Network Modeling
2^{nd} lecture in Equilibrium Modeling block

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Outline: three lectures

- Lecture 1 Equilibrium modeling
  - Introduction, motivation and preliminaries
  - Multi-variable optimization, Lagrangian multipliers and Karush Kuhn Tucker conditions
  - Single-agent and Cournot equilibrium problems
- Lecture 2 Network modeling - TODAY
  - Transportation problems
  - Assignment problems
- Lecture 3 Markets with transport networks
  - Multi-agent equilibrium problems
  - Equilibrium problems with embedded transport networks
  - Spatial and temporal aspects (network, investment)

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Equilibrium modeling in network economics - previous lecture

- Impact market power on equilibrium price and quantity?
- Finding equilibria analytically
  - Perfect competition, monopoly, Cournot oligopoly
- Equilibrium problems: KKT & MC-conditions
  - System of variable-equation pairs
  - Implicitly via Lagrange Multiplier Method
  - Write as minimization
  - Reorder restrictions and assign duals
  - KKT for each variable;
    - duals get '-' stationarity conditions
    - Include the restrictions
- GAMS Implementations

$$0 \le q \perp \frac{\partial z}{\partial q} \ge 0$$

$$q > 0 \Rightarrow \frac{\partial z}{\partial q} = 0$$

$$\frac{\partial z}{\partial q} > 0 \Rightarrow q = 0$$

$$0 \le \lambda \perp CAP - q \ge 0$$

$$\lambda > 0 \Rightarrow q = CAP$$

$$q < CAP \Rightarrow \lambda = 0$$

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Today

- NETWORKS - continuous
  - Assignment*
  - Transportation
  - Transshipment
  - min cost flow
- Assignment - integer
  - Basic Assignment
  - Facility location
  - Coverage-Location
- Focus on problem formulation and implementation, not on solution algorithms
- Illustrative problems and current research
- Implementations in GAMS and XPRESS

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What is a network

- Networks: Nodes connected by arcs
- Roads, Energy grids, Data, Contacts, …
- Flows via arcs, from supply nodes to demand nodes, possibly via transshipment nodes
- Arcs can be directed, or bi-directional
- Demand requirements, costs and losses connected to flows, capacity restrictions to flow and production.
- In a multi-period setting there may be storage, capacity investment, uncertainty, etc.
- Structure of linear network problems allows solving by highly efficient network simplex methods

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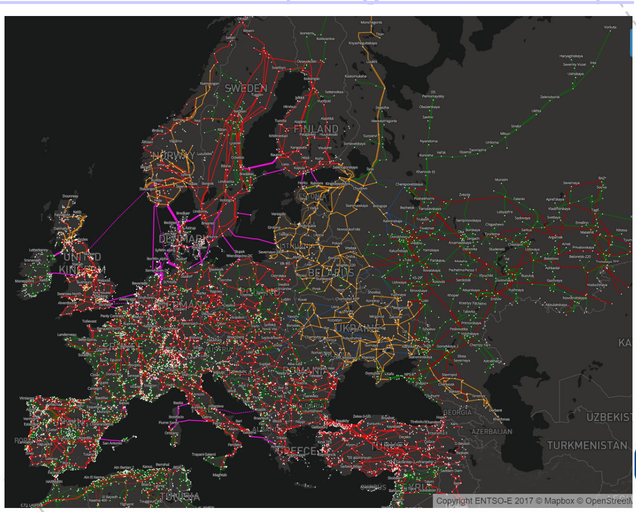
European Electric Power Network

www.entsoe.eu/map/Pages/default.aspx



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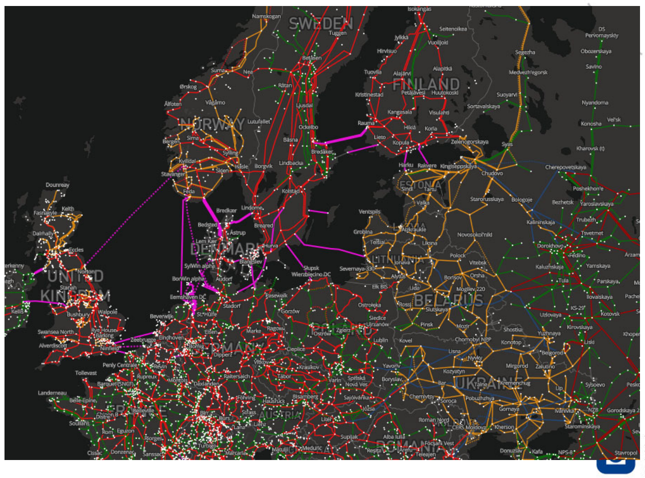

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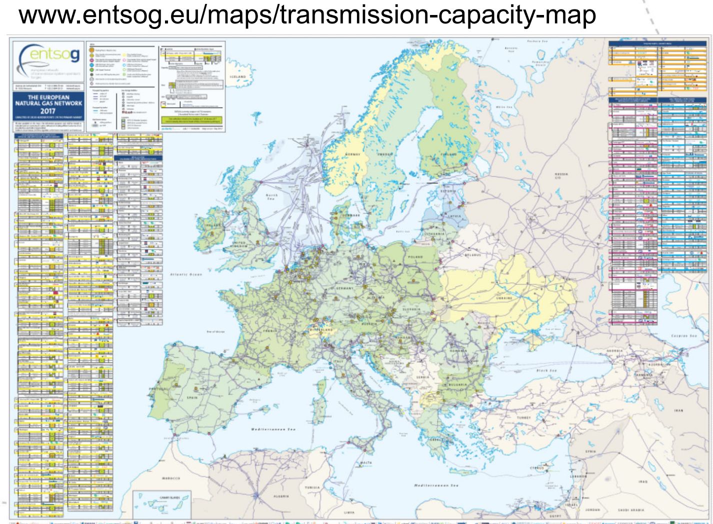
European Natural Gas NETWORK

www.entsog.eu/maps/transmission-capacity-map



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European Natural Gas Market

www.entsog.eu/maps/transmission-capacity-map

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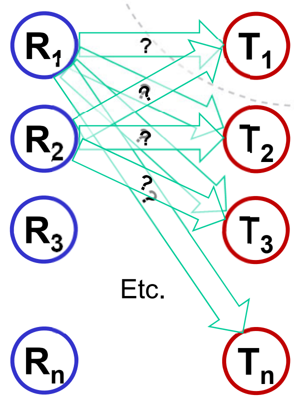
ASSIGNMENT PROBLEM (1)

- Assignment of resources to tasks. People to tasks, machines to jobs, …
- In fact binary – but ignore integer constraints initially
- Single period problem
- *bipartite graph: two sets of nodes that each have no internal connections*



Hillier & Lieberman 2015 Ed 10. Ch 9 P. 348

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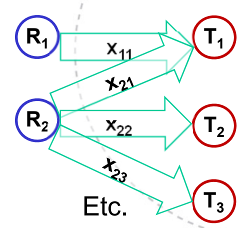
Assignment problem



$$x_{ij} = \begin{cases} 1 & resource\ i\ assigned\ to\ task\ j \\ 0 & otherwise \end{cases}$$

c_{ij}: *cost if resource i performs task j*

- Minimize costs to get all the tasks done

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

s.t.

- Subject to
- All tasks must be done:

$$\sum_{i=1}^{n} x_{ij} = 1$$

- All resources must be used:

$$\sum_{j=1}^{n} x_{ij} = 1$$

- Non-negativity, (Binary decisions)

$$x_{ij} \geq 0, (binary)$$

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Unimodular coefficient matrix: continuous solutions guaranteed integer

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Assignment problem in FICO repository

- https://examples.xpress.fico.com/example.pl?id=assignmentgr
- A set of projects is assigned to persons with the objective to maximize the overall satisfaction.
- A preference rating per person and project is given.
- 1_assignment_graph_FICO_Repository.mos
- Talk slowly through the file

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Transportation

- Demand for a good at several locations can be satisfied from a number production locations. Find the cheapest way to do so.
- m supply points/sources, n demand points/destinations/sinks
- Supply constraints: maximum supply from each plant
- Demand constraints: demand must be met
- Shipment costs: unit cost for arc flows
- Standard form: *balanced* transportation problem
  - Total supply = total demand
  - No arc capacities.

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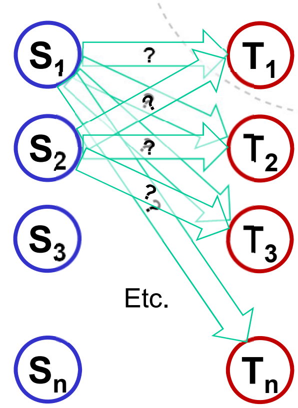
TRANSPORTATION PROBLEM

- Transport goods from supply to demand points
- Single period problem



Hillier & Lieberman 2015 Ed 10. Ch 9 P. 324

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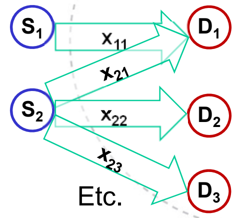
Transportation problem



x_{ij}: *supply from source i to sink j*
c_{ij}: *unit transport cost source i to sink j*
s_i: *supply at source i*
d_j: *demand at sink j*

- Minimize costs to get meet all demand

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

s.t.

- All demand must be met:

$$\sum_{i=1}^{n} x_{ij} = d_j$$

- All supply must be shipped out:

$$\sum_{j=1}^{n} x_{ij} = s_i$$

- Non-negativity

$$x_{ij} \geq 0$$

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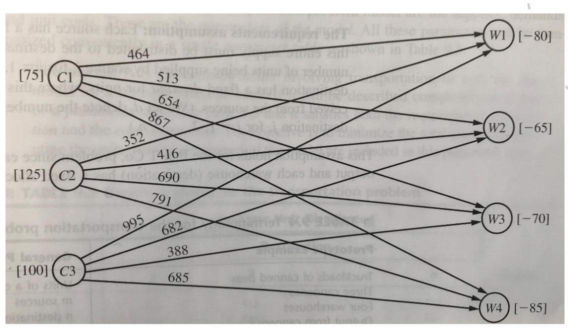
Hillier & Lieberman,p. 321



		Shipping Cost ($) per Truckload				
		Warehouse				
		1	2	3	4	Output
Cannery	1	464	513	654	867	75
	2	352	416	690	791	125
	3	995	682	388	685	100
Allocation		80	65	70	85	

Hillier & Lieberman, p. 321

- 2_transport.mos (also .gms)
- Based on file assignment_graph.mos (c) 2008 FICO
- Talk slowly through the file

Unit Cost		Destination (Warehouse)			
		Sacramento	Salt Lake City	Rapid City	Albuquerque
Source (Cannery)	Bellingham	$464	$513	$654	$867
	Eugene	$352	$416	$690	$791
	Albert Lea	$995	$682	$388	$685

Shipment Quantity (Truckloads)		Destination (Warehouse)						
		Sacramento	Salt Lake City	Rapid City	Albuquerque	Total Shipped		Supply
Source (Cannery)	Bellingham	0	20	0	55	75	=	75
	Eugene	80	45	0	0	125	=	125
	Albert Lea	0	0	70	30	100	=	100
	Total Received	80	65	70	85			
		=	=	=	=			Total Cost
	Demand	80	65	70	85			$ 152,535

Standardizing a transportation problem

- To allow network simplex
- (Linear) production costs: can be added to the flow costs
- Supply surplus: add a dummy demand point with zero shipment costs
- Supply shortage: add a dummy supply point with shipment costs higher than all other.
- Sales prices don’t matter – fixed demand

- Note: can model inventory problems as transportation problems.

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Transshipment

- Network more complex than a bipartite graph
- Transshipment nodes are intermediate nodes that receive goods and send them further
- Represent these with *(nodal)* flow (*or mass*) balance equations:
  - Supply nodes: Production = outflows
  - Transshipment nodes: Inflows = outflows
  - Demand nodes: Inflows = demand
  - General node: Production + inflows = outflows + demand
  - *Flows from Sources = Flows into Sinks*

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Minimal cost (network) flow

- General network problem, generalizes many other problems.
- Nodes can have mixed functions (supply, demand, transshipment)

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Min cost flow - transshipment

N: set of nodes i,j
c_{ij}*: unit cost for flow from node i to node j*
s_i: *production at node i*
d_i: *demand at node i*
x_{ij}: *flow from node i to node j*

- Min total costs

$$\min \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

s.t.

- Production + inflows = demand + outflows

s.t.

$$s_i + \sum_{j \in N} x_{ji} = d_i + \sum_{j \in N} x_{ij}, i \in N$$

- Non-negative flows

$$x_{ij} \geq 0$$

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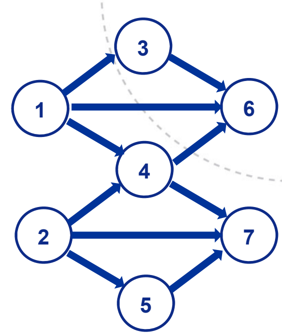
MIN COST FLOW

To / From	Sup	3	4	5	6	7
Dmd					4	6
1	5	2	1		8	
2	5		2	3		4
3					2	
4					3	1
5						2



Flow cost matrix with production & demand
Empty cells: no connection
XPRESS implementation (GAMS too)

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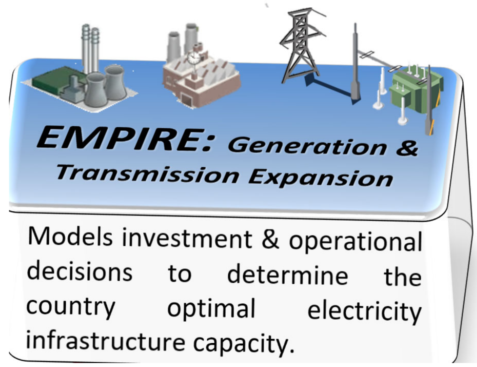
EMPIRE – Stian Backe, Christian Skar

- European power system
  - Short-term uncertainty
  - Long-term investments
  - Short-term operations
- Capacity expansion
  - Transmission
  - Generators
  - Storages
- Stochastic linear program



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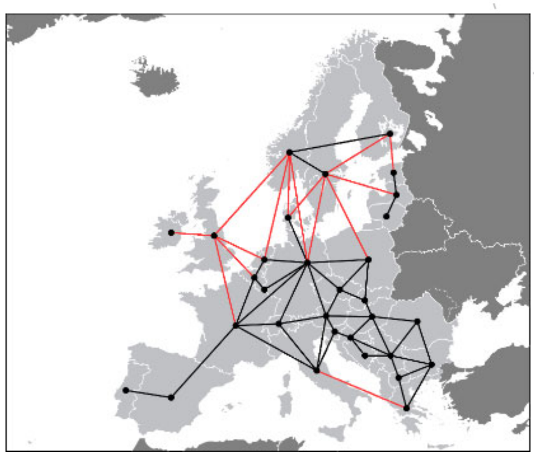

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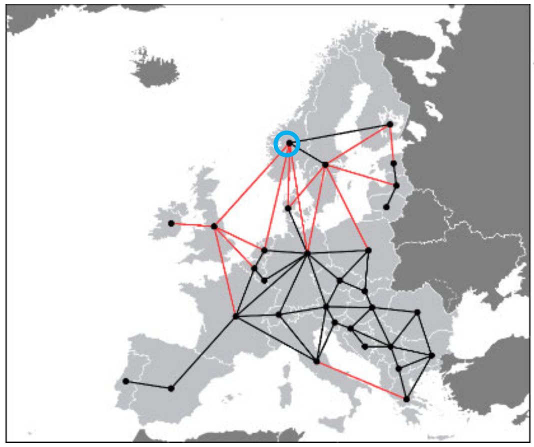
Every node:
- Demand
- Generation
- Storage



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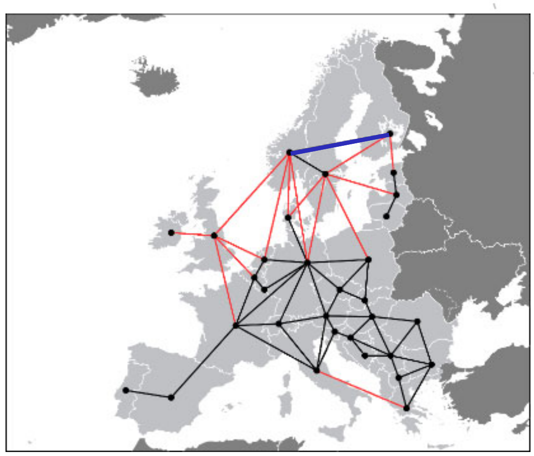
Every arc:
- Transmission



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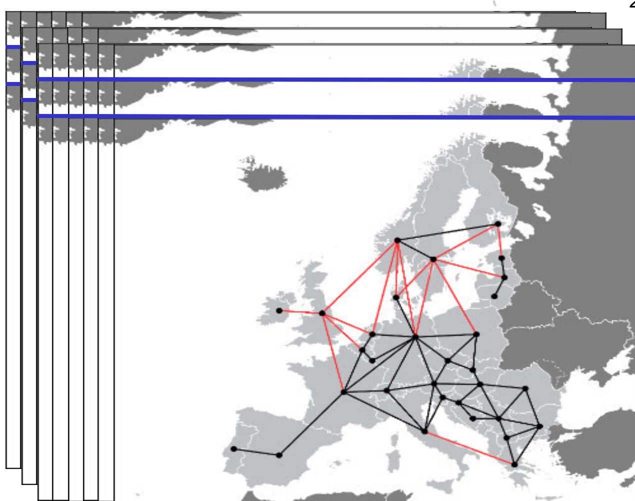
Demand = Supply

Hourly resolution

Different:
- Seasons
- Scenarios

Investment periods



Input data

Long-term data (every investment step)
Fuel prices (for each fuel)
Annual demand for electricity (by country/node)
Technology overnight costs
Thermal generator efficiency (by technology)
Technology operation and maintenance costs
Storage technology capital costs
Storage technology characteristics
Interconnector investment costs
Investment limits (generator technology, intercom.)

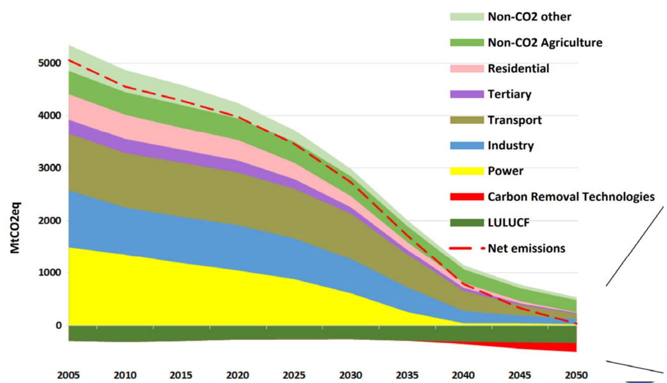
Operational data (hourly resolution)
Load profiles
Normalized wind production profiles
Normalized solar production profiles
Run-of-the-river hydro power production
Regulated hydro power production

Initialization data
Initial generating/interconnector/storage capacity

Policies
Emissions and technology policies

Research questions

How to meet emission reduction targets in the power sector at least cost?



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Results

Long-term data (every investment step)
Investments in transmission
Investments in generation
Investments in storage
Installed capacity of transmission
Installed capacity of generation
Installed capacity of storage

Operational data (hourly resolution)
Hourly generator output
Hourly transmission exchange
Hourly storage operation
Losses during transmission
Losses related to storage (charge/discharge)

Policies
Hourly CO2eq-emissions

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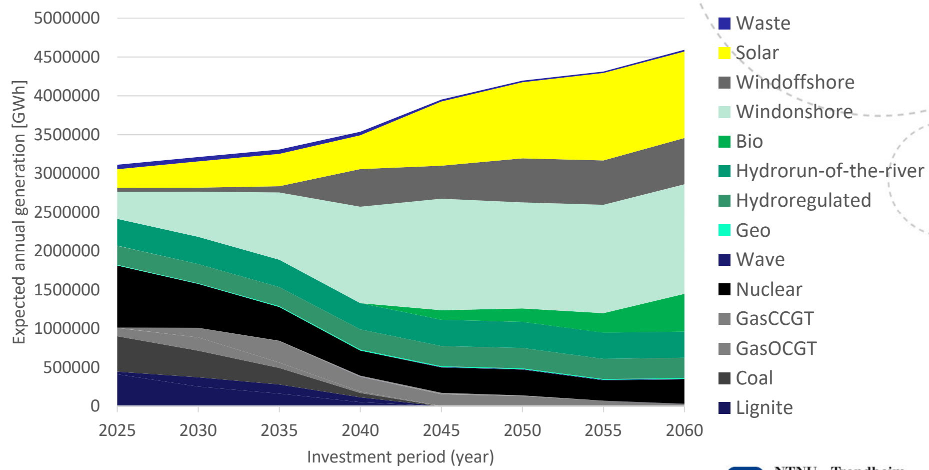
Example results: power generation mix



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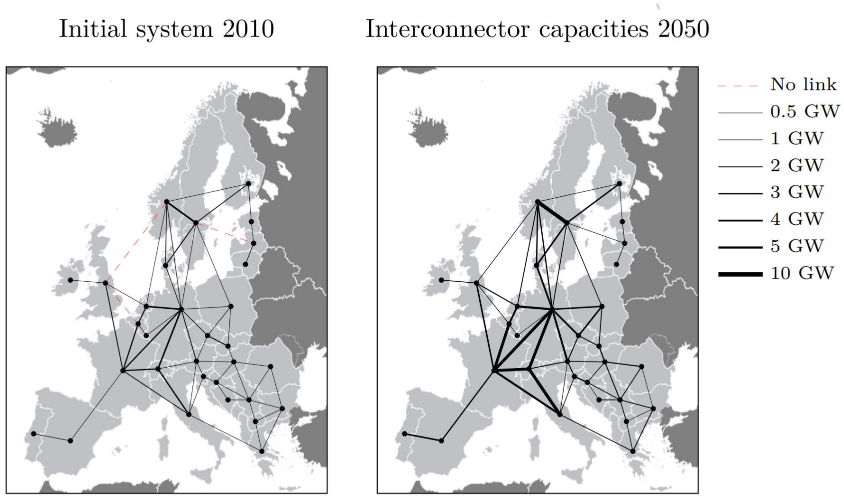
Example results



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Techno-economic renewable energy planning in smart grids - Pedro Crespo del Granado

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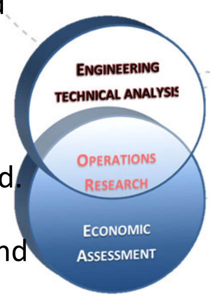
Objectives

- Overcoming the mismatch in time of local solar and wind electricity production and demand.
- Assess the ***value of energy storage technologies*** in the deployment of renewable energy in a smart grid.
- Model energy systems with storage technologies and renewable supply ***from end user perspective*** (e.g., family houses, office buildings, or communities).



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Questions

- What is the value of energy storage for the **end-user** in a smart grid?
  - Independence from the main power grid
  - Possibilities for demand shifting
  - Portfolio effects in systems with more than one household

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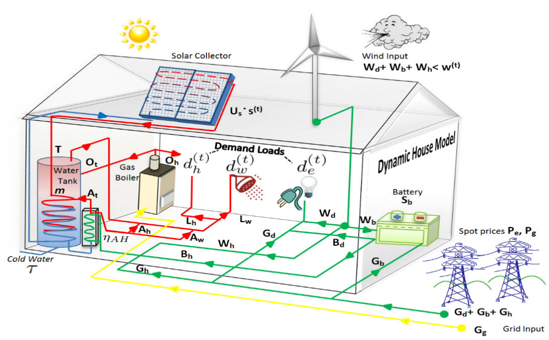
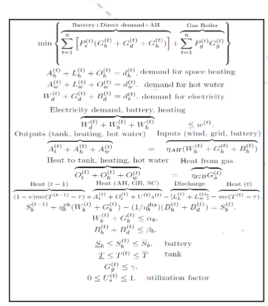
A smart house model and energy storage





Objective:
✓Minimize cost of grid consumption
Constraints:
✓Units' Capacity & Efficiency
✓Demand
✓Supply Availability from Renewables
✓Storage Intertemporality

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Community energy storage

- Point of view: A large end-user (industrial site, office building complex, university campus).
  - security of supply and CO2 emission reduction
- Complex energy mix (hybrid generation system): Local heating requirements are important
  - Specially flexibility in CHP operations and wind surplus
  - Heating and electricity systems interactions
- Analyze interactions of heat and electricity storage in the operation of the energy supply-demand balance.

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Case study - University campus distributed generation

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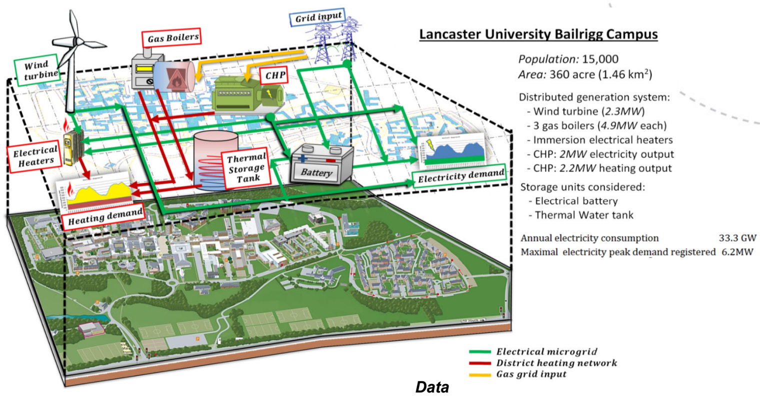
Case study features



Storage considered
- Vanadium redox battery (VRB), Capacity: 2, 4, 8MW, roundtrip efficienfy:0.75

Data
- Used the campus half hourly data electricity consumption in 2010
- National Grid sport market prices in 2010
- Historical wind data 2006-2010

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NATURAL GAS NETWORKS

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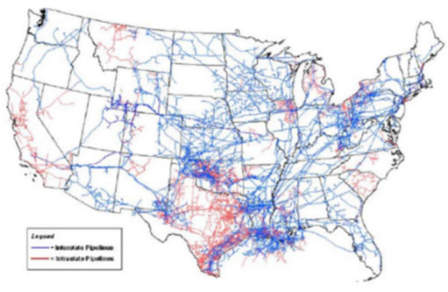

Figure 2. U.S. Natural Gas Pipeline Network

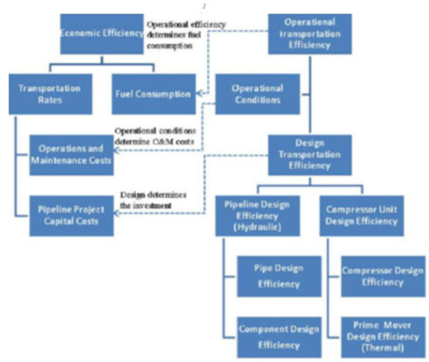

Figure 1. Linkage Between Economic and Transportation Efficiency

- Natural gas is an odorless transparent gas, primarily composed of methane.
- Most economical and efficient way to transport gas is via pipelines.
- Gas compressor stations keep the gas flowing by boosting the pressure of the gas to compensate for pressure losses along the pipeline. Friction causes pressure to drop as the compressed gas moves through the pipeline. The pressure losses and corresponding decrease in transportation efficiency are related to many factors such as pipeline diameter, operating pressure, throughput, and internal roughness of the pipeline. Other transportation efficiency losses occur in the compression process.

Interstate Natural Gas Association of America, *Interstate Natural Gas Pipeline Efficiency*, White Paper,, Washington, D.C. Oct 2010

Operational gas flow planning: NETWORK

- Ruhrgas Germany 11000 km of pipes, 26 compressor stations requiring about 32MW each (when used)
- Low demand about half the average, peak day double the average.
- Controlling network load distribution over the next 24 to 48 hours to satisfy demand subject to physical, technical, and contractual constraints as well as target values for gas production, storage, purchase, and sale determined by the mid-term planning.
- minimize variable operating costs - dominated by cost for gas consumption by compressors.
- reliable temperature forecasts (neglect demand uncertainty): deterministic model
- flow in pipes governed by thermodynamic conservation laws. Conservation of mass (continuity equation) and of momentum (pressure loss equation) form a hyperbolic PDE system that is coupled with the equation of state for a real gas: nonlinear, non-convex.
- switching compressors on or off, opening or closing valves: integer decisions

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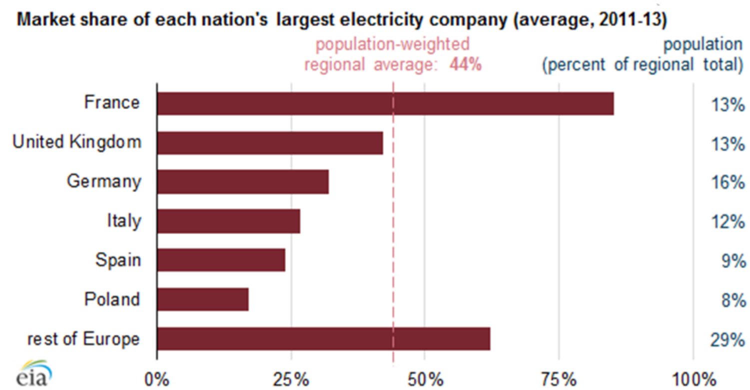
Klaus Ehrhardt and Marc C. Steinbach (2003), *Nonlinear Optimization in Gas Networks. ZIB-Report 03-46 (December 2003)*

Market shares largest power companies by country: MARKET POWER



Source: https://www.eia.gov/todayinenergy/detail.php?id=21732

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Sector coupling

- Two separate problems: each min cost over a feasible region
- Connection of problems: larger feasible region → lower min cost possible
- Intermittency and uncertainty in renewables generation reduces feasibility of power system problems.
- Coupling with other sectors (e.g., transport) & energy carriers (e.g., natural gas, hydrogen)
- Connecting networks into larger ones

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Assignment 2

Basic Assignment

- In fact: an integer problem
- Unimodular coefficient matrix and integer supply and demand guarantees integer solutions

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

s.t.

$$\sum_{i=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{n} x_{ij} = 1$$

$$x_{ij} \geq 0, (binary)$$

Facility Location

- A chain of stores needs to plan warehouses. There is a tradeoff between number of warehouses and total transport times.
- There is a network with two types of nodes: demand nodes and potential production nodes
- Establish one or more facilities so that demand can be satisfied
- Fixed cost connected to establishing facilities
- Operational costs connected to flows
- Complication: supply at potential production nodes only available if a facility is established

Lundgren 3.3 p61 (but story starts at p55)

Facility location

SETS

N : *nodes i,j*

PARAMETERS

c_{ij} : *unit transport cost node i to node j*

f_i : *costs of establishing a facility at node i*

s_i : *supply capacity at node i*

d_j : *demand at node j*

VARIABLES

y_i : *establish facility at node i*

x_{ij} : *supply from source i to sink j*

- Min total costs

s.t

- Supply is only used for established facilities; and supply limits respected
- Demand is satisfied
- Binary facilities
- Non-negative flows

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Facility location – Kahoot https://kahoot.it/ or App pin 9102017

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Facility location – https://kahoot.it/ or App – pin 9102017

	A	B	C
1 Costs for establishing facilities			
2 Costs for transporting commodities			
3 Supply only used for established facilities; supply limits respected			
4 Demand is satisfied			
5 Binary facilities			
6 Non-negative flows			

When done – unhide next slide

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Set Coverage

- *Ambulances / fire engines should be able to reach any house in the region within a maximum number of minutes. A smart choice of emergency service locations means that fewer will be needed*
- There is a network with two types of nodes: demand nodes and potential service / production nodes
- *A production node can only satisfy a demand node, if the distance is below a certain threshold*
- Establish one or more facilities so that demand can be satisfied
- Fixed cost connected to establishing facilities
- Complication: supply at potential production nodes only available if a facility is established *and if the distance is small enough*
- *We only consider coverage, not different demand levels*
- *(the blue lines differ from facility location)*

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Set coverage

SETS

N : nodes i,j

PARAMETERS

f_i: costs of establishing a facility at node i

$$c_{ij} = \begin{cases} 1 & \text{a facility at node i covers node j} \\ 0 & \text{otherwise} \end{cases}$$

VARIABLES

$$y_i = \begin{cases} 1 & \text{location i is selected} \\ 0 & \text{otherwise} \end{cases}$$

Min total costs

s.t

All locations must be covered

Binary facilities

$$\min \sum_{i \in N} f_i y_i$$

s.t.

$$\sum_{j=1}^{n} c_{ij} y_j \geq 1, i \in N$$

$$y_i \text{ binary}$$

Wolsey1998 Integer programming

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Coverage location - *emergency service station location problem*

- Covering all demand locations?
- Covering all demand locations just once?
- Covering all demand?
- Resources only used once?
  - During response not available for a new emergency
  - Free up after some amount of time.
- Demand development:
  - Longer response times may imply: more ill people, a larger fire, deterioration of patient condition,…
  - Response personnel may end up getting infected
- Uncertainty:
  - Travel times
  - Response durations
  - Actual status (vs reported status)

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Northern Sea Route: longer ice-free periods

Russia to open two additional emergency rescue centers along Northern Sea Route

currently four centers in the Arctic, which opened between 2013-2016.

New centers in the Far East will provide additional search and rescue capabilities along the Arctic shipping route.

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https://www.highnorthnews.com/en/two-new-arctic-emergency-centers-open-along-northern-sea-route

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Fire management

Bush Heritage Australia

- great diversity in fire regimes (frequency and severity of fires) across Australia.
- Fire Management Program incl. site-specific fire management and response plans
- Fire-preparedness has greatly enhanced ability to respond to and control the fires on our reserves this summer, including helping to reduce the intensity of fires and safeguarding ecological, cultural and structural assets.

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Ambulance response times

July 2017, NHS England *new performance targets for the ambulance service*.

- Category 1 - life-threatening
  - Around 9% of incidents fall under Category 1
  - Response in 7 minutes on average, and to 90% in 15.
- Category 2 - potentially serious condition
  - Response in 18 minutes on average, and to 90% in 40.
- Category 3 – urgent
  - respond to 90% in 120 minutes (no average target.
- Category 4 - not urgent but need assessment
  - Respond to 90% within 180 minutes.
- How to take all this into account when setting up centers and resources

https://www.nuffieldtrust.org.uk/resource/ambulance-response-times

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maximal coverage location models

- *Location of emergency service facilities*. Toregas et al. (1971)
- *Maximal covering location problem* - Church & ReVelle (1974) .
- *Modified maximal covering location model maximizes the covered population and includes a second objective which maximizes the demand points covered multiple times. Daskin & Stern (1981) "A hierarchical objective set covering model for emergency medical service vehicle deployment."*
- *Maximal expected coverage relocation problem* for emergency vehicles Gendreau et al. 2006

- Need estimates (incl. uncertainty margins) for different types of demand and task durations (transport, response,…)
- Coverage location models aim at providing multiple coverages of demand locations maximizing some coverage score.
- Tactical-Strategical aspects: where to put the centers?
- Operational-Tactical aspects: prioritization of responses; send response team from a further away center to improve coverage during the response;…

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Daskin & Stern (1981) A hierarchical objective set covering model for emergency medical service vehicle deployment.

$$\text{minimize} \quad Z_1 = \sum_j X_j$$

subject to

$$\sum_j a_{ij} X_j \geq 1 \quad \text{for all } i$$
$$X_j = 0, 1 \quad \text{for all } j$$

where

$$X_j = \begin{cases} 0 \text{ if an ambulance is not located in zone } j \\ 1 \text{ if an ambulance is located in zone } j \end{cases}$$
$$a_{ij} = \begin{cases} 0 \text{ if } d_{ij} > T \\ 1 \text{ if } d_{ij} \leq T \end{cases}$$

and

Z_1 = the number of ambulances required.

$$\text{minimize} \quad Z_2 = W \sum_j X_j - \sum_i S_i$$

subject to

$$\sum_j a_{ij} X_j - S_i \geq 1 \quad \text{for all } i$$
$$X_j = 0, 1 \quad \text{for all } j$$
$$S_i \geq 0 \quad \text{for all } i$$

where

S_i = number of additional EMS units capable of responding zone i in a time less than or equal to T
W = some positive weight

Emergency response for infectious disease outbreaks

- developing countries lack means to facilitate population with basic sanitation and health care; prevention and treatment not readily available.
- Past research has focused on epidemiological modelling of disease spreading
- Few models optimize response planning.
- Tactical- strategic: optimal allocation of personnel, transport means, vaccine stocks etc., given resources, population densities, seasonal patterns, …
- Operational: optimal response plans for given outbreak situations
- based on real data from World Health Organisation and Norwegian Institute of Public Health.

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Networks in complementarity problems 1

Min cost network flow problem – c.f. slide 19:

N: set of nodes i,j

c_{ij}: *unit cost for flow from node i to node j*

s_i: *production at node i*

d_i: *demand at node i*

x_{ij}: *flow from node i to node j*

Add capacity restrictions:

cap_{ij} : *capacity of arc* (i,j)

- Min total costs

  s.t.
- Production + inflows = demand + outflows
- Capacity constraints are respected
- Non-negative flows

- Define this as complementarity problem (do this on the board along the slides)

$$\min Z = \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

s.t.

$$s_i + \sum_{j \in N} x_{ji} = d_i + \sum_{j \in N} x_{ij}, i \epsilon N$$

$$x_{ij} \leq cap_{ij}, i \epsilon N, j \epsilon N$$

$$x_{ij} \geq 0$$

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Networks in complementarity problems

1. Minimization objective
2. Restrictions "≥0" and Sources-Sinks=0
3. F.O.C. wherein duals of restrictions get a '-'
4. Include the restrictions

$$\min Z = \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

s.t.

$$s_i + \sum_{j \in N} x_{ji} = d_i + \sum_{j \in N} x_{ij}$$

$$x_{ij} \leq cap_{ij}$$

$$x_{ij} \geq 0$$

$$\min Z = \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

s.t.

$$s_i + \sum_{j \in N} x_{ji} - d_i - \sum_{j \in N} x_{ij} = 0$$

$$cap_{ij} - x_{ij} \geq 0$$

$$x_{ij} \geq 0$$

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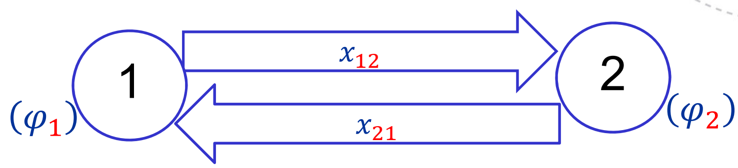
Dual variables on mass balances: tricky!

$$s_i + \sum_{j \in N} x_{ji} - d_i - \sum_{j \in N} x_{ij} = 0 \; (\varphi_i \; f.i.s.)$$



$$\begin{matrix} \varphi_1 \\ \varphi_2 \end{matrix} : \begin{cases} s_1 + x_{21} - d_1 - x_{12} = 0 \\ s_2 + x_{12} - d_2 - x_{21} = 0 \end{cases} \Rightarrow \begin{cases} 0 \le x_{12} \perp \cdots - (-1)\varphi_1 - \varphi_2 + \cdots \\ 0 \le x_{21} \perp \cdots - (-1)\varphi_2 - \varphi_1 + \cdots \end{cases}$$

More general: $s_1 + x_{21} + x_{31} + x_{41} + \cdots - d_1 - x_{12} - x_{13} - \cdots$

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Networks in complementarity problems

1. Minimization objective
2. Restrictions "≥0" and Sources-Sinks=0
3. F.O.C. wherein duals of restrictions get a '-'
4. Include the restrictions

$$\min Z = \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

s.t.

$$s_i + \sum_{j \in N} x_{ji} - d_i - \sum_{j \in N} x_{ij} = 0 \; (\varphi_i \; f.i.s.)$$

$$cap_{ij} - x_{ij} \ge 0 \; (\lambda_{ij} \ge 0)$$

$$x_{ij} \ge 0$$

$$0 \le x_{ij} \perp c_{ij} - (-1)\varphi_i - \varphi_j - (-1)\lambda_{ij} \ge 0$$

$$\varphi_{ij} \; f.i.s., s_i + \sum_{j \in N} x_{ji} - d_i - \sum_{j \in N} x_{ij} = 0$$

$$0 \le \lambda_{ij} \perp cap_{ij} - x_{ij} \ge 0$$

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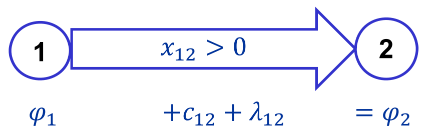
Intuition φ_i: *value at node i*

$0 \le x_{ij} \perp c_{ij} - (-1)\varphi_i - \varphi_j - (-1)\lambda_{ij} \ge 0$
$0 \le x_{ij} \perp c_{ij} + \varphi_j - \varphi_j + \lambda_{ij} \ge 0$
φ_i: *value of a unit of commodity at node i*

$x_{ij} > 0 \Rightarrow c_{ij} + \varphi_i + \lambda_{ij} = \varphi_j$



If flow from 1→2 the value at node 2 must compensate the value at node 1 + the flow costs + dual price on capacity.

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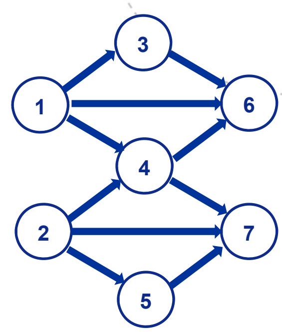
MIN COST FLOW – slide 20

From \ To	Sup	3	4	5	6	7
Dmd					*4*	*6*
1	*5*	*2*	*1*		*8*	
2	*5*		*2*	*3*		*4*
3					*2*	
4					*3*	*1*
5						*2*



Flow cost matrix with production & demand
Empty cells: no connection
GAMS LCP implementation

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Network investment

- Capacitated network
- Multi-period problem
- Investment possible to expand capacities
- Assume continues capacities
- Minimize sum of investment costs, production costs and transport costs
- s.t., in all time periods, capacity constraints are respected and mass balances hold

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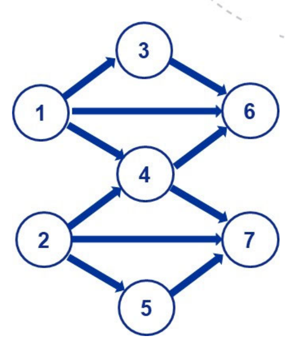
Network investment

- Consider the same network as before
- Impose a two period structure:
  - Period 1, investment
  - Period 2, operation (transports)
- Assume initial capacities zero & investment costs 1 per unit of capacity
- Formulate the cost minimization problem
- Discount rate?



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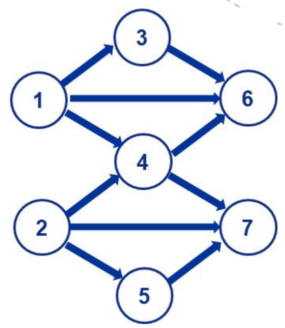
Network investment – multi-period

- Consider the same network as before
- Impose a multi period structure:
  - Period 1, investment
  - Later periods, transports
- Initial capacities zero
- Formulate the cost minimization problem
- Discount rate?
- Complementarity formulation?



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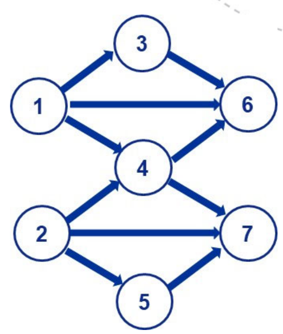
Network investment under uncertainty

- Consider the same network as before
- Impose a two stage structure:
  - Stage 1, investment
  - Stage 2, operation (transports)
- Uncertainty: transit at node 4 blocked with some probability
- initial capacities zero & investment costs 1 per unit
- Formulate the expected cost minimization problem
- Discount rate?



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Complementarity problems

- Signs of mass balance duals are tricky.
- Work consistently through derivation steps, and check logic:

$$x_{ij} > 0 \Rightarrow \varphi_i + c_{ij} + \lambda_{ij} = \varphi_j$$

- Complementarity problem is meaningless if there are integer decisions
  - (But can *implement* them using binary variables)

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Today

- Networks are a part of very many real-world problems and subject of research in many fields (not just operations research and economics)
- Modeling of Transportation in Networks
- Minimum cost flow
- Assignment, Facility location, set coverage
- Optimization and complementarity formulations and implementations

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Outline: three lectures

- Lecture 1 Equilibrium modeling
  - Introduction, motivation and preliminaries
  - Multi-variable optimization, Lagrangian multipliers and Karush Kuhn Tucker conditions
  - Single-agent and Cournot equilibrium problems
- Lecture 2 Network modeling
  - Transportation problems
  - Assignment problems
- Lecture 3 Markets with transport networks: next week
  - Combining lectures 1 & 2
  - Multi-agent equilibrium problems with embedded transport networks
  - Spatial and temporal aspects (network, investment)
  - Time permitting: uncertainty, storage

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Sources

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