NTNU
Department of Industrial Economics and Technology Management
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TIØ4285 Production and Network Economics
Assignment 1 – Proposed Solution

Out: Thursday 16 January
In: Thursday 23 January 6pm
Supervision: Monday 20 January, 4:15pm A31

Note that late answers will not be approved.

Exercise 1

Three brothers – Jim, Jack, and Johnnie – have each inherited a whisky distillery. They are experts on distilling whisky, but know nothing about the whisky market. They therefore signed an agreement with long-time family friend Glen who is running a blending and bottling facility.

Glen is using all of the whisky produced by Jim, Jack, and Johnnie, blends and bottles it and sells it to the customers. For his signature product, he uses 30% of Jim's whisky, 25% of Jack's whisky, and 45% of Johnnie's.

The three brothers and Glen want to expand their business and sell their product to Wales. For this expansion, they need to set up a new distribution center. Help them decide which distribution center to open. Cost and demand information is given in spreadsheet `exercise01.xlsx`.

a) Formulate the supply chain design problem as a mathematical programming problem. Remember to explain your formulation.

Let us introduce the following notation: We use the following notation:

Sets

$\mathcal{J}$	Set of distilleries.
$\mathcal{B}$	Set of bottling facilities.
$\mathcal{D}$	Set of possible locations for distribution centers.
$\mathcal{C}$	Set of customer locations.

Indices

k, n, m	location indices.

Parameters

D_m	Demand of customer m.
F_m	Fixed costs of opening a distribution center at location m.
S_m	Share of whisky produced at m used in the final blend.
T_{mn}	Transportation cost between location m and location n.

Decision variables

x_{mn}	Amount of whisky transported from location m to location n.
y_m	1 if DC opens at location m, 0 otherwise.

With that notation we can set up the following model:

$$\min \sum_{m \in \mathcal{D}} F_m y_m + \sum_{m \in \mathcal{J}} \sum_{n \in \mathcal{B}} T_{mn} x_{mn} + \sum_{m \in \mathcal{B}} \sum_{n \in \mathcal{D}} T_{mn} x_{mn} + \sum_{m \in \mathcal{D}} \sum_{n \in \mathcal{C}} T_{mn} x_{mn} \qquad (1)$$

subject to

$$
\begin{aligned}
\sum_{k\in\mathcal{J}} x_{km} - \sum_{n\in\mathcal{D}} x_{mn} &= 0 \qquad m \in \mathcal{B}, && (2)\\
\sum_{k\in\mathcal{B}} x_{km} - \sum_{n\in\mathcal{C}} x_{mn} &= 0 \qquad m \in \mathcal{D}, && (3)\\
\sum_{k\in\mathcal{D}} x_{km} &= D_m \qquad m \in \mathcal{C}, && (4)\\
x_{km} - S_k \sum_{n\in\mathcal{J}} x_{nm} &= 0 \qquad k \in \mathcal{J}, m \in \mathcal{B}, && (5)\\
\sum_{k\in\mathcal{B}} x_k m &\leq y_m \sum_{n\in\mathcal{C}} D_n \qquad m \in \mathcal{D}, && (6)\\
x_{mn} &\geq 0, \qquad m \in \mathcal{J}\cup\mathcal{B}\cup\mathcal{D}, n \in \mathcal{B}\cup\mathcal{D}\cup\mathcal{C}, && (7)\\
y_m &\in \{0,1\} \qquad m \in \mathcal{D}. && (8)
\end{aligned}
$$

The objective function (1) is the sum of the fixed facility costs for opening a distribution center and the transportation costs between the different levels of the supply chain. Constraints (2) and (3) are mass balance constraints for the bottling facility and the distribution centers, repesctively. Equation (4) ensures that all customer demand is satisfied. Constraints (5) describe the blending of whisky. The amount of a particluar type whisky used at the blending facility has to be equal to a particular share of the total amount of whisky used there. Constraint (6) links the flow of whisky into a distribution center to the decision whether or not it is opened. We use the sum of all demands as *'Big M'*, a number large enough not to limit the flow of whisky. The final constraints (7) and (8) are the non-negativity and binary requirements for the decision variables.

b) Implement and solve the problem using commercial software.

Please see file `exercise01_solution.xlsx` for the implemented model and the solution.

c) Discuss the advantages and disadvantages of a centralized distribution system vs. a decentralized system. When would you prefer a centralized solution over a decentralized one and vice versa?

Topics that can be discussed here include (but are not limited to):

- Risk pooling
- Safety stocks
- Correlated vs. uncorrelated demands
- Customer closeness
- Warehousing costs (both facility and product)
- ...

The above points can also be used to illustrate when a centralized solution is preferable (e. g. in a situation with negatively correlated or uncorrelated demands) or a decentralized solution is the chosen one (e. g. cheap facility costs plus a requirement to be close to the customers).

Exercise 2

a) Formulate the Newsboy Problem as a two-stage stochastic programming problem.

Let us introduce the following notation: We use the following notation:

Sets

$\mathcal{S}$	Set of demand scenarios.

Indices

s	Scenario index.

Parameters

D^s	Customer demand in scenario s.
R	Revenue of a newspaper.
W	Wholesale price of a newspaper.
S	Salvage value of a newspaper.
p^s	Probability of scenario s.

Decision variables

x	Number of newspapers to buy every day.
y^s	Number of newspapers sold in scenario s (at price R).
z^s	Number of newspapers salvaged in scenario s (at salvage value S).

With that notation we can set up the following model:

$$\max -Wx + \sum_{s\in\mathcal{S}} p^s \cdot (Ry^s + Sz^s) \tag{9}$$

subject to

$$y^s \leq x \qquad s \in \mathcal{S}, \tag{10}$$

$$y^s \leq D^s \qquad s \in \mathcal{S}, \tag{11}$$

$$y^s + z^s = x \qquad s \in \mathcal{S}, \tag{12}$$

$$x \geq 0 \tag{13}$$

$$y^s, z^s \geq 0 \qquad s \in \mathcal{S}. \tag{14}$$

The objective function (9) maximizes the Newboy's expected profits. The first term represents the wholesale cost of buying newspapers. This is also the first stage cost. The second term, i. e. the sum, are the expected revenues from selling and salvaging the newspapers in each scenario.

Constraints (10) make sure that we do not sell more newspapers in a given scenario than we have available. Similarly, constraints (11) limit the number of sold newspapers, by preventing selling more than we have demand for. Equations (12) ensure that all newspapers are either sold or salvaged. Finally, constraints (13) and (14) are non-negativity constraints for the decision variables.

b) Implement the stochastic programming Newsboy formulation and solve it using the values provided in spreadsheet `exercise02.xlsx`. Assume that all scenarios are equally likely. How many newspapers should the newsboy buy and what are her expected profits?

Please see file `exercise02_solution.xlsx` for the implemented model and the solution. Optimal order quantity is $Q^* = 127$ and the newsboy's expected profits are given as $\Pi_r = 5280.6$.

c) Solve the Newsboy Problem analytically. Use the same values for R, W and S as in b), but assume that demand is uniformly distributed on the interval [50,150].

With $R = 100$, $W = 40$, and $S = 20$ we get the following underage cost $C_u = R - W = 60$ and overage cost $C_o = W - S = 20$. The Newsboy optimality criterion is thus given as

$$\Pr(D \leq Q) = \frac{C_u}{C_u + C_o} = \frac{60}{80} = 0.75.$$

Note that $\Pr(D \leq Q) = F(Q)$, where $F(\cdot)$ is the cumulative distribution function of D. The optimal order quantity can therefore be calculated as as $Q^* = F^{-1}\left(\frac{C_u}{C_u + C_o}\right)$.

The probability density for a uniform distribution is given as $f(x) = \frac{1}{b-a}$ for $x \in [a, b]$ and 0 otherwise. The cumulative distribution function (cdf) $F(x)$ is 0 for $x < a$, 1 for $x > b$ and $\frac{x-a}{b-a}$ for $x \in [a, b]$. We find the optimal order quantity by inverting the cdf for the given optimality criterion (here: 0.75):

$$
\begin{aligned}
\frac{x-a}{b-a} &= 0.75 \\
\frac{x-50}{150-50} &= 0.75 \\
x &= 125
\end{aligned}
$$

The Newsboy's expected profits are calculated using the formula given by Rudi & Pyke:

$$\Pi_r(Q^*) = (R - W)ED - E\left[(R - W)(D - Q^*)^+ + (W - S)(Q^* - D)^+\right]. \tag{15}$$

In order to calculate the cost of uncertainty, we need to calculate the expected number of salvaged newspapers and the expected shortfall, i.e. the expected number of newspapers we could have sold exceeding the bought ones. For the order quantity $Q^* = 125$, the expected number of salvaged newspapers is uniformly distributed on the interval $[0, 75]$, resulting in a conditional expectation (given that $Q > D$) of 37.5. Note that this will happen with probability $\Pr(D < Q) = 0.75$.

Similarly, the expected shortfall is uniformly distributed on the interval $[0, 25]$. The conditional expectation is 12.5, happening with probability 0.25. With this, we can calculate the Newsboy's expected profits:

$$
\begin{aligned}
\Pi_r(Q^*) &= (R - W)ED - E\left[(R - W)(D - Q^*)^+ + (W - S)(Q^* - D)^+\right] \\
&= (100 - 40) \cdot 100 - (100 - 40) \cdot 12.5 \cdot 0.25 - (40 - 20) \cdot 37.5 \cdot 0.75 \\
&= 6000 - 187.5 - 562.5 \\
&= 5250.
\end{aligned}
$$