

$$d) \begin{cases} x' = x - y \\ y' = x + y \end{cases}$$

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 + 2 = 1 - 2\lambda + \lambda^2 + 2 = \lambda^2 - 2\lambda + 3 = 0 \Rightarrow 1-\lambda = \pm i \\ \lambda = \mp i - 1$$

$$2. \begin{cases} x' = x(1-x) = x - x^2 = f_1 \\ y' = y(3-y) = 3y - y^2 = f_2 \end{cases}$$

$$\begin{cases} x - x^2 = 0 \Rightarrow x_1 = 0 \text{ or } x_2 = 1 \\ 3y - y^2 = 0 \Rightarrow y_1 = 0 \text{ or } y_2 = 3 \end{cases}$$

\Rightarrow eq. points $(0,0)$ $(1,0)$ $(0,3)$ $(1,3)$

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 1-2x & 0 \\ 0 & 3-2y \end{pmatrix}$$

$$x' = J_f(x^*, y^*)x$$

$$\text{For } (0,0) \Rightarrow J_f = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \Rightarrow \begin{vmatrix} \lambda_1 = 1 \\ \lambda_2 = 3 \end{vmatrix} \Rightarrow \operatorname{sgn}(\lambda_1) = \operatorname{sgn}(\lambda_2) > 0 \Rightarrow \text{global attractor}$$

$$(1,0) \Rightarrow J_f = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix} \Rightarrow \begin{vmatrix} \lambda_1 = -1 \\ \lambda_2 = 3 \end{vmatrix} \Rightarrow \operatorname{sgn}(\lambda_1) \neq \operatorname{sgn}(\lambda_2) \Rightarrow \text{saddle point (unstable)}$$

$$(0,3) \Rightarrow J_f = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix} \Rightarrow \begin{vmatrix} \lambda_1 = 1 \\ \lambda_2 = -3 \end{vmatrix} \Rightarrow \text{saddle}$$

$$(1,3) \Rightarrow J_f = \begin{pmatrix} -1 & 0 \\ 0 & -3 \end{pmatrix} \Rightarrow \begin{vmatrix} \lambda_1 = -1 \\ \lambda_2 = -3 \end{vmatrix} \Rightarrow \operatorname{sgn}(\lambda_1) = \operatorname{sgn}(\lambda_2) \Rightarrow \text{attractor}$$