NTNU Department of Industrial Economics and Technology Management
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TIØ4285 Production and Network Economics

Assignment 6 – Suggested solution

Exercise 1. Hybrid market

a. Firm D: considers the inverse demand curve $p(Q)$ - immediately in standard form:

$$min_{q_D} Z_D = (c_D(q_D) - p(Q)q_D)$$
$$= (c_D q_D + d_D q_D^2 - (a - b(q_D + q_F))q_D)$$
$$= \left(c_D q_D + d_D q_D^2 - (a q_D - b(q_D + q_F)q_D)\right)$$
$$= \left((c_D - a + b q_F)q_D + (b + d_D)q_D^2\right)$$

Fringe F: considers market price $\hat{p}$ as given

$$min_{q_F} Z_F = (c_F(q_F) - \hat{p} q_F)$$
$$= (c_F q_F + d_F q_F^2 - \pi q_F)$$
$$= \left((c_F - \pi)q_F + d_F q_F^2\right)$$

b. MCP

- Firm D

$$0 \leq q_D \perp (c_D - a + b q_F) + 2(b + d_D)q_D \geq 0$$

- Fringe F:

$$0 \leq q_F \perp (c_F - \pi) + 2 d_F q_F$$
$$= \left(c_F - \left(a - b(q_D + q_F)\right)\right) + 2 d_F q_F$$
$$= (c_F - a + b q_D) + (b + 2d_F)q_F \geq 0$$

c. To facilitate a compact and intuition supporting implementation, introduce conjectural variation parameter CV_i: $CV_F = 1, CV_D = 0$ and reorder conditions to an MC=MR perspective:

$(c_D - a + b q_F) + 2(b + d_D)q_D \geq 0$	$(c_F - a + b q_D) + (b + 2d_F)q_F \geq 0$
$c_D + 2d_D q_D \geq \mathrm{a} - b q_F - 2\mathrm{b} q_D$	$c_F + (b + 2d_F)q_F \geq a - b q_D - b q_f$
$c_D + 2d_D q_D \geq \mathrm{a} - b(q_F + q_D) - \mathrm{CV_D b} q_D$	$c_F + 2d_F q_F \geq a - b(q_F + q_D) - \mathrm{CV_F} b q_D$

Both expressions can be written as: $c_i + 2d_i q_i \geq a - b\sum_j q_j - \mathrm{CV_i} b q_i$. This is implemented in file *asn_3_2_hybrid.gms*.

d. You can use the model implementation to find the answer.
For illustrative purposes, here the calculated the results are presented:
Assume $q_F > 0$. Then: $(c_F - a + bq_D) + (b + 2d_F)q_F = 0 \Rightarrow q_F = \frac{(a-c_F-bq_D)}{(b+2d_F)}$ (*note: this is the optimal response curve of the fringe, which we need in assignment 3.*)
Substitute this into the other KKT, for the dominant firm, and assume its supply is positive:

$$
\begin{aligned}
q_D > 0 \Rightarrow (c_D - a + bq_F) + 2(b + d_D)q_D &= 0 \\
&= \left(c_D - a + b\frac{(a - c_F - bq_D)}{(b + 2d_F)}\right) + 2(b + d_D)q_D \\
&= \left(c_D - a + \frac{b(a - c_F)}{(b + 2d_F)}\right) + \left(2(b + d_D) - \frac{(b^2)}{(b + 2d_F)}\right)q_D \\
\Rightarrow q_D &= \frac{\left(a - c_D - \frac{b(a - c_F)}{(b + 2d_F)}\right)}{\left(2(b + d_D) - \frac{(b^2)}{(b + 2d_F)}\right)}
\end{aligned}
$$

Substitute in parameter values $a = 100, b = 2, c_D = 10, d_D = \frac{1}{2}, c_F = 20, d_F = 1.$

$\boldsymbol{q_D} = \frac{100-10-\frac{2(100-20)}{(2+2)}}{2\left(2+\frac{1}{2}\right)-\frac{(2^2)}{(2+2)}} = \frac{50}{4} = \mathbf{12\frac{1}{2}}, \boldsymbol{q_F} = \frac{\left(100-20-2\frac{50}{4}\right)}{(2+2)} = \frac{55}{4} = \mathbf{13\frac{3}{4}}, \boldsymbol{p} = 100 - 2\left(\frac{50}{4} + \frac{55}{4}\right) = \mathbf{47\frac{1}{2}}$

Exercise 2. Dominant firm with a competitive fringe

Derive the optimal response function for the fringe (or take it from the solution from assignment 1 above) and substitute this into the optimization problem of the dominant firm.

$$q_F = \frac{(a - c_F - bq_D)}{(b + 2d_F)}$$

a. Firm D – immediately in standard form:

$$min_{q_D}\ Z_D = (c_D(q_D) - p(Q)q_D) =$$

$$= (c_D q_D + d_D q_D^2 - (a - b(q_D + q_F))q_D)$$

$$= \left(c_D + d_D q_D - \left(a - b \left(q_D + \frac{(a - c_F - bq_D)}{(b + 2d_F)} \right) \right) \right) q_D =$$

$$= \left(c_D - a + \frac{b(a - c_F)}{(b + 2d_F)} \right) q_D + \left(d_D + b - \frac{b^2}{(b + 2d_F)} \right) {q_D}^2$$

b. See file asn_3_2_dom_firm

c. MCP

- Firm D

$$0 \le q_D \perp \left(c_D - a + \frac{b(a - c_F)}{(b + 2d_F)} \right) + 2 \left(d_D + b - \frac{b^2}{(b + 2d_F)} \right) q_D \ \ge 0$$

$$q_D > 0 \Rightarrow q_D \ = \frac{a - c_D - \frac{b(a - c_F)}{(b + 2d_F)}}{2 \left(d_D + b - \frac{b^2}{(b + 2d_F)} \right)}$$

Fill out the parameter values $a = 100, b = 2, c_D = 10, d_D = \frac{1}{2}, c_F = 20, d_F = 1$:

$\boldsymbol{q_D} = \frac{50}{3} = \mathbf{16\frac{2}{3}}, \mathbf{q_F} = \frac{35}{3} = \mathbf{11\frac{2}{3}}, \boldsymbol{p} = \frac{130}{3} = \mathbf{43\frac{1}{3}}$

d. The dominant firm supplies more than in the previous assignment, because it knows that the fringe will respond by supplying less. Even though market price is lower, firm D makes higher profit.

Dominant firm's profit: $p(Q)q_D - c_D(q_D) = \left(\pi - 10 - \frac{1}{2}q_D\right) q_D =$

$$\left(\frac{130}{3} - 10 - \frac{1}{2}\frac{50}{3}\right)\frac{50}{3} = \left(\frac{260 - 60 - 50}{6}\right)\frac{50}{3} = \frac{150.50}{18} = 416\frac{2}{3}$$

Profit of market power exerting firm exercise 1: $p(Q)q_D - c_D(q_D) = \left(\pi - 10 - \frac{1}{2}q_D\right) q_D =$

$$\left(\frac{95}{2} - 10 - \frac{1}{2}\frac{25}{2}\right)\frac{25}{2} = \left(\frac{190 - 40 - 25}{4}\right)\frac{25}{2} == \frac{125.25}{8} = 390\frac{5}{8}$$

Exercise 3. Duopoly on a small network with transport losses

a. Optimization problem suppliers. We write in standard form, and assign dual variables. Account for losses by adjusting nodal inflows in the mass balance. Account for the losses on the inflows. f.i.s. means free in sign $\left(x\ f.i.s\right) \equiv \left(x \in \mathbb{R}\right)$

$$
\begin{aligned}
&\min_{q_{in}^P, q_{in}^S, f_{imn}^P} && \left\{ c_{in} q_{in}^P + \sum_{(n,m)} \left(c_{nm}^A + \tau_{nm}^A \right) f_{inm}^P - \sum_n \left(a_n - b_n \sum_j q_{jn}^S \right) q_{in}^S \right\}, && i = 1,2 \\
&s.t. && q_{in}^S + \sum_m \left(1 - l_{mn}^A\right) f_{imn}^P - q_{in}^P - \sum_m f_{inm}^P = 0 \qquad \left(\varphi_{in}^P\ f.i.s.\right), && n \in 1,2,3 \\
& && q_{in}^P, q_{in}^S, f_{inm}^P \geq 0 &&
\end{aligned}
$$

b. Optimization problem TSO

$$
\begin{aligned}
&\min_{f_{nm}^A} && -\sum_{(n,m)} \left(\tau_{nm}^A f_{nm}^A \right) && \\
&s.t. && f_{nm}^A - cap_{nm}^A \leq 0 \qquad \left(\lambda_{nm}^A \geq 0\right), && (n,m) \in \left\{(1,2),(2,3)\right\} \\
& && f_{nm}^A \geq 0 &&
\end{aligned}
$$

c. Market clearing condition for transportation services

$$
\sum_i f_{inm}^P - f_{nm}^A = 0 \qquad \left(\tau_{nm}^A\ f.is.\right), \qquad (n,m) \in \left\{(1,2),(2,3)\right\}
$$

d. Because supplier 2 cannot supply at node 1, supplier 1 is a single supplier with market power on node 1, hence a monopolist on node 1.

e. The complementarity conditions defining the equilibrium problem:

Suppliers

	$0 \leq q_{in}^P \perp c_{in}^P - \phi_{in}^P \geq 0$	(1)
	$0 \leq q_{in}^S \perp -a_n + b_n\left(\sum_j q_{jn}^S + q_{in}^S\right) + \phi_{in}^P \geq 0$	(2)
	$0 \leq f_{inm}^P \perp c_{nm}^A + \tau_{nm}^A + \phi_{in}^P - (1 - l_{nm}^A)\phi_{im}^P \geq 0$	(3)
	$0 \leq \phi_{in}^P \perp q_{in}^S + \sum_m (1 - l_{nm}^A) f_{inm}^P - q_{inm}^P - \sum_m f_{inm}^P \geq 0$	(5)

TSO

	$0 \leq f_{nm}^A \perp -\tau_{nm}^A + \lambda_{nm}^A \geq 0$	(6)
	$0 \leq \lambda_{nm}^A \perp cap_{nm}^A - f_{nm}^A \geq 0$	(7)

MCC transport services

	τ_{nm}^A f.i.s., $\sum_i f_{inm}^P - f_{nm}^A = 0$	(8)

f. See *asn_3_3_duop_netw.gms.*

node	prod	flow+	sales	flow-	price
1	7.283		4	3.283	6
2	5.934	2.955	4.889	4	5.111
3		3.6	3.6		6.4