

Lecture 9: Pricing and Incentives II – Contract Design

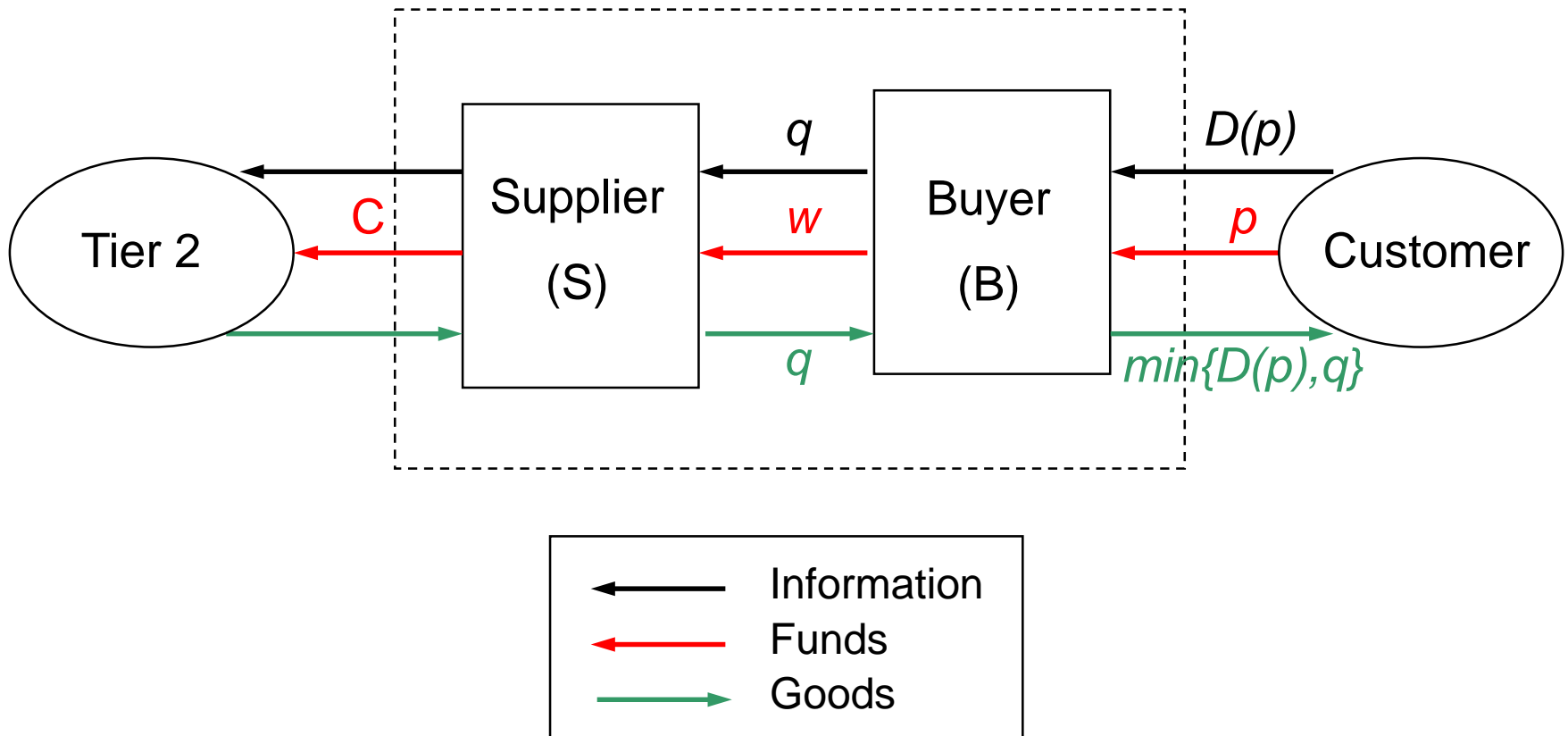
TIØ4285 Production and Network Economics

Spring 2020

Outline

- Price Only (One-Part Contracts)
- Two-Part Tariffs
- Supply Chain Contracts
 - Buy-Back Contracts
 - Profit-Sharing Contracts
 - Quantity-Flexibility Contracts
 - Sales Rebate Contracts

The Supply Chain



Price Only Contracts

- Assumptions:
 - Vertical, bilateral monopoly
 - Full information
 - Seller is a price-setter, Buyer a price-taker
- Example
 - $q := D(p) = A - Kp$
 - $\pi_S = (w - C)q$
 - $\pi_B = (p - w)q$
 - $\pi_J = (p - C)q$

Example: Two-Step Distribution

- Manufacturer producing at a constant marginal cost of 11.
- Retailer incurs no other cost than the price paid to the manufacturer (cost of retailing = p).
- Retailer faces the demand function

$$P(x) = 131 - \frac{x}{100}$$

- Global optimization results in wholesale price $p = 11$, production $x = 6000$, and profit $\pi = 360000$

Two-Step Distribution

The following questions are important:

1. What should be the wholesale price p in order to maximize manufacturer profits? What is the profit of the two firms?
2. What if the manufacturer could retail on its own, without middleman and no additional cost of retailing?
3. The manufacturer incurs now a marginal cost of k per unit sold compared to the retailer's marginal cost of 0. At what levels of k would the public prefer sales through the retailer?
4. The manufacturer decides to use a retailer. Is there a pricing scheme that coordinates the decisions better than the simple wholesale price p ?

Wholesale Price p

- The manufacturer's marginal revenue function is equal to

$$MR_M = 131 - \frac{x}{25}$$

- With constant marginal cost of 11, we get

$$131 - \frac{x}{25} = 11$$

$$x = 3000$$

$$p = 71, P = 101$$

- The profits are $\pi_M = 180000$ and $\pi_R = 90000$

Direct-to-the-Public Marketing

- With no additional costs of retailing, the optimal production quantity is determined as

$$131 - \frac{x}{50} = 11$$
$$x = 6000$$
$$P = 71$$

- The manufacturer realizes a profit of $\pi_M = 360000$.
- This exceeds the two-company solution ($\pi_J = 270000$)!

Direct-to-the-Public Marketing with Retailing Cost k

- The manufacturer has a new marginal cost function. Optimality condition is now:

$$131 - \frac{x}{50} = 11 + k \Rightarrow x = 50(120 - k)$$

- The resulting retail price is:

$$P = 131 - \frac{50(120 - k)}{100} = 71 + \frac{k}{2}$$

- With this retail price, the manufacturer's profit becomes:

$$\begin{aligned}\pi_M &= \left(71 + \frac{k}{2} - 11 - k\right) (50(120 - k)) \\ &= 25(120 - k)^2\end{aligned}$$

Comparing Direct-to-the-Public Marketing with Two-Step Distribution

- The Two-Step Distribution profit for the manufacturer was $\pi_M = 180000$,
$$\Rightarrow 25(120 - k)^2 \geq 180000$$
$$k \leq 35.148$$
- Though the retailer has no cost except for the price paid to the manufacturer, the manufacturer prefers direct-to-the-public marketing up to a retail cost of $k = 35$.
- Customers that would have to pay 101 in the two-step distribution case, would prefer direct-to-the-public marketing up to a retail cost $k = 60$. (Why?)

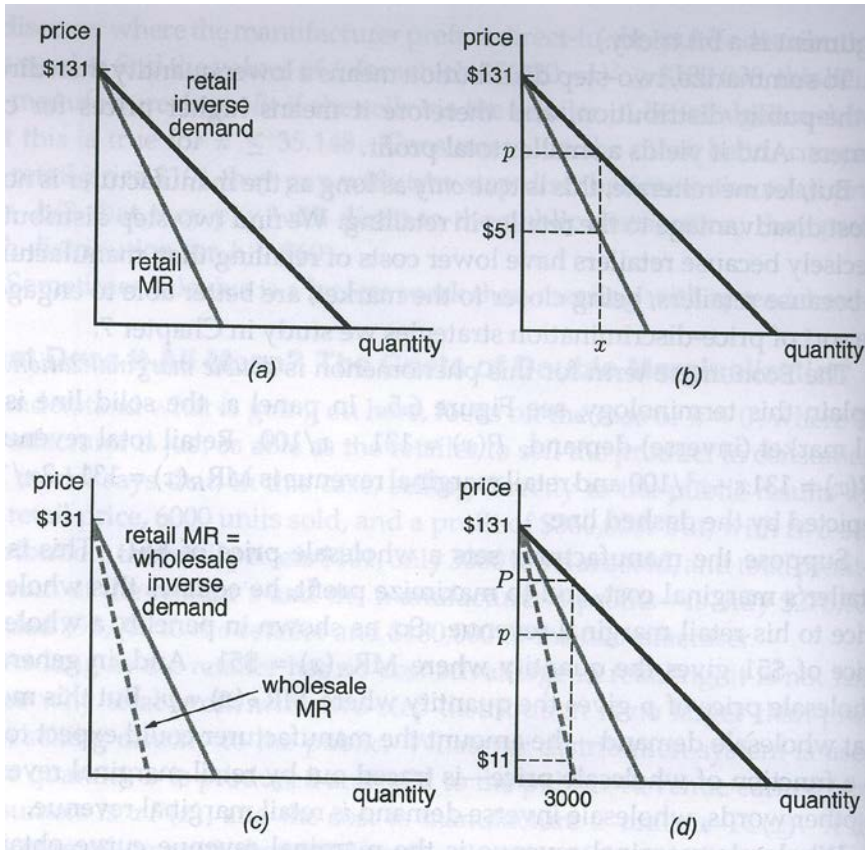
Short Comparison of Results

- Two-stage distribution results in $P = 101$, $x = 3000$, $p_f = 270000$
- Direct-to-the-public marketing results in $P = 71$, $x = 6000$, $p_f = 360000$

The optimal solution for two-stage distribution is worse than the optimal solution for direct-to-the-public marketing. Why does this happen?

- If the retailer does not have a cost advantage in retailing, it is trivial: the combined solution will have higher marginal costs, thus requesting higher prices and selling less products
- If the retailer has a cost advantage:
 - Retailer and manufacturer decide sequentially, thus deriving the demand function twice (i. e. double marginalization)
 - Every additional step in the distribution network or supply chain would lead to an additional derivation of the demand function, making the situation worse

The Effect of Double Marginalization



- We have a demand function

$$P(q) = a - bq$$

- The retailer derives $P(q)q$ to get its marginal revenue

$$MR_R = a - 2bq$$

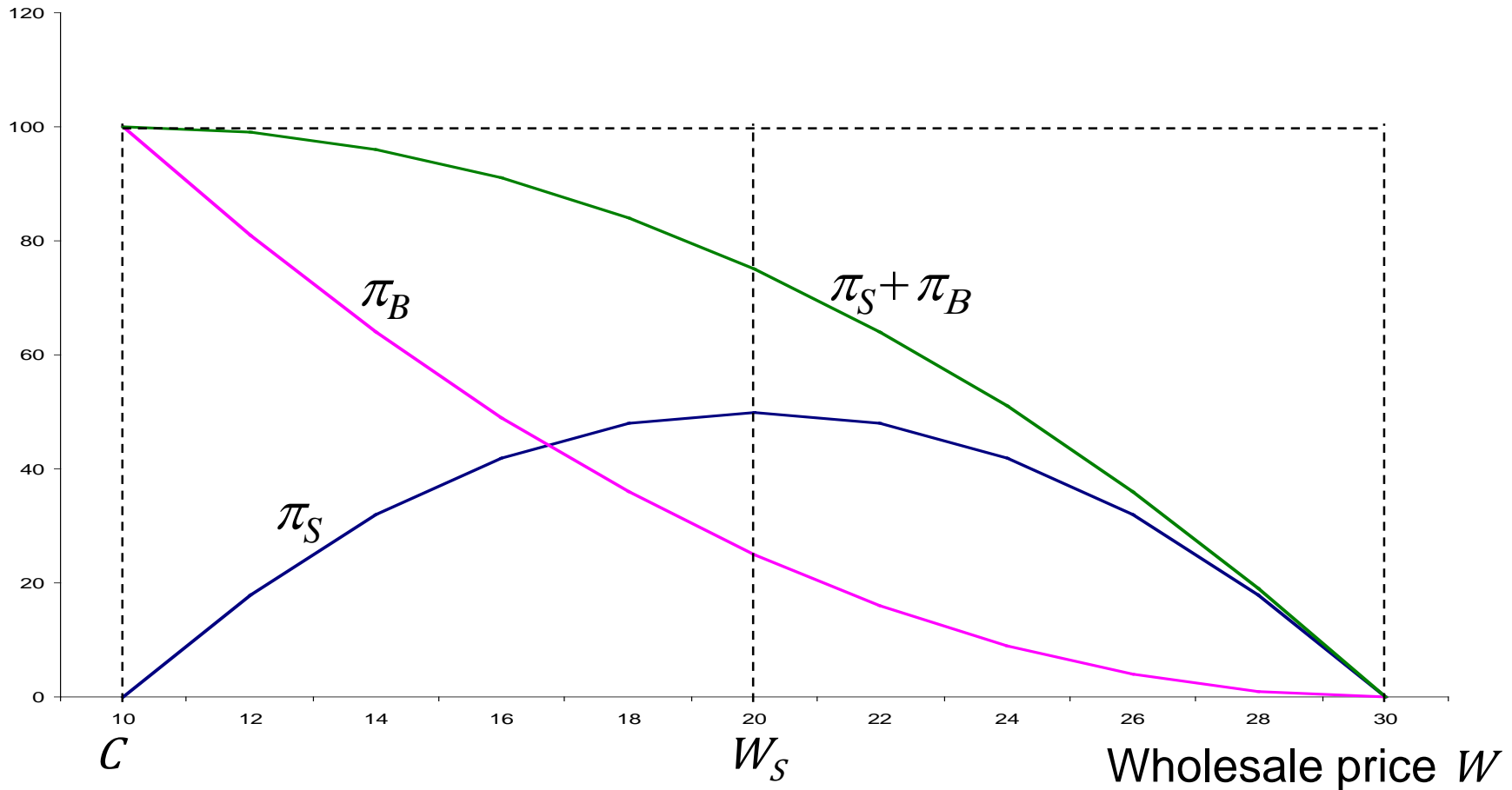
- In our case this is equal to the NMR_M , so it's the demand function for the manufacturer
- The manufacturer derives $NMR_M \cdot q$ to get its marginal revenue

$$MR_M = a - 4bq$$

Sequential Solution

Profits

$$W_S > C \Rightarrow \pi_S + \pi_B < \pi_J^*, q < q_j^*$$



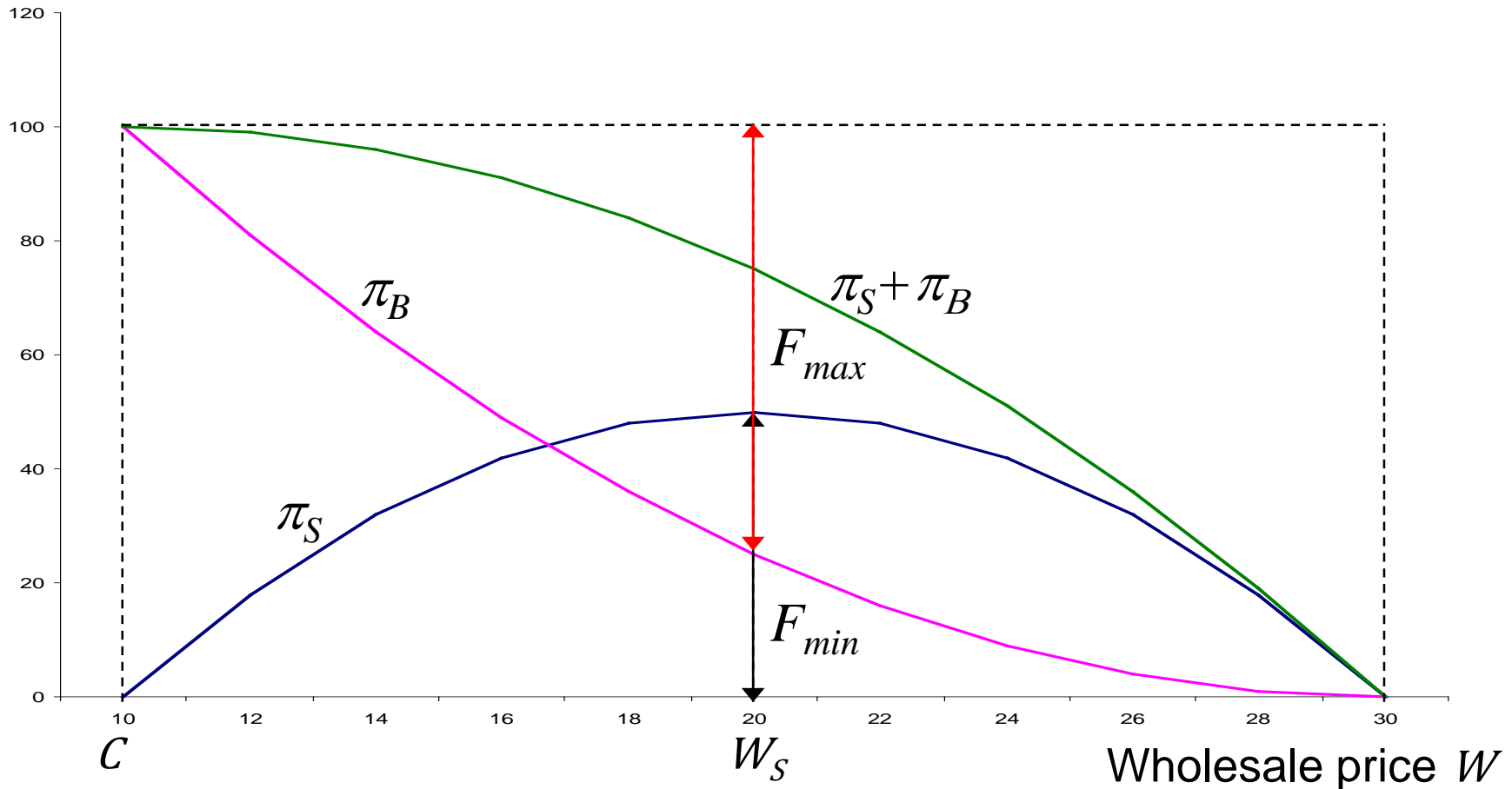
Two-Part Tariffs

- Assume the following: The retailer has marginal costs of 0 for retailing, whereas the manufacturer incurs marginal costs of retailing equal to 30
- For direct-to-the-public marketing, these figures result in a retail price of 86 and profit for the manufacturer of 202500
- Charge a fixed up-front fee F from the retailer and allow her to buy as many units at a transfer price p as she wants to
- If the retailer accepts, the fixed fee F is a sunk cost, i. e. there is no way to get the money back
- Why not ignore the fixed fee then and optimize the rest?

Global Optimization

Profits

$$W_S = C \Rightarrow \pi_S + \pi_B = \pi_J^*, q = q_j^*$$



Example: Two-Part Tariffs I

- The optimal order quantity for the retailer is

$$131 - \frac{x}{50} = p \Rightarrow x = 50(131 - p)$$

- Thus, retailer profits are

$$\begin{aligned}\pi_R &= 50(131 - p) \left(\frac{131 + p}{2} - p \right) - F \\ &= 25(131 - p)^2 - F\end{aligned}$$

- Manufacturer profits:

$$\pi_M = F + 50(131 - p)(p - 11)$$

Example: Two-Part Tariffs II

- Derivation of π_M results in the wholesale price $p=11$
- This equals the marginal cost of production. Thus, the manufacturer acquires all his profits through the fixed fee F
- The retailer is requesting the globally optimal amount of products as they are offered at marginal costs
 $\Rightarrow x = 6000, P = 71$
- The question to be answered is:
How do we have to choose F such that both manufacturer and retailer are willing to accept the two-part tariff?

The fixed fee F

- The fixed fee distributes the profit between seller and buyer
- In order to have an acceptable contract, you have to make sure that the retailer receives at least as much profit as in the double marginalization case
- The same is valid for the manufacturer, you have to remain pareto-optimal!
- This allows you to determine upper and lower bounds on F , the realized value of F is then subject to negotiations

Short Summary

- So far we were dealing with deterministic demand
- The issues addressed so far dealt with the question how to coordinate decisions in a way such that we achieve optimal profits for the supply chain despite the shortcomings of single-price approaches
- We shall now look at stochastic demand and try to find a pricing scheme/contract type that allows for coordinating supply chain decisions

Contract Design

- Contracts are a powerful tool in supply chain coordination, but they have to be designed carefully
- When designing a supply chain contract to have to consider the following points
 - You want to avoid double marginalization
 - You want to share risk
 - You want to reach the global optimum
- Designing the contract badly can result in a very poor performance of the supply chain

Profit-Sharing Contracts

- Using profit-sharing contracts, the manufacturer agrees on lowering the wholesale price in exchange for a certain percentage of the retailer's profits
- The lowered wholesale price presents an incentive to the retailer to buy more units and thus enables her to satisfy greater demand
- The sharing of retailer profits presents an incentive for the manufacturer to lower the wholesale price
- Profit-sharing is a kind of risk sharing as retailer's profits are stochastic

Example: The Newsboy Problem and Profit-Sharing Contracts I

- Let's look at the following example (Rudi/Pyke, Section 1.3):

$$R = 100, W = 50, S = 20, M = 30 \Rightarrow C_u = 50, C_o = 30$$

- Optimality condition

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = 0.625$$

- Order quantity

$$q^* = 1096$$

- Expected profits

$$\pi_R(q^*) = 40899, \pi_M(q^*) = 21920, \pi_J(q^*) = 62819$$

Example: The Newsboy Problem and Profit-Sharing Contracts II

- The manufacturer lowers the wholesale price W to 35 in exchange for 10% of the retailer's profits

$$\Rightarrow C_u = 100 - 35 = 65, C_o = 35 - 20 = 15$$

- Optimality condition

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = 0.8125$$

- Order quantity

$$q^* = 1266$$

- Expected profits

$$\pi_r(q^*) = 0.9 \cdot \pi_R(Q^*) = 52686.16$$

$$\pi_M(q^*) = (W - M) \cdot q^* + 0.1 \cdot \pi_R = 12184.02$$

$$\pi_J(q^*) = 64870.02$$

Example: The Newsboy Problem and Profit-Sharing Contracts III

- Which combinations of wholesale price W and rate of revenue shared reproduce the solution of the integrated firm?

- The optimality condition of the integrated firm is

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = \frac{R - M}{R - S} = 0.875$$

- Thus, a wholesale price $W = 30$ results in the optimal order quantity $q = 1345$. The rate of revenue shared just distributes the profit between retailer and manufacturer
- We get the solution Homework Problem #4, real option contract 2, if the retailer pays 35% of its revenues to the manufacturer

Profit-Sharing Contracts, Summary

- For a given wholesale price W , the expected profit of the retailer is constant, thus a given share of the profit is also constant
- It's as close to a two-part tariff as possible, they are difficult to realize in a stochastic setting
- Real Options are kind of a buy-back contract, but we can reproduce the Real Option-solution by carefully adjusting the parameters of our Profit-Sharing contract

Revenue-Sharing Contracts I

- Are similar to profit-sharing contracts, but here the manufacturer agrees to lower the wholesale price in exchange for a share of the revenues
- The reduced wholesale price is an incentive to the retailer to increase the order amount
- Revenue-Sharing might be easier to implement than Profit-Sharing as (sensitive) cost information does not need to be shared

Revenue-Sharing Contracts II

- Reformulate the profit function for the Newsboy Problem:

$$\pi_R = \boxed{RQ - (R - S) \cdot \int_0^Q F(x) dx} - WQ$$

Revenues

- Well-known optimality criterion (for joint profits)

$$\Pr(D < Q) = \frac{R - M}{R - S} \left(= \frac{C_u}{C_u + C_o} \right)$$

- How can we maintain joint optimality if we need to pay a share Φ of the revenues to the manufacturer?

Revenue-Sharing Contracts III

- Revenue-Sharing reduces the retailer's marginal revenue

$$\pi_R = \Phi \left[RQ - (R - S) \cdot \int_0^Q F(x) dx \right] - WQ$$

- Manufacturer profits

$$\pi_M = (1 - \Phi) \left[RQ - (R - S) \cdot \int_0^Q F(x) dx \right] + WQ - MQ$$

- Optimal wholesale price and profit distribution

$$W = \Phi M, \pi_R = \Phi \cdot \pi_J, \pi_M = (1 - \Phi) \cdot \pi_J$$

Buy-Back Contracts

- The seller agrees to buy back unsold products from the buyer at a specified price below the wholesale price ($S < B < W$, but $B > 0$ not necessary)
- The idea is to share the risk of uncertain demand between buyer and seller
- In order to evaluate buy-back contracts we have to make some assumptions on the salvage opportunities available to each player:
 - If the manufacturer can salvage products at a rate S whereas the retailer can salvage at a rate less or equal to S , offering a buy-back to a retailer costs $B-S$ per unit
 - If the retailer can salvage products at a rate S whereas the manufacturer can salvage at a rate less or equal to S , the manufacturer can pay $B-S$ per unit to the retailer, leaving him with a salvage value B
 - So it works as a buy-back no matter who is physically salvaging excess stock

Example: The Newsboy Problem and Buy-Back Contracts I

- Let's look at the following example (Rudi/Pyke, Section 1.3):

$$R = 100, W = 50, S = 20, M = 30 \Rightarrow C_u = 50, C_o = 30$$

- Optimality condition

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = 0.625$$

- Order quantity

$$q^* = 1096$$

- Expected profits

$$\pi_R(q^*) = 40899, \pi_M(q^*) = 21920, \pi_J(q^*) = 62819$$

Example: The Newsboy Problem and Buy-Back Contracts II

- Now introduce a Buy-Back contract: The manufacturer offers to buy back unsold stock for $B = 25 \Rightarrow C_u = 100 - 50 = 50, C_o = 50 - 25 = 25$

- New optimality condition

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = 0.667$$

- Order quantity

$$q^* = 1129$$

- Expected profits

$$\pi_R(q^*) = 41822.24$$

Cost of uncertain demand

$$\pi_M(q^*) = (W - M) \cdot q^* - (B - S)E(q^* - D)^+ = 21605$$

$$\pi_J(q^*) = 63427.24$$

Example: The Newsboy Problem and Buy-Back Contracts III

Which contract coordinates the supply chain such that we actually achieve optimal supply chain profits?

- Assume the manufacturer offers a contract with wholesale price W , buy-back price B and $\varepsilon \in (0, R-M]$, where

$$W(\varepsilon) = R - \varepsilon, B(\varepsilon) = R - \frac{\varepsilon(R - S)}{R - M}$$

- The optimal order quantity for the retailer is given by:

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = \frac{R - W}{R - B} = \frac{R - (R - \varepsilon)}{R - \frac{\varepsilon(R - S)}{R - M}} = \frac{R - M}{R - S}$$

Example: The Newsboy Problem and Buy-Back Contracts IV

- Optimality condition for the integrated firm:

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = \frac{R - M}{R - S}$$

- Optimal order quantity for the retailer is the same as for the integrated firm now!
- With the optimal order quantity, we also realize optimal supply chain profits
- Results are valid for each ε , so we do not have unique results
- Profit distribution (depend on ε , or negotiations):

$$\pi_R = \frac{\varepsilon}{R - M} \pi_J, \pi_M = \left(1 - \frac{\varepsilon}{R - M}\right) \pi_J$$

Example: The Newsboy Problem and Buy-Back Contracts V

- Choose for example $\varepsilon = 10$
 $\Rightarrow W = 90, B = 88.57, C_u = 10, C_o = 1.43$
- Optimality condition
$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = 0.875$$
- Optimal order quantity:
$$q^* = 1345$$
- Profit distribution:
$$\begin{aligned}\pi_R(q^*) &= 9294.13 \\ \pi_M(q^*) &= 55765.87 \\ \pi_J(q^*) &= 65060\end{aligned}$$

Quantity-Flexibility Contracts

- Quantity-flexibility contracts consist of 3 parameters:
 - Wholesale price W
 - Downward adjustment parameter $d \in [0,1)$
 - Upward adjustment parameter $u \geq 0$
- The retailer orders initially q units, observes uncertain demand ξ and buys ξ units at wholesale price W if $q(1-d) \leq \xi \leq q(1+u)$
- The manufacturer provides an "upside" coverage to the retailer of $100 \cdot u\%$ above the initial order
- The retailer has to buy at least $q(1-d)$ units. In other words: the retailer can cancel or return $100 \cdot d\%$ of its initial order
- The manufacturer refunds the retailer the full wholesale price for each unit returned (in contrast to the buy-back contract)

Example: The Newsboy Problem and Quantity-Flexibility Contracts I

- Let's look at the following example :

$$R = 100, W = 50, S = 20, M = 30 \Rightarrow C_u = 50, C_o = 30$$

- Optimality condition

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = 0.625$$

- With demand uniformly distributed between 0 and 1000, we get the optimal order quantity

$$q^* = 625$$

Example: The Newsboy Problem and Quantity-Flexibility Contracts II

- Expected profits

$$\begin{aligned}\pi_R &= (R - W)ED - E \left[(R - W)(D - Q)^+ + (W - S)(Q - D)^+ \right] \\ &= 50 \cdot 500 - 50 \cdot \frac{1}{1000} \int_{625}^{1000} (x - 625) dx - \\ &\quad 30 \cdot \frac{1}{1000} \int_0^{625} (625 - x) dx \\ &= 25000 - 50 \cdot \frac{1}{1000} \left(\frac{1}{2} 1000^2 - 625 \cdot 1000 + \frac{1}{2} 625^2 \right) - \\ &\quad 30 \cdot \frac{1}{1000} \frac{1}{2} 625^2 \\ &= 25000 - 50 \cdot 70.31 - 30 \cdot 195.31 = 15625.20\end{aligned}$$

$$\pi_M = (W - M) \cdot q = 12500$$

$$\pi_J = 28125.20$$

Example: The Newsboy Problem and Quantity-Flexibility Contracts III

- Calculate the solution for the integrated firm :
 $R = 100, W = 30, S = 20, M = 30 \Rightarrow C_u = 70, C_o = 10$
- Optimality condition
$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} = 0.875$$
- With demand uniformly distributed between 0 and 1000, we get the optimal order quantity
 $q^* = 875$
- Expected profits
 $\pi_J(q^*) = 30625$

Example: The Newsboy Problem and Quantity-Flexibility Contracts III

- Now introduce a Quantity-Flexibility contract: The manufacturer offers wholesale price $W = 50$, an upward parameter $u = 0.1$, and a downward parameter $d = 0.1$

- Retailer revenues:

$$\pi_R(q) = (R - W)q(1 + u) - (R - W) \int_{q(1-d)}^{q(1+u)} F(x)dx - (R - S) \int_0^{q(1-d)} F(x)dx$$

- Optimality condition

$$\Pr(D < Q) = \frac{(R - W)(1 + u)}{(R - W)(1 + u)^2 + (W - S)(1 - d)^2} = 0.83$$

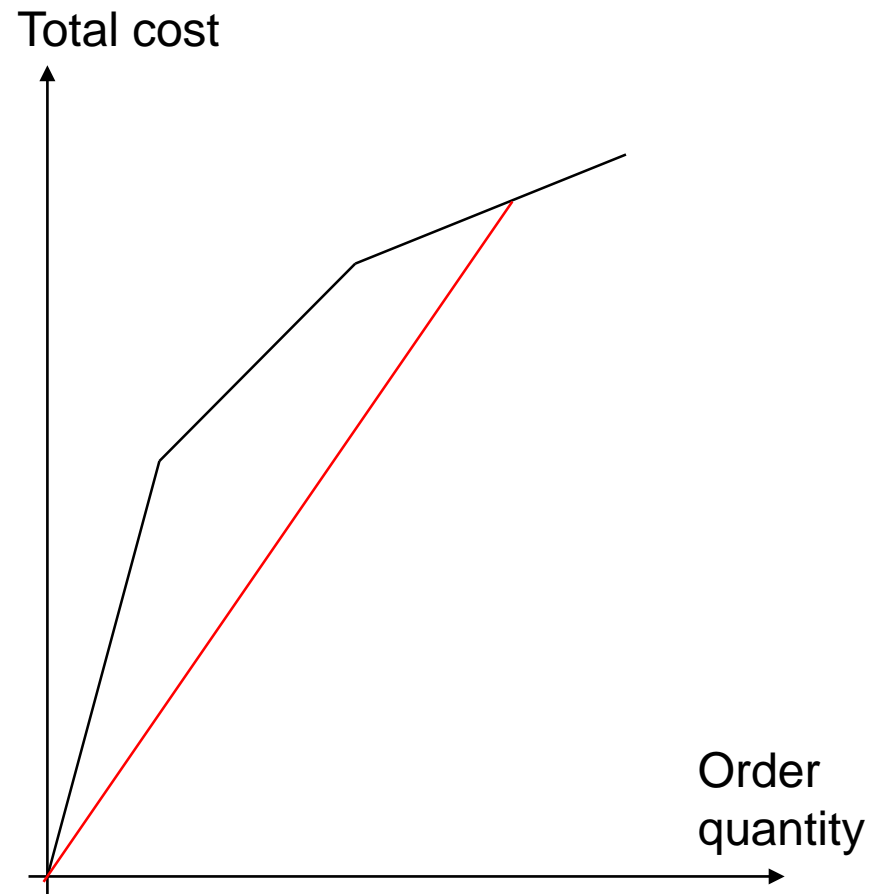
- Optimal order quantity is $q^* = 830$

Important Note

- All examples have assumed a marginal cost of retailing equal to zero
- Retailing costs can be added (fairly) easily, but lead to a slightly different optimality criterion
- (Determining this new optimality criterion is left as an exercise to the students)

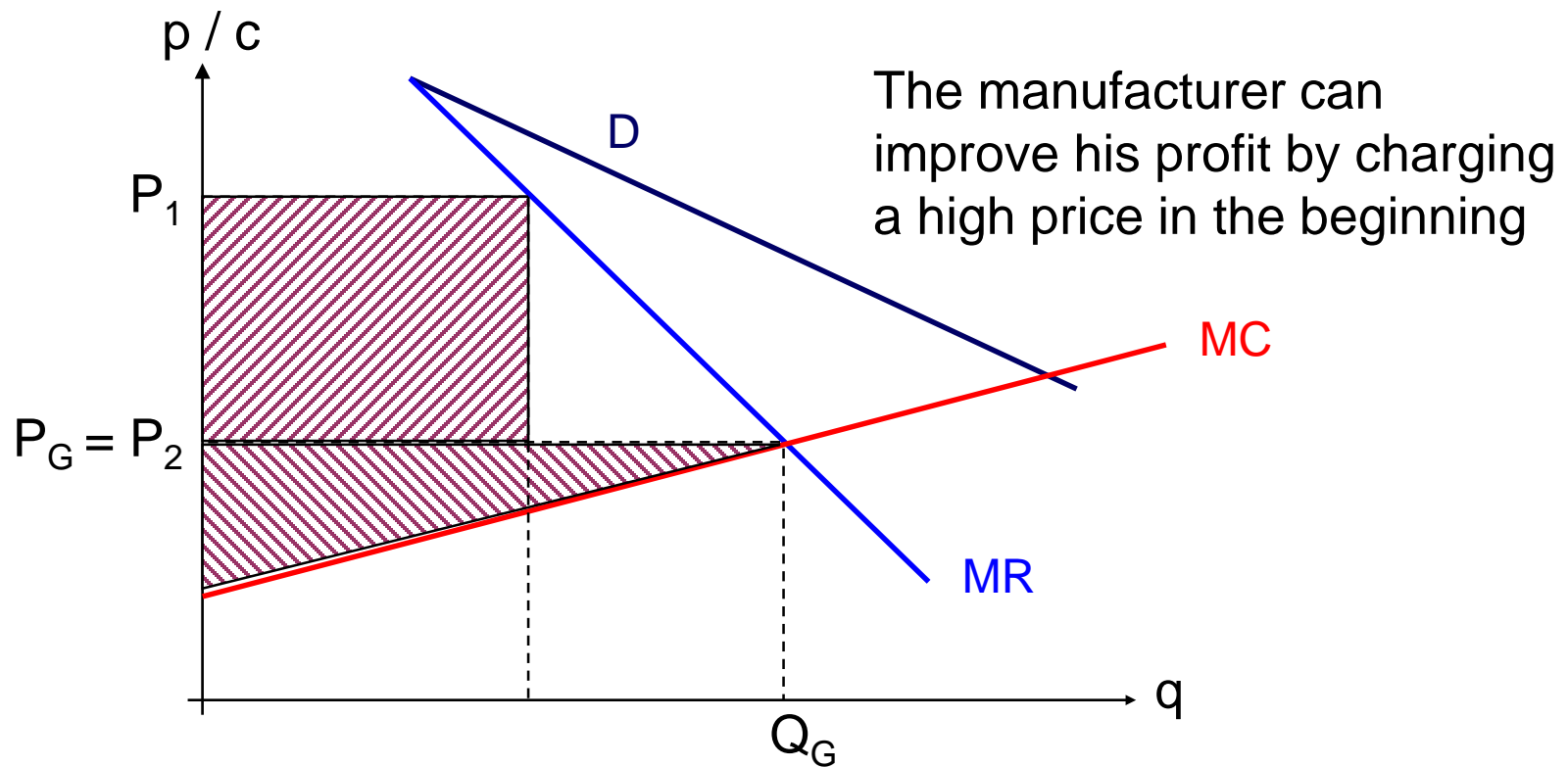
Sales Rebate Contracts

- Sales Rebate contracts provide an incentive to the retailer to increase the number of units bought by means of a rebate paid by the supplier for any item sold above a certain quantity
- The slope of the black lines represents the cost of the last unit bought
- The slope of the red line represents the average per-unit cost
⇒ the buyer's marginal costs depend on order quantity



Sales Rebates

- Sales Rebates work in the same way as price discrimination!



Supply Chain Contracts and Risk Aversion

- Supply Chain Contracts with Uncertain Demand deal with the sharing of risk
- If we have an risk-averse player, he will be willing to pay a premium for not being exposed to risk
- This means we may be able to satisfy the risk-averse player with a fixed amount of money and leave the rest of the profits to the risk-neutral player