# Lecture 8: Pricing and Incentives I – Transfer Prices

TIØ4285 Production and Network Economics

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Norwegian University of Science and Technology

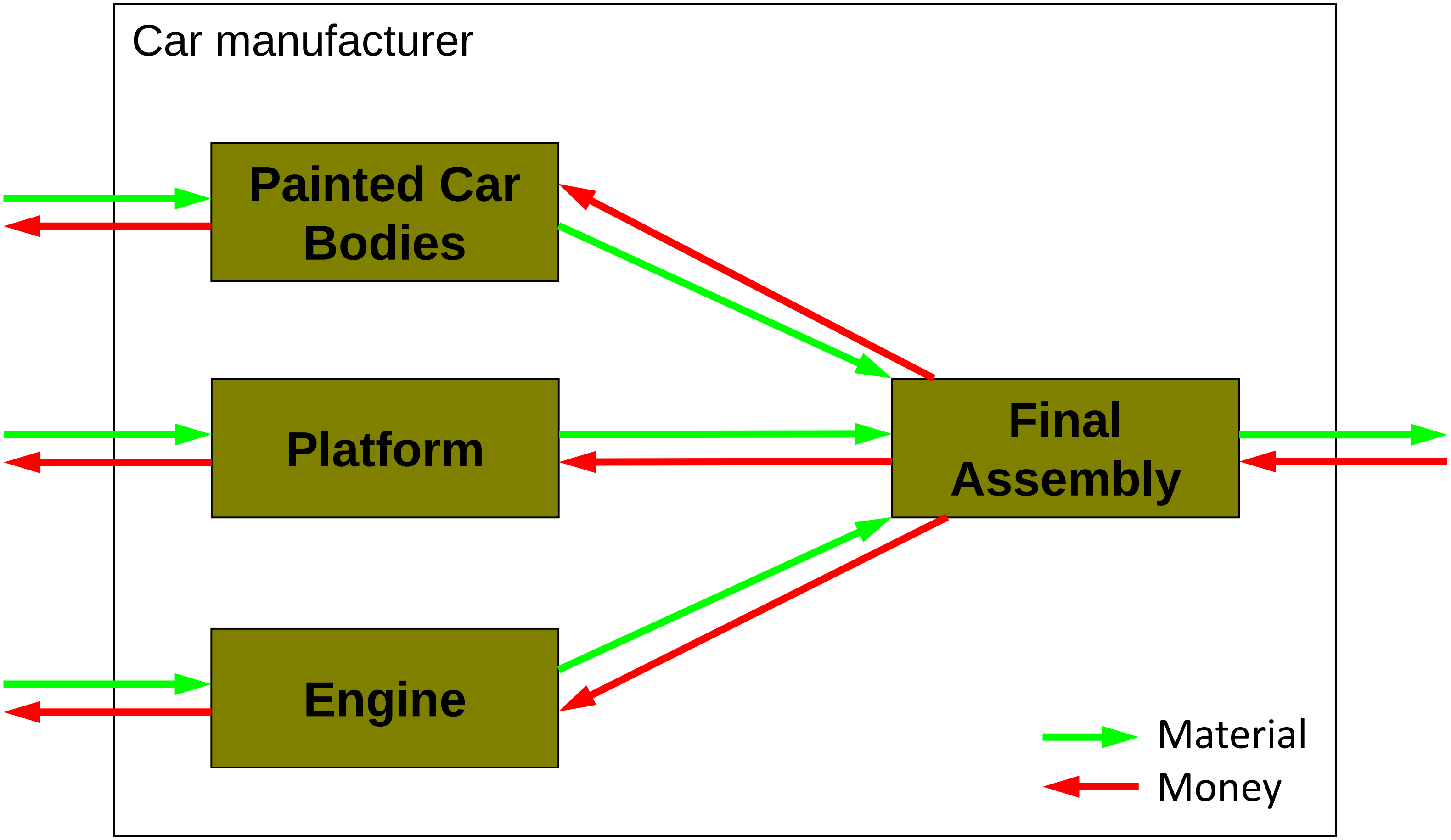
# Outline

- Economic Approach to Transfer Prices
- Operations Research Approach to Transfer Prices
- Life, the Universe and Everything
- Conclusions and Limitations

The transfer pricing problem is a difficult and frustrating one. Although there has been substantial interest in this problem among academics, many managers regard it as unsolved or unsolvable.

**Robert G. Eccles, 1985**

# Example on Transfer Prices



# Properties of Transfer Prices

Transfer Prices

- are prices for the exchange of goods or services between different divisions

- affect the distribution of the profit among the divisions, i. e. the profitability of a divisions

- influence the overall profit of a company

# Aims of Transfer Prices

Transfer prices are used for

- decentralized decision-making to balance supply and demand
- evaluating divisions as separate entities
- minimizing the worldwide taxes, duties and tariffs for multinational companies

# Economic Approach to Transfer Prices

- Microeconomic Theory
- Market-Price based Transfer Prices
- Cost based Approaches
- Negotiation based Transfer Prices

# Microeconomic Theory

- No Outside Market
- Perfect Outside Market
- Imperfect Outside Market

# No Outside Market I

Example:

- Company is producing engines at cost $C_E$ in an upstream division and assembling cars at cost $C_A$ in the downstream division

- Engines cannot be sold outside the company

- How to fix the transfer price for engines to maximize total profit?

- Total profit: $\pi(Q_A) = R(Q_A) - C_A(Q_A) - C_E(Q_E)$

# No Outside Market II

Some necessary formulas:

$$MC_E = \frac{\Delta C_E}{\Delta Q_E}$$

$$MP_E = \frac{\Delta Q_A}{\Delta Q_E}$$

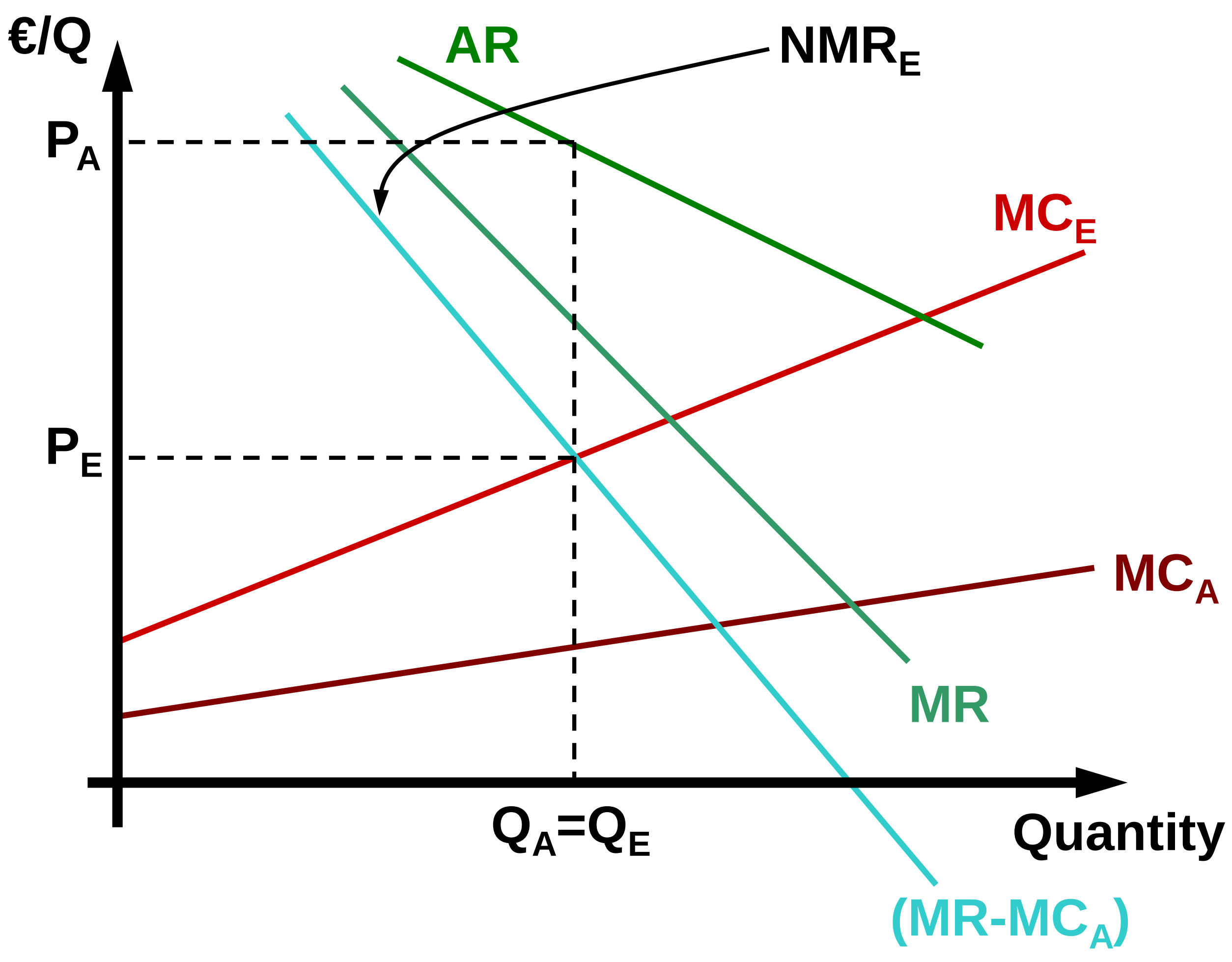
$$MC_A = \frac{\Delta C_A}{\Delta Q_A}$$

$$MR = \frac{\Delta R}{\Delta Q_A}$$

$$NMR_E = (MR - MC_A) \cdot MP_E$$

Total profit maximization for: $NMR_E = MC_E$

# No Outside Market III



# Example: Retailer + manufacturing division

- Retailer

$$
\begin{aligned}
p(q) &= 100 - 4q \\
TC_R(q) &= 15q \\
MR_R &= 100 - 8q \\
\pi_R &= p(q)q - TC_R(q) = 85q - 4q^2
\end{aligned}
$$

- Manufacturer

$$
\begin{aligned}
NMR_R &= MR_R - MC_R \\
&= 100 - 8q - 15 = 85 - 8q \\
TC_M(q) &= 10 + 0.25q^2 \\
MC_M &= 0.5q
\end{aligned}
$$

# Optimal Profits

$NMR_M = MC_M$

$(p_M(q) = MR_R - MC_R = NMR_M))$

$100 - 8q - 15 = 0.5q$

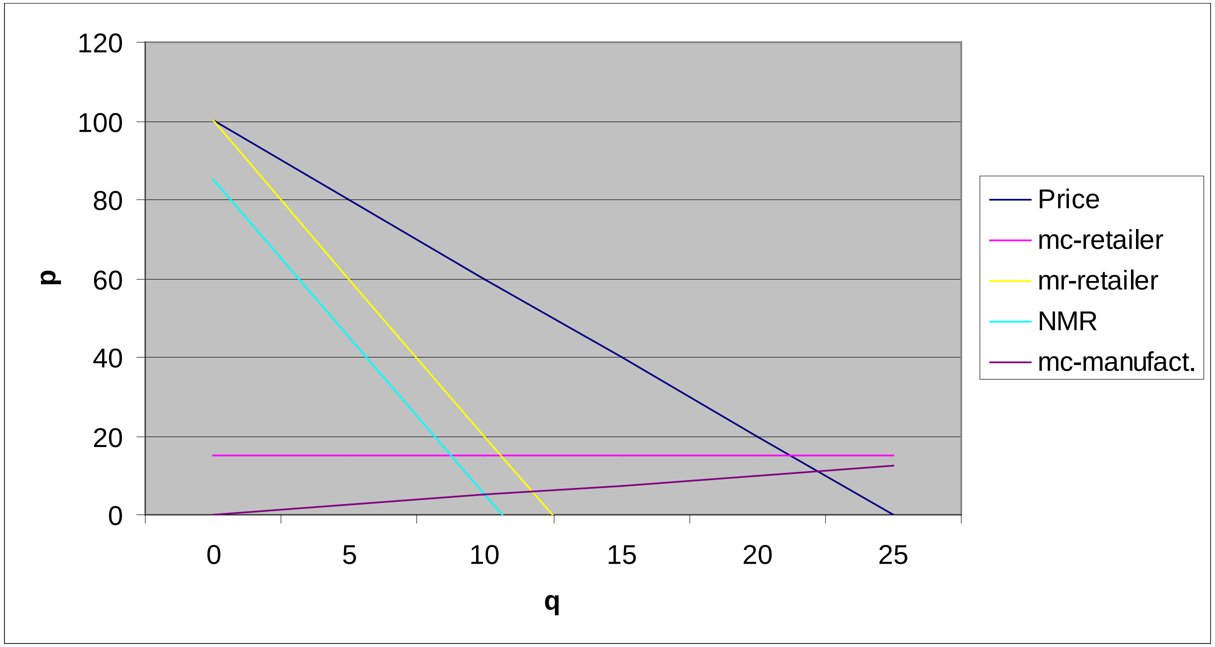
$\Rightarrow \quad 8.5q = 85, \quad q = 10, p = 60$

$\pi_R = 60 \cdot 10 - 4 \cdot 10^2 - 5 \cdot 10 = 400$

$\pi_M = 5 \cdot 10 - 10 - 0.25 \cdot 10^2 = 15$

$\pi = \pi_R + \pi_M = 415$

$MC_M = 5 = \text{transfer price}$




The NMR line is net marginal revenue for the company, seen from the manufacturer.

It can also be interpreted as demand as seen by the manufacturer for different transfer prices (because the retailer will order *q* so that marginal revenue = local marginal cost + transfer price)

# No Outside Market IV

- Transfer Price equals the marginal cost of producing engines

- All engines produced are delivered to the downstream division, i. e. the number of engines equals the number of cars assembled

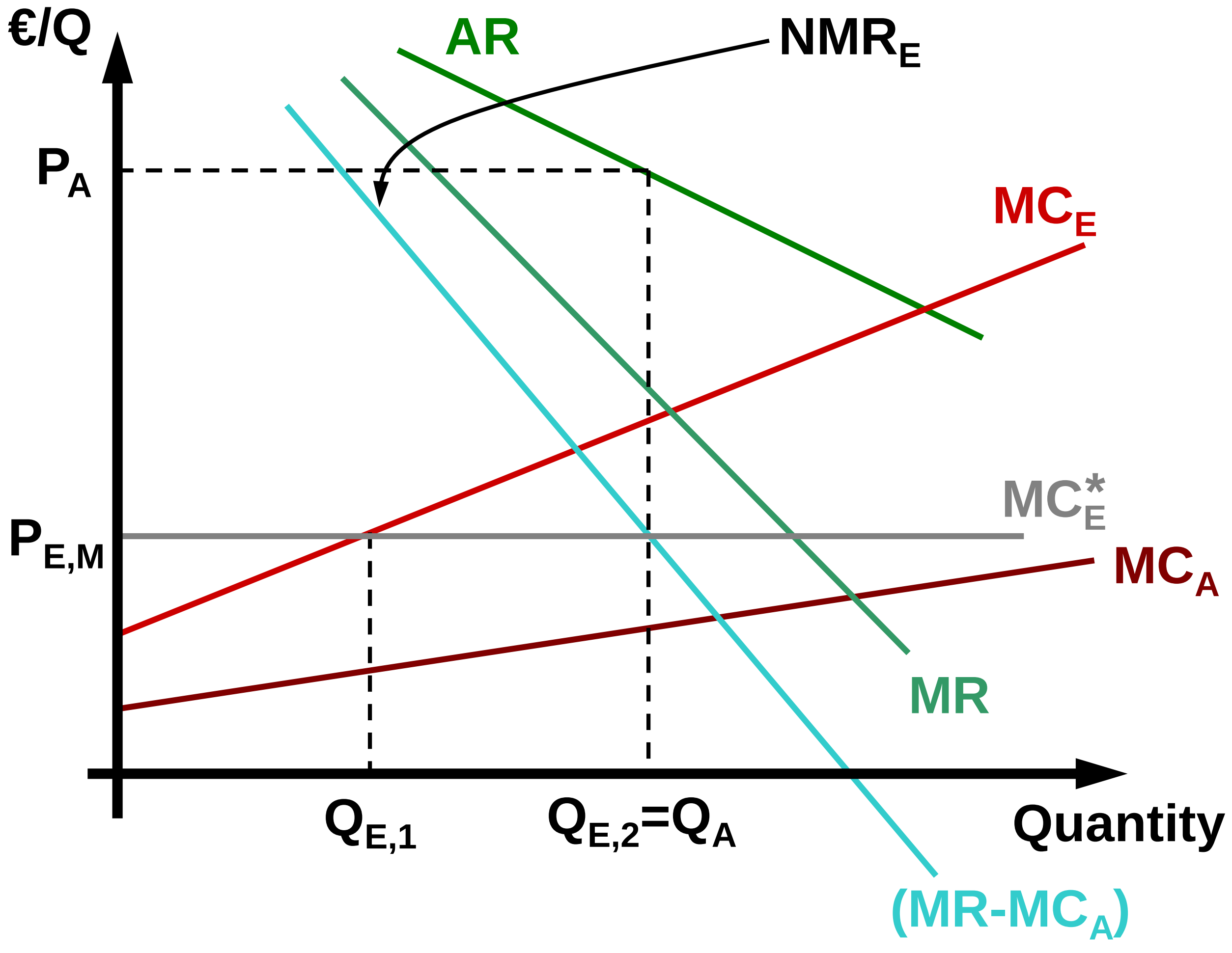
- Is this model valid for a monopoly-monopsony situation?

# Perfect Outside Market I

- Two scenarios: low market price and high market price
- Remember: in a perfect market, the market price equals the marginal cost of production!

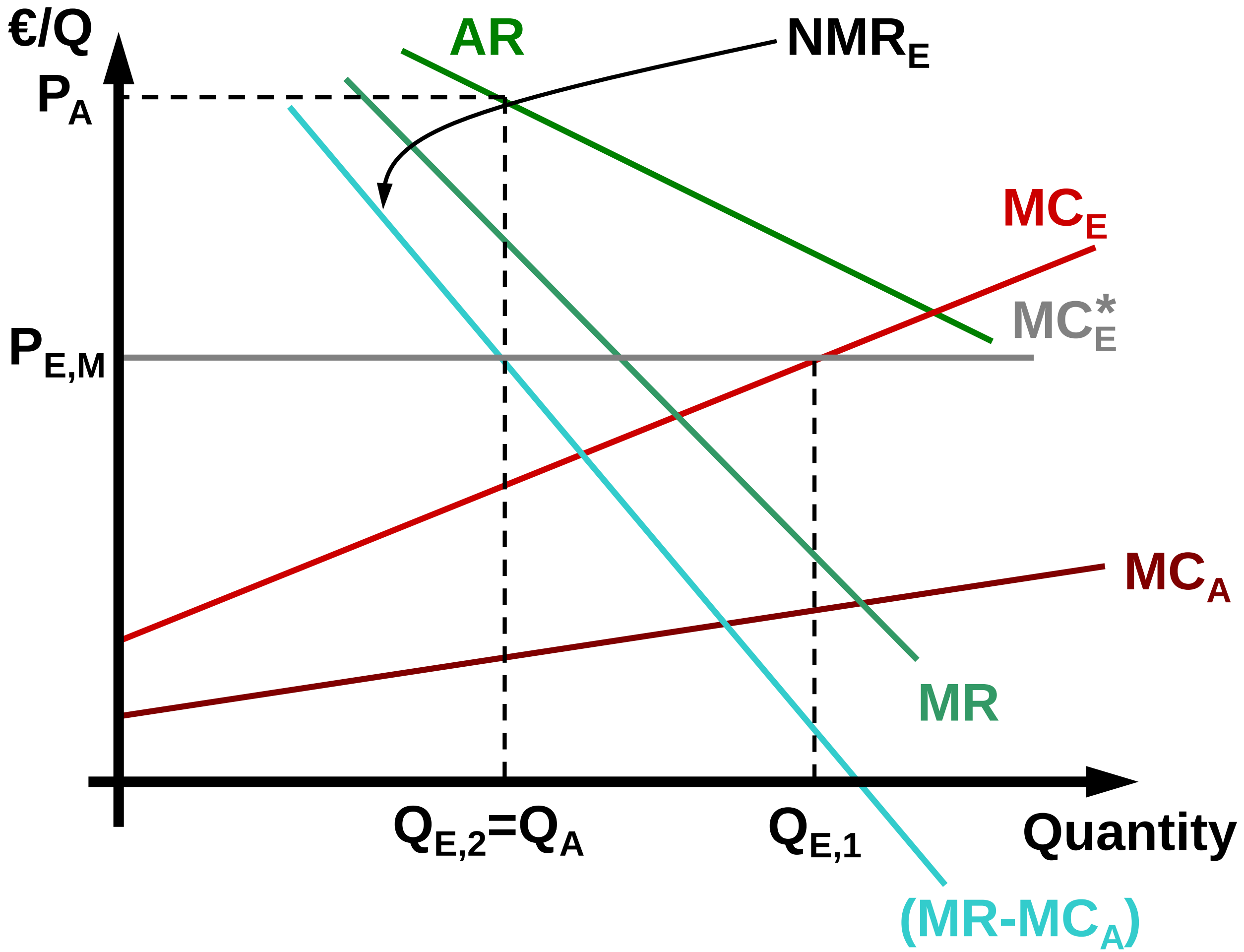
# Perfect Outside Market II

## Low Market Price



# Perfect Outside Market III

## High Market Price



# Perfect Outside Market IV

- Transfer Price equals the market price, i.e. the marginal cost of producing engines

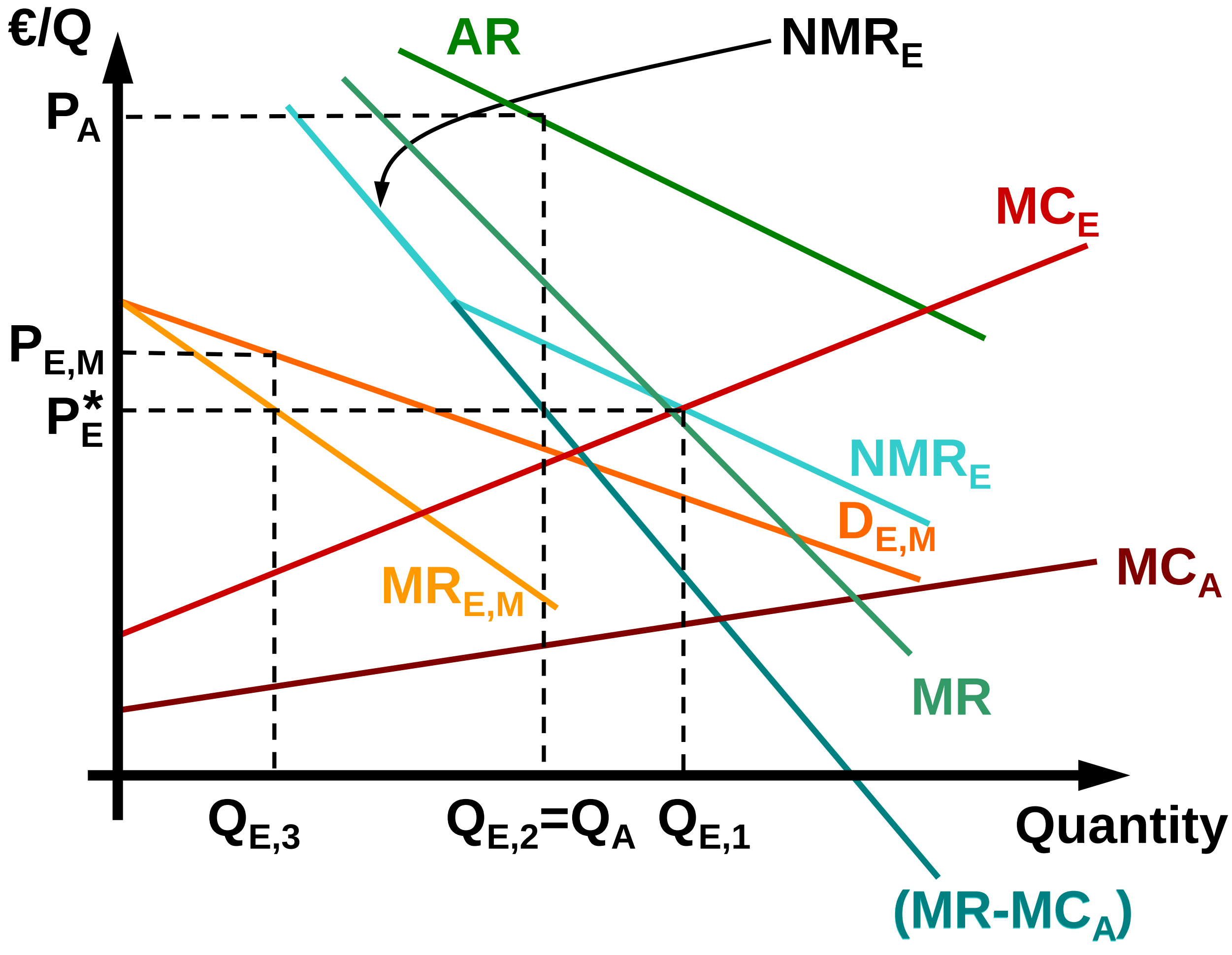
- Amount of cars assembled does not equal the amount of engines produced

- If engines are produced at constant marginal cost (= market prices), shall this be done in-house or should the production be outsourced?
  ($\rightarrow$ Transaction cost theory)

# Imperfect Outside Market I

- There is external demand for engines
- Company has monopoly power in the external engine market
- Market prices change according  to the output of engines

# Imperfect Outside Market II



# Imperfect Outside Market III

- Transfer price is equal to the marginal cost of producing engines
- Amount of cars assembled is smaller than the amount of engines produced
- Why shouldn't the engine division be paid price $P_{E,M}$ for deliveries to the assembly division?
- Who is charging the prices, so what happens to the difference between $P_E^*$ and $P_{E,M}$?

# Microeconomic Theory Observations

- Transfer prices always equals marginal costs of producing engines
- The amount of final products depends on the transfer price
- Production capacities are not taken into account
- What about fixed costs?

# Microeconomic Theory Limitations

- Central management can by assumption induce divisional managers to act in a manner consistent with corporate interests

- The division selling the intermediate product may not recover fixed costs if marginal costs are used as transfer prices

- Divisions are by assumption independent of each other with regard both to production technology and demand

# Market-Price based Transfer Prices

- Applicable if there is a competitive outside market for the intermediate product
- Interdependencies should not exist between subunits
- Market prices might be corrected due to the synergies of internal production
- Markets do not exist for every intermediate product and not every market is perfectly competitive

# Marginal Costs

- Avoid distortions in the optimal allocation of resources, result in optimal profits
- Difficult to determine
- Can vary over the range of output, especially when reaching capacity restrictions
- Selling division may not recover fixed costs
- Provide incentives to misrepresent the cost function

# Variable Costs

- May be appropriate if there is no external market or if market prices are inapplicable or not to determine
- Often used to approximate marginal costs
- Results in similar problems as marginal costs
- Provide incentives to convert fixed costs into variable costs (e. g. using high-priced outsourcing of parts instead of cheaper internal production)

# Variable Costs plus Markup

- Markup can be a fixed fee or a percentage on the variable costs
- Selling division can recover fixed costs
- Buying division pays just for the fixed costs of the required capacity
- Optimal allocation of resources is distorted
- Problems, if there is just one buying department

# Full Costs

- No incentive to reclassify fixed costs to variable costs
- Inefficiencies of the producing division will be transferred to the buying division
- Distortion of the allocation of resources, buying division buys too few units
- Able to deal with changes in capacity
- Very simple approach, resulting in low cost of implementation

# Negotiation based Transfer Prices I

In highly decentralized organizations, transfer prices might be subject of negotiations between the selling and the buying division.

If both quantity and price are subject of the negotiations, the result is probably closer to a global optimum compared to negotiations just focus on the price.

# Negotiation based Transfer Prices II

- Guidelines for determining good transfer prices
  - First the number of units has to be agreed upon to maximize total profit
  - The transfer price then only determines the distribution of total profit among the divisions

- Negotiations about the price only do not result automatically in the right amount of products exchanged

# Negotiation based Transfer Prices III

Negotiations can be successful if:

- there is some form of outside market for the intermediate product
- all market information is available to all negotiators
- the freedom to buy or sell outside exists
- the negotiations are supported by top management

# Negotiation based Transfer Prices IV

Negotiations involve the following limitations:

- It is a time consuming process
- Negotiations may lead to conflicts between divisions
- Measurement of divisional profitability becomes sensitive to negotiating skills
- Top management has to oversee the process and mediate disputes
- A global optimum is not automatically reached

# Operations Research Approach to Transfer Prices

- The Dantzig-Wolfe Decomposition Algorithm
- Example to illustrate the Decomposition Algorithm
- Limitations of Dantzig-Wolfe

# Decomposition

- Decomposition is a method of exploiting the structure of a problem to reduce it from one large problem to a series of smaller problems
- Different types of decomposition have been developed, according to the structure of the constraints section
- Methods known are e. g. decomposition by allocation or decomposition by pricing ($\rightarrow$ Dantzig-Wolfe)

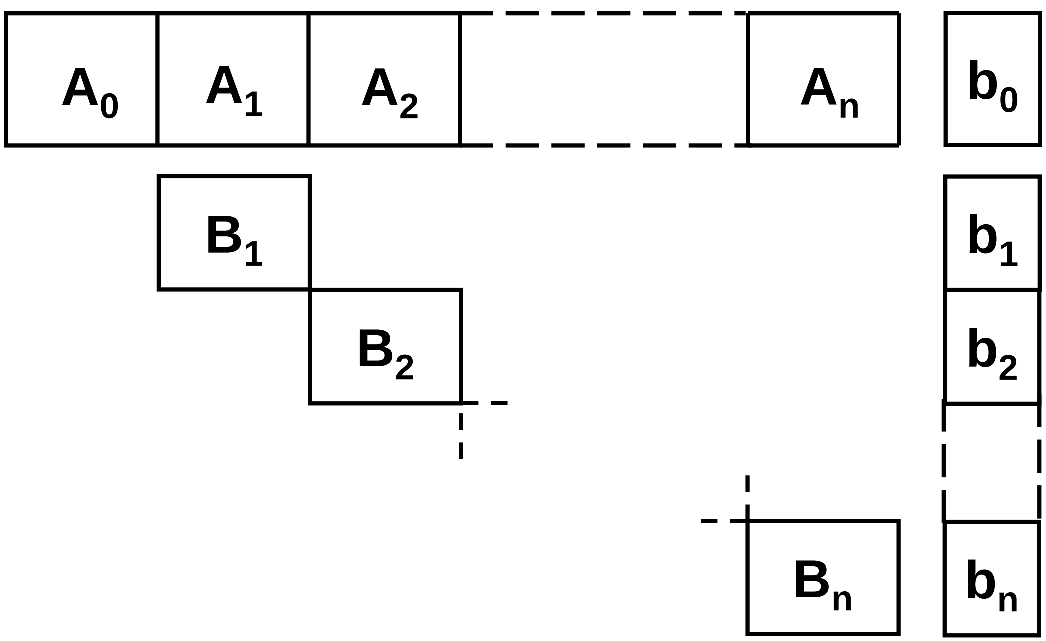
# A Few Words on Dantzig-Wolfe

- Decomposition method developed by George Dantzig and Philip Wolfe in the 1950's

- Closely related to Benders Decomposition (= Primal Decomposition), dual pair in the linear case

# Model Structure

Dantzig-Wolfe decomposition is applicable, if we have a general block angular model, i. e. the constraint section is structured like:



# General Model formulation

A problem with this structure may be written as:

$$\max P = \sum_{i=1}^{n} \mathbf{c}_i^T \mathbf{x}_i$$

subject to

$$\begin{aligned}
\sum_{i=1}^{n} \mathbf{A}_i \mathbf{x}_i &= \mathbf{b}_0 && (m_0 \text{ rows}) \\
\mathbf{B}_i \mathbf{x}_i &= \mathbf{b}_i && (m_k \text{ rows}), i = 1 \ldots n \\
\mathbf{x}_i &\geq 0 && i = 1 \ldots n
\end{aligned}$$

# Interpretation of the Structure

- Models of this structure can arise from multi-time period planning or multi-plant operation.

- If the structure arises by way of a structured organization, the decomposition procedure mirrors a method of decentralized planning

- Every model $\boldsymbol{B}_i\boldsymbol{x}_i = \boldsymbol{b}_i$ can be considered as semi-autonomous submodel with its own constraints being subject to overriding common constraints $\sum_{i=1}^{n} \boldsymbol{A}_i\boldsymbol{x}_i = \boldsymbol{b}_0$

# Considering the Submodels

- Dantzig-Wolfe assumes that a solution $\boldsymbol{x}_i$ satisfying $\boldsymbol{B}_i\boldsymbol{x}_i = \boldsymbol{b}_i, \boldsymbol{x}_i \geq 0$, is a closed and bounded set, denoted $S_i$.

- Any point in $S_i$ can then be represented by

$$\mathrm{x}_i = \sum_{j=1}^{r_i} \lambda_{ij}\mathrm{x}_{ij}$$

with

$$\sum_{j=1}^{r_i} \lambda_{ij} = 1$$

$$\lambda_{ij} \geq 0$$

and $x_{ij}, j = 1 \ldots r_i$ being the vertices of $S_i$.

# Reformulating the Master Model I

Using the above formulation in the original problem results in:

$$P = \sum_{i=1}^{n} \sum_{j=1}^{r_i} \left(\mathbf{c}_i^T \mathbf{x}_{ij}\right) \lambda_{ij} \text{ and } \sum_{j=1}^{r_i} \left(\mathbf{A}_i \mathbf{x}_{ij}\right) \lambda_{ij} = \mathbf{b}_0$$

With $\boldsymbol{p}_{ij} = \boldsymbol{A}_i \boldsymbol{x}_{ij}$ and $u_{ij} = \boldsymbol{c}_i^T \mathbf{x}_{ij}$
the original problem can be reformulated

# Reformulating the Master Model II

$$\max P = \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij}\lambda_{ij}$$

subject to

$$\sum_{i=1}^{n} \sum_{j=1}^{r_i} \mathbf{p}_{ij}\lambda_{ij} = \mathbf{b}_0$$

$$\sum_{j=1}^{r_i} \lambda_{ij} = 1 \qquad i = 1 \ldots n$$

$$\lambda_{ij} \geq 0 \qquad i = 1 \ldots n, j = 1 \ldots r_i$$

# Properties of the new Master Model

- Model consists of just $m_0+n$ rows, the number of constraints is reduced
- However, a column might be inserted for every vertex of all $S_i$, resulting a  very large model
- Dantzig-Wolfe therefore use a column-generating approach
- A column is only added when it seems worthwhile

# Solving the Master Model

- Solving the reformulated master model, we obtain price vector $\pi$ from the common constraints, containing the shadow prices of these constraints.

- These prices are used as transfer prices and result in a reformulation of the submodels

# Reformulating the Submodel

- With the price vector $\boldsymbol{\pi}$ the each submodel *i* is solved for

$$\max P_i = \left(\mathbf{c}_i - \pi^T \mathbf{A}_i\right) \mathbf{x}_i$$

subject to

$$\mathrm{B}_i \mathrm{x}_i = \mathbf{b}_i$$
$$\mathrm{x}_i \geq 0$$

- This can result in a new optimal solution $\boldsymbol{x}_{ij}^{*}$ for this submodel.

# Generating a new Column

- If the new solution $\boldsymbol{x}_{ij}^{*}$ has not yet been taken into account, a new column for the master model is generated:

$$\begin{pmatrix} u_{ij}^{*} \\ \mathbf{p}_{ij}^{*} \\ \xi_k \end{pmatrix} \lambda_{ij} = \begin{pmatrix} \mathbf{c}_{ij}^{T}\mathbf{x}_{ij}^{*} \\ \mathbf{A}_i\mathbf{x}_{ij}^{*} \\ \xi_k \end{pmatrix} \lambda_{ij}$$

with $\xi_k = 1$ for $k = i$ and 0 otherwise.

# Iteration of the Process

- The new master model is solved and a new price vector $\pi$ is determined

- The submodels are modified according to these prices and solved

- If there are no more unconsidered optimal solutions, the iteration process stops and the optimal solution can be determined

# Example on Dantzig-Wolfe

We consider one company with two production facilities. Two products are manufactured in these two facilities using the same input.

The aim is to determine the optimal production plan for both plants utilizing transfer prices for assigning the input.

# The Original Model

The following LP-model describes the problem:

$$
\begin{array}{llrcrcrcrcr}
\text{Maximize} & \text{Profit} & 10x_1 & + & 15x_2 & + & 10x_3 & + & 15x_4 & & \\
\\
\text{subject to} & \text{Raw} & 4x_1 & + & 4x_2 & + & 4x_3 & + & 4x_4 & \leq & 120 \\
& \text{Grinding A} & 4x_1 & + & 2x_2 & & & & & \leq & 80 \\
& \text{Polishing A} & 2x_1 & + & 5x_2 & & & & & \leq & 60 \\
& \text{Grinding B} & & & & & 5x_3 & + & 3x_4 & \leq & 60 \\
& \text{Polishing B} & & & & & 5x_3 & + & 6x_4 & \leq & 75 \\
& & x_1 & , & x_2 & , & x_3 & , & x_4 & \geq & 0
\end{array}
$$

# Solving the Original Model

- Solving the original model directly:

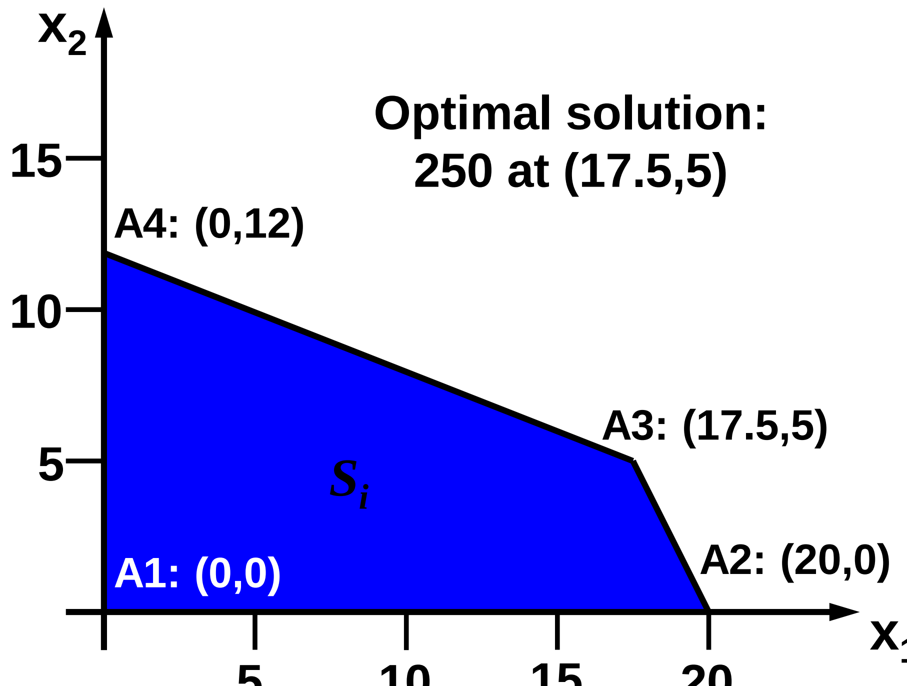
$$x_1 = 9.17 \qquad x_3 = 0$$
$$x_2 = 8.33 \qquad x_4 = 12.5$$
$$p = 1.67$$

- This assumes full information about all constraints in the divisions!
- Dantzig-Wolfe can be seen as planning under asymmetric information

# Submodel A

**Transfer price *p*=0**

$$\begin{array}{lrcrcr} \text{Maximize} & 10x_1 & + & 15x_2 & & \\ \text{subject to} & 4x_1 & + & 2x_2 & \leq & 80 \\ & 2x_1 & + & 5x_2 & \leq & 60 \\ & x_1 & , & x_2 & \geq & 0 \end{array}$$



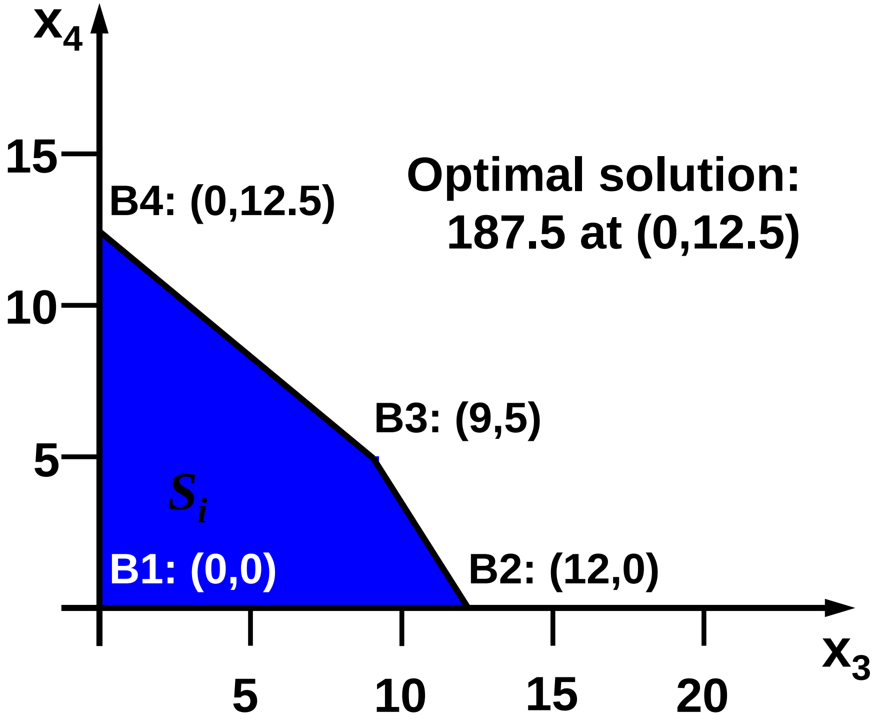

Introduce transfer price p for raw material:

$$\max(10 - 4p)x_1 + (15 - 4p)x_2$$

# Submodel B

**Transfer price *p*=0**

$$
\begin{array}{llcllcl}
\text{Maximize} & 10x_3 & + & 15x_4 & & \\
\text{subject to} & 5x_3 & + & 3x_4 & \leq & 60 \\
& 5x_3 & + & 6x_4 & \leq & 75 \\
& x_3 & , & x_4 & \geq & 0
\end{array}
$$




Introduce transfer price p for raw material:

$$\max(10-4p)x_3+(15-4p)x_4$$

# The Master Model

Beginning with a transfer price *p* of 0 results in the following master model:

$$\max 250\lambda_{A3} + 187.5\lambda_{B4}$$

$$\begin{aligned} \text{subject to} \qquad 90\lambda_{A3} + 50\lambda_{B4} &\leq 120 \\ \lambda_{A1} + \lambda_{A3} &= 1 \\ \lambda_{B1} + \lambda_{B4} &= 1 \\ \lambda_{A1}, \lambda_{A3}, \lambda_{B1}, \lambda_{B4} &\geq 0 \end{aligned}$$

# Solving the Master Model

Solving the master model results in

$$\lambda_{A1} = {}^2\!/_9 \qquad \lambda_{B1} = 0$$
$$\lambda_{A3} = {}^7\!/_9 \qquad \lambda_{B4} = 1$$
$$p = 2.78$$

The two new objective functions for submodels A and B:

$$\begin{aligned} A: &\quad -1.12x_1 + 3.18x_2 \quad \Rightarrow \quad (0, 12) \\ B: &\quad -1.12x_3 + 3.18x_4 \quad \Rightarrow \quad (0, 12.5) \end{aligned}$$

# Adding a New Column

Submodel A's solution is new. The following column (corresponding to $\lambda_{A4}$) has to be added to the master problem:

$$\begin{pmatrix} 180 \\ 48 \\ 1 \\ 0 \end{pmatrix}$$

# The New Master Model

The transfer price *p=2.78* and the added column result in the following master model:

$$\max 250\lambda_{A3} + 180\lambda_{A4} + 187.5\lambda_{B4}$$

$$\begin{aligned} \text{subject to} \quad 90\lambda_{A3} + 48\lambda_{A4} + 50\lambda_{B4} &\leq 120 \\ \lambda_{A1} + \lambda_{A3} + \lambda_{A4} &= 1 \\ \lambda_{B1} + \lambda_{B4} &= 1 \\ \lambda_{A1}, \lambda_{A3}, \lambda_{A4}, \lambda_{B1}, \lambda_{B4} &\geq 0 \end{aligned}$$

# Solving the Master Model

Solving the master model results in

$$\lambda_{A3} = 0.52 \qquad \lambda_{B1} = 0$$
$$\lambda_{A4} = 0.48 \qquad \lambda_{B4} = 1$$
$$p = 1.67$$

The two new objective functions for submodels A and B:

$$A: \quad 3.32x_1 + 8.32x_2 \quad \Rightarrow \quad (17.5, 5)$$
$$B: \quad 3.32x_3 + 8.32x_4 \quad \Rightarrow \quad (0, 12.5)$$

# Optimal Solution I

- Both solutions are already considered. Hence, the optimal solution can now be determined

- With $\lambda_{A3}$=*0.52,* $\lambda_{A4}$=*0.48* and $\lambda_{B4}$=*1*, the solution becomes:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0.52 \begin{pmatrix} 17.5 \\ 5 \end{pmatrix} + 0.48 \begin{pmatrix} 0 \\ 12 \end{pmatrix} = \begin{pmatrix} 9.17 \\ 8.33 \end{pmatrix}$$

$$\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = 1 \begin{pmatrix} 0 \\ 12.5 \end{pmatrix} = \begin{pmatrix} 0 \\ 12.5 \end{pmatrix}$$

# Optimal Solution II

- Value of the objective function is 404.15

- Profit of facility A: 99.75
- Profit of facility B: 104.00
- Profit of raw material division: 200.40

# Limitations of Dantzig-Wolfe

- Planning is supposed to be decentralized, but decisions have to be imposed by the central management

- If there is more than one common input, all profit will be assigned to the scarce resource

- Transfer prices for non-scarce resources will be zero

- Can the transfer price be called fair, if it leaves one division without profit?

# Life, the Universe and Everything I

The optimal transfer price is given by opportunity costs, i. e. the value forgone by not using the transferred product in its next best alternative use!

Why is this transfer price not implemented then?

# Life, the Universe and Everything II

- With competitive external markets, the market price represents the opportunity costs
- Using scarce resources, shadow prices become proper opportunity costs
- Operating below capacity causes variable costs to be opportunity costs
- What if some of these factors occur together?
- What about fair distribution of profit and incentives?

# Conclusions and Limitations I

- Traditional approaches can determine an optimal transfer price
- Implementation of the optimal transfer price is difficult
- Divisions cannot operate completely autonomous. Central management must have the opportunity to enforce the transfer price
- The goal of corporate optimality is dominant

# Conclusions and Limitations II

- All models assume goal consensus

- The price mechanism is considered to be a proper tool to regulate economic activities

- Questions regarding the “fairness” of transfer prices are not answered using existing models

- Traditional models don’t consider uncertainty