NTNU
Department of Industrial Economics and Technology Management
Spring 2020

TIØ4285 Production and Network Economics

Assignment 5

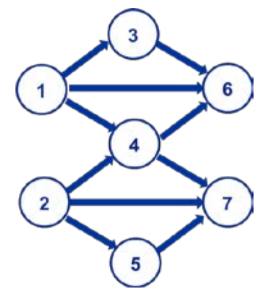
Out: Thursday 13 February
In: Thursday 20 February 6pm
Supervision: Monday 17 February 4:15pm A31

Note that late exercises will not be approved.

Submit your GAMS and XPRESS code as part of your delivery.

Exercise 1. Minimum cost flow with quadratic costs

Consider the seven node example network presented in class and included in the lecture slides, but with some modifications. Instead of a fixed supply amount from nodes 1 and 2, supply is variable with a capacity limit cap_i. The costs for production are $c_i^P(s_i) = c_i s_i + \frac{1}{2} d_i (s_i)^2$.The flow costs are $c_{ij}^F(x_{ij}) = f_{ij}(x_{ij})^2$. The values of all parameters are given in the table:



			To node:	3	4	5	6	7
From node			Dmd:				4	6
	cap_i	c_i	d_i					
1	10	1	0.25	0.2	0.5		0.8	
2	10	2	0.25		0.5	0.3		0.6
3							0.2	
4							0.3	0.2
5								0.2

a. Formulate the cost-minimizing problem to satisfy demand respecting all constraints.
b. Implement and solve the model in XPRESS as well as in GAMS.
c. What are the production levels in nodes 1 and 2 respectively? Explain why the production at one node is higher than at the other.
d. List all the positive flow values. What is the effect of using quadratic costs instead of linear (*constant per unit*) costs?
e. Formulate the complementarity problem for the cost-minimizing problem.
f. Implement and solve the complementarity problem in GAMS.
g. Explain / discuss the value of the dual variable of the mass balance at node 4.

Exercise 2. Facility location

We are given a bipartite graph consisting of ten potential locations *i* to establish a facility and 30 demand nodes *j*.

Capacities cap(i)

200, 250, 300, 350, 400, 450, 500, 550, 600, 650

Demand dmd(j)

10, 20, 30, 40, 50, 42, 49, 56, 63, 70, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90

Fixed Costs f(i)

500, 550, 600, 650, 700, 750, 800, 850, 900, 950

Transport costs c(i,j)

3,4,5,6,7,8,9,10,11,12,12,13,14,15,16,17,18,19,20,21,20,21,22,23,24,25,26,27,28,29,
5,6,7,8,9,10,11,12,13,14,13,14,15,16,17,18,19,20,21,22,19,20,21,22,23,24,25,26,27,28,
7,8,9,10,11,12,13,14,15,16,14,15,16,17,18,19,20,21,22,23,18,19,20,21,22,23,24,25,26,27,
9,10,11,12,13,14,15,16,17,18,15,16,17,18,19,20,21,22,23,24,17,18,19,20,21,22,23,24,25,26,
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13,14,15,16,17,18,19,20,21,22,17,18,19,20,21,22,23,24,25,26,15,16,17,18,19,20,21,22,23,24,
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19,20,21,22,23,24,25,26,27,28,20,21,22,23,24,25,26,27,28,29,12,13,14,15,16,17,18,19,20,21,
21,22,23,24,25,26,27,28,29,30,21,22,23,24,25,26,27,28,29,30,11,12,13,14,15,16,17,18,19,20

a. Formulate the optimization problem to meet all demand at minimal cost.
b. Implement and solve the model using XPRESS.
c. At what locations will facilities be established? How much does each produce? What is the total cost?
d. Increase the fixed costs for establishing a facility by a factor two, five and ten. Briefly discuss the logic behind the optimal number of facilities to be established for each of these three "cases".

Exercise 3. Set Coverage

We are given the same bipartite graph as in Exercise 2 with ten potential facility locations and 30 demand points. The cut-off distance so that a location covers a demand point is a parameters $cutoff$.

Formulate the Set Coverage Problem and implement it in XPRESS.

a. Solve this Set Coverage Problem for maximum coverage distances $cutoff = 25,24,23,22,21$, and 20. How many facilities have to be established in each case, and at what locations?
b. What happens for maximum coverage distances larger than 25?
c. What happens for maximum coverage distances larger than 29?
d. What happens for maximum coverage distances smaller than 21?