**Problem a1:** Solve the system: x + y = 1 and x + 3y = 9

1. *Please re-read the question carefully.*
2. *(Summary) This problem is a straight-forward math problem that is asking you to use your knowledge of the solving a system of linear equations to find the solution.*
3. *(Meta Scaffolding) What does the solution to a system of linear equations mean?*
4. *(Scaffolding) A solution to the linear system is found when you have values of x and y that make both of the equations true.*
5. *(Scaffolding) You need to find the point (values of x and y) that make both of the equations true.*
6. *(Meta Scaffolding) What methods are there to solve this equation?*
7. *(Scaffolding) Because the coefficients for x in both equations are 1, we can use the substitution method to find the solution.*
8. *(Meta Scaffolding) What is the substitution method?*
9. *(Scaffolding) The substitution method is when you solve for one variable in one of the equations and substitute in the answer you got for that variable into the other equation.*
10. *(Meta Scaffolding) Which variable should we solve for first?*
11. *(Scaffolding) Let’s solve for x in the first equation*
12. *(Meta Scaffolding) How can we solve for x?*
13. *(Scaffolding) Move everything but x to the other side of the equation.*
14. *(Scaffolding) So x + y = 1 becomes x = 1 - y*
15. *(Meta Scaffolding) How can we substitute this value in?*
16. *(Scaffolding) The second equation becomes 1 - y + 3y = 9*
17. *(Meta Scaffolding) How can we simplify this equation to get a solution for y?*
18. *(Scaffolding) Combine like terms.*
19. *(Scaffolding) So it becomes: 2y = 8 which means that y = 4*
20. *(Meta Scaffolding) Now that we have y, how can we use this information to obtain a value for x?*
21. *(Scaffolding) We can substitute in 4 for y in the first equation and solve for x*
22. *(Scaffolding) So the first equation becomes x + 4 = 1*
23. *(Scaffolding) Simplify to get x = -3*
24. *(Meta Scaffolding) Now that you have both x and y, what is your final answer?*
25. *(Scaffolding) Your answer is (-3, 4)*

**Problem a2:** For what value of ***a*** would the solution of the system of equations, ax - 5y = 8 and -2ax + 2y = 0. be (-⅓, -2) ? Check your answer.

1. *Please re-read the question carefully.*
2. *(Summary) This problem is a backwards reasoning problem that asks you to use your knowledge of a system of linear equations to find the variable that makes the system true.*
3. *(Meta Scaffolding) What does a solution to a linear system mean?*
4. *(Scaffolding) A solution to the linear system is found when you have values of x and y that make both of the equations true.*
5. *(Scaffolding) You have the point (x, y) that makes both of the equations true*
6. *(Meta Scaffolding) What does the a in the equations mean?*
7. *(Scaffolding) This a has a single value that makes the equations both true with the given solution*
8. *(Meta Scaffolding) How would we solve this problem?*
9. *(Scaffolding) We can use the values for x and y given to us to plug those into an equation and solve for a*
10. *(Meta Scaffolding) Which equation should we use first?*
11. *(Scaffolding) Let’s use the first equation. we can plug (-⅓, -2) in for x and y*
12. *(Scaffolding) The first equation becomes (-⅓ a ) - 5 (-2) = 8*
13. *(Meta Scaffolding) How can we simplify this to solve for a?*
14. *(Scaffolding) This becomes (-⅓ a ) + 10 = 8 which simplifies to -⅓ a = -2*
15. *(Meta Scaffolding) What is the value for a?*
16. *(Scaffolding) a = 6*
17. *(Meta Scaffolding) How can we check our answer?*
18. *(Scaffolding) Plug the values for a , x, and y into the second equation to see if it makes the equation true.*
19. *(Scaffolding) The second equation becomes -2 (6) (-⅓ ) + 2 (-2) = 0*
20. *(Meta Scaffolding) Simplify this equation to see if both sides are equal*
21. *(Scaffolding) We get 4 + -4 = 0, which is true*
22. *(Meta Scaffolding) What does this mean?*
23. *(Scaffolding) It means your value for a is correct. a = 6*

**Problem a3:** The equations 5x + 2y = 48 and 3x + 2y = 32 represent the money collected from school concert tickets sales during two class periods in dollars. If x represents the cost for each adult ticket and y represents the cost for each student ticket, what is the cost for each adult ticket?

1. *Please re-read the question carefully.*
2. *(Summary) This problem is a hybrid math problem that combines math and common language. It is asking for you to use your knowledge about how to solve linear equations to find a solution for the system.*
3. *(Meta Scaffolding): What kind of problem is this?*
4. *(Scaffolding) This is a system of linear equations problem.*
5. *(Meta Scaffolding) What is the problem asking for?*
6. *(Scaffolding) It is asking for the solution of the linear system.*
7. *(Meta Scaffolding) What does the solution to a system of linear equations mean?*
8. *(Scaffolding) A solution to the linear system is found when you have values of x and y that make both of the equations true.*
9. *(Scaffolding) You need to find the point (values of x and y) that make both of the equations true.*
10. *(Meta Scaffolding) What methods are there to solve this equation?*
11. *(Scaffolding) Because the coefficients for y are both 2, we can use the method of elimination to solve.*
12. *(Meta Scaffolding) What is the elimination method?*
13. *(Scaffolding) The substitution method is when one of the variables has coefficients that are opposite so that you can add the two equations together and eliminate one of the variables to solve for the other.*
14. *(Meta Scaffolding) How can we use the method of elimination here?*
15. *(Scaffolding) Both of the y coefficients are 2. You can multiply one whole equation by -1 and then add the two equations to solve*
16. *(Meta Scaffolding) Make sure you multiply the entire equation by negative one.*
17. *(Scaffolding) So the first equation becomes -5x - 2y = -48*
18. *(Meta Scaffolding) How can we use this equation to solve?*
19. *(Scaffolding) Add the two equations together. So you have (-5x -2y = -48) + 3x + 2y = 32)*
20. *(Meta Scaffolding) How can we simplify this addition?*
21. *(Scaffolding) After the addition you get -2x = -16*
22. *(Scaffolding) So this means that x = 8*
23. *(Meta Scaffolding) What did the problem ask for?*
24. *(Scaffolding) The cost of the adult tickets*
25. *(Meta Scaffolding) Which variable is used for the cost of the adult tickets?*
26. *(Scaffolding) x . So we have found the right value.*
27. *(Meta Scaffolding) What is the unit that the problem is asking for?*
28. *(Scaffolding) Dollars. So the answer is “The cost of an adult ticket is $8”*

**Problem a4:** Kendrick wrote a business plan for an entrepreneurship class, and now he has to make bound copies. Kendrick could use a printer who charges a setup fee of $50 and $5 for every copy printed. Another possibility is to go to the office supply store, where he could pay an up-front fee of $30 and $7 per copy. There is a certain number of copies that makes the two options equivalent in terms of cost. How much would the copies cost?

1. *Please re-read the question carefully.*
2. *(Summary) This is a complete word problem that asks you to use the techniques you know to solve linear systems in a real life example.*
3. *(Meta Scaffolding): What kind of problem is this?*
4. *(Scaffolding) This is a system of linear equations problem.*
5. *(Meta Scaffolding) How can you translate this written problem into a math problem?*
6. *(Scaffolding) The question gives you enough information to create a set of two linear equations.*
7. *(Meta Scaffolding) What are the variables in this problem?*
8. *(Scaffolding) For this problem, let’s use P for the total price and c for the number of copies.*
9. *(Meta Scaffolding) How could you turn the information about the printer into an equation?*
10. *(Scaffolding) “Setup fee of $50” means that the y intercept is 50*
11. *(Scaffolding) “$5 for every copy printed” means that the slope is 5*
12. *(Meta Scaffolding) You can combine these two pieces of information to create an equation for this relationship*
13. *(Scaffolding) So our first equation is P = 5c + 50*
14. *(Meta Scaffolding) How could you turn the information about the office supply store into an equation?*
15. *(Scaffolding) “Up-front fee of $30” means that the y intercept is 30*
16. *(Scaffolding) “$7 per copy” means that the slope is 7*
17. *(Meta Scaffolding) You can combine these two pieces of information to create an equation for this relationship*
18. *(Scaffolding) So our first equation is P = 7c + 30*
19. *(Meta Scaffolding) How can we use the techniques of solving linear equations to find the solution?*
20. *(Scaffolding) The easiest way to solve this problem is to graph the two equations and find their point of intersection.*
21. *(Meta Scaffolding) How should you arrange our two variables on an x-y coordinate plane?*
22. *(Scaffolding) You can use c for x and P for y*
23. *(Scaffolding) When you do that, you get an answer of (10, 100)*
24. *(Meta Scaffolding) What does this answer mean in the context of the problem?*
25. *(Scaffolding) It means that when you make 10 copies, it costs $100*
26. *(Meta Scaffolding) What is the problem asking for?*
27. *(Scaffolding) The problem just wants to know how much it will cost.*
28. *(Scaffolding) So the answer is $100*

**Problem b1:** Find the distance between A(2,0) and B(5,4)?

1. *Please re-read the question carefully.*
2. *(Summary) This is a straight-forward math problem that is asking to find the distance between two points.*
3. *(Meta Scaffolding) Graph these two points to get a better understanding of the distance*
4. *(Meta Scaffolding) What is the equation for finding the distance between two points?*
5. *(Scaffolding)The distance between two points can be found by d2 = (x1 –x2 )2 + (y1 – y2)2*
6. *(Scaffolding) Now plug in values from the two points to solve.*
7. *(Scaffolding) d2 = (5– 2)2 + (4 – 0)2*
8. *(Meta Scaffolding) You can simplify this equation to solve for the distance*
9. *(Scaffolding) Simplify the equation to get: d2 = 9 + 16 = 25*
10. *(Scaffolding) d = 5*

**Problem b2:** There exists two points A(2,4) and B(5,v), the distance between A and B is 5. What are values of v?

1. *Please re-read the question carefully.*
2. *(Summary) This is a straight-forward math problem that is asking if you know the distance between two points to find an unknown component about one of the points.*
3. *(Meta Scaffolding) Graph these two points to get a better understanding of the distance*
4. *(Meta Scaffolding) What is the equation for finding the distance between two points?*
5. *(Scaffolding)The distance between two points can be found by d2 = (x1 –x2 )2 + (y1 – y2)2*
6. *(Scaffolding) Since you know the distance between the two points, be careful with how you plug the values into the formula*
7. *(Scaffolding) 52 = (5– 2)2 + (v – 4)2*
8. *(Meta Scaffolding) You can simplify this equation to solve for the unknown quanity*
9. *(Scaffolding) Simplify the equation to get: 25 = 9 + (v – 4)2*
10. *(Scaffolding) Simplify the equation even more to get 16 = (v – 4)2*
11. *(Scaffolding) Simplify the equation even more to get +- 4 = v – 4*
12. *(Meta Scaffolding) What should you do now that you have two possibilities for v?*
13. *(Scaffolding) Solve for both separately.*
14. *(Scaffolding) So if 4 = v – 4, then 8 = v*
15. *(Scaffolding) If -4 = v – 4, then 0 = v*
16. *(Meta Scaffolding) What is the question asking for?*
17. *(Scaffolding) The question is asking for the values of v that make the distance between the point 5.*
18. *(Scaffolding) So v = 0 or 8*

**Problem b3:** The class of math is mapped on a coordinate grid with the origin being at the center point of the hall. Mary’s seat is located at the point (-4, 7) and Betty’s seat is located at (-2, 5). How far is it from Mary’s seat to Betty’s seat?

1. *Please re-read the question carefully.*
2. *(Summary) This is a hybrid math problem that combines both math elements and common language asking to find the difference between the students’ seats*
3. *(Meta Scaffolding) What kind of problem is this?*
4. *(Scaffolding) This is a distance problem.*
5. *(Scaffolding) The problem gives you the students’ seats as points on a coordinate plane*
6. *(Scaffolding) Graph these two points to get a better understanding of the distance*
7. *(Meta Scaffolding) What is the equation for finding the distance between two points*
8. *(Scaffolding) The distance between two points can be found* by *d2 = (x1 –x2 )2 + (y1 – y2)2*
9. *(Scaffolding) Now plug in values from the two points to solve.*
10. *(Scaffolding) d2 = (-2– (-4))2 + (5 – 7)2*
11. *(Meta Scaffolding) You can simplify this equation to solve for the distance*
12. *(Scaffolding) Simplify the equation to get: d2 = 4 + 4 = 8*
13. *(Scaffolding) d = sqrt(8), which you can simplify to d = sqrt(4) \* sqrt(2) = 2 \* sqrt(2)*
14. *(Meta Scaffolding) What did the problem ask for? What are the units given in the problem?*
15. *(Scaffolding) It asks for the distance between their seats, but does not give units*
16. *(Scaffolding)So the answer is “ Mary’s seat is 2 \* sqrt(2) units from Betty’s seat”*

**Problem b4:** You're leading the Shmoopville Beefalos in the championship football game  against your bitter rivals, the Yooda City Wildcats. You're 3 yards from the end zone and 4 yards from the sideline, and you throwed the ball 5 yards to Othello to complete the big play. Othello is 7 yards from the end zone. How far does Othello stand from the sideline?

1. *Please re-read the question carefully.*
2. *(Summary) This is a word problem that asks to use your knowledge on the distance formula to find the distance Othello was from the endzone.*
3. *(Meta Scaffolding) What kind of problem is this?*
4. *(Scaffolding) This is a distance problem.*
5. *(Meta Scaffolding) How can we use the distance formula with this information given in the problem?*
6. *(Scaffolding) The distances can be converted into coordinates*
7. *(Scaffolding) This is a somewhat backwards problem- it gives us d = 5*
8. *(Scaffolding) Let’s make the distance from the end zone x and the distance from the sideline y*
9. *(Scaffolding)The coordinate that you are standing at is (3, 4).*
10. *(Scaffolding) The coordinate that Othello stands at is (7, Y), where Y is the unknown distance that Othello is from the end zone*
11. *(Meta Scaffolding) What is the equation for finding the distance between two points*
12. *(Scaffolding) The distance between two points can be found* by *d2 = (x1 –x2 )2 + (y1 – y2)2*
13. *(Scaffolding) Now plug in values from the two points to solve.*
14. *(Scaffolding) 52 = (7– 3)2 + (Y – 4)2*
15. *(Meta Scaffolding) You can simplify this equation and solve for Y*
16. *(Scaffolding) Simplify the equation to get: 25 = 16 + (Y – 4)2*
17. *(Scaffolding) Simplify the equation to get: 9 = (Y – 4)2*
18. *(Scaffolding) Find the square root of both sides to simplify: 3 = Y – 4*
19. *(Scaffolding) Y = 7*
20. *(Meta Scaffolding) What did the problem ask for? What are the units of the distance?*
21. *(Scaffolding) Since the problem asked for the distance that Othello is from the end zone. The units are yards.*
22. *(Scaffolding) So the answer is “Othello is 7 yards from the end zone”*