NTNU
Department of Industrial Economics and Technology Management
Spring 2020

TIØ4285 Production and Network Economics
Assignment 6

Out: Thursday 20 February
In: Thursday 27 February 6pm
Supervision: Monday 24 February 4:15pm A31

Note that late exercises will not be approved.

Exercise 1. Hybrid market

Company D enjoys much lower production cost than its competitors in the market for commodity X. Effectively, company D is able to exert market power in the market for X. The other companies in the market act as a single competitive, price-taking agent, F. Company D has to decide how much to produce and sell to maximize its profit.

Supply cost for firm D and the others F have the same functional form, but with different values for the parameters c_i and d_i: $c_i^P(q_i) = c_i q_i + d_i q_i^2,\ i \in \{D, F\}\ with\ c_i, d_i \geq 0$. Market demand is given by $p(Q) = a - bQ,\ with\ a, b > 0$ and $Q = q_F + q_D$.

a. Set up the profit maximization problems for company D and for fringe F.
b. Derive the mixed complementarity problem for market X.

Use parameter values: $a = 100, b = 2, c_D = 10, d_D = \frac{1}{2}, c_F = 20, d_F = 1$.

c. Implement the model in GAMS.
d. What are the equilibrium price and quantities supplied by D and F?

Exercise 2. Dominant firm with a competitive fringe

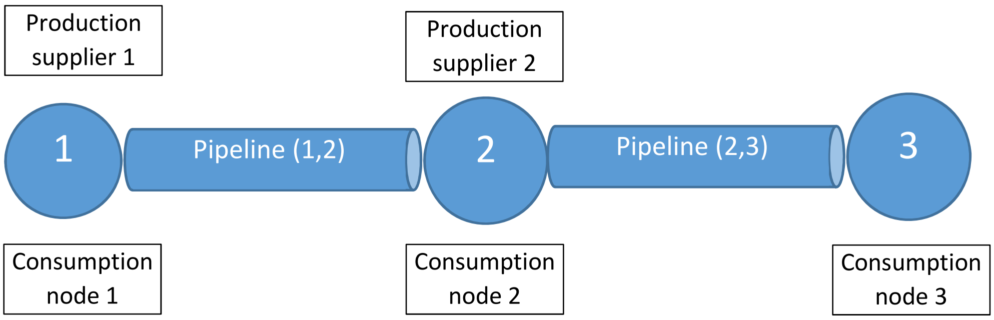
(Ref. Pindyck and Rubinfeld 2013 "Microeconomics" Internat Ed. 8, Ch 12 p. 476-477)

See previous assignment, but now company D decides on its own supply while correctly anticipates the supply by the others agents.

a. Set up the profit maximization problem for company D
b. Implement the model in appropriate software of your choice.
c. What are the equilibrium price and quantities supplied by D and F?
d. Discuss the differences in supply quantities between exercise 1 and 2.

Exercise 3. Duopoly on a small network with transport losses

Two oligopolistic suppliers. Both produce at one node (producer 1 at node 1, and producer 2 at node 2). There is demand at all three nodes. Network arcs (pipelines) are directional. Supplier 1 has access to all pipelines and nodes, supplier 2 does have access to pipeline (2,3) and nodes 2 and 3, but not to pipeline (1,2) or node 1. The pipeline network is managed and operated by a transmission system operator (TSO), who maximizes congestion revenues (as discussed during the lecture).



Supply cost for firm *i* are $c_{in}^{P}\left(q_{in}^{P}\right) = c_{in}^{P},\ i \in \{1,2\}, with\ c_i > 0$. Ignore production capacity restrictions.

Market demand in node n is given by $p_n(Q_n) = a_n - bQ_n,\ with\ a_n, b_n > 0$ and $Q_n = \sum_i q_{in}$.

Pipelines have capacity $CAP_{nm}^{A} > 0$ The base cost for pipelines is $c_{nm}^{A} \geq 0$ per unit. The loss rate for flows in pipelines is $l_{nm}^{A}: 1 > l_{nm}^{A} \geq 0$.

Use f_{imn}^{P} for supplier's arc flows and f_{nm}^{A} for the flow capacity offered by TSO.

a. Set up the optimization problems for the supply firms. Use a general formulation for the objective functions and mass balances that do not depend on the specific structure and characteristics of the network. Think about the pipeline congestion rents in the objective, and the losses in the mass balance. You can choose to write (immediately) in *standard form*: minimization objective, and the needed order for restrictions to facilitate KKT derivation.
b. Set up the optimization problem for the TSO. You can choose to write in *standard form*.
c. Give the market clearing condition for transportation services.
d. Explain why supplier 1 is a monopolist on node 1.
e. Derive the equilibrium problem.

Use parameter values $a_n = 10, b = 1, l_{nm}^{A} = 0.1, c_{in}^{P} = 2, c_{nm}^{A} = 1, CAP_{12}^{A} = 100, CAP_{23}^{A} = 4$ for the following questions.

f. Implement and solve the model in GAMS. What are the produced and supplied quantities at each node?