NTNU
Department of Industrial Economics and Technology Management
Spring 2019

TIØ4285 Production and Network Economics
Assignment 8 – Solution

Out: Thursday 5 March
In: Thursday 12 March, 6pm
Supervision: Monday 9 March, 4:15pm A31

Note that late exercises will not be approved.

Exercise

PetrolAir offers fast and very luxurious flight connections for the top managers of Arabian oil companies. They plan to set up a new route between Dubai and Riyadh using small jets (capacity per jet is one passenger). In particular, they have to decide how many planes to use before they know how many tickets they will sell eventually.

The planes will be rented from Planes-R-Us for 10,000 Petrodollars (P$) per plane. PetrolAir charges 30,000 P$ for the exclusive journey from Dubai to Riyadh. As all airlines, they will sell tickets to everyone who wants one and deal with overbooking later. Daily ticket sales are uncertain and have a discrete probability distribution. The distribution has the following realizations that are equally likely:

$$6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25$$

In case they sell more tickets than they have planes, PetrolAir incurs a cost of 15,000 P$ in order to transfer the managers to other flights. To prevent planes from flying empty, PetrolAir sells free seats at Last-Minute prices of 5,000 P$. This Last-Minute market may be viewed as having perfect competition.

Use the following notation:

- r – Revenues from selling tickets to the customers
- p – Costs of overbooking and transferring passengers
- w – Rental price per plane
- s – Revenue per Last-Minute ticket in the perfect competition market
- D – Demand / Number of tickets sold
- C – Capacity / Number of planes rented from Planes-R-Us

The optimality condition for maximizing Petrol Air's profit is given by $\Pr(D < C) = \frac{p-w}{p-s}$.

You are put in charge of planning the new connection.

a) Please express the formula for PetrolAir's expected daily profit as a function of D and C.

$$\Pi = E\left[r \cdot D + s \cdot (C - D)^{+} - p \cdot (D - C)^{+} - w \cdot C\right]$$

b) How many planes should you rent from Planes-R-Us? What is the expected profit per day?

$$\Pr(D < C) = \frac{p-w}{p-s} = \frac{15-10}{15-5} = \frac{5}{10} = 0.5 \quad \Rightarrow \quad C = 15$$
$$\Pi(C = 15) = 285'.$$

Note that $C = 16$ will result in the same expected profits.

c) Your next task is to convince the board of directors that you did a good job determining the capacity requirements. Show that the optimality criterion is indeed given by $\Pr(D < C) = \frac{p-w}{p-s}$.

Using $C = D - (D-C)^+ + (C-D)^+$, we can reformulate the formula for expected profit:

$$\begin{aligned}
\Pi &= E\left[r \cdot D + s \cdot (C-D)^+ - p \cdot (D-C)^+ - w \cdot C\right] \\
&= E\big[r \cdot D + s \cdot (C-D)^+ - p \cdot (D-C)^+ - \\
&\quad w \cdot \left(D - (D-C)^+ + (C-D)^+\right)\big] \\
&= E\big[r \cdot D + s \cdot (C-D)^+ - p \cdot (D-C)^+ - w \cdot D + \\
&\quad w \cdot (D-C)^+ - w \cdot (C-D)^+\big] \\
&= (r-w)ED + E\big[s \cdot (C-D)^+ - p \cdot (D-C)^+ - w \cdot (D-C)^+ - \\
&\quad (C-D)^+\big] \\
&= (r-w)ED + E\left[(s-w) \cdot (C-D)^+ - (p-W) \cdot (D-C)^+\right] \\
\frac{d\Pi}{dC} &= (s-w)\Pr(D < C) + (p-w) \cdot \Pr(C < D) \\
&= (s-w)\Pr(D < C) + (p-w) \cdot (1 - \Pr(D < C)) \\
&= (s-w)\Pr(D < C) + (p-w) - (p-w) \cdot \Pr(D < C) \\
&= (p-w) + (s-w-p+w) \cdot \Pr(D < C) \\
&= (p-w) - (p-s) \cdot \Pr(D < C)
\end{aligned}$$

$$\frac{d\Pi}{dC} = 0 \quad \Rightarrow \quad \Pr(D < C) = \frac{p-w}{p-s}.$$

The board is impressed by your work. However, they believe the company should order 20 planes to capture a market share big enough to prevent competitors from entering into the market. You, of course, tell them that this would only be optimal if there is a shift in the price in the last-minute market.

d) What price would you need in the Last-Minute market to make an order of 20 planes optimal?

With $C = 20$, we have $\Pr(D < C) = 0.75 \quad \Rightarrow \quad = 8.333'$.

The board of directors considers such a price shift for Last-Minute tickets as unlikely. They suggest instead that PetrolAir and Planes-R-Us should coordinate their decisions and ask you to perform the analysis. Planes-R-Us supports the idea and shares their variable cost function with you. The variable operating costs of the planes are 7,500 P$. You don't have to consider their fixed costs.

e) Please suggest an optimal Profit-Sharing contract that both companies are willing to accept. Determine the capacity that PetrolAir should book in order to maximize the supply chain profits.

Optimal supply chain profits given for $\Pr(D < C) = \frac{p-c}{p-s}$, with c being the variable operating costs of the plane. Optimal order quantity and profits are $C = 20, \Pi = 330'$ ($C = 21$ results in the same profits).

Expected profits for Planes-R-Us:

$$\pi(C = 15) \qquad = 37.5'$$
$$\pi(C = 16) \qquad = 40'$$

Expected supply chain profits:

$$\Pi(C = 15) \qquad = 322.5'$$
$$\Pi(C = 16) \qquad = 325'$$

The Profit-Sharing contract uses marginal costs as wholesale price and distributes profits by PetrolAir paying a percentage of her profits to Planes-R-Us. In order to accept the contract, both companies must make at least as much profit as in the situation without the contract.

PetrolAir has to make at least $285'$, restricting the share paid to Plane-R-Us to

$$\left(1 - \frac{285}{330}\right) \cdot 100\% = 13.64\%$$

Planes-R-Us needs at least a profit of $37.5'$, thus requiring a share of at least

$$\frac{37.5}{330} \cdot 100\% = 11.36\%$$

Using a reservation profit of $40'$ (based on $C = 16$), is also correct.

f) Explain how Planes-R-Us' fixed costs influence the optimal number of planes.

Fixed costs do not affect the optimal solution.