NTNU
Department of Industrial Economics and Technology Management
Spring 2020

TIØ4285 Production and Network Economics
Assignment 7 – Proposed Solution

Out: Thursday 27 February
In: Thursday 5 March, 6pm
Supervision: Monday 2 March, 4:15pm A31

Note that late exercises will not be approved.

Exercise 1

The company Bare Blomster AS has 2 divisions, the market garden that produces tulips and the retail division that sells these to the end customers. The market garden has the following cost function per batch of tulips q:

$$TC_G = 50 \cdot q + 2 \cdot q^2.$$

The retail division observes the following demand for batches of tulips in the market:

$$p = 150 - 0.5 \cdot q.$$

Its total cost function is given as

$$TC_B = 10 \cdot q.$$

a) Find the company's net marginal revenue curve as seen from the market garden. How should this curve be interpreted?

The net marginal revenue curve is given as $NMR = MR - MC_B = 150 - q - 10 = 140 - q$

Net Marginal Revenues represent Bare Blomster's marginal income of producing one additional unit as seen by the market garden (i. e. after the shop's marginal costs are deducted). Alternatively, it can be interpreted as the shop's demand function for "buying" tulips from the market garden.

b) Maximize the company's profits. What are the optimal profits? Which transfer price is used between the different divisions and what is the price for the end customer?

We get the following:

$$TC_T = TC_G + TC_B = 60 \cdot q + 2 \cdot q^2 \quad \Rightarrow \quad MC_T = 60 + 4 \cdot q$$
$$MR = 150 - q$$

Profit maximization for $MR = MC_T$: $150 - q = 60 + 4 \cdot q \quad \Rightarrow q^* = 18$
End-customer price: $P = 150 - 0.5 \cdot q = 141$.

Transfer price equal marginal cost of production: $MC_G(q^*) = 122$
Profits market garden: $\Pi_G = 122 \cdot 18 - 50 \cdot 18 - 2 \cdot 182 = 648$
Profits shop: $\Pi_B = 141 \cdot 18 - (10 + 122) \cdot 18 = 162$
Total profit: $\Pi_T = 810$

Exercise 2

Saftbolaget has a monopoly in the Nordic juice market. It consists of two departments: a production department and a retailing department. The manufacturing cost is given as

$$C_P = 250 + 8 \cdot q + \frac{1}{20} \cdot q^2,$$

whereas the retailing cost can be described by the following function

$$C_R = 225 + 9 \cdot q.$$

Demand in the Nordic juice market is given as

$$p(q) = 62 - \frac{1}{10} \cdot q.$$

a) How much juice should the company sell in the Nordic market to maximize profits? What are the optimal profits?

Marginal revenues are given as $MR = 62 - \frac{1}{5}q$. Saftbolagets marginal costs are simply the sum of the marginal cost of production and retailing: $MC = MC_P + MC_R = 8 + \frac{1}{10}q + 9 = 17 + \frac{1}{10}q$.

Using the optimality criterion ($MR = MC$), we get

$$\begin{aligned} 62 - \frac{1}{5}q &= 17 + \frac{1}{10}q \\ \frac{3}{10}q &= 45 \\ q &= 150 \end{aligned}$$

Selling $q^* = 150$ units of juice results in overall profits equal to $\Pi = p(q^*) \cdot q * -C_R(q^*) - C_P(q^*) = 2\,900$.

b) What is the transfer price for juice used between the production department and the retailing department?

The market price that end customers have to pay is $p(q^*) = 47$. To achieve optimal profits, we need to exchange juice at marginal cost of production, i. e. $TP = MC_P(q = 150) = 23$. Any other transfer price will distort the amount of juice ordered by the retailing division.

To increase profits, Saftbolaget considers the possibility of the production department selling juice to retailers in Southern markets. The demand function of these external retailers is given as

$$p_e(q) = 33 - \frac{1}{30} \cdot q_e.$$

Note that the demand of the external retailers does not influence the demand in the Nordic market. Saftbolaget does not incur retailing costs for juice sold to external retailers.

c) How much juice (if any) should be sold in the different markets? What are the optimal profits and which transfer prices should Saftbolaget use?

Marginal revenues from internal sales are given by the Net Marginal Revenue for producing juice, $NMR_P = MR - MC_R = 53 - \frac{1}{5}q_i$. Marginal revenues from external sales are derived from the external demand function $p_e(q_e)$, $MR_e = 33 - \frac{1}{15}q_e$. We determine the total net marginal revenue by horizontally summing NMR_P and MR_e. Note that we do not sell juice to the external market for outputs $q = q_i + q_e < 100$:

$$\left.\begin{aligned} q_i &= 265 - 5NMR_P \\ q_e &= 495 - 15MR_e \end{aligned}\right\} \quad \Rightarrow NMR_{Total} = 38 - \frac{1}{20}q$$

Setting these marginal revenues equal to marginal cost of production, we determine the optimal amount of produced juice:

$$
\begin{aligned}
NMR_{Total} &= MC_P \\
38 - \frac{1}{20}q &= 8 + \frac{1}{10}q \\
\frac{3}{20}q &= 30 \qquad \Rightarrow \qquad q^* = 200
\end{aligned}
$$

Sales to the external market

$$
\begin{aligned}
MR_e = MC_P(q = 200) \qquad \Rightarrow \qquad 33 - \frac{1}{15}q_e &= 28 \\
\frac{1}{15}q_e &= 5 \qquad \Rightarrow \qquad q_e = 75 \\
& \qquad\qquad\qquad\quad p_e = 30.5
\end{aligned}
$$

Sales to the internal market

$$
\begin{aligned}
NMR_P = MC_P(q = 200) \qquad \Rightarrow \qquad 53 - \frac{1}{5}q_i &= 28 \\
\frac{1}{5}q_i &= 25 \qquad \Rightarrow \qquad q_i = 125 \\
& \qquad\qquad\qquad\quad TP = 28 \\
& \qquad\qquad\qquad\quad p = 49.5
\end{aligned}
$$

resulting in the following overall profit

$$
\begin{aligned}
\Pi(q^*) &= 30.5 \cdot 75 + 49.5 \cdot 125 - 75 - 6 \cdot 125 - 150 - 3 \cdot 125 - 250 - 8 \cdot 200 - \frac{1}{20} \cdot 200^2 \\
&= 3\,275.
\end{aligned}
$$

Another way of determining the optimal production and sales quantities is to start by setting up the equation for total company profits:

$$\Pi(q) = p(q_i) \cdot q_i + p_e(q_e) \cdot q_e - C_R(q_i) - C_P(q_i + q_e).$$

Then you have to derive the profit function with respect to both q_i and q_e, set both equal to 0 and solve the resulting set of 2 equation with 2 unknowns.