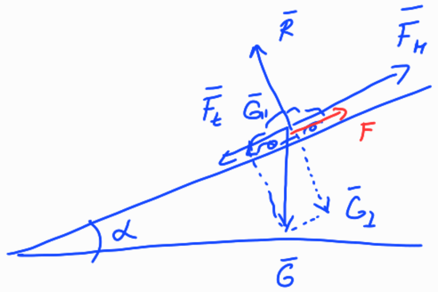
8. cvičenie - FZPH

Domáca úloha:



$$F = ma = F_H - F_t - G_{\parallel}$$

$$ma = \frac{P}{v} - (218 + 0{,}7 v^2) - mg \sin\alpha$$

$P = \ ....$

$P = 126\ kW$

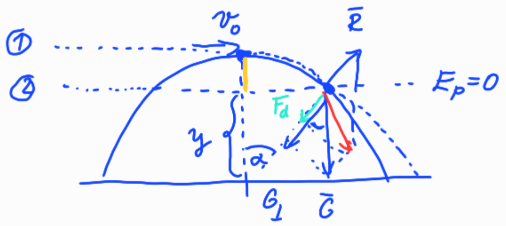
$$F = \frac{P}{v}$$

$$P = \frac{W}{t}\left(\frac{\text{práca}}{\text{čas}}\right) = \frac{F s}{t} = F v$$

sila ... F ... jedn. Newton

výkon ... P .... jedn. Watt

Príklad z minulého cvičenia - dokončenie:



$$F_d = G_{\perp} - R$$

$$m a_d = mg\cos\alpha - R$$

$$m\frac{v^2}{r} = mg\cos\alpha - R$$

dostred. zrýchl. $a_d = \frac{v^2}{r}$

V momente oddelenia: $\bar{R} = 0$

$$m\frac{v_k^2}{r} = mg\cos\alpha_k \quad /:m$$

$$(1)\quad v_k^2 = rg\cos\alpha_k$$

← 2 nezn. v_k, α_k

Zákon zachovania energie:

$$\Delta E_k + \Delta E_p = 0$$

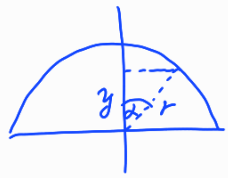
$$E_{k2} - E_{k1} + E_{p2} - E_{p1} = 0$$

$$\frac{1}{2} m v_k^2 - \frac{1}{2} m v_0^2 + 0 - mg(r - y) = 0 \quad /:m \quad /\cdot 2$$

$(E_{p0} \to \text{zem}: mgy - mgr)$

$$v_k^2 - v_0^2 - 2g(r - y) = 0$$

$$(2)\quad v_k^2 = v_0^2 + 2gr(1 - \cos\alpha_k)$$



$y = r\cos\alpha$

(1) = (2)

$$rg\cos\alpha_k = v_0^2 + 2gr(1 - \cos\alpha_k)$$

$$r g \cos\alpha_k = v_0^2 + 2gr - 2gr\cos\alpha_k$$

$$3gr\cos\alpha_k = v_0^2 + 2gr$$

$$\cos\alpha_k = \frac{v_0^2 + 2gr}{3gr}$$

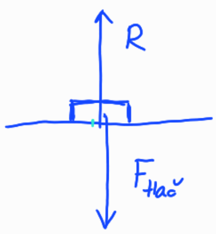
$$\underline{\underline{\alpha_k = \arccos\left(\frac{2}{3} + \frac{v_0^2}{3gr}\right)}}$$

$\alpha_k \sim 47^\circ$

$y_k \sim 1{,}7\,m$

Akou silou tlačí puk na podložku?

$F_{tlač} = ?$



Zákon akcie a reakcie

R – podl. na puk

$F_{tlač}$ – puk na podl.

$R = -F_{tlač}$

$\cancel{\bar{R} + \bar{F}_{tlač} = 0}$

$$m\frac{v^2}{r} = mg\cos\alpha - R = \underline{mg\cos\alpha + F_{tlač}}$$

zo ZZE $v^2 = v_0^2 + 2gr(1-\cos\alpha)$

$$F_{tlač} = m\frac{v^2}{r} - mg\cos\alpha$$

$$\underline{F_{tlač} = \frac{m v_0^2}{r} - mg(3\cos\alpha - 2)}$$

? Oddelenie na vrchole:

$R = 0$; $\alpha = 0$

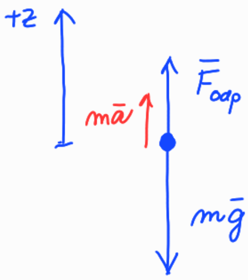
$$m\frac{v^2}{r} = mg$$

$\underline{\underline{v \geq \sqrt{rg}}}$ ... počiatoč. udel. rýchl., pri ktorej puk odletí ihneď

Aerodynamická odporová sila pri pádoch a vrhoch

$$F_{odp} = \frac{1}{2} C S \rho v^2$$

C – koef. odp. vzduchu
S – ploš. rez v smere $\perp \bar{v}$
ρ – hustota vzduchu



pohyb nadol

Newton. pohyb. rovnica:

$$ma = -mg + \frac{1}{2} C S \rho v^2 \,!$$

$v > 0$ nahor
$v < 0$ nadol

$$ma = -mg - \frac{1}{2} C S \rho |v| v \qquad /:m$$

$$a = \frac{dv}{dt}$$

$$\frac{dv}{dt} = -g - \underbrace{\frac{1}{2}\frac{CS\rho}{m}}_{K} |v| v$$

$$\dot{v} \equiv \boxed{\frac{dv}{dt} = -g - K|v|v}$$

DR – rieš. metódou separácie premenných

Špec. prípad: pád nadol $v_0 \le 0$

$$\frac{dv}{dt} = -g + Kv^2 \qquad /:(-g+Kv^2) \qquad /\cdot dt$$

$$\int_{v_0}^{v} \frac{dv}{-g+Kv^2} = \underbrace{\int_0^t dt}_{t} \qquad / \int$$

$$\boxed{\int \frac{dx}{1-x^2} = \operatorname{atanh}(x) = \frac{1}{2}\ln\frac{1+x}{1-x} + C}$$

$$\int_{v_0}^{v} \frac{dv}{g\left(\frac{K}{g}v^2-1\right)} = \frac{1}{g}\int \frac{dv}{\left(\frac{K}{g}v^2-1\right)} = \frac{1}{g}\int_{v_0\sqrt{\frac{K}{g}}}^{v\sqrt{\frac{K}{g}}} \sqrt{\frac{g}{K}}\,\frac{dx}{x^2-1} =$$

Substit.: $\frac{K}{g}v^2 = x^2 \longrightarrow v = \sqrt{\frac{g}{K}}\,x$

$$\frac{K}{g}2v\,dv = 2x\,dx$$

$$dv = \frac{g}{K}\frac{1}{v}x\,dx = \frac{g}{K}\cdot\sqrt{\frac{K}{g}}\frac{1}{x}x\,dx = \sqrt{\frac{g}{K}}\,dx$$

Hranice: $v = v_0 \;\dots\; x = v\sqrt{\frac{K}{g}} = v_0\sqrt{\frac{K}{g}}$

$v = v \;\dots\; x = v\sqrt{\frac{K}{g}}$

$$\frac{1}{g}\sqrt{\frac{g}{K}}\,\left[\operatorname{atanh}(x)\right]_{\sqrt{\frac{K}{g}}v_0}^{\sqrt{\frac{K}{g}}v} = t$$

DÚ ... úprava

$$\tanh(x+y) = \frac{\tanh(x)+\tanh(y)}{1+\tanh(x).\tanh(y)}$$

Výsledok :

$$v(t) = \frac{v_0 - v_\infty \tanh\left(\frac{gt}{v_\infty}\right)}{1 - \frac{v_0}{v_\infty}\tanh\left(\frac{gt}{v_\infty}\right)}$$

term. rýchl. v_∞

$$mg = \frac{1}{2} C S \varrho v_\infty^2$$

$$v_\infty = \sqrt{\frac{2mg}{\varrho C S}}$$

Zjednodušenie : $v_0 = 0$ poč. rýchl.

- $v(t) = -\,v_\infty \tanh\left(\frac{gt}{v_\infty}\right)$

Závislosť výšky od času $z(t) = ?$

$$z(t) = z(0) + \int_0^t v_z(t)\,dt \qquad v(t) \equiv v_z(t)$$

$\uparrow z$

$$z(t) = z_0 + \int_0^t \left[-v_\infty \tanh\left(\frac{gt}{v_\infty}\right)\right] dt =$$

tab

$$\int \operatorname{tang}(ax)\,dx = ?$$

D.Ú. výpočet integrálu

$$z(t) = z_0 - \frac{v_\infty}{g}\ln\cosh\left(\frac{gt}{v_\infty}\right)$$

Čas, za kt. teleso dopadne : $t_D = ?$

$$z(t_D) = 0$$

D.Ú výpočet času dopadu

$$t_D =$$

Veľkosť okamž. rýchlosti v čase dopadu :

$$v(t_D) = v_D =$$

DÚ

$$\tanh x = \frac{\sinh x}{\cosh x}\ ; \qquad \cosh^2 x - \sinh^2 x = 1$$

Čo sme počítali: pád nadol s odporom vzduchu
ρ – konšt.

- $v(t)$
- $z(t)$
- t_D čas dopadu
- v_D rýchl. v č. dopadu

$\boxed{v(z)}$

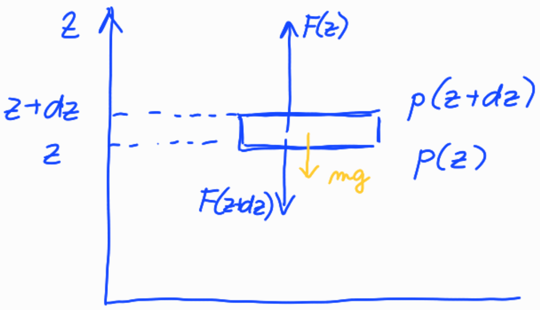
$$v_z(t) = -v_\infty \tanh\left(\frac{gt}{v_\infty}\right)$$
$$z(t) = z_0 - \frac{v_\infty^2}{g} \ln\left[\cosh\left(\frac{gt}{v_\infty}\right)\right]$$

$$\rightarrow \quad v_z(z) = -\sqrt{1-\exp\left[-\frac{CS\rho}{m}(z_0-z)\right]}\; v_\infty$$

D.Ú.

Atmosféra s poklesom tlaku s výškou

teplota T = konšt., vzduch = ideálny plyn



tlak: $p = \frac{F}{S}$ $\quad [p]$ = Pascal = Pa

$$F = pS$$

$$F(z) = F(z+dz) + G$$

$$\circledast \quad p(z)S = p(z+dz)S + \underbrace{g\rho(z)S\,dz}_{mg} \quad /:S$$

mg
$\rho V g$
$\rho S dz g$

Tlak na hornom okraji vrstvy:

$$p(z+dz) = p(z) + \frac{dp}{dz}dz$$

dos. do $\circledast$

$$\cancel{p(z)} = \cancel{p(z)} + \frac{dp}{dz}dz + g\rho\,dz$$

$$\boxed{\frac{dp}{dz} = -\rho(z)g}$$