Lecture 4: Auctions

TIØ4285 Production and Network Economics

Spring 2020

Norwegian University of Science and Technology

Outline

- Types of auctions
- Bidding behavior
- Buyer’s and seller’s problem
- Introducing risk aversion

Properties of an Auction

- An auction is a method of allocating scarce goods
  - based upon competition:
    - A seller wishes to obtain as much money as possible
    - A buyer wants to pay as little as possible
- An auction offers the advantage of simplicity in determining market-based prices
- It is efficient in the sense that it usually ensures that
  - resources accrue to those who value them most highly
  - sellers receive the collective assessment of the value
- The price is set by the bidders
- The seller sets the rules by choosing the type of auction to be used

Observations

- The higher the bid, the higher probability of winning
- The lower the bid, the higher payoff in case the bid wins

Types of Auction Mechanisms

Taxonomy of Auctions

- William Vickrey established the basic taxonomy of auctions based upon the **order in which prices are quoted** and the **manner in which bids are given**

- He established four major auction types
  - English: Ascending-price, open-cry
  - Dutch: descending-price, open-cry
  - First-price, sealed bid
  - Vickrey or second-price, sealed bid

English Auction

- An ascending sequential bid auction
- Bidders observe the bids of others and decide whether or not to increase the bid
- The item is sold to the highest bidder

English auctions (procedure)

- All bidders are initially *active*
- Start price and increment are fixed
- At each stage of the bidding:
  - Auctioneer calls out last price + increment
  - Zero or more bidders may become inactive
  - If at least 2 bidders are still active, auction proceeds to the next stage
  - If only one auctioneer is active, then he wins at the current price

Dutch Auction

- A descending price auction
- The auctioneer begins with a high asking price
  - if no bidder accepts price within a given time period (e. g. 15 seconds), then price is lowered
- The bid decreases until one bidder is willing to pay the quoted price
- Called Dutch auction, because procedure is used to sell flowers in the Netherlands

Dutch auctions (procedure)

- All bidders are initially *inactive*
- Start price and decrement are fixed
- At each stage of the bidding:
  - Auctioneer calls out last price - decrement
  - If at least one bidder says yes, then the first bidder to respond wins at the current price
  - Else auctioneer proceeds to the next round

First-Price, Sealed-bid

- An auction where bidders simultaneously submit bids on pieces of paper
  - Bidders *do not* know the bids of other players

- Once bidding period is closed, offers are revealed and highest valuation bidder receives the item at stated price

- Often used for procurement of goods and services, e. g. constructing a new highway (bidder with the lowest price wins)

Second Price, Sealed-bid

- The same bidding process as a first price sealed-bid auction

- However, the high bidder pays the amount bid by the 2nd highest bidder

- Auctions also called Vickrey Auctions

Objectives

- Sellers
  - wish to maximize profit
  - can influence structural parameters through auction rules
  - How does auction design influence revenues?

- Buyers
  - wish to maximize their profit/ utility
  - determine the price in the auction
  - How does auction design influence their strategies?

Model assumptions

- Bidders are symmetric:
  - Bid chosen from a distribution of possible values
  - Symmetric bidders choose their bid from the same distribution
    - The distribution is common knowledge
- Bidders are risk-neutral
  - Maximize expected values, not utility
- Signals are independent
  - Private-value auctions: reservation prices are a function of private information and utility
  - Common-value auctions: all bidders value the items similarly, but the true value of the good is unknown (ex.: oil-fields)

Definitions

- Reservation price
  - Seller: the minimum price he is willing to accept
  - Buyer: maximal price the buyer is willing to pay

- Number of bidders: N
- Bidder number i values the object at v_i
  - The valuation is drawn from the interval [lower, upper]
  - Distribution function (cumulative distribution): $F_i(v_i)$
  - Density function: $f_i(v_i)$
- Winning bid / price of object: p
- Probability of winning auction: P_w

- Expected profit (utility)
  - $u_i(P,b,p)= P_w(v_i-p)$

Bidding strategies

How should the bidders behave in the different auction settings

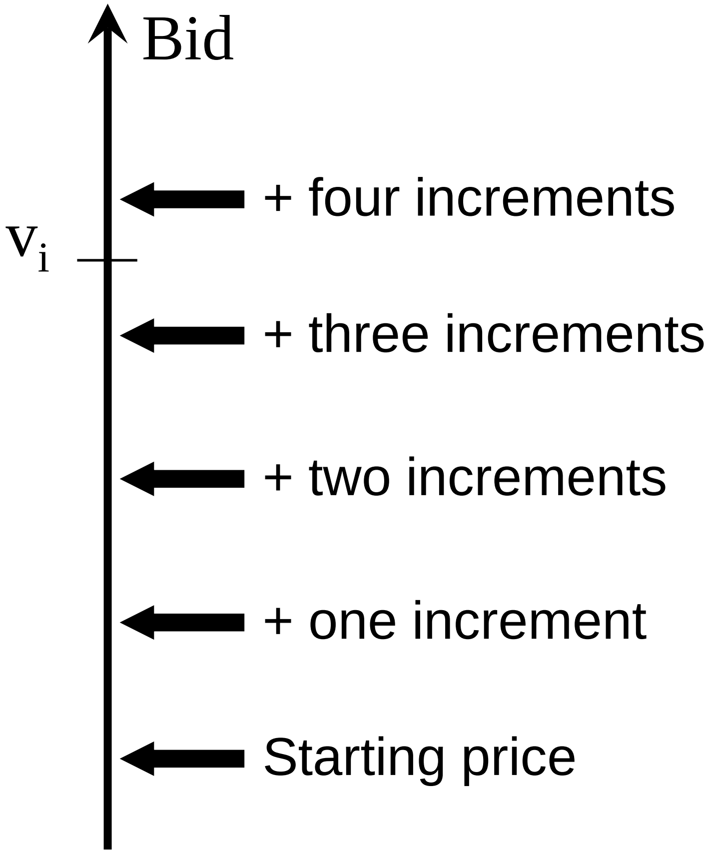
Bidding strategies: English

- Only auction where you gain information about the other bidder's valuation of the object

- Important issues (for the buyer)
  - Want to maximize profit $(v_i - p)$

  - What is the optimal strategy?
  - What is the equilibrium price?
  - Why?

Bidding strategies: English Illustration



- How long would you stay in the auction?
- What determines whether or not you will win the auction?
- Which price would you have to pay if you win?
- What is the optimal strategy?

Bidding strategies: English

- Winning bid is equal to the second-highest reservation price (+epsilon)

- Dominant strategy is to take part in bidding until your own reservation price, but with epsilon increases!

- This is not influenced by information of other bids!

Classification

English auction Optimal strategy: *Bid up to* v_i Price: *second highest* v *(+ epsilon)*	**Dutch**
Second price, sealed bid	**First price, sealed bid**

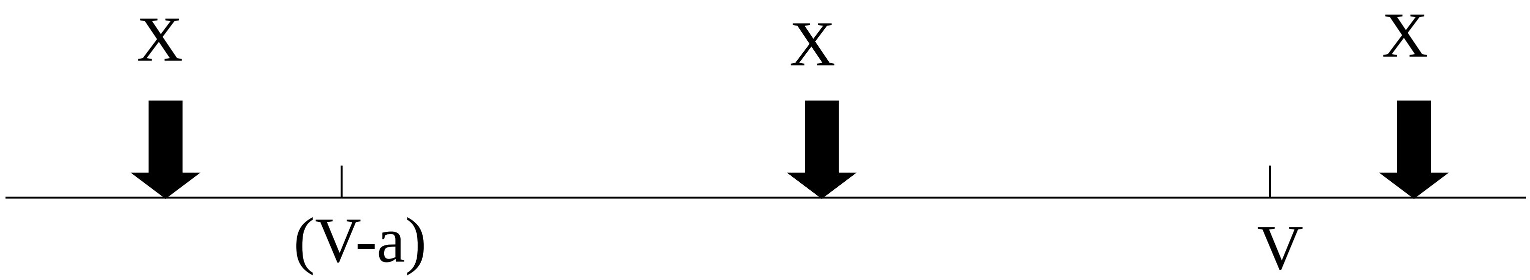
Second-Price Sealed-Bid Auction

- Again: bidders naturally want to maximize profit $(v_i - p)$

- What is now the optimal strategy?
- What is the equilibrium price?
- Why?

Bidding strategies: Second price, sealed Illustration

- Suppose I bid *(V-a)*.
- Let the value of the highest bidder (other than mine) be X.
- Three cases:



NTNU

Second-Price Sealed-Bid Auction

- Bid your reservation price
- Pay the second highest bid
- Mechanism to reveal true reservation price of bidders
  - Incentive compatible
- What is the difference between this auction and the English auction?
- Does information about other participants reservation price influence your decision in this auction?

Classification

English auction Optimal strategy: *Bid up to* v_i Price: *second highest* v *(+ epsilon)*	Dutch
Second price, sealed bid Optimal strategy: *Bid* v_i Price: *second highest* v	**First price, sealed bid**

Dutch auction: how to calculate the bid

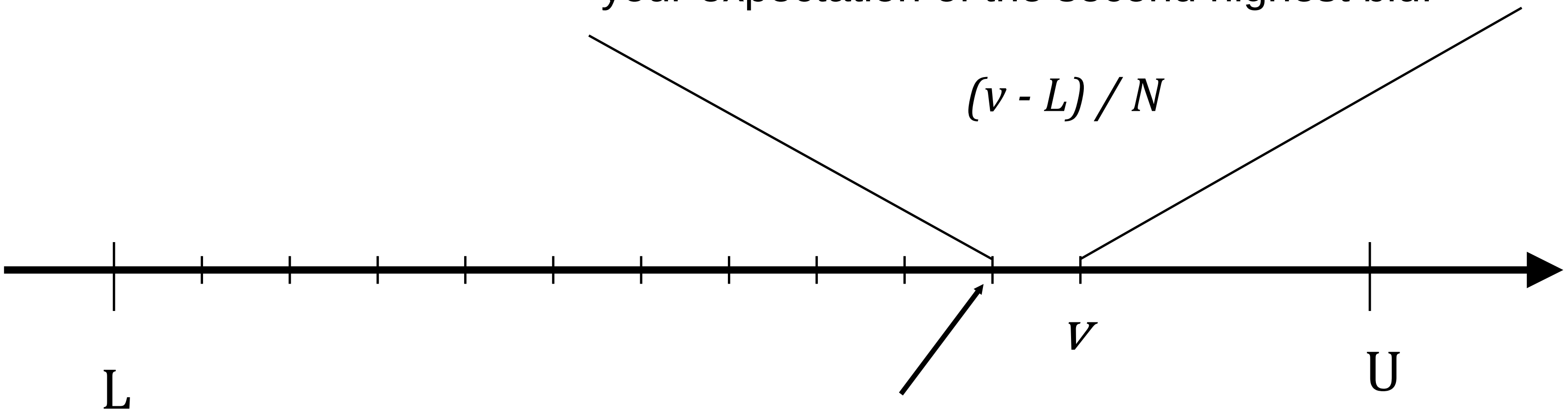
- Why not simply bid your reservation price?
- Assume we are doing our best, given the actions of the others
- We must base our bid on the expectations for the second-highest bidder
- The procedure is as follows:
  - Assume we have the highest reservation price
  - Estimate the value of the second highest bid, given your knowledge about the distribution

- Our belief about the reservation price of the second highest bidder is influenced of the number of bidders
  - The greater the number of bidders, the closer to our reservation price we should bid

Bidding strategies: Dutch Illustration

- Assume a linear distribution of bids: [L, U]
- Your reservation price is v

Difference between your reservation price and your expectation of the second highest bid:



Optimal bid: $b = v - ((v - L)/ N)$

Dutch auction: observations

- Bid less than your reservation price
  - Bid the expectation of the reservation price of the second-highest bidder, conditional on winning the auction

- Our belief about the reservation price of the second highest bidder is influenced of the number of bidders
  - The greater the number of bidders, the closer to our reservation price we should bid

Classification

English auction Optimal strategy: *Bid up to* v_i Price: *Second highest* v *(+ epsilon)*	**Dutch** Optimal strategy: *Bid* $E(2nd\ highest\ v)$*, conditional on winning* Price: *On expectation it is equal to second highest* v
Second price, sealed bid Optimal strategy: *Bid* v_i Price: *Second highest* v	**First price, sealed bid**

First price, sealed:
How to calculate the bid

- Identical situation as for the Dutch auction – same results apply

Classification

English auction Optimal strategy: *Bid up to* v_i Price: *Second highest* v *(+ epsilon)*	**Dutch** Optimal strategy: *Bid* $E(2nd\ highest\ v)$*, conditional on winning* Price: *On expectation it is equal to second highest* v
Second price, sealed bid Optimal strategy: *Bid* v_i Price: *Second highest* v	**First price, sealed bid** Optimal strategy: *Bid* $E(2nd\ highest\ v)$*, conditional on winning* Price: *On expectation it is equal to second highest* v

Strategies for sellers

Which auction design should the sellers choose in order to maximize profits?

Revenue Equivalence Theorem

- When bidders in an auction are risk-neutral and have independent private values, any auction format will generate on average the same revenue for the seller

- Intuition: In the first-price sealed bid auction, each bidder estimates how far below his own valuation the next highest valuation is on average, and then submits a bid that is this amount below his own valuation
  - Hence, on average, the price reached in a first-price auction is the same as in a second-price auction

Optimal bidding strategy, FPA

- How do bidder’s behave, i.e. determine their bid?
  - Bidder’s want to maximize their utility

- Remember:
  - A winning bid below reservation price guarantees a positive payoff
  - The lower the bid, the higher the payoff (in case of winning bid)
  - The higher the bid, the higher the probility of winning

- Assume all bidders are symmetric, i.e.
  - All reservation prices come from the same distribution
  - That **does not** mean all bidders have the same reservation price

Maximizing bidder utility I

- Assumption: bidders will determine bid $b(r)$ based on valuation r

- A single bidder's utility is given by

$$u(r, v) = F^{N-1}(r)\,(v - b(r))$$

  where
  - $u(r, v)$ bidder's utility given reservation price v and valuation r
  - $F^{N-1}(r)$ probability that all other bidders have lower bids
  - $v - b(r)$ payoff for the bidder if $b(r)$ is winning bid

Maximizing bidder utility II

- To maximize utility, derive $u(r, v)$ with respect to r and set to zero

$$\frac{dF^{N-1}(r)(v-b(r))}{dr} = (N-1)F^{N-2}(r)f(r)(v-b(r)) - F^{N-1}(r)b'(r) = 0$$

- In equilibrium, bidder will maximize expected payoff when $r = v$, i.e. when bidding $b(v)$

- Why?

Finally: optimal bidding strategy

- Evaluate expression from previous slide for *r =v* and rearrange

$$(N-1)F^{N-2}(v)f(v)b(v) + F^{N-1}(v)b'(v) = (N-1)vf(v)F^{N-2}(v)$$

$$\frac{dF^{N-1}(v)b(v)}{dv} = (N-1)vf(v)F^{N-2}(v)$$

$$F^{N-1}(v)b(v) = (N-1)\int_0^v xf(x)F^{N-2}(x)dx$$

$$b(v) = \frac{N-1}{F^{N-1}(v)}\int_0^v xf(x)F^{N-2}(x)dx$$

$$b(v) = \frac{1}{F^{N-1}(v)}\int_0^v x dF^{N-1}(x)$$

Expected revenue: FPA

- Assume bidder's value is uniform on [0,1]
  - $F(v)$ is then equal to v, and
  - $f(v)$ is equal to 1
  - $b(v)$ is the bidding strategy, here: $v - v/N$
  - $g(v)$ density of highest value, in general NfF^{N-1}

- Expected revenue in a first-price auction is then:

$$R_{FPA} = \int_0^1 b(v)g(v)dv$$

Expected revenue: SPA

- Assume bidder's value is uniform on [0,1]
  - *F(v)* is then equal to v, and
  - *f(v)* is equal to 1
  - *h(v)* is density of second-highest value, in general $N(N-1)F^{N-2}f(1-F)$

- Expected revenue in a second-price auction is then:

$$\begin{aligned}
R_{SPA} &= \int_0^1 vh(v)dv \\
&= N(N-1)\int_0^1 vF^{N-2}(v)f(v)(1-F(v))dv \\
&= N(N-1)\int_0^1 v^{N-1}(1-v)dv \\
&= N(N-1)\left[\frac{1}{N}-\frac{1}{N+1}\right] \\
&= \frac{N-1}{N+1}
\end{aligned}$$

Optimal Auctions

- Revenue equivalence says that the form of the auction does not affect how much money the seller makes

- Other factors might however influence the outcome of the auction
  - Number of bidders
  - Risk profile

Strategies for Sellers

- The optimal price is determined the distribution of reservation prices for the different bidders

- To maximize surplus, sellers have to sell to buyers with high reservation prices

- Auctions guarantee highest reservation price at which customers are still willing to buy a product

Value of Information

- Auctions are preference-revealing

- Managers can use auctions to collect information about unknown demand before announcing a price schedule

- Applications and Problems
  - Repurchase Tender Offers
  - Number of Bidders
  - Risk Aversion
  - Winner's Curse

Example: Market vs. Auction

- A seller has 4 units of output at a marginal cost of $0. 6 customers (reservation prices: $40, $20, $15, $90, $60, $50) want to buy the product. Compare an auction with a posted price scheme (price $40, maximizing total available surplus)

Fixed Market Price

Consumers	Reservation Price	Win bid
1	$40	40
2	20	
3	15	
4	90	40
5	60	40
6	50	40
Total Consumer surplus		80
Total Seller Surplus		**160**
Total Available Surplus		240

Using an Auction

Consumers	Reservation Price	Win bid
1	$40	21
2	20	
3	15	
4	90	61
5	60	51
6	50	41
Total Consumer surplus		66
Total Seller Surplus		**174**
Total Available Surplus		240

Repurchase Tender Offers (RTO)

- Used by managers to buy back stock shares paying a price above market price as incentive for shareholders to sell

- Procedure:
  - Managers announce a price range at which they are willing to repurchase tendered shares
  - Shareholders willing to sell then send back a pricing schedule
  - Managers create a supply schedule, determine the amount of shares needed and fix a price

- Since 1981, modified Dutch auctions are used to buy back shares. Average premium per share dropped from 15-20% using fixed prices to 10-15% using modified Dutch auctions

- This illustrates the value of information

3

Shareholder Supply Schedule

Price	Strong	Profit	Medium	Profit	Weak	Profit
$15	400,000	$2,000,000	310,000	$1,550,000	280,000	$1,400,000
16	415,000	1,660,000	400,000	1,600,000	315,000	1,260,000
17	600,000	1,800,000	415,000	1,245,000	400,000	1,200,000
Probability of shareholder's willingness to tender						
	0.40		0.30		0.30	

If managers choose a fixed price RTO, they must set price before knowing the supply schedule. They might then choose a price based on an expected value basis (EV):

$$EV(\$15) = \$2{,}000{,}000(0.40) + \$1{,}550{,}000(0.30) + \$1{,}400{,}000(0.30) = \$1{,}685{,}000$$
$$EV(\$16) = \$1{,}660{,}000(0.40) + \$1{,}600{,}000(0.30) + \$1{,}260{,}000(0.30) = \$1{,}522{,}000$$
$$EV(\$17) = \$1{,}800{,}000(0.40) + \$1{,}245{,}000(0.30) + \$1{,}200{,}000(0.30) = \$1{,}453{,}500$$

With a RTO information is revealed. How will this affect profit?

RTO

- Managers get the shareholders to reveal their valuation

$$\begin{aligned} EV(\text{auction}) &= \$2{,}000{,}000(0.40) + \$1{,}600{,}000(0.30) + \$1{,}400{,}000(0.30) \\ &= \$1{,}700{,}000 \end{aligned}$$

So, managers are generally better off using a modified Dutch auction RTO. They get shareholders to reveal their valuations and, hence, can buy back shares at a lower price than if they used a fixed price. This creates value for the remaining shareholders, since some shares are retired at a lower cost. And, the expected number of shares tendered is

$$0.4(400{,}000) + 0.3(400{,}000) + 0.3(280{,}000) = 364{,}000$$

Number of Bidders

- Markets: in perfect competition, equilibrium price = marginal cost

- Auctions: expected bid is given by second highest reservation price

- Number of buyers increases the price paid for a product

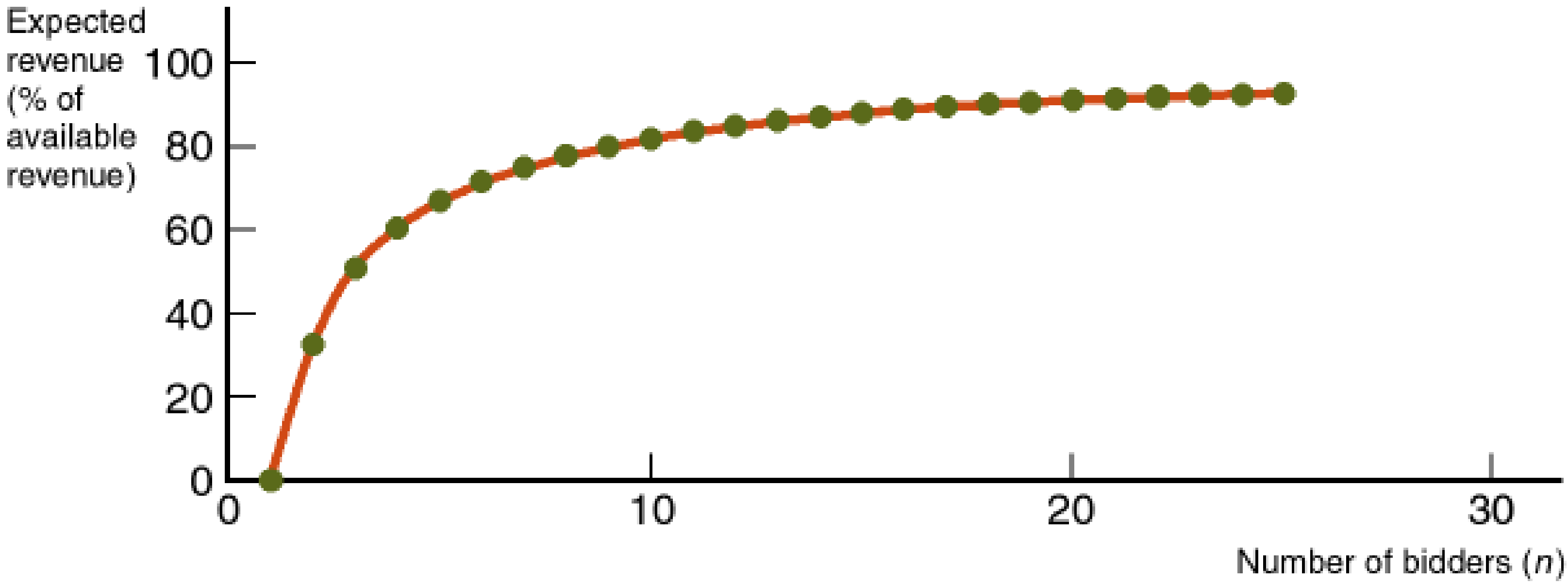


FIGURE 15.2 Expected Revenue versus Number of Bidders

Risk aversion

- What is risk aversion in this case?

- Auctions generally confront bidders with risk
  - A bidder obtains nothing and pays nothing if he loses
  - Earns a positive rent if he wins
  - Thus a bidder is facing risk
  - The extent of bidders' risk aversion will influence bidding behavior

Risk Aversion

- Higher bids increase the probability of winning the auction
- Risk-averse bidders bid higher relative to risk-neutral ones (they pay a premium in order to avoid loss)
- To exploit risk aversion, first-price auction should be used
- Rising the bid increases the possibility to win. The bidder pays an insurance premium to increase the chances of winning.
- What if the seller is risk-averse? The revenues from the four formats are equal, on expectation, but the spreads on second-price auctions is higher. Hence the seller should use first-price auctions

Winner’s Curse

- In some auctions the value of the good auctioned is not known with certainty (e. g. mining rights, oil drilling rights), although it has common value to all bidders

- When seller uses a first-price sealed-bid auction, bidder’s are exposed to the winner’s curse: price paid may be higher than true value of the object

- Other bids are unknown, so the value estimate of others is unknown

- One’s own bid might be extreme, but this is not known. Hence, it is likely to win the auction but pay a price that exceeds the true value

Winners curse

- **Winners curse** is widely recognized as being that phenomenon when a "lucky" winner pays more for an item than it is worth.

- Auction winners are faced with the sudden realization that their valuation of an object is higher than that of anyone else.

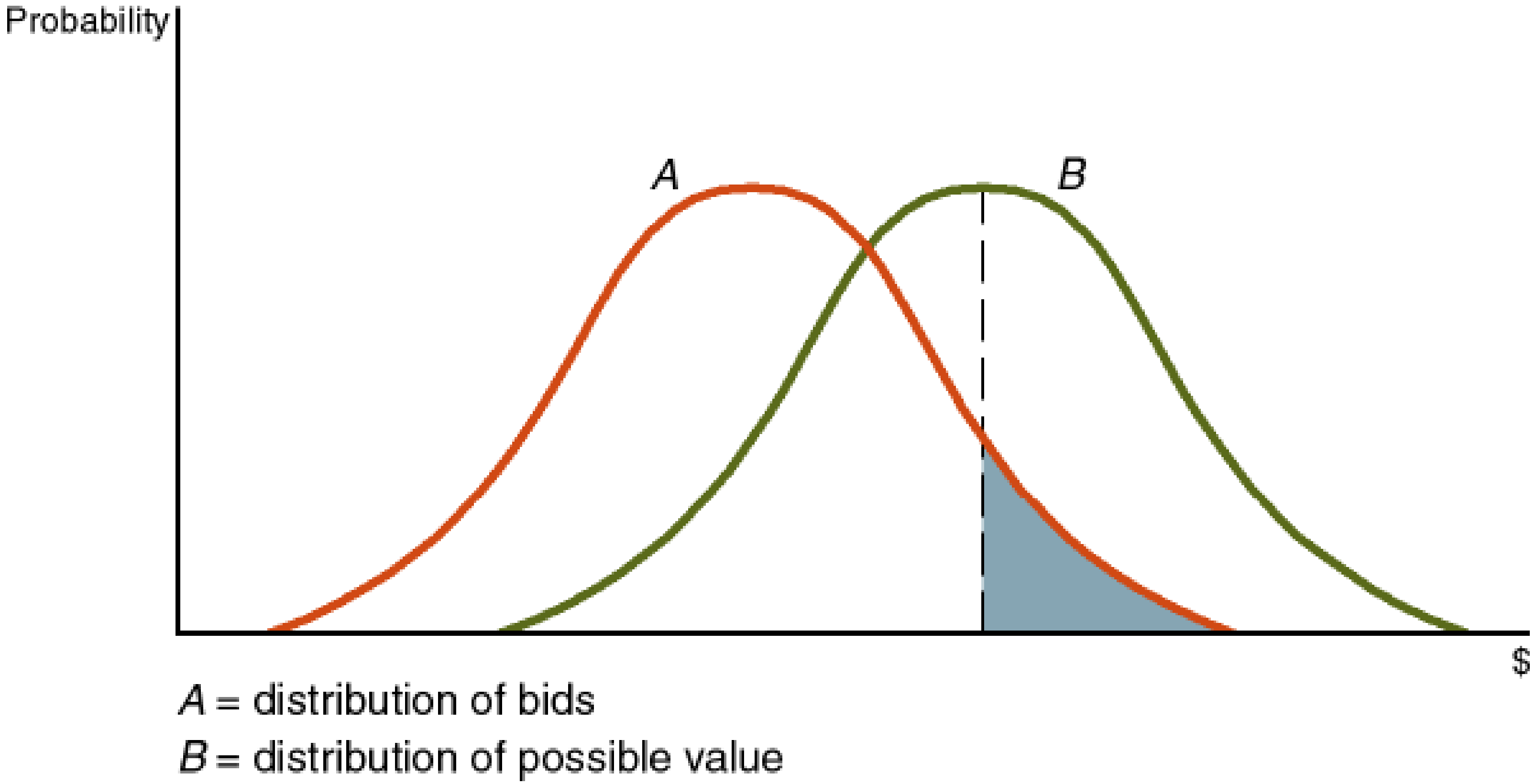


FIGURE 15.3 The Winner's Curse

Important issues in winners curse

- How much information do you have relative to others about the object’s true value?
  - The less information you have the more you should lower your bid

- How confident are you in your estimate of the object’s true value?
  - The less confident you are, the more you should lower your bid

Summary (basic auction types)

English auction	**Dutch**
Optimal strategy: *Bid up to* v_i Price: *Second highest* v *(+ epsilon)* E[Revenue]: *Same in all auctions*	Optimal strategy: *Bid* $E(2nd\ highest\ v)$*, conditional on winning* E[Price]: *Second highest* v E[Revenue]: *Same in all auctions*
Second price, sealed bid	**First price, sealed bid**
Optimal strategy: *Bid* v_i Price: *Second highest* v E[Revenue]: *Same in all auctions*	Optimal strategy: *Bid* $E(2nd\ highest\ v)$*, conditional on winning* E[Price]: *Second highest* v E[Revenue]: *Same in all auctions*

Conclusions

- For English and Second-price sealed auctions dominant strategies exist: bid your reservation price
- For Dutch and First-price sealed auctions no dominant strategies exist: there are multiple equilibriums

- For the seller in an auction, the auction design does not matter
  - Revenue equivalence theorem:
    - Bidders valuation is private information
    - Valuations are independently drawn from a probability distribution that is common knowledge among the bidders
    - Bidders are symmetric
    - Bidders are risk-neutral
- The number of bidders however matters
- So does the risk profile of the participants

NTNU