NTNU
Department of Industrial Economics and Technology Management
Spring 2020

TIØ4285 Production and Network Economics
Assignment 2 – Proposed Solution

Out: Thursday 23 January
In: Thursday 30 January 6pm
Supervision: Monday 27 January, 4:15pm A31

Note that late answers will not be approved.

Exercise 1

The manager of the local football club is putting together the budget for next season. Attendance accounts for the largest portion of the revenues and the manager believes that attendance is directly related to the number of the team's wins. For the past 8 seasons, the following attendance figures are given:

Wins	Attendance
14	3,630
16	4,010
16	4,120
18	5,300
16	4,400
17	4,560
15	3,900
17	4,750

a) Given the players on the team, the manager strongly believes that the team will win at least 17 matches next season. Use a linear regression model to forecast next season's level of attendance.

With a linear regression, we describe a linear relationship, $y = a + b \cdot x$, between the number of wins as explanatory variable, x and attendance as dependent variable, y. Using the equations from Hillier & Lieberman (2015) for the method of least squares, we can estimate the parameters a and b:

$$b = \frac{\sum_{t=1}^{8}(x_t - \bar{x})(y_t - \bar{y})}{\sum_{t=1}^{8}(x_t - \bar{x})^2} = 406.09,$$
$$a = \bar{y} - b \cdot \bar{x} = -2214.5,$$

with $\bar{x} = \sum_{t=1}^{8} \frac{x_t}{8}$ and $\bar{y} = \sum_{t=1}^{8} \frac{y_t}{8}$.

The resulting least-squares estimate of attendance is thus $y = 406.09 \cdot x - 2214.5$. For next season, the manager forecasts an attendance level of 4689.

b) Analyze and discuss the quality of the forecast. Would you trust the forecast? (Do not discuss whether or not the manager is right in expecting at least 17 wins.)

We first plot the data for a visual examination of the data series, see Figure 1.

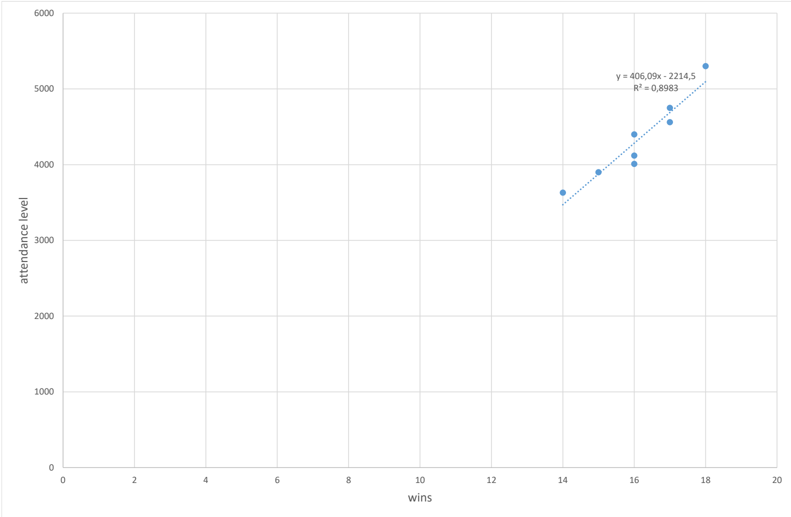

Figure 1: Attendance vs. number of wins, incl. regression line

In this case, the figure doesn't tell much. But keep in mind that the data here is not a time series, these are just realizations of wins and attendance levels. Trends and seasonality will not show up here. Still it might be possible to identify outliers (which in this case we don't have). What we do see is that the data points are somewhat nicely clustered, giving us a good chance that a linear regression produces a decent fit. The $R^2 = 0.8983$ confirms that.

Whether or not an $R^2 = 0.8983$ is good enough to use the regression for forecasting is somewhat relative. It is a good fit to the data, and a forecasted attendance level of 4689 for next season is definitely plausible. However, the linear regression has a negative intercept with the y-axis. In practice, negative attendance is not possible. As a consequence, the linear regression model will perform poorly when forecasting attendance levels in seasons with a very low number of wins. If the local football club never has fewer than, say, 7 or 8 wins, the model might still be (very) good. If low win seasons are likely, the linear regression might not be the best forecasting model for the club any longer.

(Note that checking the quality of a linear regression with mean squared error does not help with respect to finding a better linear regression. Remember that the parameters of the linear regression are usually estimated such that the sum of squared deviations is minimized. Therefore, use R^2 instead.)

Exercise 2

Good Bread AS is a large, successful bakery, selling bread to supermarkets all over the country. The company is organized in an independent sales division, distribution division and baking division. Over the past months, Good Bread AS has found itself more often in a situation with high inventory costs while at the same time being unable to deliver the ordered products in time. The figure below shows the customer demand for bread and the production orders issued by the baking division.

a) Explain to the CEO af Good Bread AS what might be causing the problems regarding inventory costs and service level.

   Good Bread's problems are could caused by the Bullwhip Effect. The variability of end-customer demand is amplified up the supply chain. This effect is influenced by the following factors (each point has to be discussed briefly):

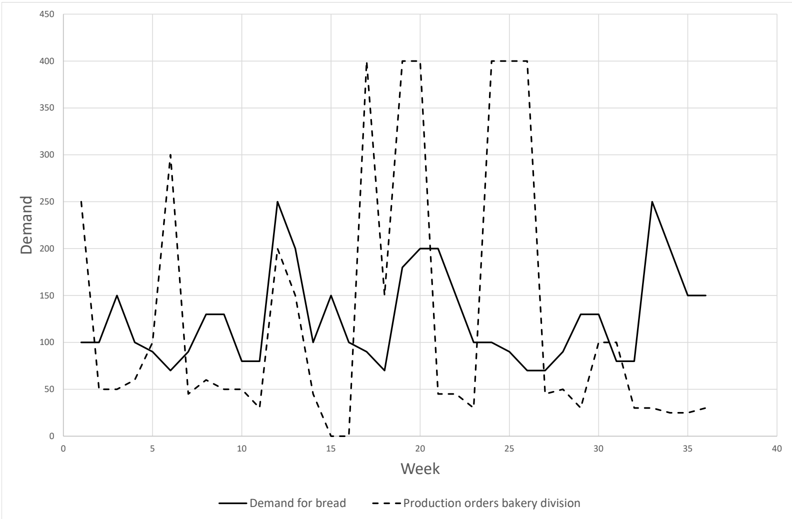

Figure 2: Customer demand and production orders by the baking division

- Demand forecasting
- Lead time
- Batch ordering
- Price fluctuations
- Inflated orders

After explaining the problem to the CEO, she wants to know more about the observed phenomenon and provides you with additional information: The demand for bread is independently and identically distributed in each week. The inventory is controlled by a policy using a safety buffer based on the variance of demand. Orders can be issued in each week.

b) There exist theoretic models to quantify the ratio between the variance of bread demand and the variance of orders issued by the bakery division. Please use one of these models to determine this ratio. Make sure you name all assumptions. Suggest and discuss measures to reduce this ratio.

We make the following assumptions:

- Decentralized demand information
- Moving-average forecasts with p observations
- Order-up-to-point determined according to SKS:

$$L \times AVG + z \times STD \times \sqrt{L}$$

- With
  - $Var\left(Q^k\right)$ – variance of orders placed by kth stage of supply chain
  - $Var(D)$ – variance of customer demand
  - L_i – lead time between stages i and $i+1$

  we get the following relationship (see SKS)

$$\frac{Var\left(Q^k\right)}{Var(D)} \geq \prod_{i=1}^{k}\left[1 + \frac{2L_i}{p} + \frac{2L_i^2}{p^2}\right]$$

The baking division is the third stage in the supply chain (the sales division being the first). The ratio between the variance of orders issued by the baking division and the variance of customer demand is therefore given as

$$\frac{Var\left(Q^3\right)}{Var(D)} \geq \prod_{i=1}^{3}\left[1+\frac{2L_i}{p}+\frac{2L_i^2}{p^2}\right]$$

If we use centralized information instead of decentralized information, we can reduce the increase in the ratio to

$$\frac{Var\left(Q^2\right)}{Var(D)} \geq 1+\frac{2\sum_{i=1}^{3} L_i}{p}+\frac{2\left(\sum_{i=1}^{3} L_i\right)^2}{p^2}$$