In [95]:

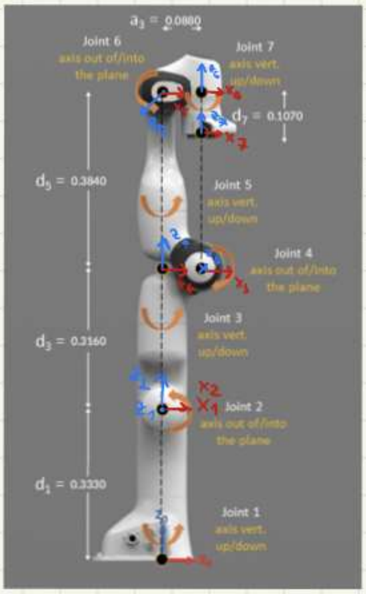
```
from sympy import*
from IPython.display import Image, display, HTML
from scipy import optimize
import matplotlib.pyplot as plt
import numpy as np
%matplotlib qt
```

Homework 4.

In [96]:

```
Image("Doc.png")
```

Out[96]:



DH Spronge

i	Θ_i (joint variable)	d_i	a_i (geometric parameters)	α_i
1	q_1	d_1	0	90
2	q_2	0	0	-90
3	q_3	d_3	a_3	-90
4	q_4	0	$-a_3$	90
5	q_5	d_5	0	90
6	q_6	0	a_3	-90
7	q_7	$-d_7$	0	0

$x_i \perp z_{i-1}$

$x_i \cap z_{i-1}$

D-H parameters:

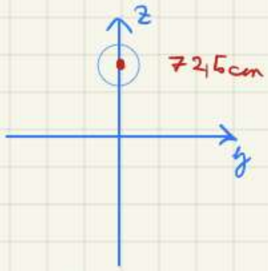
d_i: x_i distance from x_{i-1} along z_{i-1}

Θ_i: x_i angle from x_{i-1} around z_{i-1}

a_i: z_i distance from z_{i-1} along x_i

α_i: z_i angle from z_{i-1} around x_i

According to the Homework 4 $q_3 = 0$ and since the pen is 10cm long I include it in the d_7 shift so:



DH Spronge

i	Θ_i (joint variable)	d_i	a_i (geometric parameters)	α_i
1	q_1	d_1	0	90
2	q_2	0	0	-90
3	0	d_3	a_3	-90
4	q_4	0	$-a_3$	90
5	q_5	d_5	0	90
6	q_6	0	a_3	-90
7	q_7	$-(d_7+10)$	0	0

→ the frame can stay, it just not been used in the Jacobi

circle equation:

$$y^2 + (z - 72{,}5)^2 = r^2 = 10^2$$

Polar coordinates:

$y = r \cdot \cos(\Theta + 90°)$ // 90° shift since we start

$z = r \cdot \sin(\Theta + 90°)$ from the top

$x = 67{,}9$

$\Theta = \left(\frac{2\pi}{5} \cdot t\right)$ where t goes from [0;5]

since the pen does not rotate its φ, θ, ψ = constant :

So $\dot{X} = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} -r \sin(\Theta + 90°) \cdot \dot{\Theta} \\ r \cos(\Theta + 90°) \cdot \dot{\Theta} \\ \vdots \end{bmatrix}$

For the inverse kinematics the Jacobian is calculated by the second methode using the ${}^0z_i$ and $\frac{{}^0x_p}{\partial q_i}$, where ${}^0z_i$ is the z rotation of each ${}^0T_i$ transformation and ${}^0x_p$ is the translation of ${}^0T_7$, where i goes from 1...7, except 3, and $q_3 = 0$

$$J = \begin{bmatrix} \frac{\partial\, {}^0x_p}{\partial q_1} & \frac{\partial\, {}^0x_p}{\partial q_2} & \frac{\partial\, {}^0x_p}{\partial q_4} & \frac{\partial\, {}^0x_p}{\partial q_5} & \frac{\partial\, {}^0x_p}{\partial q_6} & \frac{\partial\, {}^0x_p}{\partial q_7} \\ {}^0z_1 & {}^0z_2 & {}^0z_4 & {}^0z_5 & {}^0z_6 & {}^0z_7 \end{bmatrix}$$

$$\begin{cases} \dot{q} = J^{-1}(q)\,\dot{X} \\ q_{next} = q_{current} + \dot{q}\,\Delta t \end{cases}$$

and y, z can be plotted by calculating the transformation matrix in each q iteration, ${}^0T_7(q) \rightarrow$ last column $\rightarrow$ y, z

In [97]:

```
q1,q2,q3,q4,q5,q6,q7,d1,d3,d5,d7,a3=symbols('q_1 q_2 q_3 q_4 q_5 q_6 q_7 d_1 d_3 d_5 d_7 a_
```

The transformation matrixes are calculated.

In [98]:

```
def DH_Tr_sym_UMD(theta=0, d=0 , a=0, alpha=0):
    Tr_theta=Matrix([[cos(theta),-sin(theta),0,0],
                     [sin(theta),cos(theta),0,0],
                     [0,0,1,0],
                     [0,0,0,1]])
    Tr_d=Matrix([[1,0,0,0],
                 [0,1,0,0],
                 [0,0,1,d],
                 [0,0,0,1]])
    Tr_a=Matrix([[1,0,0,a],
                 [0,1,0,0],
                 [0,0,1,0],
                 [0,0,0,1]])
    Tr_alpha=Matrix([[1,0,0,0],
                     [0,cos(alpha),-sin(alpha),0],
                     [0,sin(alpha) ,cos(alpha),0],
                     [0,0,0,1]])


    Tr_KHALIL=Tr_theta@Tr_d@Tr_a@Tr_alpha

    return Tr_KHALIL
```

In [99]:

```
H_01=DH_Tr_sym_UMD(theta=q1, d=d1 , a=0, alpha=pi/2)
H_01
```

Out[99]:

$$\begin{bmatrix} \cos(q_1) & 0 & \sin(q_1) & 0 \\ \sin(q_1) & 0 & -\cos(q_1) & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [100]:

```
H_12=DH_Tr_sym_UMD(theta=q2, d=0 , a=0, alpha=-pi/2)
H_02=H_01@H_12
H_12
```

Out[100]:

$$\begin{bmatrix} \cos(q_2) & 0 & -\sin(q_2) & 0 \\ \sin(q_2) & 0 & \cos(q_2) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [101]:

```
H_23=DH_Tr_sym_UMD(theta=q3, d=d3 , a=a3, alpha=-pi/2)
H_03=H_01@H_12@H_23
H_23
```

Out[101]:

$$\begin{bmatrix} \cos(q_3) & 0 & -\sin(q_3) & a_3\cos(q_3) \\ \sin(q_3) & 0 & \cos(q_3) & a_3\sin(q_3) \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [102]:

```
H_34=DH_Tr_sym_UMD(theta=q4, d=0 , a=-a3, alpha=pi/2)
H_04=H_01@H_12@H_23@H_34
H_34
```

Out[102]:

$$\begin{bmatrix} \cos(q_4) & 0 & \sin(q_4) & -a_3\cos(q_4) \\ \sin(q_4) & 0 & -\cos(q_4) & -a_3\sin(q_4) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [103]:

```
H_45=DH_Tr_sym_UMD(theta=q5, d=d5 , a=0, alpha=pi/2)
H_05=H_01@H_12@H_23@H_34@H_45
H_45
```

Out[103]:

$$\begin{bmatrix} \cos(q_5) & 0 & \sin(q_5) & 0 \\ \sin(q_5) & 0 & -\cos(q_5) & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [104]:

```
H_56=DH_Tr_sym_UMD(theta=q6, d=0 , a=a3, alpha=-pi/2)
H_06=H_01@H_12@H_23@H_34@H_45@H_56
H_56
```

Out[104]:

$$\begin{bmatrix} \cos(q_6) & 0 & -\sin(q_6) & a_3\cos(q_6) \\ \sin(q_6) & 0 & \cos(q_6) & a_3\sin(q_6) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [105]:

```
H_67=DH_Tr_sym_UMD(theta=q7, d=-d7 , a=0, alpha=0)
H_07=H_06*H_67
H_67
```

Out[105]:

$$\begin{bmatrix} \cos(q_7) & -\sin(q_7) & 0 & 0 \\ \sin(q_7) & \cos(q_7) & 0 & 0 \\ 0 & 0 & 1 & -d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [106]:

```
H_07.subs({q1:0,q2:0,q3:0,q4:pi/2,q5:0,q6:pi,q7:0})
```

Out[106]:

$$\begin{bmatrix} 0 & 0 & -1 & a_3 + d_5 + d_7 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2a_3 + d_1 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [107]:

```
H_07_inv=H_07 #made a copy to use it later on
```

In [108]:

```
H_07
```

Out[108]:

$$\left[\begin{array}{c}((((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1 \\ (q_5))\cos(q_6)+((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\sin(\\ +(-((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1)) \\ (q_7) \\ \\ ((((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4) \\ (q_5))\cos(q_6)+((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\sin(\\ +(-((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q \\ (q_5))\sin(q_7) \\ \\ (((\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\cos(q_5)-\sin(q_2)\sin(q_3)\sin(q_5))\mathrm{co} \\ (q_7)+(-(\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\sin \\ \\ 0\end{array}\right.$$

Generating X_p

In [109]:

```
X_p=(H_07.col(-1))
X_p
```

Out[109]:

$$a_3\left(\left(\left(-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3)\right)\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1)\right)\mathrm{c}\right.$$

$$(q_5))\cos(q_6)+a_3\left(\left(-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3)\right)\sin\right.$$

$$-a_3\left(-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3)\right)\cos(q_4)-a_3\sin(q_1)\sin(q_3)-a$$

$$-d_3\sin(q_2)\cos(q_1)+d_5\left(\left(-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(\right.\right.$$

$$-d_7($$

$$-\left(\left(\left(-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3)\right)\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1)\right)\mathrm{cc}\right.$$

$$(q_5))\sin(q_6)+\left(\left(-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3)\right)\sin(q\right.$$

$$a_3\left(\left(\left(\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1)\right)\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4)\right)\cos\right.$$

$$(q_5))\cos(q_6)+a_3\left(\left(\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1)\right)\sin(\right.$$

$$-a_3\left(\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1)\right)\cos(q_4)-a_3\sin(q_1)\sin(q_2)\sin(q_4)+$$

$$(q_1)\sin(q_2)+d_5\left(\left(\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1)\right)\mathrm{s}\right.$$

$$-d_7($$

$$-\left(\left(\left(\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1)\right)\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4)\right)\cos\right.$$

$$(q_5))\sin(q_6)+\left(\left(\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1)\right)\sin(q_4\right.$$

$$a_3\left(\left(\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2)\right)\cos(q_5)-\sin(q_2)\sin(q_3)\sin(q_5)\right)\cos$$

$$(q_6)-a_3\sin(q_2)\cos(q_3)\cos(q_4)+a_3\sin(q_2)\cos(q_3)+a_3\sin(q_4)\cos(q_2)+d_1+d_3$$

$$-d_7\left(-\left(\left(\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2)\right)\cos(q_5)-\sin(q_2)\sin(q_3)\sin(q_5)\right)\right.$$

$$(q_6))$$

$$1$$

In [110]:

```
X_p.row_del(-1)
```

In [111]:

```
X_p
```

Out[111]:

$$
\begin{bmatrix}
a_3 (((- \sin(q_1) \sin(q_3) + \cos(q_1) \cos(q_2) \cos(q_3)) \cos(q_4) + \sin(q_2) \sin(q_4) \cos(q_1)) \mathrm{co} \\
(q_5)) \cos(q_6) + a_3 ((- \sin(q_1) \sin(q_3) + \cos(q_1) \cos(q_2) \cos(q_3)) \sin \\
- a_3 (- \sin(q_1) \sin(q_3) + \cos(q_1) \cos(q_2) \cos(q_3)) \cos(q_4) - a_3 \sin(q_1) \sin(q_3) - a \\
- d_3 \sin(q_2) \cos(q_1) + d_5 ((- \sin(q_1) \sin(q_3) + \cos(q_1) \cos(q_2) \cos(\\
- d_7 (\\
- (((- \sin(q_1) \sin(q_3) + \cos(q_1) \cos(q_2) \cos(q_3)) \cos(q_4) + \sin(q_2) \sin(q_4) \cos(q_1)) \mathrm{co} \\
(q_5)) \sin(q_6) + ((- \sin(q_1) \sin(q_3) + \cos(q_1) \cos(q_2) \cos(q_3)) \sin(q \\
a_3 (((\sin(q_1) \cos(q_2) \cos(q_3) + \sin(q_3) \cos(q_1)) \cos(q_4) + \sin(q_1) \sin(q_2) \sin(q_4)) \cos \\
(q_5)) \cos(q_6) + a_3 ((\sin(q_1) \cos(q_2) \cos(q_3) + \sin(q_3) \cos(q_1)) \sin(\\
- a_3 (\sin(q_1) \cos(q_2) \cos(q_3) + \sin(q_3) \cos(q_1)) \cos(q_4) - a_3 \sin(q_1) \sin(q_2) \sin(q_4) + \\
(q_1) \sin(q_2) + d_5 ((\sin(q_1) \cos(q_2) \cos(q_3) + \sin(q_3) \cos(q_1)) \mathrm{s} \\
- d_7 (\\
- (((\sin(q_1) \cos(q_2) \cos(q_3) + \sin(q_3) \cos(q_1)) \cos(q_4) + \sin(q_1) \sin(q_2) \sin(q_4)) \cos \\
(q_5)) \sin(q_6) + ((\sin(q_1) \cos(q_2) \cos(q_3) + \sin(q_3) \cos(q_1)) \sin(q_4 \\
a_3 ((\sin(q_2) \cos(q_3) \cos(q_4) - \sin(q_4) \cos(q_2)) \cos(q_5) - \sin(q_2) \sin(q_3) \sin(q_5)) \cos \\
(q_6) - a_3 \sin(q_2) \cos(q_3) \cos(q_4) + a_3 \sin(q_2) \cos(q_3) + a_3 \sin(q_4) \cos(q_2) + d_1 + d_3 \\
- d_7 (- ((\sin(q_2) \cos(q_3) \cos(q_4) - \sin(q_4) \cos(q_2)) \cos(q_5) - \sin(q_2) \sin(q_3) \sin(q_5)) \\
(q_6))
\end{bmatrix}
$$

Canculating the velocity part of the Jacobian, by differentiating X_p, you can see the components by order in the matrix.

In [112]:

```
J_v = X_p.diff(q1).row_join(X_p.diff(q2)).row_join(X_p.diff(q4)).row_join(X_p.diff(q5)).row
```

In [113]:

```
J_v
```

Out[113]:

$$a_3\left(\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\cos\left(q_4\right)-\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q_4\right)\right)\cos\left(q\right.\right.$$
$$\left(q_5\right)\right)\cos\left(q_6\right)+a_3\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\sin\left(q_4\right)$$
$$-a_3\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\cos\left(q_4\right)+a_3\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q_4\right)$$
$$+d_3\sin\left(q_1\right)\sin\left(q_2\right)+d_5\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\right.$$
$$-d_7\left(\left(-\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\cos\left(q_4\right)-\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q_4\right)\right)\cos\left(q_5\right)\right.\right.$$
$$\left(q_6\right)+\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\sin\left(q_4\right)+\sin\left(\right.\right.$$
$$a_3\left(\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\cos\left(q_4\right)+\sin\left(q_2\right)\sin\left(q_4\right)\cos\left(q_1\right)\right)\cos\left(q_5\right.\right.$$
$$\left(q_5\right)\right)\cos\left(q_6\right)+a_3\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\sin\left(q_4\right)$$
$$-a_3\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\cos\left(q_4\right)-a_3\sin\left(q_1\right)\sin\left(q_3\right)-a_3\sin$$
$$-d_3\sin\left(q_2\right)\cos\left(q_1\right)+d_5\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\right.$$
$$-d_7\left(\left(-\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\cos\left(q_4\right)+\sin\left(q_2\right)\sin\left(q_4\right)\cos\left(q_1\right)\right)\cos\left(q_5\right)\right.\right.$$
$$\left(q_6\right)+\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\sin\left(q_4\right)-\sin\left(\right.\right.$$

Generating the angular velocity part of the Jacobian by collceting the Z-rotations from the corresponding transformation matrices. You can see the conponents by order in the matrix.

In [114]:

```
J_w=H_01.col(2).row_join(H_02.col(2)).row_join(H_04.col(2)).row_join(H_05.col(2)).row_join(
J_w
```

Out[114]:

$$\left[\begin{matrix}\sin\left(q_1\right) & -\sin\left(q_2\right)\cos\left(q_1\right) & \left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\sin\left(q_4\right) \\ -\cos\left(q_1\right) & -\sin\left(q_1\right)\sin\left(q_2\right) & \left(\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)+\sin\left(q_3\right)\cos\left(q_1\right)\right)\sin\left(q_4\right) - \\ 0 & \cos\left(q_2\right) & \sin\left(q_2\right)\sin\left(q_4\right)\cos\left(q_3\right)+\cos\left(q_2\right)c \\ 0 & 0 & 0\end{matrix}\right.$$

Building the Jacobian called M

In [115]:

```
J_w.row_del(-1)
```

In [116]:

```
M=J_v.col_join(J_w)
```

In [126]:

```
M=M.subs({q3:0})
M
```

Out[126]:

$$-20.7\left(-\left(-\sin\left(q_{1}\right)\sin\left(q_{2}\right)\sin\left(q_{4}\right)-\sin\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\right.$$
$$+\,8.8\left(\left(-\sin\left(q_{1}\right)\sin\left(q_{2}\right)\sin\left(q_{4}\right)-\sin\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\mathrm{cc}\right.$$
$$+\,8.8\left(\sin\left(q_{1}\right)\sin\left(q_{2}\right)\cos\left(q_{4}\right)-\sin\left(q_{1}\right)\sin\left(q_{4}\right)\cos\left(q_{2}\right)\right)\sin\left(q_{6}\right)-20.7\left(\sin\left(q_{1}\right)\sin\left(\right.\right.$$
$$\left(q_{1}\right)\sin\left(q_{2}\right)\sin\left(q_{4}\right)+38.4\sin\left(q_{1}\right)\sin\left(q_{2}\right)\cos\left(q_{4}\right)+31.6\sin\left(q_{1}\right)\sin\left(q_{2}\right)-38.4\sin\left(q\right.$$
$$\left(q_{1}\right)\cos\left(q_{2}\right)$$
$$-20.7\left(-\left(\sin\left(q_{2}\right)\sin\left(q_{4}\right)\cos\left(q_{1}\right)+\cos\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\mathrm{c}\right.$$
$$+\,8.8\left(\left(\sin\left(q_{2}\right)\sin\left(q_{4}\right)\cos\left(q_{1}\right)+\cos\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\mathrm{co}\right.$$
$$+\,8.8\left(-\sin\left(q_{2}\right)\cos\left(q_{1}\right)\cos\left(q_{4}\right)+\sin\left(q_{4}\right)\cos\left(q_{1}\right)\cos\left(q_{2}\right)\right)\sin\left(q_{6}\right)-20.7\left(-\sin\left(q\right.\right.$$
$$-\,8.8\sin\left(q_{2}\right)\sin\left(q_{4}\right)\cos\left(q_{1}\right)-38.4\sin\left(q_{2}\right)\cos\left(q_{1}\right)\cos\left(q_{4}\right)-31.6\sin\left(q_{2}\right)\cos\left(q_{1}\right)+3$$
$$+\,8.8\cos\left(q_{1}\right)\cos\left(q_{2}\right)$$
$$0$$
$$\sin\left(q_{1}\right)$$
$$-\cos\left(q_{1}\right)$$
$$0$$

Inserting the distances and the fixed joint, the pen's length is added to the d_7 into M

In [118]:

```
M=M.subs({d1:33.3,d3:31.6,d5:38.4,a3:8.8,d7:20.7})
K=M #Making a copy just in case
```

In [119]:

```
K
```

Out[119]:

$$\begin{bmatrix} -20.7\left(-\left(-\sin\left(q_{1}\right)\sin\left(q_{2}\right)\sin\left(q_{4}\right)-\sin\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\cos\left(q_{5}\right)+\sin \right. \\ +8.8\left(\left(-\sin\left(q_{1}\right)\sin\left(q_{2}\right)\sin\left(q_{4}\right)-\sin\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\cos\left(q_{5}\right)-\sin \right. \\ +8.8\left(\sin\left(q_{1}\right)\sin\left(q_{2}\right)\cos\left(q_{4}\right)-\sin\left(q_{1}\right)\sin\left(q_{4}\right)\cos\left(q_{2}\right)\right)\sin\left(q_{6}\right)-20.7\left(\sin\left(q_{1}\right)\sin\left(q_{2}\right)\cos\left(q_{4}\right)\right. \\ \left(q_{1}\right)\sin\left(q_{2}\right)\sin\left(q_{4}\right)+38.4\sin\left(q_{1}\right)\sin\left(q_{2}\right)\cos\left(q_{4}\right)+31.6\sin\left(q_{1}\right)\sin\left(q_{2}\right)-38.4\sin\left(q_{1}\right)\sin\left(q_{4}\right)\cos \\ \left(q_{1}\right)\cos\left(q_{2}\right) \\ -20.7\left(-\left(\sin\left(q_{2}\right)\sin\left(q_{4}\right)\cos\left(q_{1}\right)+\cos\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\cos\left(q_{5}\right)+\sin \right. \\ +8.8\left(\left(\sin\left(q_{2}\right)\sin\left(q_{4}\right)\cos\left(q_{1}\right)+\cos\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\cos\left(q_{5}\right)-\sin\left(\right.\right. \\ +8.8\left(-\sin\left(q_{2}\right)\cos\left(q_{1}\right)\cos\left(q_{4}\right)+\sin\left(q_{4}\right)\cos\left(q_{1}\right)\cos\left(q_{2}\right)\right)\sin\left(q_{6}\right)-20.7\left(-\sin\left(q_{2}\right)\cos\left(q_{1}\right)c\right. \\ -8.8\sin\left(q_{2}\right)\sin\left(q_{4}\right)\cos\left(q_{1}\right)-38.4\sin\left(q_{2}\right)\cos\left(q_{1}\right)\cos\left(q_{4}\right)-31.6\sin\left(q_{2}\right)\cos\left(q_{1}\right)+38.4\sin\left(q_{4}\right) \\ +8.8\cos\left(q_{1}\right)\cos\left(q_{2}\right) \\ 0 \end{bmatrix}$$

Inserting the distances and the fixed joint, the pen's length is added to the d_7 into T_{07}

In [120]:

```
H_07_inv=H_07_inv.subs({d1:33.3,d3:31.6,d5:38.4,a3:8.8,d7:20.7,q3:0})
H_07_inv
```

Out[120]:

$$\begin{bmatrix} \left(\left(\left(\sin\left(q_{2}\right)\sin\left(q_{4}\right)\cos\left(q_{1}\right)+\cos\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\cos\left(q_{5}\right)-\sin\left(q_{1}\right)\sin\left(q_{5}\right)\right)\cos\left(q_{6}\right.\right. \\ \left(q_{7}\right)+\left(-\left(\sin\left(q_{2}\right)\sin\left(q_{4}\right)\cos\left(q_{1}\right)+\cos\left(q_{1}\right)\cos\left(q_{2}\right)\cos\right.\right. \\ \\ \left(\left(\left(\sin\left(q_{1}\right)\sin\left(q_{2}\right)\sin\left(q_{4}\right)+\sin\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\cos\left(q_{5}\right)+\sin\left(q_{5}\right)\cos\left(q_{1}\right)\right)\cos\left(q_{6}\right.\right. \\ \left(q_{7}\right)+\left(-\left(\sin\left(q_{1}\right)\sin\left(q_{2}\right)\sin\left(q_{4}\right)+\sin\left(q_{1}\right)\cos\left(q_{2}\right)\cos\left(\right.\right.\right. \\ \\ \left(\left(\sin\left(q_{2}\right)\sin\left(q_{4}\right)+\cos\left(q_{2}\right)\cos\left(q_{4}\right)\right)\sin\left(q_{6}\right)+\left(\sin\left(q_{2}\right)\cos\left(q_{4}\right.\right.\right. \\ -\left(\sin\left(q_{2}\right)\cos\left(q_{4}\right)-\sin\left(q_{4}\right)\cos\left(q\right.\right. \\ \\ 0 \end{bmatrix}$$

Running a loop to calculate to do the inverse kinematics, firrst calculate the velocity vector, from that using the inverse Jacobian we get $\dot{q}$, by numerical integration we get q and using the transformation matrix we calculate the Y, Z coordiantes in each iteration. The loop runs almost for couple of minutes due to the small timestep, but the circle is nice.

In [121]:

```
## 2D plot essentials
#
#plt.ylim(60,90)
#plt.xlim(-15,15)
#plt.rcParams['figure.figsize'] = [10, 10]
#plt.axis([-15, 15, 60, 90])
#

#Initializing the start values, and making some matrices to reduce the computation time in
theta_dot=2*pi/5
t=0
V=Matrix([[0],[0],[0],[0],[0],[0]])
Q=Matrix([[0.0],[0.0],[pi/2],[0.0],[pi],[0.0]])
i=0
r=10
x=[]
y=[]
z=[]
Plot=Matrix([[0],[0],[0],[0]])
A=H_07_inv.col(-1)
while(i<=5):

    V[1]=(-r*sin(pi/2+theta_dot*i)*theta_dot).evalf()
    V[2]=(r*cos(pi/2+theta_dot*i)*theta_dot).evalf()
    #print(Plot[2])
    #Plot=(H_07_inv.col(-1)).subs({q1:Q[0],q2:Q[1],q4:Q[2],q5:Q[3],q6:Q[4],q7:Q[5]}).evalf(
    K=M.subs({q1:Q[0],q2:Q[1],q4:Q[2],q5:Q[3],q6:Q[4],q7:Q[5]}).evalf()
    Q_dot=K.inv().evalf()*V
    Q=Q+0.01*Q_dot
    #Q[0]=Q[0]+0.25*Q_dot[0]
    #Q[1]=Q[1]+0.25*Q_dot[1]
    #Q[2]=Q[2]+0.25*Q_dot[2]
    #Q[3]=Q[3]+0.25*Q_dot[3]
    #Q[4]=Q[4]+0.25*Q_dot[4]
    #Q[5]=Q[5]+0.25*Q_dot[5]
    Plot=(A.subs({q1:Q[0],q2:Q[1],q4:Q[2],q5:Q[3],q6:Q[4],q7:Q[5]})).evalf()
    x.append(Plot[0])
    y.append(Plot[1])
    z.append(Plot[2])
    ## 2D plot essentials
    #print(Plot[2])
    #plt.plot(Plot[0],Plot[1], Plot[2], color='green', linestyle='solid', linewidth = 3,
     #    marker='o')
    #

    i=i+0.01
```

3D plot

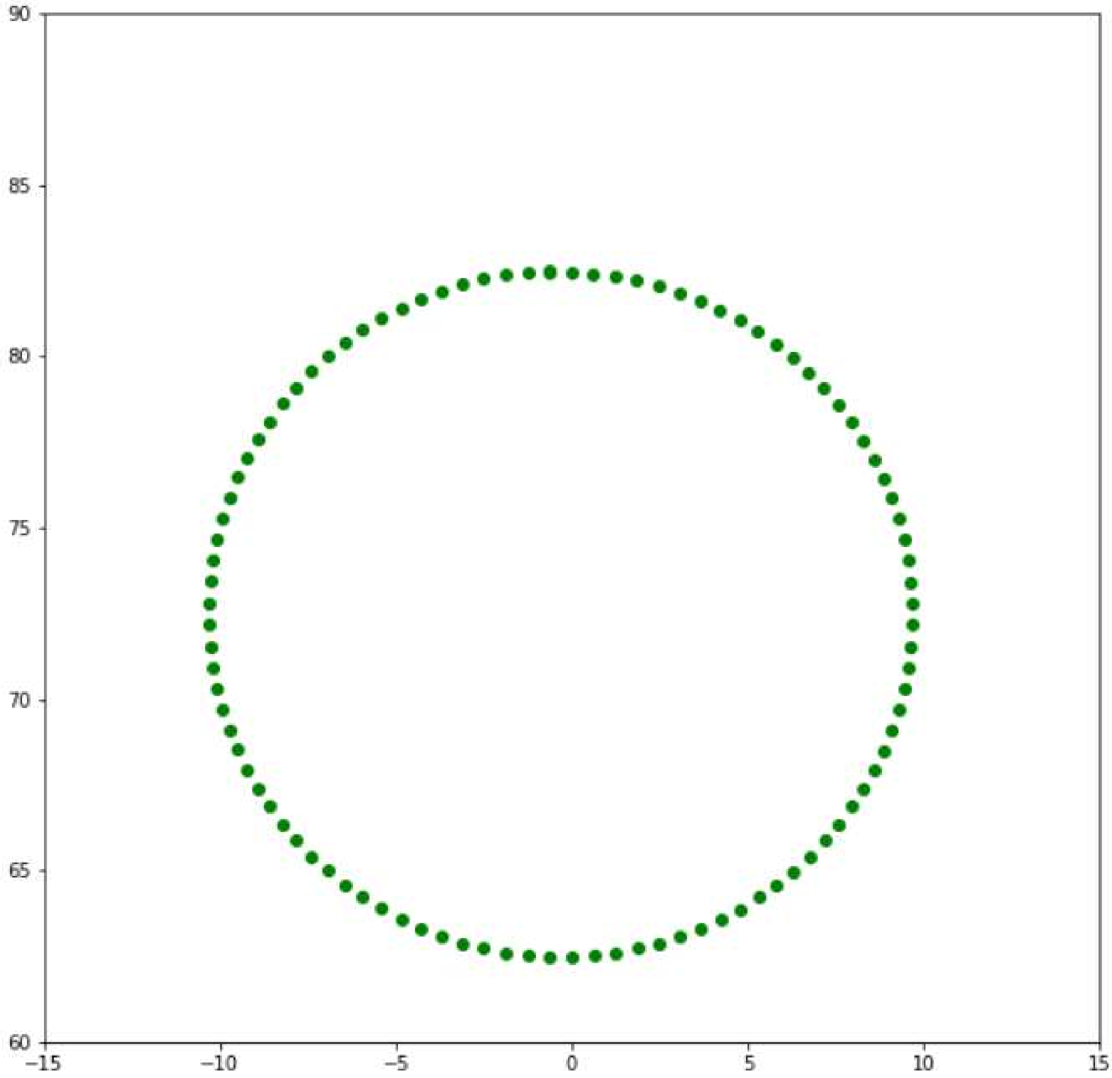
In [125]:

```
ax = plt.axes(projection='3d')
ax.set_aspect('equal','box')
ax.axes.set_xlim3d(left=40, right=80)
ax.axes.set_ylim3d(bottom=-20, top=20)
ax.axes.set_zlim3d(bottom=55, top=85)
ax.scatter3D(x, y, z, color='green');
```

In [123]:

```
Image("2d_circle.png")
```

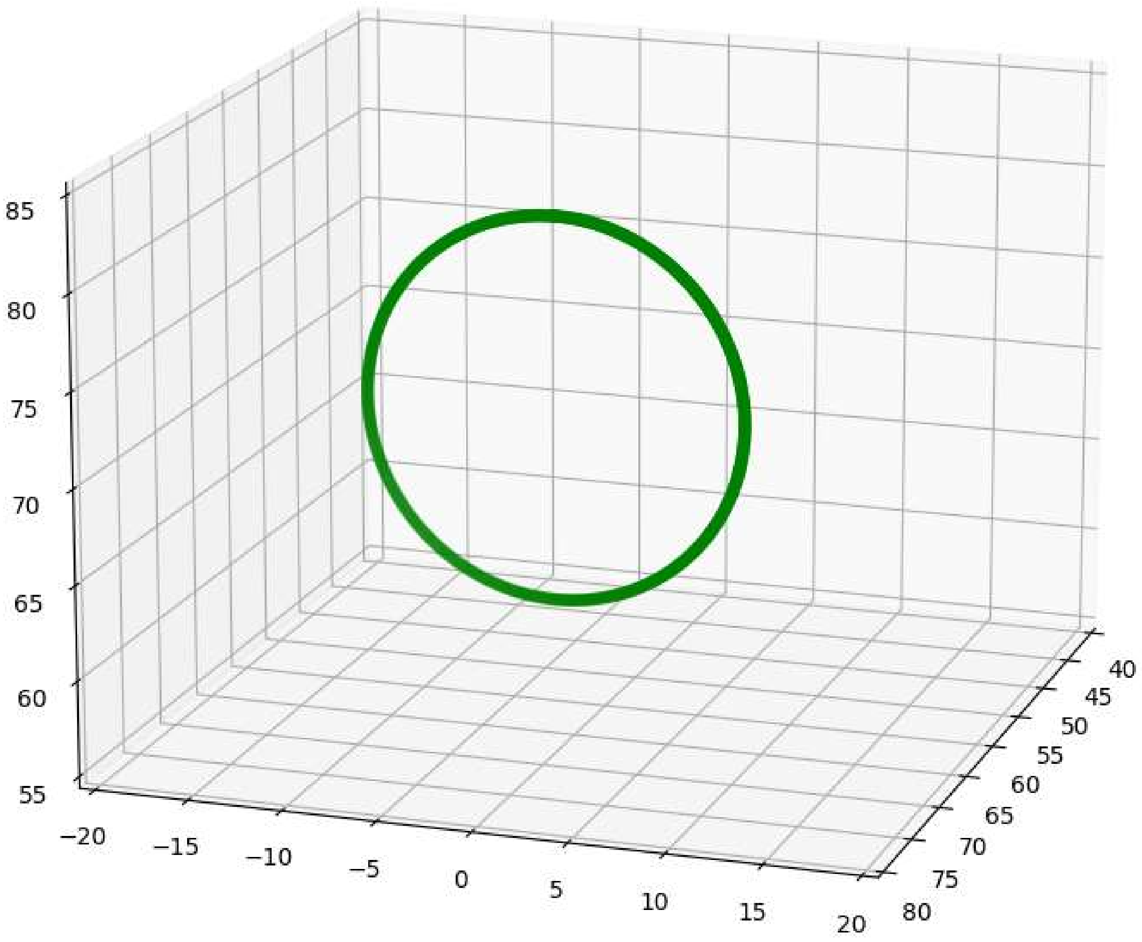
Out[123]:



In [124]:

```
Image("3d_circle.png")
```

Out[124]:



In [ ]:

```

```