Lecture 1: Planning Problems and Supply Chain Design

TIØ4285 Production and Network Economics

Spring 2021

Note that the lecture is recorded and streamed through Panopto.

Norwegian University of Science and Technology

Outline

- Production and Network Economics vs. Industrial Optimization and Optimization Methods
- Hierarchical Planning
- Supply Chain Design
- Dealing with Uncertainty

Production and Network Economics

- Focus on economical analysis and decision making in single companies and networks

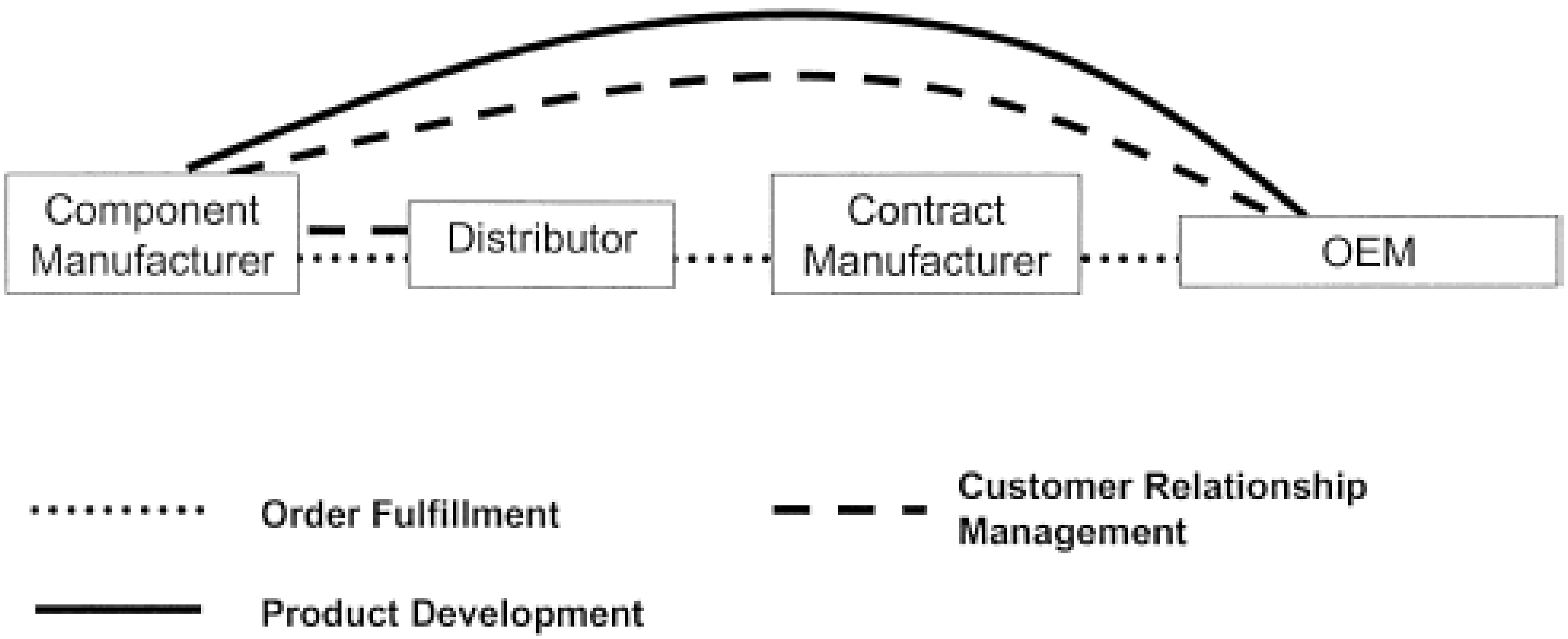
- Strongly related to supply chain management, but with focus on the economic issues

- We will address mainly coordination and control issues with a particular focus on the interfaces between companies or divisions within companies.

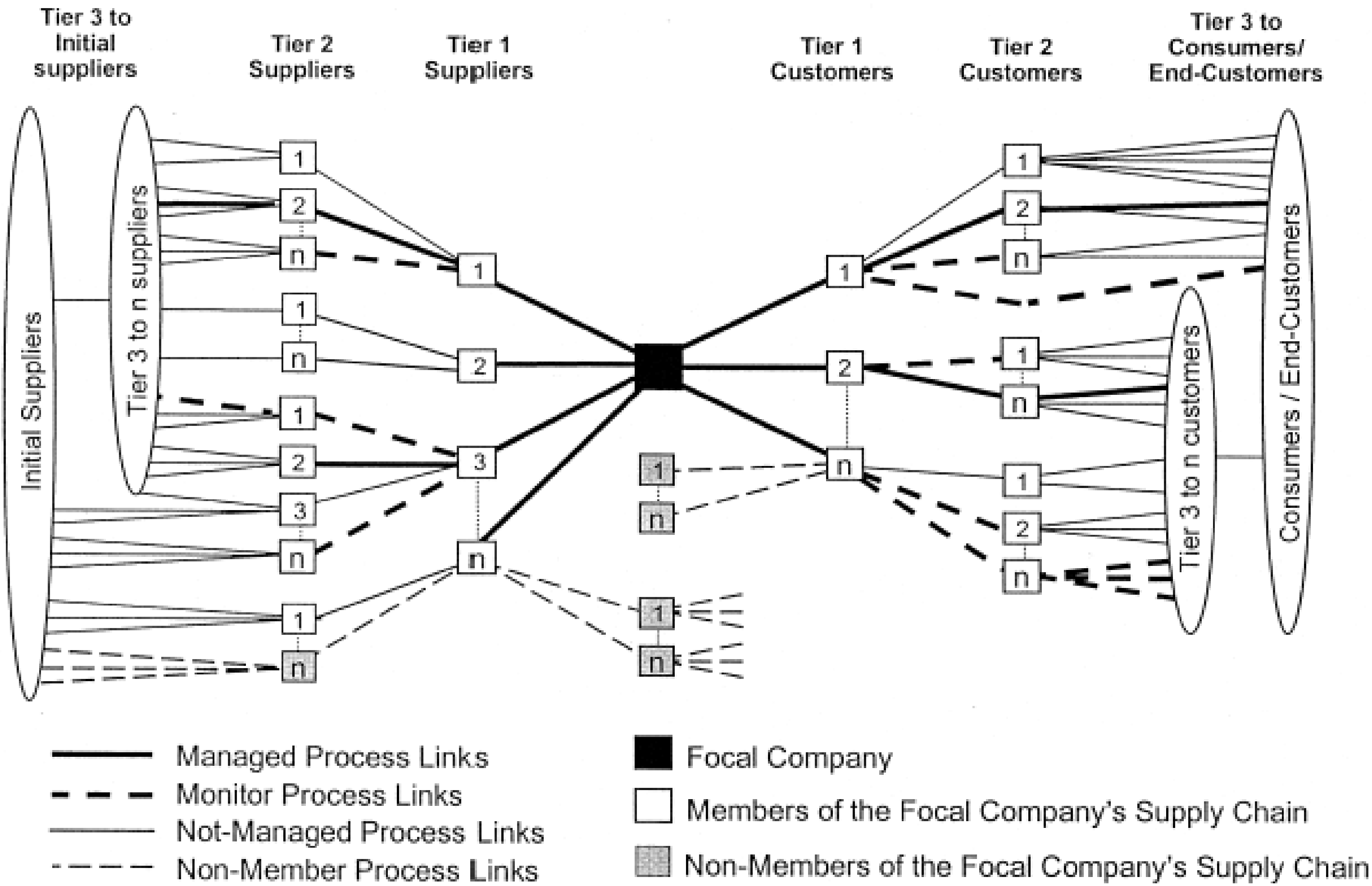
Table 1
A sample of definitions of supply chain management

Authors	Definition
Tan et al. (1998)	Supply chain management encompasses materials/supply management from the supply of basic raw materials to final product (and possible recycling and re-use). Supply chain management focuses on how firms utilise their suppliers' processes, technology and capability to enhance competitive advantage. It is a management philosophy that extends traditional intra-enterprise activities by bringing trading partners together with the common goal of optimisation and efficiency.
Berry et al. (1994)	Supply chain management aims at building trust, exchanging information on market needs, developing new products, and reducing the supplier base to a particular OEM (original equipment manufacturer) so as to release management resources for developing meaningful, long term relationship.
Jones and Riley (1985)	An integrative approach to dealing with the planning and control of the materials flow from suppliers to end-users.
Saunders (1995)	External Chain is the total chain of exchange from original source of raw material, through the various firms involved in extracting and processing raw materials, manufacturing, assembling, distributing and retailing to ultimate end customers.
Ellram (1991)	A network of firms interacting to deliver product or service to the end customer, linking flows from raw material supply to final delivery.
Christopher (1992)	Network of organisations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the hands of the ultimate consumer.
Lee and Billington (1992)	Networks of manufacturing and distribution sites that procure raw materials, transform them into intermediate and finished products, and distribute the finished products to customers.
Kopczak (1997)	The set of entities, including suppliers, logistics services providers, manufacturers, distributors and resellers, through which materials, products and information flow.
Lee and Ng (1997)	A network of entities that starts with the suppliers' supplier and ends with the customers' custom the production and delivery of goods and services.

Theory: Chain of Actors



Practice: Network of Actors

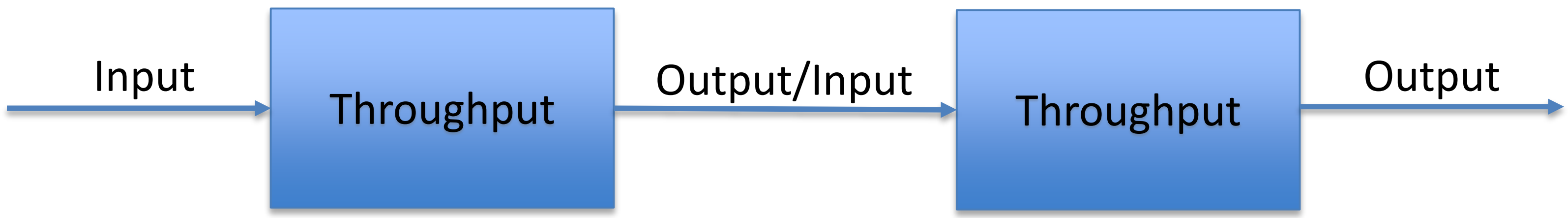


Optimization Perspective

- Usually either maximize output or minimize input (given the other)

Extending the System

- Let's extend the system under consideration (make it more of a network)



- What changes from an optimization point of view?
  - More variables
  - More constraints
  - Not that much else…
- Who is making decisions?
  - Still a single centralized decision maker

What happens if…

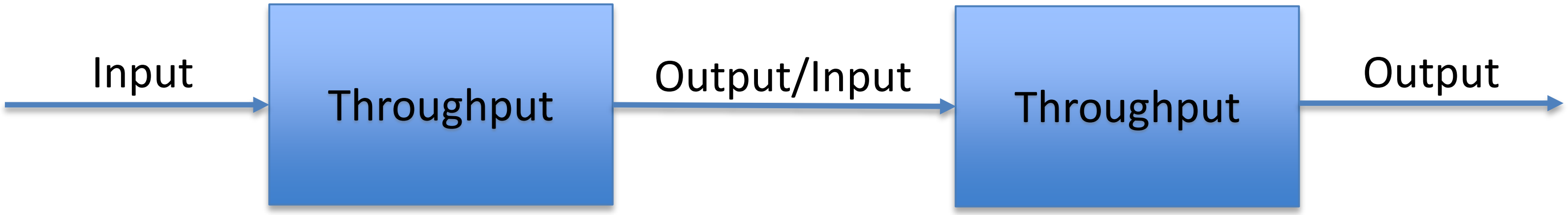
- The output depends on your decisions?
  - Market prices may change according to changes in supply
  - How do you model market power?
- The two different «Throughputs» belong to different decision makers?
  - Different independent companies?
  - Same company, different divisions (e.g. Norway vs. USA)?
- How do you change the decision makers' behaviour?
  - To improve another company's result and/or joint profits?
  - For the benefit of «the greater good»?

Focus in this course I

- This course will consider problems for single decision makers
  - Supply Chain Design
  - Inventory Management
  - Distribution Planning

Focus in this course II

- But we will also consider decisions taken by different decision makers
  - Incentive mechnisms
  - Value of information
  - Asymmetric information
  - Principal-Agent problems



- And we will look in to how to model markets as this is where companies interact
  - Equilibrium modelling

Models and methods

- This course builds on (and has a certain overlap) with
  - TIØ4118 Industrial Economic Analysis
  - TIØ4130 Optimization Methods
- We will therefore use models and methods from both optimization and economics
  - Advantages and Disadvantages
  - Limitations

Planning and Decision Support

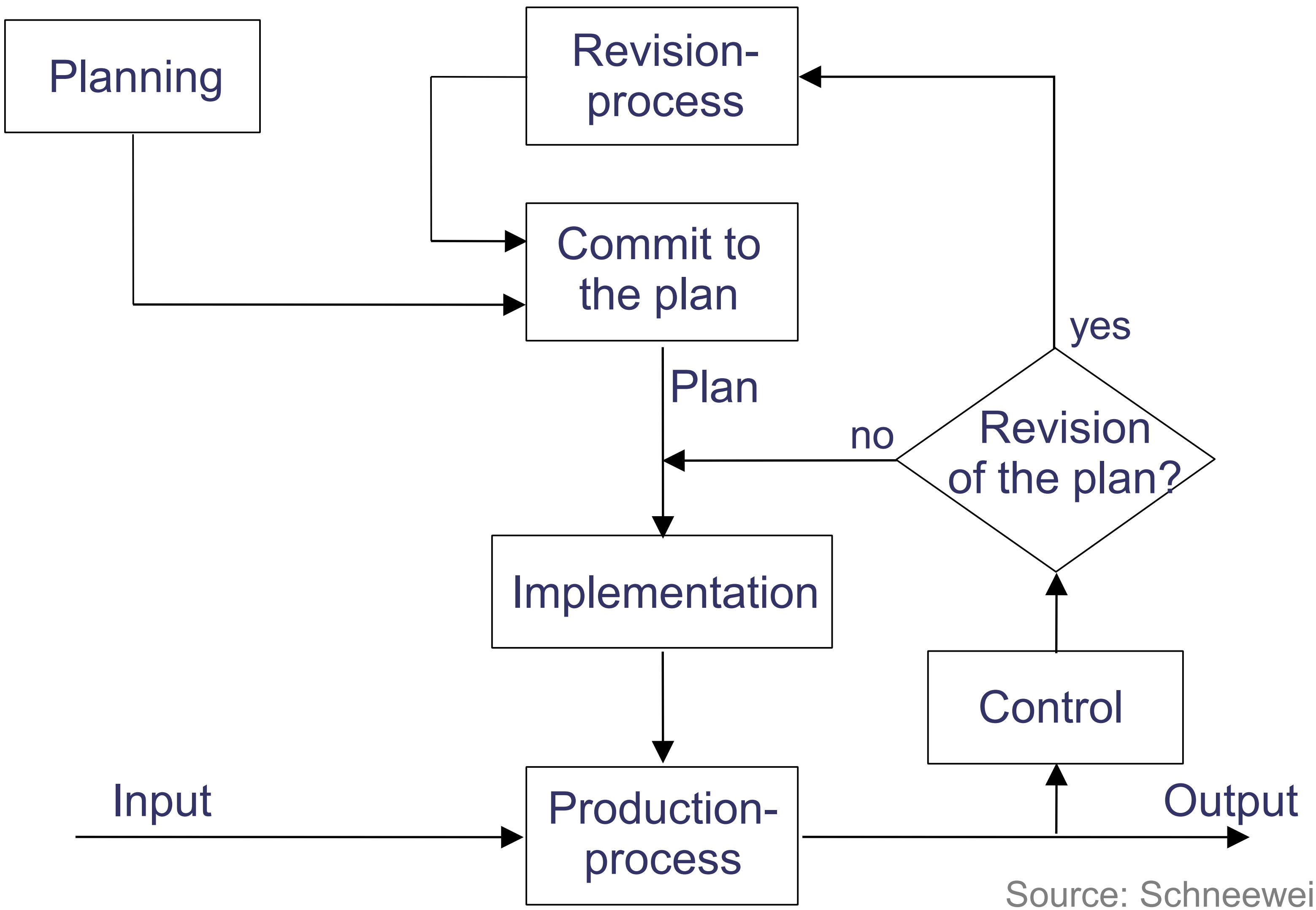
What is Planning?

- First statement:
  - Planning is a target-directed decision to take future action, it is therefore a decision process
  - Result of the decision process is a plan

- Second statement:
  - Planning is the activity of establishing goals over some future time period, called the planning horizon

- Third statement:
  - Planning is a complex social activity that cannot be simply structured by rules of thumb or quantitative procedures
  - Essence of planning is to organize, in a disciplined way, the major tasks that the firm has to address to maintain an operational efficiency in its existing businesses and to guide the organization into a new and better future

The Planning Process

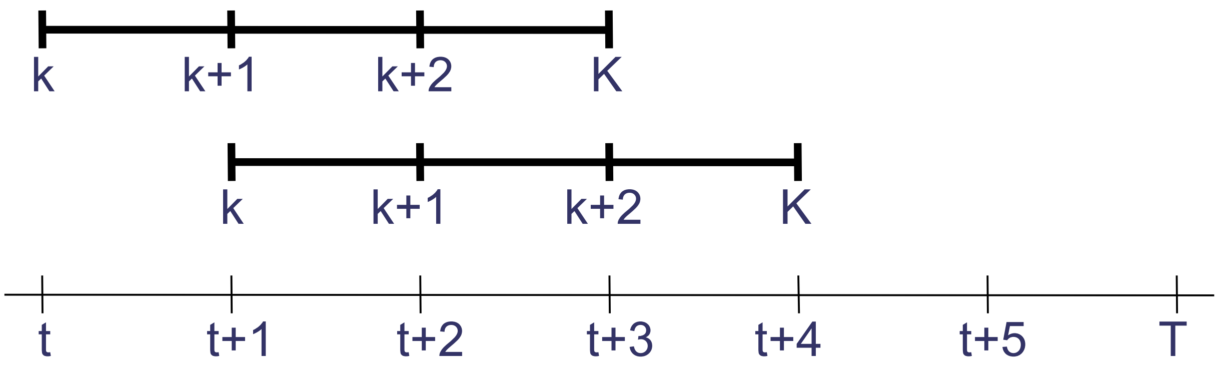


Source: Schneeweiß 1997

The Planning Horizon

- The planning horizon should be adjusted according to the effects of the decisions and the availability of reliable forecasts

- Rolling horizons can be used to incorporate dynamics and the possibility of learning in the decision process



- NB: optimal policies are very likely to turn out to be suboptimal in an ex-post analysis

Problems of the Deterministic Setting

- Rolling horizons are presented in every textbook on planning, usually in a deterministic setting

- Deterministic setting can not capture uncertainties in the future: flexible decisions are not made

- Flexibility (and options as well) never shows up in a deterministic solution, unless it comes free

- "You never buy an insurance in a deterministic world!"

Anthony's Framework

- Developed by Robert N. Anthony in 1965
- First framework to classify different planning and decision levels
- Originally, Anthony distinguished between
  - Strategic Planning
  - Management Control
  - Operational Control
- Today known as
  - Strategic Planning
  - Tactical Planning
  - Operational Planning

Popular picture of Anthony's framework

- Top management
- Long term perspective (5 – 10 years)
- Highly aggregated data

- Middle management
- Medium time horizon (3 months – 2 years)
- Less aggregated data

- Lower management
- Short time horizon (Days – 3 months)
- Disaggregated data

Comparing the 3 Planning Levels I

FACTOR	STRATEGIC PLANNING	TACTICAL PLANNING	OPERATIONAL PLANNING
Purpose	Management of change, resource acquisition	Resource utilization	Execution, evaluation, and control
Implementation instruments	Policies, objectives, capital investments	Budgets	Procedures, reports
Planning horizon	Long	Medium	Short
Scope	Broad, corporate level	Medium, plant level	Narrow, job shop level
Level of management involvement	Top	Middle	Low
Frequency of Replanning	Low	Medium	High

Comparing the 3 Planning Levels II

FACTOR	STRATEGIC PLANNING	TACTICAL PLANNING	OPERATIONAL PLANNING
Source of information	Largely external	External and internal	Largely internal
Level of aggregation of information	Highly aggregated	Moderately aggregated	Detailed
Required accuracy	Low	Medium	High
Degree of uncertainty	High	Medium	Low
Degree of risk	High	Medium	Low

Source: Hax/Candea 1984

Operational Control

- Some kind of fourth level within the framework
- Deals with the material flow in the plant, basically no planning horizon
- Focuses basically on “error management”, deviation from the plan due to unforeseen events (breakdown of production line, etc)
- Probably best covered by the Norwegian terms “styring” and “drift”

Implications of the Framework

- Decisions on the different levels cannot be made isolated of each other, there is a strong interaction

- Decisions at one level are linked to the decisions at a higher level. Lower level decisions have to satisfy constraints given from a higher level and allow in turn to evaluate the decisions from a higher level.

- Integrated approach is needed to avoid suboptimal solutions

- Decomposition (here: in terms of organization) is necessary to perform planning on the different levels, as one global approach will fail due to the complexity

Decision Support Traditions

- Data processing/manipulation
  - MRP, DRP (little optimization)
- Mathematical programming
  - LP, MIP, Heuristics, Stochastic Programming
- Simulation
  - Discrete time simulation, system dynamics
- AI
  - Expert systems (rule based), constraint optimization, knowledge based rules, neighborhood search, machine learning

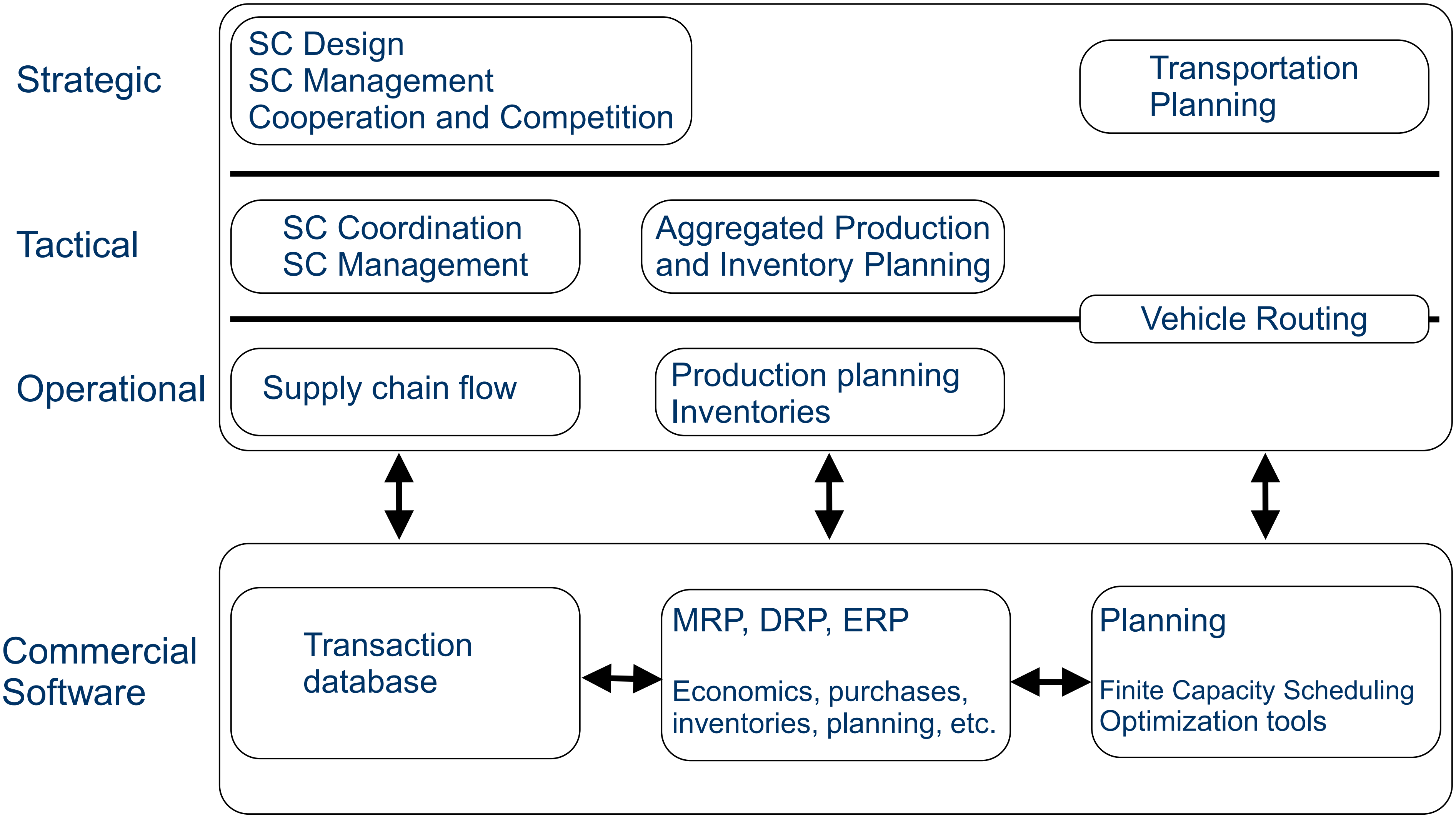
Hierarchical Planning and Operations Research

- Decisions are very complex, especially in a setting with many locations, plants, warehouses, machines, products, customers, etc. (not to talk about uncertainties of the future...)

- Decision support tools are therefore used to provide decision-makers with necessary support

- Most of today's decision support tools are based on optimization and methods of Operations Research

Supply Chain Management Packages



Optimization in supply chain network design

An example from Simchi-Levi et al. (2007)

Solution Techniques

- Mathematical optimization techniques:
  - Exact algorithms: find optimal solutions
  - Heuristics: find “good” solutions, not necessarily optimal

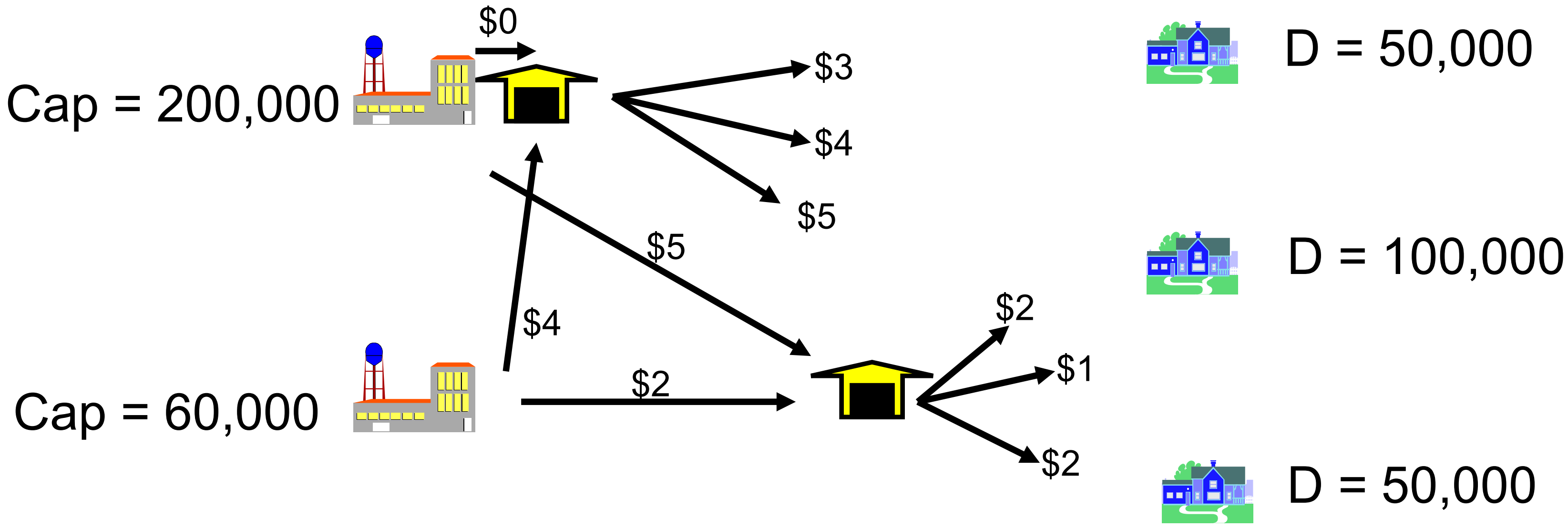
- Simulation models: provide a mechanism to evaluate specified design alternatives created by the designer.

Heuristics or exact algorithms?

Consider the production network for a single product:

- Two plants p1 and p2
  - Plant p1 has an annual capacity of 200,000 units.
  - Plant p2 has an annual capacity of 60,000 units.
- The two plants have the same production costs.
- There are two warehouses w1 and w2 with identical warehouse handling costs.
- There are three markets areas c1,c2 and c3 with demands of 50,000, 100,000 and 50,000, respectively.

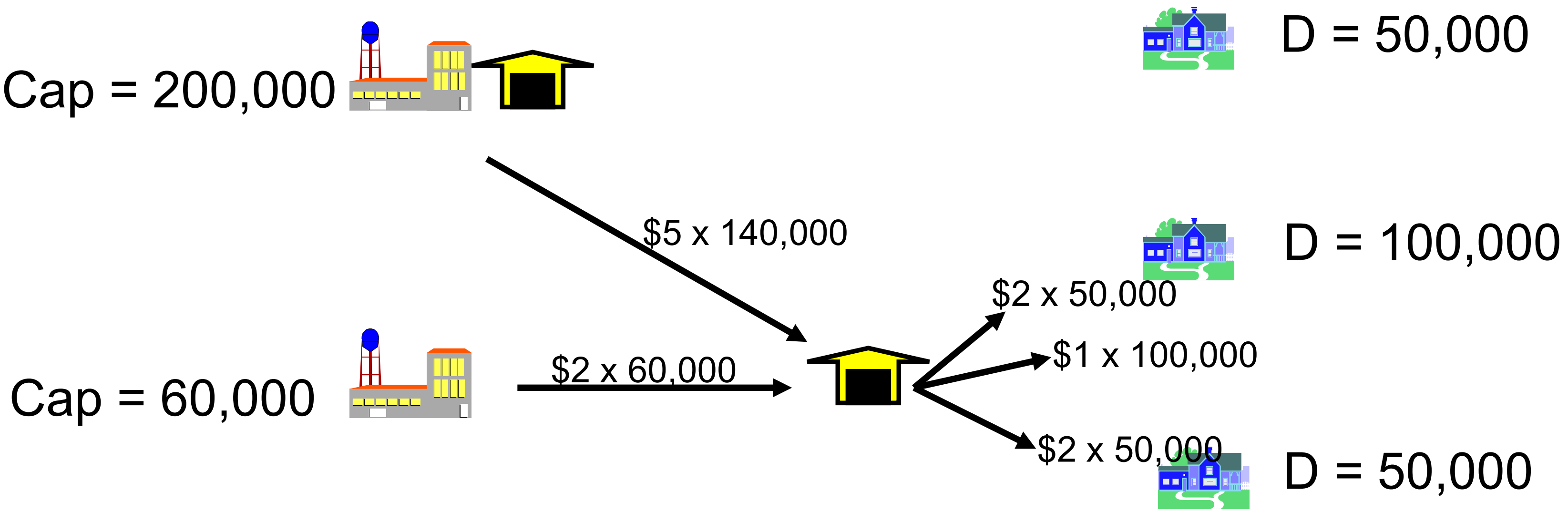
Why Optimization Matters?



Production costs are the same, warehousing costs are the same

Traditional Approach #1:

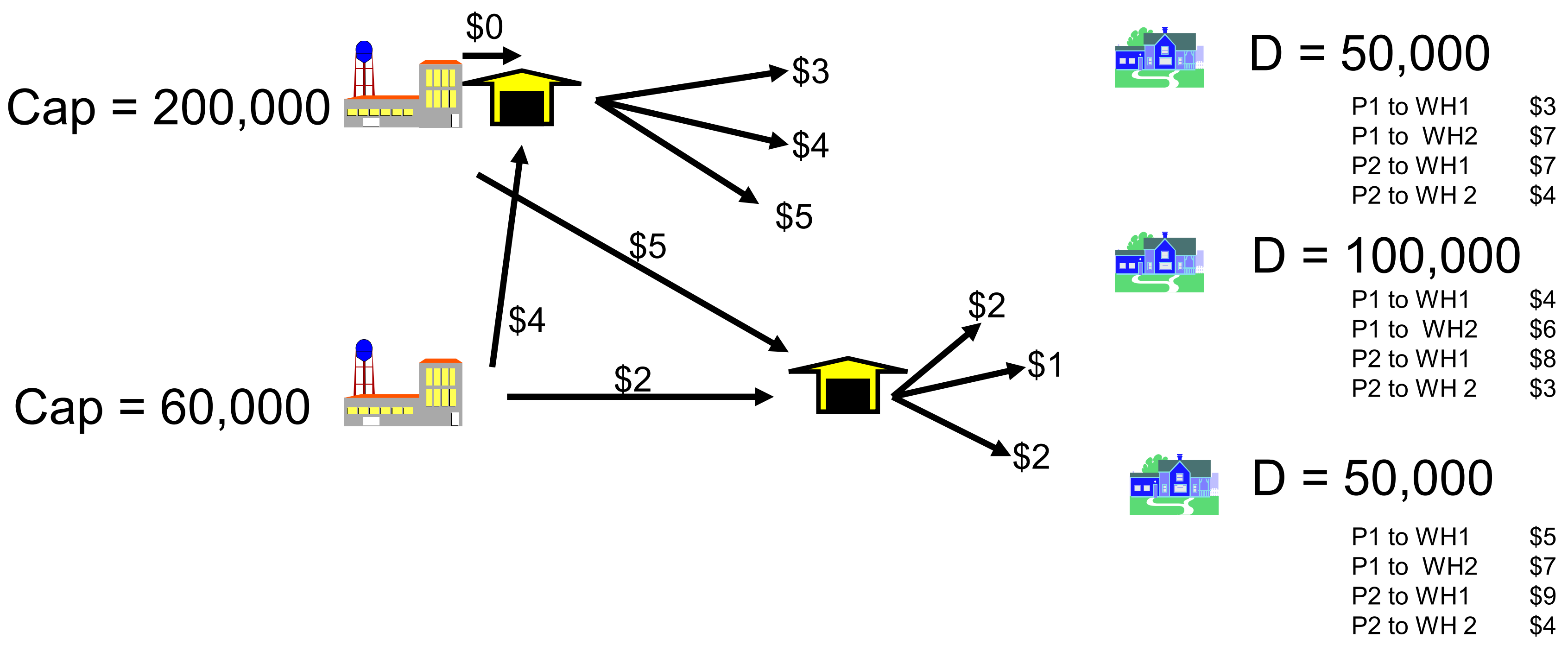
Assign each market to closet WH. Then assign each plant based on cost.



Total Costs = $1,120,000

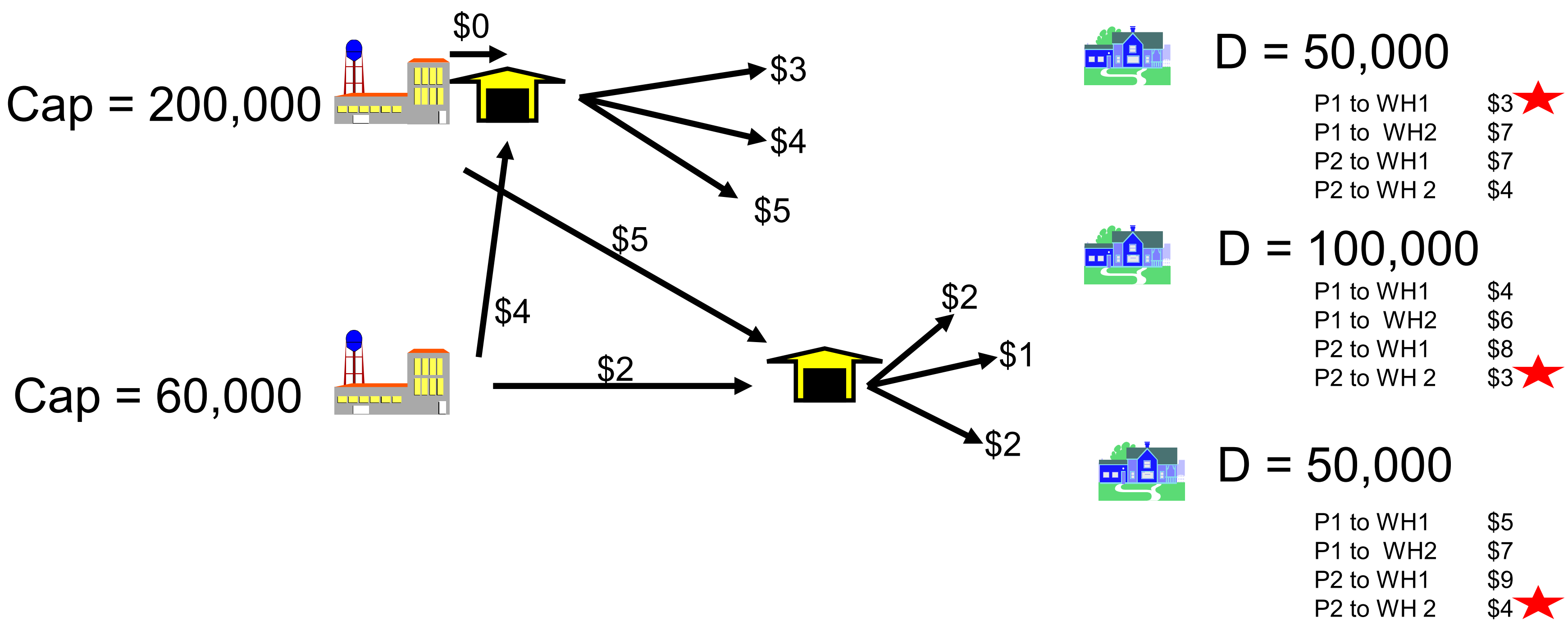
Traditional Approach #2:

Assign each market based on total landed cost



Traditional Approach #2:

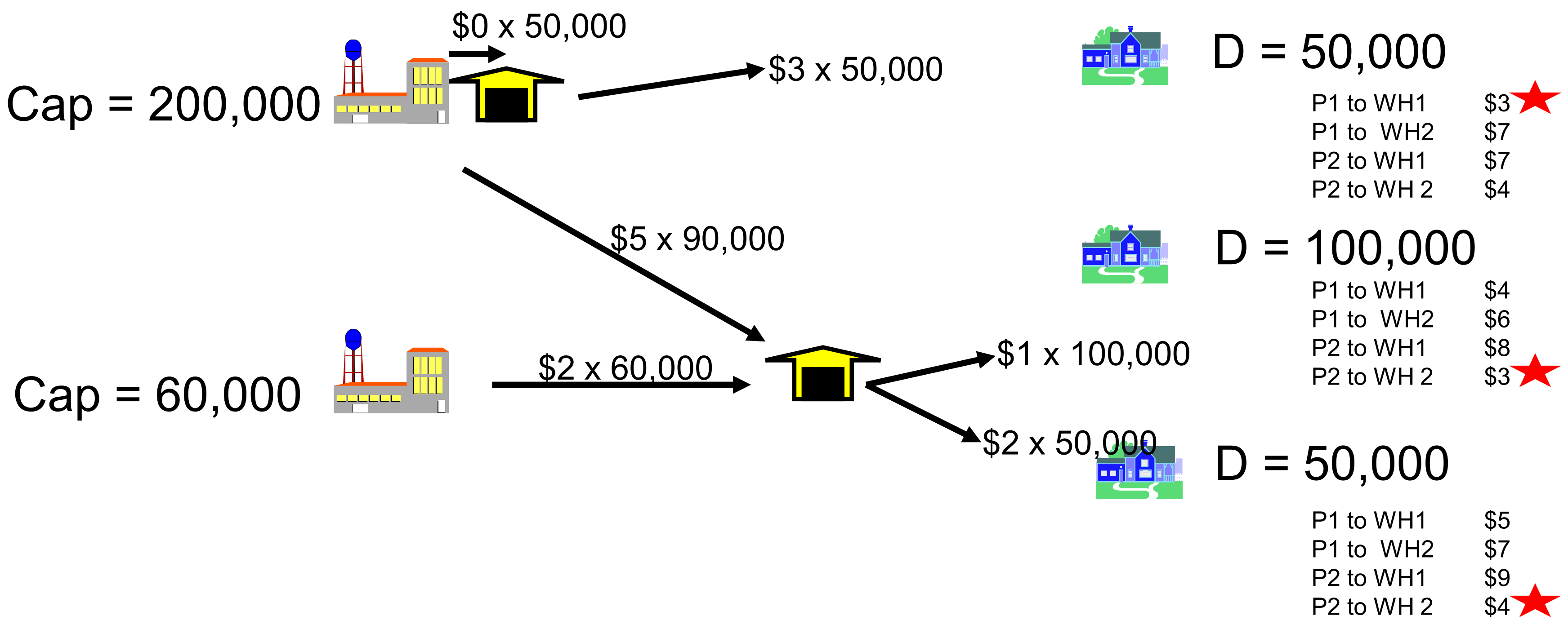
Assign each market based on total landed cost



Market #1 is served by WH1, Markets 2 and 3 are served by WH2

Traditional Approach #2:

Assign each market based on total landed cost



Total Cost = $920,000

The Optimization Model

- The problem can easily be formulated as a linear programming problem.

- Let
  - $x(p1, w1), x(p1, w2)$, $x(p2, w1)$, $x(p2, w2)$ be the flows from the plants to the warehouses.
  - $x(w1, c1), x(w1, c2)$, $x(w1, c3)$ be the flows from the warehouse w1 to customer zones c1, c2 and c3.
  - $x(w2, c1), x(w2, c2)$, $x(w2, c3)$ be the flows from warehouse w2 to customer zones c1, c2 and c3

Formulation

- We want to solve the following problem

$$\begin{aligned}\min\ & 0x(p1,w1)+5x(p1,w2)+4x(p2,w1)+2x(p2,w2)\\ &+3x(w1,c1)+4x(w1,c2)+5x(w1,c3)\\ &+2x(w2,c1)+1x(w2,c2)+2x(w2,c3)\end{aligned}$$

subject to

$$x(p2,w1)+x(p2,w2)\leq 60\,000$$
$$x(p1,w1)+x(p2,w1)=x(w1,c1)+x(w1,c2)+x(w1,c3)$$
$$x(p2,w1)+x(p2,w2)=x(w2,c1)+x(w2,c2)+x(w2,c3)$$
$$x(w1,c1)+x(w2,c1)=50\,000$$
$$x(w1,c2)+x(w2,c2)=100\,000$$
$$x(w1,c3)+x(w2,c3)=50\,000$$

and all flows greater than or equal to zero

Optimal Solution

- Distribute product according to the following plan

to / from	W1	W2	C1	C2	C3
P1	140 000				
P2		60 000			
W1			50 000	40 000	50 000
W2				60 000	

- Total cost for the optimal solution is 740,000.

Facility Location under Economics of Scale

A Strategic Case from Gilde (Nortura)

Background

- Slaughterhouse location analysis

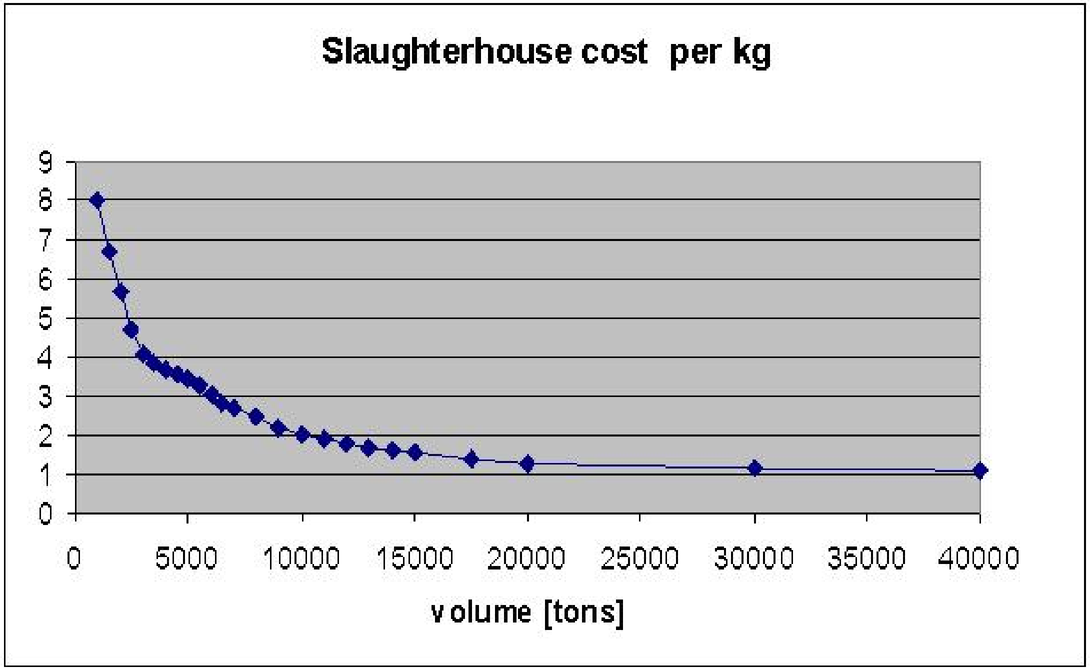
- The company has approximately 25 slaughterhouses for cattle

- What would be the optimal number and locations if they were free to replace them today (using the animal populations they are currently serving)?
  - Location
  - Size

- Requirements
  - All demand (from farmers) should be met
  - No animal should stay more than 8 hours in the car on its way to the slaughterhouse

Slaughterhouse Cost I

- Nonlinearities in objective
- Economics of Scale in slaughterhouses
- Numbers are from a German best practice study



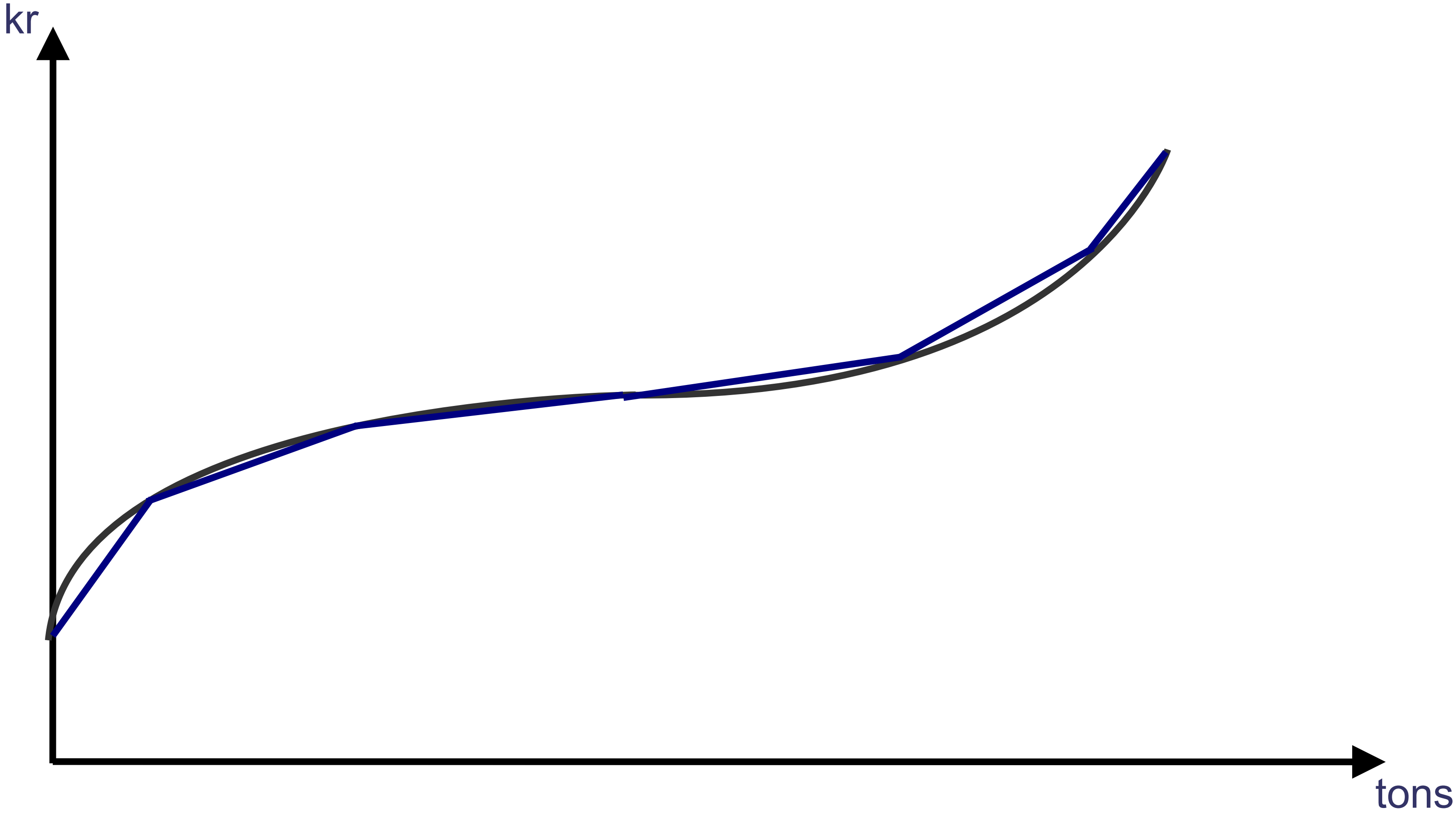
Slaughterhouse Cost II

- Includes
  - Fixed costs
    - Capital cost, Personal, Insurance
  - Variable costs
    - Energy, Personal, Water, Cleaning, Repairs, Classification, Material, Waste management,

- Broken down to yearly numbers and further down to cost per kilo

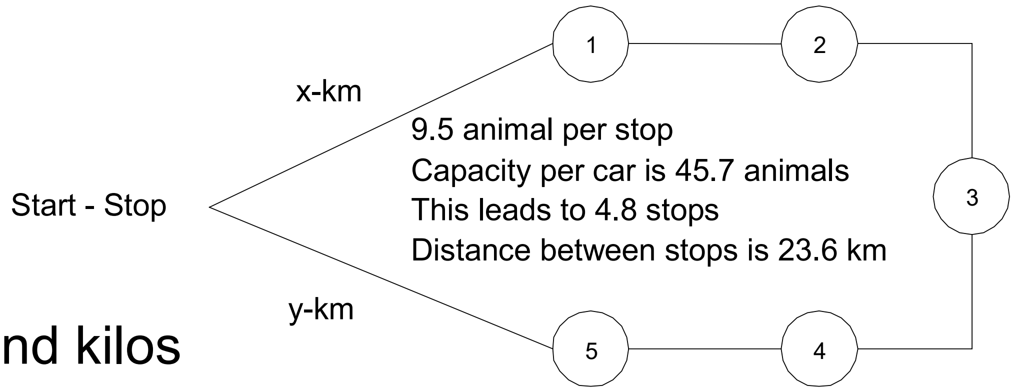
Total Slaughterhouse Costs

(without transportation)



Transportation Time and Cost Estimates

- Time
  - Time between slaughterhouse and region
  - Time on roundtrip within region
  - Terminal time
  - Max limit for transportation is 8 hours



- Cost
  - Linear in travel distance and kilos

- Both time and cost depend on the size of the car used

Notation

Sets

$\mathcal{B}$	Set of breakpoints of piecewise linear function approximating facility costs (i.e. blue line in slide 43)
$\mathcal{M}$	Set of municipalities

Parameters

A_i	Number of animals to be picked up in municipality i, $i \in \mathcal{M}$
C_{ij}	Cost of transporting one animal from municipality i to municipality j, $i \in \mathcal{M}, j \in \mathcal{M}$
g	Average weight of an animal
P_b	Total cost of breakpoint b of the piecewise linear function, $b \in \mathcal{B}$
Q_b	Total weight of breakpoint b of the piecewise linear function, $b \in \mathcal{B}$
T_{ij}	1 if animal can be transported within 8 hours from municipality i to municipality j, 0 otherwise, $i \in \mathcal{M}, j \in \mathcal{M}$

Decision Variables

x_{ij}	Number of animals transported from municipality i to municipality j, $i \in \mathcal{M}, j \in \mathcal{M}$
λ_{jb}	Weight of breakpoint b for municipality j, $j \in \mathcal{M}, b \in \mathcal{B}$

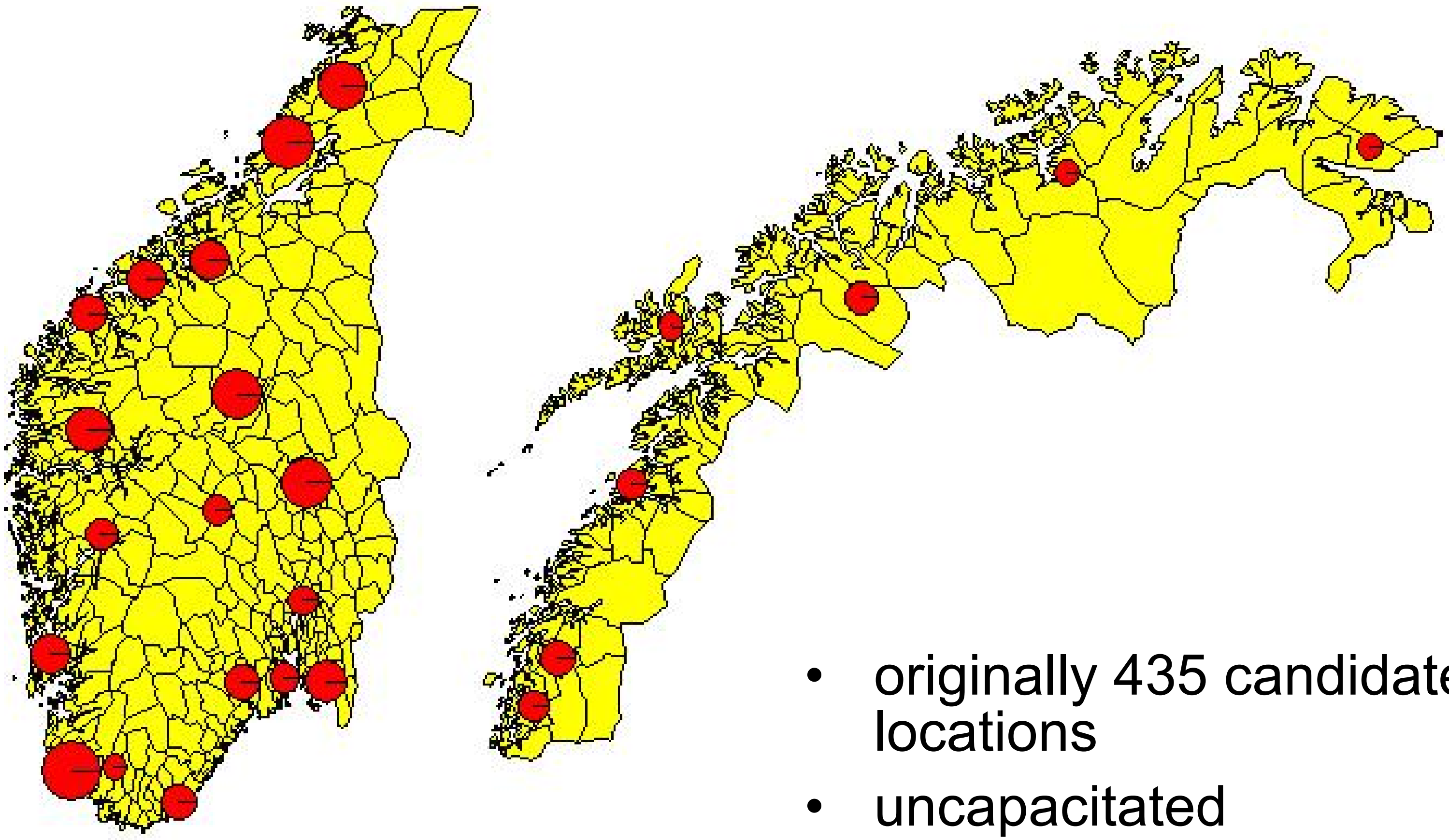
Facility Location Model

$$\min \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{M}} C_{ij}x_{ij} + \sum_{j\in\mathcal{M}}\sum_{b\in\mathcal{B}} Q_b P_b \lambda_{jb}$$

subject to

$$
\begin{aligned}
\sum_{b\in\mathcal{B}} Q_b\lambda_{jb} &= g\sum_{i\in\mathcal{M}} x_{ij} && j\in\mathcal{M},\\
\sum_{b\in\mathcal{B}} \lambda_{jb} &= 1 \quad \text{(S2)} && j\in\mathcal{M},\\
\sum_{j\in\mathcal{M}} x_{ij} &= A_i && i\in\mathcal{M},\\
x_{ij} &\le T_{ij}A_i && (i,j)\in(\mathcal{M}\times\mathcal{M}),\\
x_{ij} &\ge 0 && (i,j)\in(\mathcal{M}\times\mathcal{M}),\\
\lambda_{jb} &\ge 0 && j\in\mathcal{M}, b\in\mathcal{B}.
\end{aligned}
$$

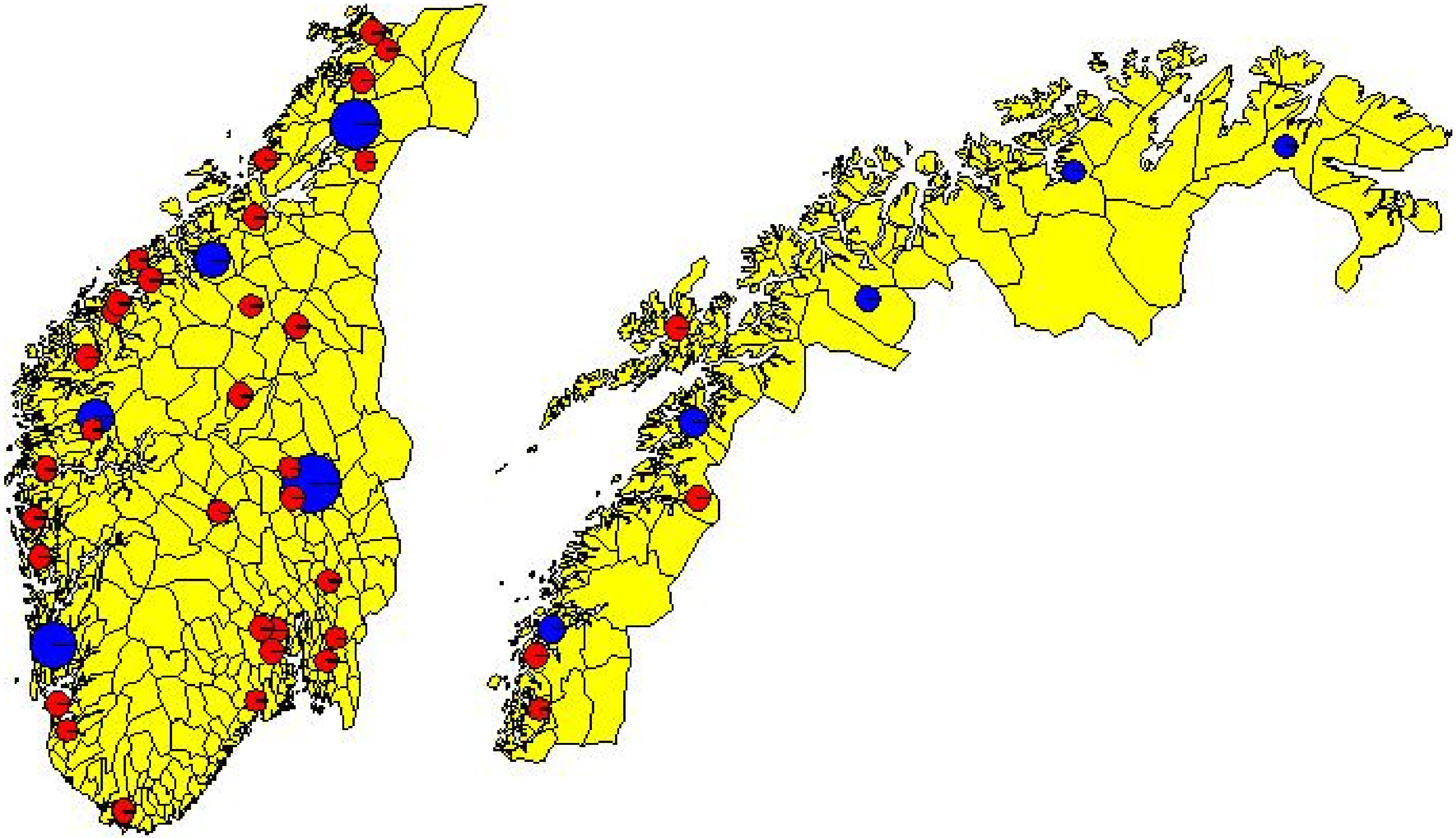
Original Locations



- originally 435 candidate locations
- uncapacitated

Typical Result

(in this case on artificial data)



Results

- Best solution found has 11 slaughterhouses
  - Original solution has 25

- Reduces costs with 30%

- Optimality gap :(UB-LB)/UB =27%

- Solution time about 12 hours!

How should the model be used?

- The models indicates a potential for saving in today's situation
- It does not indicate what should be the future structure
- It indicates that the number of slaughterhouses is more important to the overall cost than the exact location is
- There are many almost equally good solutions with the same number of slaughterhouses, but where the geographical distribution is different.

How to deal with data: Aggregating Customers

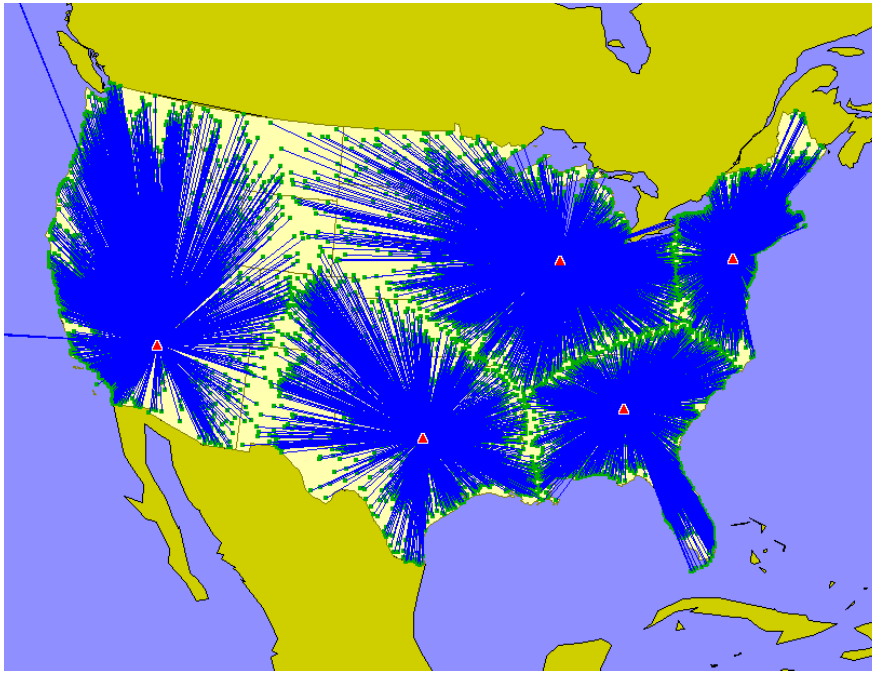
- Customers located in close proximity are aggregated using a grid network or clustering techniques. All customers within a single cell or a single cluster are replaced by a single customer located at the centroid of the cell or cluster.

  We refer to a cell or a cluster as a customer zone.

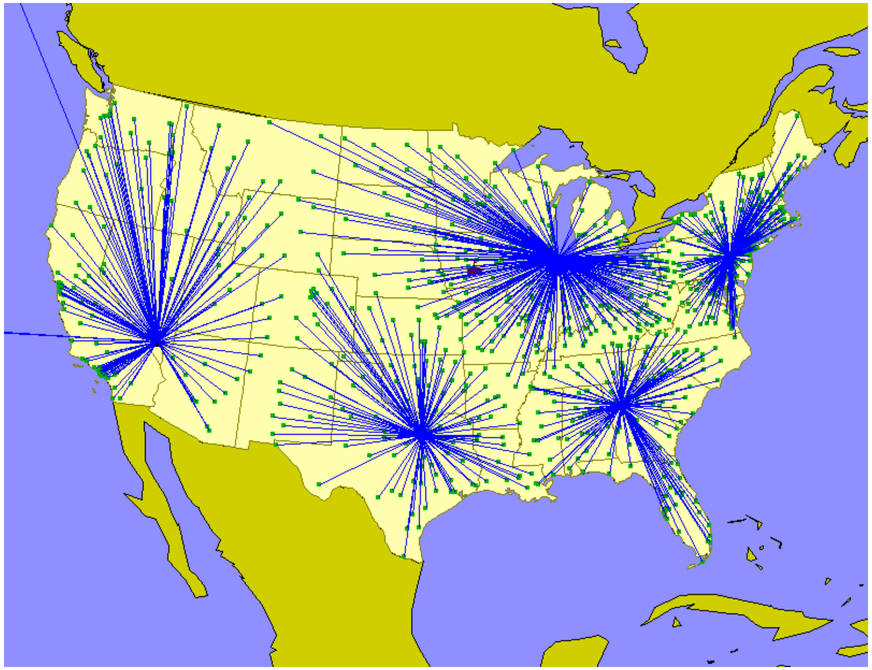
- Why?
  - The cost of obtaining and processing data
  - The form in which data is available
  - The size of the resulting location model
  - The accuracy of forecast demand

Comparing Output

Total Cost:$5,796,000
Total Customers: 18,000



Total Cost:$5,793,000
Total Customers: 800



Cost Difference < 0.05%

How to deal with data: Product Grouping

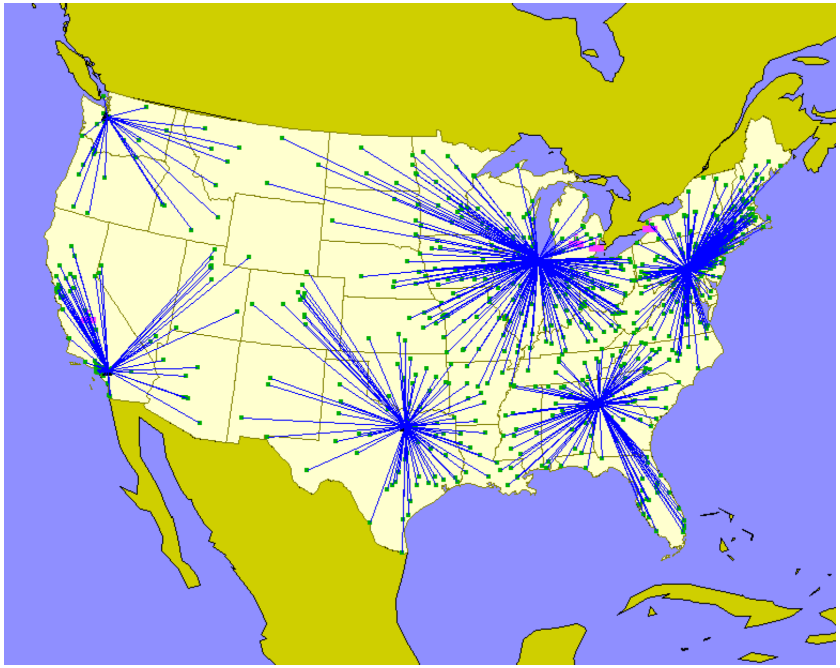
- Companies may have hundreds to thousands of individual items in their production line
  - Variations in product models and style
  - Same products are packaged in many sizes

- Collecting all data and analyzing it is impractical for so many product groups

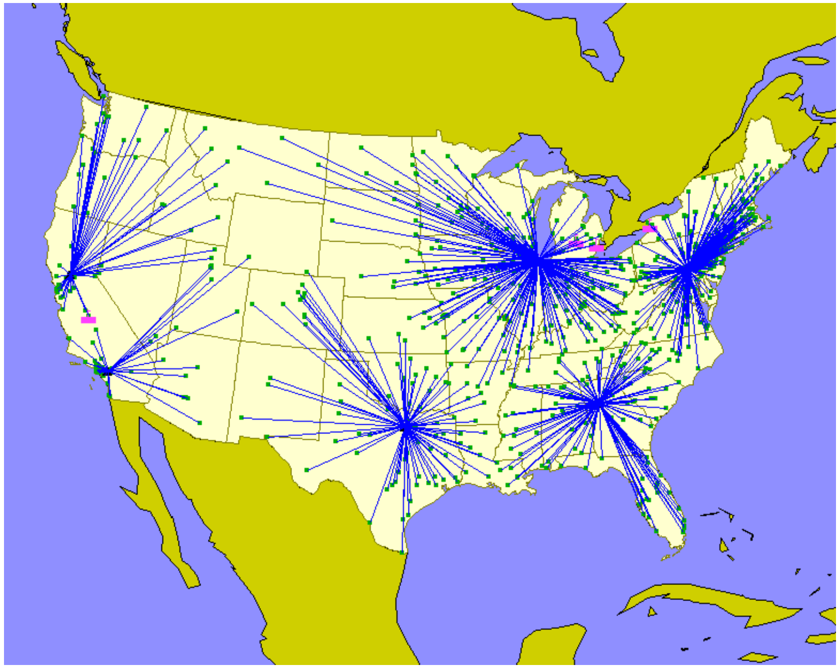
- Aggregate the products by similar logistics characteristics
  - Weight
  - Volume
  - Holding Cost

Sample Aggregation Test: Product Aggregation

Total Cost:$104,564,000
Total Products: 46



Total Cost:$104,599,000
Total Products: 4



Cost Difference: 0.03%

Returning to the slaughterhouses…

- Facility location problems are usually strategic problems
  - Long time horizons
  - Highly aggregated data

- Yet, we only consider 1 scenario: the present

- What happens if the future is different from the present?
  - Would it help to consider more scenarios?

A small example

- Consider the following optimization problem:

$$\max 3x + 2y + z$$

subject to

$$\begin{aligned} x &\leq \xi \\ y &\leq 1 - \xi \\ x + y + z &\leq 1 \\ x, y, z &\geq 0 \end{aligned}$$

where ξ is (for now) a known parameter.

- What is the optimal solution for any given ξ?

The impact of uncertainty I

- Assume now, you have to decide upon x, y and z before you know ξ

- Assume further that ξ is uniformly distributed on [0,1]

- Any suggestions for how to solve this problem?
  - Let's first solve the problem for every realization
  - Then check the solutions for other realizations and pick the one the performs best

- What happens if we evaluate a solution for any other possible realization (i.e. all scenarios)?

The impact of uncertainty II

- The solution is infeasible for every realization that is different from ξ (i.e. the value used to find the solution)
  - This is valid for any ξ!
- What if we had chosen z=1?
  - Why is this happening?
- In face of uncertainty: scenario analysis (deterministic optimization) may not produce good solutions, but it can evaluate the performance of solutions (= simulation)!

Modeling for uncertainty

- Let’s provide some context for the small example:
  - ξ is uncertain demand
  - *x*, *y* and *z* are production decisions (taken before demand is known)

- Can you fix the model?

Lecture 2: Inventory Models and Risk Pooling

TIØ4285 Production and Network Economics

Spring 2021

Note that the lecture is recorded and streamed through Panopto.

Information about privacy can be found at https://s.ntnu.no/video-recording

Norwegian University of Science and Technology

Outline

- Classic inventory models
  - Economic Order Quantity (aka Economic Lot Size)
  - The (s,S) Inventory Policy
- The Newsboy Problem
- Risk Pooling
- Supply Chain Design under Uncertainty

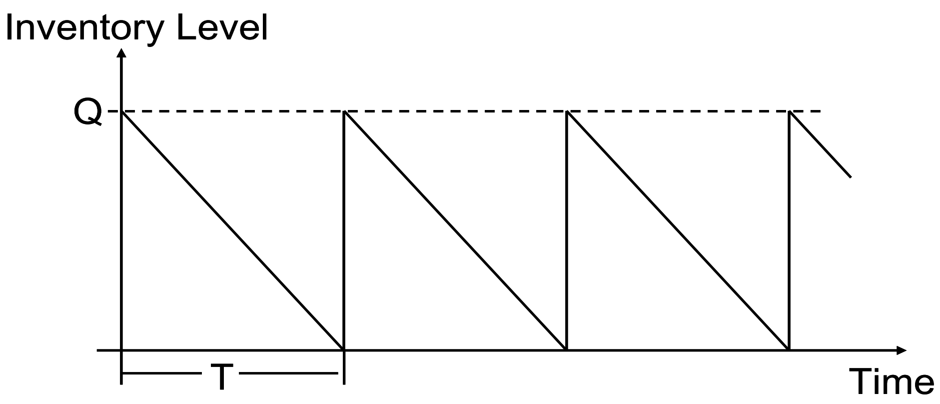
Classic Inventory Theory

The Economic Order Quantity

Assumptions:

- Constant demand rate of D items per day
- Fixed order quantity Q
- Fixed cost K every time an order is placed
- Inventory carrying cost h (holding cost) per unit and day
- Lead time between placing an order an receipt of goods is zero
- Initial inventory is zero
- Planning horizon is infinite

Determining the EOQ



- Total inventory cost per cycle *T*

$$K + \frac{hTQ}{2}$$

- Average cost per unit of time

$$\frac{KD}{Q} + \frac{hQ}{2}$$

- Cost minimizing quantity

$$Q^* = \sqrt{\frac{2KD}{h}}$$

Q* is known as the Economic Order Quantity (EOQ)

The (s,S) inventory policy, introduction

Basic description of the (s,S) inventory policy:

Whenever our inventory level drops below a certain level, say *s,* we order (or produce) in order to increase the inventory level to *S*. Such a policy is referred to as an (*s,S*) or *min max* policy.

s is called the reorder point and *S* the order-up-to-level

The (s,S) inventory policy, random demand

Additional Assumptions:

- Daily demand is random and normally distributed
- If an order is placed, it arrives after the appropriate lead time
- All orders that can't be satisfied from stock are lost
- A service level is defined as the probability that the retailer is not stocking out during lead time

We also have:

- Fixed cost K for placing an order
- Inventory holding cost h is charged per item per unit time

The (s,S) policy with random demand

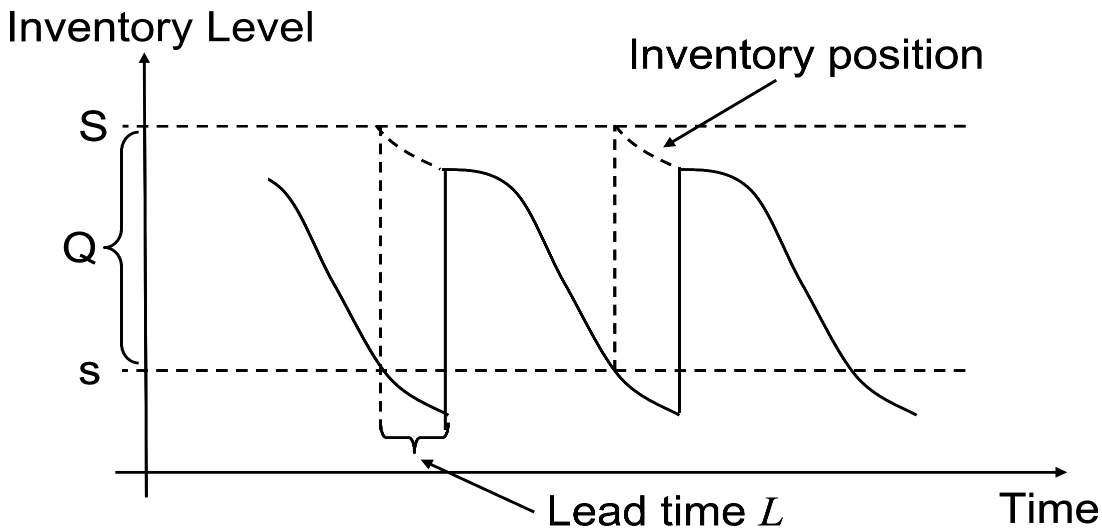
Additional information needed to calculate the inventory policy

AVG	=	Average daily demand
STD	=	Standard deviation of daily demand
L	=	Replenishment lead time
$L \times STD^2$	=	Cumulative variance over time

α = Service level (implying that the probability of stocking out is 1- α)

z = Safety factor associated with the service level (can be taken from tables, e. g. Table 2-2, p. 43 in SKS)

Reorder point s and Order-up-to-level S



- Average demand during lead time

$$L \cdot AVG$$

- Safety stock

$$z \cdot STD \cdot \sqrt{L}$$

- Reorder point *s* $L \cdot AVG + z \cdot STD \cdot \sqrt{L}$

- *z* is taken from a statistical table to ensure that the probability of a stock-out during lead time is 1-α

$$\Pr\{\text{demand during lead time} \geq L \cdot AVG + z \cdot STD \cdot \sqrt{L}\} = 1 - \alpha$$

Reorder point *s* and Order-up-to-level *S*

- Recall from EOQ

$$Q^* = \sqrt{\frac{2K \cdot AVG}{h}}$$

- Order-up-to-level S

$$S = Q^* + s$$

- Expected level of inventory

before receipt	after receipt	on average
$z \cdot STD \cdot \sqrt{L}$	$Q^* + z \cdot STD \cdot \sqrt{L}$	$\frac{Q^*}{2} + z \cdot STD \cdot \sqrt{L}$

Example 2-8 I

Define the inventory policy for a TV distributor given the following assumptions:

- Whenever an order is placed the distributor incurs a fixed cost of $4500 (independent of order size)
- The cost of the TV to the distributor is $250 and annual inventory holding cost is about 18% of the product cost
- Replenishment lead time is 2 weeks
- Desired service level is 97%
- Demand data for the last 12 months:

Month	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.
Sales	200	152	100	221	287	176	151	198	246	309	98	156

Example 2-8 II

- Average monthly demand is 191.17, standard deviation of monthly demand is 66.53

- Lead time is two weeks, transform monthly demand data into weekly demand data:

$$\text{Average weekly demand} = \frac{\text{Average monthly demand}}{4.3}$$

$$\text{Standard deviation of weekly demand} = \frac{\text{Monthly standard deviation}}{\sqrt{4.3}}$$

- Table 2-2 (SKS, p. 43) for determining the safety factor z for a 97% service level as 1.88

Example 2-8 III

Parameter	Average weekly demand	Standard deviation of weekly demand
Value	44.58	32.08

Calculate

- Average demand during lead time

$$L \cdot AVG = 2 \cdot 44.58 = 89.16$$

- Safety stock

$$z \cdot STD \cdot \sqrt{L} = 1.88 \cdot 32.08 \cdot \sqrt{2} = 85.29$$

- Reorder point

$$L \cdot AVG + z \cdot STD \cdot \sqrt{L} = 89.16 + 85.29 = 174.45 \approx 175$$

Example 2-8 IV

- Weekly inventory holding cost

$$\frac{0.18 \cdot 250}{52} = 0.87$$

- Order quantity Q

$$Q^* = \sqrt{\frac{2K \cdot AVG}{h}} = \sqrt{\frac{2 \cdot 4500 \cdot 44.58}{0.87}} = 679.1 \approx 679$$

- Order-up-to-level

$$S = Q^* + s = 679 + 175 = 854$$

- Average inventory level

$$\frac{Q^*}{2} + z \cdot STD \cdot \sqrt{L} = \frac{679}{2} + 85.29 = 424.79 \approx 425$$

The Newsboy Problem

The Newsboy Problem

Short problem description:

You are selling newspapers. Each morning you go and buy the amount of newspapers you think you are going to sell in the course of the day. Unfortunately, you don't know exactly how many newspapers you are going to sell, your demand is uncertain.
In addition, you have no chance of buying additional newspapers if you realize you face a higher demand than what you anticipated. So all demand you cannot satisfy from stock is lost.
In the other case – you have bought too many newspapers – you will not be able to sell them later, because no one will be interested in yesterday's news. (But you can sell the papers with a loss as recycling material.)

How many newspapers do you buy every morning?

The Newsboy Problem, Formulation by Rudi and Pyke

We use the following notation here:+

W	-	cost of buying a newspaper
R	-	selling price of the newspaper ($R > W$)
S	-	return price (salvage value) of a newspaper ($S < W$)
Q	-	number of newspapers bought in the morning
D	-	stochastic demand

In addition, we will use the following definition:

$$y^+ = \begin{cases} y & \text{if } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The Newsboy Problem, Formulation by Rudi and Pyke

- Define expected profit as function of Q

$$\Pi_r(Q) = E\left[R\min(D,Q) + S(Q-D)^+ - WQ\right]$$

- With

$$\min(D,Q) = D - (D-Q)^+$$

and

$$Q = D - (D-Q)^+ + (Q-D)^+$$

we reformulate $\Pi_r(Q)$

$$\Pi_r(Q) = (R-W)ED - \underbrace{E\left[(R-W)(D-Q)^+ + (W-S)(Q-D)^+\right]}_{G_r(Q)}$$

and call $G_r(Q)$ the cost of uncertainty

The Newsboy Problem, Formulation by Rudi and Pyke

- The first term

  $$(R-W)(D-Q)^{+}$$

  represents expected number of units short times the opportunity cost per unit short. Thus, define the unit underage cost C_u as: $C_u = (R-W)$

- The second term

  $$(W-S)(Q-D)^{+}$$

  represents expected number of left over units times the opportunity cost per left over unit. Thus, define the unit overage cost C_o as: $C_o = (W-S)$

- Rewrite $G_r(Q)$ as $G_r(Q) = E\left[C_u(D-Q)^{+} + C_o(Q-D)^{+}\right]$

The Newsboy Problem, Formulation by Rudi and Pyke

In order to determine the order quantity that results in the highest expected profit, derive $G_r(Q)$ and find Q such that the derivative equals zero.

$$G_r(Q) = E\left[C_u(D-Q)^+ + C_o(Q-D)^+\right]$$

- Derivative of the first term of $G_r(Q)$:

$$-C_u \Pr(D > Q)$$

- Derivative of the second term of $G_r(Q)$:

$$C_o \Pr(Q > D)$$

- The derivative of $G_r(Q)$:

$$G_r'(Q) = -C_u \Pr(D > Q) + C_o \Pr(Q > D)$$

The Newsboy Problem, Formulation by Rudi and Pyke

- With $\Pr(D>Q) = 1 - \Pr(D<Q)$, we can rewrite $G'_r(Q)$ as

$$G'_r(Q) = -C_u[1 - \Pr(D < Q)] + C_o \Pr(D < Q)$$

- Equating this derivative to zero, we get the following Newsboy optimality condition

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o}$$

- This allows to find the optimal order quantity Q^*

Example: Homework Problem #3.1

NoNo is a producer of high-end YoYos (they only make one product) at **unit cost $10**. The YoYos only sell during the summer and a new model is launched very year. The total production lead time is 6 months.

The YoYo is currently sold by 100 different identical retailers. They pay a **wholesale price** of **$15** and sell at **unit revenue $30**. **Unit salvage value is $8**. Each retailer faces a **normal distribution** of demand with **mean of 100** and **standard deviation of 40**.

a) How many YoYo’s should each retailer order?
b) What is the expected profit for each retailer?
c) What is the manufacturer profit?

How many to order?

- Underage cost $C_u = R - W = 30 - 15 = 15$
- Overage cost $C_o = W - S = 15 - 8 = 7$

- Optimality criterion:

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = 0.6818$$

- Invert normal distribution to find order quantity

$$Q^* = 119$$

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Expected profit for each retailer

- Retailer profit given as

$$\Pi_r(Q) = (R-W)ED - \underbrace{E\left[(R-W)(D-Q)^+ + (W-S)(Q-D)^+\right]}_{G_r(Q)}$$

- Cost of uncertainty $G_r(Q)$ requires calculating expected shortage and expected salvage
  - Complex for normal distribution (use software!)
  - Fairly easy for uniform distribution
  - Here: $G_r(Q)$=314

- Retailer profits 1500-314=1186

Expected profit NoNo

- Profit margin per YoYo: $W - M = 15 - 10 = 5$
- Each retailer order 119 YoYos, 100 retailers
- Manufacturer profits
$$\Pi_m = (W - M) \cdot 119 \cdot 100 = 59500$$

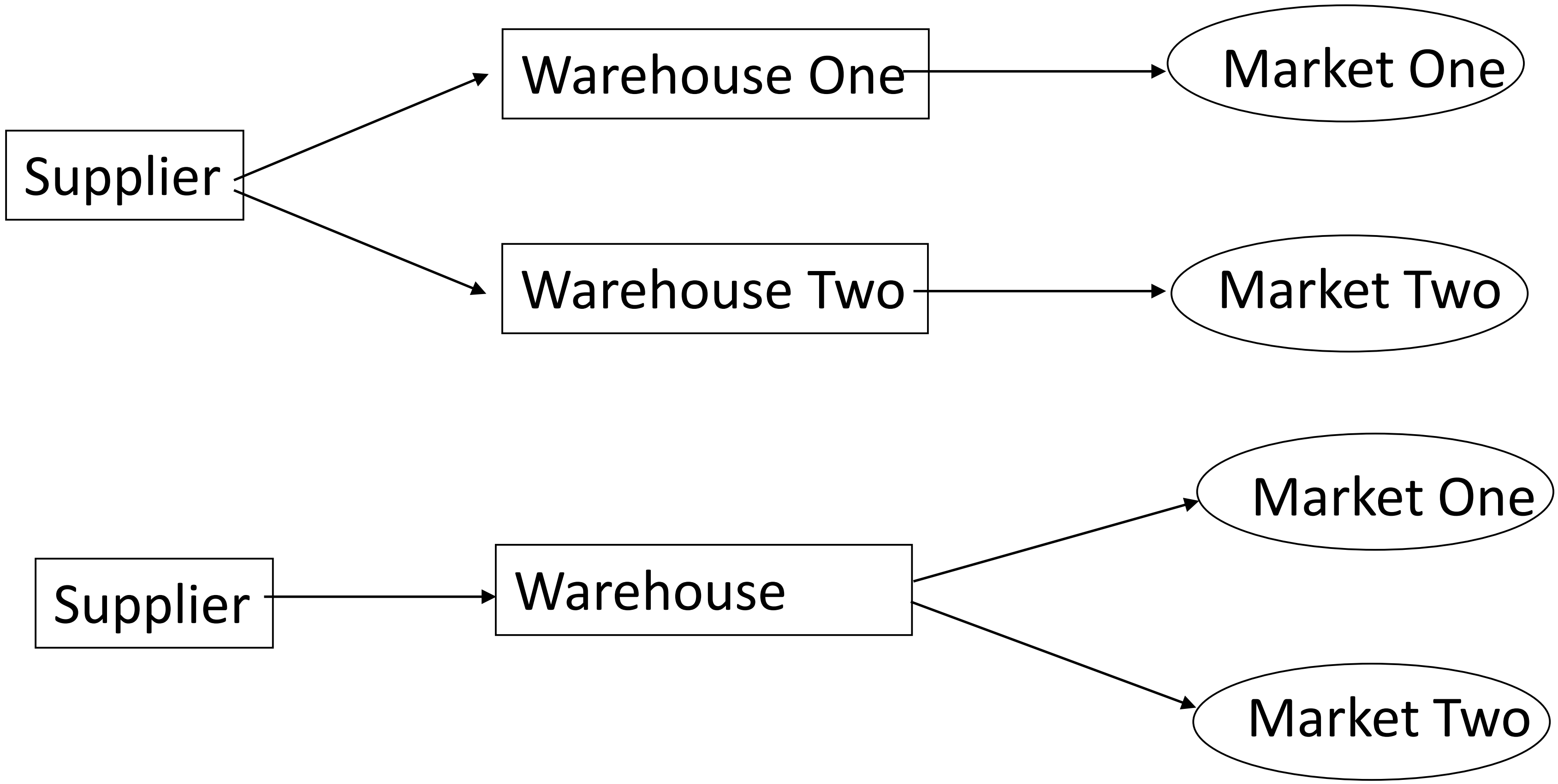
Risk Pooling

Central versus local facilities in supply chains

- Overhead: Economics of scale suggests few sites
- Lead time: more warehouses normally leads to shorter lead times
- Transportation costs: usually increases with fewer warehouses
- Service: Depends on how service is defined. Shipping time increases with fewer warehouses, while the probability that the products are in stock increases even with lower total inventory levels.
- Safety stock: risk pooling in centralized systems

Risk Pooling

- Consider these two systems:

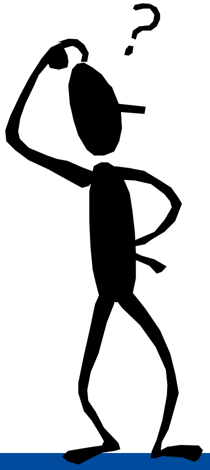


Risk Pooling

- For the same service level, which system will require more inventory? Why?

- For the same total inventory level, which system will have better service? Why?

- What are the factors that affect these answers?



Risk Pooling Example

- Compare the two systems:
  - two products
  - maintain 97% service level
  - $60 order cost
  - $.27 weekly holding cost
  - $1.05 transportation cost per unit in decentralized system, $1.10 in centralized system
  - 1 week lead time

Risk Pooling Example

Week	1	2	3	4	5	6	7	8
Prod A, Market 1	33	45	37	38	55	30	18	58
Prod A, Market 2	46	35	41	40	26	48	18	55
Prod B, Market 1	0	2	3	0	0	1	3	0
Product B, Market 2	2	4	0	0	3	1	0	0

Risk Pooling Example

Warehouse	Product	AVG	STD	CV	s	S
Market 1	A	39.3	13.2	.34	65	197
Market 2	A	38.6	12.0	.31	62	193
Market 1	B	1.125	1.36	1.21	4	29
Market 2	B	1.25	1.58	1.26	5	29

$$\text{CV} = \text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Average demand}}$$

Risk Pooling Example

Warehouse	Product	AVG	STD	CV	s	S	Avg. Inven.	% Dec.
Market 1	A	39.3	13.2	.34	65	197	91	
Market 2	A	38.6	12.0	.31	62	193	88	
Market 1	B	1.125	1.36	1.21	4	29	14	
Market 2	B	1.25	1.58	1.26	5	29	15	
Cent.	A	77.9	20.7	.27	118	304	132	26%
Cent	B	2.375	1.9	.81	6	39	20	33%

$$\text{CV} = \text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Average demand}}$$

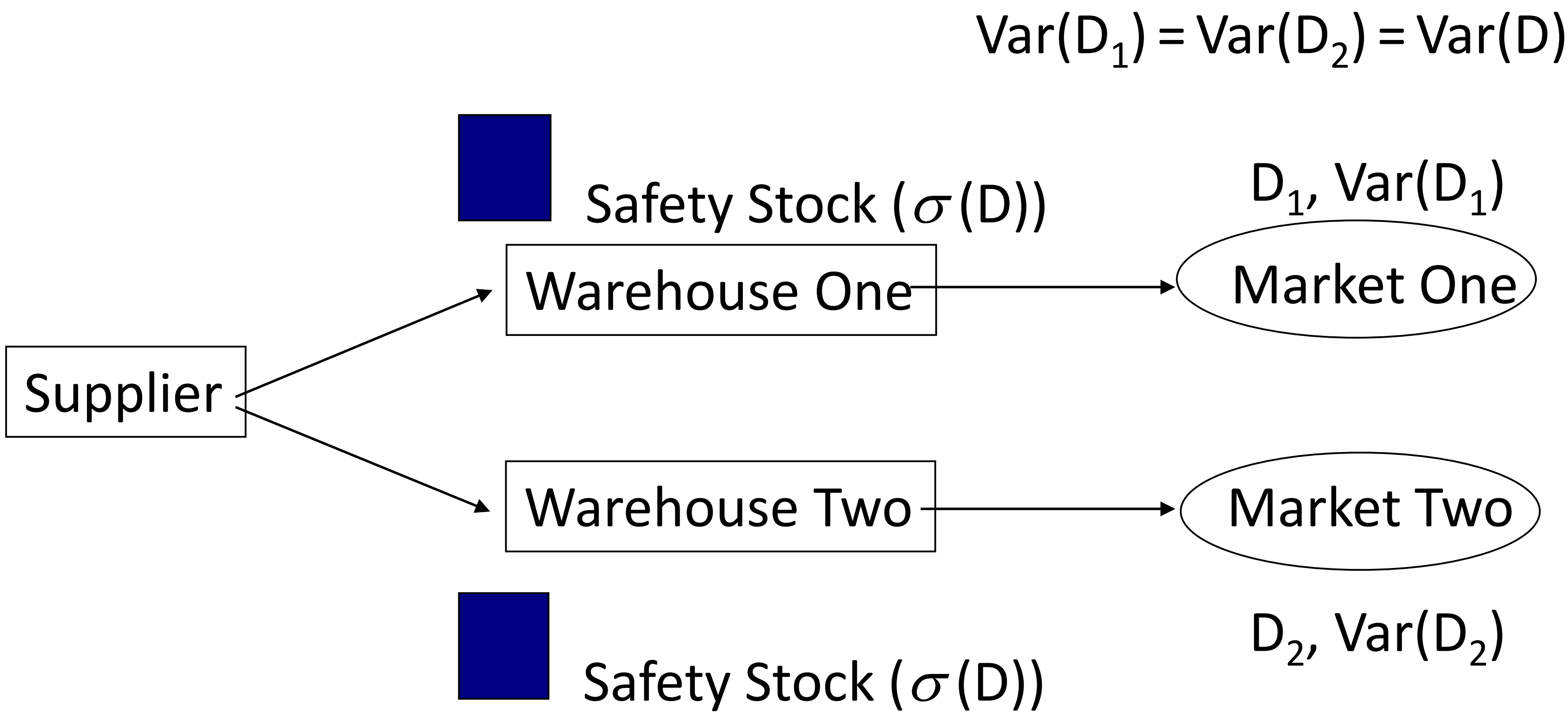
Important Observations

- Centralizing inventory control reduces both safety stock and average inventory level for the same service level.

- This works best for
  - High coefficient of variation, which reduces required safety stock.
  - Negatively correlated demand. Why?

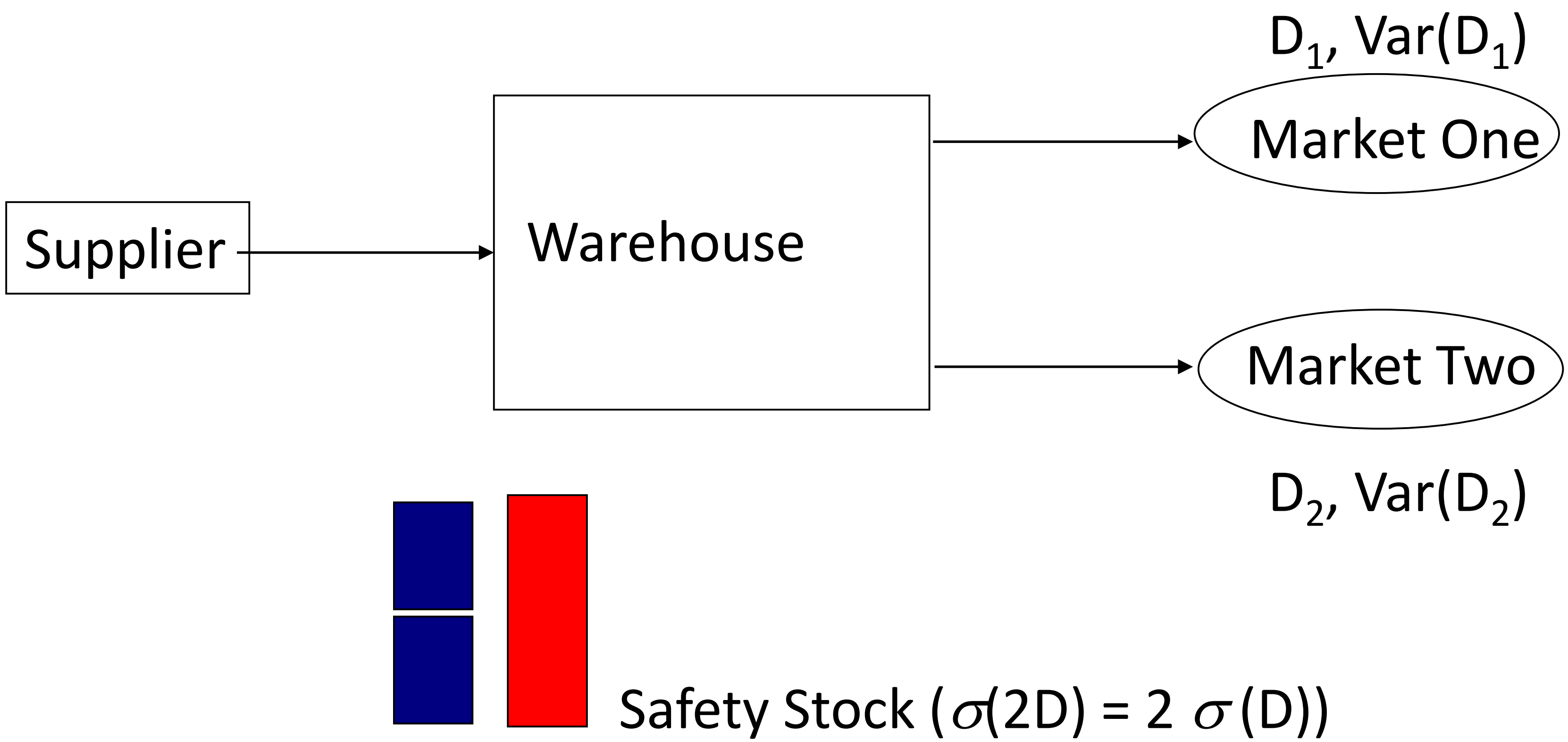
- Risk Pooling only focuses on inventory levels, there might be a trade-off between inventory levels and transportation cost

The Decentralized System



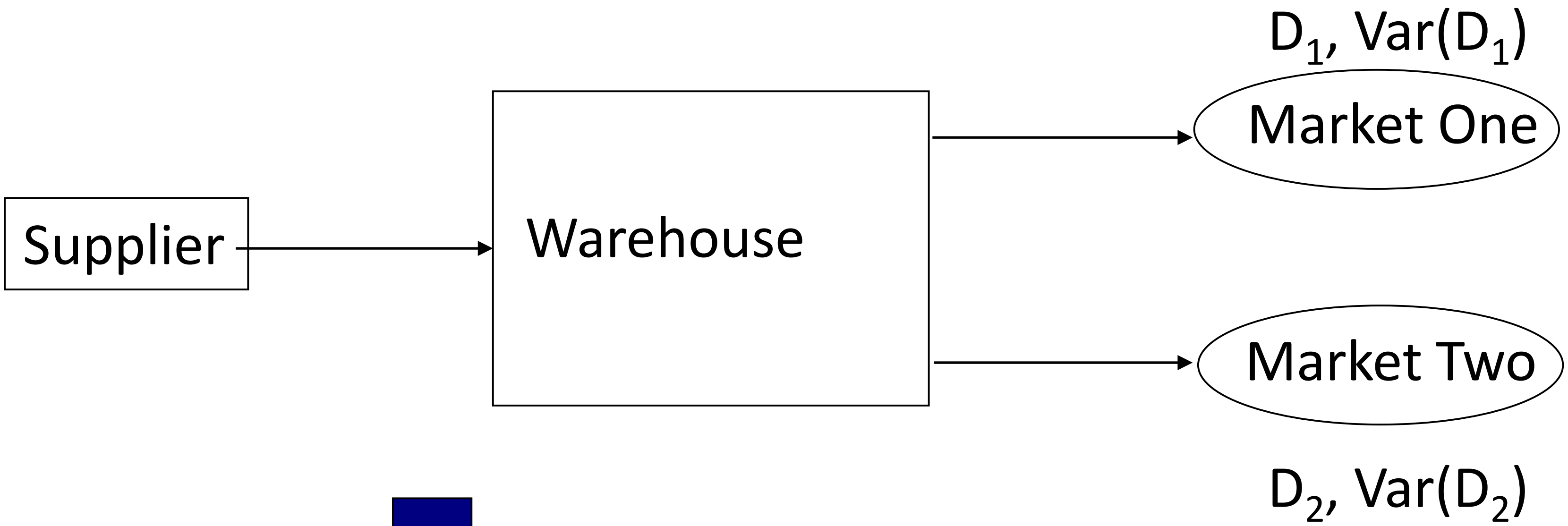
Perfectly Correlated Demand, $\rho_{12} = 1$
Centralized

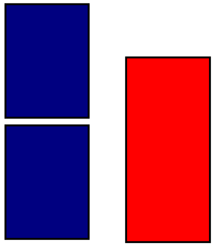
$Var(D_1) = Var(D_2) = Var(D)$



Uncorrelated Demand, $\rho_{12} = 0$
Centralized

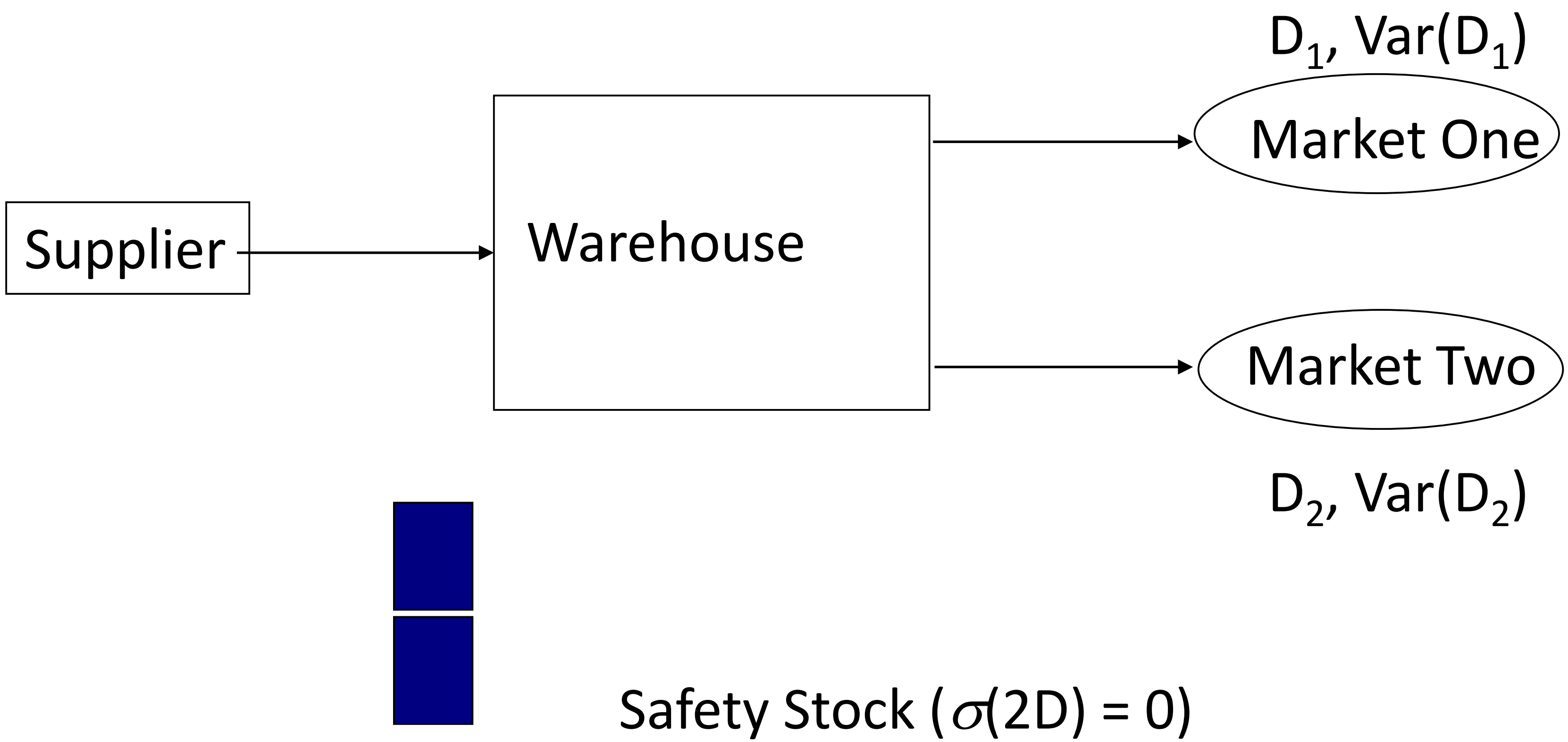
$Var(D_1) = Var(D_2) = Var(D)$





Negatively Correlated Demand, ρ_{12}= -1 Centralized

$\mathrm{Var}(D_1) = \mathrm{Var}(D_2) = \mathrm{Var}(D)$



Supply Chain Design under Uncertainty

What is Stochastic Programming?

- Mathematical programming is about decision making
- Stochastic programming is about decision making under uncertainty
  - Mathematical programming with uncertain parameters
- Fundamental assumption: we know a (joint) probability distribution
  - We may not need to know the whole joint distribution (the focus is normally on some random variables)
  - A subjective specification of the joint distribution can also provide useful information / insight to the problem

Why Stochastic Programming?

- To find all the *explicit and implicit options* worth paying for
  - A spare bus in case of breakdowns
  - Extra ground time for a plane to absorb small delays
  - A financial instrument to reduce variations in income from trade in several currencies
  - A different schedule which is simply more robust in light of delays

- In order to do this, we must

  Model stages …

  … and thereby the information structure

  Study random phenomena …

  … and decide how to represent them

  Update our old modeling skills …

  … not the least how we treat constraints

Recourse Decisions

- Consider a problem with two stages. The following sequence of events occurs:
  1. We make a decisions now (first-stage decision)
  2. Nature makes a random decision (high/low, wet/dry, etc…)
  3. We make a second stage decision that attempts to correct some of the problems caused by (2)
  4. (in a multi-stage problem, nature makes another decision, we make another corrective action, and so on…)
- The second stage decision are called recourse decisions

- The goal of a two-stage model is to identify a first-stage decision that is well positioned against all possible realizations of the random variables

Remember last week?

- Consider the following optimization problem:

$$\max 3x + 2y + z$$

subject to

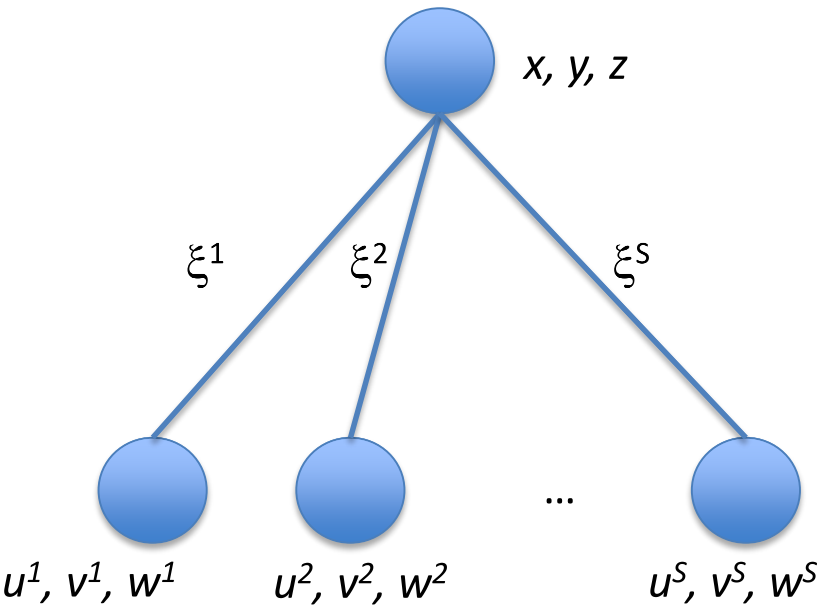
$$\begin{aligned} x &\leq \xi \\ y &\leq 1 - \xi \\ x + y + z &\leq 1 \\ x, y, z &\geq 0 \end{aligned}$$

- ξ is an unknown parameter (demand) and *x, y, z* have to be chosen before ξ is known (production)

What happens in the real world?

- We first choose a production level (*x*,*y*,*z*)
- We then observe demand ξ
- We then sell our products according to demand (*u*, *v*, *w*)

Two-stage structure



Stochastic Programming formulation

- General two-stage SP model

$$\min c^{\mathsf{T}} x + \mathcal{Q}(x)$$

subject to

$$Ax = b,$$

$$x \geq 0,$$

First stage (valid for all scenarios)

where

$$\mathcal{Q}(x) = \sum_s p^s Q(x, \xi^s)$$

Second stage (valid for each scenario, *S* different problems)

and

$$Q(x, \xi) = \min\{q(\xi)^{\mathsf{T}} y | W(\xi) y = h(\xi) - T(\xi) x, y \geq 0\}$$

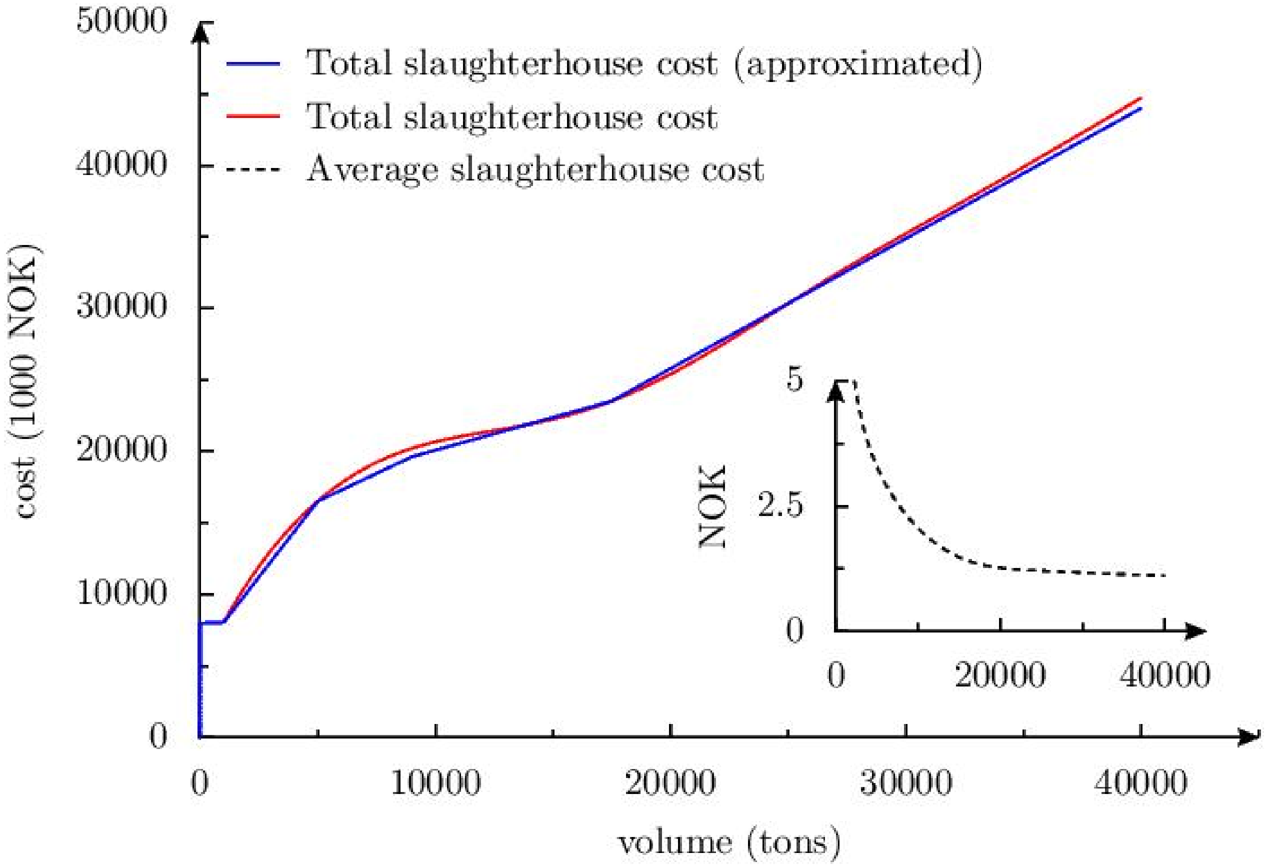
The slaughterhouses again…

- Let's assume that future demand for slaughtering services is uncertain

- How does the decision-information structure look like?
  - First, we need to build slaughterhouses
  - Then, we observe demand
  - Last, we satisfy demand

- This is a two stage structure, but what does this new structure mean for our model?
  - What happens if realized demand deviates from the one we planned for?

Total Slaughterhouse Costs
(without Transportation)



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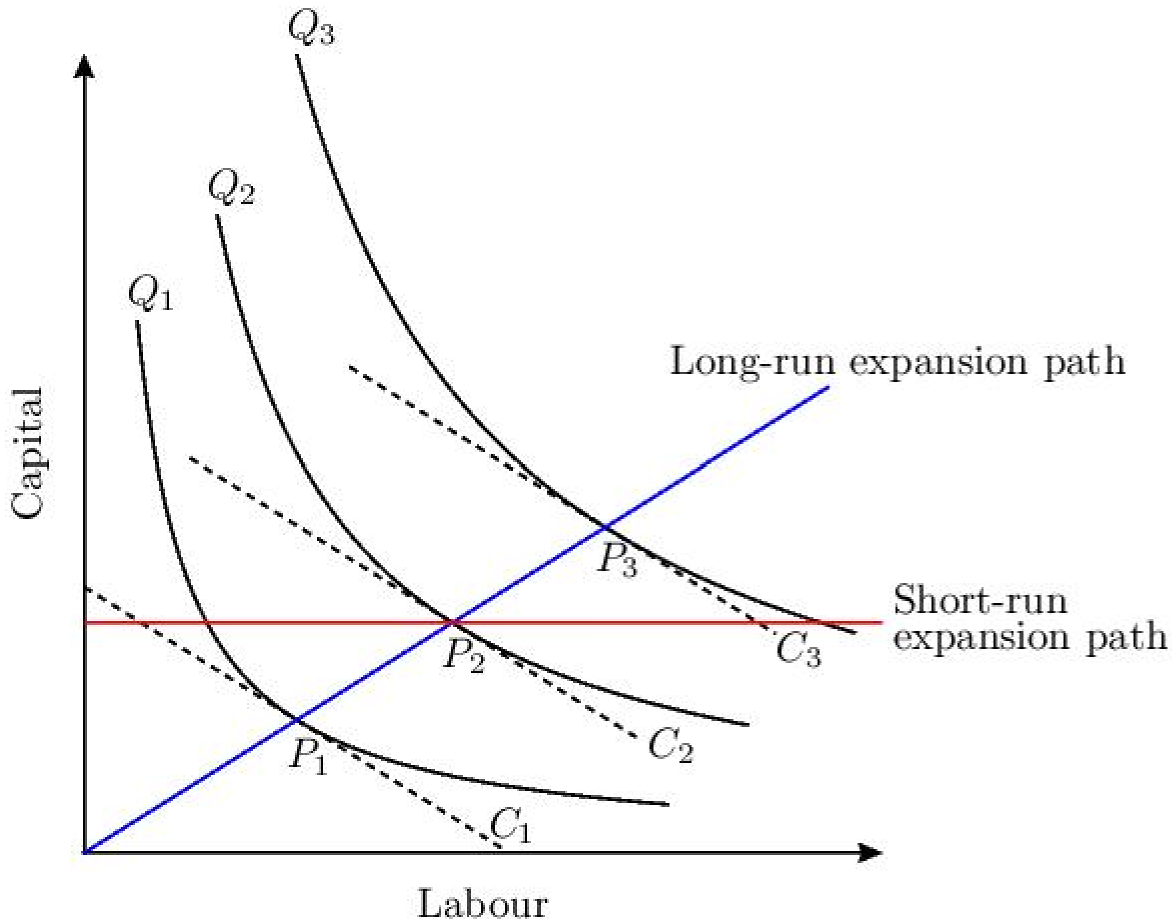
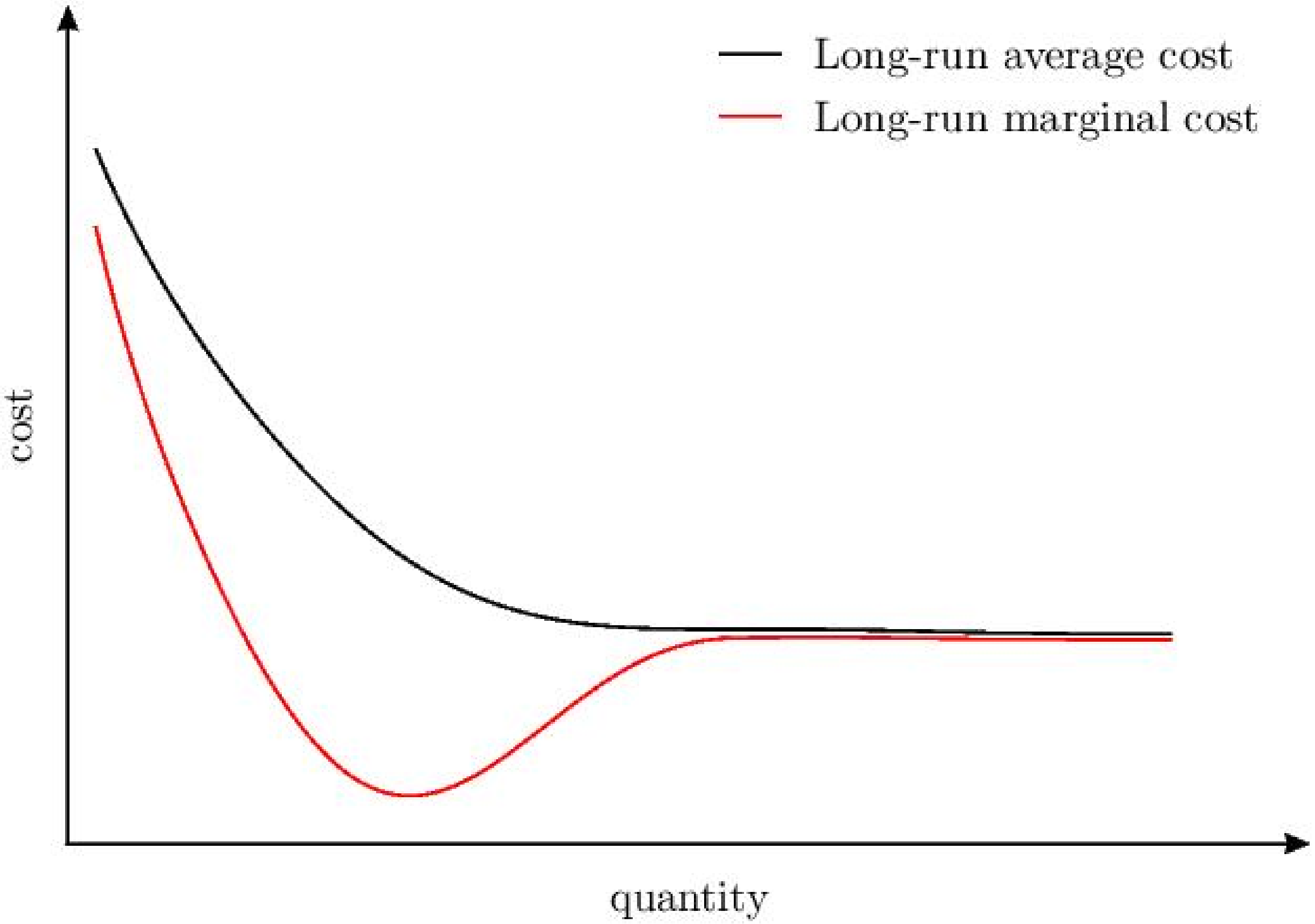
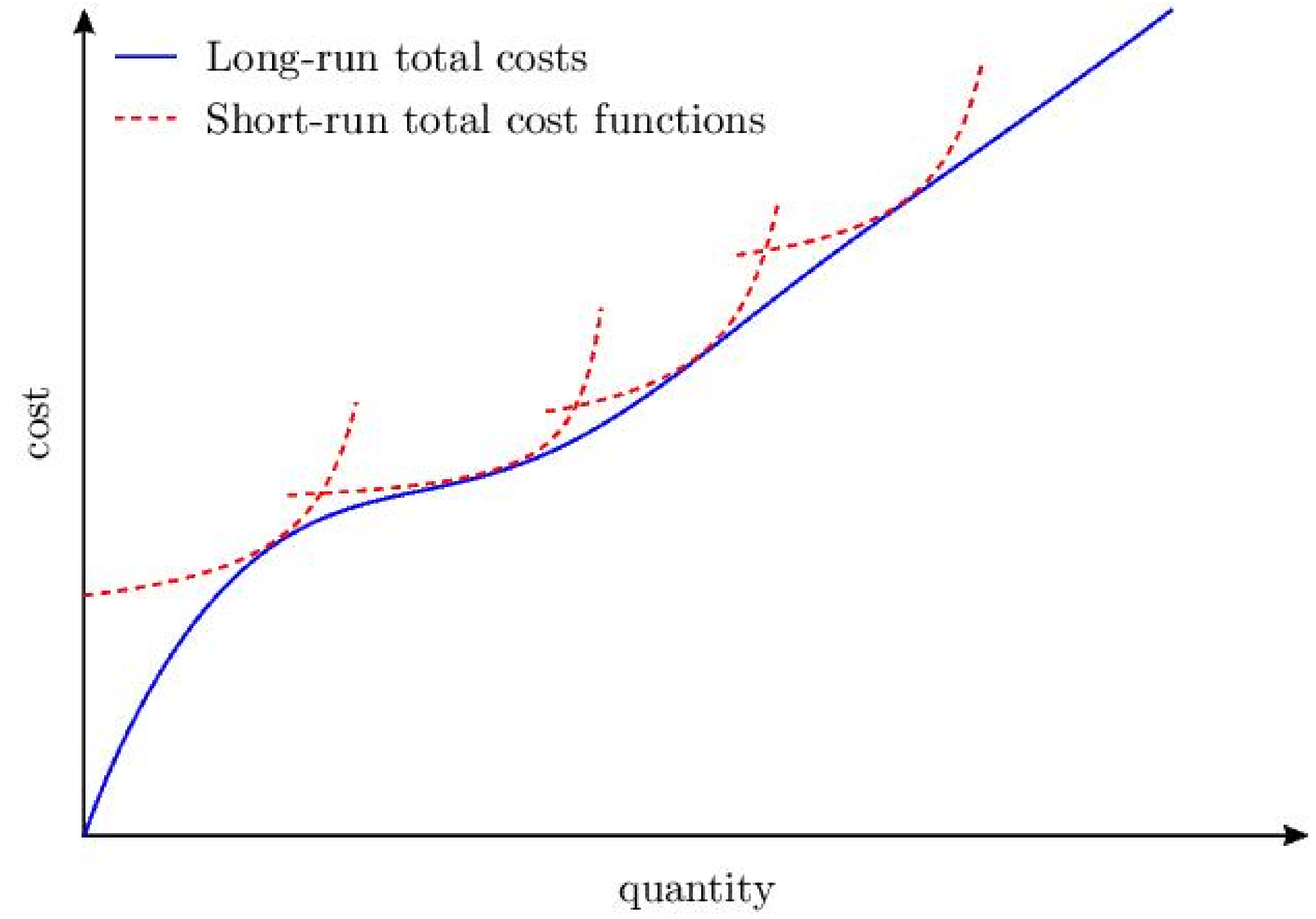
Long-Run and Short-Run Capacity Expansion



Economies of Scale



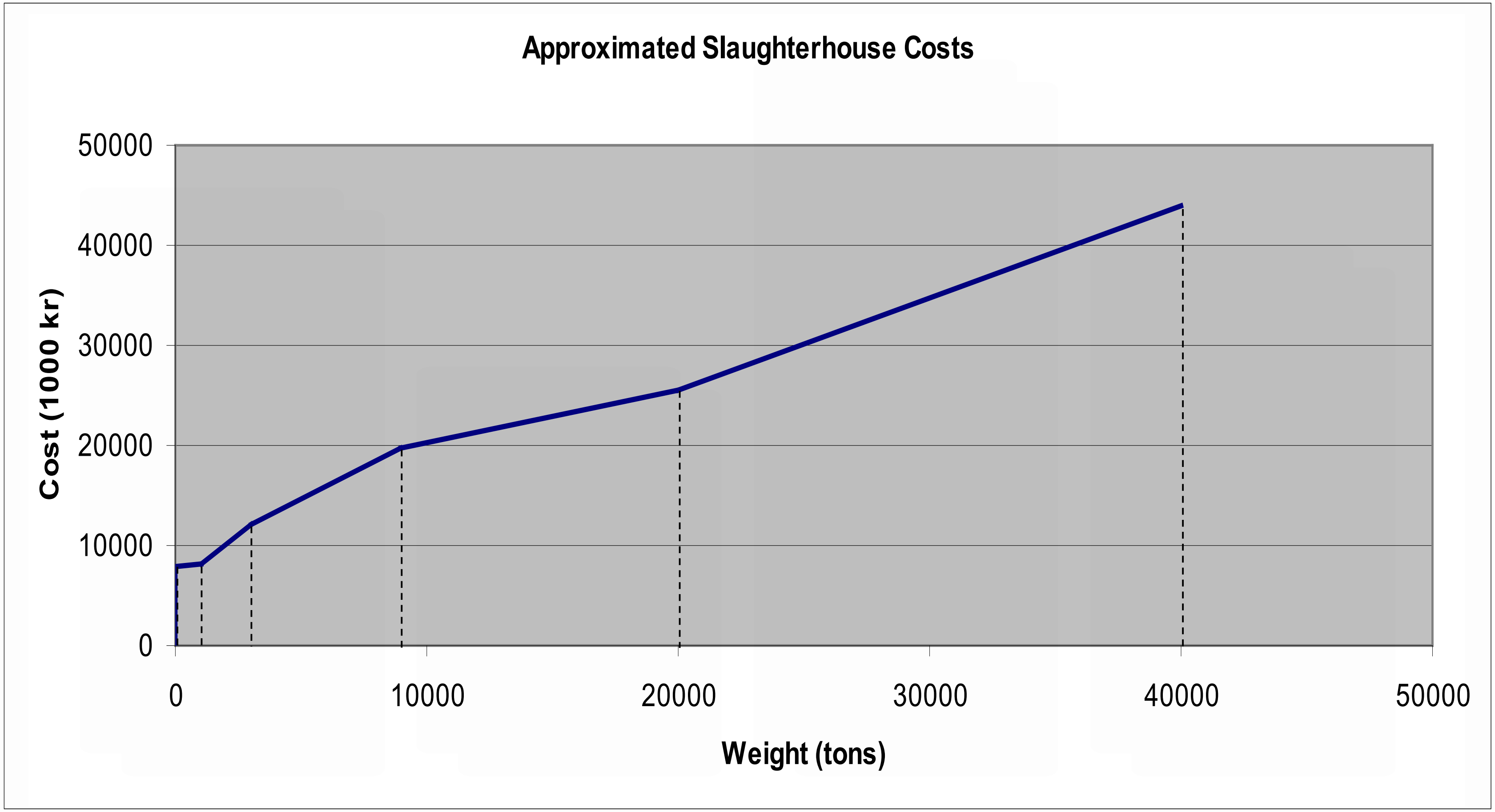
Long-Run vs. Short-Run Costs



Uncertain Demand

- 420 locations with stochastic demand
- Data generated to test the algorithm based on real-world animal population
- Demand is assumed to be normal distributed
- 2 datasets with 100 scenarios each
  - 1 set with low standard deviation
  - 1 set with high standard deviation

First stage: Building a facility… or choose a capacity at location *j*



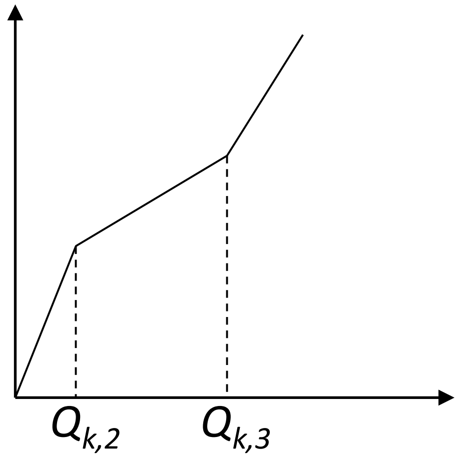
First Stage Problem

$$\min E\left[Q\left(y, \tilde{D}\right)\right]$$

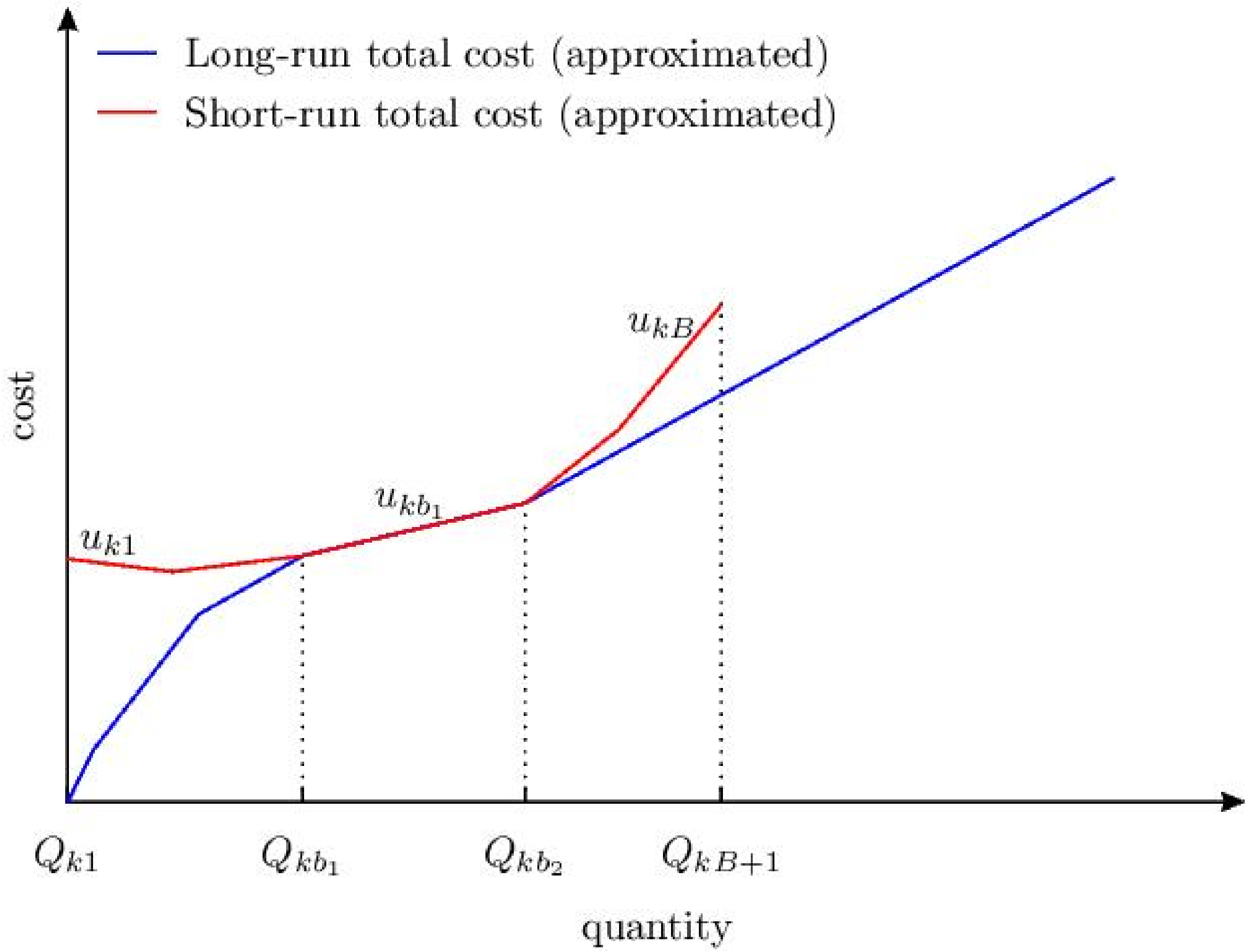
subject to

$$\sum_{k \in \mathcal{K}} y_{jk} = 1, \qquad SOS1, j \in \mathcal{J}$$

$$y_{jk} \in \{0, 1\}, \qquad j \in \mathcal{J}, k \in \mathcal{K}$$



Second stage: Satisfy demand… or allocating animals given a capacity



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Second Stage Problem (for a given scenario)

$$Q(y,D) = \min \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} T_{ij}x_{ij} + \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} \sum_{b\in\mathcal{B}} C_{kb}\mu_{jkb}$$

subject to

$$\sum_{j\in\mathcal{J}} x_{ij} = D_i \qquad i \in \mathcal{I}$$

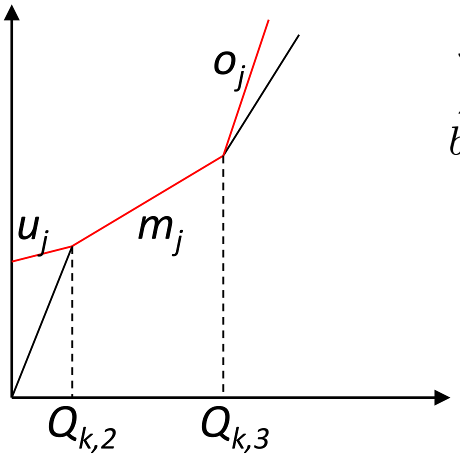
$$\sum_{i\in\mathcal{I}} x_{ij} = \sum_{k\in\mathcal{K}} \sum_{b\in\mathcal{B}} Q_{kb}\mu_{jkb} \qquad j \in \mathcal{J}$$

$$x_{ij} \leq L_{ij}D_i \qquad i \in \mathcal{I}, j \in \mathcal{J}$$

$$\sum_{b\in\mathcal{B}} \mu_{jkb} = y_{jk} \qquad SOS2, j \in \mathcal{J}, k \in \mathcal{K}$$

$$x_{ij} \geq 0 \qquad i \in \mathcal{J}, j \in \mathcal{J}$$

$$\mu_{jkb} \geq 0 \qquad j \in \mathcal{J}, k \in \mathcal{K}, b \in \mathcal{B}$$



Results (using Lagrangean Relaxation)

- Results for $K = 5$ and initial $\lambda_i^s = 650$

η	IWI	S	σ	# of slaughterhouses in best feasible solution	Gap	Runtime [hh:mm]
2	150	1	---	10	5.3%	0:25
1	50	10	low	11	7.9%	3:51
1	50	10	high	10	8.4%	3:55
1	50	50	low	11	7.5%	18:14
1	50	50	high	10	7.6%	19:18
1	50	100	low	11	8.3%	39:53
1	50	100	high	10	6.8%	39:59

Lecture 3: Forecasting and Value of Information

TIØ4285 Production and Network Economics

Spring 2021

Note that the lecture is recorded and streamed through Panopto.

Information about privacy can be found at https://s.ntnu.no/video-recording

Check-in registration is mandatory when physically attending the lecture in G1.

Norwegian University of Science and Technology

Outline

- Forecasting Methods
  - Qualitative methods
  - Quantitative methods
- The Value of Information

Forecasting

Forecasting

Basic Principles of Forecasting:

- The forecast is always wrong
- The longer the forecast horizon, the worse the forecast
- Aggregate forecasts are more accurate

Forecasting Methods

Forecasting methods can be split into four general categories:

- Judgment methods
- Market research methods
- Causal methods
- Time-series methods

Judgment Methods

- Panels of experts
  A group of experts is assembled in order to agree upon a forecast. It is assumed that sharing information and communicating allows for reaching a consensus.

- Delphi method
  A group of experts makes independent forecasts. The results of all forecasts is presented to the experts and they are asked to refine their forecasts based on the now available knowledge.
  The process is repeated a specified number of times or until the expert's forecast are within a certain margin.

Market Research Methods

- Market testing
  Groups of potential customers are assembled and tested for their response to products. Their response is extrapolated to the whole market and used for estimating demand.
  When new models are launched in the automotive industry, potential customers are assembled to check their response to the planned design long before production starts.

- Market survey
  Customer response is estimated by gathering data from various potential customer groups. Data is usually collected through interviews, telephone-based surveys, written surveys, etc.

Causal Methods

Causal methods try to generate forecast based on data other than the data being predicted.

Using a causal forecasting method, you may try to forecast sales for the next three months based on

- inflation
- GNP
- advertising/marketing activities
- weather
- etc.

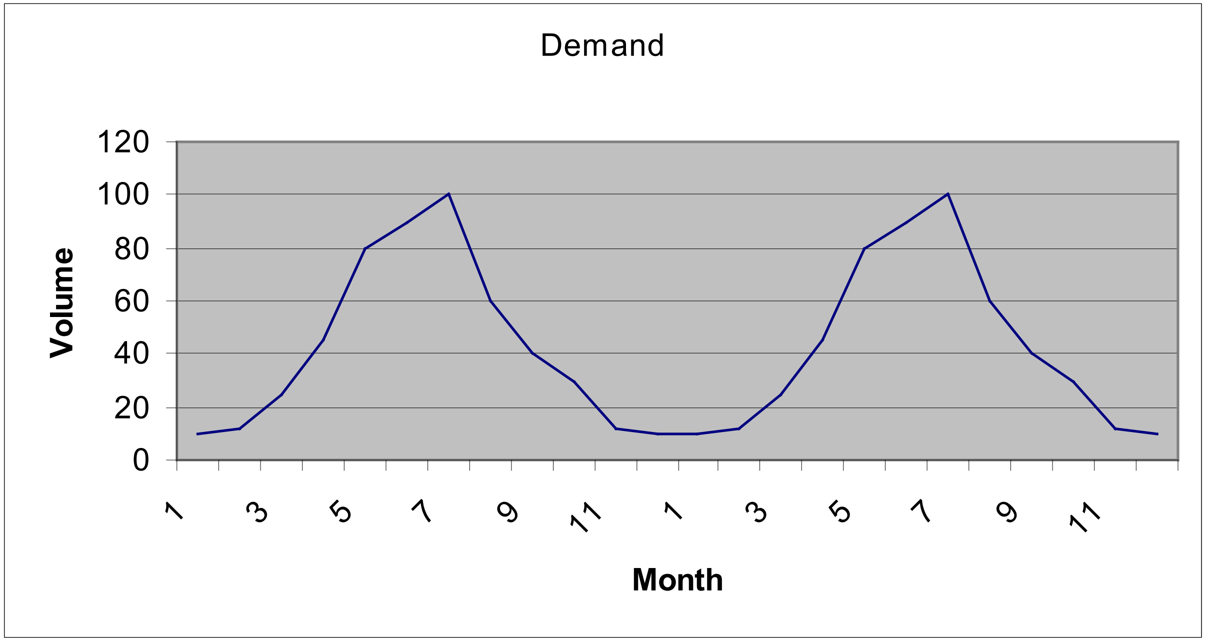
Time-Series Methods

- Time-series based methods use a variety of past data to estimate future data.

- Methods usually require stationary data series (no trends, no seasonality)

- Examples of time-series methods:
  - Last-Value forecasting
  - Averaging forecast
  - Moving average
  - Exponential smoothing
  - Methods for data with trends
  - Methods for seasonal data
  - etc.

Example of time series data



Last-Value forecasting

$$F_{t+1} = x_t$$

- Variance is large because of small sample size (1 data point)

- Worth considering if
  - Constant-level is not likely
  - World is changing so fast that most of the previous observations are irrelevant
  - Often called the naive method (by statisticians)
  - May be the only relevant value (under rapid changes)

Averaging Forecasting

$$F_{t+1} = \sum_{i=1}^{t} \frac{x_i}{t}$$

- Works ok if the process is stable

- Normally limited to young processes (i. e. processes with few observations)

Moving Average

$$F_{t+1} = \sum_{i=t-n+1}^{t} \frac{x_i}{n}$$

- Calculates the average of previous demand over a given number of time periods
- Every past demand point is weighted equally
- Determining the number of time periods n to calculate the average demand for is very important
- Only recent history and multiple observations

Exponential Smoothing

$$
\begin{aligned}
F_{t+1} &= \alpha x_t + (1-\alpha)F_t \\
&= \alpha x_t + \sum_i \alpha(1-\alpha)^i x_{t-i}
\end{aligned}
$$

- α – smoothing constant (between 0 and 1)
- Forecast is a weighted average of the previous forecast and the last demand
- More recent value receive more weight than past values
- Similar to the Moving Average Forecast, but tracks changes in the process faster
- Lags behind a continuous trend
- Choice of the smoothing constant α (weight of the last demand) is important

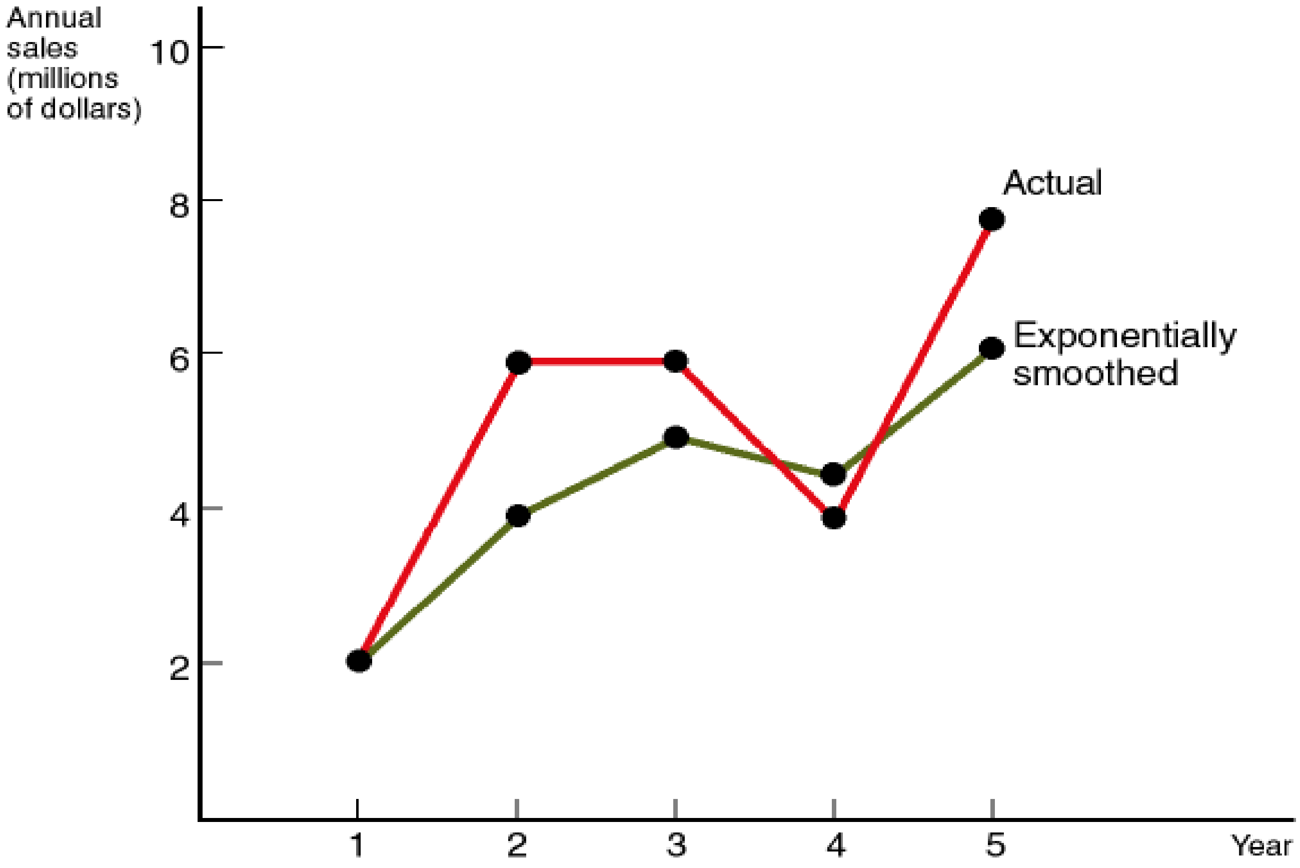


FIGURE 6.9 Sales of Firm, Actual and Exponentially Smoothed

Methods for Data with Trends

- Regression Analysis,
  tries to fit a straight line into data points. Thus, it's identifying linear trends.

- Holt's Method,
  combines the concepts of exponential smoothing with the ability to capture linear trends.

Box-Jenkins method

Box-Jenkins

- Alternative name: ARIMA method
- Box-Jenkins is a methodology for identifying, estimating, and forecasting ARIMA models
- ARIMA stands for “AutoRegressive Integrated Moving-Average"
  - Made up of sums of autoregressive and moving-average components
- Requires a large amount of data (historical observations of a variable)
- Iterative process

ARIMA models

- Describes a stochastic process or a model of one

- Explains the variable Y based only on the history of Y
  - No other explanatory variables
  - Not derived from economic theory

- An ARIMA process is stationary
  - mean, variance and autocorrelation structure do not change over time

ARIMA components

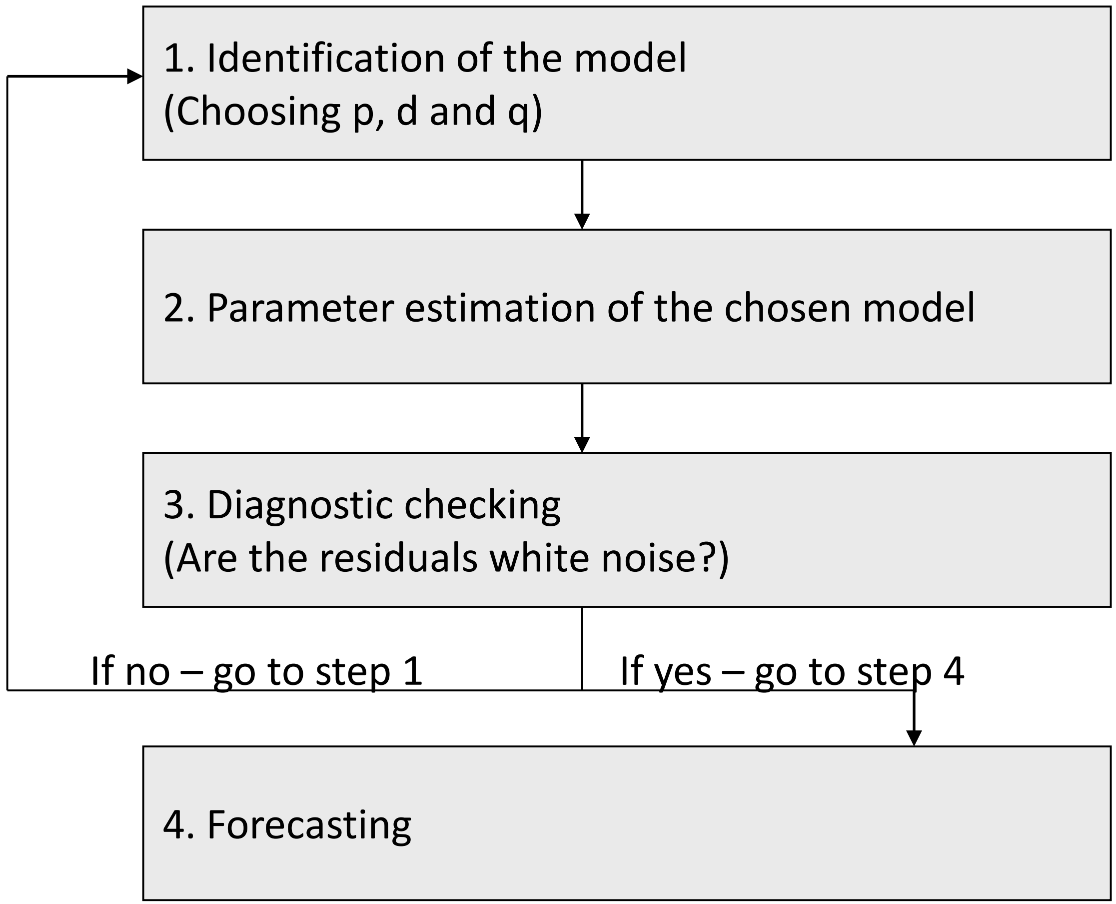
- AR (Autoregressive)
  - Process that can be described by a weighted sum of its previous values and a white noise error
  - AR(1): $Y_t = \alpha Y_{t-1} + \theta_t$
  - AR(p): $Y_t = \sum_{p=1}^{n} \alpha_p Y_{t-p} + \theta_t$
- MA (Moving Average)
  - Process that can be described by a constant plus a moving average of the current and past error terms
  - MA(1): $Y_t = \mu + \beta\theta_t$
  - MA(q): $Y_t = \mu + \sum_{n=0}^{q} \beta_{t-n}\theta_{t-n}$

ARIMA components (2)

- ARMA (p, q)
  - May be non-stationary (integrated)
    - Non-constant mean, variance or covariance
  - The *d*-differences of a time series that are integrated of order *d* is stationary
    - 1-differences of Y: $Y_t - Y_{t-1}$

- ARIMA (p, d, q)
  - *p* AR terms, integrated of order *d* and *q* MA terms

The Box-Jenkins Methodology



2. Parameter estimation

- Estimate the value of the *p* AR and the *q* MA parameters
  - Simple problems: use least squares method
  - Otherwise: nonlinear (in parameter) estimation methods
  - Done by several statistical packages
    - (SPSS, MiniTab, S-Plus, R,…..)

3. Diagnostic checking

- Test if the estimated model fits the historical data

- In particular: test if the residuals are white noise
  - If not – the model is not appropriate and a new one must be estimated
  - If they are – the model may be good and used for forecasting

4. Forecasting

- Use the model to forecast new values

- Can be used as a test of the goodness of the model
  - Out-of-sample tests
  - Cross validation

Conclusions

- Requires no economic theory
  ÷ Cannot make use of any economic theory
  \+ Cannot misuse economic theory/ make false assumptions

- Usually provides accurate forecasts

- Requires a large amount of data

- Only needs data for one variable

Regression analysis

- Applicable if causal link is strong
- Assume that a company's demand function can be written on the form

$$Y = A + B_1x_1 + B_2x_2 + B_3I + B_4P_r$$

- Y is the dependent variable
- x_1, x_2, I, P_r are examples of independent variables.
- We wish to estimate

$$A, B_1, B_2, B_3, B_4$$

based on historical data

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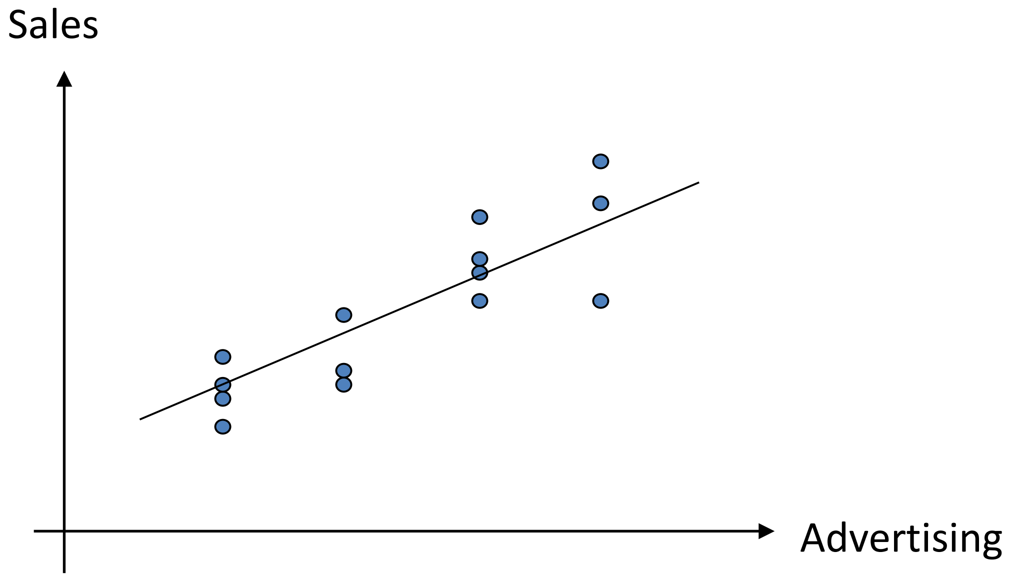
Population regression curve



- Line shows the value of E(Y|X)

$$Y = A + B_1 x_1$$

- The line is fitted so that squared deviation is minimized

Explained variance and the constant of determination

- Variance in dependent variable

$$\sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

and

$$Y_i - \bar{Y} = (Y_i - \hat{Y}_i) + (\hat{Y}_i - \bar{Y})$$

- Explained variance

$$\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$$

- Unexplained variance

$$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

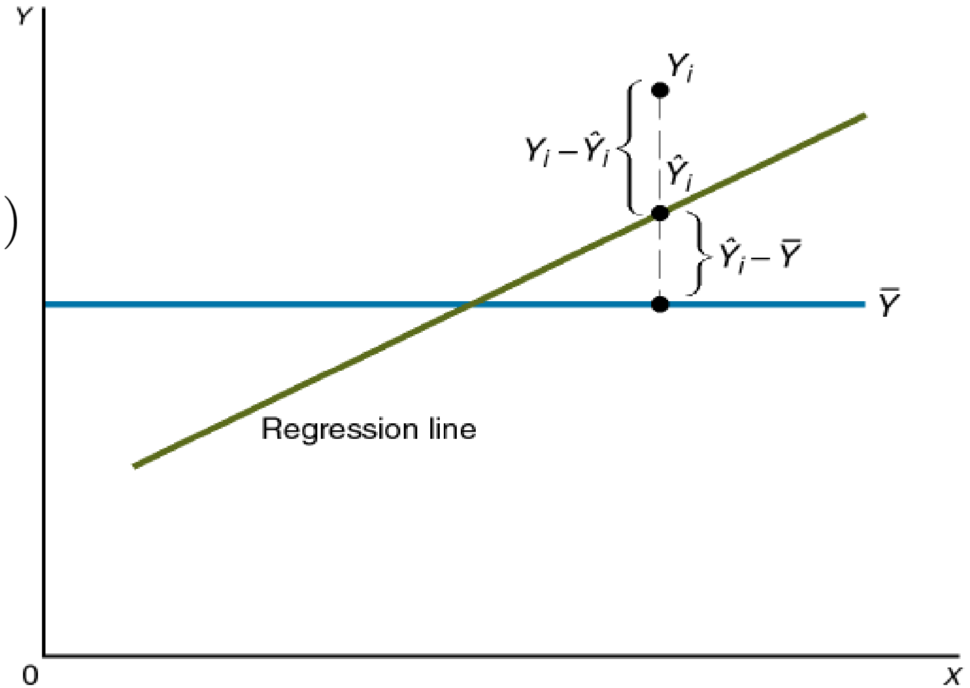


FIGURE 5.13 Division of $(Y_i - \bar{Y})$ into Two Parts: $(Y_i - \hat{Y}_i)$ and $(\hat{Y}_i - Y)$

Explained variance and the constant of determination

$$R^2 = 1 - \frac{\text{Variation not explained by regression}}{\text{Total variation}}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$$

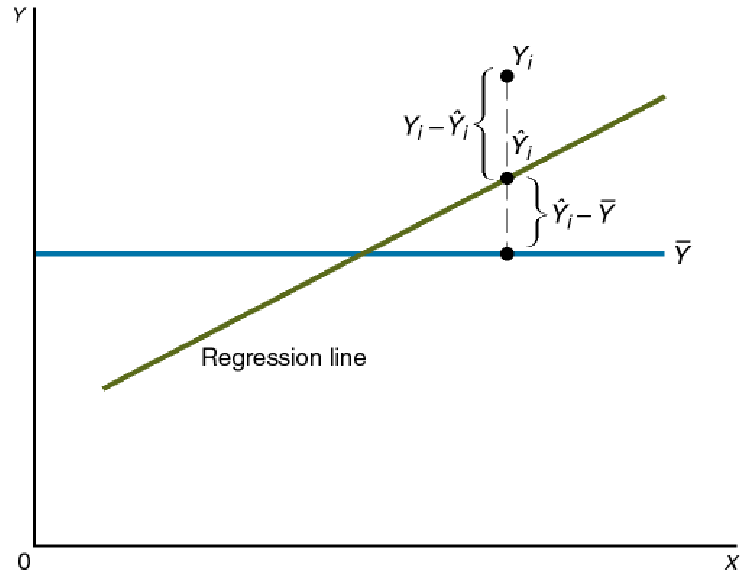


FIGURE 5.13 Division of $(Y_i - \bar{Y})$ into Two Parts: $(Y_i - \hat{Y}_i)$ and $(\hat{Y}_i - Y)$

Methods for Seasonal Data

Seasonal data has to be accounted for.

1. Calculate seasonal factor

$$\text{Seasonal factor} = \frac{\text{average for the period}}{\text{overall average}}$$

2. Calculate seasonally adjusted values

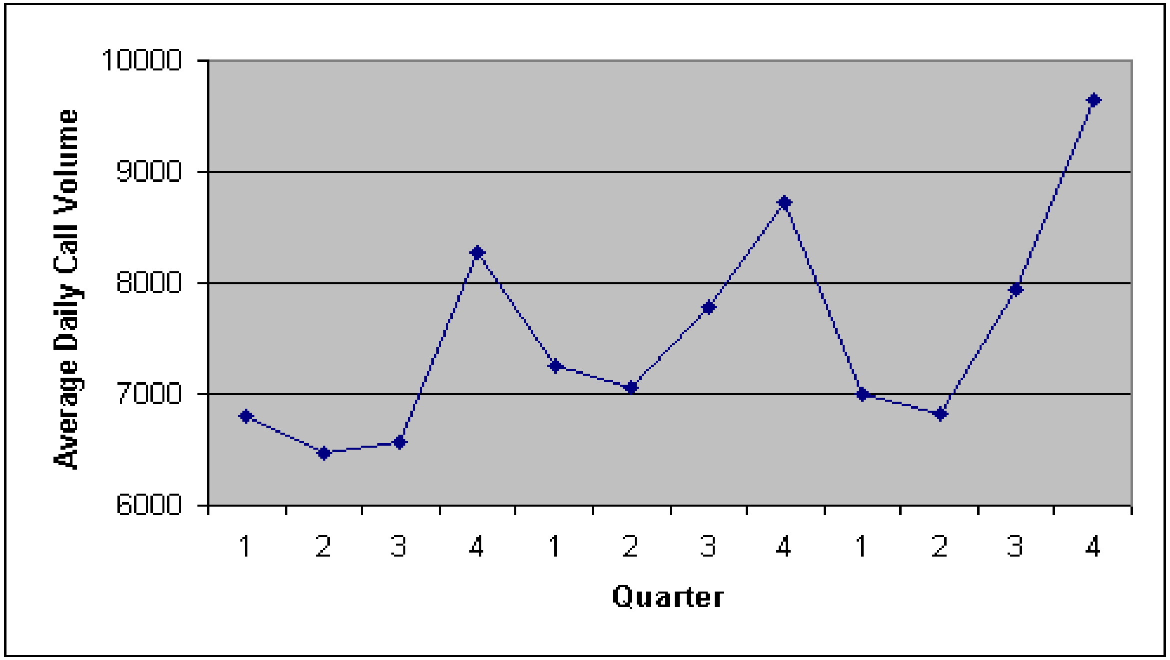
$$\text{Seasonal adjusted value} = \frac{\text{actual value}}{\text{seasonal factor}}$$

3. Select time-series forecasting method
4. Apply this method to the seasonally adjusted data
5. Multiply this forecast with the seasonal factor in order to get the next actual value

Example on Forecasting Seasonal Demand I

CCW's Average Daily Call Volume

Year	Quarter	Call Volume
1	1	6809
1	2	6465
1	3	6569
1	4	8266
2	1	7257
2	2	7064
2	3	7784
2	4	8724
3	1	6992
3	2	6822
3	3	7949
3	4	9650



Example on Forecasting Seasonal Demand II

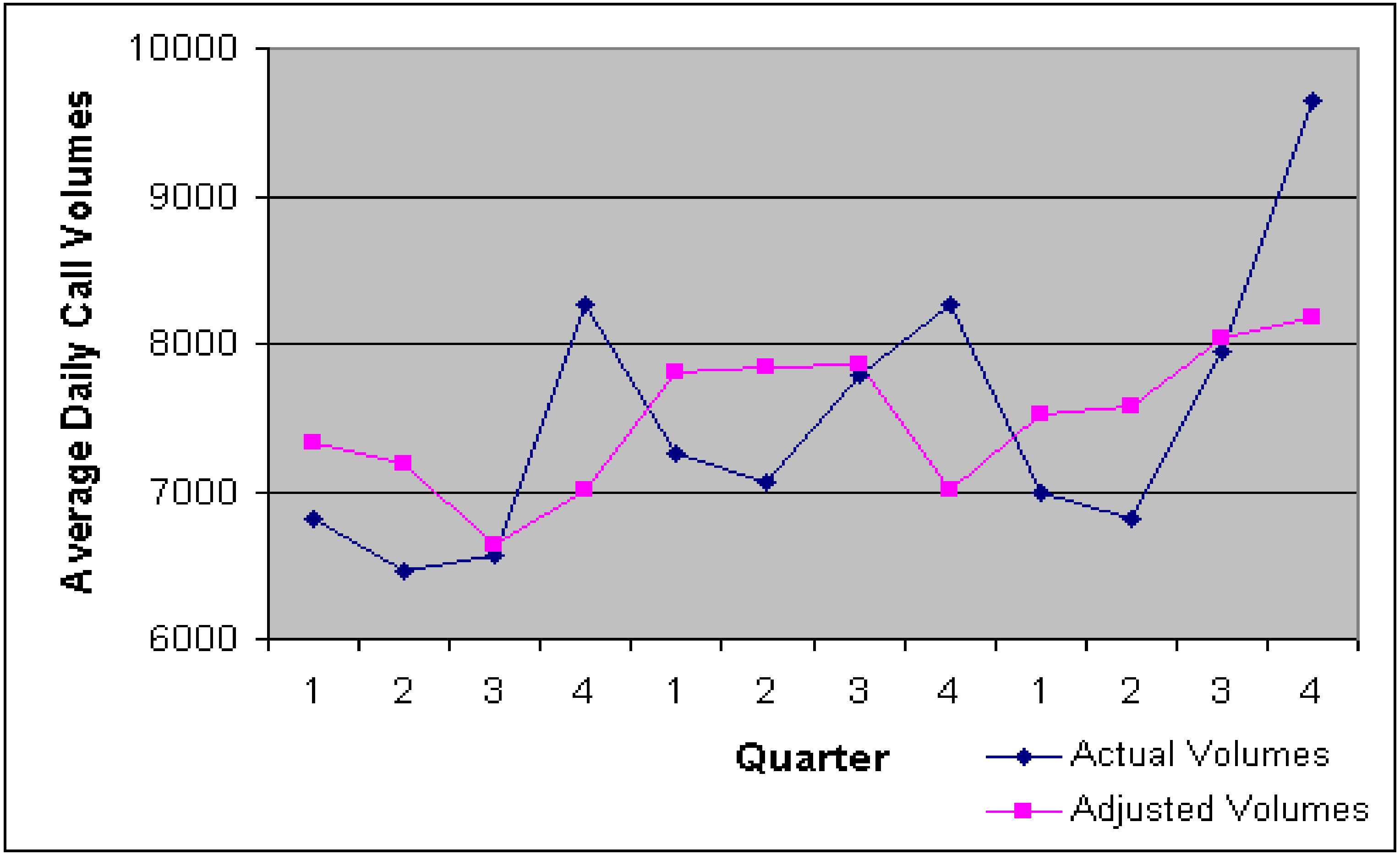
Calculating seasonal factors

Quarter	Three-Year Average	Seasonal Factor
1	7019	$\frac{7019}{7529} = 0.93$
2	6784	$\frac{6784}{7529} = 0.90$
3	7434	$\frac{7434}{7529} = 0.99$
4	8880	$\frac{8880}{7529} = 1.18$
Total=30117		
Average= $\frac{30117}{4} = 7529$		

Example on Forecasting Seasonal Demand III

Year	Quarter	Seasonal Factor	Actual Call Volume	Seasonally Adjusted Call Volume
1	1	0.93	6809	7322
1	2	0.90	6465	7183
1	3	0.99	6569	6635
1	4	1.18	8266	7005
2	1	0.93	7257	7803
2	2	0.90	7064	7849
2	3	0.99	7784	7863
2	4	1.18	8274	7012
3	1	0.93	6992	7518
3	2	0.90	6822	7580
3	3	0.99	7949	8029
3	4	1.18	9650	8178

Example on Forecasting Seasonal Demand III



Demand Forecasting and Scenario Generation

- Assume demand can be forecasted perfectly using an autoregressive process

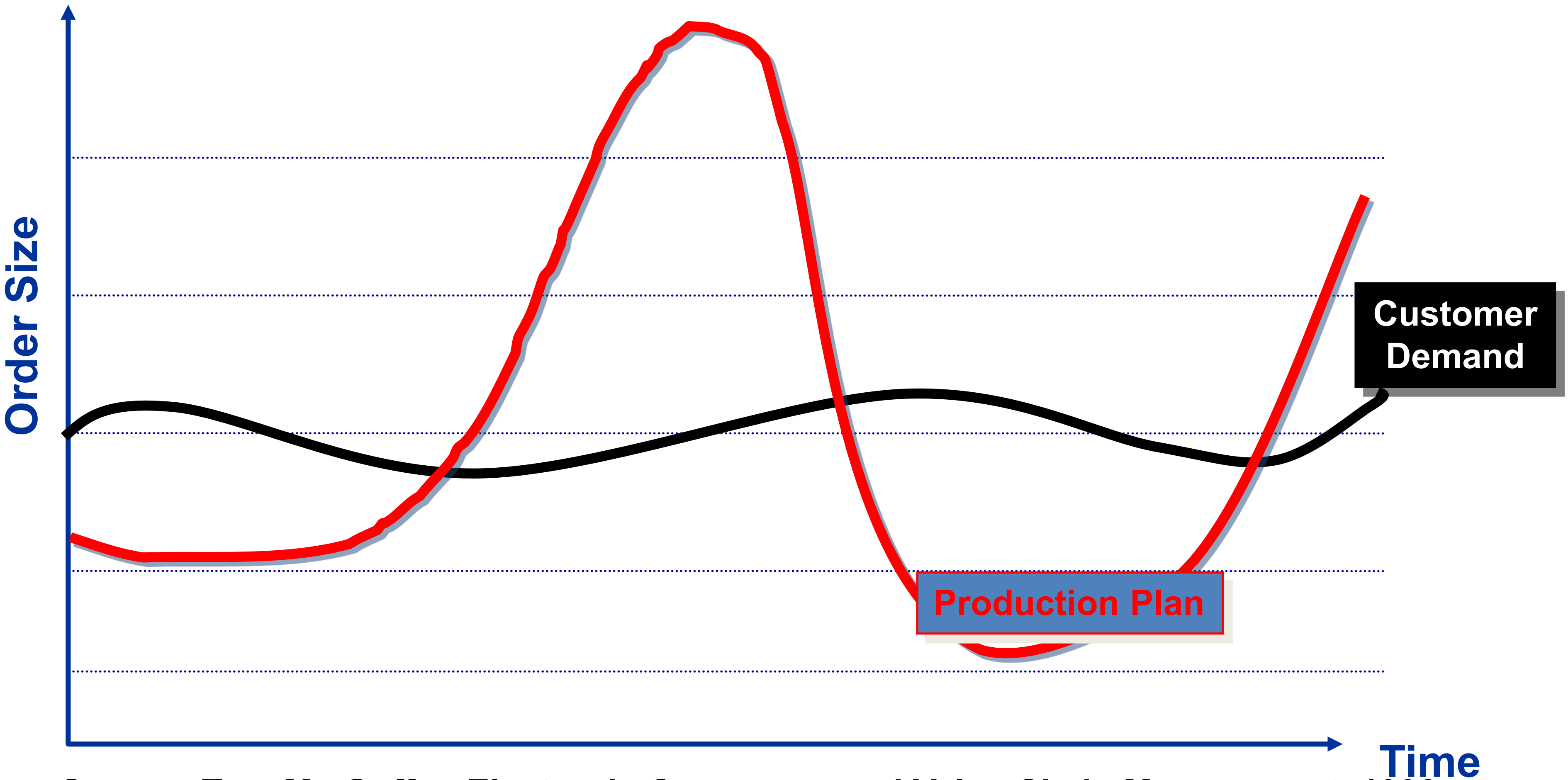
$$\hat{d}_{t+1} = \alpha + \sum_{i=1}^{N} \beta_i \cdot d_{t-i+1} + \varepsilon(\omega)$$

- But: The forecast is always wrong!
  - So there is an error term.

- Put all uncertainty in error term.
  - White noise $\rightarrow$ normal distribution with mean 0
  - Generate scenario tree for your error term and combine tree with forecasting model

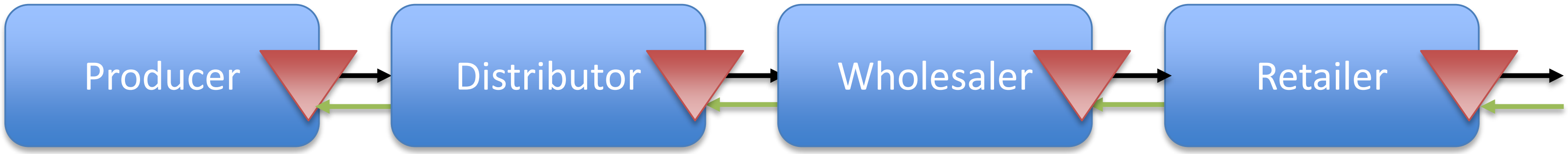
The Value of Information

The Dynamics of the Supply Chain



Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

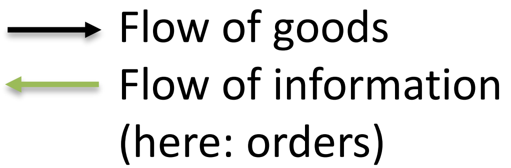
The Supply Chain



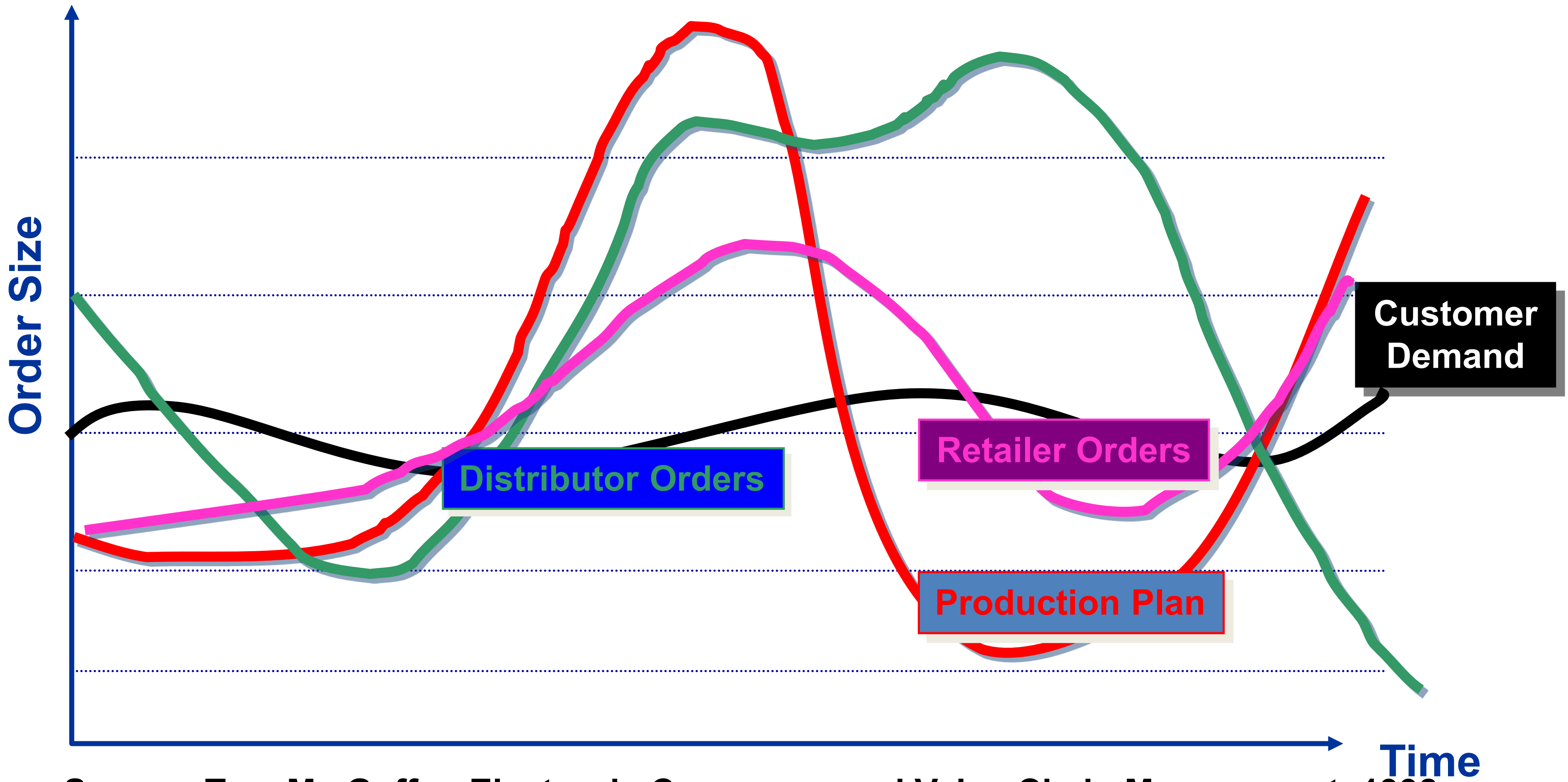
- Upstream receives orders from downstream actor

- Order is satisfied from stock or backlogged

- Leadtimes in both flow of goods <u>and</u> flow of information



The Dynamics of the Supply Chain



Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

What are the causes…

- For example
  - Promotional sales
  - Price fluctuations
  - Volume and transportation discounts
    - Batching
  - Inflated orders
  - Long cycle times
  - Demand forecasts
  - Lack of visibility of demand information

The Bullwhip Effect and its Impact on the Supply Chain

- Consider the order pattern of a single color television model sold by a large electronics manufacturer to one of its accounts, a national retailer.

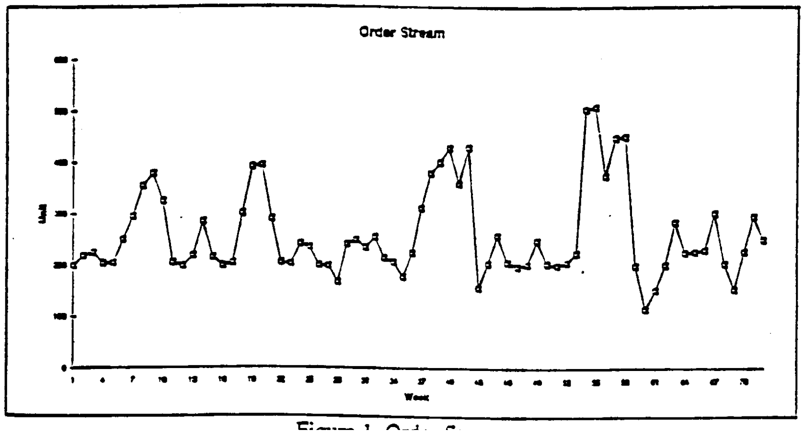


Figure 1. Order Stream

Huang at el. (1996), Working paper, Philips Lab

The Bullwhip Effect and its Impact on the Supply Chain

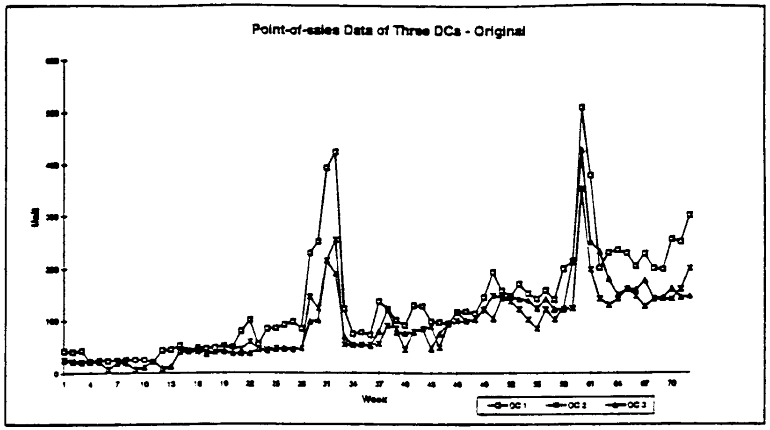


Figure 2. Point-of-sales Data-Original

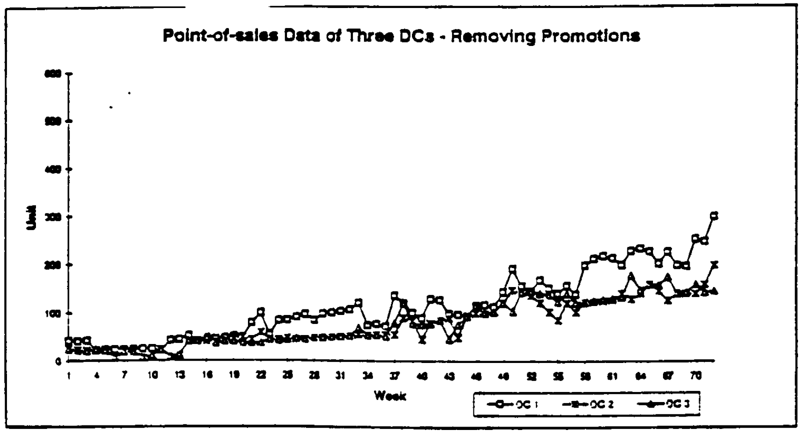


Figure 3. POS Data After Removing Promotions

The Bullwhip Effect and its Impact on the Supply Chain

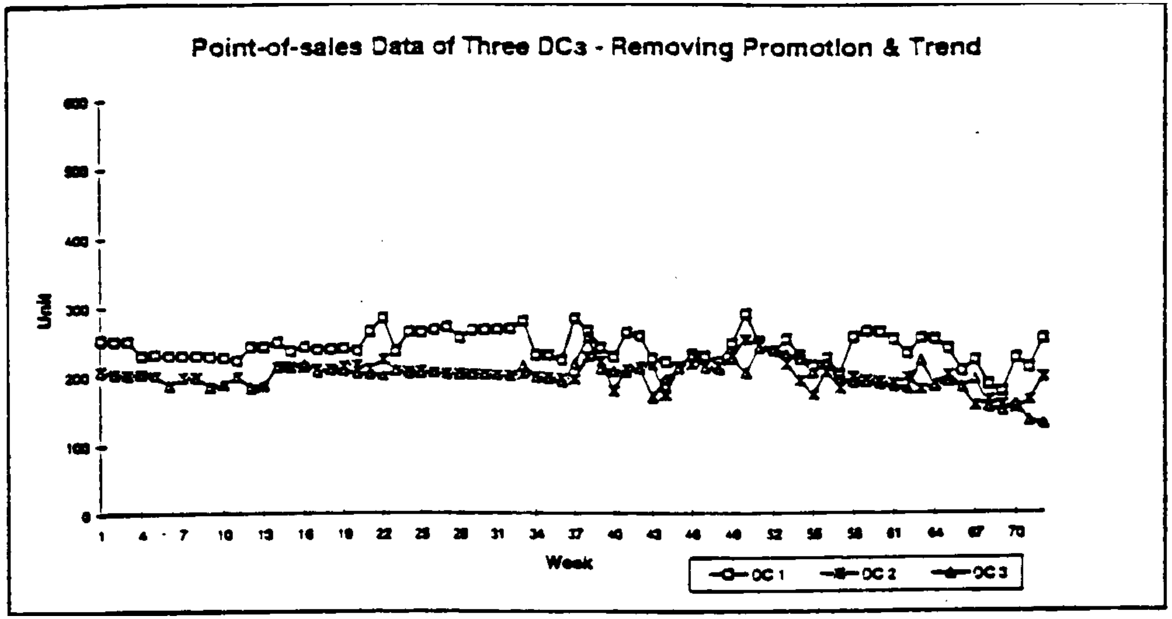
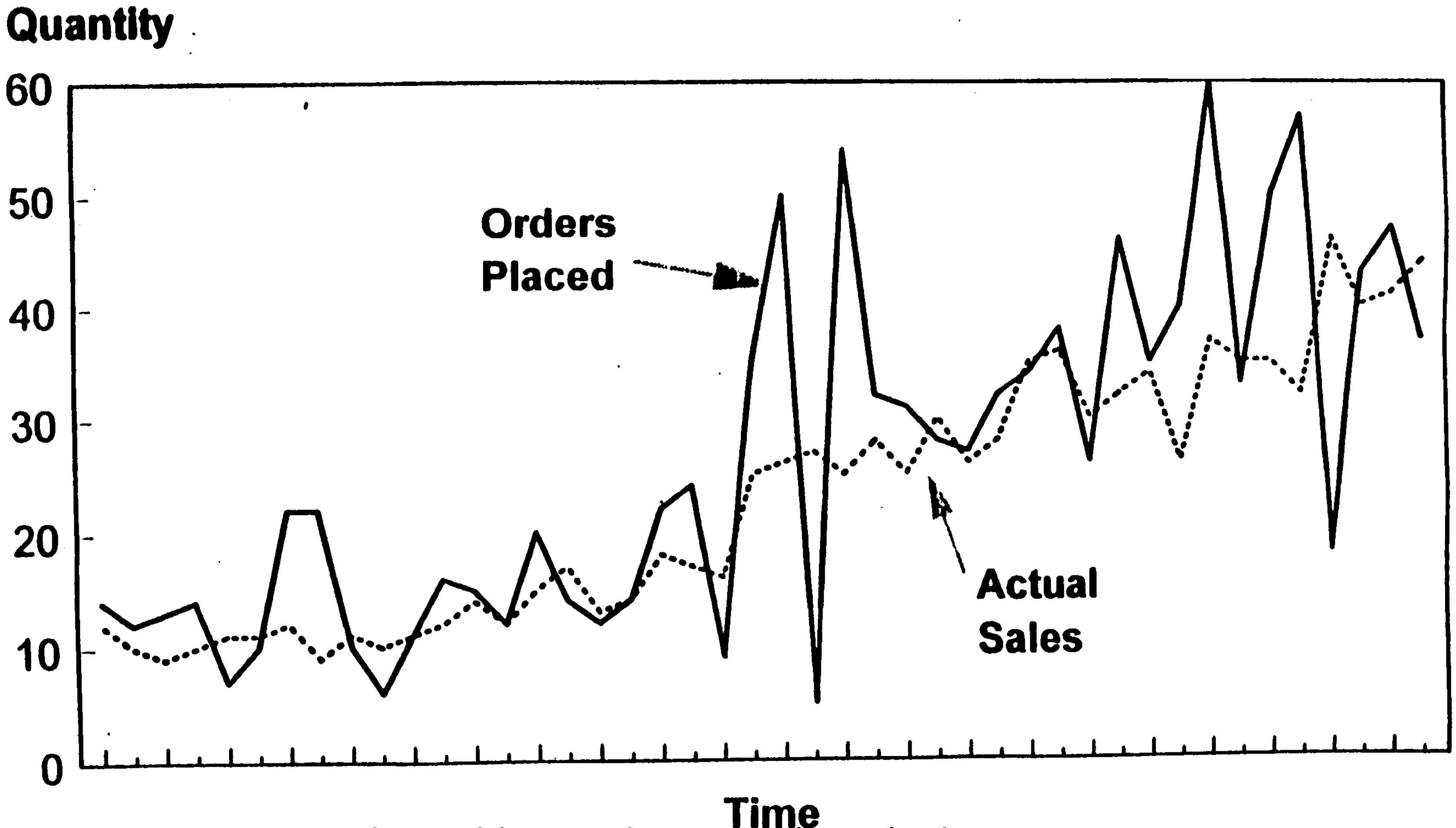


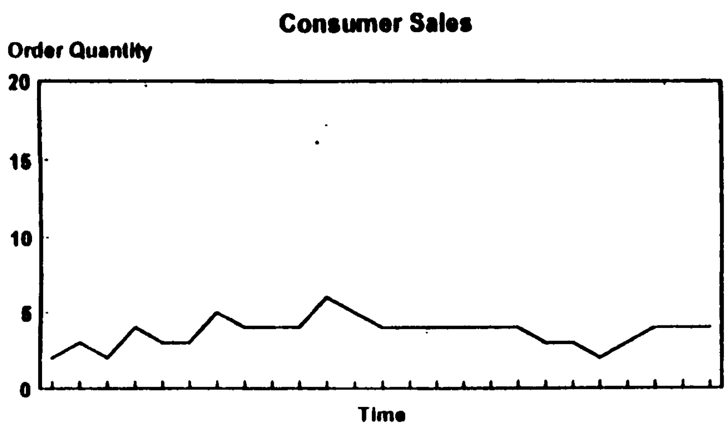
Figure 4. POS Data After Removing Promotion & Trend

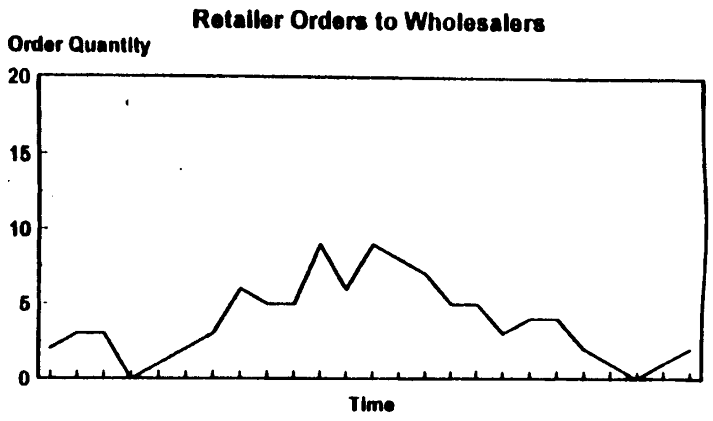
Higher Variability in Orders Placed by Computer Retailer to Manufacturer Than Actual Sales

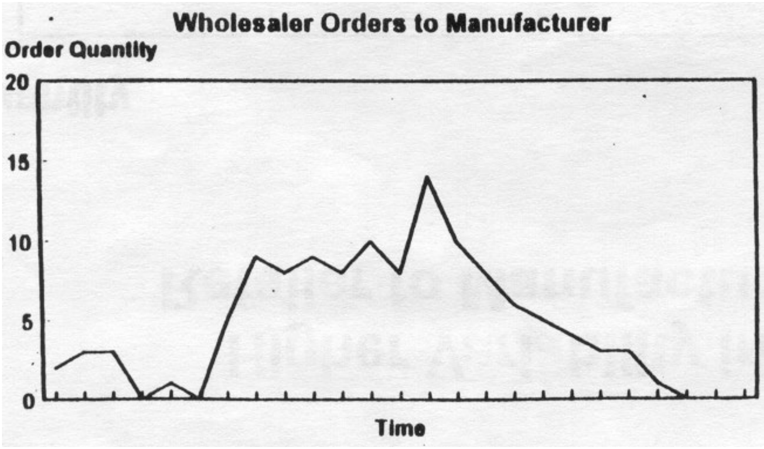


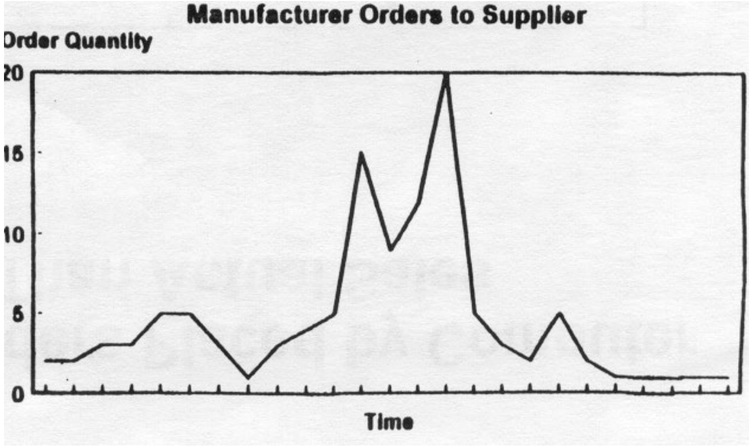
Lee, H, P. Padmanabhan and S. Wang (1997), Sloan Management Review

Increasing Variability of Orders up the Supply Chain









Lee, H, P. Padmanabhan and S. Wang (1997), Sloan Management Review

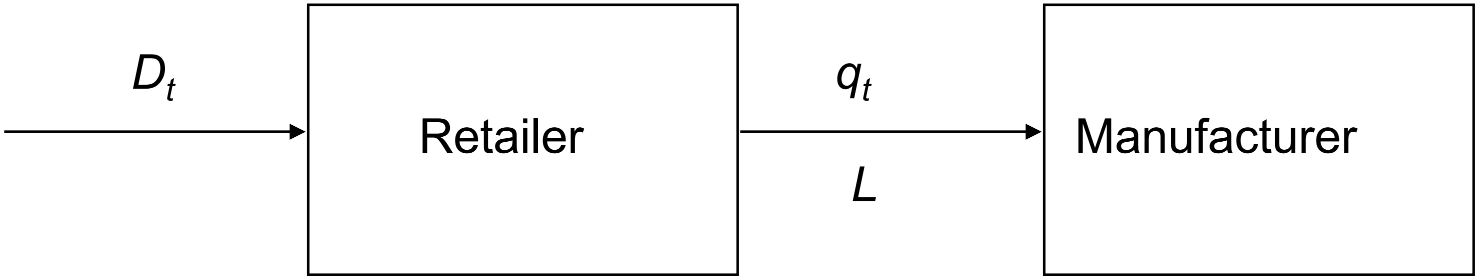
We Conclude ….

- Order Variability is amplified up the supply chain; upstream echelons face higher variability.

- What you see is not what they face.

What are the Causes….

- Single retailer, single manufacturer.
  - Retailer observes customer demand, D_t.
  - Retailer orders q_t from manufacturer.



Formulas

- Target inventory

$$y_t = \widehat{\mu}_t^L + z\widehat{\sigma}_t^L$$

$\widehat{\mu}_t^L$ – estimate of mean leadtime demand

$\widehat{\sigma}_t^L$ – estimate of s.d. of forecasting error over leadtime demand

z – service level parameter

- Assume demand is

$$D_t = \mu + \rho D_{t-1} + \varepsilon_t$$

$$|\rho| < 1$$

Forecasting

- Moving average forecast

$$\widehat{\mu}^t = \frac{\sum_{i=1}^{p} D_{t-i}}{p}$$

$$\varepsilon_t = D_t - \widehat{\mu}_t$$

- Let q_t be the order quantity

$$q_t = y_t - y_{t-1} + D_{t-1}$$

(the difference in target inventory + demand)

$$\begin{aligned} q_t &= \widehat{\mu}_t^L - \widehat{\mu}_{t-1}^L + z\left(\widehat{\sigma}_t^L - \widehat{\sigma}_{t-1}^L\right) + D_{t-1} \\ &= \left(1 + \frac{L}{p}\right) D_{t-1} - \left(\frac{L}{p}\right) D_{t-p-1} + z\left(\widehat{\sigma}_t^L - \widehat{\sigma}_{t-1}^L\right) \end{aligned}$$

Variance of the orders

$$
\begin{aligned}
Var(Q_t) &= \left(1+\frac{L}{p}\right)^2 Var(D_{t-1}) + \left(\frac{L}{p}\right)^2 Var(D_{t-p-1}) \\
&\quad - 2\left(\frac{L}{p}\right)\left(1+\frac{L}{p}\right) Cov(D_{t-1}, D_{t-p-1}) \\
&\quad + 2z\left(\frac{L}{p}\right)\left(1+\frac{L}{p}\right) Cov(D_{t-1}, \sigma_t^L) \\
&\quad + z^2 Var\left(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L\right) \\
&= \left[1+\left(\frac{2L}{p}+\frac{2L^2}{p}\right)(1-\rho^p)\right] Var(D) \\
&\quad + 2z\left(1+\frac{2L}{p}\right) Cov\left(D_{t-1}, \sigma_t^L\right) \\
&\quad + z^2 Var\left(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L\right)
\end{aligned}
$$

where

$$Var(D) = \frac{\sigma^2}{1-\rho^2}, Cov(D_t, D_{t-p-1}) = \frac{\rho^p}{1-\rho^2}\sigma^2$$

Variance of the orders

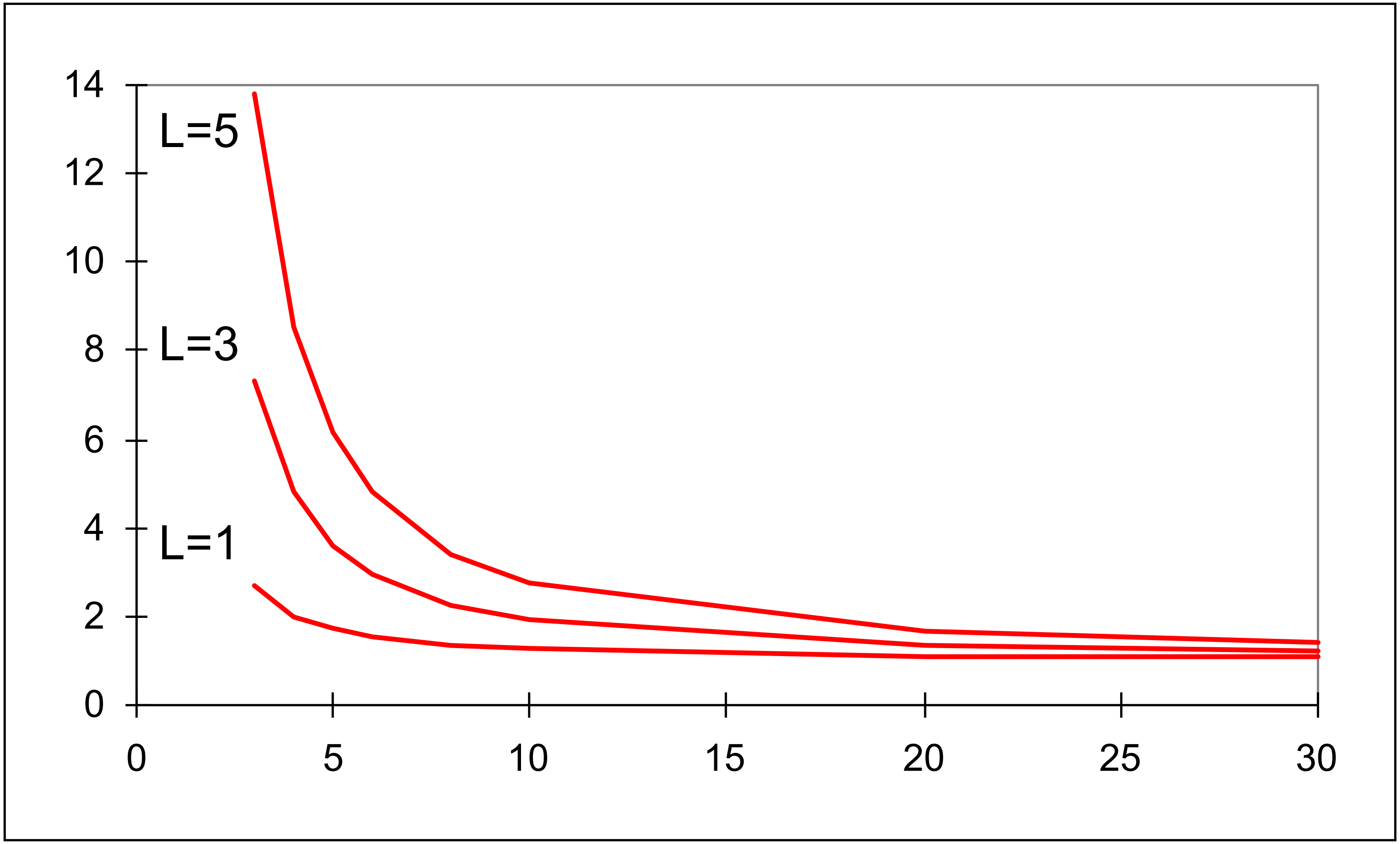
- For moving average forecasts and for the demand process we have assumed

$$Cov(D_{t-1}, \hat{\sigma}_t^L) = 0$$

- Variance of orders placed by the retailer to the manufacturer (upstream unit), relative to variance in demand

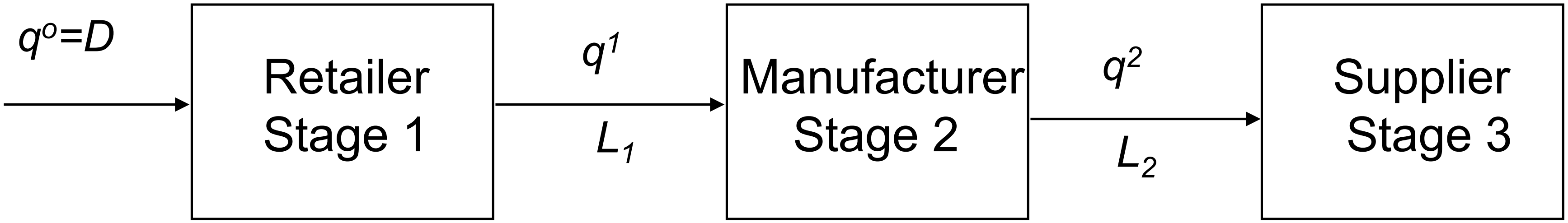
$$\frac{Var(q)}{Var(D)} \geq 1 + \frac{2L}{p} + \frac{2L^2}{p^2}$$

Var(q)/Var(D): For Various Lead Times (as a function of p)



Multi-Stage Supply Chains

- Consider a multi-stage supply chain:
  - Stage i places order qi to stage i+1.
  - Li is lead time between stage i and i+1.



Multistage centralised

- Each stage has complete information about the customer demand and uses the same estimate.
- The second level receives the orders from the first level and the demand forecast from the first level.
- The third level receives the order form the second level and the demand forecast from the first level
- ……….

$$\hat{\mu}_t = \frac{\sum_{i=1}^{p} D_{t-i}}{p}$$

Each stage k follow an order-up-to policy with target ($z = 0$)

$$y_t^k = L_k \hat{\mu}_t$$

Then

$$\frac{Var(q_t^k)}{Var(D)} \geq 1 + \left(\frac{2 \sum_{i=1}^{k} L_i}{p} + \frac{2 \left(\sum_{i=1}^{k} L_i \right)^2}{p^2} \right)$$

1) All demand information is centralized
2) Every stage of the supply chain uses the same forecast technique

Coping with the Bullwhip Effect

- Reduce Variability and Uncertainty
  - POS
  - Sharing Information
  - Year-round low pricing

- Reduce Lead Times
  - EDI
  - Cross Docking

- Alliance Arrangements
  - Vendor managed inventory
  - On-site vendor representatives

Distribution Strategies

- Warehousing

- Direct Shipping
  - No DC needed
  - Lead times reduced
  - “smaller trucks”
  - no risk pooling effects

- Cross-Docking

Cross Docking

- In 1979, Kmart was the king of the retail industry with 1891 stores and average revenues per store of $7.25 million

- At that time Wal-Mart was a small niche retailer in the South with only 229 stores and average revenues about half of those Kmart stores.

- Ten years later, Wal-Mart transformed itself; it has the highest sales per square foot, inventory turnover and operating profit of any discount retailer. Today Wal-Mart is the largest and highest profit retailer in the world.

The Bullwhip Effect: Managerial Insights

- Exists, in part, due to the retailer's need to estimateand update the mean and variance of demand.
- The increase in variability is an increasing function of the lead time.
- The more complicated the demand models and the forecasting techniques, the greater the increase.
- Centralized demand information can reduce the bullwhip effect, but will not eliminate it.

Lecture 4: Asymmetric Information

TIØ4285 Production and Network Economics

Spring 2021

Note that the lecture is recorded and streamed through Panopto.

Information about privacy can be found at https://s.ntnu.no/video-recording

Check-in registration is mandatory when physically attending the lecture in G1.

Norwegian University of Science and Technology

Outline

- Asymmetric information
  - Definition
  - Why is it a problem?

- Adverse selection
  - Definition
  - Problems arising from adverse selection
    - Market for “Lemons”
    - Price discrimination
    - Market power

- Principal-Agent problems / Moral Hazard
  - Definition
  - Production efficiency
  - Risk sharing/ trade-off with production efficiency
  - Contract design

Definition – asymmetric information

- Some player has useful private information
  - An information partition that is different and not worse than another player's
- In contrast: in the case of symmetric information no player has an informational advantage
- Example: seller knows the quality of a product whilst the buyer does not
- In a competitive market with full information, consumers can buy whatever quality good they want at its marginal cost
  - This may not be the case when we have asymmetric information

Opportunistic behavior

- Taking selfish advantage of circumstances
  - little regard for principles

- The more informed party exploits the less informed party
  - Takes advantage of the information asymmetry

- Leads to market failures

Problems arising from asymmetric information

- There are two main form of problems arising from asymmetric information
  - Adverse selection
  - Moral hazard

- Both exist because of opportunistic behavior

Adverse selection

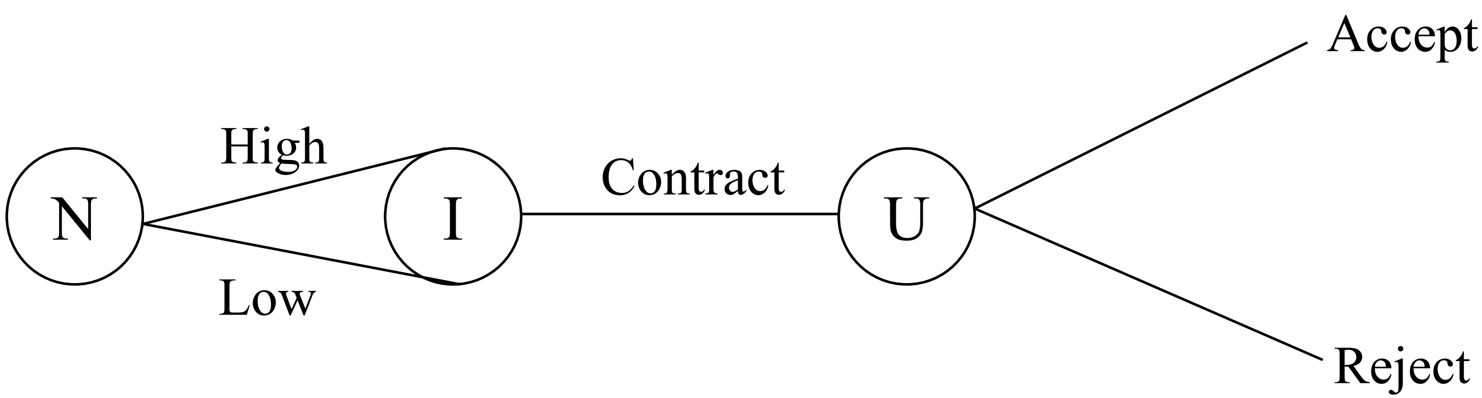
- An informed person benefiting from trading with a less informed person through an unobserved characteristic of the informed person

- Example:
  - Insurance
  - Market for “Lemons”
  - Maternity leave

- Creates market failure by reducing the size of a market
  - Prevents desirable transactions

Adverse selection – game tree

- N = Nature
- I = Informed player
- U = Uninformed player



Moral hazard

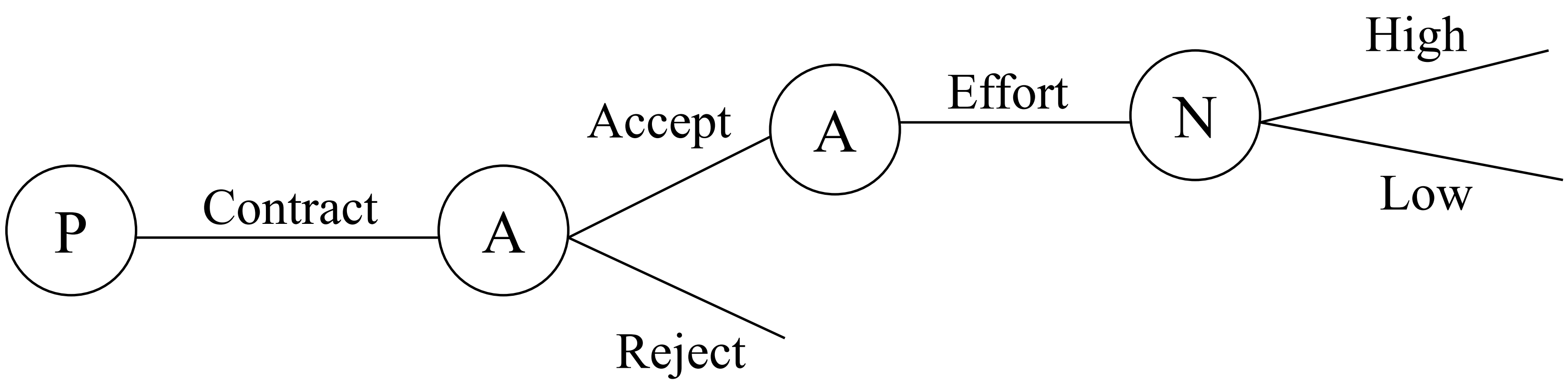
- An informed person benefiting from trading with a less informed person through an unobserved action or through unobserved information

- Example:
  - Insurance
  - Employee

- Creates market failures by reducing efficiency/ harm society

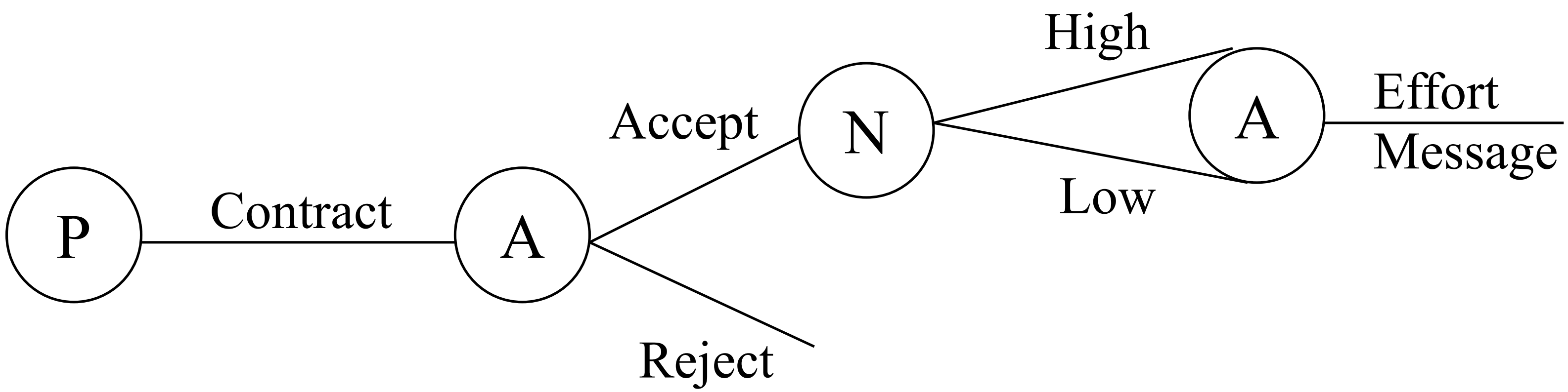
Moral hazard with hidden action

- N = Nature
- P = Principal
- A = Agent



Moral hazard with hidden information

- N = Nature
- P = Principal
- A = Agent



Example: Difference between moral hazard and adverse selection

- George and Marge both enjoy skydiving (which is unknown to the insurance company)
- Both wish to sign a life insurance because of the high risk associated with skydiving
- George will skydive whether or not he has a life insurance
- Marge will only skydive if she has a life insurance

- Consider the insurance company:
  - Is adverse selection a problem here?
  - What about moral hazard?

Consequences of adverse selection

Drive out high quality goods

- Sellers have more information than buyers
- Good quality products are driven out of the market by lower quality products

- Example:
  - Used cars (Lemons)

Example: Market for Lemons

- Used cars market
- Assumptions:
  - All cars look the same (you can not see the quality by studying the car)
  - There are two groups of qualities: good cars and lemons

  - Many potential buyers, each will pay
    - 1000 $ for a lemon
    - 2000 $ for a good car

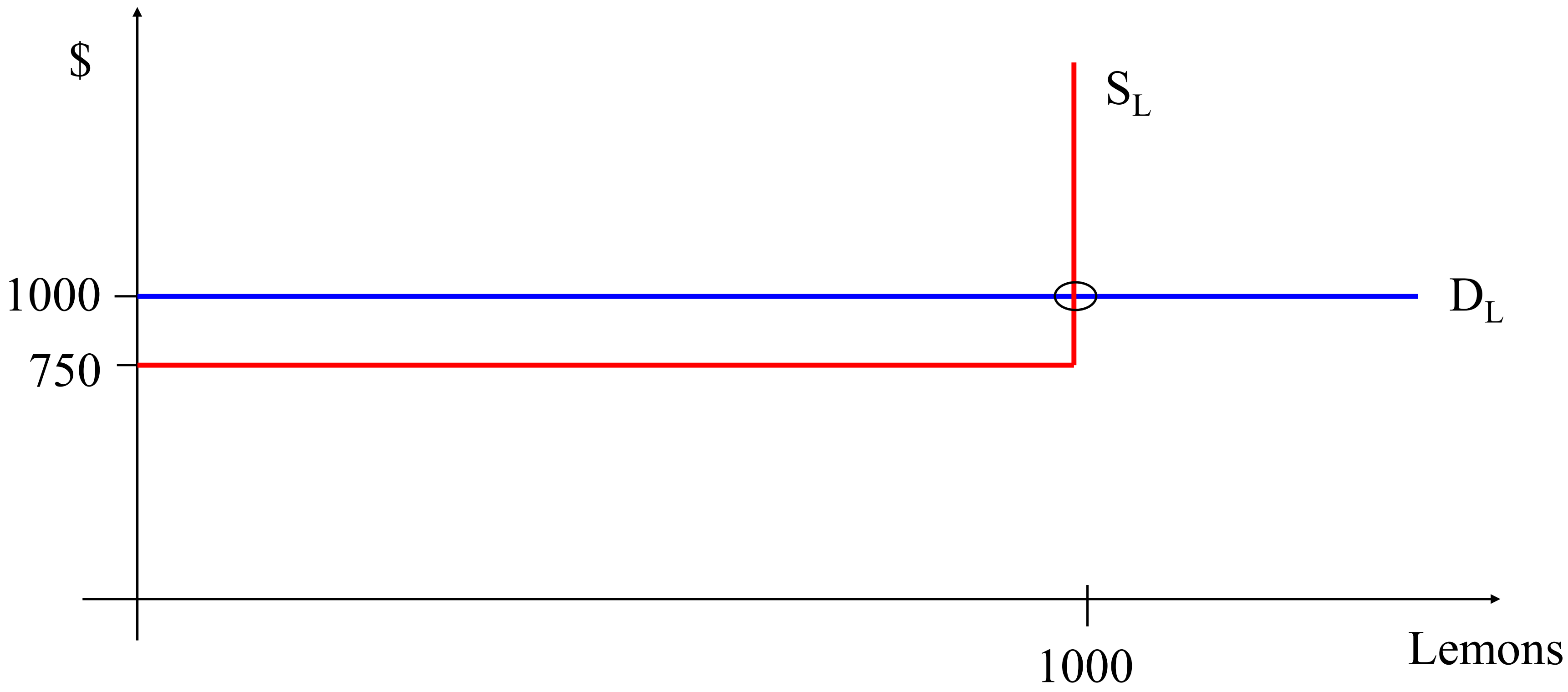
  - There are 1000 lemons and 1000 good cars for sale
  - The reservation price for the sellers are
    - 750 $ for lemons
    - v $ for good cars (v is less than 2000 $)

Example (cont.)
- symmetric information

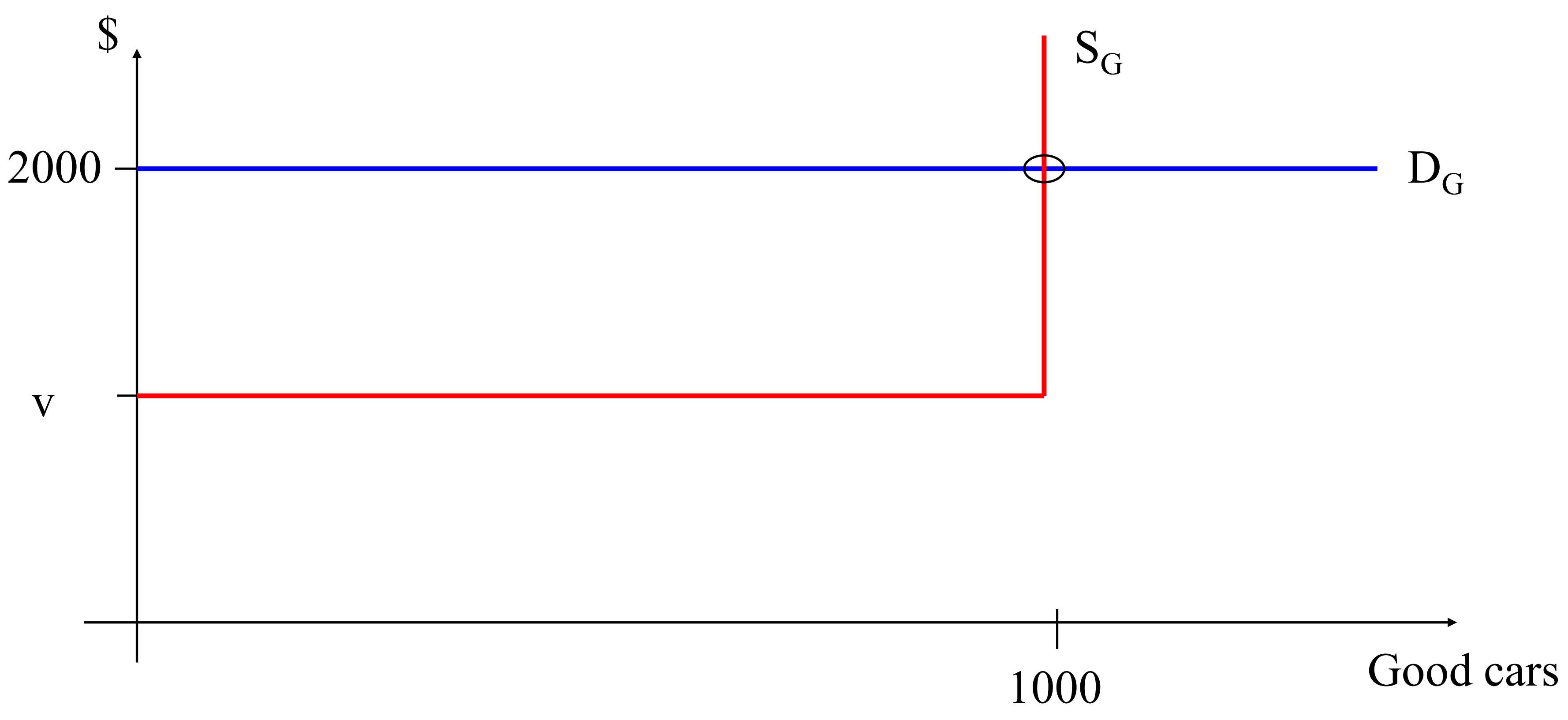
- Both sellers and buyers know the quality of the cars

- Efficient market
  - The goods go to the people who value them the most

Example – symmetric information (equilibrium in the market for lemons)



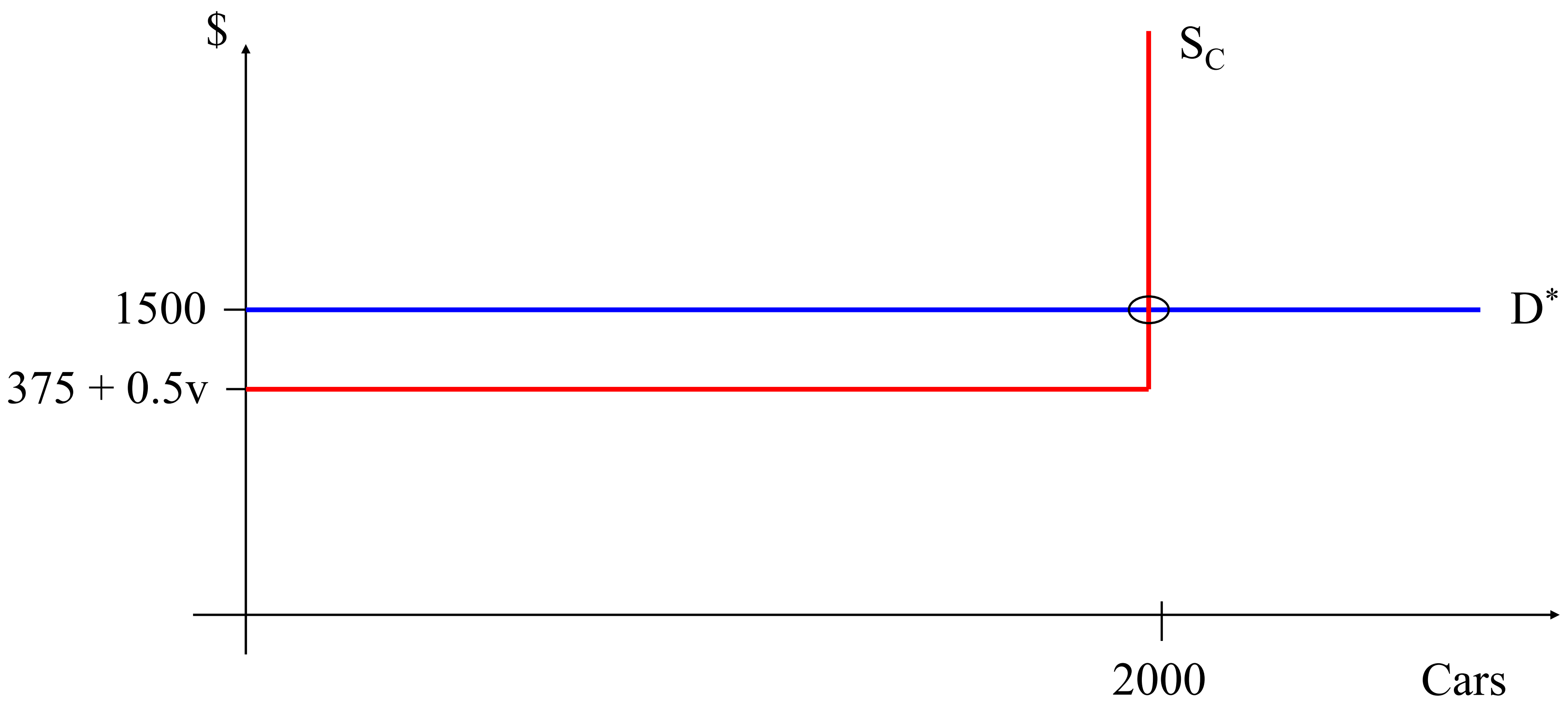
Example – symmetric information (equilibrium in the market for good cars)



Example (cont.) - symmetric information

- Neither sellers nor buyers know the quality of the cars
- Assume sellers and buyers are risk neutral
- Expected value is 1500 $

Example – symmetric information (equilibrium when quality is unknown)



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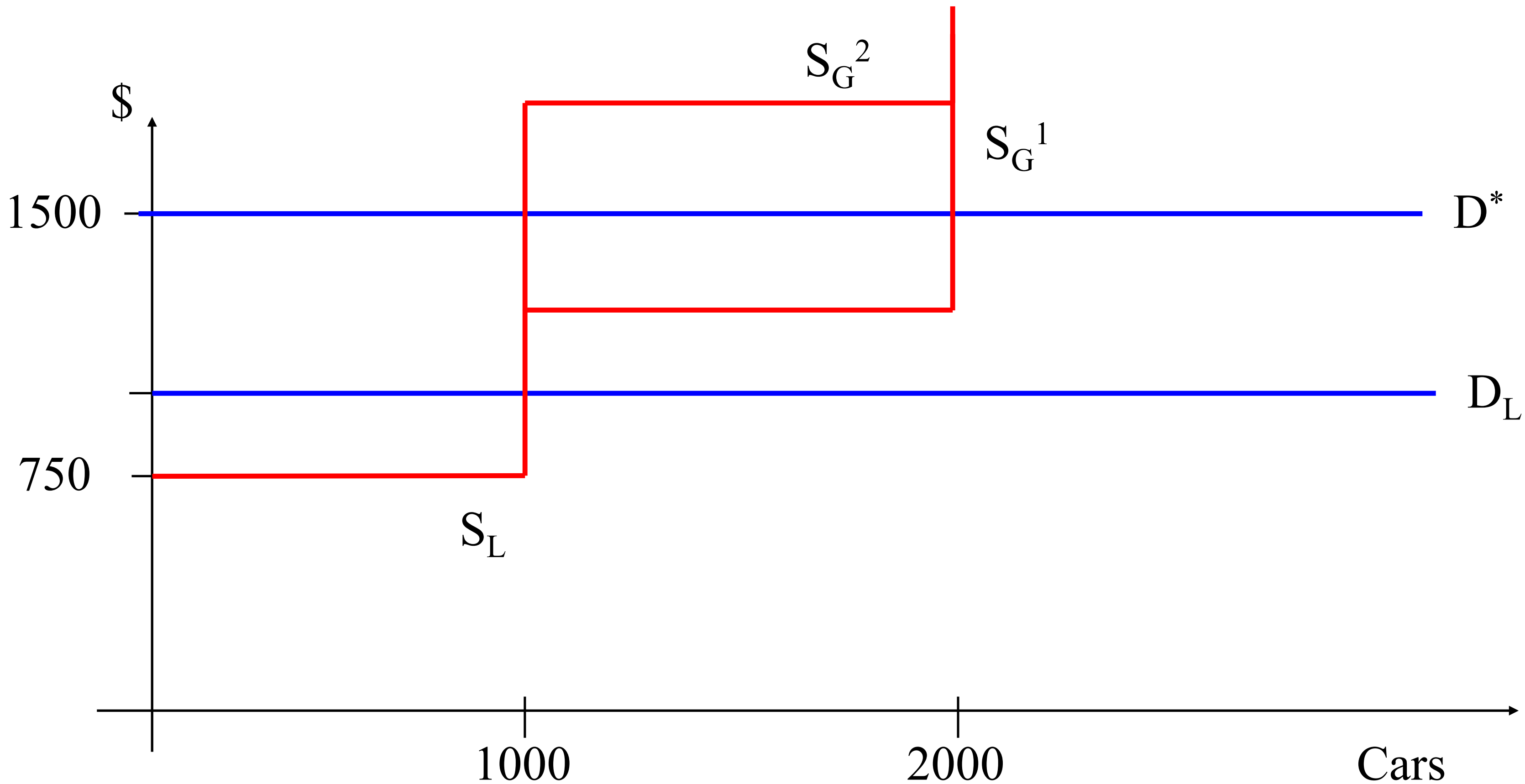
Example (asymmetric information)

- Only sellers know the exact quality of a car
- Two possible solutions/ equilibriums:
  - All cars sell at the same price
  - Only the lemons are sold
- What determines which equilibrium we reach?

Example
(equilibrium with asymmetric information)



Summary, Example

- If the reservation prices of the sellers with good cars are below 1500, all cars are sold at a price of 1500
- If the reservation prices of the sellers of good cars are above 1500, no cars of good quality will be sold
  - In this case, the buyers realize that only lemons are on the market
  - The market price will then be 1000, and all the lemons will be sold at this price
  - As a consequence, the market decreases – there are potential transactions with buyers willing to buy and the sellers willing to sell that are not carried out due to information asymmetry

Lemons market with variable quality

- Social value is not necessarily maximized simultaneously with private value

- Example:
  - Five firms produce a product
  - The per unit cost of production is C
  - The price per unit is R
  - One firm considers increasing the quality of their product, giving it a value of R + Q
  - The cost of this new production is q
  - Given that there exists a market for higher quality than currently produced, will the firm increase the quality of their product in a market with asymmetric information?

Price discrimination

- Same quality is sold to different product groups for different prices

- Consumers are willing to pay more for higher quality
  - When information asymmetry exists the seller can exploit this fact

- Company creates uncertainty by adding noise
  - Different names
  - Different design

Market power

- Even in a highly competitive market, asymmetric information can give the companies market power

- Consider the following example:
  - Many stores in a town sell the same product
  - The competitive price of the product (MC) is equal to p*
    - What happens to a store charging more than p* in a market with symmetric information?
  - The consumers have limited information and a searching/ traveling cost of c (cost of going from store to store)
    - What happens to a store charging more than p* now?

Example (cont.)

- Is the price p* an equilibrium price?

- Which price is the equilibrium price (given that enough stores are present)?

- If there are an insufficient amount of firms there might be either no equilibrium or equilibriums with multiple prices

Example: Earthquake Insurance

- The state of California set up its own earthquake insurance program for homeowners in 1997. The rates vary by ZIP code, depending on the proximity of the nearest fault line. However, critics claim that the people who set the rates ignored soil type. Some houses rest on bedrock; others sit on unstable soil

- What can be the implications of such a policy?

- What kind of a problem is this?

Example: Safety Investment Game

- Firm have more information regarding job safety than potential employees
  - Injury rate for the industry as a whole is available, but not for individual firms
- A risk premium is required to attract employees to high risk jobs
- Each firm must decide how safe it wants to make it's plant

	No investment	Investment
No investment	$200 / $200	$250 / $100
Investment	$100 / $250	$225 / $225

Example: Cheap Talk

- Workers have more information regarding their own ability than firms do
- When does it work to send inexpensive signals?
- Two jobs are available – one demanding and one undemanding
- The worker has one of two ability levels: High or Low
- Before the company decides on which job to offer the worker, the worker can send a signal

Cheap talk works	**Demanding**	**Undemanding**
High	$\boxed{3}$ / 2	1 / 1
Low	1 / 1	$\boxed{2}$ / 4

Cheap talk fails	**Demanding**	**Undemanding**
High	$\boxed{3}$ / 2	1 / 1
Low	$\boxed{3}$ / 1	2 / 4

Limiting lemons

- Laws
  - Merchandise Marks Act 1887
- Consumer screening
  - Objective experts
    - A mechanic to appraise a used car
  - Learn of a firm's reputation from other consumers
  - Third party comparisons
- Standard and certification
  - Climbing equipment / safety equipment
  - What is the potential reaction to a standard with only a high versus low quality rating?
- Signaling by firms
  - Brand names
  - Guarantess and warranties
  - The signals must be credible

Responses to adverse selection

- There are two main approaches
  - Restrict opportunistic behavior
  - Equalize information

- Examples:
  - Mandatory insurance
    - All car owners must have an insurance
  - Health insurance as benefit
    - Companies pay a lower wage and include health insurance
      - Firms reduce the adverse selection problem and can buy the insurance at a low price
      - Both healthy and unhealthy people are insured
  - Signaling
    - The informed person takes an action to send information to the uninformed person
  - Screening
    - The uninformed person takes an action to determine the information possessed by the informed person

Screening

- Equalize information
- Collect more information
  - Uncover hidden information
- Possible to uncover all hidden information?
- Beneficial to uncover all hidden information?

- Example:
  - Insurance

Signaling

- Used by informed parties to eliminate adverse selection
- The informed party try to signal information to the uninformed party

- Why would the informed parties want to share information?
- Which informed parties would want to share information?

- Example:
  - Physical examination
  - Education

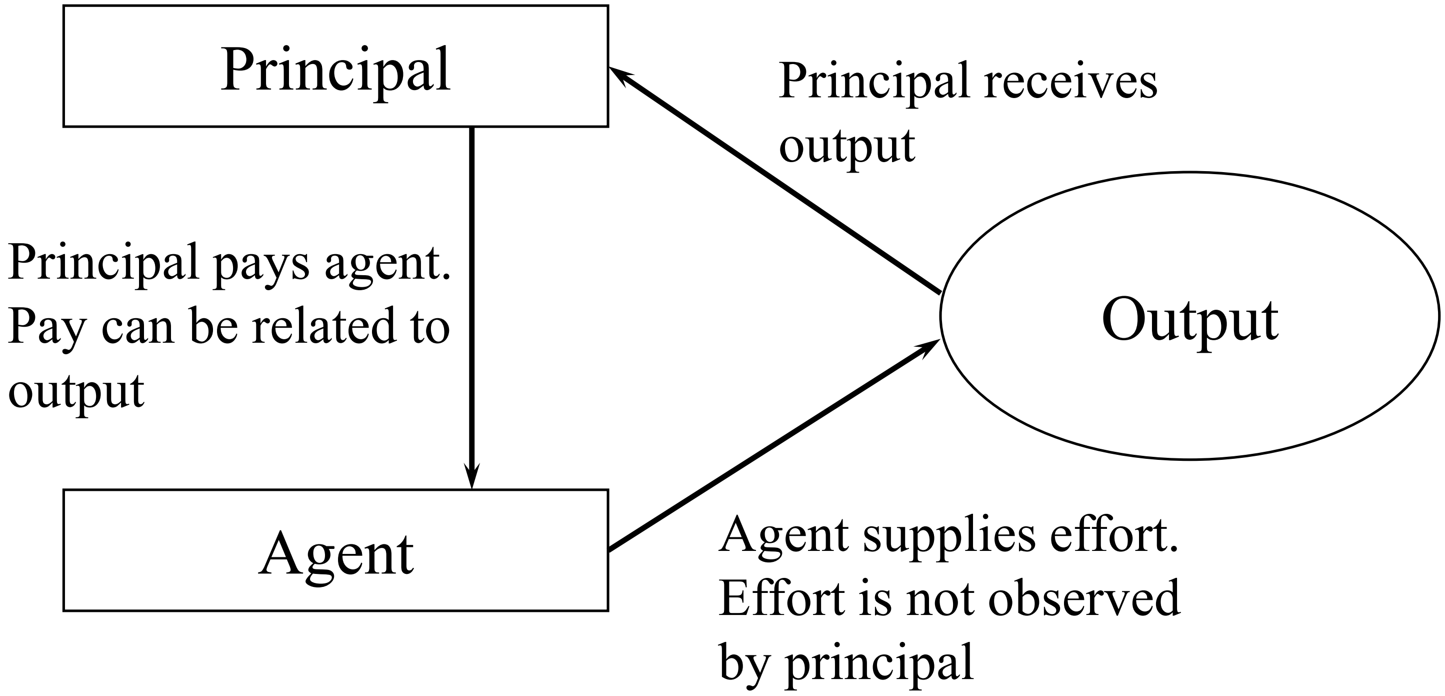
Principal-Agent theory
Moral Hazard

Contracts, production efficiency and risk-sharing

Principal-Agent setting

- A principal contracts with an agent to take some action that benefits the principal
- The actions made by the agent influences the payoff to the principal
- The actions of the agent are unobservable to the principal

Principal-Agent relationship:



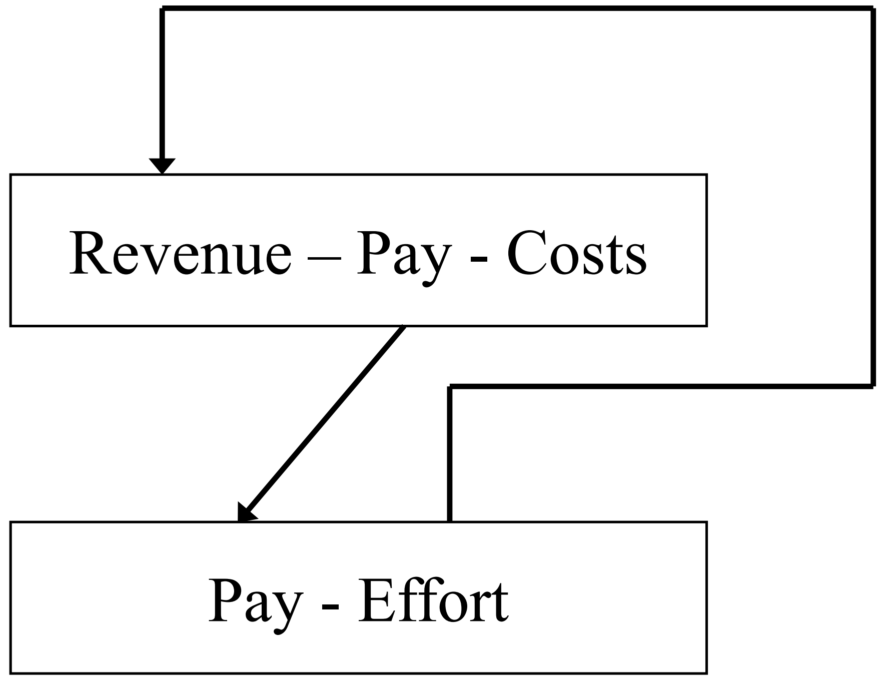
Examples

Principal	Agent
Owner	Manager
Employer	Employee
Client	Lawyer
Insurance company	Client

Principal vs Agent objectives

Principal chooses pay to maximize:

Agent chooses effort to maximize:



Model for analysis

$$\pi = \pi(a,\theta)$$

π is the payoff

a is the action taken by the agent

θ is a random variable

Efficiency

- No party can be made better off without harming the other party
- Requires both efficiency in production and in risk sharing
- Efficiency in production means that the payoff is maximized
- Efficiency in risk means that the least risk-averse person bears most of the risk

Production efficiency

- To ensure production efficiency each contract has to satisfy two criteria:
  - Provide a large enough payoff for the agent to participate
  - Be incentive compatible

Example
Buy – A – Duck

- Paula owns the store Buy-A-Duck (Principal)
- Arthur is the manager of the store (Agent)
- The store sells wood carvings of ducks
- The demand and joint profit function is:

$$p = 24 - 0.5a$$
$$\pi(a) = 24a - 0.5a^2 - 12a$$

- Arthur has a cost of 12 \$ in obtaining and selling each duck
- What is the optimal amount of carvings for the joint profit function?

Example
Buy – A – Duck

- What kind of contract should Paula offer Arthur?

- Alternatives:
  - Fixed-fee rental contract
    - Arthur rents the store from Paula for a fixed fee
  - Hire contract
    - Paula contracts to pay Arthur for each carving he sells
  - Revenue-Sharing contract
    - Paula and Arthur share the revenue from the store
  - Profit-Sharing contract
    - Paula and Arthur share the economic profit π

Example (symmetric information) Buy – A – Duck

1. Fixed-fee rental contract
   - Arthur gets the difference between the profit and the rent F
   - Paula gets the profit F
   - Leads to an efficient solution

2. Hire contract
   - Payment lower than 12$
     - Leads to Arthur refusing the contract
   - Payment equal to 12$
     - Can give an efficient solution if Arthur is supervised
   - Payment higher than 12$
     - Leads to an inefficient solution
   - Will generally not lead to an efficient solution

Example (symmetric information) Buy – A – Duck

3. Revenue sharing contract
   - MR for Arthur is lower than the original MR
   - Will not lead to an efficient solution

4. Profit sharing contract
   - Is incentive compatible and will lead to an efficient solution

Example (asymmetric information) Buy – A – Duck

- Paula has less information than the agent – she cannot observe sales or revenues
  - Moral hazard problem

1. Fixed-fee rental contract
   - Will the solution be efficient in this case?

2. Hire contract
   - Efficient solution?
   - What will happen with pay equal to, lower than or higher than 12$?

Example (asymmetric information)
Buy – A – Duck

3. Revenue sharing contract
   - Can this contract lead to an efficient solution?
   - What will influence the potential underproduction?

4. Profit sharing contract
   - Under which assumptions can this contract be efficient?

- With the assumptions in this example, what is the only efficient solution?

What is the best contract?

- Varies from case to case
- Depends on risk profile of the participants
- Difficulties of monitoring

Efficiency in risk bearing

- The least risk-averse party should bear most of the risk
- Usually there is no optimal solution that ensures efficiency both in production and in risk sharing
- A trade-off is needed

Example

- A company's future value is either 10 or 20 million dollars
- The probability of each outcome is equally likely
- The utility function of the manager is $(\text{Income})^{0.5}$
  - He is risk averse
- He need a utility level of a least 1000 in order to accept a contract
  - Otherwise he will accept an offer from a different company

Example (cont.)

- Alternative 1: fixed payment of 1 mill. dollars
  - Gives the manager a utility of 1000
  - Expected value of the company is 14 mill. Dollars

- Alternative 2: an owner share in the company
  - Solves the following equation:

$$0.5(10.000.000x)^{0.5} + 0.5(20.000.000x)^{0.5} = 1000$$

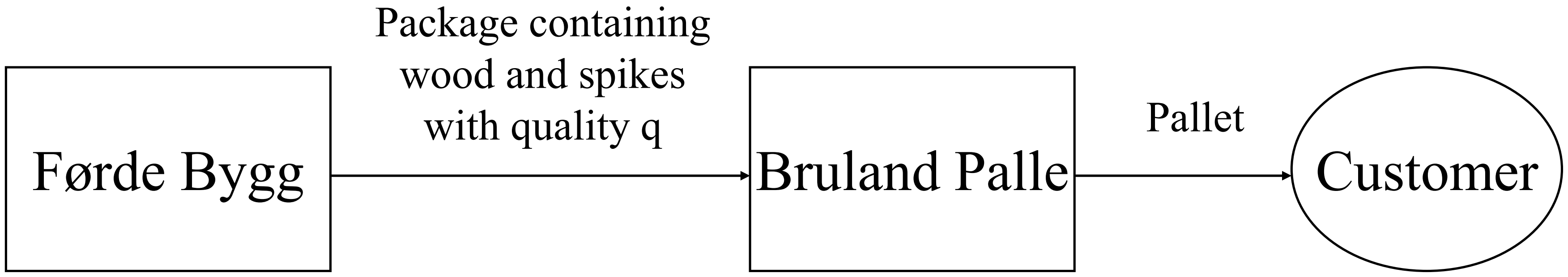
  - Gets the following solution: x = 0.06863

Example (cont.)

- Expected value of the owner share: 1.029.450
- Why is this value higher than the fixed payment?
- Expected value of the company (for the owners): 13.970.550
- Which alternative would you have chosen
  - …if you were the manager?
  - …if you were the owners of the company?

Example – risk sharing

- Setting:
  - Two firms
    - Bruland palle AS
    - Førde Bygg AS



NTNU

Example 2 (cont.)

- Quality q of the package depends on two parameters: e and ε ($q = q(e, \varepsilon)$)
  - The parameter ε is exogenous
  - e is controlled by Førde Bygg (high e leads to high quality)
  - Førde Bygg has a cost related to producing high quality; $H(e)$
  - Førde Bygg is risk-averse while Bruland Palle is risk-neutral
- Bruland Palle can only observe q
- Bruland Palle gets revenue $r(q)$

Example 2 (cont.)

- If BP could observe e → contract establishing e* and price p*
- What happens if they agree upon a fixed e* and p* under the assumptions in this example?
- The contract will have a price structure of p(q)
- Problem of optimal solution versus risk sharing

Example – family farm

- You inherit the family farm, but after completion of your education at NTNU you would rather work as a consultant
- However, you do not wish to sell of the farm since it has been in the family for generations
- The solution is to hire someone to run it for you
- What kind of contract should you offer the person you hire?

Example 3 (cont.)

- The payoff from the farm depends both on the effort by the person hired to run the farm and the price of grain

	Low price (p = 0.5)	High price (p = 0.5)
Low Effort	50.000	150.000
High Effort	100.000	200.000

Example 3 (cont.)

- The person you hire to run your farm has the following utility function:

$$U = W^{0.5} - u(e)$$

- With low effort:

$$U = W^{0.5}$$

- With high effort:

$$U = W^{0.5} - 46.3$$

Example (cont.)

- You are considering two alternatives:

1. Fixed yearly payment of 50.000 (the reservation price of the person you hire)
2. Payment varying with the profitability of the farm

1. Calculate the expected utility:
   - Low effort: E(U) = 223.6
   - High effort: E(U) = 177.3
   - Which level of effort will be chosen?

   - Your expected profit = 50000

Example (cont.)

- E(U) with fixed payment and low effort
  = E(U) with x % of the profit and high effort
  - Need to offer utility of at least 223.6
  - Corresponds to 50% of the profits

- Your expected profit: 75000
  - (assuming high effort)

Example (cont.)

- Will the person you hire actually choose high effort?
  - Calculates E(U) in both cases:
    - Low effort: E(U) = 216
    - High effort: E(U) = 223.6
- Which effort will be chosen?

Example: Client – Lawyer Contracts

- Especially challenging situation: the agent knows more than the principal, the principal never learns the truth and both face risk
  - Situation often arise when contracting with an expert
- Here: Pam is injured in a traffic accident and Alfredo is her lawyer
- Uncertainty regarding the attitudes of the jury
- Asymmetric information regarding Alfredo's work effort

Type of contract	**Fixed fee to the lawyer**	**Fixed payment to Client**	**Lawer paid by the hour**	**Contingent contract**
Lawyer's payoff				
Client's payoff				
Production efficiency				
Who bears risk				

How to reduce moral hazard

- Piece rates / bonuses
  - Linked to a workers individual output, or linked to the firms profitability
  - Measuring output?
  - Accept contract?
- Monitoring
  - Punch a time clock
  - Installing videocameras
  - Assembly lines (dictates the work pace)
  - Record employees' e-mail, phone calls, review computer files, etc.
- Bonding
  - Require agents to deposit funds guaranteeing their good behavior
  - Security deposits when renting an apartment
  - Costly training / sabbatical
- Deferred payments
  - Pension funds
  - Raise the cost of being fired (as with bonding)
- Efficiency wages
  - Unusually high wage – discourages shirking (loss of wage if fired)

Example: Contract Choice

- So far the focus has been on single contracts
- A menu of contracts can be used to screen job applicants

- A firm wants to hire a salesperson
  - The potential employees are risk neutral
- A hard working salesperson generates sales of $100.000 a year, while a lazy salesperson generates sales of $60.000

	30 % of sales	$25 000 salary
Sales:	100.000	100.000
Salary:	-30.000	-25.000
Fixed cost:	-50.000	-50.000
Profit:	20.000	25.000

	30 % of sales	$25 000 salary
Sales:	60.000	60.000
Salary:	-18.000	-25.000
Fixed cost:	-50.000	-50.000
Profit:	-8.000	-15.000

Conclusion

- Asymmetric information and opportunistic behavior can lead to market failures
- Adverse selection
  - Bad goods drive good ones out of the market
  - Price discrimination
  - Market power
  - Prevents desirable transactions
- Moral hazard
  - Poses challenges for contract design in order to achieve efficiency
    - Production
    - Risk
  - Monitoring/bonding/etc can reduce the problem