Understanding Geometry Wordy Problem under Cartesian Coordinate System using Compositional Constrain Semantic

Juncheng Pan
Department of Computer Science
University of Central Florida, Orlando, FL 32716
email: patrician.lighten@knights.ucf.edu
UCF student ID: 3951426

Abstract

The main contribution of the project is that we propose a computational framework for computer to understand geometry wordy problem under Cartesian Coordinate system. We formulate such geometry problem understanding task as a task of modeling the inter-constrains among different geometry entities and the inner-constrain that lies within innate compositional structure of geometry concept itself. In order to minimize the semantic gap lies in the natural language description that prevent the statistic learning method from directly capturing the correct geometric constraints, we proposed Two-phase learning framework, Local Match and Global Merge. I test my idea on a new geometry problem Data Set based on Cartesian Coordinate system, showing a promising result.
To our best knowledge, this project is the first effort to model the geometry wordy understanding problem under the Cartesian Coordinate system.

1 Introduction

Compositional Semantic are prevalent in natural language description text. let's take a forged but realistic example, "the car with 2 vents consumes more gas than the car with 1 vent does". In the toy example, Vent is one of component of Car so it is composed by car, the statement "will the need more gas" is the relationship between cars that have different configurations. Some Domain free Compositional Semantic parsing has been proposed by many researchers (Socher et al., 2013), (Liang et al., 2013) however, apply such general model to domain specific task either needing a large training effort or end with a low accuracy performance.

In this project, we focus on understanding the Geometry problem described in natural language without the diagram as input. This is different with the current art of state method proposed by (Seo et al., 2014), (Seo et al., 2015) which solve the geometry problem combining the natural language and the diagram. However, pure natural language as input could be more challenge depend on the data set. we try to understand the pure natural language input Geometry problem, and such kind of problem arise a new dimension of NLP challenge.
Firstly lets take a look at the work of (Seo et al., 2015) which represent the state of art of solving the geometry problem with diagram. The basic idea of (Seo et al., 2015) to solve the problem is, firstly the algorithm create a hyper-graph that contains all of the entity in the diagram and the text, initially every entity is a node without edge among them. Next, every time when computer encounter a geometry entity in the NLP description, match the entity to one of entity of in the diagram and then through the diagram map the entity to the entity in the hypergraph. After that, incremental embed the property of the entity or the relationship between entity that described by natural language description to the hyper graph. Finally the hypergraph will include all the relationships among entities as the edges linking corresponding entity nodes, and encode properties of entities as nodes pointing to the corresponding entities.

However one big feature also the limitation of the work (Seo et al., 2014) and (Seo et al., 2015) is that they only model the inter relationship between different geometry entity not the inner constrain of the geometry entity itself. The inner constrain of geometry entity here is refer to the geometry's property described by Cartesian Coordinate system. For example, the line's slope, the line's equation and so on.

We argue that inner constrains of geometry entity need the compositional structure to represent. The argument is based on the fact that all the geometry entity is represent by vectors or points in Cartesian Coordinate with some constraints on the points, for example, a point can be represented as a vector$(x, y)^T$ in 2D space, while a line in 2D space can be represented as a set of vector (xi,yi) that satisfies certain constrain,like $(xi, yi) \times (a, b)^T$=C, where a,b,c $\in$ R. Based on above understanding, we propose new frame work that use the compositional semantic for Geometry problem understanding and representation. Because the Geometry entity and their relationship in the Cartesian Coordinate System is in fact a set of constraints on the vectors, we name our framework the C$ompositional\ Constrain\ Semantic$. So the task of understand the wordy geometry problem is transfer to a task of constructing a C$ompositional\ Constrain\ Semantic\ Graph$ (CCSG) for the problem. The figure 1 shows an example of CCSG representation of a geometry wordy problem.

However as we will see, the NLP described problem has many coreference relationship which form a semantic gap which could not solved by statistic learning method by one pass. In order to tackle the problem, I propose a two phase learning strategy,Local Match and Global Merge, to minimize the semantic gap.

2 Problem Formulation

2.1 Two Phase Learning for CCSG

We formulate the geometry problem Parsing(understanding) as the construction of CCSG for the problem, and it is easy to show that once the CCSG is determined the solving will be determin-

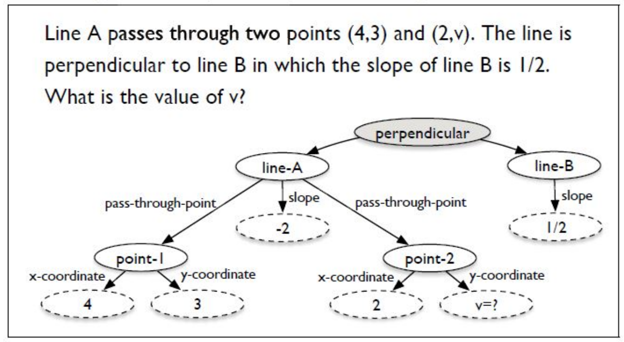

Figure 1: Example of Compositional Constrain Semantic Graph for Geometry Problem .

stic, the proof is not in the scope of the report.

Before Formally formulate our problem, let's run an example in the figure1 to get an understanding of our two phase learning for CCSG construction, Local Match and Global Merge.

"Line A passes through two points (4,3) and (2,v). The line A is perpendicular to line B in which the slope of line B is 1/2. What is the value of v?"

The Local Match of the problem can be presented as:

$$\begin{cases} (LineA, point(4,3)) \quad (LineA, Point(2, V)) \\ (Perpendicular, LineA1, LineB) \quad (LineB1, slope) \\ (v, ?) \quad (slope, 1/2) \end{cases}$$

The idea behind the local match is two-fold:

(1) Even a geometry concept is semantically composed by or matches with another, from statistical point of view they should not match.

(2) The two concepts appear in different place of the description that are semantic same object should be consider as different object during the local match stage.

The motivation for this measurement is to tackle the semantic gap in global reference.

For example, the line A and second the line A are semantically same object, however, the points(4,3) and (2,v) are more statistically related to the first Line A, rather than to the second Line a, and symmetrically, the perpendicular is more statistical related to the second line A rather than the first line A.So the semantically information in the NLP description will served as a noise for statistic learning. So in order to make the statistic learning accurate and easy, one way is to decompose the problem into the a set of sub problems where the semantic noise is minimize, and we simply adopt

this idea.
Because we decompose the problem into set of sub local match problem, after solving them, we need to merge the outcomes from each of the sub problem to form a global inference result, and this is what the meaning of the next stage, Global Merge.
So the Global Merge is to form a link between semantically same object merge their outcome, remove redundance and finally form a global inference.The Global Merge for above example is hence, to form a link between (line A1, line A) and a link between (line B, line B1). so the final graph should be look like the figure1, and its annotation should be like following:

$$\begin{cases} (LineA, point(4,3)) \quad (LineA, Point(2,V)) \\ (Perpendicular, LineA, LineB) \quad (LineB, slope) \\ (slope, 1/2)(v?) \end{cases}$$

In the following two subsection, I give a formulation for the Local Match and the Global Merge(also call Global Inference) respectively.

2.2 Formulation of Local Match

2.2.1 Some Definitions

In general, there are two main type of Local Match, namely, C$ompose$ and M$atch$ respectively. The C$ompose$ is a Knowledge Based Guided Match, for each of geometry concept, the classes of other geometry elements it can compose depend on its attribution. For example, we define the line's attribution as $\{on\ line\ points,\ line's\ slope,\ lines'\ equation,$ $themiddle\ points\ of\ the\ line\}$. We refer all the geometry concepts that are used to define the attribution of the target concept as **Attrbution Set** of the target concept.
Based on the defined attribution of the line, the line could only match with the geometry entity that belong to the classes of its Attribution Set. And in the report, we will call the two geometry entity is **Composable** if one entity is in another entities Attribution Set.
So, different with the (Seo et al., 2014) and (Seo et al., 2015), the our proposed Compose match is a knowledge based match, which is the basis of the compositional semantic.
The second type match is to match concept with its possible value property, for example, the the slope is 4, the equation is "Ax+BY=C".
For the compose match, different concepts may have different features for statistic learning, and different concept may have different match structure. for example, the geometry relationship distance will compose other two concept, this is a three elements wise compose, while for the non relation geometry concept, like circle and line,their match is two element wise match.
However, regardless the type of the local match, they have the same optimization model.The model is presented in the following section.

2.2.2 Optimization Model for Local Match

we use logistic regression model to learn the parameters for different local match task. The object Function is as follow:

$$\begin{cases} \text{Min: J(theta) =} \\ 1/m \sum [y^i log(h_\theta(X^i1, X^i2)) + \\ (1 - y^i log(1 - h_\theta(X^i1, X^i2))] + \gamma/2m \sum \theta^2 \\ \text{s.t. compose(} X^i1, X^i2 \text{)} \end{cases}$$

where the $h_\theta(X^i1, X^i2))isdefinedas$:
$P(Y=1|x_1, x_2, \theta) = \frac{1}{1+(1+e^{-\theta' F_{cps}(x_1,x_2)})}$ (1)
And the compose(x_1, x_2) is defined as:

$$\begin{cases} 1, & ifconceptx_1 andX_2 iscomposable; \\ 0, & otherwise; \end{cases}$$

In the equation (1) , the $F_{cps}(x_1, x_2)$ is a feature function for concept X_1, and concept x_2. As I have mentioned, different match will need different feature as input of the logistic function, in the next section, I will list some representative features for the match.

2.3 Formulation of Global Match

The key part of Global match(merge) is two find correspond concept in different sub match problems. The Global Merge can be defined as follow:

$$\begin{cases} \underset{x1,x2}{argmax}\ h_\theta(x1, x2); \\ s.t. \\ h_{relation}(x1, y1, ri) < 0.5; \\ ri \in \{geometry\ relationship\} \end{cases}$$

The constrain $ri \in \{geometry\ relationship\}$ define if two concepts in different sub match problems can be merged with each other. This is a rule based threshold, it says if two concept is linked by a geometry property then, they would not be same object, for example, if the there are two points is linked by the geometry relation distance, then these two points should be different object, it also means the algorithm should not merge them.

3 The Feature for Match

In our dataset, there are two type of Local compose in terms of number of elements involved in one match, namely two elements wise compose and three elements wise compose.
The representative two element wise compose is the compose of "line", the feature for it is as follow.

The feature is computed for match x1 with x2, where one of x1 and x2 is Line concept

Two element wise Compose for "Line" concept
if same sentence
if split By Verb
if x1 is of x2
if element is of another concept
if x1 before Of
if x2 before Of
distance in chunker
distance in concept
If there is same type concept with X1 between
If there is same type concept with X2 between

The example features for three element wise compose is listed as below.
the feature is for compose the distance concept d with x1 and x2.

Three element wise Compose for "distance"
if same sentence
if split By Verb
if split By To
if x1 and x2 are after Between
if x1 and x2 Both After Distance concept d
distance between x1 and x2 in chunker
distance between x1 and x2 in concept
If there is same type concept with X1 between
If there is same type concept with X2 between
distance between x1 and d
distance between x2 and d

4 Experiments

Our experiment is based on 186 real world geometry wordy problems collected by my collaborator, most of them are under Cartesian coordinator, and about 120 problems of them are in our scope.
We test our idea on 60 problems, where there are in average about 20 compose samples per every problem, 20 merge samples per problem, and in total, we collect 200 samples for training the line concept compose, 200 for circle concept compose, 89 for distance concept compose, 69 for parallel concept compose, and total 200 samples for global merge. note that the compose of geometry relation concept is three element wise compose.
And the result shows that for two element compose, line and circle in our experiment, the accuracy achieves around 89% in average, while for the two element merge, the accuracy achieve around 95%, while for the three element wise compose, the result is not very good, for parallel compose the accuracy is around 70%, for the distance, the accuracy is 83%, the reason is that the data for training the three element wise compose is sparse and also the feature is not discriminative enough for three element wise compose. Because the three element wise has a more flexible structure than two element wise's which need to be captured by more complex features.

During error checking, I also find that many error occur in the local match is because of that the feature is not discriminative enough, feature for false positive match is same with true positive.

5 Related Work

The most related work is the compositional semantic (Liang et al., 2013) and the work on geometry problem with diagram input (Seo et al., 2014), (Seo et al., 2015). For the compositional semantic proposed by [2] is a general one which not only need huge amount of data to train the system but also could not represent complex compositional structure, such as geometry problem, in a correct and light way.Whereas for the existing representative work (Seo et al., 2015) on geometry problem, they don't model the geometry entity in a compositional way which make it could not represent geometry problem under the Cartesian Coordinate System. In this project, by analyzing the weakness of related work on geometry problem under Cartesian Coordinate system we propose new scheme that not only efficiently represent geometry problem under Cartesian system but also make it possible the understanding complex geometry problem without diagram input.

6 Conclusion

In this project, we propose a new dimension for computer to automatically parse geometry problem: semantically parse the geometry problem under Cartesian Coordinate System without diagram input. It is first work that try to automatically solve the geometry problem under the Cartesian Coordinate System. We argue that the compositional semantic structure is the basis for such kind of task.Based on this understanding We propose a frame work that use the compositional semantic structure to represent as well as to understand the geometry problem. The result is a Compositional Constrain Semantic Graph representation. In order construct correct Semantic Graph representation for geometry problem described in natural language, I propose two phase learning to minimize the semantic gap that is innate in the natural language description. I test our idea on a new data set proposed by my collaborate, and it shows a promising result.

7 Future Work

We find that most of the error is result from the non discriminative feature. Currently we design the feature in rule based way, in the future we will try to combine features that learned by unsupervised method with the rule based feature hoping it can make the feature more discriminative.

References

Percy Liang, Michael I Jordan, and Dan Klein. 2013. Learning dependency-based compositional semantics. *Computational Linguistics*, 39(2):389–446.

Min Joon Seo, Hannaneh Hajishirzi, Ali Farhadi, and Oren Etzioni. 2014. Diagram understanding in geometry questions. In *AAAI*, pages 2831–2838.

Minjoon Seo, Hannaneh Hajishirzi, Ali Farhadi, Oren Etzioni, and Clint Malcolm. 2015. Solving geometry problems: Combining text and diagram interpretation. *Proceedings of EMNLP, Lisbon, Portugal*.

Richard Socher, John Bauer, Christopher D Manning, and Andrew Y Ng. 2013. Parsing with compositional vector grammars. In *ACL (1)*, pages 455–465.