Comparison of PD, PID and Sliding-Mode Position Controllers for V–Tail Quadcopter Stability

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***Abstract*—This paper explains the stability analysis of the dynamic model of a V-tail quadcopter for a PD, PID, and SMC position controllers. The customized design of the V-tail quadcopter showing its coordinate frames for earth (fixed) frame and mobile frame is compared to the known parameters of this structure from other papers. The dynamic analysis of the V-tail quadcopter is done using the Newton–Euler formulation method. The main aspect of this paper is in the corrected design from the original paper and Lyapunov stability analysis of the PD, PID, and SMC position controllers for the V-tail quadcopter. We have included minor changes to make it simpler to understand and execute. The simulation results validate our proposed controller models and algorithms for the V-tail quadcopter. Hence, three controllers for the V-tail quadcopter designed using the robot manipulator theory are presented.**

Keywords—V-tail quadcopter, PID controller, PD controller, sliding-mode controller, Lyapunov stability analysis.

I. Introduction

Drones or Unmanned Aerial Vehicles (UAVs) have been around since the late 1700s, but the invention of the modern-day drone only came in the 1900s. In the last decade, there have been huge developments in drone research especially in military and agriculture. This is mainly due to their capability to carry out several complex tasks, in addition to their low–cost production and relatively simple operation [1].

Most of the research is focused mainly on Quadcopters. Quadcopters, as the name suggests, is a type of an aerial vehicle or helicopter with four motors. The most commonly researched are the cross-structure quadcopters (x structure), due to their reduced dimensions, lightweight, mechanical structure, autonomy, stability and their outstanding ability to efficiently complete assigned tasks [1]. They can also be classified as VTOL (Vertical Take Off and Landing), due to their ability to take off, land and hover vertically. The quadrotor has six degrees of freedom, three translational and three rotational movements. By adjusting the angular velocities of each rotor, with respect to the other three, and with two of the rotors, on the same axis, rotate clockwise while the other two rotate counterclockwise, the net yaw becomes zero. A difference in speeds between the two pair of motors creates a net yaw, whereas different speeds on opposite pair create net roll or pitch. The vertical motion is produced by the rotation of propellers that generates vertical upward lifting force and lifts the quadcopter body in the air [1].

Various fields of science use quadcopters for their explicit research. These fields are not even necessarily centered on aerospace or defense. Some of the fields include agriculture, anthropology, fluid dynamics and microprocessors. In agriculture, due to the size and ability of the quadcopter to traverse over the fields, with the help of a navigational framework it can be used to detect the areas of low yield and poor crop health, with the help of a multispectral camera [2]. Research on human behavior is being conducted by use of drones in social gatherings and or during the agricultural or hunting activities of human populations. Even when drones are not deliberately used to study human populations, the use of drones (to study wildlife, for example) may inadvertently capture data on human beings. Collecting data in such a manner raises a lot of ethical issues in which again there is research being conducted as well [3].

Our paper focuses on a specific type of quadcopter known as V-tail quadcopter. Unlike a normal quadcopter, the V-tail has an unconventional design which incorporates two slanted tail surfaces. The two fixed tail surfaces act as both horizontal and vertical stabilizers and each has a moveable flight control surface referred to as a ruddervator. These ruddervators perform the combined functions of both a rudder and an elevator. When the pilot moves the control column forward or aft, the ruddervators move symmetrically in the same manner as a conventional elevator. Conversely, when the rudder pedals are displaced, the ruddervator surfaces move differentially to emulate the movement of a conventional rudder [4].

There has not been a lot of research on V-tail quadcopters. There has been experimental analysis of its dynamics as its configuration is able to achieve better performance in maneuvering control and is able to achieve higher performance than regular quadcopters at the same altitude it is in.

For example, a robust flight control system for Y-shaped V-tail quadcopter to model wind gusts acting on the vehicle and which includes disturbances in the dynamic model of the vehicle, by means of simple addition to stream velocities developed by Shawndy Michael Lee Jin Lun, Sarul Sakulthong and Sutthiphong Srigrarom [5]. With the newly developed wind disturbance control, the model can be flown on course despite strong wind, which helps future researchers when working on this problem.

For this paper, we are applying the Newton-Euler formulation and the Euler-Lagrange formulation to obtain our dynamic models for the system. The former is most used to obtain the dynamic model of quadcopters, due to the simplicity of its equations, but this is not the case for robot manipulators, where the Euler–Lagrange formulation is more convenient to implement [6].

In this paper, the dynamic analysis using Newton-Euler formulation for customized design of a V–tail quadcopter is presented to know the parameters of this structure and compare them with the × structure quadcopter as used in most papers. The main contribution of this paper remains in the design and Lyapunov stability analysis of the PD, PID and SMC position controllers for the V–tail quadcopter because the robot manipulator methodology was used for that purpose [6]. The simulation results validate the proposed controllers and algorithms for the V–tail quadcopter when the three controllers reach the desired position. Also, a nonconventional variable is introduced to study the stability analysis of unmanned aerial vehicles when controlled by a PID position controller.

Finally, a comparison between the three designed controllers for a V–tail quadcopter is presented, where the differences between each can be appreciated. It is important to mention that in previous studies reviewed by the authors and given in the References section, this theory has never been implemented for UAV control [7].

II. COORDINATE REFERENCE FRAMES AND GEOMETRIC DESCRIPTION OF THE V–TAIL QUADCOPTER

First, we describe the coordinate reference frames which will be used later in obtaining the dynamic model of the system.

In mobile robotics, two different coordinate reference frames are needed to control the robot. The first reference frame is the inertial reference frame (O_{XYZ}), whose origin stays fixed to a specific point in space. The second one is a mobile reference frame ($O_{X'Y'Z'}$), and its origin is fixed to the center of mass of the vehicle (Figure. 1), which implies that this reference frame moves with the quadcopter. It is used to determine the position and orientation of the V–tail in space as found in [7],[8], [13] and [15].

The position of the quadcopter is described in the earth–fixed reference frame by the vector $\xi = [x\ y\ z]^T \in \mathrm{IR}^3$. Here, x,y and z refers to the x,y and z axis respectively, I is the moment of inertia and R is the radius. The orientation is fully described by the Euler angles contained in the vector $\eta = [\varphi\ \theta\ \psi]^T \in \mathrm{IR}^3$, also known as roll, pitch and yaw angles, respectively [7],[8] and [23] .

The geometric parameters are shown in Fig. 2 and 3, which represent the rear view and the top view, respectively. The descriptions of these parameters are introduced in Table 1.

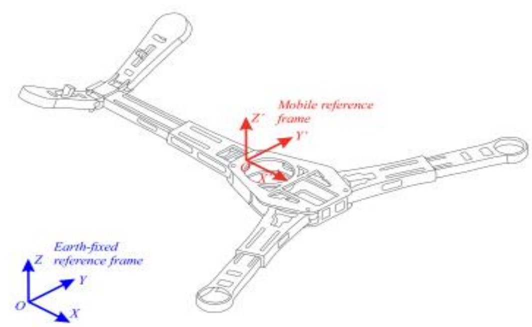


Figure 1: Co-ordinate reference frames

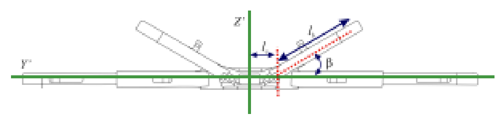


Figure 2: Back view of the V-tail quadcopter

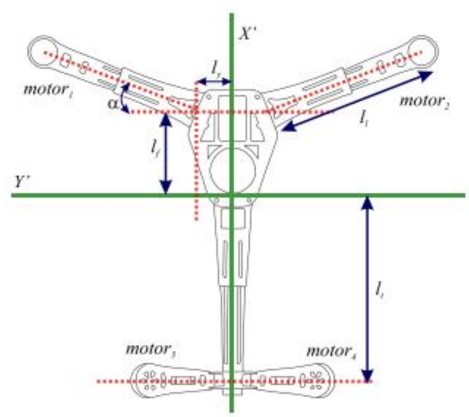


Figure 3: Top view of the V-tail quadcopter

Parameter	Description
α	Orientation angle of the front links
β	Orientation angle of the back links
l_b	Length of the back link
l_c	Distance in Y' from the origin of $O_{X'Y'Z'}$ to the beginning of the back link
l_f	Distance in X' from the origin of $O_{X'Y'Z'}$ to the beginning of the front link
l_l	Length of the front link
l_t	Distance in X' from the origin of $O_{X'Y'Z'}$ to motors 3 and 4
l_y	Distance in Y' from the origin of $O_{X'Y'Z'}$ to the beginning of the front link

Figure 4: Table of geometric parameters for V-tail

III. NEWTON–EULER FORMULATION FOR A RIGID BODY WITH 6 DOFS

In this section, the Newton–Euler formulation to obtain the dynamic model of a rigid body with 6 DOFs is introduced.

The Newton–Euler formulation offers equations that describe the translational and rotational dynamics of a rigid body. It is derived by the direct interpretation of Newton's Second Law of Motion, which describes dynamic systems in terms of force and momentum. The equations incorporate all the forces and moments acting on the individual robot links, including the coupling forces and moments between the links. The equations obtained from the Newton-Euler method include the constraint forces acting between adjacent links. Thus, additional arithmetic operations are required to eliminate these terms and obtain explicit relations between the joint torques and the resultant motion in terms of joint displacements [6] and [7].

Let I_b be the body inertia and R be the 3X3 rotation matrix for the drone. Now, angular momentum vector ($\vec{H}$) at center of gravity is given as:

$$\vec{H}=(RI_bR^T)\omega$$

Also, linear momentum can be given as:

$$\vec{L}=\widetilde{M}v_i$$

Now, the mass moment of inertia tensor along the world coordinates on the center of gravity is $I= RI_bR^T$, which transforms the rotational velocity ω into local coordinates, multiplies by I_b and transforms backs into world coordinates. Hence,

$$\vec{H}=\tilde{I}\ \omega \qquad (1)$$

Now the equations of motion on the center of gravity are defined from the sum of forces and moments equals the rate of change of momentum, i.e.,

$$\Sigma F=\dot{\vec{L}} \text{ and } \Sigma M=\dot{\vec{H}}$$

Now, sum of forces can be given as:

$$\Sigma F=\dot{\vec{L}} = \widetilde{M}\dot{v}_i \qquad (2)$$

Now, to represent inertial motion in the drone coordinate frame, we must use the chain rule of derivation to account for both the change due to the time derivative of the vector within the coordinate frame, as well as the time derivative of the coordinate frame's rotation. First, the scalar expression of the time differentiation of the body-fixed velocity vector is given as follows:

$$v^b=\begin{bmatrix} u \\ v \\ w \end{bmatrix}^b = u\hat{b}_1+v\hat{b}_2+w\hat{b}_3$$

Here, u, v and w are the linear velocities acting on $\hat{b}_1$,$\hat{b}_2$ and $\hat{b}_3$ axes (X',Y' and Z-axes on mobile reference frame). Hence, by chain rule [11],

$$\left(\frac{dv}{dt}\right)_{\text{inertial}} = \left(\frac{du}{dt}\hat{b}_1 + \frac{dv}{dt}\hat{b}_2 + \frac{dw}{dt}\hat{b}_3\right) + \left(u\frac{d\hat{b}_1}{dt} + v\frac{d\hat{b}_2}{dt} + w\frac{d\hat{b}_3}{dt}\right)$$

The components in the first set of parentheses represent the change in inertial velocity within the body frame coordinate system. The components in the second set of parentheses represent perceived velocity change due to the rotation of the coordinate frame. This equation (2) can be rewritten using Coriolis' Theorem, where the frame rotation is represented by the angular velocity crossed with the velocity vector. Coriolis acceleration is given by [11] and [12]:

$$\left(\frac{dv}{dt}\right)_{\text{inertial}} = \dot{v} + \omega \times v \qquad (3)$$

Hence, on comparing (2) and (3) we get,

$$\Sigma F=\widetilde{M}(\dot{v} + \omega \text{ X } v)= \widetilde{M}\dot{v} + \omega \times \widetilde{M}v \qquad (4)$$

Also, the sum of moments can be given as:

$$\begin{aligned}\Sigma M&=\dot{\vec{H}}\\ &= \frac{\partial\vec{H}}{\partial t} + \omega \times \vec{H}\\ &= \frac{\partial(\tilde{I}\omega)}{\partial t} + \omega \times \tilde{I}\omega \quad \text{[From (1)]}\end{aligned}$$

$$\Sigma M = \tilde{I}\dot{\omega} + \omega \times \tilde{I}\omega \qquad (5)$$

Now, from (4) and (5) it can be represented in the matrix form as follows [6],[8] and [9]:

$$\begin{bmatrix} \widetilde{\boldsymbol{M}} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{I}} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{v}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega} \times \widetilde{\boldsymbol{M}}\cdot\boldsymbol{v} \\ \boldsymbol{\omega} \times \tilde{\boldsymbol{I}}\cdot\boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\kappa} \\ \boldsymbol{\tau} \end{bmatrix} \qquad (6)$$

where $\widetilde{M}$, $\tilde{I} \in IR^{3X3}$ represent the mass and inertia tensors, respectively ($\tilde{.}$ is only used to represent a tensor), $0 \in IR^{3X3}$ is the zero matrix. The vectors $\kappa \in IR^3$ and $\tau \in IR^3$ describe the forces and moments acting on the body, respectively, and are given in the mobile reference frame. The vector $\nu = [u\ v\ w]^T \in IR^3$ represents the linear velocity of the body expressed in the mobile reference frame, which is related to the velocity of the earth–fixed reference frame $\dot{\xi}$, by multiplying its rotation matrix with its respective velocities. It can be given as [15]:

$$\boldsymbol{\dot{\xi} = R\nu}$$

Here, $R \in IR^{3\times3}$ is the rotational matrix corresponding to an absolute representation in relation to the earth–fixed reference frame. For a UAV, its rotation is taken around the z-axis and hence, it can be given as [5], [13] and [22]:

$$R=R_zR_yR_x$$

$$=\begin{bmatrix} C\Psi & -S\Psi & 0 \\ S\Psi & C\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & C\varphi & -S\varphi \\ 0 & S\varphi & C\varphi \end{bmatrix}$$

$$=\begin{bmatrix} C\theta C\Psi & S\varphi S\theta C\Psi - C\varphi S\Psi & C\varphi S\theta C\Psi + S\varphi S\Psi \\ C\theta S\Psi & S\varphi S\theta S\Psi + C\varphi C\Psi & C\varphi S\theta S\Psi - S\varphi C\Psi \\ -S\theta & C\theta S\varphi & C\theta C\varphi \end{bmatrix}$$

Here, C(a) = Cos(a) and S(a)=Sin(a), throughout the paper. Hence, the vector can be represented as:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}C\theta C\Psi & S\varphi S\theta C\Psi - C\varphi S\Psi & C\varphi S\theta C\Psi + S\varphi S\Psi\\ C\theta S\Psi & S\varphi S\theta S\Psi + C\varphi C\Psi & C\varphi S\theta S\Psi - S\varphi C\Psi\\ -S\theta & C\theta S\varphi & C\theta C\varphi\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix}$$

In the same way, the angular velocity of the mobile reference frame is expressed by the vector $\omega = [p\ q\ r]^T \in IR^3$. According to figure 1, as we assume the rotation on the x-axis (as its on the same plane), the angular velocity ω can be defined as the sum of the rotations of each axis with respect to z [5], i.e.,

$$\omega = \begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}\dot{\varphi}\\ 0\\ 0\end{bmatrix} + R_x\begin{bmatrix}0\\ \dot{\theta}\\ 0\end{bmatrix} + R_xR_y\begin{bmatrix}0\\ 0\\ \dot{\Psi}\end{bmatrix}$$

$$=\begin{bmatrix}\dot{\varphi}\\ 0\\ 0\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\ 0 & C\varphi & -S\varphi\\ 0 & S\varphi & C\varphi\end{bmatrix}\begin{bmatrix}0\\ \dot{\theta}\\ 0\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\ 0 & C\varphi & -S\varphi\\ 0 & S\varphi & C\varphi\end{bmatrix}\begin{bmatrix}C\theta & 0 & -S\theta\\ 0 & 1 & 0\\ S\theta & 0 & C\theta\end{bmatrix}\begin{bmatrix}0\\ 0\\ \dot{\Psi}\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & -S\varphi\\ 0 & C\varphi & S\varphi C\theta\\ 0 & -S\varphi & C\varphi C\theta\end{bmatrix}\begin{bmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}$$

which can be related to the roll, pitch and yaw velocities and, it depends on $\dot{\eta}$ (the time derivative of vector η). It can be simplified as shown by the following relation [7]:

$$\boldsymbol{\omega = T\dot{\eta}}$$

where $T \in IR^3$ is a transformation matrix and is given as:

$$T = \begin{bmatrix}1 & 0 & -S\varphi\\ 0 & C\varphi & S\varphi C\theta\\ 0 & -S\varphi & C\varphi C\theta\end{bmatrix} \quad (7)$$

Equation (6) corresponds to a dynamic model of a rigid body with 6 DOFs that depend on the linear and angular velocities and accelerations of the body as described in the mobile reference frame. We can simplify this by rearranging this equation to an equation involving linear velocities and accelerations of the body with respect to the earth-fixed reference frame, and the angular velocities and accelerations of the body expressed in the mobile reference frame. This is done so that the inertia tensor $\tilde{I}$ is not dependent on time. After modifying this equation, the following result is obtained [6] and [10]:

$$\begin{bmatrix}\boldsymbol{\tilde{M}} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{\tilde{I}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\ddot{\xi}}\\ \boldsymbol{\dot{\omega}}\end{bmatrix} + \begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{\omega \times \tilde{I}\cdot\omega}\end{bmatrix} = \begin{bmatrix}\boldsymbol{f}\\ \boldsymbol{\tau}\end{bmatrix} \qquad (8)$$

Equation (8) combines the dynamics of the body in the earth–fixed reference frame and in the mobile reference frame. Here, the vector $f \in IR^3$ appears, representing the external forces acting on the body described in the earth–fixed reference frame, the vector $0 \in IR^3$ represents the zero vector, and $\ddot{\xi}$ is the vector of linear accelerations in the earth–fixed reference frame.

IV. DYNAMIC MODEL OF THE V–TAIL QUADCOPTER

Until this point, the Newton–Euler formulation was used to obtain the dynamic model of a rigid body. Now we will adapt it to the V–tail quadcopter, considering the effects and the phenomena that affect the movement of the vehicle.

First, we need to consider the effect of gravity. For this we need to define equation (8) in terms of gravitational forces. The Lagrangian (L) of a robot of n-degrees of freedom (DOF) can be defined as the difference between its kinetic energy (K) and potential energy(U) [6], i.e.,

$$L=K-U \qquad (9)$$

Now, the kinetic energy for such a mechanism is given as:

$$K = \frac{1}{2}[\dot{q}^T\ M\ \dot{q}]$$

Hence, equation (9) becomes,

$$L = \frac{1}{2}[\dot{q}^T\ M\ \dot{q}] - U \qquad (10)$$

Now, the quadrotor dynamics are obtained from the Euler Lagrangian equations with external generalized force $F=(f, \tau)$ as follows [6]:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} = (f, \tau) \qquad (11)$$

Where, f is the translational force applied to the quadrotor due to the throttle control input in the body frame and τ is the external forces and torque acting at each state vector (q) as well as other non-conservative forces. They can be defined as:

$$f = [F_x\ F_y\ F_z]^T$$
$$\tau = [\tau_\phi\ \tau_\theta\ \tau_\psi]^T$$

Now, from equation (10) and (11), we get:

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}}\ [\frac{1}{2}\dot{q}^T\ M\ \dot{q}]\right] - \frac{\partial}{\partial q}\ [\frac{1}{2}\dot{q}^T\ M\ \dot{q}] + \frac{\partial U}{\partial q} = \tau$$

Now, $\frac{\partial}{\partial \dot{q}}\left[\frac{1}{2}\dot{q}^T\ M\ \dot{q}\right] = M\dot{q}$ and

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}}\ [\frac{1}{2}\dot{q}^T\ M\ \dot{q}]\right] = \frac{d}{dt}[\ M\ \dot{q}]] = M\ddot{q} + \dot{M}\dot{q} \quad (12)$$

Now, substituting equation (12) in the above Lagrangian equation, we get [6]:

$$M\ddot{q} + \dot{M}\dot{q} - \frac{1}{2}\frac{\partial}{\partial q}\ [\dot{q}^T\ M\ \dot{q}] + \frac{\partial U}{\partial q} = \tau$$

Now, making this equation in compact form, the dynamic model of the vehicle can be expressed as [6]:

$$M\ddot{q} + C\dot{q} + G = \tau$$

Where, $C\dot{q} = \dot{M}\dot{q} - \frac{1}{2}\frac{\partial}{\partial q}\ [\dot{q}^T\ M\ \dot{q}]$

$$G = \frac{\partial U}{\partial q}$$

Now, in matrix form it can be expressed as :

$$\widetilde{Mt}\begin{bmatrix}\ddot{\boldsymbol{\xi}}\\ \dot{\boldsymbol{\omega}}\end{bmatrix} + \mathrm{C}(\dot{\boldsymbol{\xi}}, \boldsymbol{\omega})\begin{bmatrix}\dot{\boldsymbol{\xi}}\\ \dot{\boldsymbol{\omega}}\end{bmatrix} + \begin{bmatrix}\boldsymbol{G}\\ \boldsymbol{0}\end{bmatrix} = \begin{bmatrix}\boldsymbol{f}\\ \boldsymbol{\tau}\end{bmatrix} \quad (13)$$

Where,

$$\widetilde{Mt} = \begin{bmatrix}\widetilde{M} & 0\\ 0 & \tilde{I}\end{bmatrix} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0\\ 0 & m & 0 & 0 & 0 & 0\\ 0 & 0 & m & 0 & 0 & 0\\ 0 & 0 & 0 & I_{xx} & I_{xy} & I_{xz}\\ 0 & 0 & 0 & I_{yx} & I_{yy} & I_{yz}\\ 0 & 0 & 0 & I_{zx} & I_{zy} & I_{zz}\end{bmatrix}$$

$$\mathrm{C}(\dot{\xi}, \omega) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & A & -B\\ 0 & 0 & 0 & -A & 0 & C\\ 0 & 0 & 0 & B & -C & 0\end{bmatrix}$$

$$\mathrm{G} = \begin{bmatrix}0\\ 0\\ mg\end{bmatrix} \quad (14)$$

where $\widetilde{Mt} \in \mathrm{IR}^{6\times6}$ is the total mass tensor of the system, which includes the mass tensor $\widetilde{M}$ and the inertia tensor $\tilde{I}$, $\mathrm{C}(\dot{\xi}, \omega) \in \mathrm{IR}^{6x6}$ is the Coriolis and centripetal effects matrix, m is the quadcopter mass, I_{ii} are the moments of inertia and g is the gravitational constant in the International System. Now, A, B and C are the total moment acting on the body of the vehicle, it can be expanded as follows:

$$A = I_{zx}p + I_{zy}q + I_{zz}r$$
$$B = I_{yx}p + I_{yy}q + I_{yz}r$$
$$C = I_{xx}p + I_{xy}q + I_{xz}r$$

The vector of external forces f includes the vector of lifting forces produced by each motor $f_{\mathrm{m}} \in \mathrm{IR}^3$, which is described in the mobile reference frame, and the vector of drag forces $f_{\mathrm{D}} \in \mathrm{IR}^3$ that acts in the opposite direction of the movement, as described in the earth–fixed reference frame. Now, from the figure, f can be defined as:

$$f = Rf_m - f_D$$

For transferring the aerodynamic forces to the blade coordinate frame, we need to consider that each motor produces a lifting force f_{mi}. Also, considering the geometry of the model and characteristics of the quadcopter, the vector f_{m} can be defined by the addition of forces, as follows [7] and [11]:

$$f_{\mathrm{m}} = \begin{bmatrix}\boldsymbol{0}\\ (f_{m4} - f_{m3})\mathrm{Sin}\beta\\ f_{m1} + f_{m2} + (f_{m3} + f_{m4})\mathrm{Cos}\beta\end{bmatrix} \quad (15)$$

where the magnitudes of the forces f_{mi} are computed from the lift theory for the quadcopter's propellers. The equations for lift forces acting on the drone is given as follows [17] and [21]:

$$f_{\mathrm{mi}} = \frac{\rho A_s C_T R^2 \omega_i^2}{2} = \mathrm{b}\omega_i^2$$

where ρ is the air density, A_s the effective blades area, C_T the lift coefficient, R is blades radius, b is the thrust factor and ω_i is the angular velocity of the blades of motor i.

The drag forces are expressed in the earth–fixed reference frame because they depend directly and only on the linear velocities of the vehicle given in this reference frame. It can be defined as follows [21]:

$$f_D = \frac{\rho}{2}\left\|\dot{\xi}\right\|^{\mathrm{T}} \mathrm{AC_D}\dot{\boldsymbol{\xi}} = \begin{bmatrix}\boldsymbol{fD_x}\\ \boldsymbol{fD_y}\\ \boldsymbol{fD_z}\end{bmatrix}$$

where $A \in \mathrm{IR}^{3x3}$ is the diagonal matrix of contact areas and $CD \in \mathrm{IR}^{3x3}$ is the diagonal matrix of drag coefficient.

Now, three main effects compose the vector of moments acting on the vehicle, with respect to the mobile reference frame: moments produced by lifting forces (generated by motors), τ_{fm} ; torques produced by the motors, τ_{m}; and gyroscopic effects, τ_{g}. This can be defined as:

$$\tau = \tau_{\mathrm{fm}} + \tau_{\mathrm{m}} + \tau_{\mathrm{g}}$$

The vector τ_{fm} contains the moments produced by the lifting force related to the center of mass of the vehicle and can be expressed using the above geometric assumptions and factors as [7]:

$$\tau_{\mathrm{fm}} = \begin{bmatrix}(f_{m1} - f_{m2})(l_y + l_l C_\alpha) + (f_{m3} - f_{m4})(l_c C_\beta + l_b)\\ -(f_{m1} + f_{m2})(l_f + l_l S_\alpha) + (f_{m3} + f_{m4})(l_t + l_l C_\beta)\\ (f_{m3} - f_{m4}))(l_l S_\beta)\end{bmatrix} \quad (16)$$

It is known that the motors produce a free moment due to the rotation around their own axes, which is a function of the angular velocity. This phenomenon can also be represented with design considerations in the vector τ_{m}, as follows:

$$\tau_{\mathrm{m}} = \begin{bmatrix}\boldsymbol{0}\\ (\tau_{m3} - \tau_{m4})S_\beta\\ \tau_{m1} - \tau_{m2} + (-\tau_{m3} + \tau_{m4})C_\beta\end{bmatrix} \quad (17)$$

The magnitude of the i–th moment τ_{m} is:

$$\tau_{\mathrm{mi}} = \frac{\rho A_t C_Q R^3 \omega_i^2}{2} = \mathrm{d}\omega_i^2$$

where A_t is the transverse section area of the blade, d is the drag factor and C_Q is the torque coefficient. The sign of the moment is selected according to the sense of rotation of the blades. We will also consider the gyroscopic effects acting on the vehicle. This can be defined using the equation for gyroscopic torque (τ_{g}) [24]:

$$\tau_{\mathrm{g}} = \frac{L}{\Delta t} = \frac{I\omega}{\Delta t}$$

According to right hand rule, as our axis rotates in the clockwise direction, it will be defined as:

$$\tau_g = -\frac{I\omega}{\Delta t}$$

Also, considering all the yaw moments acting on the motor in unit time Δt, we get:

$$\tau_g = -\sum_{i=1}^{4} I\omega$$

If we take rotational moment of inertia of motor around its own axis as J_{tp} and define omega along the z-axis (rotates about z-axis) , we get [17]:

$$\tau_g = -\sum_{i=1}^{4} J_{tp}\ (\omega \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix})\ \omega_i$$

Notice that all these vectors are expressed in the mobile reference frame, so the vector τ is expressed in the same reference frame. Due to the non–linearity of the system and the number of unknown parameters and terms, it is necessary to simplify the dynamic model when designing the controllers.

A. Simplified Dynamic Model of the V–Tail Quadcopter

It is necessary to simplify the dynamic model presented in equation (13) when designing the controllers. These simplifications are examined in this section.

First, the product of the Coriolis and centripetal effects matrix C ($\dot{\xi}$, ω) , the velocity vector [$\dot{\xi}$ ω] ^{T} and the gyroscopic effects is neglected due to the magnitude of the resultant vector is smaller than the other terms of the dynamic model. Assuming the V–tail quadcopter operates in a quasi–stationary region, i.e., $\varphi \approx 0$ and $\theta \approx 0$, it can be established that $\dot{\eta} = \omega$ and $\ddot{\eta} = \dot{\omega}$, which can be easily proved if $\varphi = 0$ and $\theta = 0$ are substituted into the transformation matrix T, derived in equation (7). The third and last simplification consists of assuming that the quadcopter operates at low speeds, so that comparison of PD, PID and Sliding-Mode Position Controllers drag effects can be neglected, i.e., $f_D = 0$. Also, for this reason, coupling effects and gyroscopic effects (τ_g) are not taken into consideration. Finally, and considering the dynamic model of equation (13), the modified dynamic model of the vehicle is given as follows [7].

$$\widetilde{\boldsymbol{Mt}} \begin{bmatrix} \ddot{\xi} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{G} \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Rf_m} \\ \boldsymbol{\tau_{fm}} + \boldsymbol{\tau_m} \end{bmatrix}$$

To simplify the vector f_m, given in (15), and the vectors of external moments τ_{fm} and τ_m, introduced in (16) and (17), respectively, the angle β is set to 0, so the modified vectors are [7]:

$$f = \begin{bmatrix} 0 \\ 0 \\ f_{m1} + f_{m2} + f_{m3} + f_{m4} \end{bmatrix} \quad (18)$$

$$\tau = \begin{bmatrix} (f_{m1} - f_{m2})(l_y + l_l C_\alpha) + (f_{m3} - f_{m4})(l_c + l_b) \\ -(f_{m1} + f_{m2})(l_f + l_l S_\alpha) + (f_{m3} + f_{m4})(l_t) \\ \tau_{m1} - \tau_{m2} - \tau_{m3} + \tau_{m4} \end{bmatrix} \quad (19)$$

Besides the simplification of the vectors, β is set to 0 to get an approximation of the V–tail quadcopter behavior. We are assuming this case as there is no information about it.

Now, let U1,U2,U3,U4 , be taken as the inputs of the four independent control channels. They can be defined as [7], [14], [16] and [22]:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_4 - F_2 \\ F_3 - F_1 \\ F_2 + F_4 - F_3 - F_1 \end{bmatrix}$$

Where, U_1 is vertical speed control input; U_2 is roll control input; U_3 is pitch control input; U_4 is yaw control input and F is the tension on the rotor. Considering equations (18) and (19), the control variables of the quadcopter are defined as follows [7] and [9]:

$U_1 = f_{m1} + f_{m2} + f_{m3} + f_{m4}$ (20)

$U_2 = (f_{m1} - f_{m2})(l_y + l_l C_\alpha) + (f_{m3} - f_{m4})(l_c + l_b)$ (21)

$U_3 = -(f_{m1} + f_{m2})(l_f + l_l S_\alpha) + (f_{m3} + f_{m4})(l_t)$ (22)

$U4 = \tau_{m1} - \tau_{m2} - \tau_{m3} + \tau_{m4}$ (23)

The inertia tensor $\tilde{I}$ in (14), is simplified if it is considered that the main axis of inertia coincides with the axis of the mobile reference frame, as defined in section II. This simplification is proved by the data obtained from the results found by the authors, when creating the prototype of the quadcopter. It was found that the main inertia moments are of the order of 10^{-3} kg · m^2 . The other components of the inertia tensor are of the order of 10^{-7} kg · m^2 , which can be mathematically expressed in matrix form as:

$$\tilde{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

Now, to obtain quadcopter dynamic model in terms of control inputs, we apply Newton-Euler method again. From equation (12), its matrix form can be defined as:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] = \begin{bmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \\ I_{xx}\ddot{\varphi} \\ I_{yy}\ddot{\theta} \\ I_{zz}\ddot{\psi} \end{bmatrix} \quad (24)$$

Also, from equation (18), as force acts in the z-direction,

$$\frac{\partial L}{\partial q} = [0 \quad 0 \quad -mg \quad 0 \quad 0 \quad 0]^T$$

Now, by combining and comparing equations (18), (19), (20), (21), (22), (23) and (24), we can get the final dynamic model as [18]:

$$\begin{bmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \\ I_{xx}\ddot{\varphi} \\ I_{yy}\ddot{\theta} \\ I_{zz}\ddot{\Psi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ (-mg) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} U_x U_1 \\ U_y U_1 \\ U_z U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \quad (25)$$

where $U_x = C_\varphi S_\theta C_\psi + S_\varphi S_\psi$, $U_y = C_\varphi S_\theta S_\psi - S_\varphi C_\psi$ and $U_z = C_\varphi C_\theta$, and which are used to design the controller and carry out the corresponding simulations.

V. DESIGN OF THE PID POSITION CONTROLLER BASED ON THE CONTROL THEORY OF ROBOT MANIPULATORS

In this section, the procedure to design the PID controller is described [6] and [23].

Now it is essential to define the vector $q \in IR^6$ as [7]:

$$q=\begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix}$$

The PID control law may be expressed via the two following equations:

$$\tau = K_p q + K_v\dot{q} + K_i \zeta \quad (26)$$

$$\zeta = q \quad (27)$$

The closed-loop equation is obtained by substituting the control action τ from (26) in the robot model (13) from [6], i.e

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = K_p q + K_v\dot{q} + K_i \zeta$$

which may be written in terms of the state vector $[\,\zeta^T \quad q^T \quad \dot{q}^T]^T$:

$$\frac{d}{dt}\begin{bmatrix} \zeta \\ q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \\ \ddot{q}_d - M(q)^{-1}[Kpq + Kv\dot{q} + K_I\zeta - C(q,\dot{q})\dot{q} - g(q)] \end{bmatrix} \quad (28)$$

Now lets define the system state using a vector $Z \in IR^{18}$,i.e.,

$$Z = \begin{bmatrix} \zeta \\ q \\ \dot{q} \end{bmatrix} \quad (29)$$

such that $\dot{\zeta} = q$, $q_i = q_{di} - q_i$ and $\dot{q} = -\dot{q}$ where qdi is the set point.

Now, from equation (13), the PID control law can also be written in terms of forces, i.e.,

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = K_p q + K_v\dot{q} + K_i \zeta$$

where $K_p, K_v, K_i \in IR^{6x6}$ are the diagonal matrices of proportional, derivative and integral gains, respectively.

Thus, the closed–loop equation of the system (29) is obtained using equations (25) and (26):

$$\ddot{q} = \begin{bmatrix} (K_{p_x}\tilde{x} - K_{v_x}\dot{x} + K_{i_x}\zeta_x)/m \\ (K_{p_y}\tilde{y} - K_{v_y}\dot{y} + K_{i_y}\zeta_y)/m \\ (K_{p_z}\tilde{z} - K_{v_z}\dot{z} + K_{i_z}\zeta_z - mg)/m \\ (K_{p_\varphi}\tilde{\varphi} - K_{v_\varphi}\dot{\varphi} + K_{i_\varphi}\zeta_\varphi)/I_{xx} \\ (K_{p_\theta}\tilde{\theta} - K_{v_\theta}\dot{\theta} + K_{i_\theta}\zeta_\theta)/I_{yy} \\ (K_{p_\Psi}\tilde{\Psi} - K_{v_\Psi}\dot{\Psi} + K_{i_\Psi}\zeta_\Psi)/I_{zz} \end{bmatrix} \quad (30)$$

which corresponds to the control law to be implemented on the quadcopter based on the dynamic model of equation (25).

A. Stability analysis using a Lyapunov Function

It is convenient to adopt the following global change of variables as we move forward with the stability analysis [7]:

$$\mu = \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_z \\ \mu_\varphi \\ \mu_\theta \\ \mu_\Psi \end{bmatrix}$$

Now, replacing these values with equation (30),

$$\mu = \begin{bmatrix} \zeta_x \\ \zeta_y \\ \zeta_z - K_{iz}{}^{-1}mg \\ \zeta_\varphi \\ \zeta_\theta \\ \zeta_\Psi \end{bmatrix}$$

The new vector of states $Z \in IR^{18}$ is given by:

$$Z = \begin{bmatrix} \mu \\ \tilde{q} \\ \dot{q} \end{bmatrix}$$

Hence, equation (28) becomes:

$$\frac{d}{dt}\begin{bmatrix} \mu \\ \tilde{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \tilde{q} \\ -\dot{q} \\ \ddot{q} \end{bmatrix} \quad (31)$$

Also, equation (30) becomes:

$$\ddot{q} = \begin{bmatrix} (K_{p_x}\tilde{x} - K_{v_x}\dot{x} + K_{i_x}\mu_x)/m \\ (K_{p_y}\tilde{y} - K_{v_y}\dot{y} + K_{i_y}\mu_y)/m \\ (K_{p_z}\tilde{z} - K_{v_z}\dot{z} + K_{i_z}\mu_z)/m \\ (K_{p_\varphi}\tilde{\varphi} - K_{v_\varphi}\dot{\varphi} + K_{i_\varphi}\mu_\varphi)/I_{xx} \\ (K_{p_\theta}\tilde{\theta} - K_{v_\theta}\dot{\theta} + K_{i_\theta}\mu_\theta)/I_{yy} \\ (K_{p_\Psi}\tilde{\Psi} - K_{v_\Psi}\dot{\Psi} + K_{i_\Psi}\mu_\Psi)/I_{zz} \end{bmatrix}$$

This guarantees that the only equilibrium point of the system is the origin as verified by comparing the above matrix with the one defined in (30). Regarding the stability analysis of the PID controller, the next global change of variables is established:

$$\begin{bmatrix}\gamma\\ \tilde{q}\\ \dot{q}\end{bmatrix}=\begin{bmatrix}\epsilon I & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{bmatrix}\begin{bmatrix}\mu\\ \tilde{q}\\ \dot{q}\end{bmatrix}=\begin{bmatrix}\epsilon\mu+\tilde{q}\\ \tilde{q}\\ \dot{q}\end{bmatrix} \quad (32)$$

with $\epsilon > 0$ being a new parameter, introduced to analyze the stability of the UAV. Here, $I \in IR^{6\times 6}$ is the identity matrix. Also, the parameter ϵ is a characteristic used in control theory of robot manipulators, which is applied in this paper. The new variable vector γ is newly introduced in equation (32), and it belongs to the control theory of robot manipulators. We are taking this variable to obtain the closed loop equation of the quadcopter.

Comparing equations (31) and (32), we get the closed loop equation as [6]:

$$\frac{d}{dt}\begin{bmatrix}\gamma\\ \tilde{q}\\ \dot{q}\end{bmatrix}=\begin{bmatrix}\epsilon\tilde{q}-\tilde{q}\\ -\dot{q}\\ \ddot{q}\end{bmatrix} \quad (33)$$

Now, from equation (28), $\ddot{q}$ can be redefined as:

$$\ddot{q}= \widetilde{\mathrm{M}}_t^{-1}[\mathrm{K}_p\tilde{q} - \mathrm{K}_v\dot{q} + \frac{1}{\epsilon}\mathrm{K}_i(\epsilon\tilde{q}-\tilde{q})]$$

$$= \widetilde{\mathrm{M}}_t^{-1}[\mathrm{K}_p\tilde{q} - \mathrm{K}_v\dot{q} + \frac{1}{\epsilon}\mathrm{K}_i(\epsilon\tilde{q}-\tilde{q})] \quad (34)$$

Equation (34) is autonomous, as its invariant with time and its origin is the only equilibrium point. Also, due to the globality of the variable change in (32), the attributes of stability of this equilibrium correspond to the ones of the equilibrium of equation (31).

The origin is stable and at least for sufficiently small values of the initial states z(0), $\tilde{q}(0)$ and $\dot{q}(0)$, the state – particularly $\tilde{q}$(t) – tend asymptotically to zero, then we are able to conclude that the position control objective is achieved, at least locally. Therefore, based on this argument we use Lyapunov's direct method via La Salle's theorem to establish conditions under which the choice of design matrices for the PID controller guarantee asymptotic stability of the origin of the closed-loop equation.

This equation governs the behaviour of n-DOF robot manipulators under PID control in the case of a constant desired position q_d. The Lyapunov candidate function is positive–definite if ϵ is chosen in such a way that the following condition is true:

$$\frac{\lambda_{min}\{K_v\}\lambda_{min}\{\widetilde{\mathrm{M}}_t\}}{\lambda_{max}{}^2\{\widetilde{\mathrm{M}}_t\}} > \epsilon > \frac{\lambda_{max}\{K_i\}}{\lambda_{min}\{K_p\}} \quad (35)$$

We ignore K_g as, $\lambda_{min}\{K_p\}$ >>> K_g, in this case.

Now, we use Lyapunov's direct method by proposing the following Lyapunov function candidate [6]:

$$\mathrm{V}(\tilde{q},\dot{q},\gamma)=\frac{1}{2}\begin{bmatrix}\gamma\\ \tilde{q}\\ \dot{q}\end{bmatrix}^T\begin{bmatrix}\frac{1}{\epsilon}\mathrm{K}_i & 0 & 0\\ 0 & \epsilon\mathrm{K}_v & -\epsilon\widetilde{\mathrm{M}}_t\\ 0 & -\epsilon\widetilde{\mathrm{M}}_t & \widetilde{\mathrm{M}}_t\end{bmatrix}\begin{bmatrix}\gamma\\ \tilde{q}\\ \dot{q}\end{bmatrix}$$

$$+\frac{1}{2}\tilde{q}^T[\mathrm{K}_p-\frac{1}{\epsilon}\mathrm{K}_i]\tilde{q}+\mathrm{U}(q_d-\tilde{q})-\mathrm{U}(q_d)+\tilde{q}^T\begin{bmatrix}\boldsymbol{G}\\ \boldsymbol{0}\end{bmatrix} \quad (36)$$

where U(q) represents the potential energy of the quadcopter.

For the Lyapunov function candidate to be a Lyapunov function we must verify first that it is a positive definite. For this, consider the following terms from equation (36),

$$\frac{1}{2}\tilde{q}^T[\mathrm{K}_p-\frac{1}{\epsilon}\mathrm{K}_i]\tilde{q}+\mathrm{U}(q_d-\tilde{q})-\mathrm{U}(q_d)+\tilde{q}^T\begin{bmatrix}\boldsymbol{G}\\ \boldsymbol{0}\end{bmatrix}$$

Now, we conclude that the above function defined as the sum of these terms is positive definite, globally for all $\tilde{q}$ if,

$$\lambda_{min}\{\mathrm{K}_p-\frac{1}{\epsilon}\mathrm{K}_i\} > \mathrm{K}_g$$

$$\lambda_{min}\{\mathrm{K}_p\} - \lambda_{min}\{\frac{1}{\epsilon}\mathrm{K}_i\} > \mathrm{K}_g$$

which holds due to the lower-bound condition imposed on ϵ , in accordance with equation (35). Therefore, we may claim that the Lyapunov function candidate satisfies

$$\mathrm{V}(\tilde{q},\dot{q},\gamma)\geq\frac{1}{2}\begin{bmatrix}\gamma\\ \tilde{q}\\ \dot{q}\end{bmatrix}^T\begin{bmatrix}\frac{1}{\epsilon}\mathrm{K}_i & 0 & 0\\ 0 & \epsilon\mathrm{K}_v & -\epsilon\widetilde{\mathrm{M}}_t\\ 0 & -\epsilon\widetilde{\mathrm{M}}_t & \widetilde{\mathrm{M}}_t\end{bmatrix}\begin{bmatrix}\gamma\\ \tilde{q}\\ \dot{q}\end{bmatrix}$$

Now, on expanding, we get the following inequalities:

$$\frac{1}{\epsilon}\gamma^T\mathrm{K}_i\gamma\geq\frac{1}{\epsilon}\lambda_{min}\{\mathrm{K}_i\}\,||\gamma||^2$$

$$\epsilon\tilde{q}^T\mathrm{K}_v\tilde{q}\geq\epsilon\,\lambda_{min}\{\mathrm{K}_v\}||\tilde{q}||^2$$

$$\dot{q}^T\widetilde{\mathrm{M}}_t\,\dot{q}\geq\lambda_{min}\{\widetilde{\mathrm{M}}_t\}||\dot{q}||^2$$

$$-\epsilon\tilde{q}^T\mathrm{K}_v\,\dot{q}\geq-\epsilon\,\lambda_{max}\{\widetilde{\mathrm{M}}_t\}||\tilde{q}||\,||\dot{q}||$$

Using the above inequalities, we can get the lower bound Lyapunov function candidate as:

$$\mathrm{V}(\tilde{q},\dot{q},\gamma)\geq$$

$$\frac{\epsilon}{2}\begin{bmatrix}||\gamma||\\ ||\tilde{q}||\\ ||\dot{q}||\end{bmatrix}^T\begin{bmatrix}\frac{1}{\epsilon^2}\lambda_{min}\{\mathrm{K}_i\} & 0 & 0\\ 0 & \lambda_{min}\{\mathrm{K}_v\} & -\lambda_{max}\{\widetilde{\mathrm{M}}_t\}\\ 0 & -\lambda_{max}\{\widetilde{\mathrm{M}}_t\} & \frac{1}{\epsilon}\lambda_{min}\{\widetilde{\mathrm{M}}_t\}\end{bmatrix}\begin{bmatrix}||\gamma||\\ ||\tilde{q}||\\ ||\dot{q}||\end{bmatrix}$$

This shows that V (q˜, q˙ , w) is positive definite globally. Now, we can obtain its time derivative.

Taking the time derivative of equation (36) and substituting values in (33), and taking close estimations, we get:

$$\dot{V}(\tilde{q}, \dot{q}, \gamma) = - \dot{\tilde{q}}^{T}(\mathrm{K}_v - \epsilon \widetilde{\mathrm{M}}_t)\, \dot{q} - \tilde{q}^T(\epsilon \mathrm{K}_p - \mathrm{K}_i)\, \tilde{q} - \epsilon \tilde{q}^T \mathrm{C}^{\mathrm{T}} \dot{q} - \epsilon \tilde{q}^T [\mathrm{g}(\mathrm{q}_{\mathrm{d}}) - \mathrm{g}(\mathrm{q})] \quad (37)$$

where we also used, $\mathrm{g}(\mathrm{q}) = \frac{\partial U(q)}{\partial q}$

From standard properties of symmetric positive definite matrices, we conclude also that the first two terms of the derivative of the Lyapunov function candidate above satisfy the following inequalities [6]:

$$- \dot{\tilde{q}}^{T}(\mathrm{K}_v - \epsilon \widetilde{\mathrm{M}}_t)\, \dot{q} \leq [\lambda_{min}\{\mathrm{K}_v\} - \epsilon\, \lambda_{max}\{\widetilde{\mathrm{M}}_t\}] ||\dot{q}||^2 \quad (38)$$

$$- \tilde{q}^T(\epsilon \mathrm{K}_p - \mathrm{K}_i)\, \tilde{q} \leq [\epsilon \lambda_{min}\{\mathrm{K}_p\} - \lambda_{max}\{\mathrm{K}_i\}] ||\tilde{q}||^2 \quad (39)$$

Now, for our quadcopter, as it has exclusively revolute joints, there exists a number $k_{C1} > 0$ such that:

$$||C(q, x)y|| \leq k_{C1}\, ||x|| \, ||y||$$

Also, the vector g(q) is Lipschitz, that is, there exists a constant $k_g > 0$ such that:

$$||g(x) - g(y)|| \leq k_g\, ||x - y||$$

Now, using these two properties, we can write (38) and (39) as:

$$-\epsilon \dot{\tilde{q}}^T C(q, \dot{q})^{\mathrm{T}} \dot{q} \leq \epsilon\, k_{C1}\, ||\tilde{q}|| \, ||\dot{q}^2||$$

$$-\epsilon \dot{\tilde{q}}^T [g(q_d) - g(q)] \leq \epsilon\, k_g\, ||\tilde{q}||^2$$

Consequently, it also holds that,

$$\dot{V}(\tilde{q}, \dot{q}, \gamma) \leq - \begin{bmatrix} ||\tilde{q}|| \\ ||\dot{q}|| \end{bmatrix}^T \begin{bmatrix} \wedge_{11} & 0 \\ 0 & \wedge_{22} \end{bmatrix} \begin{bmatrix} ||\tilde{q}|| \\ ||\dot{q}|| \end{bmatrix}$$

Where, $\wedge_{11} = \epsilon \lambda_{min}\{\mathrm{K}_p\} - \lambda_{max}\{\mathrm{K}_i\}$ and

$\wedge_{22} = \lambda_{min}\{\mathrm{K}_v\} - \epsilon\, \lambda_{max}\{\widetilde{\mathrm{M}}_t\}$

These values are chosen so that the time derivative of the Lyapunov candidate function is negative–definite if ϵ is selected to satisfy the following condition in (35).

1.

If all the conditions over ϵ are satisfied, it can be ensured that the origin of the closed–loop equation (32) is a stable equilibrium point of the system. The LaSalle theorem needs to be applied in order to prove the asymptotic stability of the origin [7]. So, the set Ω is defined as:

$$\Omega = \{ Z_n \in \mathrm{IR}^{18} : \dot{V}(Z_n) = 0 \}$$

$$= \{ \gamma \in \mathrm{IR}^6, \tilde{q} = 0 \in \mathrm{IR}^6, \dot{q} = 0 \in \mathrm{IR}^6 \}$$

Now, $\dot{V}(\tilde{q}, \dot{q}, \gamma) = 0$, if and only if $\tilde{q} = 0$ *and* $\dot{q} = 0$. For the solution $Z_n(t)$ to belong to Ω for all $t \geq 0$, it is necessary that both $\tilde{q} = 0$ *and* $\dot{q} = 0$, for all $t \geq 0$. Hence, $\ddot{q} = 0$ also needs to be satisfied for the same.

As it can be concluded that $Z_n(t) \in \Omega$ for all $t \geq 0$, then, $\gamma = 0$, for all $t \geq 0$. Hence, the only initial condition in Ω for which $Z_n \in \Omega$ for all $t \geq 0$ can be given as $[\gamma^T \quad \tilde{q}^T \quad \dot{q}^T] = 0^{\mathrm{T}}$. 0. Therefore, it can be assumed that the origin of the closed– loop equation (32) is an asymptotic stable equilibrium point.

VI. DESIGN OF THE PD POSITION CONTROLLER BASED ON THE CONTROL THEORY OF ROBOT MANIPULATORS

In this section, the design procedure for the PD controller is described [23]. It is important to mention that the procedure followed is based on the control theory of robot manipulators similar to what has been shown in the previous section.

Considering the vectors q and q˜ previously established, the state of the system can be defined as [7]:

$$\Gamma = \begin{bmatrix} \tilde{q} \\ \dot{q} \end{bmatrix} \in \mathrm{IR}^{12} \quad (40)$$

By knowing that the PD control law is given as:

$$\tau = \mathrm{K}_p \tilde{q} - \mathrm{K}_v \dot{q} \quad (41)$$

This can be written in matrix form as:

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = \mathrm{K}_p \tilde{q} - \mathrm{K}_v \dot{q}$$

with K_p and K_v as defined in the previous section.

Now, we can redefine equation (30) for the above equation as [6]:

$$\ddot{q} = \begin{bmatrix} (\mathrm{K}_{p_x}\, \tilde{\mathrm{x}} - \mathrm{K}_{v_x}\, \dot{\mathrm{x}})/\mathrm{m} \\ (\mathrm{K}_{p_y}\, \tilde{\mathrm{y}} - \mathrm{K}_{v_y}\, \dot{\mathrm{y}})/\mathrm{m} \\ (\mathrm{K}_{p_z}\, \tilde{\mathrm{z}} - \mathrm{K}_{v_z}\, \dot{\mathrm{z}} - \mathrm{mg})/\mathrm{m} \\ (\mathrm{K}_{p_\varphi}\, \tilde{\varphi} - \mathrm{K}_{v_\varphi}\, \dot{\varphi})/\mathrm{I}_{xx} \\ (\mathrm{K}_{p_\theta}\, \tilde{\theta} - \mathrm{K}_{v_\theta}\, \dot{\theta})/\mathrm{I}_{yy} \\ (\mathrm{K}_{p_\psi}\, \tilde{\Psi} - \mathrm{K}_{v_\psi}\, \dot{\Psi})/\mathrm{I}_{zz} \end{bmatrix} \quad (42)$$

This equation corresponds to the control to be implemented on the quadcopter based on the dynamic model of equation (25).

A. Stability analysis using a Lyapunov Function

Using the time derivative of equation (40) and letting this be equal to $0 \in \mathrm{IR}^{12}$, the equilibrium of the system can be found. Since this equilibrium point is not equal to the origin of the system, it is necessary to make a change of variables in the manner that:

$$\delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \delta_\varphi \\ \delta_\theta \\ \delta_\psi \end{bmatrix}$$

Now, replacing these values with those in (42):

$$\delta = \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} - {K_{pz}}^{-1} mg \\ \varphi \\ \theta \\ \psi \end{bmatrix}$$

So now, it is convenient to define a new vector of states $\Gamma_n \in IR^{12}$ as:

$$\Gamma_n = \begin{bmatrix} \delta \\ \dot{q} \end{bmatrix} \quad (43)$$

where the origin of the system is the only equilibrium point of the system. We consider the dynamic model which does not contain the gravitational g(q), that is [6] :

$$\widetilde{M}_t + C\,(q,\dot{q})\,\dot{q} = \tau \quad (44)$$

Also, its closed loop equation is given by:

$$\frac{d}{dt}\begin{bmatrix} \tilde{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -\dot{q} \\ \ddot{q} \end{bmatrix} \quad (45)$$

Where, $\ddot{q} = {\widetilde{M}_t}^{-1}[K_p\tilde{q} - K_v\dot{q} - C\,(q_d - q, \dot{q})\,\dot{q}\ ]$ (46)

Let the Lyapunov candidate function, as defined in (36), be taken as:

$$V\,(\delta, \dot{q}) = \frac{1}{2}\begin{bmatrix} \delta \\ \dot{q} \end{bmatrix}^T \begin{bmatrix} K_p & 0 \\ 0 & \widetilde{M}_t \end{bmatrix} \begin{bmatrix} \delta \\ \dot{q} \end{bmatrix}$$
$$= \frac{1}{2}\,\dot{q}^T\,\widetilde{M}_t\,\dot{q} + \frac{1}{2}\,\tilde{q}^T K_p\,\tilde{q} \quad (47)$$

which is positive–definite since K_p and $\widetilde{M}_t$ are positive– definite matrices. The derivative is given as:

$$\dot{V}\,(\delta, \dot{q}) = \frac{1}{2}\,\dot{q}^T\,\widetilde{M}_t\,\ddot{q} + \frac{1}{2}\,\dot{q}^T\,\dot{\widetilde{M}}_t\,\dot{q} + \tilde{q}^T K_p\,\dot{\tilde{q}}$$

Now, from (41), (44), (45) and (46), we can equate the above equation as:

$$\dot{V}\,(\delta, \dot{q}) = -\frac{1}{2}\,\dot{q}^T\,\widetilde{M}_t\,{\widetilde{M}_t}^{-1}[K_p\tilde{q} - K_v\dot{q} - C\,(q_d - q, \dot{q})\,\dot{q}\ ]$$

$$+ \frac{1}{2}\,\dot{q}^T\,[K_p\tilde{q} - K_v\dot{q} - C\,(q_d - q, \dot{q})\,\dot{q}]\,\dot{q} - \dot{q}^T K_p\,\dot{\tilde{q}}$$

$$\dot{V}\,(\delta, \dot{q}) = -\,\dot{q}^T K_v\,\dot{\tilde{q}}$$

This can be written in matrix form as:

$$\dot{V}\,(\delta, \dot{q}) = -\begin{bmatrix} \delta \\ \dot{q} \end{bmatrix}^T \begin{bmatrix} 0 & 0 \\ 0 & K_v \end{bmatrix} \begin{bmatrix} \delta \\ \dot{q} \end{bmatrix}$$

where K_v is a positive–definite matrix.

With these two properties of the function shown in equation (46), it can be ensured that the origin of the system (43) is a stable equilibrium point. It is necessary to apply the LaSalle theorem in the same way that it was used for the PID controller to prove asymptotic stability [7].

Let the set Π be:

$$\Pi = \{\ \Gamma_n \in IR^{12} : \dot{V}(\Gamma_n) = 0\}$$
$$= \{\ \delta \in IR^6\ ,\ \dot{q} = 0 \in IR^6\ \}$$

Now, we can say that $\dot{V}$ (δ, $\dot{q}$) = 0 0 if and only $\dot{q}$ = 0. For the solution Γ_n(t) t) to belong to Π for all t ≥ 0, it is necessary that $\dot{q}$ = 0 for all t ≥ 0. Hence, also $\ddot{q}$=0 0 must be satisfied for all t ≥ 0. Taking this into consideration, it can be concluded that if Γ_n(t) belongs to Π for all t ≥ 0, then δ =0 for all t ≥ 0. From these statements, it can be assumed that the origin of the closed– loop equation (43) is also an asymptotic stable equilibrium point.

VII. DESIGN OF THE SLIDING–MODE POSITION CONTROLLER (SMC)

Sliding–mode controllers have been successfully adapted to UAVs. In this section, the design of a controller of SMC is shown, while the stability analysis is carried out using the control theory of robot manipulators.

SMC objective involves two parts: in the first one, a control law to enforce the error vector toward a decision rule, called sliding surface, during the reaching phase is designed, this part the control is switching on the different sides of this sliding surface and secondly, once the error vector is restricted in the sliding surface, it tracks the dynamics imposed by the equations describing the sliding surface, this second part of the controller is called equivalent control [20].

Let the sliding surfaces be defined and given in the vector S ∈ IR^6 as [7] :

$$S = \begin{bmatrix} S_x \\ S_y \\ S_z \\ S_\varphi \\ S_\theta \\ S_\psi \end{bmatrix} \quad (48)$$

Now, the sliding surface equation can be written as:

$$S = \dot{e}\_ + \lambda e \quad (49)$$

Where λ is a tuning parameter design greater than zero and e is the sliding surface.

Now, (48) can be written in terms of (49), i.e.,

$$S = \begin{bmatrix} \dot{x} + \lambda\,\tilde{x} \\ \dot{y} + \lambda\,\tilde{y} \\ \dot{z} + \lambda\,\tilde{z} \\ \dot{\varphi} + \lambda\,\tilde{\varphi} \\ \dot{\theta} + \lambda\,\tilde{\theta} \\ \dot{\Psi} + \lambda\,\tilde{\Psi} \end{bmatrix}$$

with $\lambda_i > 0$.

By knowing that $\dot{q} = -\dot{q}$, the sliding surfaces can be expressed as:

$$S = \begin{bmatrix} -\dot{x} + \lambda\,\tilde{x} \\ -\dot{y} + \lambda\,\tilde{y} \\ -\dot{z} + \lambda\,\tilde{z} \\ -\dot{\varphi} + \lambda\,\tilde{\varphi} \\ -\dot{\theta} + \lambda\,\tilde{\theta} \\ -\dot{\Psi} + \lambda\,\tilde{\Psi} \end{bmatrix}$$

$$= -\dot{q} + K_l\tilde{q} \;(50)$$

with $K_l \in IR^{6\times6}$ being the diagonal matrix of gains λ_i . It is necessary to define the attractive sliding surfaces, which depend on the vector $\dot{S} \in IR^6$ such that [20] :

$$\dot{S} = -K_k \text{sign}(S) \;(51)$$

Where, $K_k \in IR^{6\times6}$ 6 is a diagonal matrix that depends on the gains of the controller. Satisfying this, the sliding condition $S^T\dot{S} \leq 0$.

By considering equation (50), the time derivative of the sliding surfaces vector S can also be expressed as [20]:

$$\dot{S} = -\ddot{q} - K_l q \;(52)$$

Substituting equation (51) into equation (52) and solving for $\ddot{q}$, the following result can be obtained:

$$\ddot{q} = K_k \text{sign}(S) - K_l q \;(53)$$

or in an extended form:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\Psi} \end{bmatrix} = \begin{bmatrix} K_{k_x}\text{sign}(S_x) - \lambda_x\,\dot{x} \\ K_{k_y}\text{sign}(S_y) - \lambda_y\,\dot{y} \\ K_{k_z}\text{sign}(S_z) - \lambda_z\,\dot{z} \\ K_{k_\varphi}\text{sign}(S_\varphi) - \lambda_\varphi\,\dot{\varphi} \\ K_{k_\theta}\text{sign}(S_\theta) - \lambda_\theta\,\dot{\theta} \\ K_{k_\Psi}\text{sign}(S_\Psi) - \lambda_\Psi\,\dot{\Psi} \end{bmatrix} (54)$$

A. Stability analysis using a Lyapunov Function

To analyze the stability of the system, let the Lyapunov candidate function be given as:

$$V(S) = \frac{1}{2} S^T S \;(55)$$

where S was established in equation (50) and is also used to describe the state of the system.It can be proved that the Lyapunov candidate function (55) is a local positive–definite function while the time derivative of it is local negative–definite function and is given as:

$$\dot{V}(S) = S^T\dot{S}$$

So, it can be guaranteed that the origin of the system is a stable equilibrium point. Now, to apply the LaSalle theorem, let the set Φ be defined as:

$$\Phi = \{S \in IR^6 : \dot{V}(S) = 0\} = \{S=0\}$$

Herein, the only element of the set is the zero vector 0, which ensures the asymptotic stability of the origin.

VIII. SIMULATIONS AND RESULTS

The three controllers were designed and analyzed; however, we got the conclusion that the implementation that was provided is either not held to water or some important parts were not published, therefore we had to make modifications based on the physics of the drone and control theory principles. The following equations are our corrections and implementations:

$$S_\varphi = \frac{\ddot{x}S_\Psi - \ddot{y}C_\Psi}{\sqrt{\ddot{x}^2+\ddot{y}^2+(\ddot{z}+g)^2}} \;(56)$$

$$T_\theta = \frac{\ddot{x}C_\Psi + \ddot{y}S_\Psi}{\ddot{z}+g} \;(57)$$

$$m\,\ddot{\xi} + m\,[0\ 0\ g]^T = R\cdot f \;(58)$$

$$U_1 = R^T\,(m\ddot{\xi}_{ref} + m\,[0\ 0\ g]^T \;(59)$$

The modified block diagrams can be seen below, the main modifications are the following:

- The non-holonomic constraints are fed with the calculated Ψ instead of the desired.
- An additional subsystem was added to calculate the U_1 based on the equation (58), since the force acting on the drone must be perpendicular to the plane of the propellers, therefore U_1 can be calculated by the means of (59).
- The SMC controller in our implementations is just the modified version of the PD/PID, since the provided block diagram did not seem right, the two controller parts had to be modified, therefore additional feedback was needed to recreate the equations from equation (54).
- The non-holonomic constraints sadly also were not detailed or referred to in the original paper, so we acknowledge them but we use Atan in (57).

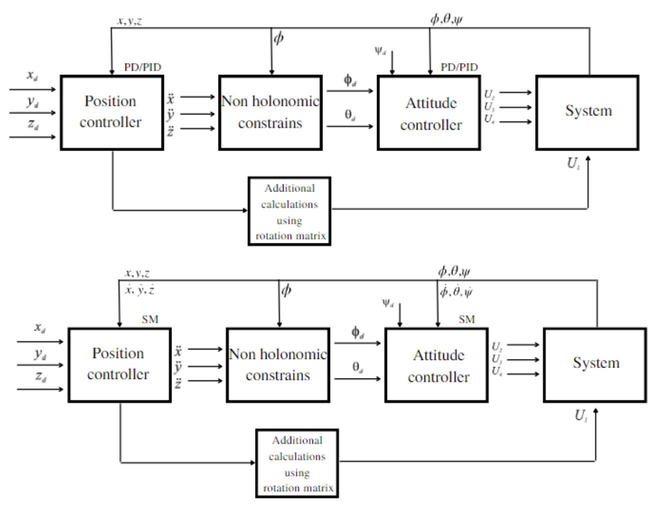
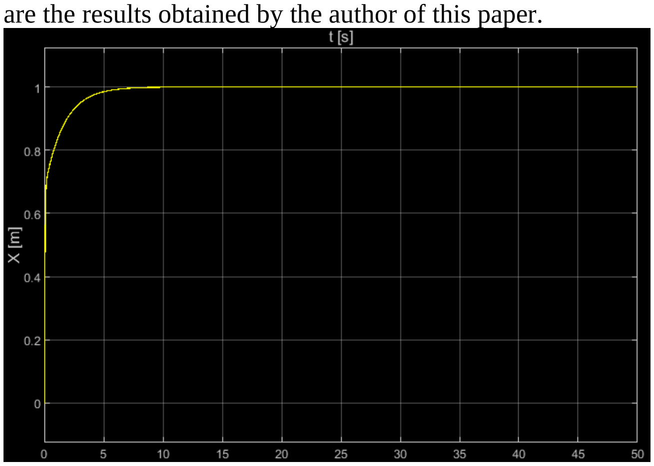


Figure 5: Corrected Block diagrams

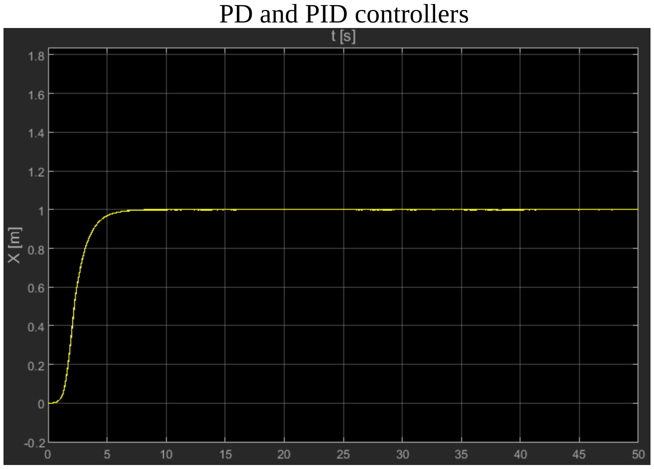
There is the system block that contains the dynamic equations of our system, and there are two controllers: an inner that is responsible for the orientation of the quadcopter and an outer that deals with the position of it.

By observing the graphs resulted from the simulation the exponential behavior of the PD & PID can be seen, since we built our own controllers and the gain values were not given the results are not complete the same especially the angular values of PD and PID, we tried many different combinations but they were not working out on the way that they should have it can be due to the integrations and the trigonometric functions, but since we had to build it up from scratch it shows something similar that it should.

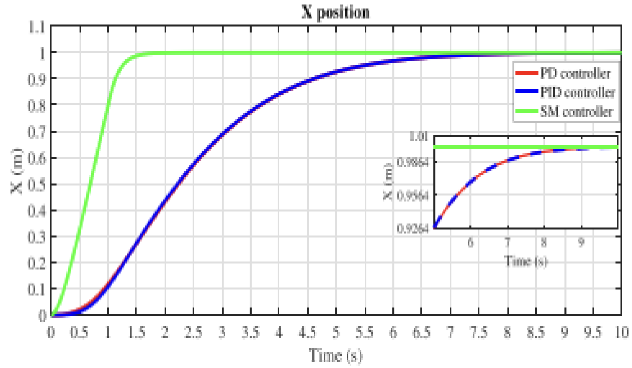
The figures below show our output results and the results obtained by the author of the paper. The figures with a black background are our results and the ones in white background are the results obtained by the author of this paper.



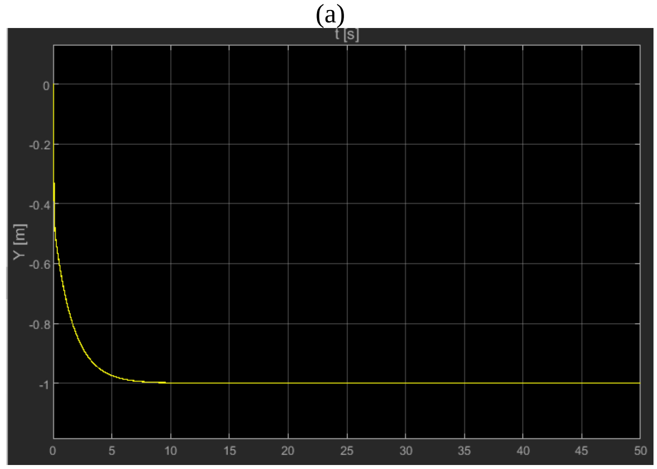
PD and PID controllers



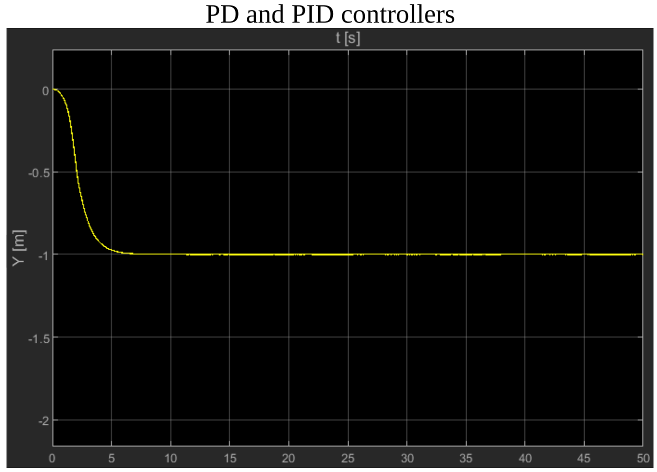
SMC controller



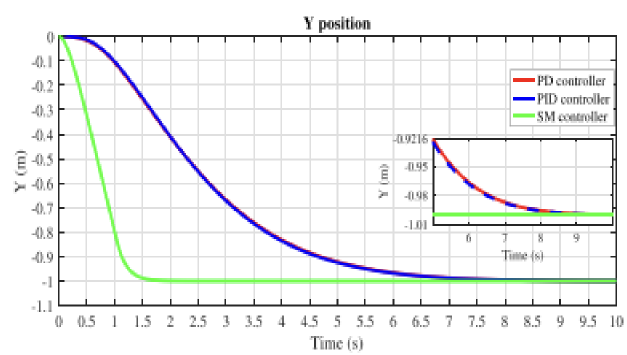
(a)



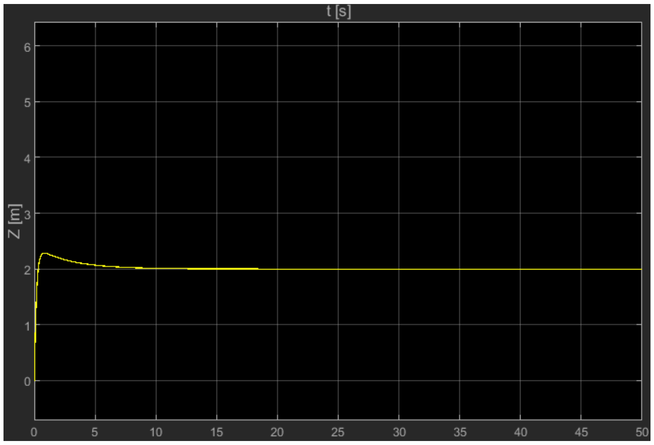
PD and PID controllers



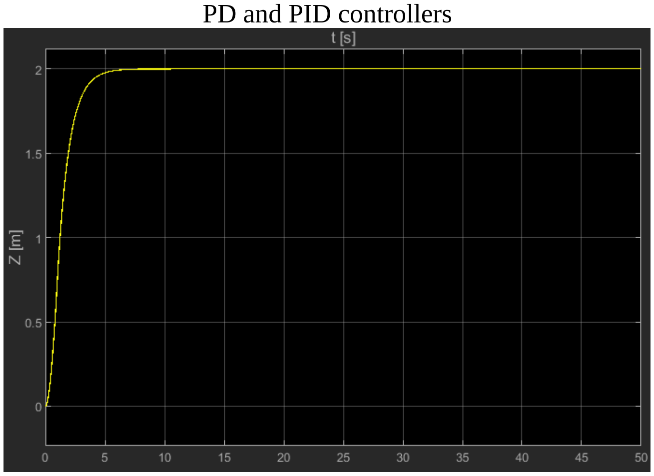
SMC controller



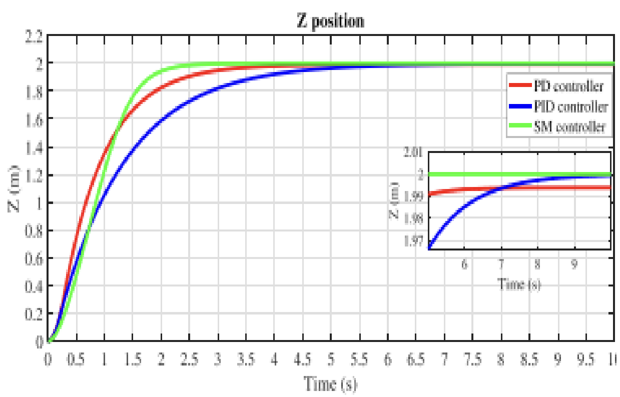
(b)



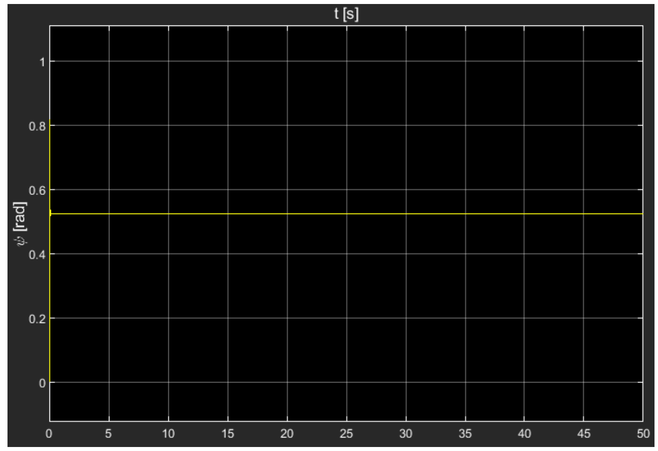
PD and PID controllers



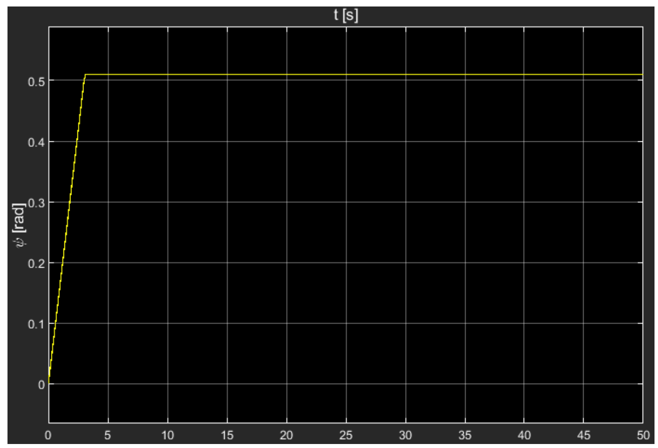
SMC controllers



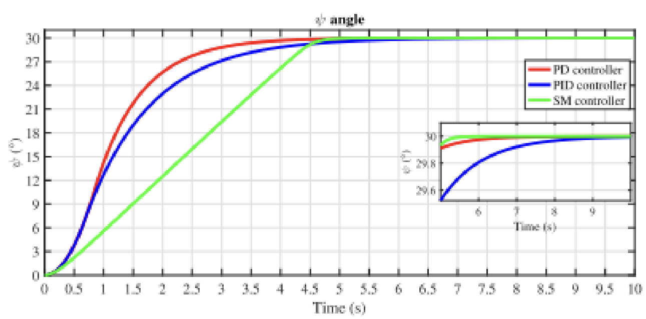
(c)



PD and PID controllers

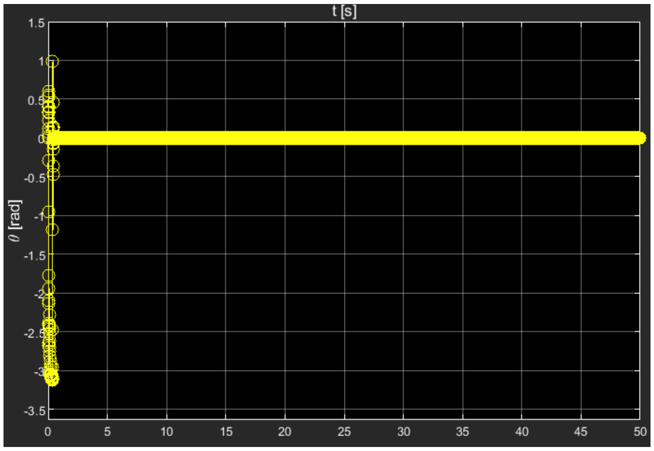


SMC controller

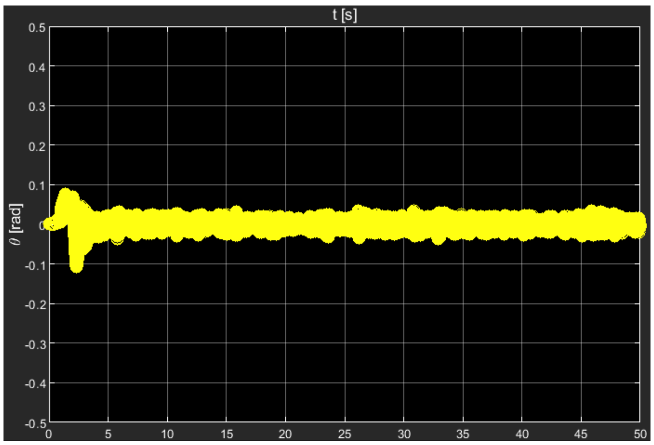


(a)

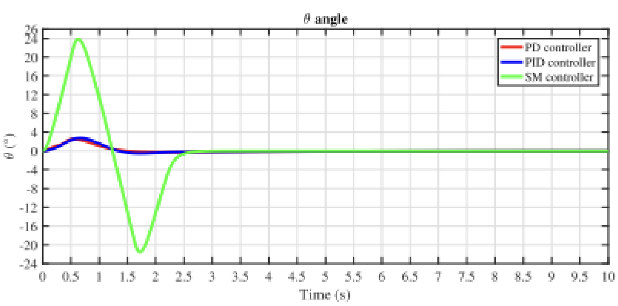
Figure 6: Position of the quadcopter for the three controllers: (a) X–position , (b) Y–position, (c) Z–position



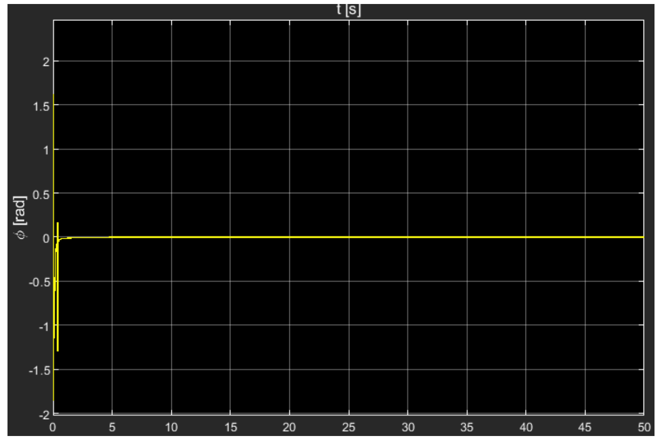
PD and PID controllers



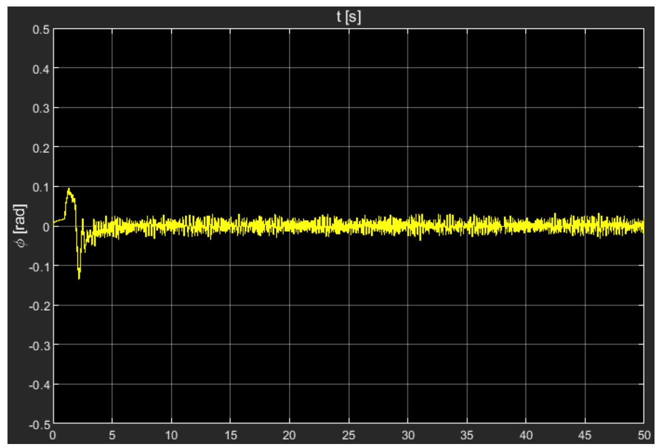
SMC controller



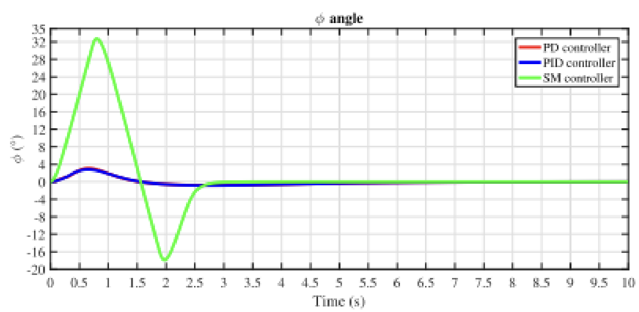
(b)



PD and PID controllers



SMC controller



(c)

Figure 7: Angles of the quadcopter for the three controllers; (a) Yaw (ψ), (b) Pitch (θ) and (c) Roll (φ)

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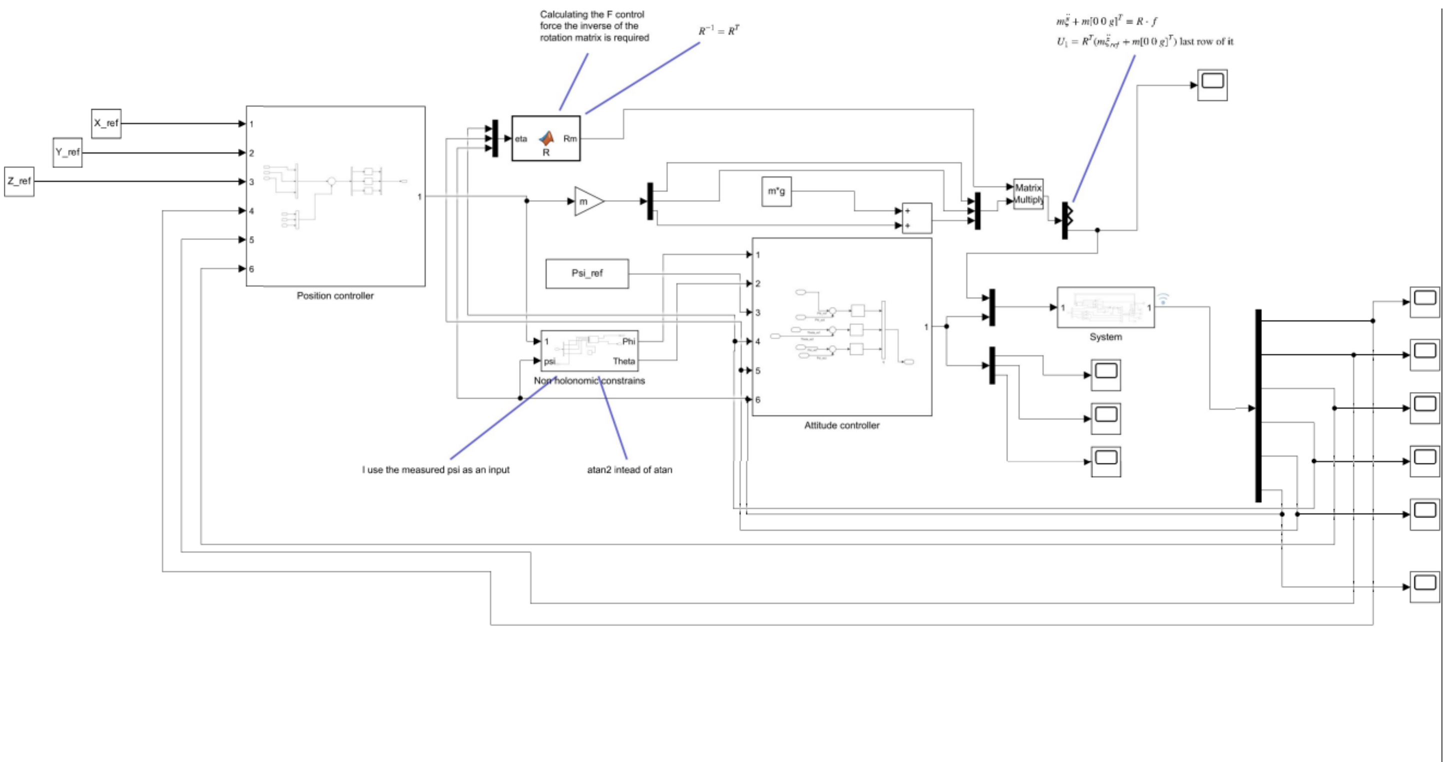
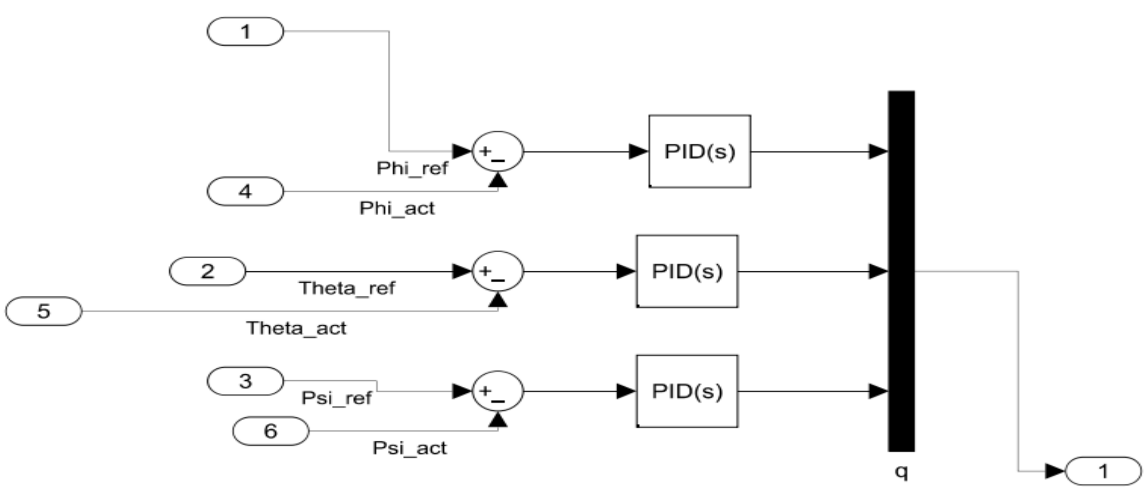
Code:

1) Workspace

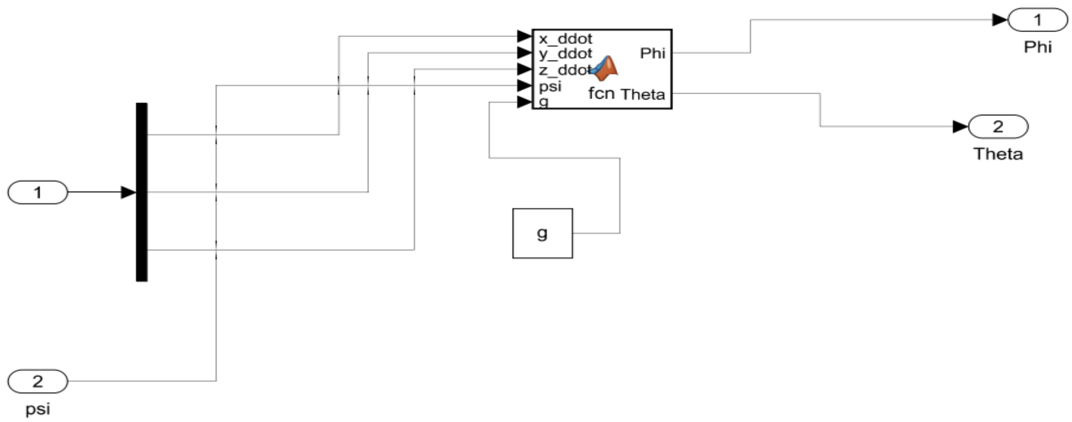
```
I_xx=2.547*10^-3;
I_yy=3.613*10^-3;
I_zz=1.074*10^-3;
l_b=0.083;
l_c=0.04;
l_f=0.093;
l_l=0.185;
l_t=0.2;
l_y=0.4;
b=4.95*10^-5;
d=7.5*10^-7;
m=2.03;

g=9.8; % it should be positive
X_ref=1;
Y_ref=-1;
Z_ref=2;
Psi_ref=pi/6;


%%%% SMC
lambda_x=1;
lambda_y=1;
lambda_z=1;
lambda_phi=6;
lambda_theta=6;
lambda_psi=6;

K_k_x=1;
K_k_y=1;
K_k_z=0.1*20;
K_k_phi=1*20;
K_k_theta=1*20;
K_k_psi=0.1*10;
```

2)PID Controller

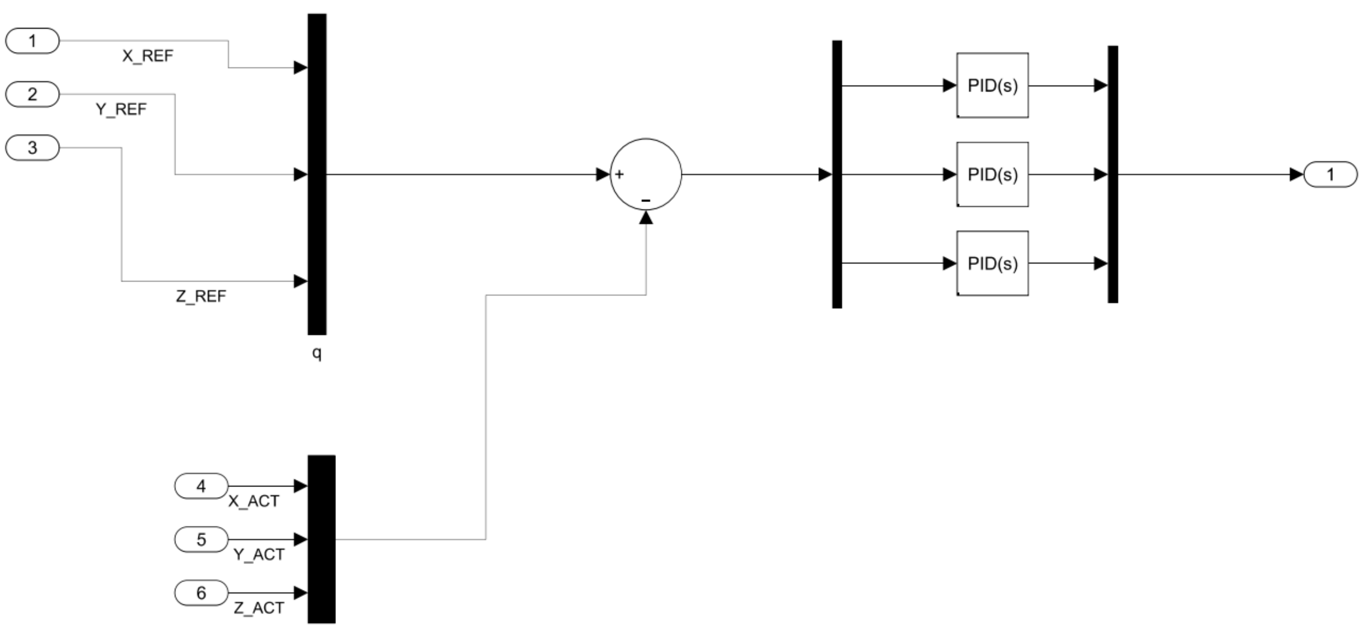


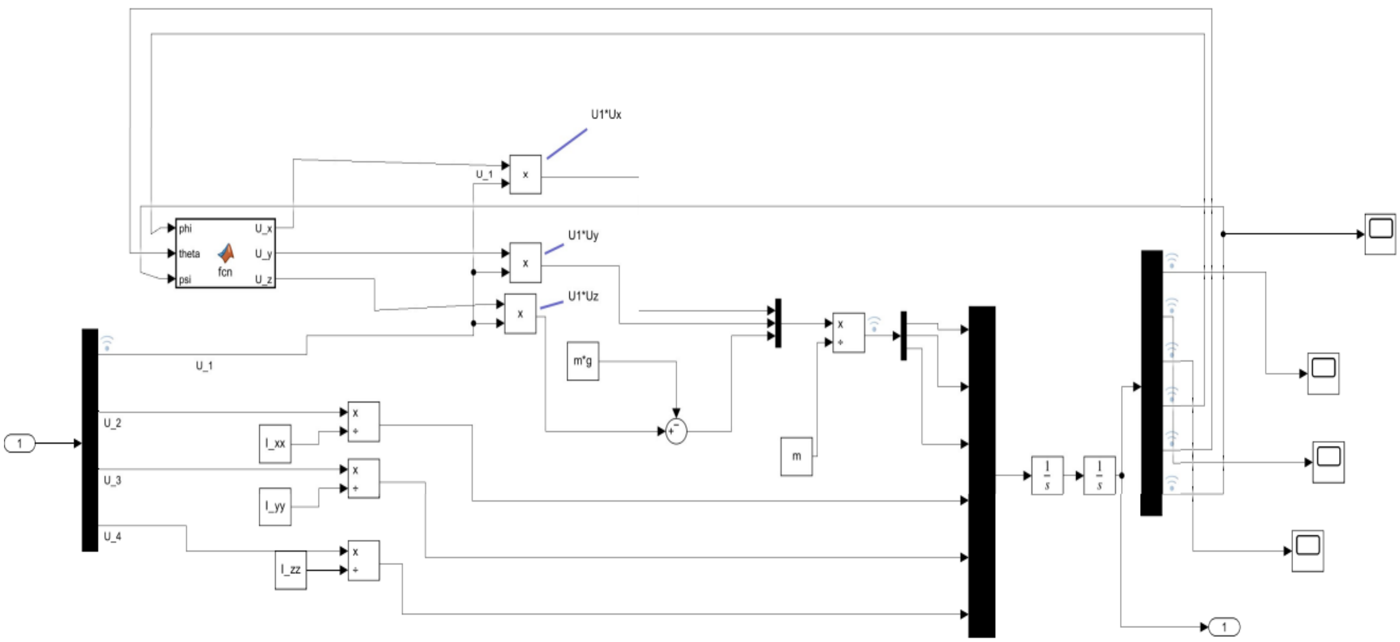


```
function Rm = R(eta)
phi = eta(1);
theta = eta(2);
psi = eta(3);

Rx = [1 0 0; 0 cos (phi)  -sin(phi); 0 sin(phi) cos(phi)];
Ry = [cos(theta) 0 sin(theta); 0 1  0; -sin(theta) 0 cos(theta)];
Rz = [cos(psi) -sin(psi) 0; sin(psi) cos (psi)  0; 0 0 1];
Rm = (Rx*Ry*Rz) ‘;
```




```
function [Phi, Theta] = fcn(x_ddot, y_ddot, z_ddot, psi, g)

Phi = asin((x_ddot*sin(psi)-y_ddot*cos(psi))/(sqrt(x_ddot^2+y_ddot^2+(z_ddot+g)^2))) ;
Theta = atan2((x_ddot*cos(psi) + y_ddot*sin(psi)) , (z_ddot+g)) ;
```

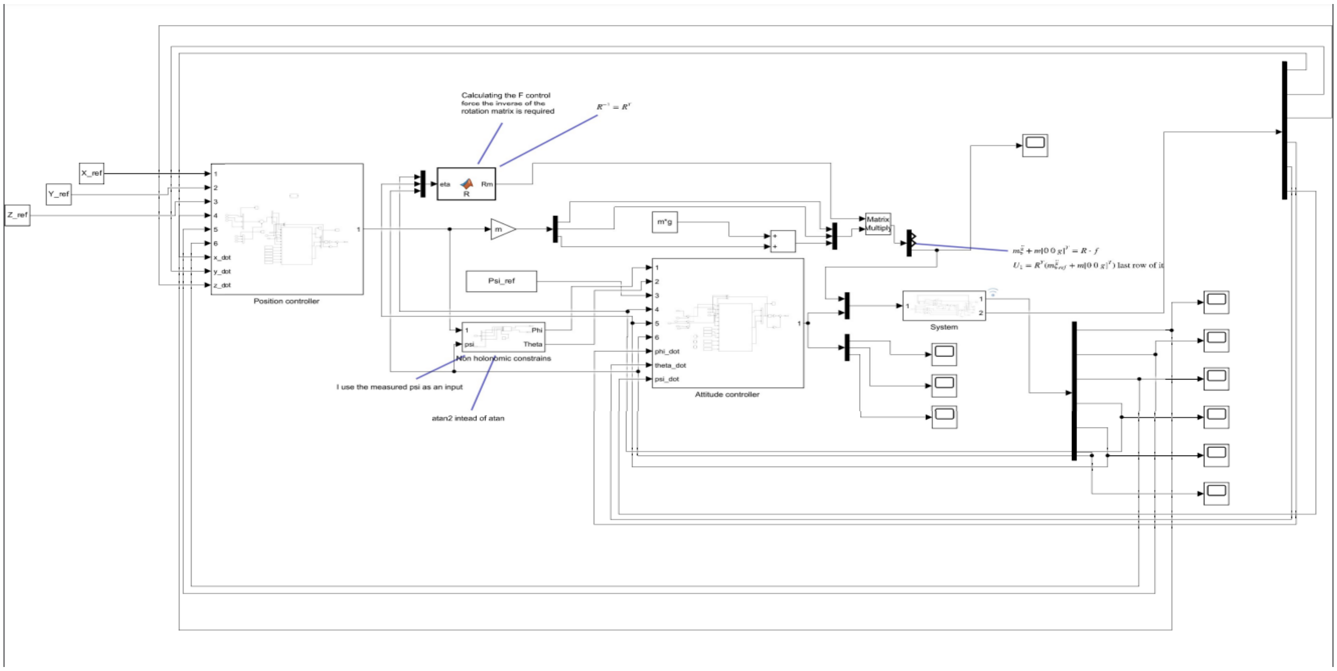


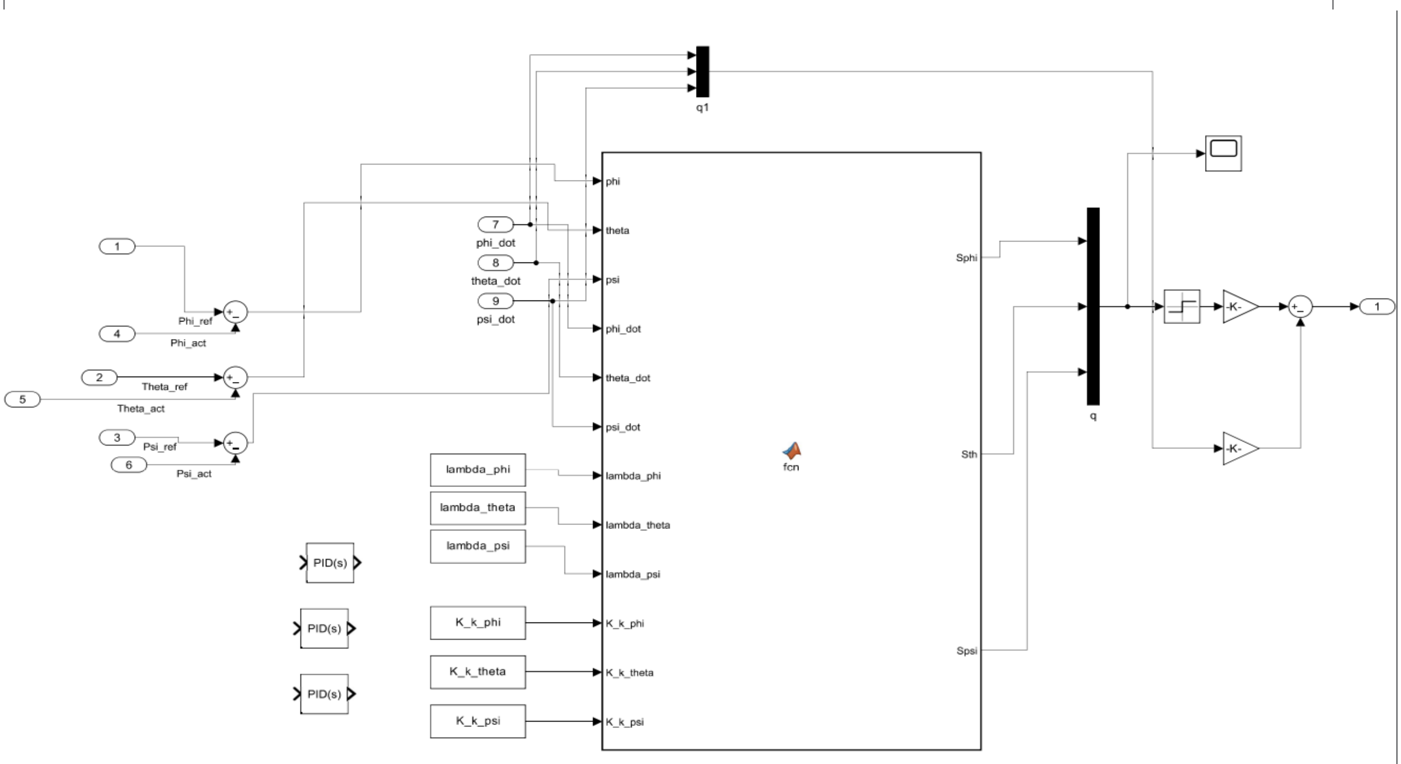


function [U_x,U_y,U_z] = fcn(phi,theta,psi)

*U_x = cos(phi)*sin(theta)*cos(psi)+sin(phi)*sin(psi);*
*U_y = cos(phi)*sin(theta)*sin(psi)-sin(phi)*cos(psi);*
*U_z = cos(phi)*cos(theta);*

3)SMC Controller



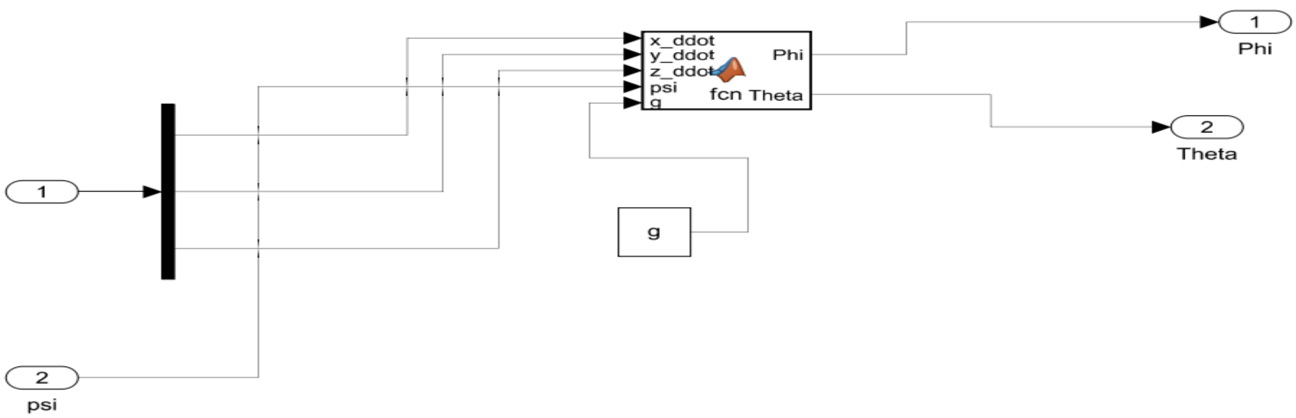


function [Sphi,Sth,Spsi] = fcn(phi, theta, psi, phi_dot, theta_dot, psi_dot, lambda_phi, lambda_theta, lambda_psi, K_k_phi, K_k_theta, K_k_psi)

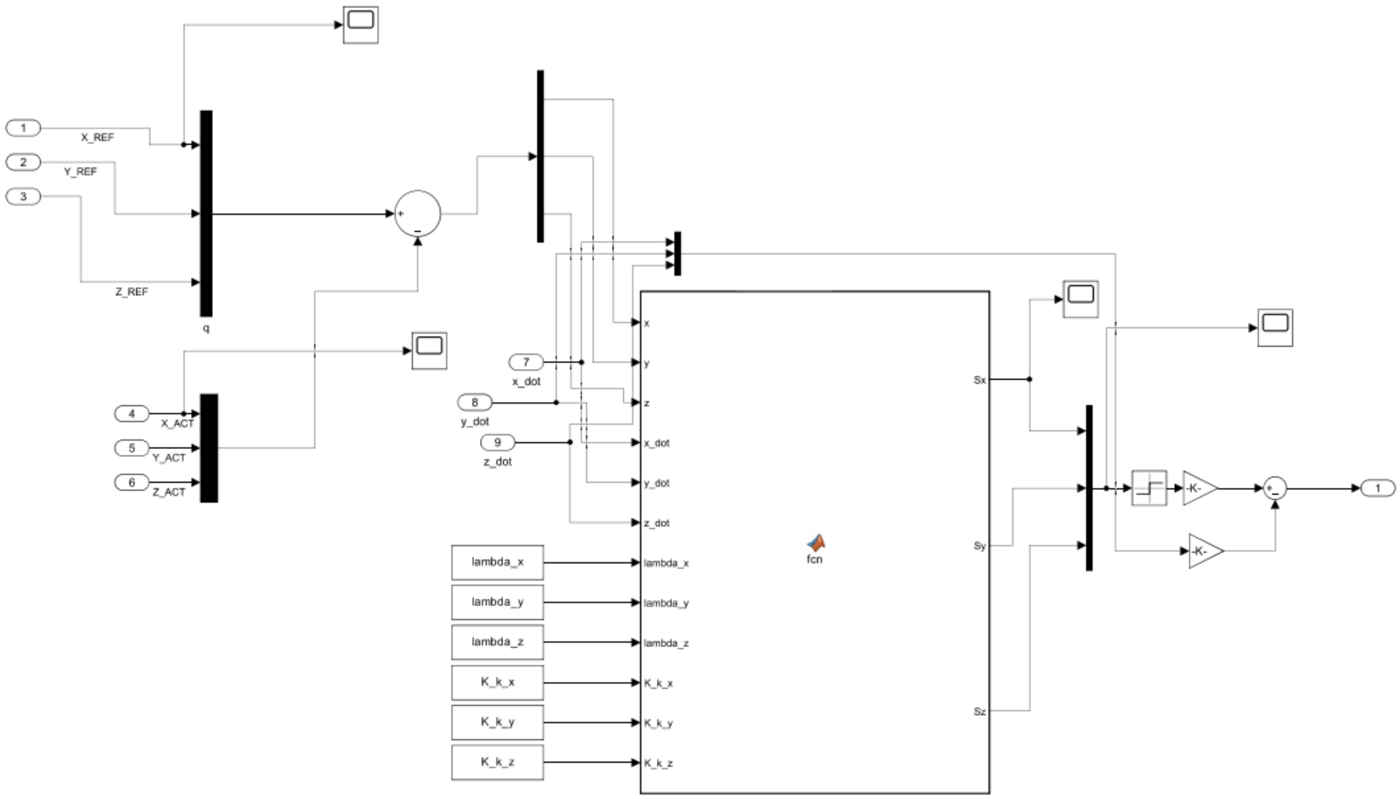
*Sphi = (-phi_dot+lambda_phi*phi);*
*Sth = (-theta_dot+lambda_theta*theta);*
*Spsi = (-psi_dot+lambda_psi*psi);*

function Rm = R(eta)
phi = eta(1);

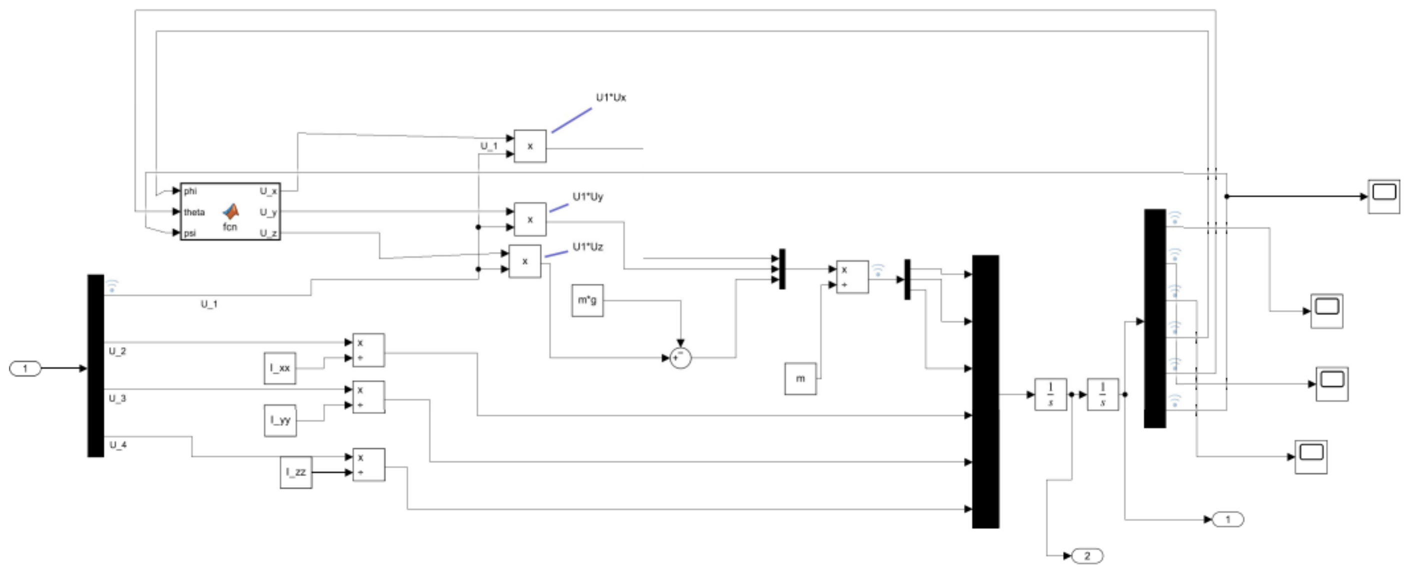
```
theta = eta(2);
psi = eta(3);

Rx = [1 0 0; 0 cos (phi)  -sin(phi); 0 sin(phi) cos(phi)];
Ry = [cos(theta) 0 sin(theta); 0 1  0; -sin(theta) 0 cos(theta)];
Rz = [cos(psi) -sin(psi) 0; sin(psi) cos (psi)  0; 0 0 1];
Rm = (Rx*Ry*Rz) ';
```




```
function [Phi, Theta] = fcn(x_ddot, y_ddot, z_ddot, psi, g)

Phi = asin((x_ddot*sin(psi)-y_ddot*cos(psi))/(sqrt(x_ddot^2+y_ddot^2+(z_ddot+g)^2))) ;
Theta = atan2((x_ddot*cos(psi) + y_ddot*sin(psi)) , (z_ddot+g)) ;
```




```
function [Sx,Sy,Sz] = fcn(x, y, z, x_dot, y_dot, z_dot, lambda_x, lambda_y, lambda_z, K_k_x, K_k_y, K_k_z)

Sphi = (-x_dot+lambda_x*x);
Sth = (-y_dot+lambda_y*y);
Spsi = (-z_dot+lambda_z*z);

function Rm = R(eta)
phi = eta(1);
theta = eta(2);
psi = eta(3);

Rx = [1 0 0; 0 cos (phi)  -sin(phi); 0 sin(phi) cos(phi)];
Ry = [cos(theta) 0 sin(theta); 0 1  0; -sin(theta) 0 cos(theta)];
Rz = [cos(psi) -sin(psi) 0; sin(psi) cos (psi)  0; 0 0 1];
Rm = (Rx*Ry*Rz) ‘;
```



function [U_x,U_y,U_z] = fcn(phi,theta,psi)

*U_x = cos(phi)*sin(theta)*cos(psi)+sin(phi)*sin(psi);*
*U_y = cos(phi)*sin(theta)*sin(psi)-sin(phi)*cos(psi);*
*U_z = cos(phi)*cos(theta);*