TIØ4285 Production and Network Economics

Assignment 4 – Suggested solution

Exercise 1. Cournot oligopoly with capacity constraints

a. The optimization problems:

$$\forall\, i: \ \min\, z_i = \left(c_i q_i + d_i q_i^2\right) - \left(a - b\sum_j q_j\right) q_i$$

$$s.t. \quad cap_i - q_i \geq 0$$

Note: $\left(a - b\sum_j q_j\right) q_i = \left(a q_i - b q_i \sum_j q_j\right) = a q_i - b q_i \sum_{j \neq i} q_j - b q_i^2$

b. The complementarity problem:

$$\forall i: \qquad 0 \leq q_i \perp \left(c_i + 2 d_i q_i\right) - \left(a - b\sum_{j\neq i} q_j - 2 b q_i\right) \geq 0$$

$$0 \leq \lambda_i \perp cap_i - q_i \geq 0$$

Which is equivalent to:

$$\forall i: \qquad 0 \leq q_i \perp \left(c_i + 2 d_i q_i\right) - \left(a - b\sum_{j} q_j - b q_i\right) \geq 0$$

$$0 \leq \lambda_i \perp cap_i - q_i \geq 0$$

c. The equilibrium price and quantities for $N = 3, c_i = 2, d = \frac{1}{2}, a = 20, b = 1, cap_i = 5$ can be calculated via the optimal response function (see lecture slides), but also taking into account the capacity restrictions: $q_i = \min\left(\dfrac{a - c_i - b\sum_{j\neq i} q_j}{2\left(b + d_i\right)}, cap_i\right)$

Assume first that the capacities are not binding:

$$q_i = \frac{a - c_i - b\sum_{j\neq i} q_j}{2\left(b + d_i\right)} = \frac{20 - 2 - 1\sum_{j\neq i} q_j}{2\left(1 + 0.5\right)} = \frac{18 - \sum_{j\neq i} q_j}{3} = 6 - \frac{\sum_{j\neq i} q_j}{3}$$

Use symmetry: $q_i = q_j \Rightarrow q_j = \dfrac{18 - \left(N - 1\right) q_j}{3} \Rightarrow \left(N + 2\right) q_j = 18 \Rightarrow q_j = \frac{18}{5} = 3.6$

Since this is lower than the respective capacity limits of 5, this is the optimal solution.

d. See separate GAMS code

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*Capacity-constrainted oligopoly on a single node. Partial solution assignment 4
*Ruud Egging-Bratseth, 2020. NTNU TIØ4285, Production and Network Economics.

set i suppliers /1*4/;
alias(i,j);

parameter a      intercept of the inv demand curve                        /20/
          b      negative slope of inverse demand curve                   / 1/
          c(i)   constant cost term in productoin cost   c*q + d*q^2 /1*3  2/
          d(i)   increasing cost term in productoin cost c*q + d*q^2 /1*4  0.5/
          cap(i) capacity of supplier i                              /1*3  5/
;
c('4')= 99;

positive variable Q(i)    quantity supplied by supplier i
                  lam(i)  capacity dual price i
;
equations stat_q, eq_cap
;

stat_q(i) .. c(i) + 2*d(i)*Q(i) + lam(i) - (a - b*(sum(j,Q(j)) + Q(i))) =G= 0
;
eq_cap(i) ..  cap(i) - Q(i) =G= 0;
;
model olig / stat_q.Q
             eq_cap.lam
/
;
solve olig using mcp
;
parameter rep(*,*,*)  output report;
```

```
parameter rep(*,*,*)  output report;

rep('p','mkt',  '1')= a-b*sum(j,Q.l(j));
rep('q', i,     '1')= Q.l(i);
rep('lam',i,    '1')= lam.l(i);

c(i)$(ord(i)<4)= 1;
solve olig using mcp;
rep('p','mkt',  '2')= a-b*sum(j,Q.l(j));
rep('q', i,     '2')= Q.l(i);
rep('lam',i,    '2')= lam.l(i);

cap(i)$(ord(i)<4)=3;
solve olig using mcp;
rep('p','mkt',  '3')= a-b*sum(j,Q.l(j));
rep('q', i,     '3')= Q.l(i);
rep('lam',i,    '3')= lam.l(i);

cap('2')=4;
cap('3')=5;
solve olig using mcp;
rep('p','mkt',  '4')= a-b*sum(j,Q.l(j));
rep('q', i,     '4')= Q.l(i);
rep('lam',i,    '4')= lam.l(i);

c('4')=1;
cap(i)=3;
solve olig using mcp;
rep('p','mkt',  '5')= a-b*sum(j,Q.l(j));
rep('q', i,     '5')= Q.l(i);
rep('lam',i,    '5')= lam.l(i);

c(i)= 2;
solve olig using mcp;
rep('p','mkt',  '6')= a-b*sum(j,Q.l(j));
rep('q', i,     '6')= Q.l(i);
rep('lam',i,    '6')= lam.l(i);

option rep:2:2:1
display rep;
```

```
                1           2           3           4           5           6
p   .mkt     9.20        8.60       11.00        9.00        8.00        8.00
q   .1       3.60        3.80        3.00        3.00        3.00        3.00
q   .2       3.60        3.80        3.00        4.00        3.00        3.00
q   .3       3.60        3.80        3.00        4.00        3.00        3.00
q   .4                                                       3.00        3.00
lam.1                                4.00        2.00        1.00
lam.2                                4.00                    1.00
lam.3                                4.00                    1.00
lam.4                                                        1.00
```

	1	**2**	**3**	**4**	**5**	**6**
parameter	**value**	**value**	**value**	**value**	**value**	**value**
N	3	3	3	3	4	4
a	20	20	20	20	20	20
b	1	1	1	1	1	1
c_i	2	1	1	1	1	2
cap_1	5	5	3	3	3	3
cap_2	5	5	3	4	3	3
cap_3	5	5	3	5	3	3
cap_4					3	3
variable						
p	9.2	8.6	11	9	8	8
q_1	3.6	3.8	3	3	3	3
q_2	3.6	3.8	3	4	3	3
q_3	3.6	3.8	3	4	3	3
q_4					3	3
λ_1	0	0	4	2	1	0
λ_2	0	0	4	0	1	0
λ_3	0	0	4	0	1	0
λ_4					1	0

Discuss briefly for each supplier how MR=MC in each outcome.

Column 1

- MR_i=a-(N+1)bq_I=20-4x3.6=5.6
- MC_i=c+2dq_I=2+q_i=2+3.6=5.6

Column 2

- MR_i=a-(N+1)bq_I=20-4x3.8=4.8
- MC_i=c+2dq_I=1+q_i=1+3.8=4.8

Column 3

- MR_i=20-4*3=8
- MC=1+3=4
- The difference between MR and MC = lambda = 4!

Column 4

- MR(1)=20-(2x3+4+4)=6
- MC(1)=1+3=4
- Lam(1)=2
- MR(2,3)= 20-(2x4+3+4)=5
- MC(2,3)=1+4=5

Column 5

- MR(i)=20-(3x3+2x3)=5
- MC=1+3=4
- Lam=1

Column 6

- MR(i)=20-(3x3+2x3)=5
- MC(i)2+3=5
- Lam=0

Is the capacity limit restrictive in the last column?

No. The shadow price on the constraint is zero. This means that increasing the capacities will not change the solution.

Exercise 2. Social welfare maximization

SW_assignment.mos

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!SW maximization on a single node. Solution assignment 4.2
!Ruud Egging-Bratseth, 2020. Lecture notes NTNU TIØ4285, Production and Network Economics.

model "SW_opt"
 uses "mmxprs","mmquad"

parameters
   a=  20     ! intercept of the inverse demand curve
   b=   1     ! negative slope of inverse demand curve
   c=   2     ! constant cost term in productoin cost   c*q + d*q^2
   d=   0.5   ! increasing cost term in productoin cost c*q + d*q^2
end-parameters

declarations
    Q: mpvar;
    Z: qexp;
end-declarations

Q>=0

!def_sw :=
Z:= 0.5*b*Q*Q + (a - b*Q)*Q  - (c + d*Q)*Q

maximize(Z);

writeln("Supply        ", getsol(Q))
writeln("Social welfare: ", getobjval)
writeln("Market price  ", a - b*getsol(Q))

end-model
```

Supply 9
Social welfare: 81
Market price 11

MR(i)=20-9=11

MC(i)=2+2x0.5x9=11