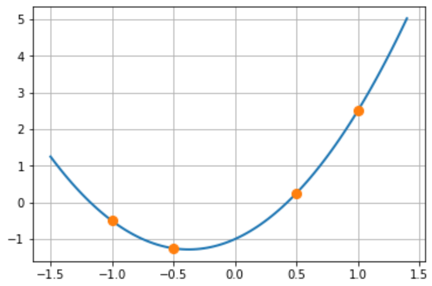
Oppgave 1

In [2]:
```
def cardinal(xdata, x):
    """
    cardinal(xdata, x):
    In: xdata, array with the nodes x_i.
        x, array or a scalar of values in which the cardinal functions are evaluated.
    Return: l: a list of arrays of the cardinal functions evaluated in x.
    """
    n = len(xdata)              # Number of evaluation points x
    l = []
    for i in range(n):          # Loop over the cardinal functions
        li = ones(len(x))
        for j in range(n):      # Loop to make the product for l_i
            if i is not j:
                li = li*(x-xdata[j])/(xdata[i]-xdata[j])
        l.append(li)            # Append the array to the list
    return l

def lagrange(ydata, l):
    """
    lagrange(ydata, l):
    In: ydata, array of the y-values of the interpolation points.
         l, a list of the cardinal functions, given by cardinal(xdata, x)
    Return: An array with the interpolation polynomial.
    """
    poly = 0
    for i in range(len(ydata)):
        poly = poly + ydata[i]*l[i]
    return poly
```

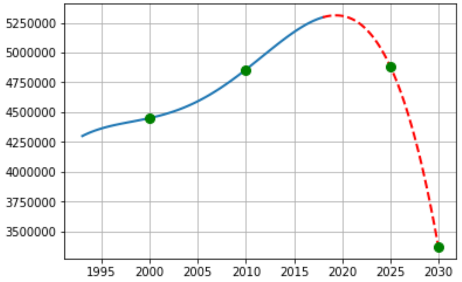
In [3]:
```
x_data = [-1, -1/2, 1/2, 1]
y_data = [-1/2, -5/4, 1/4, 5/2]

x = linspace(-1.5, 1.4, 101)

plot(x, 2*x**2 + (3/2)*x - 1)
plot(x_data, y_data, 'o');

#l = cardinal(x_data, x)
#p = lagrange(y_data, l)

#plot(x, p);
```



Oppgave 2

In [4]:
```
years = [1993, 1998, 203, 2008, 2013, 2018]
population = [4299167, 4417599, 4552252, 4737171, 5051275, 5295619]

# Interpolation
x1 = linspace(1993, 2018, 101)
l1 = cardinal(years, x1)
p1 = lagrange(population, l1)

# Extrapolation
x2 = linspace(2018, 2030, 101)
l2 = cardinal(years, x2)
p2 = lagrange(population, l2)

# Test points
x3 = array([2000, 2010, 2025, 2030])
l3 = cardinal(years, x3)
p3 = lagrange(population, l3)

points = list(zip(x3, p3))

# Print plots
plot(x1, p1)
plot(x2, p2, 'r--')
plot(x3, p3, 'og')

# Print points
for point in points:
    print('Year = {:.0f}\t Population = {:.0f}'.format(*point));
```

```
Year = 2000      Population = 4449960
Year = 2010      Population = 4857131
Year = 2025      Population = 4884810
Year = 2030      Population = 3369091
```



Kommentar

Som vi ser, vil polynominterpolasjonen gi uønskede resultater utenfor interpolasjonsdomenet. Interpolasjon er egnet til approksimasjon av funksjoner innenfor interpolasjonsdomenet, ikke utenfor (såkalt ekstrapolasjon).

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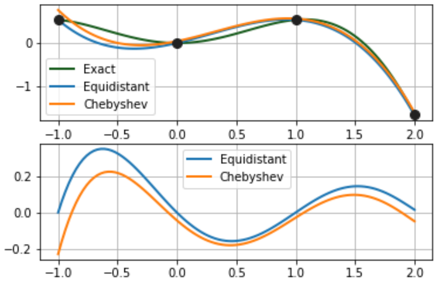
Oppgave 3

In [5]:
```
x_data = [-1, 0, 1, 2]
y_data = [0.54, 0, 0.54, -1.66]
x = linspace(-1, 2, 101)

def f(x):
    return [x**2 * cos(x) for x in x]
def g(x):
    return -0.64*x**3 + 0.54*x**2 + 0.64*x
def h(x):
    return -0.66*x**3 + 0.625*x**2 + 0.56*x + 0.0432

# Print interpolation error
print("Max error for equidistant nodes is {:.2e}".format(max(abs(f(x)-g(x)))))
print("Max error for chebyshev nodes is {:.2e}".format(max(abs(f(x)-h(x)))))

# Plot functions and interpolation points
subplot(2, 1, 1)
plot(x, f(x), '#1B5E20')
plot(x, g(x))
plot(x, h(x))
plot(x_data, y_data, 'o', color="#212121")
legend(['Exact','Equidistant','Chebyshev'])

# Plot interpolation error
subplot(2, 1, 2)
plot(x, f(x) - g(x))
plot(x, f(x) - h(x))
legend(['Equidistant','Chebyshev']);
```

```
Max error for equidistant nodes is 3.49e-01
Max error for chebyshev nodes is 2.28e-01
```



In [ ]:

In [ ]:

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1)

x_i	-1	$-1/2$	$1/2$	1
$f(x_i)$	$-1/2$	$-5/4$	$1/4$	$5/2$

a) Finner kardinalfunksjonene $l_i(x)$.

$$l_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x-x_j}{x_i-x_j}$$

$$l_0(x) = \frac{(x+1/2)(x-1/2)(x-1)}{(-1+1/2)(-1-1/2)(-1-1)} = \frac{-2}{3}\left(x^3 - x^2 - \frac{1}{4}x + \frac{1}{4}\right) = \frac{-2}{3}x^3 + \frac{2}{3}x^2 + \frac{1}{6}x - \frac{1}{6}$$

$$l_1(x) = \frac{(x+1)(x-1/2)(x-1)}{(-1/2+1)(-1/2-1/2)(-1/2-1)} = \frac{4}{3}\left(x^3 - \frac{1}{2}x^2 - x + \frac{1}{2}\right) = \frac{4}{3}x^3 - \frac{2}{3}x^2 - \frac{4}{3}x + \frac{2}{3}$$

$$l_2(x) = \frac{(x+1)(x+1/2)(x-1)}{(1/2+1)(1/2+1/2)(1/2-1)} = \frac{-4}{3}\left(x^3 + \frac{1}{2}x^2 - x - \frac{1}{2}\right) = \frac{-4}{3}x^3 - \frac{2}{3}x^2 + \frac{4}{3}x + \frac{2}{3}$$

$$l_3(x) = \frac{(x+1)(x+1/2)(x-1/2)}{(1+1)(1+1/2)(1-1/2)} = \frac{2}{3}\left(x^3 + x^2 - \frac{1}{4}x - \frac{1}{4}\right) = \frac{2}{3}x^3 + \frac{2}{3}x^2 - \frac{1}{6}x - \frac{1}{6}$$

$$
\begin{aligned}
p(x) &= -\frac{1}{2}l_0(x) - \frac{5}{4}l_1(x) + \frac{1}{4}l_2(x) + \frac{5}{2}l_3(x) \\
&= \frac{1}{3}x^3 - \frac{1}{3}x^2 - \frac{1}{12}x + \frac{1}{12} \\
&\quad - \frac{5}{3}x^3 + \frac{5}{6}x^2 + \frac{5}{3}x - \frac{5}{6} \\
&\quad - \frac{1}{3}x^3 - \frac{1}{6}x^2 + \frac{1}{3}x + \frac{1}{6} \\
&\quad + \frac{5}{3}x^3 + \frac{5}{3}x^2 - \frac{5}{12}x - \frac{5}{12} \\
&= 0 + 2x^2 + \frac{3}{2}x - 1
\end{aligned}
$$

Innsatt for $x = 0$ har vi:

$$f(0) = 2(0)^2 + \frac{3}{2}(0) - 1 = -1$$

③ $f(x) = x^2 \cos x$

a) Equidistant nodes

x	-1	0	1	2
$f(x)$	0,54	0	0,54	-1,66

$$l_0(x) = \frac{(x-0)(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} = \frac{-1}{6}\cdot x\cdot(x^2-3x+2) = \frac{-1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{3}x$$

$$l_1(x) = \frac{(x+1)(x-1)(x-2)}{(0+1)(0-1)(0-2)} = \frac{1}{2}\cdot(x^2-1)(x-2) \quad \cancel{= \frac{1}{2}x^3 - x^2 - \frac{1}{2}x + 1}$$

$$l_2(x) = \frac{(x+1)(x-0)(x-2)}{(1+1)(1-0)(1-2)} = \frac{-1}{2}\cdot x\cdot(x^2-x-2) = \frac{-1}{2}x^3 + \frac{1}{2}x^2 + x$$

$$l_3(x) = \frac{(x+1)(x-0)(x-1)}{(2+1)(2-0)(2-1)} = \frac{1}{6}\cdot x\cdot(x^2-1) = \frac{1}{6}x^3 - \frac{1}{6}x$$

$$p(x) = 0{,}54\, l_0(x) + \cancel{0\cdot l_1(x)} + 0{,}54\, l_2(x) - 1{,}66\, l_3(x)$$

$$= -\frac{0{,}54}{6}x^3 + \frac{0{,}54}{2}x^2 - \frac{0{,}54}{3}x$$
$$+ 0$$
$$- \frac{0{,}54}{2}x^3 + \frac{0{,}54}{2}x^2 + 0{,}54x$$
$$- \frac{1{,}66}{6}x^3 \qquad + \frac{1{,}66}{6}x$$
$$= -0{,}64x^3 + 0{,}54x^2 + 0{,}64x$$

Chebyshev nodes:

$$\tilde{x}_i = \cos\left(\frac{(2i+1)\pi}{2(n+1)}\right) \text{ for } i = 0, 1 \ldots, n$$

$$x_i = \frac{b-a}{2}\tilde{x}_i + \frac{b+a}{2}$$

x	1,89	1,07	-0,0740	-0,886
$f(x)$	-1,12	0,550	0,00546	0,497

$$l_0(x) = \frac{(x-1{,}07)(x+0{,}0740)(x+0{,}886)}{(1{,}89-1{,}07)(1{,}89+0{,}0740)(1{,}89+0{,}886)} = 0{,}224x^3 - 0{,}0246x^2 - 0{,}215x - 0{,}0157$$

$$l_1(x) = \frac{(x-1{,}89)(x+0{,}0740)(x+0{,}886)}{(1{,}07-1{,}89)(1{,}07+0{,}0740)(1{,}07+0{,}886)} = -0{,}545x^3 + 0{,}507x^2 + 0{,}953x + 0{,}0675$$

$$l_2(x) = \frac{(x-1{,}89)(x-1{,}07)(x+0{,}886)}{(-0{,}0740-1{,}89)(-0{,}0740-1{,}07)(-0{,}0740+0{,}886)} = 0{,}546x^3 - 1{,}13x^2 - 0{,}329x + 0{,}982$$

$$l_3(x) = \frac{(x-1{,}89)(x-1{,}07)(x+0{,}0740)}{(-0{,}886-1{,}89)(-0{,}886-1{,}07)(-0{,}886+0{,}0740)} = -0{,}227x^3 + 0{,}654x^2 - 0{,}409x - 0{,}0340$$

$$\Rightarrow p(x) = -1{,}12\cdot l_0(x) + 0{,}550\cdot l_1(x) + 0{,}00546\cdot l_2(x) + 0{,}497\cdot l_3(x)$$

$$\approx -0{,}660x^3 + 0{,}625x^2 + 0{,}560x + 0{,}0432$$

Equidistant nodes:

b) $|e_n(x)| \leq \frac{h^{n+1}}{4(n+1)} \cdot \max_{x\in[a,b]} |f^{\{n+1\}}(x)|$ for $h = \frac{b-a}{n}$ | Setter $n=3$ og $a=-1$, $b=2$

$$\leq \frac{1^4}{16} \cdot \max\left\{(-1)^2\left(x^2\cos(x) + 2\cdot(4)x\sin(x) - 4\cdot(4-1)\cos(x)\right)\right\}$$

$$\leq \frac{1}{16} \cdot \max\left\{x^2\cos x + 8x\sin x - 12\cos x\right\}$$

$$\leq \frac{1}{16} \cdot 17{,}878 \approx \underline{1{,}117}$$

Chebyshev nodes:

$|e_n(x)| \leq \frac{(b-a)^{n+1}}{2^{2n+1}(n+1)!} \cdot \max_{x\in[a,b]} |f^{\{n+1\}}(x)|$ | setter $n=3$ og $a=-1$, $b=2$

$$\leq \frac{3^{(3+1)}}{2^{2\cdot3+1}\cdot(3+1)!} \cdot 17{,}878$$

$$\approx \underline{\underline{0{,}4714}}$$