NTNU
Department of Industrial Economics and Technology Management

Examination paper for TIØ4285 Production & Network Economics
Solution Proposal

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Examination date: 21 May 2019
Examination time: 09:00 – 13:00
Permitted examination support material: A – All printed and handwritten support material is allowed. All calculators are allowed.

Language: English

Deadline for examination results: 12 June 2019

Some of these exercises may have alternative ways to solve them. Just because they are not mentioned here, does not mean they do not exist. Any comprehensible (and correct) way to solve the exercises is accepted, as long as the questions in the exercises are properly answered. Not using methods specified in the exercises or failing to address all questions will count as an incomplete answer.

Exercise 1 (25%)

Jürgen is selling football jerseys over the internet. He is currently preparing his order for the upcoming season and needs help with the task of determining how many to order from his supplier, Mauricio. He assumes that demand for his jerseys will be uniformly distributed between 33,000 and 158,000. Before the end of the season, the jerseys sell for 125 each, but once the season is over, he can only get 50 for each of them as the fans then eagerly wait for the following season's jerseys.

Jürgen also tells you that he has to pay 80 for each jersey to Mauricio. In addition, he has to pay 25 in shipping for each jersey sold online. Any unsold jersey will be put in Jürgen's (physical) discount store at the end of the season, where customers eventually pick it up at the reduced price. The discount store incurs fixed costs of 125,000.

a) Express Jürgen's expected profit as a function of the order quantity and demand. Explain your profit function.

Let us first introduce the following notation:

R – Revenues from selling a jersey at full price, 125.
W – Wholesale price for jerseys, 80.
S – Salvage value of a jersey at end of season, 50.
C_s – Cost of shipping a jersey sold online, 25. This cost only applies to online sales.
F_D – Fixed cost of the operating the discount store.
q – Number of jerseys Jürgen orders.
D – Realized demand (uniformly distributed between 33 and 158)

We can express Jürgen's expected profits based on the formulation proposed by Rudi & Pyke (2002) or Lariviere (1999). While the approaches to calculating expected profits are slightly different, both the results and the optimality citerion are the same.

- *According to Rudi & Pyke (2002):* We first define Jürgen's profit margin from selling regular-priced jerseys online as the underage cost, $C_u = R - C_s - W$. The cost of the jerseys that Jürgen has to sell at a discounted price is the pverage cost, $C_o = W - S$. With these definitions, we can express Jürgen's expected operational profits as

$$\pi_J(q) = C_u \cdot ED - E\left[C_u \cdot (D-q)^+ + C_o \cdot (q-D)^+\right] - F_D.$$

  The first term in the expected profits are the profits from selling expected demand online at regular prices. The second and third term correspond to the cost of uncertainty, where the second term accounts for the expected shortfall, while the third term considers the expected number of leftover jerseys at the end of the season. The last term are the fixed costs the discount store incurs.

- *According to Lariviere (1999):* Expected profits can be expressed as

$$\pi_J(q) = (R - C_s - W) \cdot q - (R - C_s - S) \cdot \int_{33}^{q} F(\xi) d\xi - F_D,$$

  where $F(\xi) = \frac{\xi - 33}{158-33}$ is the cumulative distribution function of demand. The first term in this slightly simpler expression are the profits from selling all jerseys online. The second term corrects these profits by subtracting the online profits and adding the salvage value of the expected number of jerseys that have to sold in the discount store. With the last term we account for the fixed costs of the discount store.

b) How many jerseys should Jürgen order from Mauricio? What will be his expected profits?

The optimality criterion for maximizing profits is for both expressions given as

$$\Pr(D \leq q) = \frac{R - C_s - W}{R - C_s - S} = 0.4.$$

This results in an optimal order quantity $q = 83$ and expected profits $\pi_J = 1,035$ (both in 1000).

c) Jürgen considers to close down the discount store and also sell the jerseys at reduced prices over the internet. Can Jürgen increase his expected profits by only selling over the internet?

The shipment cost C_s will in this case also apply to the jerseys sold at the end of the season, which will affect Jürgen's ordering decision and his expected profits. On the other hand, Jürgen won't incur the fixed costs of operating the discount store. Jürgen's new expected profits given all online sales are given as:

$$\pi_J(q) = (R - C_s - W) \cdot q - (R - S) \cdot \int_{33}^{q} F(\xi) d\xi,$$

where $F(\xi) = \frac{\xi - 33}{158 - 33}$ is the cumulative distribution function of demand. Note that we no longer correct for the shipping cost in the second term and that we no longer subtract the discount store's fixed cost.

The new optimality criterion is given as

$$\Pr(D \leq q) = \frac{R - C_s - W}{R - S} = 0.267.$$

This results in a new order quantity of $q = 66.3$ and expected profits of $\pi_J = 993.3$. Jürgen will be better off keeping the discount store, instead of also selling the jerseys at reduced prices online.

d) How would Jürgen's order quantity change, if he were risk-averse?

Jürgen's profits are stochastic as they depend on the realized demand. Even though we can calculate the order quantity that will maximize his expected profits, these profits are not guaranteed. The variation in his realized profits is the risk Jürgen faces. In order to reduce this risk, a risk-averse Jürgen will reduce the number of jerseys he orders: for example, at an order quantity of 33,000, Jürgen's profits will be lower, but deterministic.

Mauricio's cost of producing and selling a jersey is actually just 60. He sees an opportunity to increase profits and after having been impressed by the work you did for Jürgen, he asks for your assistance.

e) Design an optimal Buy-Back contract that is acceptable to both Jürgen and Mauricio. What are the expected profits?

Introducing the following additional notation:

M – Mauricio's cost of producing all selling a jersey, 60.
B – Buy-Back price at which Mauricio will take back unsold jerseys.

Mauricio's profits under the original agreement are given as

$$\pi_M = (W - M) \cdot q = 1,660.$$

If Jürgen and Mauricio coordinate their decisions, we need to replace W with M in the optimality criterion and get

$$\Pr(D \leq q) = \frac{R - C_s - M}{R - C_s - S} = 0.8$$

The new optimal order quantity is then $q = 133$ with optimal joint profits $\pi = 3,195$.

An optimal Buy-Back contract will distribute these profits, ensuring that Jürgen makes at least 1,035, whereas Mauricio will earn at least 1,660. Following Lariviere (1999), we introduce the parameter $\varepsilon \in (0, R - C_s - M]$ and define the following:

$$W(\varepsilon) = R - C_s - \varepsilon \text{ and } B(\varepsilon) = R - C_s - \frac{\varepsilon(R - C_s - S)}{R - C_s - M}.$$

The optimality criterion ensures that Jürgen will order the optimal amount:

$$\Pr(D \leq q) = \frac{R - C_s - W(\varepsilon)}{R - C_s - B(\varepsilon)} = \frac{R - C_s - M}{R - C_s - S} = 0.8.$$

Profits will be distributed such that

$$\pi_J = \frac{\varepsilon}{R - C_s - M}\pi' - F_D, \text{ and}$$
$$\pi_M = \left(1 - \frac{\varepsilon}{R - C_s - M}\right)\pi',$$

where $\pi' = \pi + F_D$ are the gross joint profits including the fixed costs for Jürgen's discount store. Any ε satisfying $13.98 \leq \varepsilon \leq 20$ will then lead to a Buy-Back contract that is acceptable to both Jürgen and Mauricio.

f) Which of the two agreements (the original or the Buy-Back contract) would Mauricio prefer, if he were risk-averse?

Before entering the Buy-Back contract (i.e. under the original agreement), Mauricio has deterministic profits. After entering the Buy-Back contract, his expected profits will be higher than the original deterministic profits, but they will now depend on realized demand for jerseys. A risk-averse Mauricio might therefore prefer the original agreement over the Buy-Back contract as he will not be exposed to the risk of having profits lower than the expected profits.

Exercise 2 (25%)

José and Pep are playing a series of games. As usual, they are trying to win, i.e. maximize their payoff. For the remainder of this exercise, the first number in a pair of payoffs denotes José's payoff, while the second number denotes Pep's payoff.

1. In their first game, both have 3 strategies to choose from:

		Pep		
		L	C	R
	T	$4,2$	$3,0$	$1,1$
José	M	$1,2$	$2,4$	$0,3$
	B	$1,1$	$4,2$	$2,4$

Note that – due to an unfortunate typoo – Pep's actions were denoted {L,C,M} in the original exam set. Answers referring to Pep's action "M" are of course correct, despite this solution proposal presenting the originally intended solution.

a) Find the set of pure strategy Nash equilibria.

The pure strategy NE are $(T;L)$ and $(B;R)$.

b) Find the set of mixed strategy Nash equilibria.

First, notice that M is a strictly dominated action for Player 1; therefore, it will not be played with positive probability in any Nash equilibrium, and we can eliminate it. Once we eliminate it, notice that C is also strictly dominated for Player 2 and can be eliminated. We are left with the game:

	L	R
T	$4,2$	$1,1$
B	$1,1$	$2,4$

Let p be Player 1's probability on T, and q be Player 2's probability on L. For Player 1, the expected payoffs to T and B must be the same:

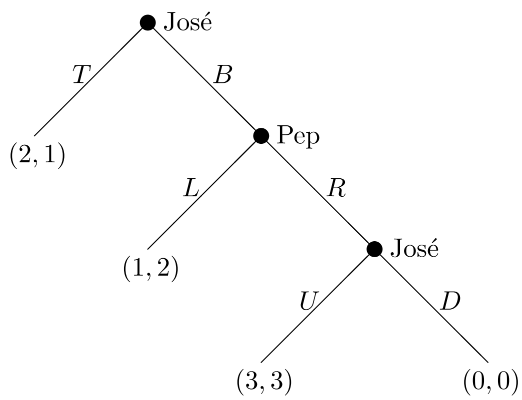
$$q \cdot 4 + (1-q) \cdot 1 = q \cdot 1 + (1-q) \cdot 2,$$

which gives $q = 1/4$. For Player 2, the expected payoffs to L and R must be the same:

$$p \cdot 2 + (1-p) \cdot 1 = p \cdot 1 + (1-p) \cdot 4,$$

which gives $p = 3/4$. The only non-pure strategy Nash equilibrium is $((3/4; 1/4); (1/4; 3/4))$.

2. Their second game is given in extensive form below. The name next to the node indicates which of the two players moves.



c) Apply backward induction.

B, R, U

d) Write the game in strategic form.

		Pep L	Pep R
José	TU	$2,1$	$2,1$
	TD	$2,1$	$2,1$
	BU	$1,2$	$3,3$
	BD	$1,2$	$0,0$

e) Find all pure strategy Nash equilibria. Which ones are sub-game perfect?

All pure NE: $(TU; L)$, $(TD; L)$, $(BU; R)$. Only the last is sub-game perfect.

f) Is there a pure Nash equilibrium which Pareto-dominates the other pure Nash equilibria?

Yes, $(BU; R)$.

3. Arsène joins José and Pep for a third game and each of the three needs to decide whether he should start participate or not. The rules for the third game are the following: a player that does not play has a payoff of 0. A player that chooses to play has to pay a cost of 62 up front in order to participate, and will receive a payoff equal to $\frac{150}{n}$, where n is the number of players that choose to participate.

g) Find the set of pure strategy Nash equilibria.

Let the action set be {*Join*; *Not Join*}. If all three players choose *Not Join*, they each get a payoff of 0. If exactly one player chooses *Join*, he gets a payoff of 88, while the other two get 0. If exactly two players choose *Join*, they both get a payoff of 13, while the other player gets 0. If all three players choose *Join*, they all get a payoff of -12.

- Suppose all three players choose *Not Join*. Any player can increase its payoff from 0 to 88 by switching to em Join, so this is not a NE.
- Suppose exactly one player chooses *Join*. One of the players that chose *Not Join* can increase his payoff from 0 to 13 by switching to *Join*, so this is not a NE.
- Suppose exactly two players choose *Join*. The remaining player that chose *Not Join* will decrease his payoff from 0 to -12 by switching to *Join*. Each player that chose *Join* will decrease its payoff from 13 to 0 by switching to *Not Join*. Therefore, this is a NE.
- Suppose all three players choose *Join*. Each player can increase his payoff from -12 to 0 by switching to *Not Join*, so this is not a NE.

Therefore, there are three pure strategy NE, where player i = 1; 2; 3 chooses *Not Join*, and the remaining two players choose *Join*.

h) Find the symmetric mixed strategy Nash equilibrium, where all three player join the game with the same probability.

Let p be the probability of choosing *Join*. Suppose we are player 1; the probability that $n = 1$ (if both other players choose *Not Join*) is $(1-p)^2$. The probability that $n = 2$ (if exactly one other player chooses *Join*) is $2p(1-p)$. The probability that $n = 3$ (if both other players choose *Join*) is p^2. The expected payoffs to *Join* and *Not Join* must be the same:

$$EV(Join) = (1-p)^2 \cdot 88 + 2p(1-p) \cdot 13 + p^2 \cdot (-12) \tag{1}$$

$$EV(Not\ Join) = 0 \tag{2}$$

$$EV(Join) = EV(Not\ Join) \text{ when } p = \frac{4}{5}. \tag{3}$$

Exercise 3 (10%)

Thomas sells footballs. The demand for footballs is given as 7,300 boxes per year (365 days). It costs Thomas 40 per order of footballs, and it costs 2.34 per box per year to keep the balls in stock. Once Thomas has placed an order for footballs, it takes 4 days to receive the order from the wholesaler.

a) What is the cost-minimizing order quantity?

We use the Economic Order Quantity (EOQ) to calculate the optimal order quantity. The optimal order quatity q is given as

$$q = \sqrt{\frac{2KD}{h}},$$

where K is the fixed order cost, D is the demand, and h is the inventory holding cost. We can either use the annual data provided in the exercise text or convert demand and inventory holding into daily data. The optimal order quantity will be the same either way.

D – Demand, 7300 (annual) or 20 (daily).
h – Inventory holding cost, 2.34 (annual) or 0.0064 (daily).
K – Fixed order cost,40.

With the formula given above, we determine the optimal order quantity as $q = 500$.

b) How high are the minimum total annual costs (order plus inventory holding costs)?

One order lasts for 25 days. Therefore, we have to place 14.6 orders every year to cover annual demand. Minimum annual total costs are then

$$TC = 14.6 \cdot K + \frac{hq}{2} = 1169.$$

c) At which inventory level does Thomas have to place the next order, i.e. what is the reorder point?

Average daily demand is 20 boxes of footballs. With a delivery lead time of 4 days, Thomas needs to place his next order when he has 80 boxes left in storage.

Exercise 4 (20%)

Ernesto owns a company that produces football shoes. The company consists of 2 divisions: an upstream division that prepares and cuts the leather, and a downstream division that manufactures and sells the shoes. Demand for Ernesto's football shoes is given as

$$p_S = 1,700 - 0.35q_S,$$

where q_S is the number of pairs of shoes sold and p_S is the price for each pair of shoes.

The cost function for preparing and cutting the leather in the upstream division is

$$C_L = 0.65q_L^2 + 150q_L + 75,000,$$

where q_L is the quantity of leather prepared and cut in m^2. The downstream division needs 1m^2 of leather per pair of shoes. Excluding leather, the cost function for manufacturing a pair of football shoes is

$$C_S = 200q_S.$$

a) What is the optimal transfer price for leather? How many pairs of shoes does Ernesto produce?

Ernesto will maximize his profits when $MC = MR$. His marginal costs are given as $MC = MC_L + MC_S = 1.3q + 350$. The marginal revenues are $MR = 1,700 - 0.7q$. Ernesto will therefore maximize his profits when producing $q_S = 675$ pairs of football shoes. The optimal transfer price for leather, w_L, is equal to the marginal cost of producing leather and therefore given as $w_L = 1027.5$.

b) Suppose that there is a competitive market for prepared and cut leather where leather can be bought and sold for $p_L = 800$ per m^2.

How much leather should Ernesto's company prepare and cut? How much leather does the downstream division use to produce shoes and what is the optimal transfer price?

Ernesto will produce leather until his marginal cost of production is equal to market price of leather, i.e. $MC_L = 1.3q_L = 150 = 800$. Ernesto therefore chooses to produce $q_L = 500$. As the marginal cost of producing leather is capped at 800, the marginal cost of of producing football shoes is given as $MC = 1000$. With these marginal costs, it is optimal to produce $q_S = 1000$ pairs of football shoes.

The downstream division thus uses a total of $1{,}000\text{m}^2$ of leather, of which 500 are produced by the upstream division and 500 are bought from the market. The optimal transfer price is given as the market price for leather, i.e. $w_L = 800$.

c) Assume now that the leather that the upstream division of Ernesto's company produces is unique and of extremely high quality. The division can therefore act both as a supplier to the downstream division and as a monopoly supplier to an outside market. Demand in the outside market for Ernesto's leather is given as $p_L^{out} = 1,380 - 0.125q_L^{out}$, where q_L^{out} is the number of m^2 sold in the outside market and p_L^{out} is the price per m^2 in the outside market.

What is the optimal transfer price paid by the downstream division? At what price, if any, should leather be sold in the outside market?

For these new market conditions, we can express Ernesto's profit function as

$$\begin{aligned}\pi_E = {} & (1,700 - 0.35q_S) \cdot q_S + \left(1,380 - 0.125q_L^{out}\right) \cdot q_L^{out} - 200 \cdot q_S \\ & - 0.65 \cdot \left(q_S + q_L^{out}\right)^2 - 150 \cdot \left(q_S + q_L^{out}\right) - 75,000,\end{aligned}$$

where the first and second term are the revenues from selling shoes and leather in the outside market, respectively. The third term are the cost from manufacturing shoes, while the last line represents the total costs of producing leather, both for further processing and sales in the outside market.

We derive the profit function with respect to q_S and q_L^{out} and solve the resulting set of equations:

$$\begin{aligned}\frac{\partial \pi_E}{\partial q_S} &= 1,700 - 0.7q_S - 200 - 1.3q_S - 1.3q_L^{out} - 150\\ &= 1,350 - 2q_S - 1.3q_L^{out} = 0\\ \frac{\partial \pi_E}{\partial q_L^{out}} &= 1,380 - 0.25q_L^{out} - 1.3q_L^{out} - 1.3q_S - 150\\ &= 1,230 - 1.55q_L^{out} - 1.3q_S = 0\end{aligned}$$

Solving these two equations results in $q_S = 350$ and $q_L^{out} = 500$ (minor differences due to rounding may occur). The optimal transfer price for leather used in the production of shoes is $w_L = 1,255$. Leather is sold in the outside market at $p_L^{out} = 1317,5$.

An alternative way of solving the problem is by horizontally adding the marginal revenue curves (from selling to the downstream division and from selling to the outside market) and setting the resulting marginal revenues equal to marginal cost of producing leather. The net marginal revenues of producing leather are given as

$$NMR_L = MR - MC_S = 1,500 - 0,7q_s.$$

The marginal revenues from selling leather to the outside market are given as

$$MR_L^{out} = 1,380 - 0.25q_L^{out}.$$

For the first 171.43m^2 of leather, the marginal revenues of producing shoes are higher than the marginal revenues from selling the leather in the outside market. For $q_L > 171.43$ we have:

$$\begin{aligned} NMR_L &= 1,500 - 0,7q_S &\Rightarrow&& q_S &= \frac{15000}{7} - \frac{10}{7}NMR_L\\ MR_L^{out} &= 1,380 - 0.25q_L^{out} &\Rightarrow&& q_L^{out} &= 5,520 - 4MR_L^{out}\end{aligned}$$

This results in the following joint marginal revenues:

$$q = \frac{53640}{7} - \frac{38}{7}MR \qquad \Rightarrow \qquad MR_{joint} = 1,411.58 - \frac{7}{38}q$$

Setting $MR_{joint} = MC$, we get $q = 850$. The marginal costs of producing 850m^2 of leather are 1,255. Using $MR = 1,255 + MC_S$ and $MR_L^{out} = 1,255$, we get $q_S = 350$ and $q_L^{out} = 500$, respectively (and the same transfer/market prices as above).

Exercise 5 (20%)

In this exercise, you are given three different discussion questions. Answer each question short and concise. If you think the questions are imprecise or vague, state necessary assumptions and, in each case, discuss the relevant aspects of the imprecise formulation(s).

a) Consider a game with a finite number of players and a finite number of actions. Will there always be a pure strategy Nash equilibrium?

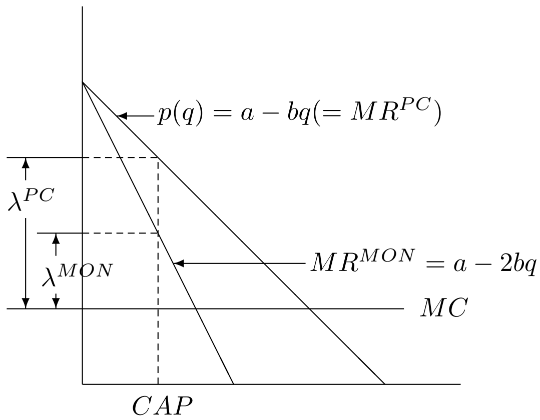
No. For example, Matching Pennies has no pure strategy Nash equilibrium. It has been proven, however, that a mixed strategy Nash equilibrium always exists in a game with finite players and actions.

b) We are given two nodes, 1 and 2, connected by a pipeline. Production is located at node 1. Demand at node 2. Demand is given by an inverse demand curve, $p(q) = a - bq$. The pipeline has a capacity limit. Compare the following situations: we have located at node 1 either (A) a monopoly supplier or (B) many perfectly competitive suppliers.

Assume that marginal production cost are constant and the same in both situations (A) and (B), and that in both situations, (A) and (B), the pipeline capacity restricts the supply. In which of the two situations, (A) or (B), is the shadow price on the pipeline capacity the highest?

In situation (B), the shadow price is highest.

The shadow price on capacity indicates the marginal value of capacity: "By how much would the objective function value change if there would be an additional unit of capacity available." The marginal change of an objective function value is given by the marginal revenues minus the marginal costs. The marginal costs are the same in both situations. The marginal revenue, however, is lower for a monopolist compared to that of perfectly competitive suppliers. Compare in the Figure below.



c) If the value of an item that is auctioned off is not known with certainty, the bidders can be exposed to what is known as "Winner's Curse", i.e. the winner has to pay more than the true value of the item. Is the probability of being exposed to the Winner's Curse greater in an English auction or in a first-price, sealed bid auction? Explain.

The main difference between a first-price, sealed-bid auction and an English auction is the auction format. In a first-price, sealed-bid auction, we can only submit 1 bid and will have no chance to submit a new bid (or change the existing one), before the auction is over. In an English auction, there will be successive bidding rounds, which allows us to observe the other bidders' valuation of the item for sale.

We can now consider two cases:

(a) *Only 1 bidder overestimates the item's value:* In an English auction, the bidder overestimating the item's value has to chance to adjust his valuation during the auction, based on the bid's submitted by the other bidders. This reduces the risk of being exposed to the "Winner's Curse". In addition, even when not adjusting his own valuation, he will only pay slightly above the second highest valuation and as such reduce the extent of the "Winner's Curse".

(b) *All bidders overestimate the item's value:* If all bidders overestimate the value of the item equally, we cannot extract any information from their bidding behaviour and it is equally likely in both auctions to be exposed to the "Winner's Curse".