TIØ4285 Production and Network Economics
Deliverable 1

Group 97
Adrian Kjærran, Anders Bennæs and Simen Strømstad

February 22, 2021

Exercise 1

a) *Discuss suitable forecasting methods with respect to the time series given in the Excel-file exercise01.xlsx.*

Before we could decide on which forecasting methods are suitable we need to understand the underlying properties of the time series. From previous courses we know that there are three important properties to consider: **stationarity**, **seasonality**, and **autocorrelation**.

i Is the time series **stationary**?

A time series is said to be stationary if it has constant statistical metrics, i.e. a constant variance and mean. From Figure 1 we conclude the variance to be constant, but the moving average indicates that there is a downsloping trend, hence time series is **not stationary**

ii Is the time series **seasonal**?

A time series is said to be seasonal if there is a regular pattern that repeats itself for a given time period. From Figure 1 it seems to be a large variance, but there is **not a significant seasonality**. There are some periods with higher and lower variance, but we assume this is not significant enough to utilize seasonal forecasting methods.

iii Is the time series **autocorrelated**?

A time series is said to be autocorrelated if there is a relation between two a measurement its previous measurements. From the autocorrelation plot in Figure 2 we observe that with some variance that there is a slight positive correlation with a measurement and its precessors. i.e if there has been a trend of negative deviation the last measurements, we could expect the next measurement to have a negative deviation.

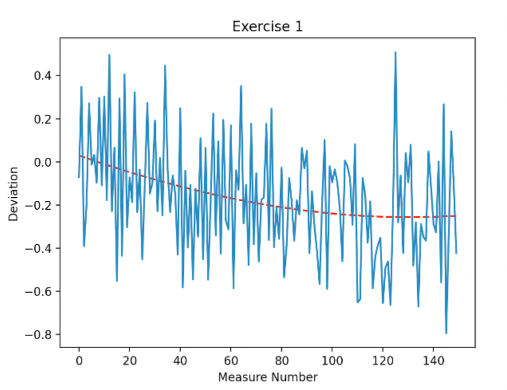

Figure 1: Input deviations with rolling average.

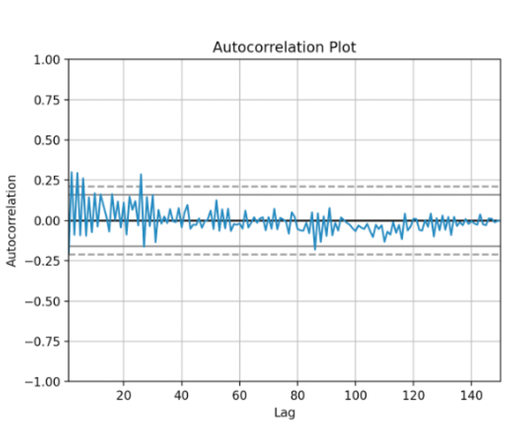

Figure 2: Autocorrelation plot

Based on these properties, which forecasting methods could be suitable?

From Figure 1 we observe that there the variance of the series seems constant but there is a slight negative trend in the deviation and following the time series is not a stationary time series . Some time series method that require a stationary input such as,

Suitable forecasting methods:

Feed Frowar average

b) *Select a forecasting method and create a forecast for the next measurement. Discuss the quality of your forecast.*

...

c) *Could you use your forecasting model to not only predict the next measurement but the next 3 measurements? Explain how you would do that and reflect upon the model's possibilities and limitations.*

...

Exercise 2

a) *What type of planning problem does the distribution planning problem represent? Explain briefly.*

The distribution planning problem represents a transshipment problem, which is an extension of the standard transportation problem since the problem allows for either direct transportation from factories to customers or through distribution centers. Its deterministic structure allows it to be solved in a single stage.

b) *Formulate an optimization problem that finds the cost minimizing distribution pattern for satisying demand. Remember to introduce your notation and explain the model. Find the optimal solution for the demand information on worksheet "Exercise 2b" in Excel-file exercise02.xlsx*

We introduce the following notation:

Sets

$\mathcal{F}$	Set of factories
$\mathcal{D}$	Set of distribution centres
$\mathcal{C}$	Set of customers

Parameters

L_{fd}	Distribution cost from factory f to distribution centre d
M_{dc}	Distribution cost from distribution centre d to customer c
N_{fc}	Distribution cost from factory f to customer c
P_f	Production capacity factory f
T_d	Maximum throughput at distribution centre d
D_c	Demand of customer c

Decision variables

x_{fd}	The amount of product transported from factory f to distribution centre d
y_{dc}	The amount of product transported from distribution centre d to customer c
z_{fc}	The amount of product transported from factory f directly to customer c

Using this notation, the formulation of the problem becomes:

$$\min \quad z = \sum_{f\in\mathcal{F}}\sum_{d\in\mathcal{D}} L_{fd}x_{fd} + \sum_{d\in\mathcal{D}}\sum_{c\in\mathcal{C}} M_{dc}y_{dc} + \sum_{f\in\mathcal{F}}\sum_{c\in\mathcal{C}} N_{fc}z_{fc} \tag{1}$$

subject to:

$$\sum_{d\in\mathcal{D}} x_{fd} + \sum_{c\in\mathcal{C}} z_{fc} \leq P_f \qquad \forall f \in \mathcal{F} \tag{2}$$

$$\sum_{f\in\mathcal{F}} z_{fc} + \sum_{d\in\mathcal{D}} y_{dc} = D_c \qquad \forall c \in \mathcal{C} \tag{3}$$

$$\sum_{f\in\mathcal{F}} x_{fd} - \sum_{c\in\mathcal{C}} y_{dc} = 0 \qquad \forall d \in \mathcal{D} \tag{4}$$

$$\sum_{f\in\mathcal{F}} x_{fd} \leq T_d \qquad \forall d \in \mathcal{D} \tag{5}$$

$$x_{fd} \geq 0, integer \qquad \forall f \in \mathcal{F}, \forall d \in \mathcal{D} \tag{6}$$

$$y_{dc} \geq 0, integer \qquad \forall d \in \mathcal{D}, \forall c \in \mathcal{C} \tag{7}$$

$$z_{fc} \geq 0, integer \qquad \forall f \in \mathcal{F}, \forall c \in \mathcal{C} \tag{8}$$

We assume that everything produced is delivered, and since there is *one* product, we only need decision variables in two dimensions to control the transportation volumes. There are three types of transportation ($f \rightarrow d$, $d \rightarrow c$ and $f \rightarrow c$), each with a distribution cost and corresponding decision variable. To deny the prohibited routes without distribution cost, we set the relevant parameters to an extremely large value ($\sim inf$). This is a computationally effective method to punish invalid routes to the extent that they cannot be selected by the model. We conduct this preprocessing step manually when setting up the data files, but if this was a larger problem with many routes, the mentioned preprocessing could be implemented with a simple script

The objective (1) seeks to minimize total cost, given by the sum of distribution costs multiplied by transportation volumes. Constraint (2) ensures that the distribution from each factory does not exceed the given production capacity. Constraint (3) makes sure that every customer's demand is satisfied. Constraint (4) controls that the total flow in to a distribution centre equals the total flow out of the distribution centre, while constraint (5) ensures that the maximum allowed throughput of each distribution centre is not violated. Constraints (6)-(8) are integer- and non-negativity constraints for each decision variable.

The model is implemented and solved using *Xpress Mosel*. We get an optimal objective value of $z =$ **198 500**. All constraints are satisfied, and none of the prohibited routes are used. The cost minimizing distribution pattern is given in table 1 below. See the Mosel file "model2b.mos" in the appended zipped file for full setup and results.

to	from Liverpool	Brighton	Newcastle	Birmingham	London	Exeter
Newcastle	0	-				
Birmingham	0	50 000				
London	0	55 000				
Exeter	40 000	0				
C1	50 000	0	-	0	-	-
C2	-	-	0	10 000	0	-
C3	0	-	0	0	0	40 000
C4	0	-	0	35 000	-	0
C5	-	-	-	5000	55 000	0
C6	20 000	-	0	-	0	0

Table 1: Value of decision variables in exercises 2b.

c) *How does the new situation change the assessment you made in Exercise 2a? If it changes your assessment, explain why and how. If it does not change your assessment, explain why not.*

The unknown customer demand introduces uncertainty into the model and needs to be handled appropriately in order to plan strategically. The deterministic transshipment planning problem transforms into a two-stage stochastic recourse problem, in which some decisions must be fixed before information about customer demand is available, and certain decisions can be delayed until after the the customer demand is known.

In this particular case, the distribution amounts between factories and distribution centers are fixed decisions that must be determined before demand information is revealed. The decisions for distribution amounts between distribution centers and customers and express delivery between factories and customers can wait until after the customer demand is known, and are therefore "flexible" or "adaptive".

d) *Discuss how you have to change your model from Exercise 2b to provide decisions support for the distribution planners of this production network. Which decisions would be implemented first by them?*

The model from Exercise 2b needs modification as we have to separate between the first- and second stage decision problems. The formulation of the first stage problem remains quite similar to the one from 2b, with two important modifications. Firstly, the decision regarding distribution from distribution centers to customers is moved to the second stage and is dependent on scenario. Secondly, in the two-stage problem, products can be stored in the distribution centers until demand is known, so the flow constraint, (4) will be altered to a sign to ensure that the amount of product distributed from a distribution center is less than or equal to the amount that was transported to the distribution center.

The introduction of a second stage is done by summing up over the relevant decision variables for all scenarios with a probability weight for each scenario. The second stage variables that depend of scenario will be the amounts distributed between distribution centers and customers, and express delivery from factories directly to customers to meet demand. The new decision variable for the express distribution between factories and customers will need to be included in the maximum production constraint, constraint (2), and the the demand satisfaction constraint, constraint (3).

This means that the decisions for the amounts that should be transported between the factories and the distribution centers and the decisions for the amounts that should be transported directly between factories and customers (at normal distribution cost in the first stage) will be implemented first by the distribution planners of this production network.

e) *Suggest and formulate a stochastic programming model, taking into account the changes you have discussed in Exercise 2d.*

We assume that:

- The planning horizon is only one period, so that the only consideration taken regarding excess goods at distribution centers will be a sunk cost without storage cost.
- The first stage problem is solved based on the values provided for the different scenarios, each with an equal probability of 0.2 of occurring.
- The factory cannot produce more than the given capacity in total (both first and second stage).

We need to add a new set $\mathcal{S}$ to represent the multiple scenarios, and a new parameter P_s which is the probability of scenario s. The demand parameter and first stage decision variable y get an additional subscript s to account for the different scenarios. The variables x and z are both first stage decision variables, thus they are not dependent on scenarios. We also add a new variable o_{fcs} which is the amount of product transported from factory f directly to customer c given scenario s using express service at twice the regular cost. The formulation of the problem then becomes:

$$
\begin{aligned}
\min \quad z = & \sum_{f\in\mathcal{F}}\sum_{d\in\mathcal{D}} L_{fd}x_{fd} + \sum_{f\in\mathcal{F}}\sum_{c\in\mathcal{C}} N_{fc}z_{fc} \\
& + \sum_{s\in\mathcal{S}} P_s \left(\sum_{d\in\mathcal{D}}\sum_{c\in\mathcal{C}} M_{dc}y_{dcs} + \sum_{f\in\mathcal{F}}\sum_{c\in\mathcal{C}} 2N_{fc}o_{fcs} \right) \qquad (9)
\end{aligned}
$$

subject to:

$$
\begin{aligned}
\sum_{d\in\mathcal{D}} x_{fd} + \sum_{c\in\mathcal{C}}(z_{fc} + o_{fcs}) \leq P_f & \qquad \forall f \in \mathcal{F}, \forall s \in \mathcal{S} && (10)\\
\sum_{f\in\mathcal{F}}(z_{fc} + o_{fcs}) + \sum_{d\in\mathcal{D}} y_{dcs} = D_{cs} & \qquad \forall c \in \mathcal{C}, \forall s \in \mathcal{S} && (11)\\
\sum_{f\in\mathcal{F}} x_{fd} - \sum_{c\in\mathcal{C}} y_{dcs} \geq 0 & \qquad \forall d \in \mathcal{D}, \forall s \in \mathcal{S} && (12)\\
\sum_{f\in\mathcal{F}} x_{fd} \leq T_d & \qquad \forall d \in \mathcal{D} && (13)\\
x_{fd} \geq 0, integer & \qquad \forall f \in \mathcal{F}, \forall d \in \mathcal{D} && (14)\\
y_{dcs} \geq 0, integer & \qquad \forall d \in \mathcal{D}, \forall c \in \mathcal{C}, \forall s \in \mathcal{S} && (15)\\
z_{fc} \geq 0, integer & \qquad \forall f \in \mathcal{F}, \forall c \in \mathcal{C} && (16)\\
o_{fcs} \geq 0, integer & \qquad \forall f \in \mathcal{F}, \forall c \in \mathcal{C}, \forall s \in \mathcal{S} && (17)
\end{aligned}
$$

The objective (9) seeks to minimize total expected distribution cost. We have separated the first- and second stage objectives, as the second stage ones are .... Constraint (10) has been modified to make sure that the total production of factory f does not exceed the limit in any scenario. Constraint (11) ensures that each customer gets the correct demand. Constraint (12) has been altered, as the flow into a distribution centre can be greater than the flow out. Constraint (13) is the same as previously and makes sure that the maximum throughput at each distribution centre is maintained. Constraints (14)-(17) are integer- and non-negativity constraints for each decision variable. If there were more than two stages in the problem we would have to include nonanticipativity constraints. Since this is not the case, we chose to not let the first stage decision variables depend on s, thus ending up with fewer constraints.

f) *Solve the problem you formulated with the two different scenario set demand data on worksheet "Exercise 2f" in Excel-file exercise02.xlsx. Discuss the results. What is the impact of correlation in demand?*

We implemented the stochastic model in Mosel, see "model2f.mos" in the appended zipped file. It uses one of the demand data sets at a time, which is selected at the start of the model. After solving, we end up with optimal objective values of **xxx** and **yyy** for the uncorrelated- and correlated dataset respectively. See tables table **??** and table **??** for an overview of the values of the decision variables.

The model yielded a lower objective value for the uncorrelated scenarios (202 793) compared to the correlated scenarios (210 987,12). We have identified two factors that we believe influenced this. First of all, the total demand over all scenarios is 13 096 lower for the uncorrelated demand than for the correlated demand, which means that there are fewer units that need to be distributed in the uncorrelated scenarios. This factor is not related to whether the scenarios are correlated or uncorrelated, but rather the totals of the values chosen for the different sets of scenarios.

However, given the equal probability of all scenarios, the variability of demands matter. Specifically, the higher the variability in the demands for different scenarios, the greater the need to UFERDIG production has to

Although the uncorrelated scenarios might have differing demands, the total demand for each scenario is far more likely to be more even than for the correlated scenarios. For the correlated scenarios, although the demand behaviour in each scenario is more predictable and less varying, which scenario that will occur is still not predictable. This means that for the correlated scenarios, there is a greater likelihood of getting either a much higher or much lower total demand than the average total demand over all scenarios, making it harder to produce an appropriate quantity. The variability of the total demands for the two sets of scenarios is illustrated in figure (LEGG INN GRAF). Since the uncorrelated scenarios have more stable total demands, it is easier to accurately predict the quantity that will be suitable in all these scenarios.