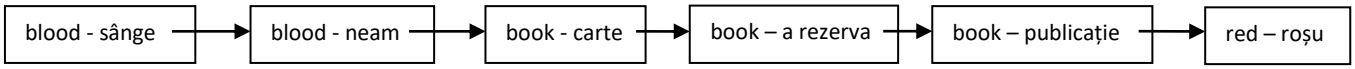
DSA – Seminar 3
Sorted MultiMap (SMM)

- Map – contains key-value pairs. Keys are unique, each key has a single associated value.
- MultiMap – a key can have multiple associated values (can be considered a list of values).
- Sorted MultiMap – there is a relation R defined on the keys and they are ordered based on the keys. There is no particular order of the values belonging to a key (we do not order based on the values)

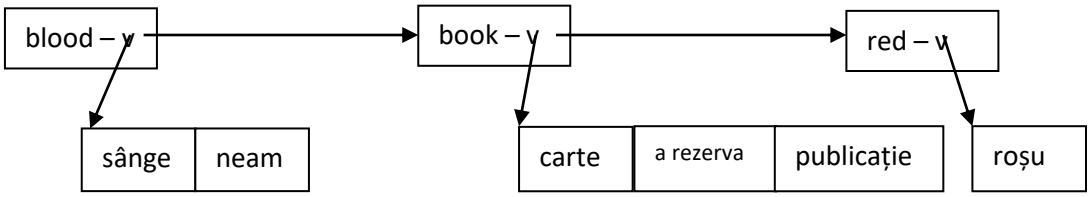
Problem: Implement the SortedMultiMap ADT – use a singly linked representation with dynamic allocation

Ex. a multimap with the translation of different English words in Romanian
- book – carte, a rezerva, publicație
- red – roșu
- blood – sânge, neam

Representation 1: Singly linked list of <key, value> pairs. There might be multiple nodes with the same key, they will be placed one after the other (since the nodes are sorted based on the keys).



Representation 2: Singly linked list of <key, list of values> pairs. The keys are unique and sorted.



No matter which representation we choose, the content of the SMM is the same: we have 6 key-value pairs.

How could we represent the *list of values* from the second representation?

- Data structure level:
  - Dynamic array, SLL, DLL
- ADT level:
  - List, Bag

We will consider that the *list of values* is actually an ADT List, already implemented (together with the ListIterator)

TElem:	Node:	SMM:
k: TKey	info: TElem	head: ↑Node
vl: List	next: ↑Node	R: Relation

$$R(k_1, k_2) = \begin{cases} true, if\ "k_1 \leq k_2"\ (k_1 comes\ before\ k_2) \\ false, otherwise \end{cases}$$

Iterator:

We need to keep in the iterator:

- the SMM
- a reference to the current node from the SMM
- an iterator for the list of values associated to the current node

Obs 1: In a SMM we have key-value pairs, so current element from the iterator has to be a key-value pair. Even if we chose representation 2, we cannot say that our current element is a key and a list of values.

Obs 2: Instead of an iterator over the list of values associated to the current node we could have used the index/position of the current element from the list of values (since it is a list and it has positions). But, working with an iterator over the value list is more efficient.

IteratorSMM:
smm: SMM
current: ↑Node
itL: IteratorList

Iterator operations: init, valid, next, getCurrent (returns a <key, value> pair).

Printing the elements of a SMM using the iterator:

```
Subalgorithm print(smm) is:
	iterator(smm, it)
	while valid(it) execute:
		getCurrent(it, <k,v>)
		@print k and v
		next(it)
	end-while
end-subalgorithm
```

The print subalgorithm looks in the same way independently of the representation of the iterator and the representation of the map!

Operations for the iterator

```
subalgorithm init (it, smm) is:
```

```
        it.smm ← smm
        it.current ← smm.head
        if it.current ≠ NIL then:
                iterator([it.smm.head].info.vl, it.itL)
        end-if
end-subalgorithm
Complexity: θ(1)


subalgorithm getCurrent(it) is: // result will be a <k, v> pair
        if it.current = NIL then
                @throw exception
        end-if
        k ← [it.current].info.k
        v ← getCurrent(it.itL)
        getCurrent ← <k,v>
end-subalgorithm
Complexity: θ(1)


function valid(it):
        if it.current ≠ NIL then
                valid ← true
        else
                valid ← false
end-function
Complexity: θ(1)



subalgorithm next(it) is:
        if it.current = NIL then
                @throw exception
        end-if
        next(it.itL)
        if not valid(it.itL) then
                it.current ← [it.current].next
                if it.current ≠ NIL then
                        iterator ([it.current].info.vl, it.itL)
                end-if
        end-if
end-subalgorithm
Complexity: θ(1)

subalgorithm first(it) is:
        it.current ← it.smm.head
        if it.current ≠ NIL then:
                iterator([it.smm.head].info.vl, it.itL)
        end-if
end-subalgorithm
Complexity: θ(1)
```

Operations for the sorted multi map

Notations for the complexities:
n – number of distinct keys
smm – total number of elements

```
subalgorithm init(smm, R) is:
	smm.R ← R
	smm.head ← NIL
end-subalgorithm
Complexity: θ(1)


subalgorithm destroy(smm) is:
	while smm.head ≠ NIL execute:
		aux ← smm.head
		smm.head ← [smm.head].next
		destroy([aux].info.vl)
		free(aux)
	end-while
end-subalgorithm
Complexity:
If destroy for list is θ(1) => θ(n)
If destroy for list is θ(length of list) => θ(smm)

//auxiliary function that will help us with the other operations (private function,
it is not part of the interface).
//pre: smm is  SMM, k is a Tkey
//post: kNode is a ↑Node, prevNode is a ↑Node. If there is a node with k as key,
kNode will be that node and prevNode will be the previous node. If there is no node
with k as key, kNode will be NIL and prevNode will be the node after which the key k
should be.

For the previous example (the one with the words and translations):
searchNode for „book” -> kNode the node with book, prevNode the node with blood
searchNode for „blood” -> kNode the node with blood, prevNode will be NIL
searchNode for „day” -> kNode will be NIL, prevNode the node with book
searchNode for „air” -> kNode will be NIL, prevNode will be NIL

subalgorithm searchNode(smm, k, kNode, prevNode) is:
	aux ← smm.head
	prev ← NIL
	found ← false
	while aux ≠ NIL and smm.R([aux].info.k, k) and  not found execute
		if [aux].info.k = k then
			found ← true
		else
			prev ← aux
			aux ← [aux].next
		end-if
	end-while
	if found then
		kNode ← aux
		prevNode ← prev
	else
		kNode ← NIL
```

```
            prevNode ← prev
      end-if
end-subalgorithm
Complexity: O(n)

subalgorithm search(smm, k, list) is:
      searchNode (smm, k, kNode, prevNode)
      if kNode = NIL then
            init(list) // return an empty list
      else
            list ← [aux].info.vl
      end-if
end-subalgorithm
Complexity: O(n)


subalgorithm add(smm, k, v) is:
      searchNode(smm, k, kNode, prevNode)
      if kNode = NIL then
            addANewKey (smm, k, v, prevNode)
      else
            addEnd([kNode].info.vl, v)
      end-if
end-subalgorithm
Complexity:
//searchNode is O(n)
//addANewKey is θ(1) operation (we will use the prevNode)
//instead of addEnd another add function can be used (so it can have θ(1) complexity)
If addEnd (or whatever function is used for values) is θ(1) => O(n)
If addEnd (or whatever function is used for values) is θ(length of the list) =>
O(smm)

//auxiliary operation (not part of interface)
//pre: smm is a SMM, k is a TKey, v is a TElem/ TValue,  prevNode is a ↑Node (the
node after which the new node should be added)
//post: a new node with key k and value v is added to the smm. The order of the keys
will respect the relation.
subalgorithm addANewKey (smm, k, v, prevNode) is:
      allocate(newNode)
      [newNode].info.k ← k
      init ([newNode].info.vl)
      addEnd([newNode].info.vl, v)
      if prevNode = NIL then
            [newNode].next ← smm.head
            smm.head ← newNode
      else
            [newNode].next ← [prevNode].next
            [prevNode].next ← newNode
      end-if
end-subalgorithm
Complexity: θ (1) //supposing addToEnd it θ(1) – which is true since in this
situation we will always add an element into an empty list


function remove(smm, k, v) is:
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```
        searchNode(smm, k, kNode, prevNode)
        if kNode ≠ NIL then
                pos ← indexOf([kNode].info.vl, v)
                if pos ≠ -1 then
                        remove([kNode].info.vl, pos, e)
                end-if
                if isEmpty([kNode].info.vl) then
                        removeKey(smm, k, prevNode)
                end-if
                remove ← true
        end-if
        remove ← false
end-subalgorithm
Complexity: O(smm)

//auxiliary operation (not part of the interface)
//pre: smm is a SMM, k is a TKey, prevNode is a ↑Node, smm contains a node with key k
after the node prevNode (if prevNode is NIL, then the first node if smm contains the
key k). The value list of the node with key k is empty.
//post: the node containing key k is removed from smm
subalgorithm removeKey(smm, k, prevNode) is:
        if prevNode = NIL then
                deleted ← smm.head
                smm.head ← [smm.head].next
                destroy([deleted].info.vl)
                free(deleted)
        else
                deleted ← [prevNode].next
                [prevNode].next ← [[prevNode].next].next
                destroy([deleted].info.vl)
                free(deleted)
        end-if
end-subalgorithm
Complexity: 0(1)
Destroy will destroy an empty list => 0(1)
```