if this constraint is to be kept, it needs to be checked in a different manner (e.g., through the program)

Superkey -> group that contains a key



- Shares[PirateId, TreasureId, Value, CNP, FName, LName, Reputation, Salary, CrtCity, Country, Description]
- Reputation -> Salary

- Shares'[PirateId, TreasureId, Value]
- Pirates[PirateId, CNP, FName, LName, Reputation, Salary, CrtCity]
- Treasures[TreasureId, Country, Description]

2NF

- PirateId->Reputation->Salary

PirateID -> CNP -> LName

Not trasitive Because CNP -> Pirate ID

- Pirates'[PirateId, CNP, FName, LName, Reputation, CrtCity]
- Salaries[Reputation, Salary]

3NF

BCNF ?

No trivial dependencies

everything on the left is a key

- R[A] - a relation
- F - a set of functional dependencies
- α – a subset of attributes

- problems

I. compute the closure of F: F^+

II. compute the closure of a set of attributes under a set of functional dependencies, e.g., the closure of α under F: α^+

- R[A] - a relation
- F - a set of functional dependencies
- problems

I. compute the closure of F: F^+

- the set F^+ contains all the functional dependencies implied by F
- F implies a functional dependency *f* if *f* holds on every relation that satisfies F
- the following 3 rules can be repeatedly applied to compute F^+ (Armstrong's Axioms):
  - α, β, γ - subsets of attributes of A

  1. reflexivity: if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
  2. augmentation: if $\alpha \rightarrow \beta$, then $\alpha\gamma \rightarrow \beta\gamma$
  3. transitivity: if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
- these rules are complete (they compute the closure) and sound (no erroneous functional dependencies can be derived)

- R[A] - a relation
- F - a set of functional dependencies
- problems

I. compute the closure of F: F$^+$

- the following rules can be derived from Armstrong's Axioms:

4. union: if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\alpha \to \beta\gamma$

$$\left.\begin{array}{l} \alpha \to \beta \underset{\text{augmentation}}{=>} \alpha\alpha \to \alpha\beta \\ \\ \alpha \to \gamma \underset{\text{augmentation}}{=>} \alpha\beta \to \beta\gamma \end{array}\right\} \underset{\text{transitivity}}{=>} \quad \alpha \to \beta\gamma$$

5. decomposition: if $\alpha \to \beta\gamma$, then $\alpha \to \beta$ and $\alpha \to \gamma$

$$\left.\begin{array}{l} \alpha \to \beta\gamma \\ \beta\gamma \to \beta \text{ (reflexivity)} \end{array}\right\} \underset{\text{transitivity}}{=>} \alpha \to \beta \text{ (}\alpha \to \gamma \text{ can similarly be shown to hold)}$$

- R[A] - a relation
- F - a set of functional dependencies
- problems

I. compute the closure of F: F^+

- the following rules can be derived from Armstrong's Axioms:

6. pseudotransitivity: if $\alpha \to \beta$ and $\beta\gamma \to \delta$, then $\alpha\gamma \to \delta$

$$\left.\begin{array}{l} \alpha \to \beta \text{ => } \alpha\gamma \to \beta\gamma \\ \beta\gamma \to \delta \end{array}\right\} \underset{\text{transitivity}}{\text{=>}} \quad \alpha\gamma \to \delta$$

- $\alpha, \beta, \gamma, \delta$ - subsets of attributes of A

- R[A] - a relation
- F - a set of functional dependencies
- α – a subset of attributes
- problems

II. compute the closure of a set of attributes under a set of functional dependencies

- determine the closure of α under F, denoted as α^+
- α^+ - the set of attributes that are functionally dependent on attributes in α (under F)

- R[A] - a relation
- F - a set of functional dependencies
- α – a subset of attributes
- problems

II. compute the closure of a set of attributes under a set of functional dependencies

- algorithm

```
closure := $\alpha$;
repeat until there is no change:
  for every functional dependency $\beta \rightarrow \gamma$ in F
    if $\beta \subseteq$ closure
      then closure := closure $\cup$ $\gamma$;
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- see lecture problems
  - R - a relation, F - a set of functional dependencies, f - a functional dependency
    - show that f is in F^+
  - R - a relation, F - a set of functional dependencies, α - a subset of the set of attributes of R
    - compute α^+

References

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