NTNU
Department of Industrial Economics and Technology Management
Spring 2021

TIØ4285 Production and Network Economics
Deliverable 2

This deliverable counts for 10% of the final grade. Please note that there will not be a separate score for this deliverable.

Out: **Thursday 4 March**
In: **Thursday 18 March 6pm**

Late submissions will not be accepted.

General Advice

- You can work together in groups of up to 3 students. Collaboration between groups is not allowed and considered cheating.
- Please read the questions carefully and answer to what is asked. Keep you answer short (but as long as necessary) and concise.
- Base your answers on the contents taught in this course (or previous courses).
- If you make any assumptions that are necessary to reproduce your results, make sure you mention them in your report.
- Document how you find your results. You have to provide either calculations or sound logical reasoning.

Exercise 1. COVID Game

A disease and a group of human beings have recently met. The disease has to decide how easily it may infect human beings, choosing to be either lowly or highly contagious. The human beings have to decide to keep society open, or impose a lockdown.

Naturally, the contagiousness and the openness of society affect the number of human beings becoming sick, as well as how much humans enjoy their life. The disease aims to maximize the number of infected humans. The humans aim to maximize fun.

- For an open society, if the disease is
  - highly contagious, it will infect 7 humans, and human fun will be 10
  - lowly contagious, it will infect 5 humans, and human fun will be 50
- For a locked down society, if the disease is
  - highly contagious, it will infect 4 humans, and human fun will be 30
  - lowly contagious, it will infect 1 humans, and human fun will be 70

a. Set up the single-period payoff matrix. Determine the single-period equilibrium for the interaction between the disease and the human beings.

b. Given the payoff matrix that you set up in 1.a., consider an infinitely repeated game.

   i. Design a trigger strategy for both players that will result in higher payoffs for the infinitely repeated game than just repeating the outcome of part 1.a. in every period.

   ii. What is the lowest discount factor for which the trigger strategies are still optimal?

Exercise 2. Equilibrium

An equilibrium in a market is characterized by the following optimization problem:

$$\max\left[Z = \left(100 - \sum_j q_j\right)\sum_i q_i + \frac{1}{2}\left(\sum_i q_i\right)^2 - \sum_i \left(c_i q_i + \frac{1}{2}(q_i)^2\right) - \frac{1}{2}\sum_i (q_i)^2\right]$$

$$s.t.\ 0 \leq q_i \leq cap_i$$

Here, $i, j \epsilon \{1,2,3\}$ are suppliers, q_i supplied amounts, c_i supplier specific cost parameters, cap_i supplier specific capacities, and Z the objective. Parameter values are listed in the table:

i	c_i	cap_i
1	5	16
2	10	20
3	10	20

Derive and implement a mixed integer problem using disjunctive constraints to determine the equilibrium in this market. Use the software of your choice, e.g., GAMS, XPRESS, Python…

Beside the derived problem, report equilibrium price, and for each company optimal quantity, profit and costs, as well as the dual price of capacity. Explain the dual price of capacity for each supplier.

Hint: you will need to derive the complementarity conditions as an intermediate step.

Include your implementation as part of your deliverable.

Exercise 3. Vaccine Competition

Two vaccine manufacturers have to determine their optimal strategies concerning the development of a vaccine against a newly discovered virus. In a first stage, each of them has to decide whether to invest an amount of 1000 to develop and test the vaccine. In the second stage, the manufacturers that decided to invest in the first stage will compete in a "game on quantities" given an inverse demand curve for units of vaccine.

Assume that the developed vaccines are identical, and that inverse demand is given by $p = 100 - q$ with q the total number of supplied vaccine units. Assume that variable supply costs are zero.

What is the Nash Equilibrium for this two-stage game?

Hint: by smartly considering symmetry you can save yourself a lot of time.