Markets with transportation networks
Lecture 3 in Equilibrium Modeling Block

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TIØ4285 Production & Network Economics

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Outline: three lectures

- Lecture 1 Equilibrium modeling
  - Introduction, motivation and preliminaries
  - Multi-variable optimization, Lagrangian multipliers and Karush Kuhn Tucker conditions
  - Single-agent and Cournot equilibrium problems
- Lecture 2 Network modeling
  - Transportation problems
  - Assignment problems
- Lecture 3 Markets with transport networks
  - Combining lectures 1 & 2
  - (Multi-agent) equilibrium problems with embedded transport networks
  - Spatial and temporal aspects: network, investment, uncertainty, storage

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Last week

- Networks are a part of very many real-world problems and subject of research in many fields (not just operations research and economics)
- Modeling of Transportation in Networks
- Minimum cost network flow
- Assignment, Facility location, set coverage
- Optimization and complementarity formulations and implementations

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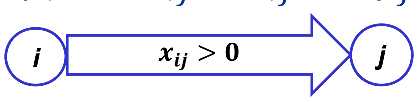
Mass balances in complementarity problems

- Signs of mass balance duals are tricky.

$$SOURCES - SINKS = 0 \ (\varphi \ f.i.s.)$$

- Work consistently through derivation steps, and check logic:

$$x_{ij} > 0 \Rightarrow \varphi_i + \quad c_{ij} + \lambda_{ij} \quad = \varphi_j$$



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Imperfectly competitive multi-agent problems

- Agents not price-takers: behavior not necessarily leads to the system-wide 'optimal' solution
- Equilibrium problems can account for market power a là Cournot, that optimization models can't
- modeled using complementarity conditions
- Linearly constrained problems with convex minimization objectives: KKT points are optimal solutions

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Equilibrium problems: agents in markets

- Suppliers
  - maximize profits (revenues-costs)
  - May face (technical) restrictions such as production capacity
  - Mass-balance equations (next)
  - May exert market power (à la Cournot, or moderated)
  - Rent infrastructure services for transportation, storage, …
  - Acquire emission permits
- Infrastructure operators / service providers
  - Maximize value of their assets (*congestion revenues*)
  - May face (technical) restrictions such as capacity limits, electricity loop-flows, gas pressure-flows, processing losses, …
  - Rent out infrastructure services / emission permits
- Market clearing
  - Inverse demand curve (can often be substituted out)
  - Infrastructure services
  - Environmental restrictions, e.g. emission permits

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ENERGY MARKETS

Energy markets

- Energy / Electricity needed for virtually every (productive) human activity
- Government: *affordable, secure, sustainable*
- Companies: making good decisions in imperfect market with lot of uncertainty
- Complexity in energy markets require methodologies that can analyze technical and economic aspects.
  - Market liberalization, climate policy, weather, planning and management of renewable energy supply, smart grids, international transmission lines, etc.

NATURAL GAS MARKET

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European Natural Gas Network

www.entsog.eu/maps/transmission-capacity-map

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Competitive markets

- Homogenous goods?
- Entry barriers?
- External effects?
- Transparency?
- All agents price takers?
- Russia, Gas Exporting Countries Forum
  - www.gecf.org/about/mission-objectives.aspx
- Market shares EU

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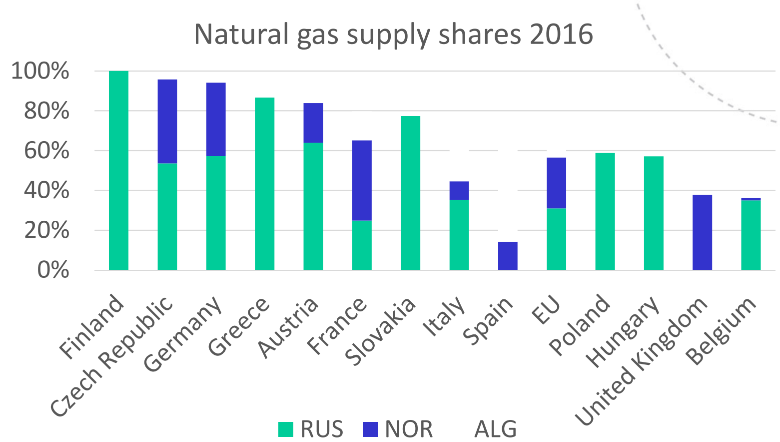
Market shares EU



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Credit: based on BP 2017 Stat Rvw of World Energy

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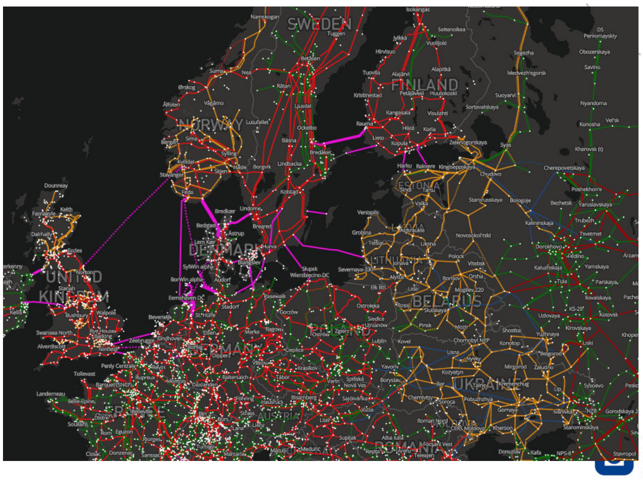
Entry barriers

- Assets for fossil fuel exploration, processing, production, transportation, storage are highly capital intensive: mln's-bln's of $$
- Natural monopoly characteristics

ELECTRICITY MARKET

European Electric Power Network

www.entsoe.eu/map/Pages/default.aspx



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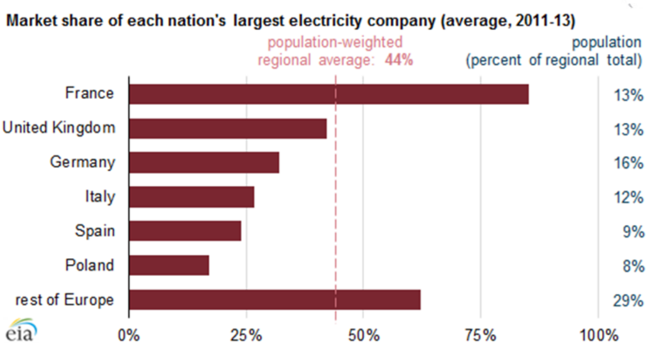
Market shares largest company



Source: https://www.eia.gov/todayinenergy/detail.php?id=21732

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Market power in energy markets

- Characteristics of energy sector imply imperfectly competitive markets
- social welfare maximization will generally not adequately reflect behavior of actors;
- Can represent imperfect competition (e.g., oligopolies) by equilibrium modeling
- Network nature of energy sector value chains make it a natural part of TIØ 4285

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COURNOT OLIGOPOLY

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Cournot Oligopoly

Supplier i: $\max_{q_i} \quad z_i = \left(a - b\sum_j q_j\right)q_i - \left(c_i q_i + d_i q_i^{\,2}\right)$

Micro-economics / Industrial Economic Analysis:
solve using *optimal response curves*

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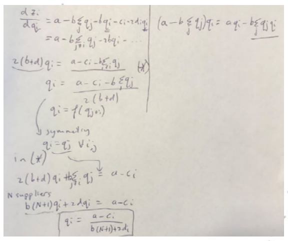
Cournot Oligopoly: optimal response curves



$$\frac{\partial z_i}{\partial q_i} = \left(a - b\sum_j q_j - bq_i\right) - \left(c_i + 2d_i q_i\right)$$

$$\frac{\partial z_i}{\partial q_i} = 0 \;\Rightarrow \left(2bq_i + 2d_i q_i\right) = a - c_i - b\sum_{j\neq i} q_j$$

$$\Rightarrow q_i = \frac{a - c_i - b\sum_{j\neq i} q_j}{2\left(b + d_i\right)}$$

Using symmetry:

$$\Rightarrow q_i = \frac{a - c_i}{\left(N+1\right)b + 2d_i}$$

Easily solved when only two, or symmetric suppliers. What if many asymmetric suppliers, production or network capacity constraints, …?

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Cournot Oligopoly: complementarity problem

- Set up complementarity problem:
  - Write as minimization
  - Stationarity KKT:

$$\forall i: \quad \max z_i = \left(a - b\sum_j q_j\right)q_i - \left(c_i q_i + d_i q_i^2\right)$$

$$\forall i: \quad \min z_i = \left(c_i q_i + d_i q_i^2\right) - \left(a - b\sum_j q_j\right)q_i$$

$$\forall i: \quad 0 \le q_i \perp \left(c_i + 2d_i q_i\right) - \left(a - b\sum_j q_j - bq_i\right) \ge 0$$

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COURNOT OLIGOPOLY ON A NETWORK

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Cournot Oligopoly on a network

- Cournot oligopoly on network with suppliers on some nodes
- Agents:
  - Suppliers
  - Consumers
  - Network operator TSO: *transmission system operator*
- Suppliers maximize profits
  - Cournot players
  - Rent transportation services from the TSO
- Consumers: inverse demand curve
- Transmission System Operator:
  - provides transportation services
  - Price-taking maximizer of value of the network
- Market clearing demand via inverse demand curve
- Market clearing transmission capacity

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Oligopoly on a network cont'd

- Network: nodes n,m connected by arcs
- Suppliers i,j
  - produce at one or more nodes amounts q_{in}^P
  - sell at one or more nodes amounts q_{in}^S at nodes reachable via the network.
  - flows over arcs: f_{inm}^P
  - Price of transmission services = costs + congestion tariff: $c_{nm}^A + \tau_{nm}^A$
- Transmission System Operator:
  - offers transport services on arcs (n,m) f_{nm}^A
- MCC demand via inverse demand $\mathrm{p_n} = \mathrm{a}_n - b_n \sum_i q_{in}^S$
- MCC transmission services: $f_{nm}^A = \sum_i f_{inm}^P \quad (\tau_{\mathrm{nm}}\ f.i.s)$

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Oligopoly on a network: producer

Supplier i:

$$\max_{q^P_{in}, q^S_{in}, f^P_{imn}} \left\{ \sum_n \left(p_n \left(\sum_i q^S_{in} \right) q^S_{in} - c^P_{in}\left(q^P_{in}\right) \right) - \sum_{(n,m)} \left(\left(c^A_{nm} + \tau^A_{nm} \right) f^P_{inm} \right) \right\}$$

$$\text{s.t.} \quad q^P_{in} \le cap^P_{in} \qquad \left(\lambda^P_{in} \ge 0\right), \qquad n \in N$$

$$q^P_{in} + \sum_m f^P_{inm} = q^S_{in} + \sum_m f^P_{inm} \qquad \left(\varphi^P_n \ f.i.s.\right), \qquad n \in N$$

- *Discuss terms & notation*
- *Restrictions: duals; “>=“ or sources - sinks= 0*
- *Stationarity conditions – duals get a “-1”*

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Oligopoly on a network: producer

Supplier i:

$$\max_{q^P_{in}, q^S_{in}, f^P_{inm}} \left\{ \sum_n \left(p_n \left(\sum_i q^S_{in} \right) q^S_{in} - c^P_{in}\left(q^P_{in}\right) \right) - \sum_{(n,m)} \left(\left(c^A_{nm} + \tau^A_{nm} \right) f^P_{inm} \right) \right\}$$

$$\text{s.t.} \quad q^P_{in} \le cap^P_{in} \qquad \left(\lambda^P_{in} \ge 0\right), \qquad n \in N$$

$$q^P_{inm} + \sum_m f^P_{inm} = q^S_{in} + \sum_m f^P_{inm} \qquad \left(\varphi^P_{in} \ f.i.s.\right), \qquad n \in N$$

$$\text{Production cost}: \quad c_{in}\left(q^P_{in}\right) = c_{in} q^P_{in} + d_{in}\left(q^P_{in}\right)^2$$

$$\text{Inverse demand}: \quad p_n\left(\sum_i q^S_{in}\right) = a_n - b_n \sum_i q^S_{in}$$

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Oligopoly on a network: TSO

T.S.O.: $\max \sum_{(n,m)} \left\{ \left(c^A_{nm} + \tau^A_{nm} \right) f^A_{nm} - c^A_{nm} f^A_{nm,} \right\}$

$s.t. \quad f^A_{nm} \le cap^A_{nm} \quad \left(\lambda^A_{nm} \right), \; (n,m) \in N \times N$

T.S.O.: $\max \sum_{(n,m)} \left\{ \tau^A_{nm} f^A_{nm} \right\}$

$s.t. \quad f^A_{nm} \le cap^A_{nm} \quad \left(\lambda^A_{nm} \ge 0 \right), (n,m) \in N \times N$

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Oligopoly on a network: MCC transport services

M.C.C. transportation services:

$$f^A_{nm} = \sum_i f^P_{inm} \quad \left(\tau^A_{nm} \; f.i.s \right), \qquad (n,m) \in N \times N$$

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Deriving the complementarity problem

- For each agent:
  - Minimization objective
  - Restrictions: Duals; >=0 & sources - sinks = 0
  - Derive KKT
- Add MCC

- *DO THIS*

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INVESTMENT

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Multi-period investment problem

- Agent decides on capacity expansion, which will become available after some time lag
- Do not forget the discount rate

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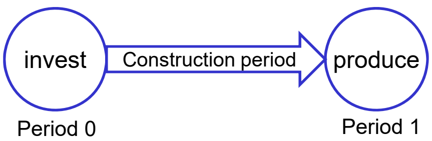
Two-stage investment problem

- Price-taking supplier
- Invest in production capacity in first period
- Produce and sell in the second period



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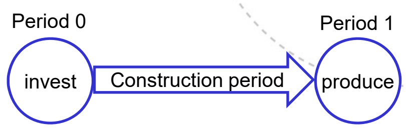
Two-stage production capacity investment problem

Supplier problem

- In-class assignment: Derive the MCP
  - Minimization
  - Duals; >=0 & sources - sinks = 0
- Intuition: how are p, c^P, c^I and r connected?



$$\max_{cap_0^{\Delta P}, q_1} \quad r_1\left(p_1 q_1 - c_1^P q_1\right) - c_0^{\Delta P} cap_0^{\Delta P}$$

$$s.t. \quad q_1 \leq cap^P + cap_0^{\Delta P} \qquad \left(\lambda_1 \geq 0\right)$$

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Network investment – multi-period

- Assume a single agent
- Capacitated network
- Multi-period problem
- Investment possible to expand capacities; it takes one period for capacity expansions are available
- Assume continues capacities
- Separate capacities and flows
- Minimize sum of investment costs, production costs and transport costs
- s.t., in all time periods, capacity constraints are respected and mass balances hold
- Discount rate
- Complementarity formulation

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Multi-period network investment

- Nodes $i, j \in N$
- Periods $t \in T = \{1,2,3,..\}$
- Unit flow costs c_{tij}^{X}
- Unit investment costs $c_{tij}^{\bar{X}}$
- Initial capacity $\bar{X}_{0ij}$
- Capacity expansions $\bar{X}_{tij}$
- Flows X_{tij}
- Minimize sum investment costs and transport costs

$$\min \sum_{t,i,j} r_t \{ c_{tij}^{\bar{X}} \bar{X}_{tij} + c_{tij}^{X} X_{tij} \}$$

$$s_{ti} + \sum_{j \in N} X_{tji} = d_{ti} + \sum_{j \in N} X_{tij}, i \in N, t \in T$$

$$X_{tij} \leq \sum_{\tau=0}^{t-1} \bar{X}_{\tau ij}, i \in N, j \in N, t \in T$$

$$\bar{X}_{tij}, X_{tij} \geq 0, i \in N, j \in N, t \in T$$

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Multi-period network investment: example with terms written out

$$X_{tij} \leq \sum_{\tau=0}^{t-1} \bar{X}_{\tau ij}$$

$$X_{1ij} \leq \bar{X}_{0ij} \qquad (\lambda_{1ij})$$

$$X_{2ij} \leq \bar{X}_{0ij} + \bar{X}_{1ij} \qquad (\lambda_{2ij})$$

$$X_{3ij} \leq \bar{X}_{0ij} + \bar{X}_{1ij} + \bar{X}_{2ij} \qquad (\lambda_{3ij})$$

$$X_{4ij} \leq \bar{X}_{0ij} + \bar{X}_{1ij} + \bar{X}_{2ij} + \bar{X}_{3ij} \qquad (\lambda_{4ij})$$

Etc.

$$\mathbf{0} \leq \boldsymbol{\lambda_{3ij}} \perp$$
$$\bar{X}_{0ij} + \bar{X}_{1ij} + \bar{X}_{2ij} - X_{3ij} \geq 0$$

$$\mathbf{0} \leq \boldsymbol{X_{3ij}} \perp$$
$$r_3 c_{3ij}^{X} + \lambda_{3ij} + \phi_{3i} - \phi_{3j} \geq 0$$

$$\mathbf{0} \leq \boldsymbol{\bar{X}_{1ij}} \perp$$
$$r_3 c_{tij}^{\bar{X}} - \lambda_{2ij} - \lambda_{3ij} - \lambda_{4ij} \geq 0$$

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Multi-period network investment: general notation

$$min \sum_{t,i,j} r_t\{c^{\bar{X}}_{tij}\bar{X}_{tij} + c^{X}_{tij}X_{tij}\}$$

$$s_{ti} + \sum_{j\in N} X_{tji} - d_{ti} - \sum_{j\in N} X_{tij} = 0\;(\phi_{it})$$

$$\sum_{\tau=0}^{t-1} \bar{X}_{\tau ij} - X_{tij} \geq 0\;(\lambda_{tij} \geq 0)$$

$$0 \leq \lambda_{tij} \perp \sum_{\tau<t} \bar{X}_{\tau ij} - X_{tij} \geq 0$$

$$\phi_{it}\ f.i.s,$$

$$s_{ti} + \sum_{j\in N} X_{tji} - d_{ti} - \sum_{j\in N} X_{tij} = 0$$

$$0 \leq X_{tij} \perp r_t c^{X}_{tij} + \lambda_{tij} + \phi_{ti} - \phi_{tj} \geq 0$$

$$0 \leq \bar{X}_{tij} \perp r_t c^{\bar{X}}_{tij} - \sum_{\tau>t} \lambda_{\tau ij} \geq 0$$

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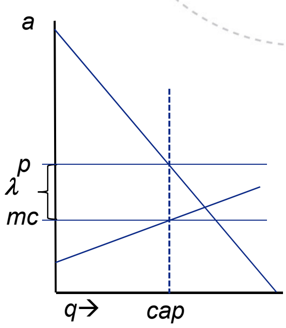
Investment: insight

$$0 \leq \bar{X}_{tij} \perp r_t c^{\bar{X}}_{tij} - \sum_{\tau>t} \lambda_{\tau ij} \geq 0$$

$$\bar{X}_{tij} > 0 \Rightarrow r_t c^{\bar{X}}_{tij} = \sum_{\tau>t} \lambda_{\tau ij}$$

- Shadow price λ_{tij} is the value of scarce capacity, and is a mark-up on the marginal cost; a profit margin.
- Investment costs = future profits: NPV=0!



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STOCHASTIC EQUILIBRIUM

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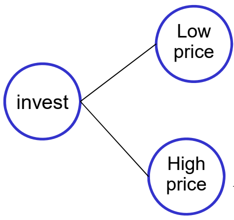
Two-stage stochastic investment: optimization

- Scenarios S={L,H}
- First stage: capacity investment
- Second stage: low or high price outcome (*scenarios*), decide what amount to supply
- Supplier problem: discounted, probably-weighted profit in each scenario, s.t. capacity limit is respected



$p_L = a_L - b_L q_L$

$p_H = a_H - b_H q_H$

$$\max_{\bar{q}, q_S} \quad r\begin{pmatrix} \Pr_L\left(p_L - c^P\right) q_L + \\ \Pr_H\left(p_H - c^P\right) q_H \end{pmatrix} - c^{inv}\,\bar{q}$$

$$s.t. \quad q_s \le \bar{q} \quad (\lambda_s \ge 0), \;\; s = L, H$$

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Two-stage stochastic investment: complementarity problem

$$\max_{\bar{q},q_s} r \begin{pmatrix} \Pr_L(p_L - c^P)q_L + \\ \Pr_H(p_H - c^P)q_H \end{pmatrix} - c^{inv}\,\bar{q}$$
$$s.t.\, q_s \le \bar{q}(\lambda_s \ge 0),\;\; s = L, H$$

$$\min_{\bar{q},q_s} c^{inv}\,\bar{q} - r\sum_s \{\Pr_s(p_s - c^P)q_s\}$$
$$s.t.\, \bar{q} - q_s \ge 0(\lambda_s \ge 0)$$

$$0 \le \lambda_s \perp \bar{q} - q_s \ge 0$$

$$0 \le q_s \perp rPr_s(c^P - p_s) + \lambda_s \ge 0$$

$$0 \le \bar{q} \perp c^{inv} - \lambda_L - \lambda_H \ge 0$$

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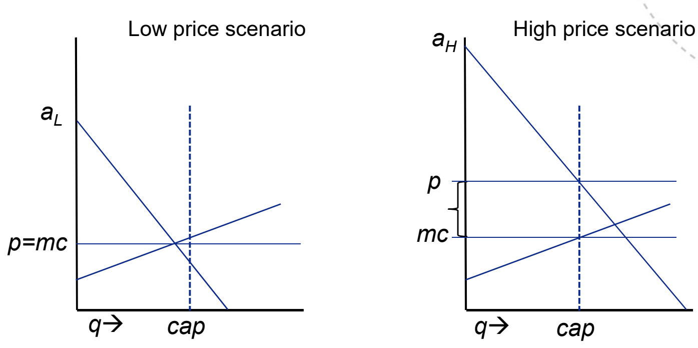
Two-stage stochastic investment: insight 1



Typically: capacity not binding in the low demand / low price scenario.

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Stochastic investment: insight

$$0 \le \lambda_s \perp \bar{q} - q_s \ge 0$$

$$0 \le q_s \perp rPr_s(c^P - p_s) + \lambda_s \ge 0$$

$$0 \le \bar{q} \perp c^{inv} - \lambda_L - \lambda_H \ge 0$$

Previous slide: capacity not binding in the low price scenario: hence only binding in the high price scenario

$$\Longrightarrow \lambda_L = 0$$
$$\Longrightarrow q_H = \bar{q}$$

GAMS

$$q_L > 0 \Longrightarrow p_L = c^P$$
$$\bar{q} > 0 \Longrightarrow c^{inv} = \lambda_H$$
$$q_H > 0 \Longrightarrow rPr_H(c^P - p_H) + \lambda_H = 0 \Longrightarrow$$
$$rPr_H(c^P - p_H) + c^{inv} = 0 \Longrightarrow$$
$$p_H = c^P + \frac{c^{inv}}{rPr_H}$$

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MIXED SUPPLIER BEHAVIOR

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Conjectural variation: mixing competitive behavior in one model

Cournot: $0 \le q_{in}^S \perp \varphi_{in}^N - \left(a_n - b_n \sum_j q_{jn} \quad -b_n q_{in} \right) \ge 0$

$0 \le q_{in}^S \perp \varphi_{in}^N - \left(\pi_n \quad -b_n q_{in} \right) \ge 0$

Bertrand: $0 \le q_{in}^S \perp \varphi_{in}^N - \left(\pi_n \right) \ge 0$

Conjectural Variation:

$0 \le q_{in}^S \perp \varphi_{in}^N - \left(\pi_n \quad -cv_n b_n q_{in} \right) \ge 0$

$0 \le cv_n \le 1$, allows for pure Bertrand and pure Cournot, mixes of competition types, but also *hybrid* forms

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STORAGE

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Storage

- Subperiods within larger periods (summer/winter, day/night) with significantly different demand levels (or production, e.g., solar)
- Storage allows carrying over goods between subperiods, often at a cost and / or losses.
  - Pumped hydro, battery, natural gas storage
- Capacity restrictions
  - Additions/Injections, withdrawals/extractions, inventory/stock
- Supplier can rent storage services
- Storage System Operator (SSO) rents out services

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Storage losses

- Losses
  - when storing (adding to storage)
  - when stored (being in storage)
  - when taking from storage
- Pumped Hydro:
  - power needed to pump up water
  - evaporation
- Seasonal storage in district heating:
  - Heat exchangers both for adding and withdrawing heat
  - Heat losses on stored heat

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Inventory accounting

Inventory at end of current sub period is inventory at end of previous sub period s_{t-1}^S *plus current sub period additions* s_t^{-S} *minus current sub period withdrawals* s_t^{+S}.

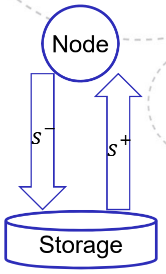
$$s_t^S = s_{t-1}^S + s_t^{-S} - s_t^{+S}$$

Considering losses and sub period lengths:

$$s_t^S = (1-l^S)s_{t-1}^S + (1-l^{-S})l_t s_t^{-S} - (1-l^{+S})l_t s_t^{+S}$$

Ignoring order of seasons – assume additions come after withdrawals: SSO inventory restriction (+ storage cycle balance individual suppliers – see next)

$$(1-l^{-S})\sum_t l_t s_t^{-S} \le cap^S$$



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Storage operator: SSO

$$\max \sum_d l_d\{\tau_d^{-S}s_d^{-S} + \tau_d^{+S}s_d^{+S} + \tau_d^S s_d^S\}$$

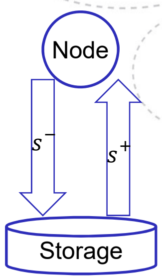
s.t.

$$s_d^{-S} \le cap_d^{-S}\left(\lambda_d^{-S} \ge 0\right) \quad \textit{Addition}$$

$$s_d^{+S} \le cap_d^{+S}\left(\lambda_d^{+S} \ge 0\right) \quad \textit{Withdrawal}$$

$$s_d^S \le cap^S \quad \left(\lambda_d^S \ge 0\right) \quad \textit{Inventory}$$

- $s_d^{+/-}$: matching suppliers' perspective
  - adding or removing commodity



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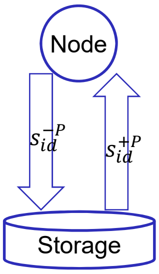
Oligopoly on a single node with storage

Supplier i:

$$\max_{q^P_{id},\, q^S_{id},\, s^{+P}_{id},\, s^{-P}_{id}} \sum_d l_d \left\{ \begin{array}{l} p_d\left(\sum_i q^S_{id}\right) q^S_{id} - c^P_i\left(q^P_{id}\right) - \\ \left(\left(c^{+S}_d + \tau^{+S}_d\right) s^{+P}_{id}\right) - \left(\left(c^{-S}_d + \tau^{-S}_d\right) s^{-P}_{id}\right) \end{array} \right\}$$

$$s.t. \quad q^P_{id} + s^{+P}_{id} = q^S_{id} + s^{-P}_{id} \quad \left(\varphi^N_{id}\ f.i.s.\right),\ d \in D$$

$$\sum_d l_d s^{+P}_{id} = \sum_d l_d s^{-P}_{id} \quad \left(\varphi^S_{id}\ f.i.s.\right)$$



- *Terms & notation*
- l_d: *if seasons have different lengths*
- $s_{id}^{+/-}$: + adds to the availability at the node / – reduces availability - from suppliers' perspective

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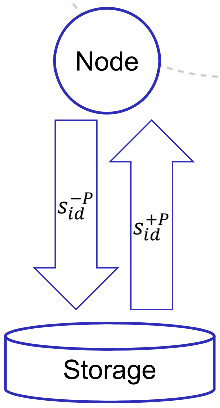
Oligopoly on a single node with storage

$$q^P_{id} + s^{+P}_{id} - q^S_{id} - s^{-P}_{id} = 0 \ \left(\varphi^N_{id}\right)$$

$$\sum_d l_d s^{-P}_{id} - \sum_d l_d s^{+P}_{id} = 0 \ \left(\varphi^S_{i}\right)$$

Storage cycle balance:

- *sources – sinks >= 0:*
  injections – withdrawals = 0
- *opposing signs compared to nodal mass balance*



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MCC storage services

- M.C.C. injection services

$$s_d^{-S} = \sum_i s_{id}^{-S} \left(\tau_d^{-S}\, f.i.s\right), d \in D$$

- M.C.C. extraction services:

$$s_d^{+S} = \sum_i s_{id}^{+S} \left(\tau_d^{+S}\, f.i.s\right), d \in D$$

- M.C.C. inventory services – accounting for season order:

$$s_d^{S} = \sum_{i,\delta \le d} l_\delta\left(s_{i\delta}^{-S} - s_{i\delta}^{+S}\right) \left(\tau_d^{S}\, f.i.s\right), d \in D$$

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Deriving the complementarity problem

- Suppliers' KKT conditions *extraction / withdrawal*

$$0 \le s_{id}^{+S} \perp l_d\left(c_{id}^{+S} + \tau_{id}^{+S}\right) + \varphi_{id}^{N} - l_d \varphi_i^{S} \ge 0$$

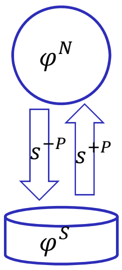
- injection / addition

$$0 \le s_{id}^{-S} \perp l_d\left(c_{id}^{-S} + \tau_{id}^{-S}\right) - \varphi_{id}^{N} + l_d \varphi_i^{S} \ge 0$$

- Insight:

$$s_{id}^{+S} > 0 \Rightarrow c_{id}^{+S} + \tau_{id}^{+S} + \frac{\varphi_{id}^{N}}{l_d} = \varphi_i^{S}$$

Addition if value stored unit φ_i^S makes up for current market value $\frac{\varphi_{id}^N}{l_d}$ plus price to add a unit $c_{id}^{+S} + \tau_{id}^{+S}$



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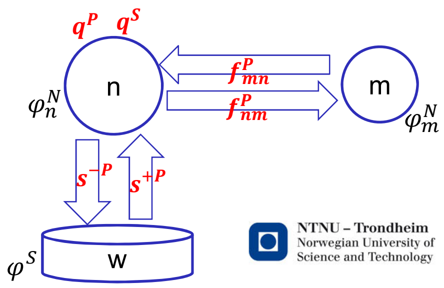
Nodal mass balance with storages and a network

$$q^P_{ind} + \sum_m f^P_{inmd} + \sum_{w,d} s^{+P}_{inwd} - q^S_{ind} - \sum_m f^P_{imnd} - \sum_{wd} s^{-P}_{inwd} = 0$$

$$\left(\varphi^N_{ind}\ f.i.s.\right), i \in I, n \in N, d \in D$$

$$\sum_d l_d s^{-P}_{inwd} - \sum_d l_d s^{+P}_{inwd} = 0 \ \left(\varphi^S_{iw}\ f.i.s.\right), i \in I, n \in N, w \in W$$

Suppliers I
Nodes N
Seasons D
Storage types W



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Multi-year nodal mass balance with storages and a network

$$q^P_{inyd} + \sum_m f^P_{inmyd} + \sum_w s^{+P}_{inwyd} - q^S_{inyd} - \sum_m f^P_{inmyd} - \sum_w s^{-P}_{inwyd} = 0$$

$$(\varphi^N_{inyd}\, f.i.s.),\ i \in I, n \in N, y \in Y, d \in D$$

$$\sum_d l_d s^{-P}_{inwyd} - \sum_d l_d s^{+P}_{inwyd} = 0\ (\varphi^S_{iwy}\, f.i.s.),\ i \in I, n \in N, w \in W, y \in Y$$

Suppliers I
Nodes N
Seasons D
Storage types W
Years Y

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Multi-year nodal mass balance with storages and a network

$$q_{inyd}^{P} + \sum_{a \in A^{+}(n)} f_{iayd}^{P} + \sum_{w} s_{inwyd}^{+P} - q_{inyd}^{S} - \sum_{a \in A^{-}(n)} f_{iayd}^{P} - \sum_{w} s_{inwyd}^{-P} = 0$$

$$\left(\varphi_{inyd}^{N}\ f.i.s.\right), i \in I, n \in N, y \in Y, d \in D$$

$$\sum_{d} l_d s_{inwyd}^{-P} - \sum_{d} l_d s_{inwyd}^{+P} = 0 \ \left(\varphi_{iwyd}^{S}\ f.i.s.\right), i \in I, n \in N, w \in W, y \in Y$$

Suppliers I
Nodes N
Seasons D
Storage types W
Years Y
Arcs A. Incoming arcs $A^{+}(n)$, Outward $A^{-}(n)$

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Master project topics

- Planning and operating smart, sustainable power and energy systems; renewable energy, smart grids, Value chains for bio-energy and bio-fuels, hydrogen, flexibility services
- Investment under uncertainty in Energy Markets – stochastic programming
- Emergency preparedness and response planning for infectious diseases in developing countries
- Analysis of external effects and policies in energy markets with complex market structures.

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Objectives

- Understand why and when equilibrium models are useful
- Able to develop and solve small-scale equilibrium problems
- Able to interpret and to have an intuition for equations and results

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Sources

- Bazaraa Sherali Shetty 1993. *Nonlinear programming theory and algorithms.*
- Cottle Pang Stone 1992. *The Linear Complementarity Problem.*
- Gabriel et al. 2013. *Complementarity Modeling in Energy markets.*
- Hillier and Lieberman 2014. *Introduction to Operations Research.*
- Korpås ET8208 - Power Market Theory - PTDFs
- Lundgren Rönnqvist Värbrand 2010. *Optimization*
- Nash and Sofer 1996. *Linear and nonlinear programming.*

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