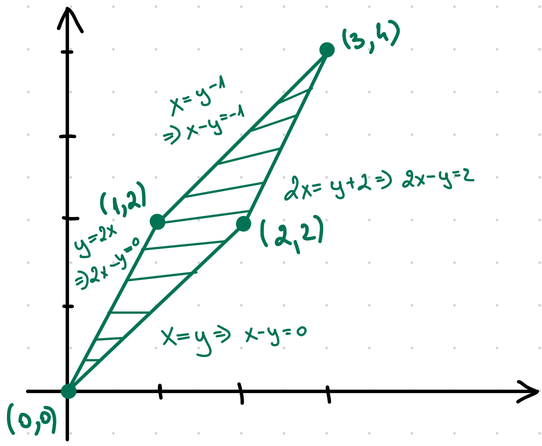
(c) ★ $\displaystyle\int_0^{\pi/2}\int_x^{\pi/2}\frac{\sin(y)}{y}\,dy\,dx.$

$$\left.\begin{matrix} x \le y \le \frac{\pi}{2} \\ 0 \le x \le \frac{\pi}{2}\end{matrix}\right\} \Rightarrow \begin{matrix} 0 \le y \le \frac{\pi}{2} \\ 0 \le x \le y \end{matrix}$$

$$\int_0^{\frac{\pi}{2}}\int_x^{\frac{\pi}{2}}\frac{\sin y}{y}\,dy\,dx = \int_0^{\frac{\pi}{2}}\left(\int_0^{y}\frac{\sin y}{y}\,dx\right)dy = \int_0^{\frac{\pi}{2}}\frac{\sin y}{\cancel{y}}\cdot\cancel{y}\,dy = \int_0^{\frac{\pi}{2}}\sin y\,dy = -\cos y\Big|_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2}+\cos 0 = 1$$

(d) ★ $\displaystyle\iint_D xy\,dx\,dy$, where D is the parallelogram with vertices $(0,0)$, $(2,2)$, $(1,2)$, $(3,4)$.

$$\Rightarrow \begin{matrix} -1 \le x-y \le 1 \\ 0 \le 2x-y \le 2 \end{matrix} \Rightarrow \begin{matrix} u = x-y & u \in [-1,1] \\ v = 2x-y & v = [0,2] \end{matrix}$$



$$\begin{cases} u = x-y \\ v = 2x-y \end{cases} \quad (-)$$

$u - v = -x \Rightarrow x = v-u$

$y = x-u = v-2u$

$$\Rightarrow \begin{cases} x = v-u \\ y = v-2u \end{cases}$$

$$\frac{\partial x}{\partial u} = -1 \qquad \frac{\partial x}{\partial v} = 1 \qquad \frac{\partial y}{\partial u} = -2 \qquad \frac{\partial y}{\partial v} = 1$$

$$J = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} \Rightarrow |\det J| = |-1+2| = 1$$

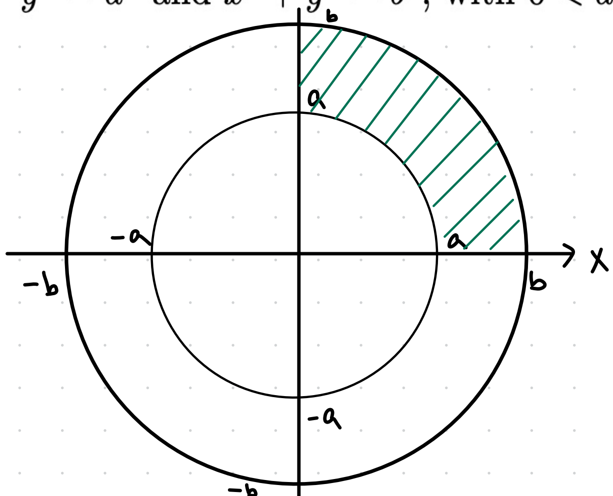
$$\iint_D xy\,dx\,dy = \iint_{D^*}(v-u)(v-2u)\,du\,dv = \int_0^2\left(\int_{-1}^{1}(v^2-3uv+2u^2)\,du\right)dv =$$

$$= \int_0^2\left(v^2 u - 3v\frac{u^2}{2}+\frac{2u^3}{3}\right)\Bigg|_{-1}^{1}dv = \int_0^2\left(2v^2+\frac{4}{3}\right)dv = \left(\frac{2v^3}{3}+\frac{4}{3}v\right)\Bigg|_0^2 = \frac{16}{3}+\frac{8}{3} =$$

$$= \frac{24}{3} = 8$$

$$v^2 - \cancel{3v} + \frac{2}{3} + v^2 + \cancel{3v} + \frac{2}{3} = 2v^2 + \frac{4}{3}$$

(d) ★ $\displaystyle\iint_D \ln(x^2+y^2)\,dx\,dy$, where D is the region in the first quadrant between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, with $0<a<b$.



$x = r\cos\theta \qquad r \in [a,b]$

$y = r\sin\theta \qquad \theta \in [0,\frac{\pi}{2}]$

$|\det J| = r$

$$\iint_D \ln(x^2+y^2)\,dx\,dy = \iint_D \ln r^2 \cdot r\,dr\,d\theta = \int_0^{\frac{\pi}{2}} d\theta \int_a^b r \ln r^2\,dr =$$

$$= \frac{\pi}{2}\cdot\left(\frac{r^2}{2}\cdot\ln r^2\Big|_a^b - \int_a^b \frac{r^2}{2}\cdot\frac{1}{r^2}\cdot 2r\,dr\right) = \frac{\pi}{2}\left(\frac{b^2}{2}\ln b^2 - \frac{a^2}{2}\ln a^2 - \frac{r^2}{2}\Big|_a^b\right)$$

$$= \frac{\pi}{2}\left(b^2\ln b - a^2\ln a - \frac{b^2}{2} + \frac{a^2}{2}\right) =$$

$$= \frac{\pi}{2}\left[a^2\left(\frac{1}{2}-\ln a\right) - b^2\left(\frac{1}{2}-\ln b\right)\right]$$