NTNU
Department of Industrial Economics and Technology Management

Examination paper for TIØ4285 Production & Network Economics
Solution Proposal

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Some of these exercises may have alternative ways to solve them. Just because they are not mentioned here, does not mean they do not exist. Any comprehensible (and correct) way to solve the exercises is accepted, as long as the questions in the exercises are properly answered. Not using methods specified in the exercises or failing to address all questions will count as an incomplete answer.

Exercise 1 (20%)

Traditionally, Bank Trønder has only given out loans to customers with a low credit risk that have a high probability of paying back the loan. Richard, the bank's new CEO, considers now to also give out loans to customers with high credit risk.

If he only gives out loans to customers with low credit risk, the bank will make a profit of 160 million. In case he also gives out loans to customers with high credit risk, the bank will earn 800 million, as long as the economy is good. If the economy is bad on the other hand, the bank will lose 320 million, as many customers will no longer be able to pay back their loans.

There is a 75% probability that the economy will be bad. Richard's work contract guarantees him a bonus of 1% of the profits as long as these are positive. Both Richard and the bank's owners are risk-neutral.

a) Which decision is Richard going to take? Will he give out loans to customers with high credit risk?

The pay-offs for Richard and the bank's owner s are given in Table 1:

Table 1: Pay-offs for Richard and the bank's owners.

		Economy			
		Good (25%)		Bad (75%)	
		Owners	Richard	Owners	Richard
Decision	low risk	158.4	1.6	158.4	1.6
	high risk	792	8	-320	0

Richard's expected bonus from only giving out loans to low risk customers is $b = 1.6$. The expected bonus once he grants loans to high risk customers is $b = 0.25 \cdot 8 + 0.75 \cdot 0 = 2$. Richard therefore decides to also grant loans to customers with high credit risk.

b) Would Richard have taken a different decision, if he had been risk-averse with the following utility function: $u = \sqrt{b}$, where u is Richard's utility and b is Richard's bonus?

We calculate Richard's expected utility for both decisions:

$$u_l = 0.25 \cdot \sqrt{1.6} + 0.75 \cdot \sqrt{1.6} = 1.26$$
$$u_h = 0.25 \cdot \sqrt{8} + 0.75 \cdot \sqrt{0} = 0.71$$

Risk-averse Richard will only give loans to customers with low credit risk.

c) Which decision would the bank's owners prefer Richard to make?

If Richard decides to only give out loans to low risk customers, the bank will make profits of $0.99 \cdot 160 = 158.4$. If high customers also get loans from Bank Trønder, the bank's expected profits are $0.25 \cdot 792 + 0.75 \cdot (-320) = -42$. The bank's owners therefore prefer a conservative loan policy with loans only granted to low risk customers.

The bank's owners consider giving a new contract to Richard. According to the new contract, Richard will get a fixed salary of 1.15 million plus 0.25% of the bank's profit.

d) Will the new contract cause Richard to take a different decision? Will Richard be happy with the new contract?

Richard's pay-offs under the new contract are given in Table 2.

Richard's expected bonus from only giving out loans to low risk customers changes to $b = 1.55$. The expected bonus from granting loans to high risk customers is

Table 2: Richard's pay-offs under the new contract.

		Economy	
		Good (25%)	Bad (75%)
Decision	low risk	1.55	1.55
	high risk	3.15	1.15

$b = 0.25 \cdot 3.15 + 0.75 \cdot 1.15 = 1.65$. Richard therefore still decides to give out loans to customers with high credit risk, but he will be unhappy with the new contract, as he is worse off in any situation.

e) Which problem do the bank's owners face?

The bank's owners face a Moral Hazard problem. They enter into a contract with Richard, where their pay-off will depend on Richard's actions. Richard however, will act to maximize his own pay-off. The incentive mechanism in the contract will determine whether or not Richard's action also maximizes the owners' pay-off.

Exercise 2 (25%)

Over the past years, Norwegian Wood has managed to establish themselves as one of Norway's most important producers of pinewood and is serving large and small customers from the furniture industry both in Norway and abroad. Fine Stoler AS needs a supplier for its newly established product line Furu Premium. We assume in this exercise, that one unit of pinewood from Norwegian Wood results in one unit of processed goods that Fine Stoler can sell.

Customer demand for Furu Premium that Fine Stoler AS faces in the market is given as:

$$p = 100 - \frac{1}{2}q,$$

where p is the price Furu Premium is sold for in the market and q is the number of products sold in the market.

Fine Stoler's cost function is given as $C_L = 75 + 40q$, while the cost function of Norwegian Wood is $C_N = 60 + 10q$. If Norwegian Wood does not enter into a contract with Fine Stoler AS, they can sell the pinewood to one of their other customers giving them an alternative profit of 540.

The initiative for suggesting contracts has been given to Fine Stoler AS. The company considers two different types of contracts: a one-part linear contract and a two-part linear contract.

a) How does the optimization problem look like that Fine Stoler AS has to solve in order to maximize its profits?

   Fine Stoler's optimization problem is given as

$$\max\ \Pi_F = \left(100 - \frac{1}{2}q\right)q - wq - 40q - 75 - L$$

   subject to

$$wq - 10q - 60 + L \geq 540,$$

   where L will be equal to 0 in case of the one-part linear contract.

b) Assume that both companies have the same information. Which two contracts will Fine Stoler offer to Norwegian Wood and how will these contracts distribute the profit between the two companies?

   The initiative for offering contracts is with Fine Stoler AS. This means that, with symmetric information, they will be able to restrict Norwegian Wood's profit to the reservation profit of 540. The following two different contracts will then be offered:

   1. One-part linear contract: Fine Stoler's optimal order quantity for any wholesale price w is given as

$$q = \frac{1}{2}(200 - 2(w + 40)) = 60 - w.$$

      We use the information that Norwegian Wood will only earn reservation profit and replace q with the expression above in the formula for calculating Norwegian Wood's profits. We then have to solve the following quadratic equation:

$$w(60 - w) - 10(60 - w) = 600$$
$$w^2 - 70w + 1200 = 0$$

      This results in wholesale prices (and corresponding order quantities) $w_1 = 40, q_1 = 20$ and $w_2 = 30, q_2 = 30$. Contract 1 results in profits for Fine Stoler of 125, whereas contract 2 generates profits of 375.

      Fine Stoler will therefore offer contract 2 to Norwegian Wood, resulting in profits of 375 for Fine Stoler and 540 for Norwegian Wood.

2. Two-part linear contract: The optimal wholesale price in a two-part linear contract is equal to the marginal cost of production, $w = 10$. The lump sum payment L has then to ensure that Norwegian Wood earns her reservation profit, resulting in $L = 600$.

   With wholesale price $w = 10$, Fine Stoler orders $q = 50$ and makes a profit of 575.

c) How would the optimal contracts look like if Norwegian Wood had taken the initiative to suggest contracts?

If Norwegian Wood has the initiative in suggesting contracts, they will be able to acquire a larger share of the profits and will under symmetric information restrict Fine Stoler's profits to Fine Stoler's reservation profits. Note that Fine Stoler's reservation profits are not given in the text and can therefore be assumed as either 0 or -75 (the fixed costs).

1. One-part linear contract: Irrespective of who has the initiative in offering contracts, Fine Stoler's optimal order quantity is given as

$$q = \frac{1}{2}\left(200 - 2(w + 40)\right) = 60 - w.$$

   Now however, Norwegian Wood will choose a wholesale price that maximizes her own profits, i. e. solve the following equation:

$$\begin{aligned} \frac{d(w^2 - 70w + 1200)}{dw} &= 0 \\ 2w - 70 &= 0 \\ w &= 35 \end{aligned}$$

   Norwegian Wood offers a contract with wholesale price $w = 35$, resulting in an order quantity $q = 25$. Norwegian Wood makes a profit of 565, while Fine Stoler earns 237,5.
2. Two-part linear contract: The optimal wholesale price in a two-part linear contract is still equal to the marginal cost of production, $w = 10$. With wholesale price $w = 10$, Fine Stoler orders $q = 50$ and the joint profits are 1115. The lump sum payment L then transfers the most of these profits to Norwegian Wood.

   If Fine Stoler's reservation profit is 0, $L = 1175$ and Norwegian Wood makes a profit of 1115. Alternatively, with Fine Stoler's reservation profit equal to -75, $L = 1250$ and Norwegian Wood earns a profit of 1190.

Assume now that Norwegian Wood's true cost function is unknown to Fine Stoler AS. Fine Stoler AS has therefore employed some consultants, who are experts on the production of pinewood, and they estimate that Norwegian Wood's variable costs are constant and most likely uniformly distributed between $\underline{c}$ and $\overline{c}$ ($\underline{c} \leq 10 \leq \overline{c}$).

d) Discuss the consequences this information asymmetry has for the contracts Fine Stoler AS is offering to Norwegian Wood.

Due to the information asymmetry, Fine Stoler can no longer know whether or not Norwegian Wood will accept the proposed contract. The only way to guarantee that Norwegian Wood might be interested in accepting the contract is to assume that their variable cost c is equal to $\overline{c}$, and then design the contract accordingly. This implies that Norwegian Wood will be able to increase her profits beyond the reservation profits if $c < \overline{c}$.

Offering contracts that involve higher payments to Norwegian Wood imply a higher probability of the contract being accepted, but also reduce Fine Stoler's profits. Note that it is possible that no feasible contracts exist (e. g. in case Fine Stoler's optimal order quantity based on $\overline{c}$ is too low to guarantee Norwegian Wood's reservation profits).

Exercise 3 (20%)

Løsøre AS needs a new main supplier and is currently negotiating with several potential candidates to determine who will be awarded the contract. The company considers to use a first-price, sealed-bid auction in order to take the final decision, but needs a little bit of help to evaluate the consequences of this process. The candidates Løsøre AS has negotiated with so far are risk-neutral, symmetric and have reservation prices that are uniformly distributed between 60 and 75.

a) Løsøre AS has so far talked to 5 possible suppliers. What is the expected price Løsøre has to pay if they award the contract to the bidder with the lowest bid?

This is a procurement auction and in contrast to the usual auction setup, the lowest bid wins. Two ways of solving this exercise are presented here:

1. We interpret the auction slightly differently: Løsøre will pay 75 minus a rebate and the supplier who submits the bid for the highest rebate will win the auction. This brings us back to a setting where the highest bid wins. Reservation prices for the rebates are uniformly distributed between 0 and 15.

   The expected revenue from a first price auction with N bidders that have uniformly distributed reservation prices between 0 and 1 as well as being risk-neutral, symmetric and independent, is given as:
   $$E(R) = \frac{N-1}{N+1}.$$
   With this information we can easily compute the expected rebate from an auction with 5 bidders and reservation prices (or reservation rebates) uniformly distributed between 0 and 15 as
   $$E(r) = 15 \cdot \frac{5-1}{5+1} = 10.$$
   The expected price Løsøre has to pay is then $E(R) = 75 - E(r) = 65$.

2. An alternative way to solve this exercise is to set up the expression for expected revenue of an auction , where the lowest bid wins. With N risk-neutral, symmetric and independent bidders with reservation prices uniformly distributed between 0 and 1, we get :
   $$E(R) = N \cdot \int b(v) \cdot f(v) \cdot (1 - F(v))^{N-1}\, dv.$$
   where
   $$b(v) = v + \frac{1-v}{N}, f(v) = 1 \text{ and } F(v) = v.$$
   Expected revenues are then given as
   $$E(R) = 1 - \frac{N-1}{N+1}.$$
   This also results in Løsøre AS paying $60 + 15 \cdot (1 - \frac{5-1}{5+1}) = 65$.

b) It is still possible for other supplier to join the auction, but Løsøre AS may have to spend some money to attract these. How much should Løsøre be willing to spend to attract one additional bidder?

The maximum amount Løsøre is willing to spend on attracting one additional bidder is equal to the change in expected revenues from the auction:

$$E(R_6) - E(R_5) = 15 \cdot \left(\frac{6-1}{6+1} - \frac{5-1}{5+1} \right) = 0.714.$$

c) The cost of attracting new bidders (in addition to the 5 Løsøre already has) is given as $c_b = 1 + 1.25n$, where n is the number of new bidders. With how many bidders should Løsøre AS carry out the auction?

From 3b), we know that Løsøre should spend no more than 0.714 to increase the number of bidders from 5 to 6. We also know that the expected auction revenues decrease less for each additional bidder that is attracted to the auction. Thus, with given c_b, Løsøre should not attract any new bidders to the auction as the cost of attracting them will be higher than the expected decrease in auction revenues.

It is also possible to argue that the marginal revenues from the auction are already lower than the marginal cost of attracting an additional bidder and that therefore no additional bidder should be attracted.

d) Could Løsøre AS have reduced its cost further by choosing a different auction type? Explain.

According to the Revenue Equivalence Theorem, all four standard auction types (English, Dutch, First-price, sealed-bid and Second-price, sealed-bid) result in the same expected auction revenue as long as bidders are risk-neutral, symmetric and independent. Hence, Løsøre AS cannot reduce their expected costs by selecting a different auction type.

Exercise 4 (20%)

A company produces goods in country A and sells the amount q_A in country A and the amount q_B in the neighboring country B. To transport goods to country B, the supplier has to rent pipeline capacity from a transportation service provider (TSP). The goods supplier is a monopolist in both countries. The TSP is a price-taking agent and offers transportation services in the amount of f^T.

Consider a single period problem wherein the producer maximizes profits consisting of sales revenues, quadratic production costs and transportation charges (rental cost + congestion fee). The TSP maximizes congestion fee revenues. Ignore losses in transportation.

We can write the producer's problem as:

$$\max \begin{pmatrix} (a_A - b_A q_A)q_A + (a_B - b_B q_B)q_B \\ -c^P(q_A + q_B) + d^P(q_A + q_B)^2 \\ -(c^T + \tau^T)q_B) \end{pmatrix}$$

subject to

$$q_A + q_B \leq cap^P \qquad (\lambda^P \geq 0).$$

We can write the TSP problem as:

$$\max \tau^T f^T$$

subject to

$$f^T \leq cap^T \qquad (\lambda^T \geq 0).$$

The following condition enforces market clearing in the market for transportation services:

$$f^T = q_B \qquad (\tau^T \in \mathbb{R}).$$

The complementarity problem for the market equilibrium is then given as:

$$
\begin{aligned}
0 \leq q_A \quad &\perp \quad c^P + 2d^P(q_A + q_B) - a_A + 2b_A q_A + \lambda^P \geq 0 && (1)\\
0 \leq q_B \quad &\perp \quad c^P + 2d^P(q_A + q_B) + (c^T + \tau^T) - a_B + 2b_B q_B + \lambda^P \geq 0 && (2)\\
0 \leq \lambda^P \quad &\perp \quad cap^P - q_A - q_B \geq 0 && (3)\\
0 \leq f^T \quad &\perp \quad \lambda^T - \tau^T \geq 0 && (4)\\
0 \leq \lambda^T \quad &\perp \quad cap^T - f^T \geq 0 && (5)\\
&\tau^T \in \mathbb{R}, f^T - q_B = 0 && (6)
\end{aligned}
$$

a) Reformulate condition (2) such that it can be read as the optimality criterion "marginal cost equals marginal revenue" (MC=MR) for the amount sold to country B, q_B.

Condition (2) tells us that if

$$q_B > 0 \Rightarrow c^P + 2d^P(q_A + q_B) + (c^T + \tau^T) - a_B + 2b_B q_B + \lambda^P = 0.$$

Collecting all cost terms on the left hand side and the marginal revenue (intercept minus double the slope times q) on the right hand side then results:

$$c^P + 2d^P(q_A + q_B) + (c^T + \tau^T) + \lambda^P = a_B - 2b_B q_B.$$

This expression can be easily read as MC=MR.

A potentially more insightful way to rearrange this is by collecting all the production related terms and all transportation related terms, and show the MR as the market price minus the "market power mark-up":

$$c^P + 2d^P(q_A + q_B) + \lambda^P + (c^T + \tau^T) = \pi_B - b_B q_B,$$

where $\pi_B(q_B) = a_B - b_B q_B$.

NOTE: Unfortunately, there is an error in sign, either in the producer's objective function or equations (1) and (2) as d^P is needs to be negative in the first or negative in the latter two to be consistent with the other. However, this mistake does not affect the way to solve this exercise. Solutions correcting for this mistake are as correct as those that use the information provided in the exercise.

b) The main difference between condition (1) and condition (2) is the term $(c^T + \tau^T)$. What is the meaning of c^T and τ^T? What can you say about the value of q_B, if $\tau^T > 0$ (strictly positive)?

- c^T reflects the operational cost / fee / charge per unit transported via the pipeline. Other terms that may be used instead of operational: base, standard, regulated, operations and maintenance.
- τ^T reflects the congestion fee on pipeline capacity: the shadow or dual price on capacity, which can be positive if the capacity is fully used.

If $\tau^T > 0$, the capacity must be restrictive, i.e., the pipeline is a bottleneck. Condition (4) gives $\tau^T = \lambda^T > 0$ which together with (5) results in $f^T = cap^T$, and according to (6) this means in $q_B = f^T = cap^T$.

c) Is it possible that both $\lambda^P > 0$ and $\lambda^T > 0$ at the same time? Explain why not or sketch a possible situation wherein that would happen.

Yes, they can both be positive at the same time. Both will be positive if

- the pipeline capacity is so restrictive that the monopoly supplier would have got higher profit if she could have sold more in country B, and
- the production capacity is limiting the total production so much that it limits sales in country A below the unrestricted optimal sales amount.

Note that in the situation with no demand in country A, and pipeline capacity not equal to production capacity, only one of them can become positive, because they can never be both restricting at the same time.

Exercise 5 (15%)

In this exercise, you are given four different discussion questions. In each case, you are requested to explain and/or discuss the statement using what you have learnt in this course. The statements may be right or wrong, and some of them may be vague or imprecise. State the necessary assumptions in each case and, when necessary, discuss the relevant aspects of imprecise formulations.

a) The Bullwhip-effect can be reduced by changing the distribution system.

   Measures that reduce lead time generally reduce the Bullwhip-effect. Changing the distribution distribution can therefore reduce the Bullwhip-effect. This can be achieved in different ways. For example by introducing a cross-docking strategy or replacing shipments through a warehouse with direct shipments from the producer to the customer. Note however, that the new distribution system is not necessarily cheaper than the old one, so that there might be a trade-off between the reduction in Bullwhip-effect and cost of the new system.

b) In a Vickrey auction (second-price, sealed-bid), risk-averse bidders will increase their buds to increase the probability of winning the auction.

   The optimal strategy of a risk-neutral bidder in a second-price, sealed-bid auction is to bid her reservation price and then (in case of winning the auction) to pay the second highest price. A risk-averse bidder will not deviate from this strategy as increasing the bid increases the risk of a negative pay-off (which is not possible with a bid equal to the reservation price).

c) Centralized distribution systems will always have lower safety stocks than decentralized distribution systems.

   Risk pooling effects can be better exploited with a centralized distribution system, compared to a decentralized distribution system. The size of the safety stock however, also depends on the demand correlation in the markets connected to the warehouse(s). For example:

   - with uncorrelated demand, safety stock in the centralized system is lower (but positive) than in the decentralized system.
   - with perfect negatively correlated demand, safety stock in the centralized warehouse is equal to 0, while it is positive in the decentralized system.
   - with perfect positively correlated demand, safety stock the same in both systems.

d) One of the challenges of Moral Hazard in the insurance industry is that is causes insurances to have the wrong prices.

   Maral Hazard in the insurance industry is due to the fact that the insurance company cannot observe actions of the insured person (or company). A homeowner insured against theft might be less diligent about locking doors or might choose to not install an alarm system. If many homeowners act like this, the expected damage due to theft might become larger than the premium the homeowners have to pay, i. e. the insurance policy is priced too cheap.

NTNU

Department of Industrial Economics and Technology Management

Solution to exam
TIØ4285 Production- and Network Economics
May 30, 2017

1. Construction kit from IKEA or exclusive design from Boffi (25 %)

1.1 Hinges and drawer glides fulfill the exact same functionality across kitchen brands. Different brands thus need the exact same or very similar parts. External suppliers producing for several brands can exploit economies of scale, scope, as well as learning curve effects in production and development. The relatively high degree of standardization across brands implies low needs for relationship specific investments so transaction costs related to buying rather than making these parts can be expected to be low. Competition in the supplier industry ensures high powered market incentives contribute to efficiency. Similar arguments apply to countertops. Even though countertops are not as standardized – they typically need to be adapted to each kitchen delivered, they are fabricated from standard materials and the adaption is specific for each final customer, not for each brand. Thus, again there is little need for the brands to commit to specific suppliers and thus little risk of the holdup problem. And the variation in materials would imply that each kitchen brand typically would not be able to produce at large scale, while producers delivering the same kind of countertop to many brands will.

1.2 Nobia can exploit economies of scale and scope by combining production for several brands in one given plant. (The case text shows that the firm has more brands than it has production facilitites.) It may also be able to exert more market power in relation to the supplier industry, thus enabling it to buy parts at lower prices since it can negotiate on behalf of several brands. Dropping brands would not reduce competition intensity in the industry significantly, and Nobia would not expect potential customers to move only towards their other brands.

1.3 By outsourcing production of b1, bulthaup gets higher production capacity without having to invest more in their own production facilities. It may be difficult to achieve as high efficiency if they expanded internally. Not renewing a contract is relatively easy compared to having to terminate own employees in case the model is not a long-term success. This can have a disciplining effect on the supplier that knows it must perform in order to get a new contract. On the other hand, there is the risk that the external supplier will not deliver the expected level of quality which can not only hurt profitability of b1, but also tarnish the bulthaup brand. And over time bulthaup might end up investing significant resources in the relationship risking a hold-up situation.

1.4 Different countertop materials require different production technology limiting economies of scope between different countertop types. Hinges and drawer slides are more standardized products but require automated production for cost efficiency. (They include moving parts that require relatively high precision, consistent quality and each kitchen delivered requires quite a few of them.) While countertops can be delivered directly to the customer, hinges and drawers are often fitted in the kitchen factory. A kitchen supplier thus can easily relate to many different suppliers of countertops, but would typically limit the number of suppliers of hinges and drawer slides. These arguments point towards lower degree of economies of scale for countertops than for hinges and drawers which explains the difference in concentration between industry niches.

2. Value of integration under asymmetric information (50 %)

2.1 Optimal order from shop perspective.

a. If demand is the stochastic variable D, ordered quantity is Q, overage cost, the cost of a unit of left over product, is C_o and the underage opportunity unit cost, i.e. the cost of not satisfying one unit of demand is C_u, optimal quantity is given by: $P(D < Q) = \frac{C_u}{C_u+C_o}$. The cumulative probability of the uniform distribution equals the proportion of the distance from the lower bound to the upper bound: $P = \frac{Q-L}{H-L}$. (This can also be more formally derived by integrating the density $1/(H-L)$ from the lower bound to the quantity Q: $P = \int_L^Q \frac{1}{H-L} dq = \frac{1}{H-L}[Q-L] = \frac{Q-L}{H-L}$.) Thus $P(D < \square) = \frac{Q-L}{H-L}$ so $\frac{Q-L}{H-L} = \frac{C_u}{C_u+C_o} \Rightarrow Q = \frac{C_u}{C_u+C_o}(H-L) + L$

Here the underage cost is $C_u = 1\,500 - 1\,000 - 100 = 400$. Overage costs are $C_o = 1000 - 750 + 100 = 350$ if we assume the store has variable costs of 100 also for these garments. (And $C_o = 1\,000 - 750 = 250$ if we assume no variable costs associated with garments that are not sold quickly.) Parameters in the uniform distribution are $L = 800$ and $H = 1600$. Hence:

$$Q = \frac{400}{400+350}(1\,600 - 800) + 800 = 1\,226{,}66 = 1\,227$$

b. The expected net income can be expressed as the profit with no demand uncertainty less expected loss of overage and underage. The profit with no demand uncertainty is the expected demand times the margin. Expected demand is the midpoint in the uniform distribution: $\mathrm{E[Q] = L + \frac{1}{2}(H - L)}$. And the profit then becomes: $\Pi^{\mathrm{R}}_{\mathrm{Q=\bar{Q}}} = \left[\mathrm{L + \frac{1}{2}(H-L)}\right]\mathrm{C_u}$. The expected losses can be expressed as the probability of e.g. an underage situation times the expected number of underage units given underage times the cost per unit of underage. The probability of ending in an underage situation is $1 - \frac{\mathrm{C_u}}{\mathrm{C_u+C_o}}$. The expected number of units underage in that situation is the expected distance from the order quantity given that the outcome is on the right hand side, i.e. half the distance from the order quantity to the upper boundary $\frac{1}{2}\mathrm{(H-Q)}$. The expected costs of underage become: $\left(1 - \frac{\mathrm{C_u}}{\mathrm{C_u+C_o}}\right)\frac{1}{2}\mathrm{(H-Q)C_u}$. Similarly, the expected costs of overage become: $\frac{\mathrm{C_u}}{\mathrm{C_u+C_o}}\frac{1}{2}\mathrm{(Q-L)C_o}$. The expected profit thus can be expressed as:

$$
\begin{aligned}
\mathrm{E}[\Pi] &= \left[\mathrm{L}+\frac{1}{2}(\mathrm{H}-\mathrm{L})\right]\mathrm{C_u}-\left(1-\frac{\mathrm{C_u}}{\mathrm{C_u}+\mathrm{C_o}}\right)\frac{1}{2}(\mathrm{H}-\mathrm{Q})\mathrm{C_u}-\frac{\mathrm{C_u}}{\mathrm{C_u}+\mathrm{C_o}}\frac{1}{2}(\mathrm{Q}-\mathrm{L})\mathrm{C_o} \\
&= \left[800+\frac{1}{2}(1600-800)\right]400-\left(1-\frac{400}{400+350}\right)\frac{1}{2}(1600-1227)400 \\
&\quad -\frac{400}{400+350}\frac{1}{2}(1227-800)350 = 480\,000-34\,813.33-39\,853.33 \\
&= 405\,333.33
\end{aligned}
$$

c. The expected costs of uncertainty in the form of expected overage and underage costs were expressed in symbols and calculated above. The total costs of uncertainty become: 34 813.33 + 39 853.33 = 74 666.67

2.2 The one-part linear contract with a unit price of 1000 gives CCI a net income of $1227(1\,000-500)=613\,500$. Total value chain profit is 1 018 833. Since the two contracts to be considered contain the one-part linear as a special case, optimal contracts should yield at least as much.

a. Since it has full information, CCI can tailor the two-part linear contract as to acquire the total value chain profit above the reservation profit of the store. There is no information regarding alternatives for the store so it is natural to assume the store would like to order as long as it achieves a non-negative expected net income related to the garment. (It is also OK to assume the store has an option to keep the contract studied in 2.1 so that the profit of 405333.33 becomes the reservation profit.) It will be optimal for CCI to set a unit wholesale price equal to the constant marginal cost and combine with a fixed transfer that transfers all profit to CCI. Unit overage and underage costs for the store, however, then becomes: $C_o = 500 - 750 + 100 = -150$ $C_u = 1500 - 500 - 100 = 900$. Direct use of these numbers in the newsvendor optimality condition would yield $P(D < Q) = \frac{900}{900-150} = 1.2$. Probabilities above 100 % do not make sense, and fact is that the optimality condition does not apply when the salvage value is above the wholesale price. If it indeed was possible to get a value of 750 for each garment with unit variable costs of 600 in total, the value chain could generate infinite profit. One natural assumption then is that the salvage value of 750 only applies as an average number in the setting where the store does not order much more than expected demand. Larger orders giving a higher expected number of garments left over after the selling season, must be expected to lead to a reduction in unit salvage value. Since the case text does not specify the exact relation, any sensible assumption is acceptable. Let us use 600 as an example. That would correspond to zero overage costs.[1] Optimal order and optimal value chain profit is as follows:

$$Q = \frac{900}{900+0}(1\,600-800)+800 = 1\,600$$

$$\mathrm{E}[\Pi] = \left[800+\frac{1}{2}(1600-800)\right]900 = 1\,080\,000$$

Note that the costs of uncertainty disappear here since the probability of underage is zero and the unit costs of overage also is zero. Value chain profit increases by about 61 167 under these assumptions, and CCI can increase its profit by this amount if the profit of the store is not to be reduced, or by $1\,080\,000 - 613\,500 =$

[1] Several students made the assumption that the store would order at the upper limit of the demand distribution, which corresponds to a salvage value of 600. Hence, this number as an example here.

466 500 if it can set new contract terms without regard to the original one-part linear contract.

b. Let the unit wholesale price be w and the buy-back price be b. The unit overage cost then is: $C_o = w - 750 - b + 100 = w - b - 650$. The unit underage cost is: $C_u = 1500 - w - 100 = 1400 - w$. The optimal order for the store then is: $Q = \frac{1400-w}{1400-w+w-b-650}800 + 800 = \frac{1400-w}{750-b}800 + 800$. The expected number of overage is $\frac{1400-w}{750-b}\frac{1}{2}\left(\frac{1400-w}{750-b}800 + 800 - 800\right) = 400\left(\frac{1400-w}{750-?}\right)^2$. The expected net income for CCI then is: $(w - 500)\left(\frac{1400-w}{750-b}800 + 800\right) - b400\left(\frac{1400-w}{750-b}\right)^2$. The first order condition that the derivatives of the expected net income with respect to w and b both are zero yield a saddle point at $w = 1400$ and $b = -150$. The key to getting a sensible solution is to realize that it will be on the boundary where $\frac{C_u}{C_u+C_o} = 1$. Then profit approaches 1 140 000 as w approaches 1400 and b approaches 750. The extra profit from increased margin then dominates the loss related to buy-back. Note that in this case the buy-back contract does not create value by transfer of risk because of assumptions of risk neutrality. [This assignment and 2.3.a were designed to be extra difficult. In hindsight, considering student answers, it seems they were more difficult than necessary. Thus, substantial credit was awarded for good attempts.]

2.3 Double marginalization and demand uncertainty.

a. In theory, the optimal contract can be found by first determining demand from the shop and then use this relation as input in the profit optimization for CCI. The expected profit for the shop assuming optimizing behavior is:

$$\mathrm{E}[\Pi] = \left[\mathrm{L} + \frac{1}{2}(\mathrm{H} - \mathrm{L})\right]\mathrm{C_u} - \left(1 - \frac{\mathrm{C_u}}{\mathrm{C_u} + \mathrm{C_o}}\right)\frac{1}{2}(\mathrm{H} - \mathrm{Q})\mathrm{C_u} - \frac{\mathrm{C_u}}{\mathrm{C_u} + \mathrm{C_o}}\frac{1}{2}(\mathrm{Q} - \mathrm{L})\mathrm{C_o}$$

The expected profit simplifies to:

$$\mathrm{E}[\Pi] = \frac{1}{2}\frac{\mathrm{C_u}(\mathrm{C_u}H + 2\mathrm{C_o}L + \mathrm{C_u}L)}{\mathrm{C_u} + \mathrm{C_o}}$$

The following relations now can be used to express expected profit in terms of price and wholesale price:

$$\mathrm{C_u} = P - w - 100$$

$$C_o = w - 750 + 100 = w - 650$$

$$L = 1050 - \frac{1}{6}\cdot P \quad H = 2100 - \frac{1}{3}\cdot P$$

$$Q = \left(\frac{C_u}{C_u + C_o}(H - L) + L\right) = \left(\frac{P - w - 100}{P - 750}\left(1050 - \frac{1}{6}\cdot P\right) + 1050 - \frac{1}{6}\cdot P\right)$$

The first order condition for profit maximization for the store is that the first order derivative with respect to price is zero. This leads to expressions that give the price for each wholesale price that in turn can be used to express the profit for CCI for any wholesale price. The optimal wholesale price then is found by forcing the first order derivative of CCI profit with respect to the wholesale price to zero. However, this approach becomes exceedingly complex in this case.

A more feasible approach is to start with the deterministic case, find optimal wholesale price and then take the uncertainty into account using a numerical procedure. The optimal wholesale price given demand equal to the expected demand is found as follows:

$$Q = 1575 - 0{,}25P \Leftrightarrow P = 6300 - 4Q \Leftrightarrow MR_{Store} = 6300 - 8Q$$

$$MC_{Store} = MR_{Store} \Rightarrow w + 100 = 6300 - 8Q \Leftrightarrow w = 6200 - 8Q$$

$$MC_{CCI} = MR_{CCI} \Rightarrow 1000 = 6200 - 16Q \Leftrightarrow Q = 325$$

$$P = 6300 - 4 \cdot 325 = 5000 \quad w = 6200 - 8 \cdot 325 = 3600$$

This price and wholesale price would lead to:

$$\frac{\mathrm{C_u}}{\mathrm{C_u} + \mathrm{C_o}} = \frac{5000 - 3600 - 100}{5000 - 3600 - 100 + 3600 - 650} = \frac{1300}{4350} = 0{,}3$$

$$Q = \frac{2 \cdot 325}{3} + 0.3 \cdot \left(\frac{4 \cdot 325}{3} - \frac{2 \cdot 325}{3}\right) = 281{,}7$$

The high wholesale price limits margin compared to salvage value and the store orders a lower amount than in the deterministic case. Then it is profitable for CCI to have a lower wholesale price. Some iterations of numerical search yield a wholesale price of 2852. [Note: This assignment was very difficult. Substantial credit should be given for good attempts. And it should not be necessary to come very close in terms of numerical solution to get practically full credit.]

b. CCI have stronger incentive to introduce a two-part tariff when the store can set the price since it then is exposed to ordinary double marginalization in addition to the effect of wholesale price above marginal costs yielding to low underage costs as seen from its perspective. The buy-back contract cannot fix this problem and is not necessarily more attractive.

2.4 Uncertain marginal costs.

a. Not knowing marginal costs exactly CCI cannot expect to set optimal contract terms and hence will expect to lose compared to a situation where it has perfect information. It cannot set optimal wholesale price in any contract, and it might be afraid setting a high fixed transfer in a two-part linear contract might lead the store to reject the contract.

b. Asymmetric information will in general lead to sub-optimal value creation.

2.5 Since the store has many garments, there will be a risk diversification effect, analogous to the risk pooling effect in inventory theory, implying the store will act almost as if risk neutral in relation to each garment even if it is risk averse in relation to overall profit. Similarly for CCI that has many stores each with many garments. Thus, risk aversion as specified in assignment text is unlikely to have any significant effects on earlier results. (Effects of risk aversion could include: Risk aversion for stores would reduce optimal orders in 2.1, 2.2 and thus reduce expected profits for both the store and CCI. Asymmetric information and risk aversion would mean CCI would be forced to set a fixed transfer in a two-part linear contract at a lower level as to not risk zero orders from the store.)

2.6 The bullwhip effect is a tendency for demand volatility to be amplified towards upstream entities in the value chain. Here it can appear only if the store updates its forecasts, inventory needs for future garments based on garment sales. In general it can be limited by sharing information about demand, i.e. downstream entities share information with upstream entities. Here, since there is two entities, the effect of information sharing will not be that big. However, CCI can benefit from knowing sales concurrently and using them to forecast demands for future garments.

3. Auction forms (15 %)

3.1 The revenue equivalence theorem says that under private value model conditions all ordinary selling mechanisms yield the same expected revenue for the seller. In e.g. second price auctions the revenue is equal to the second highest valuation. In first price auctions bidders bid the expectation of the second highest bid, conditional of that the bidder has the highest valuation. It can be shown that the expected value of the highest bid then is equal to the second highest valuation.

3.2 Note that since bids in firs price auctions are based on expectations rather than the actual sample of bidder valuations, there is less variability in actual revenues in first-price auctions than in second-price auctions. Risk aversion does not influence bidding in second-price auctions. In firs price auctions bidders bid more aggressively in order to not risk losing potential gain. This means risk averse sellers will prefer first-price auctions both because of higher expected revenue and less risk related to revenue. Bidders on the other hand face a trade-off. Compared to first-price auctions, second-price auctions give higher expected payoff, due to lower expected price to pay, but the variability in payoff is larger than in a first-price auction.

3.3 The revenue equivalence theorem applies also to all-pay auctions, so it is certainly relevant. Since it is effectively a first-price auction, it can be argued that risk aversion would tend to inflate bids. However, this is an auction form where bidders typically expect to lose so this effect is probably not important. (This auction form is not common, and seems to be used primarily related to charity fund raising.)

4. Principal agent theory and transfer pricing (10 %)

Important similarities include: Higher level in organization, principal want to set contract terms as to achieve high level of efficiency. Lower level in organization, agent typically has preferences that are not initially aligned with higher level, principal. Asymmetric information limits the possibility for higher level, principal to set optimal contract terms.

Important differences include: In the principal-agent model participation is an important constraint, while in transfer pricing we have assumed participation is given. In the principal-agent discussion risk aversion is important, whereas in the transfer-pricing discussion it has not been in focus.

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Department of Industrial Economics and Technology Management

Solution to exam
TIØ4285 Production- and Network Economics
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1. Gigafactory (25 %).

1.1 Sourcing cells from the market has enabled Tesla to exploit economies of scale and scope as well as learning curve effects of producers supplying several other firms. Being directly exposed to market forces, these producers have had strong incentives to keep costs low and squeeze margins as to sell their cells. Organizational transactions costs thus have probably been low in this production while the effects of double marginalization have been limited by competition. Since the product is standardized, there has been little need for relationship specific investments and transaction costs in the relationships have likely been very low. In total it is clear that Tesla would not have been able to produce these parts at costs lower than the price the company has paid for them in the market.

1.2 Plans to build so many cars that they would need a major part of world production of the cells, implies important changes. Now Tesla can achieve even larger scale than any single current cell producer. By locating cell production and battery pack production together in the same plant, they should be able to reduce costs. It might also be the case that Tesla can keep prices in cells down by investing in the industry instead of waiting for other firms to expand production capacity dramatically.

1.3 Tesla does not have technical expertise in cell production. Panasonic on the other hand is a leading cell producer. Thus Tesla aims to exploit the technical know-how of Panasonic. In addition this means cell production will not be an integrated part of Tesla. It is likely that Tesla hopes this will limit organizational transaction costs. Panasonic, and their work force, will know they are only tenants in the factory. Unless they produce efficiently, keeping costs for Tesla down, they may be exchanged for another partner.

1.4 The energy numbers imply that Tesla plans to source a significant amount of cells from outside the Gigafactory. It could be from Panasonic or from other producers. Thus Tesla clearly signals it will not commit to a single supplier of cells. A likely goal is to limit Panasonic's leverage in the contract relationship. The Gigafactory involves relationship specific investments and could lead to significant transaction costs. Tesla hopes this strategy will limit these costs.

2. Value of forecast improvement and flexibility (45 %)

2.1 Moving average is a pure time series approach. When adjusting using weather forecast, we are in the causal category. Moving average means each of the observations in the sample are equally weighted. Exponential smoothing typically uses all previous observations, but puts more weight on the more resent ones. If there is a high degree of correlation between days, last value forecasting might have been a good option. Both approaches fail to include trend or seasonality effects. Methods, like Winter's, that include these effects thus might be more accurate.

2.2 Optimal ordering.

a. If demand is the stochastic variable D, ordered quantity is Q, overage cost, the cost of a unit of left over product, is C_o and the underage opportunity unit cost, i.e. the cost of not satisfying one unit of demand is C_u, optimal quantity is given by: $P(D < Q) = \frac{C_u}{C_u + C_o}$. The cumulative probability of the uniform distribution equals the proportion of the distance from the lower bound to the upper bound: $P = \frac{Q-L}{H-L}$. (This can also be more formally derived by integrating the density $1/(H-L)$ from the lower bound to the quantity Q: $P = \int_L^Q \frac{1}{H-L} dq = \frac{1}{H-L}[Q-L] = \frac{Q-L}{H-L}$.) Thus $P(D < Q) = \frac{Q-L}{H-L}$ so $\frac{Q-L}{H-L} = \frac{C_u}{C_u+C_o} \Rightarrow Q = \frac{C_u}{C_u+C_o}(H-L) + L$

Using i to denote ice cream and d to denote drinks we have the overage costs, the cost of a unit of left over product, C_o, is wholesale price less the salvage value: $C_{oi} = 0.75 - (-0{,}01) = 0.76$, $C_{od} = 0.02$. The underage opportunity unit cost is the cost of not satisfying one unit of demand. Here that is the margin, i.e. the selling price less the wholesale price: $C_{ui} = 1.5 - 0.75 = 0.75$, $C_{ud} = 1 - 0.3 = 0.7$. The distributions are from 25 % below 500 to 25 % above for drinks and from 40 % below 250 to 40 % above 250 for ice cream. Consequently:

$$L_i = (1-0.40)250 = 150 \quad H_i = (1+0.40)250 = 350$$
$$L_d = (1-0.25)500 = 375 \quad H_d = (1+0.25)250 = 625$$

$$Q_i = \frac{0.75}{0.75+0.76}(350-150) + 150 = 249.3$$

$$Q_d = \frac{0.7}{0.7+0.02}(625-375) + 375 = 618.1$$

Rounding to integer units yields 249 units of ice cream and 618 units of drinks.

b. The expected net income can be expressed as the profit with no demand uncertainty less expected loss of overage and underage. The profit with no demand uncertainty is the expected demand times the margin. Expected demand is the midpoint in the uniform distribution: $E[Q] = L + \frac{1}{2}(H-L)$. And the profit then becomes: $\Pi^R_{Q=\bar{Q}} = \left[L + \frac{1}{2}(H-L)\right] C_u$. The expected losses can be expressed as the probability of e.g. a underage situation times the expected number of underage units given underage times the cost per unit of underage. The probability of ending in an underage situation is $1 - \frac{C_u}{C_u+C_o}$. The expected number of units underage in that situation is the expected distance from the order quantity given that the outcome is on the right hand side, i.e. half the distance from the order quantity to the upper boundary $\frac{1}{2}(H-Q)$.

The expected costs of underage becomes: $\left(1-\frac{C_u}{C_u+C_o}\right)\frac{1}{2}(H-Q)C_u$. Similarly the expected costs of overage becomes: $\frac{C_u}{C_u+C_o}\frac{1}{2}(Q-L)C_o$. The expected profit thus can be expressed as:

$$E[\Pi] = \left[L+\frac{1}{2}(H-L)\right]C_u - \left(1-\frac{C_u}{C_u+C_o}\right)\frac{1}{2}(H-Q)C_u - \frac{C_u}{C_u+C_o}\frac{1}{2}(Q-L)C_o$$

For ice cream $\frac{C_u}{C_u+C_o} = 0.497$. For drinks $\frac{C_u}{C_u+C_o} = 0.972$. Expected profits become:

$$E[\Pi_i] = \left[150+\frac{1}{2}(350-150)\right]0.75 - (1-0.497)\frac{1}{2}(350-249)0.75 - 0.497 \cdot \frac{1}{2}(249-150)0.76 = 187.5 - 19.05 - 18{,}70 = 149.75$$

$$E[\Pi_d] = \left[375+\frac{1}{2}(625-375)\right]0.7 - (1-0.972)\frac{1}{2}(625-618)0.7 - 0.972 \cdot \frac{1}{2}(618-375)0.02 = 350 - 0.07 - 2{,}36 = 347{,}57$$

c. The expected loss due to uncertainty has been expressed in symbols above. It becomes: $19.05 + 18{,}70 + 0.07 + 2{,}36 = 40.18$.

2.3 Improvement over current approach.

a. With the current approach the order quantities would be $250 \cdot 1.1 = 275$ for ice cream and $500 \cdot 1.2 = 600$ for drinks. The probability for overage then would be $\frac{275-150}{350-150} = 0.625$ for ice cream and $\frac{600-375}{625-375} = 0.9$ for drinks. The expected over- and underage costs then would be:

$$(1-0.625)\frac{1}{2}(350-275)0.75 + 0.625 \cdot \frac{1}{2}(275-150)0.76 = 10.55 + 29.69 = 40.24$$

$$(1-0.9)\frac{1}{2}(625-600)0.7 + 0.9 \cdot \frac{1}{2}(600-375)0.02 = 0.88 + 2.03 = 2.91$$

Total expected loss becomes 43.15, which is 2.97 more than when using the Newsboy optimality condition.

b. The distributions would have ranges between 425 and 575 for drinks and between 200 and 300 for ice cream. Order quantities would be:

$$Q_i = 0.497(300-200) + 200 = 249{,}7$$

$$Q_d = 0.972(575-425) + 425 = 570.8$$

The expected over- and underage costs then would be:

$$(1-0.497)\frac{1}{2}(300-250)0.75 + 0.497 \cdot \frac{1}{2}(250-200)0.76 = 9.43 + 9.44 = 18.87$$

$$(1-0.972)\frac{1}{2}(575-571)0.7 + 0.972 \cdot \frac{1}{2}(571-425)0.02 = 0.04 + 1.42 = 1.46$$

Total expected cost of uncertainty becomes 20.33. This is 19.85 less than without the weather adjustment.

2.4 The new equipment would imply zero overage costs and order quantities at the upper limit of distributions. Cost of uncertainty would be zero. On this typical day expected income would be 20.33 higher than when using weather-adjusted forecasts.

2.5 Assume equal storage need per unit of product (or include ratio of exchange in terms of storage in reasoning below). Consider a starting point at optimal order levels without capacity constraints.

New equipment – on boundary: Probability of selling last unit times margin is expected loss. Reduce first quantity of product with lowest. When equal reduce both as to maintain equality.

Old equipment – interior solution – marginal change of quantity gives zero effect on expected profit. Check for unit change. Reduce quantity of product with lowest effect on profit.

2.6 Often normal distributions are used even though they cannot be entirely realistic – can't have negative demand. But typically works well. Lognormal is a possibility. Normal implies closer to expectation for P close to 50 %, longer from expectation for extreme P.

3. Double marginalization and asymmetric information (15 %)

3.1 Down faces direct demand $Q = a - bP$ and thus indirect demand $P = \frac{a}{b} - \frac{1}{b}Q$. Marginal revenue then is $MR_{Down} = \frac{a}{b} - \frac{2}{b}Q$. The first order condition for profit maximization becomes $\frac{a}{b} - \frac{2}{b}Q = MC_{Down} + w \Rightarrow w = \frac{a}{b} - \frac{2}{b}Q - MC_{Down}$. This latter relationship is the indirect demand facing Up. The marginal revenue becomes $MR_{Up} = \frac{a}{b} - \frac{4}{b}Q - MC_{Down}$. The first order condition for Up becomes $MC_{Up} = \frac{a}{b} - \frac{4}{b}Q - MC_{Down}$. The optimal quantity thus is $Q = \frac{a}{4} - \frac{b}{4}\left(MC_{Up} + MC_{Down}\right)$. The optimal wholesale price thus becomes $w = \frac{a}{b} - \frac{2}{b}\left(\frac{a}{4} - \frac{b}{4}\left(MC_{Up} + MC_{Down}\right)\right) - MC_{Down} = \frac{1}{2}\left(\frac{a}{b} + MC_{Up} - MC_{Down}\right)$.

3.2 The effect for the one-part linear contract under risk neutrality is discussed in Corbett, Zhou and Tang (2004) in the reading material (and Corbett and Tang (1999) referred to in the lecture note on supply chain contracting). Downside risk is related to Up underestimating the marginal costs of Down and setting a transfer price higher than the optimal level. Risk aversion would tend to imply lower wholesale price than the formula in the paper implies. The literature mentioned also provides results for the two-part linear case under an extreme level of risk aversion – i.e. that Up insists on zero probability that the contract proposal will be rejected. Lower level of risk aversion, or risk neutrality, would imply higher fixed transfer and lower wholesale price than formulas suggest.

4. Mixture (15 %)

4.1 Risk pooling means the proportion of safety stock can be reduced due to the diversification effect when volatile demands from several markets are aggregated. This will be the case as long as demands are not perfectly correlated.

4.2 Risk aversion among bidders means the seller might prefer first price auctions (Dutch, sealed first price). Risk averse bidders will then tend to bid higher as to increase probability of winning (and reduce risk of losing profit/utility). If bidders are concerned about the "winner's curse" a process stimulation price discovery might be beneficial. Bidders uncertain about the real value of the object, can feel more certain they will not sustain a significant loss by learning (lower bounds of) the valuation of other bidders. This can in turn reduce the effect of risk aversion related to the possible loss of buying at a too high price.

4.3 A key difference between the principal-agent model as described and a typical two-stage stochastic optimization problem is that different decision-makers make decisions in the two stages in the former while in the latter the decision-maker typically is the same in the two stages. In stochastic optimization decisions in the first stage will often aim to exploit the flexibility represented by second stage decision possibilities. In the principal-agent model the aim is to set contract terms as to gain as much as possible and in the set of such problems discussed in the course, contract terms are set as to ensure the agent accepts. This means the possible decision to reject is only hypothetical and defines a restriction on the set of possible contract terms.

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Solution to exam
TIØ4285 Production- and Network Economics
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1. Oakley – Innovator in the sunglass industry (30 %).

1.1 The main disadvantage of owning the retailing part of the value chain has to do with the tendency for organizational incentives to be weaker than market incentives. Exposure to market forces – the threat of bankruptcy in particular – tends to motivate management and employees to a higher degree than traditional incentives within a large corporation. The result is a lower degree of efficiency, typically, within organizational units that feel protected from market forces by the corporation.

By controlling the retailing part of the value chain Luxottica ensures that its brands are exposed where consumers are looking for glasses. It becomes part of the brand building that is central in this line of business. At the same time Luxottica can possibly learn about customer needs, fashion trends faster by the direct contact with customers through retailing.

Double marginalization is easier to avoid by owning retailers. (And by limiting double marginalization internally, Luxottica also reduces the market power of the other stores selling its brands.) Direct control usually also enables a higher level of coordination in the value chain, which for instance can be used to limit the bullwhip effect.

The case text does not contain enough information to conclude regarding the overall profitability of this dual degree of vertical integration of retailing in this industry. Maybe it is a good choice assuming good capabilities in governing subsidiaries.

1.2 Frames and lenses are more technically demanding to produce. In particular lenses. Since Oakley builds its reputation on differentiated technology, there would be a need for significant relationship specific investments if production were to be outsourced. There might be a need for some secrecy regarding technology. And there is a marketing effect – the company's message of superior technology might not be conceived as trustworthy if other firms produced the sunglasses. Thus, it is likely that transaction costs in a contractual relationship would be high. And since production of these kinds of products to a relatively large degree is automated, the transaction costs related to low powered organizational incentives are likely to be limited.

Clothing, bags and similar products are technologically much easier to produce. Quality can be reasonably well defined, there are many possible suppliers and low needs for relationship specific investments. And this kind of production is labor intensive. Hence the disadvantages related to outsourcing are smaller and the advantages larger for clothing etc. compared to frames and lenses

1.3 The properties sellers of differentiated goods claim make their products superior, are in many cases hard for most consumers to evaluate before having substantial experience using the product. And in some cases it is difficult to distinguish precisely between the qualities of competing products even after extensive use. These products can thus be classified as experience goods or credence goods. In any case quality is to a significant degree unobserved before purchase. Thus, there is asymmetric information since the supplier knows more about product quality than buyers.

When consumers cannot distinguish between goods of varying quality, the *lemons problem* might arise. Since quality is not observable at the point of purchase it becomes difficult for sellers of high quality products that are competing with lower quality products, to charge a higher price. Production and marketing of high quality products then might become unprofitable and low quality products dominate in the market.

Typical approaches to overcome the lemons problem include extended warranties, money back return policies, certification by independent agencies, reputation building etc. Expert endorsement of products is another possibility. Athletes using the products signal that the products are of high enough quality to be useful for their activity. The credibility of athletes in this sense is of course highest for products used in the sports activity. Hence, a natural explanation considering the course syllabus is that Oakley uses athletes to overcome the lemons problem.

2. Rush Delivery Option (50 %)

2.1 In the following marginal selling costs for retailers are assumed to accrue for all ordered units. The alternative interpretation that this marginal cost of 5 only accrues for sold units is also fully acceptable.

a. If demand is the stochastic variable D and ordered quantity is Q, the optimal quantity is given by: $P(D < Q) = \frac{C_u}{C_u + C_o}$. The overage cost, the cost of a unit of left over product, C_o, is wholesale price plus the marginal cost less the salvage value: $C_o = 20 + 5 - 3 = 22$. The underage opportunity unit cost is the cost of not satisfying one unit of demand. Here that is the margin, i.e. the selling price less the sum of marginal cost and wholesale price: $C_u = 40 - 5 - 20 = 15$. Consequently: $P(D < Q) = \frac{15}{15+22} = 0{,}4054$. This is the probability of ending up in an overage situation. The cumulative probability of the uniform distribution equals the proportion of the distance from the lower bound to the upper bound: $P = \frac{Q-L}{H-L}$. (This can also be more formally derived by integrating the density $1/(H-L)$ from the lower bound to the quantity Q: $P = \int_L^Q \frac{1}{H-L} dq = \frac{1}{H-L}[Q-L] = \frac{Q-L}{H-L}$.) Thus the optimality condition becomes $\frac{Q-100}{200-100} = 0.4054 \Leftrightarrow Q = 100 + 0.4054(200-100) = 140{,}54$.

b. For the retailer the expected net income can be expressed as the profit with no demand uncertainty less expected loss of overage and underage. The profit with no demand uncertainty is the expected demand times the margin. Expected demand is the midpoint in the uniform distribution: $E[Q] = 100 + \frac{1}{2}(200 - 100) = 150$. And the profit then becomes: $\Pi^R_{Q=\bar{Q}} = 150(40 - 20 - 5) = 2\,250$. The expected losses can be expressed as the probability of e.g. a underage situation times the expected number of underage units given underage times the cost per unit of underage. The probability of ending in an underage situation is $1 - 0.4054 = 0{,}5946$. The expected number of units underage in that situation is the expected distance from the order quantity given that the outcome is on the right hand side, i.e. half the distance from the order quantity to the upper boundary. Here it is the distance from 140.54 to the midpoint between 140.54 and 200, which is 170,27 making the expected number underage 29.73. The expected underage cost becomes: $\Pi_u^- = 0.5946 \times 29.73 \times 15 = 265.16$. Similarly for the overage case we have: $\Pi_o^- = 0.4054 \times 20.27 \times 22 = 180.78$. Expected profit then is $E[\Pi] = 2\ 250 - 265.16 - 180.78 = 1\ 804.06$.

EBF will have a deterministic profit per retailer determined by the demanded quantity and the delivery cost: $140.54(20 - 10) - 250$ =1 155.4.

c. The expected loss due to uncertainty per retailer is the sum of expected underage and overage costs: $265.16 + 180.78 = 445.94$.

2.2 Some possible ways to improve profits for EBF.

a. The wholesale price that is way above marginal cost creates inefficiency in the value chain. Hence there is a value creation potential in contracts involving a lower unit price. Since EBF will want to capture some of the value potential the contract must ensure some other mechanism of value transfer from retailers to EBF. Such mechanisms could include profit sharing or a fixed transfer as in a two-part linear contract. EBF might also consider a buy back offer – i.e. offering to buy unsold units from retailers and thus reducing overage cost, which would increase order size.

A wholesale unit price equal to EBF's marginal cost of 10 implies: $C_o = 10 + 5 - 3 = 12$. $C_u = 40 - 5 - 10 = 25$. Consequently: $P(D < Q) = \frac{25}{25+12} = 0{,}6757$. $Q = 100 + 0.6757(200 - 100) = 167{,}57$. $\Pi_{Q=\bar{Q}}^R = 150(40 - 10 - 5) = 3\ 750$. $\Pi_u^- = (1 - 0{,}6757) \times 16.21 \times 25 = 131.42$. $\Pi_o^- = 0.6757 \times 33.79 \times 12 = 273.98$. Expected profit then is $E[\Pi] = 3\ 750 - 131.42 - 273.98 = 3\ 344.60$. The profit increase is: $3\ 344.60 - 1\ 804.06 = 1\ 540.54$. Thus, if EBF offers a two-part linear contract with a unit price of 10 and fixed transfer of 1 540.54 the retailer will be no worse off. EBF's profit per retailer will then equal 1 540.54, which is 385.14 more than with the current contract.

b. Rush ordering means increased flexibility and ability to adapt to actual demand. In the general setting where demands are not perfectly correlated it would not allow perfect prediction of demand after having observed pre-lunch demand. However, it would allow firms to face a lower degree of uncertainty when ordering, and hence decrease cost of uncertainty.

For EBF it becomes important to be able to plan production. Without rush orders all orders would probably be placed the day before delivery at latest so that production could be planned without uncertainty about demand. With a rush order mechanism it would have to produce before receiving actual rush order. (Assuming production takes significant amount of time.) However, given independent distributions and many retailers, relative variability becomes very low. There is a risk pooling effect.

c. Given the assumptions it would appear that retailers can order 100, which is the highest possible amount for the period before lunch and also the lowest possible for the day. After having observed demand they decide whether to order more knowing for a fact what demand will be. Then they will face no residual demand uncertainty. Since they are not paying for delivery they have no incentive to order more than 100 first thinking that it might be enough for the whole day and save a delivery trip. The expected profit becomes 2 250. (I.e. the profit without costs of under- or overage). EBF's expected profit per retailer becomes $150(20 - 9) = 1\ 650$ before delivery costs.

If we think in terms of integer units, the first delivery is certain while the other is 98 % likely giving delivery costs of: $250 + 0.98 \cdot 250 = 495$. The expected profit then is 1 155. Note that strictly speaking EBF now will face some uncertainty, but due to the risk pooling effect it is relatively low. However in this setting profits do not increase. The retailers profit increase by 445.94, i.e. the original cost of uncertainty.

d. Given the assumptions retailers now are ordering the total number of units that maximizes value chain profit when delivery costs are left out. However it is obvious that value is lost due to over utilization of the rush order delivery mechanism. Assume the retailers ordered 102 before lunch. Only if demand is exactly 50 before lunch will there be an overage situation at the end of the day. Two units will be left over with probability 2 %. The expected overage cost is $0.02 \cdot (9 + 5 - 3) = 0.22$ for the value chain while the expected reduction in delivery cost is $0.02 \cdot 250 = 5$. By charging separately for the rush delivery EBF will induce retailers to take this cost into account when ordering. They will then have an incentive to order more than 100 in order to reduce the number of rush orders necessary. But then the wholesale price above marginal cost will induce value chain loss. So a natural contract could be wholesale price at 9, delivery charge at 250 and a fixed transfer ensuring participation while providing higher profit for EBF. Thus, we have a three-part contract.

2.3 Asymmetric information.

a. With asymmetric information EBF will not be able to set a fixed sum in a two-part linear contract as to extract all surplus profit. Similar arguments apply to profit sharing and buy back as well. Thus it will have a lower profit potential.

b. Before implementation the asymmetric information will certainly make estimation of the value more difficult. So if EBF is risk averse, and particularly if it will involve some irreversible investment, the value will be lower beforehand. If EBF uses the three-part contract, the fixed fee can't be set to capture the whole surplus profit including the value of the rush order option. Thus, asymmetric information probably will reduce the value of the option.

2.4 Realism of analysis.

a. There are some assumptions that probably are unrealistic and will bias option value upwards. First, normally we would not expect demand after lunch to mirror demand before. If there is only a correlation between them observed demand will give better decisions for the rush ordering. However even without correlation, the rush order does increase flexibility and will provide some value. (But not necessarily enough to cover costs.) Second, the assumption of independent demand and many retailers limits relative variability of total orders for EBF. However we would expect some correlation giving a need for production flexibility that probably is costly.

The assumption of uniformly distributed demand is again unrealistic. Lognormal might be better. (Normal would not be realistic – demands cannot be negative.) It is not obvious how more realistic assumptions would influence calculated option value.

b. The costs of uncertainty seem large compared to the costs of the rush order delivery option. For expected demand of 150 the production costs are reduced by 150 and the extra delivery costs is 250 for a net cost of providing the option of 100 when it is actually utilized. The cost of uncertainty is more than four times this amount and with good contracts it will not be used by a very large number of retailers on a typical day.

3. Mixture (20 %)

3.1 The principal agent model gives this insight. The principal is to hire the agent to do some task. However the principal cannot perfectly monitor the effort level of the agent who is likely to shirk unless some mechanism to motivate effort is introduced. In order to motivate the agent to work hard, the principal can link compensation to some measure that is imperfectly related to the effort of the agent. Thus the agent will have an incentive to work hard in order to increase expected pay, but will be exposed to risk since pay is only imperfectly related to effort. Since the agent is risk averse the agent will require compensation to carry this risk. This risk premium becomes an expected cost for the principal. And since the principal typically is assumed approximately risk neutral the transfer of risk to the agent does not benefit the principal apart from the effect on motivation. The principal now will have to make a trade off between the level of motivation and the risk premium - in other words the efficiency in risk sharing - when deciding how much of pay to link to the imperfect performance measure.

3.2 Evaluating with the same data that was used for calibration implies the model is judged in terms of how well it fits data rather than how well it predicts unobserved outcomes. Calibration procedures tend to optimize parameters as to make the model fit as well as possible to data. Flexible models – typically with several parameters – will then often fit data well. But it might very well be a case of “over fitting” which means the model adapts to more or less random noise components instead of mirroring underlying mechanisms. Then the forecast performance might very well be poor compared to a seemingly simpler model. To conclude: This approach to forecast model evaluation will tend for favor flexible models with several parameters enabling over fitting.

3.3 Scenarios represent an approach to discretizing complex stochastic processes using randomly drawn outcome vectors, i.e. scenarios. Since this process gives an imperfect representation of stochastic processes in the model, decision variable output from the optimization might very well reflect these imperfections. By running the model with different sets of scenarios and evaluation robustness we can get an impression of the severity of this issue.

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Solution to exam
TIØ4285 Production- and Network Economics
June 2, 2014

1. Moods Of Norway (30 %)

1.1. Vertical integration

Note that production in low-cost countries is possible also in factories the brand owns. The focus of discussion here therefore should be on transaction costs, not wages.

Production in factories that also produce for other brands enables exploitation of economies of scale and scope as well as learning curve effects to a larger degree than in factories only producing for one brand. However, this is a low-tech industry, products are produced in large series so these effects are probably not strong. But the efficiency effect of factories being exposed to high-powered market incentives rather than the typical low-powered organizational incentives is probably important.

Since Moods can choose among many factories, clothes are fairly generic, well defined products that can be evaluated by objective quality standards, investments in relationship specific assets are probably relatively small. Thus it is likely that efficiency effects of external production are larger than transaction costs in contractual relations. And Moods apparently expects this net advantage to be more important than potential benefits of having command line control over factories, which may include more production flexibility, tighter quality controls etc.

1.2. RFID

Potential benefits of RFID tagging include:

NOSBOS: RFID scanning enables much easier detection of NOSBOS incidents leading to higher real service level for customers, reduced labor cost related to dealing with the problem.

Shrinkage: The RFID approach enables better identification of points in the value chain where there is shrinkage, which should improve the efficiency of measures taken to reduce the problem.

Stock level: More precise numbers for current stock levels should improve efficiency of ordering routines leading to higher service levels for given real stock level or lower needed real stock level for given service level. Increased precision also means higher real level of transparency assuming numbers are shared in the value chain. The effect on the bullwhip effect is probably minor at best since it is primarily driven by variations in demand.

1.3. Zara

Zara's business model means they have lead times of only a fraction of their main competitors. It is ok, but not required or expected to mention benefits in terms of being able to adapt quickly to changing fashion trends. The expected focus is on classical supply chain effects of short lead times, production flexibility.

Short lead times reduce bullwhip effects. But other effects are probably much more important. They can order for much shorter selling periods meaning variability will be much lower. This reduces overage costs as mentioned in the case (a much smaller proportion of products needs to be sold at markdown prices). And it also reduces underage costs. If they observe higher demand than anticipated for an item, they can order more from one of their factories and have the product in store within the selling season. Higher order frequency also means lower stock levels for given service levels.

2. Down and Up (50 %)

2.1. Value chain profit maximization

a. If demand is the stochastic variable D and ordered quantity is Q, the optimal quantity is given by: $P(D < Q) = \frac{C_u}{C_u + C_o}$. The overage cost, the cost of a unit of left over product, C_o, is the sum of marginal costs less the salvage value: $C_o = MC_{Up} + MC_{Down} - S = 200 + 300 - 100 = 400$. The underage opportunity unit cost is the cost of not satisfying one unit of demand. Here that is the margin, i.e. the selling price less the sum of marginal costs: $C_u = P - \left(MC_{Up} + MC_{Down}\right) = 1100 - (300 + 200) = 600$. Consequently: $P(D < Q) = \frac{P - (MC_{Up} + MC_{Down})}{P - (MC_{Up} + MC_{Down}) + MC_{Up} + MC_{Down} - S} = \frac{P - (MC_{Up} + MC_{Down})}{P - S} = \frac{600}{1100 - 100} = 0{,}6$. This is the probability of ending up in an overage situation. The cumulative probability of the uniform distribution equals the proportion of the distance from the lower bound to the upper bound: $P = \frac{Q - L}{H - L}$. (This can also be more formally derived by integrating the density $1/(H - L)$ from the lower bound to the quantity Q: $P = \int_L^Q \frac{1}{H - L} dq = \frac{1}{H - L}[Q - L] = \frac{Q - L}{H - L}$.) Thus the optimality condition becomes $\frac{Q - L}{H - L} = \frac{P - (MC_{Up} + MC_{Down})}{P - S} \Leftrightarrow Q = L + \frac{P - (MC_{Up} + MC_{Down})}{P - S}(H - L) = 5000 + 0{,}6(25000 - 5000) = 5000 + 12000 = 17000$.

b. The expected net income can be expressed as the profit with no demand uncertainty less expected loss of overage and underage. The profit with no demand uncertainty is the expected demand times the margin. Expected demand is the midpoint in the uniform distribution: $E[D] = L + \frac{1}{2}(H - L) = 15000$. And the profit then becomes: $\Pi_{Q=\bar{Q}} = \left(L + \frac{1}{2}(H - L)\right)\left(P - \left(MC_{Up} + MC_{Down}\right)\right) = 15000 \times 600 = 9000000$. The expected losses can be expressed as the probability of e.g. a underage situation times the expected number of underage units given underage times the cost per unit of underage. The probability of ending in an underage situation is $1 - \frac{P - (MC_{Up} + MC_{Down})}{P - S} = \frac{(MC_{Up} + MC_{Down}) - s}{P - S} = 0{,}4$. The expected number of units underage in that situation is the expected distance from the order quantity given that the outcome is on the right hand side, i.e. half the distance from the order quantity to the upper boundary: $E[Q_u] = \frac{1}{2}\left(H - \left(L + \frac{P - (MC_{Up} + MC_{Down})}{P - S}(H - L)\right)\right)$. Here it is the distance from 17 000 to the midpoint between 17 000 and 25 000, which is 21 000 making the expected number underage 4000. The expected underage cost becomes:

$\Pi_u^- = \frac{(MC_{Up}+MC_{Down})-s}{P-S} \times C_u \times E[Q_u] = 0{,}4 \times 600 \times 4000 = 960000$. Similarly for the overage case we have: $E[Q_o] = \frac{1}{2}\left(\left(L + \frac{P-(MC_{Up}+MC_{Down})}{P-S}(H-L)\right) - L\right) = 6000$, $\Pi_o^- = \frac{P-(MC_{Up}+MC_{Down})}{P-S} \times C_o \times E[Q_o] = 0{,}6 \times 400 \times 6000 = 1440000$. Expected profit then is $E[\Pi] = \Pi_{Q=\bar{Q}} - E[Q_u] - E[Q_o] = 9000000 - 960000 - 1440000 = 6600000$.

c. The components of value chain profit were presented above. They are:
Profit as if no demand uncertainty:

$$\Pi_{Q=\bar{Q}} = \left(L + \frac{1}{2}(H-L)\right)\left(P - \left(MC_{Up} + MC_{Down}\right)\right) = 9000000$$

Less expected underage and overage costs:

$$\Pi_u^- = \frac{\left(MC_{Up} + MC_{Down}\right) - s}{P - S} \times \left(P - \left(MC_{Up} + MC_{Down}\right)\right) \times \frac{1}{2}\left(H - \left(L + \frac{P - \left(MC_{Up} + MC_{Down}\right)}{P - S}(H-L)\right)\right) = 960000$$

$$\Pi_o^- = \frac{P - \left(MC_{Up} + MC_{Down}\right)}{P - S} \times \left(MC_{Up} + MC_{Down} - S\right) \times \frac{1}{2}\left(\left(L + \frac{P - \left(MC_{Up} + MC_{Down}\right)}{P - S}(H-L)\right) - L\right) = 1440000$$

2.2. *Independent companies and alternative contracts*

a. With a wholesale price W, the marginal cost of Down becomes $MC_{Down} + W$. Using the same optimality condition as in 2.1, the order quantity then is: $Q = L + \frac{P-(MC_{Down}+W)}{P-S}(H-L)$. This is the demand facing Up. The profit for Up can thus be expressed as: $\Pi_{Up} = \left(L + \frac{P-(MC_{Down}+W)}{P-S}(H-L)\right)\left(W - MC_{Up}\right)$. The first order condition for profit maximization then is: $\frac{\partial \Pi_{Up}}{\partial W} = -\frac{H-L}{P-S}\left(W - MC_{Up}\right) + L + \frac{P-(MC_{Down}+W)}{P-S}(H-L) = 0 \Rightarrow -\left(W - MC_{Up}\right) + \frac{P-S}{H-L}L + P - (MC_{Down} + W) = 0 \Rightarrow W = \frac{1}{2}\left(MC_{Up} + \frac{P-S}{H-L}L + P - MC_{Down}\right) = \frac{1}{2}\left(200 + \frac{1100-100}{25000-5000}5000 + 1100 - 300\right) = 625$. For Down the margin, and the overage cost becomes $C_u = P - MC_{Down} - W = 1100 - 300 - 625 = 175$. The overage cost is the marginal cost less salvage value: $C_o = MC_{Down} + W - S = 825$. The optimality condition becomes $P(D < Q) = \frac{175}{175+825} = 0{,}175$. The order quantity is $Q = 8500$. The profit with no demand uncertainty becomes $175 \times 15000 = 2625000$. Expected underage costs are $(1 - 0{,}175) \times 175 \times \frac{1}{2}(25000 - 8500) = 1191093{,}75$. Expected overage costs are: $0{,}175 \times 825 \times \frac{1}{2}(8500 - 5000) = 252656{,}25$. Thus, Down's expected profits are: $2625000 - 1191093{,}75 - 252656{,}25 = 1181250$. The profit for Up becomes $8500 \times (625 - 200) = 3612500$. Total value chain profit is $4793750{,}75$, or 27,4% less than the maximum.

b. *Similarity with double marginalization*

In terms of mathematics and effects on value chain efficiency, the similarity to classic double marginalization is obvious. The Newsboy optimality condition becomes a downward sloping demand curve facing Up that marginalizes in the same manner in both situations. Value chain profit is much reduced compared to the maximum due to firms maximizing their own profit rather than value chain profit.

In terms of economics the behavioral model of Down is clearly different in the two cases. In the classical double marginalization situation the Down firm faces a downward sloping demand curve. Here Down faces a fixed price, but adjusts quantity as to optimize profit under uncertainty. However, the behavior that results is similar.

c. *Alternative contracts*

Two-part linear: Given risk neutrality optimal value chain profits can be achieved if Down faces the marginal costs of the value chain. Hence the wholesale price should be 200. The order quantity will then be the value chain profit maximizing 17000. Up would have no margin but could demand a fixed amount from Down. Down will face all risk implying that this contract form might be inefficient in terms of risk sharing if firms are risk averse.

Profit sharing contract: This means for instance that the wholesale price is set at marginal costs and there is an agreement that Up and Down will share profits. This might be more efficient in terms of risk sharing than the two-part linear contract. Within the setting here it will also maximize expected value chain profit. More generally profit sharing reduces incentives for cost containment compared to e.g. the two-part linear contract since the other firm will carry a proportion of lost profits.

Buy-back contract: Up can buy back unsold items from Down at a given price. This mechanism can be integrated into e.g. the two contracts already mentioned, but in our setting the usual form would be a contract specifying a wholesale price and a buy back price only. The buy-back term increases optimal order size for given wholesale price. Thus a wholesale price above marginal costs combined with a suitable buy back can lead to supply chain efficiency. And the mechanism implies risk sharing without the mentioned drawbacks of a profit sharing contract.

2.3. Asymmetric information

a. Asymmetric information makes contracting more complicated. The players will have incentives to maintain beliefs of higher than actual marginal costs as well as fixed costs. In the typical case this will lead to something analogous to a prisoners dilemma situation, but with opposite effects of honesty. While simultaneous honesty can lead to a value chain maximizing contract and thus the highest possible profit to divide between them, both have incentives to be dishonest about the costs trying to get a larger share of value chain profit. They end up sharing less than they could.

b. An efficient two-part linear contract should contain a transfer price close to marginal cost (but no less than marginal costs and in this case probably slightly above). Thus it is typically the fixed lump sum that is used to signal demand expectations. In this case Down can offer a lump sum that is high enough to be unprofitable unless demand is high. This becomes a credible signal since only a downstream firm expecting high demand would offer it. And since Down's profit is dependent on Up making a sufficient investment to cover demand, it might well be an optimal approach for Down.

3. Auctions (30 %)

3.1. Revenue equivalence

Revenue equivalence means that expected revenue from selling an object in auction is the same regardless of which of the following forms are used: English (open ascending price auction), Sealed second price, Sealed First price or Dutch auction (open descending price auction). It builds on the private value model with risk neutral bidders and a risk neutral seller. The private value model implies that each bidder has a private valuation, but valuations are drawn from a commonly known distribution.

3.2. Risk averse bidders

In a first price auction, i.e. Dutch and Sealed first price, risk aversion leads bidders to adjust bids upwards in order not to forgo the potential profit related to winning the auction. Within the private model the behavior in the other two mentioned auction forms are not influenced by pure risk aversion among bidders. This means that expected income is higher with first price auctions if bidders are risk averse.

3.3. Risk averse bidders uncertain about value

Risk averse bidders uncertain about value will be fearful of the winner's curse, i.e. paying more than an actual value. This fear can be moderated in a setting where some information about other bidders' valuation is revealed. Thus a process of price discovery is generally expected to lead to higher bids. Of the forms mentioned, only the English auction provides this kind of effect. In this case the choice of auction form then depends on the relative strength of risk aversion related to forgoing a potential profit and related to the winner's curse.