In [1]:

```
from sympy import*
from IPython.display import Image, display, HTML
from scipy import optimize
import matplotlib.pyplot as plt
import numpy as np
#%matplotlib qt
```

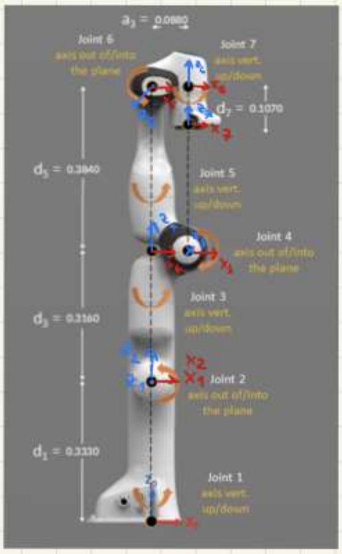
Homework 5.

Homework 4.

In [2]:

```
Image("Doc.png")
```

Out[2]:



DH Sponge

joint variable / geometric parameters

i	θ_i	d_i	a_i	α_i
1	q_1	d_1	0	90
2	q_2	0	0	-90
3	q_3	d_3	a_3	-90
4	q_4	0	$-a_3$	90
5	q_5	d_5	0	90
6	q_6	0	a_3	-90
7	q_7	$-d_7$	0	0

$x_i \perp z_{i-1}$

$x_i \cap z_{i-1}$

D-H parameters:

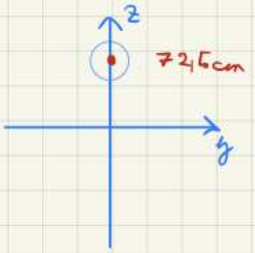
d_i: x_i distance from x_{i-1} along z_{i-1}

θ_i: x_i angle from x_{i-1} around z_{i-1}

a_i: z_i distance from z_{i-1} along x_i

α_i: z_i angle from z_{i-1} around x_i

According to the Homework 4 $q_3 = 0$ and since the pen is 10cm long I include it in the d_7 shift so:



DH Sponge

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3	0	d_3	a_3	-90
4	q_4	0	$-a_3$	90
5	q_5	d_5	0	90
6	q_6	0	a_3	-90
7	q_7	$-(d_7+10)$	0	0

→ the frame can stay, it just should not been used in the Jacobian.

circle equation:

$$y^2 + (z - 72{,}5)^2 = r^2 = 10^2$$

↳ Polar coordinates:

$y = r \cdot \cos(\theta + 90°)$ // 90° shift since we start from the top

$z = r \cdot \sin(\theta + 90°)$

$x = 67{,}9$

$\theta = \left(\frac{2\pi}{200} t\right)$ where t goes from [0; 5]

since the pen does not rotate its φ, θ, ψ = constant

So $\dot{X} = \begin{bmatrix} V \\ \omega \end{bmatrix} = \begin{bmatrix} -r \sin(\theta + 90°) \cdot \dot{\theta} \\ r \cos(\theta + 90°) \cdot \dot{\theta} \\ 0 \\ \vdots \end{bmatrix}$

For the inverse kinematics the Jacobian is calculated by the second method using the ${}^0Z_i$ and $\frac{{}^0x_p}{\partial q_i}$,

where ${}^0Z_i$ is the z rotation of each ${}^0T_i$ transformation and ${}^0x_p$ is the translation of ${}^0T_7$,

where i goes from 1–7, except 3, and $q_3 = 0$

$$J = \begin{bmatrix} \frac{\partial {}^0x_p}{\partial q_1} & \frac{\partial {}^0x_p}{\partial q_2} & \frac{\partial {}^0x_p}{\partial q_4} & \frac{\partial {}^0x_p}{\partial q_5} & \frac{\partial {}^0x_p}{\partial q_6} & \frac{\partial {}^0x_p}{\partial q_7} \\ {}^0Z_1 & {}^0Z_2 & {}^0Z_4 & {}^0Z_5 & {}^0Z_6 & {}^0Z_7 \end{bmatrix}$$

$$\begin{cases} \dot{q} = J^{-1}(q)\,\dot{X} \\ q_{next} = q_{current} + \dot{q}\,\Delta t \end{cases}$$

and y, z can be plotted by calculating the transformation matrix in each q iteration, ${}^0T_7(q)$ → last column → y, z

In [3]:

```
q1,q2,q3,q4,q5,q6,q7,d1,d3,d5,d7,a3=symbols('q_1 q_2 q_3 q_4 q_5 q_6 q_7 d_1 d_3 d_5 d_7 a_
```

The transformation matrixes are calculated.

In [4]:

```
def DH_Tr_sym_UMD(theta=0, d=0 , a=0, alpha=0):
    Tr_theta=Matrix([[cos(theta),-sin(theta),0,0],
                     [sin(theta),cos(theta),0,0],
                     [0,0,1,0],
                     [0,0,0,1]])
    Tr_d=Matrix([[1,0,0,0],
                 [0,1,0,0],
                 [0,0,1,d],
                 [0,0,0,1]])
    Tr_a=Matrix([[1,0,0,a],
                 [0,1,0,0],
                 [0,0,1,0],
                 [0,0,0,1]])
    Tr_alpha=Matrix([[1,0,0,0],
                     [0,cos(alpha),-sin(alpha),0],
                     [0,sin(alpha) ,cos(alpha),0],
                     [0,0,0,1]])


    Tr_KHALIL=Tr_theta@Tr_d@Tr_a@Tr_alpha

    return Tr_KHALIL
```

In [5]:

```
H_01=DH_Tr_sym_UMD(theta=q1, d=d1 , a=0, alpha=pi/2)
H_01
```

Out[5]:

$$\begin{bmatrix} \cos(q_1) & 0 & \sin(q_1) & 0 \\ \sin(q_1) & 0 & -\cos(q_1) & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [6]:

```
H_12=DH_Tr_sym_UMD(theta=q2, d=0 , a=0, alpha=-pi/2)
H_02=H_01@H_12
H_02
```

Out[6]:

$$\begin{bmatrix} \cos(q_1)\cos(q_2) & -\sin(q_1) & -\sin(q_2)\cos(q_1) & 0 \\ \sin(q_1)\cos(q_2) & \cos(q_1) & -\sin(q_1)\sin(q_2) & 0 \\ \sin(q_2) & 0 & \cos(q_2) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [7]:

```
H_23=DH_Tr_sym_UMD(theta=q3, d=d3 , a=a3, alpha=-pi/2)
H_03=H_01@H_12@H_23
H_03
```

Out[7]:

$$\begin{bmatrix} -\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3) & \sin(q_2)\cos(q_1) & -\sin(q_1)\cos(q_3)-\mathrm{si} \\ \sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1) & \sin(q_1)\sin(q_2) & -\sin(q_1)\sin(q_3)\cos(\\ \sin(q_2)\cos(q_3) & -\cos(q_2) & -\sin(q_2 \\ 0 & 0 & 0 \end{bmatrix}$$

In [8]:

```
H_34=DH_Tr_sym_UMD(theta=q4, d=0 , a=-a3, alpha=pi/2)
H_04=H_01@H_12@H_23@H_34
H_34
```

Out[8]:

$$\begin{bmatrix} \cos(q_4) & 0 & \sin(q_4) & -a_3\cos(q_4) \\ \sin(q_4) & 0 & -\cos(q_4) & -a_3\sin(q_4) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [9]:

```
H_45=DH_Tr_sym_UMD(theta=q5, d=d5 , a=0, alpha=pi/2)
H_05=H_01@H_12@H_23@H_34@H_45
H_45
```

Out[9]:

$$\begin{bmatrix} \cos(q_5) & 0 & \sin(q_5) & 0 \\ \sin(q_5) & 0 & -\cos(q_5) & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [10]:

```
H_56=DH_Tr_sym_UMD(theta=q6, d=0 , a=a3, alpha=-pi/2)
H_06=H_01@H_12@H_23@H_34@H_45@H_56
H_56
```

Out[10]:

$$\begin{bmatrix} \cos(q_6) & 0 & -\sin(q_6) & a_3\cos(q_6) \\ \sin(q_6) & 0 & \cos(q_6) & a_3\sin(q_6) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [11]:

```
H_67=DH_Tr_sym_UMD(theta=q7, d=-d7 , a=0, alpha=0)
H_07=H_06*H_67
H_67
H_07
```

Out[11]:

$$\left[\begin{matrix} ((((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1 \\ (q_5))\cos(q_6)+((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\sin(\\ +(-((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1)) \\ (q_7) \\ \\ ((((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4) \\ (q_5))\cos(q_6)+((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\sin(\\ +(-((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q \\ (q_5))\sin(q_7) \\ \\ (((\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\cos(q_5)-\sin(q_2)\sin(q_3)\sin(q_5))\mathrm{co} \\ (q_7)+(-(\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\sin \\ \\ 0 \end{matrix}\right.$$

In [12]:

```
H_07.subs({q3:0})[2,3]
```

Out[12]:

$$a_3(\sin(q_2)\sin(q_4)+\cos(q_2)\cos(q_4))\sin(q_6)+a_3(\sin(q_2)\cos(q_4)-\sin(q_4)\cos(q_2))\mathrm{c}$$
$$(q_4)\cos(q_2)+d_1+d_3\cos(q_2)+d_5(\sin(q_2)\sin(q_4)+\cos(q_2)\cos(q_4))$$
$$-d_7((\sin(q_2)\sin(q_4)+\cos(q_2)\cos(q_4))\cos(q_6)-(\sin(q_2)\cos(q_4)-\sin(q_4)\cos(q_2))\mathrm{s}$$

In [13]:

```
H_07_inv=H_07 #made a copy to use it later on
```

In [14]:

```
H_07
```

Out[14]:

$$((((-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4) + \sin(q_2)\sin(q_4)\cos(q_1$$
$$(q_5))\cos(q_6) + ((-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(q_3))\sin($$
$$+ (-((-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4) + \sin(q_2)\sin(q_4)\cos(q_1))$$
$$(q_7)$$

$$((((\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1))\cos(q_4) + \sin(q_1)\sin(q_2)\sin(q_4)$$
$$(q_5))\cos(q_6) + ((\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1))\sin($$
$$+ (-((\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1))\cos(q_4) + \sin(q_1)\sin(q_2)\sin(q$$
$$(q_5))\sin(q_7)$$

$$(((\sin(q_2)\cos(q_3)\cos(q_4) - \sin(q_4)\cos(q_2))\cos(q_5) - \sin(q_2)\sin(q_3)\sin(q_5))\mathrm{co}$$
$$(q_7) + (-(\sin(q_2)\cos(q_3)\cos(q_4) - \sin(q_4)\cos(q_2))\sin$$

$$0$$

Generating X_p

In [15]:

```
X_p=(H_07.col(-1))
X_p
```

Out[15]:

$$a_3\,(((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1))\,\mathrm{c}$$
$$(q_5))\cos(q_6)+a_3\,((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\sin$$
$$-a_3\,(-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)-a_3\sin(q_1)\sin(q_3)-a$$
$$-d_3\sin(q_2)\cos(q_1)+d_5\,((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos($$
$$-d_7\,($$
$$-(((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1))\,\mathrm{c}$$
$$(q_5))\sin(q_6)+((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\sin(q$$
$$a_3\,(((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4))\cos$$
$$(q_5))\cos(q_6)+a_3\,((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\sin($$
$$-a_3\,(\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)-a_3\sin(q_1)\sin(q_2)\sin(q_4)+$$
$$(q_1)\sin(q_2)+d_5\,((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\,\mathrm{s}$$
$$-d_7\,($$
$$-(((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4))\cos$$
$$(q_5))\sin(q_6)+((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\sin(q_4$$
$$a_3\,((\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\cos(q_5)-\sin(q_2)\sin(q_3)\sin(q_5))\cos$$
$$(q_6)-a_3\sin(q_2)\cos(q_3)\cos(q_4)+a_3\sin(q_2)\cos(q_3)+a_3\sin(q_4)\cos(q_2)+d_1+d_3$$
$$-d_7\,(-((\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\cos(q_5)-\sin(q_2)\sin(q_3)\sin(q_5))$$
$$(q_6))$$
$$1$$

In [16]:

```
X_p.row_del(-1)
```

In [17]:

```
X_p
```

Out[17]:

$$
\left[\begin{array}{l}
a_3 (((-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4) + \sin(q_2)\sin(q_4)\cos(q_1)) \mathrm{c} \\
(q_5))\cos(q_6) + a_3 ((-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(q_3))\sin \\
- a_3 (-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4) - a_3 \sin(q_1)\sin(q_3) - a \\
- d_3 \sin(q_2)\cos(q_1) + d_5 ((-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(\\
- d_7 (\\
- (((-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4) + \sin(q_2)\sin(q_4)\cos(q_1)) \mathrm{c} \\
(q_5))\sin(q_6) + ((-\sin(q_1)\sin(q_3) + \cos(q_1)\cos(q_2)\cos(q_3))\sin(q \\
a_3 (((\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1))\cos(q_4) + \sin(q_1)\sin(q_2)\sin(q_4))\cos \\
(q_5))\cos(q_6) + a_3 ((\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1))\sin(\\
- a_3 (\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1))\cos(q_4) - a_3 \sin(q_1)\sin(q_2)\sin(q_4) + \\
(q_1)\sin(q_2) + d_5 ((\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1)) \mathrm{s} \\
- d_7 (\\
- (((\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1))\cos(q_4) + \sin(q_1)\sin(q_2)\sin(q_4))\cos \\
(q_5))\sin(q_6) + ((\sin(q_1)\cos(q_2)\cos(q_3) + \sin(q_3)\cos(q_1))\sin(q_4 \\
a_3 ((\sin(q_2)\cos(q_3)\cos(q_4) - \sin(q_4)\cos(q_2))\cos(q_5) - \sin(q_2)\sin(q_3)\sin(q_5))\cos \\
(q_6) - a_3 \sin(q_2)\cos(q_3)\cos(q_4) + a_3 \sin(q_2)\cos(q_3) + a_3 \sin(q_4)\cos(q_2) + d_1 + d_3 \\
- d_7 (-((\sin(q_2)\cos(q_3)\cos(q_4) - \sin(q_4)\cos(q_2))\cos(q_5) - \sin(q_2)\sin(q_3)\sin(q_5)) \\
(q_6))
\end{array}\right]
$$

Canculating the velocity part of the Jacobian, by differentiating X_p, you can see the components by order in the matrix.

In [18]:

```
J_v = X_p.diff(q1).row_join(X_p.diff(q2)).row_join(X_p.diff(q4)).row_join(X_p.diff(q5)).row
```

In [19]:

```
J_v
```

Out[19]:

$$\begin{bmatrix} a_3\left(\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\cos\left(q_4\right)-\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q_4\right)\right)\cos\left(q\right.\right. \\ \left(q_5\right)\right)\cos\left(q_6\right)+a_3\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\sin\left(q_4\right)\right. \\ -a_3\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\cos\left(q_4\right)+a_3\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q_4\right) \\ +d_3\sin\left(q_1\right)\sin\left(q_2\right)+d_5\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\right. \\ -d_7\left(\left(-\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\cos\left(q_4\right)-\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q_4\right)\right)\cos\left(q_5\right)\right.\right. \\ \left(q_6\right)+\left(\left(-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)-\sin\left(q_3\right)\cos\left(q_1\right)\right)\sin\left(q_4\right)+\sin\left(\right.\right. \\ a_3\left(\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\cos\left(q_4\right)+\sin\left(q_2\right)\sin\left(q_4\right)\cos\left(q_1\right)\right)\cos\left(q_5\right.\right. \\ \left(q_5\right)\right)\cos\left(q_6\right)+a_3\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\sin\left(q_4\right)\right. \\ -a_3\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\cos\left(q_4\right)-a_3\sin\left(q_1\right)\sin\left(q_3\right)-a_3\sin \\ -d_3\sin\left(q_2\right)\cos\left(q_1\right)+d_5\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\right. \\ -d_7\left(\left(-\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\cos\left(q_4\right)+\sin\left(q_2\right)\sin\left(q_4\right)\cos\left(q_1\right)\right)\cos\left(q_5\right)\right.\right. \\ \left(q_6\right)+\left(\left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\sin\left(q_4\right)-\sin\left(\right.\right. \end{bmatrix}$$

Generating the angular velocity part of the Jacobian by collceting the Z-rotations from the corresponding transformation matrices. You can see the conponents by order in the matrix.

In [20]:

```
J_w=H_01.col(2).row_join(H_02.col(2)).row_join(H_04.col(2)).row_join(H_05.col(2)).row_join(
J_w
```

Out[20]:

$$\begin{bmatrix} \sin\left(q_1\right) & -\sin\left(q_2\right)\cos\left(q_1\right) & \left(-\sin\left(q_1\right)\sin\left(q_3\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)\right)\sin\left(q_4\right) \\ -\cos\left(q_1\right) & -\sin\left(q_1\right)\sin\left(q_2\right) & \left(\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_3\right)+\sin\left(q_3\right)\cos\left(q_1\right)\right)\sin\left(q_4\right)- \\ 0 & \cos\left(q_2\right) & \sin\left(q_2\right)\sin\left(q_4\right)\cos\left(q_3\right)+\cos\left(q_2\right)c \\ 0 & 0 & 0 \end{bmatrix}$$

Building the Jacobian called M

In [21]:

```
J_w.row_del(-1)
```

In [22]:

```
M=J_v.col_join(J_w)
```

In [23]:

```
M=M.subs({q3:0})
M
```

Out[23]:

$$a_3\left(\left(-\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q_4\right)-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_4\right)\right)\cos\left(q_5\right)\right.$$
$$+a_3\left(\sin\left(q_1\right)\sin\left(q_2\right)\cos\left(q_4\right)-\sin\left(q_1\right)\sin\left(q_4\right)\cos\left(q_2\right)\right)\sin\left(q_6\right)+a_3\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q\right.$$
$$+d_3\sin\left(q_1\right)\sin\left(q_2\right)+d_5\left(\sin\left(q_1\right)\sin\left(q_2\right)\cos\left(q_4\right)-\sin\left(\right.\right.$$
$$-d_7\left(\left(-\left(-\sin\left(q_1\right)\sin\left(q_2\right)\sin\left(q_4\right)-\sin\left(q_1\right)\cos\left(q_2\right)\cos\left(q_4\right)\right)\cos\left(q_5\right)+\sin\left(q_5\right)\cos\left(q_1\right)\right)\sin\left(q_6\right)+\right.$$
$$a_3\left(\left(\sin\left(q_2\right)\sin\left(q_4\right)\cos\left(q_1\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_4\right)\right)\cos\left(q_5\right)\right.$$
$$+a_3\left(-\sin\left(q_2\right)\cos\left(q_1\right)\cos\left(q_4\right)+\sin\left(q_4\right)\cos\left(q_1\right)\cos\left(q_2\right)\right)\sin\left(q_6\right)-a_3\sin\left(q_2\right)\sin\left(q_4\right)\text{co}$$
$$\left(q_2\right)-d_3\sin\left(q_2\right)\cos\left(q_1\right)+d_5\left(-\sin\left(q_2\right)\cos\left(q_1\right)\cos\left(q_4\right)+\text{s}\right.$$
$$-d_7\left(\left(-\left(\sin\left(q_2\right)\sin\left(q_4\right)\cos\left(q_1\right)+\cos\left(q_1\right)\cos\left(q_2\right)\cos\left(q_4\right)\right)\cos\left(q_5\right)+\sin\left(q_1\right)\sin\left(q_5\right)\right)\sin\left(q_6\right)+\left(\right.\right.$$
$$0$$
$$\sin\left(q_1\right)$$

Inserting the distances and the fixed joint, the pen's length is added to the d_7 into M

In [24]:

```
M=M.subs({d1:33.3,d3:31.6,d5:38.4,a3:8.8,d7:20.7})
K=M #Making a copy just in case
```

In [25]:

```
K
```

Out[25]:

Inserting the distances and the fixed joint, the pen's length is added to the d_7 into T_{07}

In [26]:

```
H_07_inv=H_07_inv.subs({d1:33.3,d3:31.6,d5:38.4,a3:8.8,d7:20.7,q3:0})
H_07_inv
```

Out[26]:

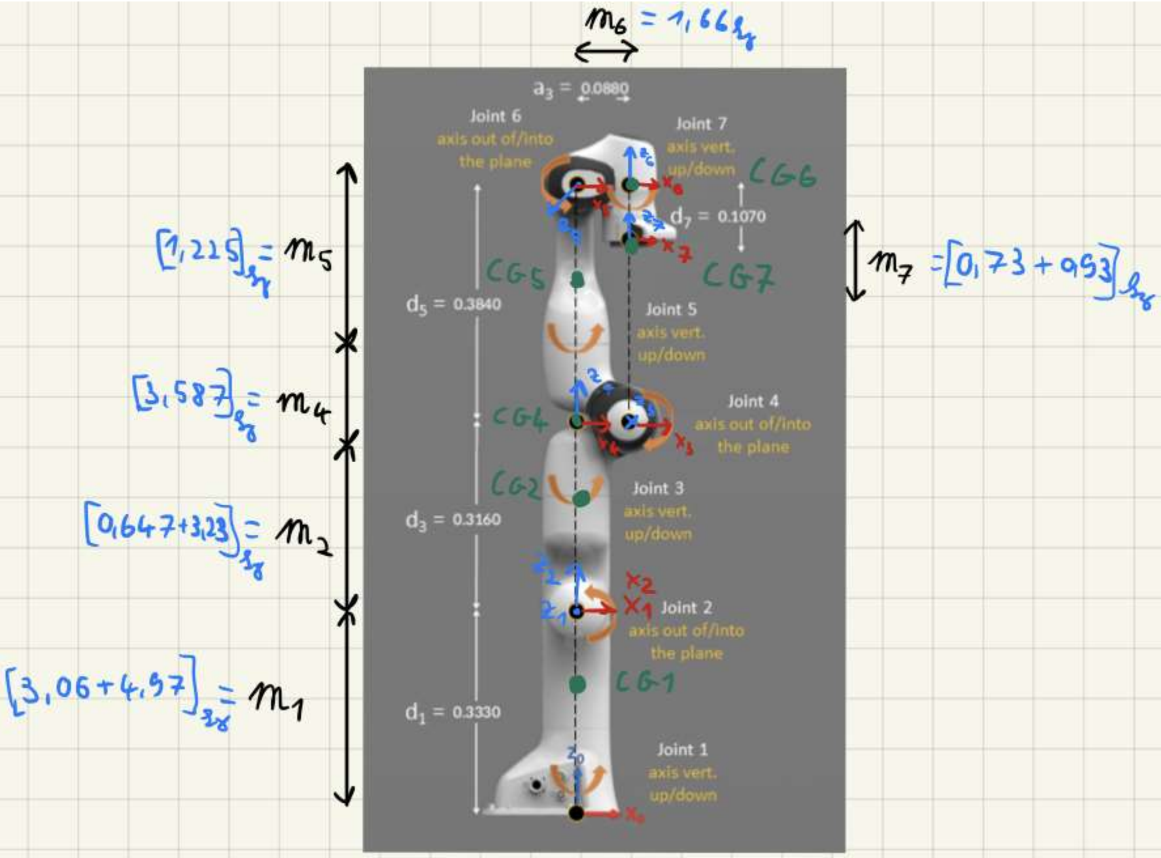
$$\left[\begin{matrix}(((\sin(q_2)\sin(q_4)\cos(q_1)+\cos(q_1)\cos(q_2)\cos(q_4))\cos(q_5)-\sin(q_1)\sin(q_5))\cos(q_6 \\ (q_7)+(-(\sin(q_2)\sin(q_4)\cos(q_1)+\cos(q_1)\cos(q_2)\cos \\ \\ (((\sin(q_1)\sin(q_2)\sin(q_4)+\sin(q_1)\cos(q_2)\cos(q_4))\cos(q_5)+\sin(q_5)\cos(q_1))\cos(q_6 \\ (q_7)+(-(\sin(q_1)\sin(q_2)\sin(q_4)+\sin(q_1)\cos(q_2)\cos(\\ \\ ((\sin(q_2)\sin(q_4)+\cos(q_2)\cos(q_4))\sin(q_6)+(\sin(q_2)\cos(q_4 \\ -(\sin(q_2)\cos(q_4)-\sin(q_4)\cos(q \\ \\ 0\end{matrix}\right.$$

Homework 5.

In [27]:

```
Image("HW5.png")
```

Out[27]:



$$P = \left(CG_1 \cdot m_1 + {}^0H_2 \cdot CG_2\, m_2 + {}^0H_4\, CG_4\, m_4 + {}^0H_5\, CG_5\, m_5 + {}^0H_6\, CG_6\, m_6 + {}^0H_7\, CG_7\, m_7\right) \cdot g$$

$$\underbrace{\begin{bmatrix} \frac{\partial P}{\partial q_1} \\ \frac{\partial P}{\partial q_2} \\ \frac{\partial P}{\partial q_4} \\ \frac{\partial P}{\partial q_5} \\ \frac{\partial P}{\partial q_6} \\ \frac{\partial P}{\partial q_7} \end{bmatrix}}_{g} - K^T \underbrace{\begin{bmatrix} -S_N \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{F} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \end{bmatrix}$$

K = Jacobian

Based on the signs above I created 6 different links, and calculated their mass based on the panda_arm.xacro and hand.xacro files. Since I could not open the given CAD files properly I picked their place and marked them in their local frames, using the corresponding transformation matrices the Potential energy was calculated, also the external force vector was written up according to the description.

In [28]:

```
m_1=4.97+3.06
m_2=3.228+0.646
m_4=3.58
m_5=1.225
m_6=1.666
m_7=0.73+0.93

m1,m2,m3,m4,m5,m6,m7=symbols('m_1 m_2 m_3 m_4 m_5 m_6 m_7') #symbols
CG1=Matrix([[0],[0],[33.3/2],[1]])
CG2=Matrix([[0],[0],[31.6/2],[1]])
CG4=Matrix([[0],[0],[0],[1]]) #the motor seems heavy so I put the CG to the origo
CG5=Matrix([[0],[38.4/2],[0],[1]]) # this choice is an assumption to make the calculations
CG6=Matrix([[0],[0],[0],[1]])
CG7=Matrix([[0],[0],[0],[1]]) #the last link and the end effector has approxiatelly the sam
g=9.8

P=((CG1*m1+H_02*CG2*m2+H_04*CG4*m4+H_05*CG5*m5+H_06*CG6*m6+H_07*CG7*m7)*g).subs({q3:0})
P
```

Out[28]:

$$
\left[\begin{matrix}
-154.84m_2 \sin(q_2)\cos(q_1) + 9.8m_4(-a_3 \sin(q_2)\sin(q_4)\cos(q_1) - a_3 \mathrm{c}\\
+ 9.8m_5(-a_3 \sin(q_2)\sin(q_4)\cos(q_1) - a_3 \cos(q_1)\cos(q_2\\
+ d_5(-\sin(q_2)\cos(q_1)\cos(q_4) + \sin(q_4)\cos(q_1)\cos(q_2)) - 19\\
+ 9.8m_6(a_3((\sin(q_2)\sin(q_4)\cos(q_1) + \cos(q_1)\cos(\\
+ a_3(-\sin(q_2)\cos(q_1)\cos(q_4) + \sin(q_4)\cos(q_1)\cos(q_2))\sin(q_6) - a_3 \sin\\
(q_2) - d_3 \sin(q_2)\cos(q_1) + d_5(-\sin(q_2)\cos(\\
+ 9.8m_7(a_3((\sin(q_2)\sin(q_4)\cos(q_1) + \cos(q_1)\cos(\\
+ a_3(-\sin(q_2)\cos(q_1)\cos(q_4) + \sin(q_4)\cos(q_1)\cos(q_2))\sin(q_6) - a_3 \sin\\
(q_2) - d_3 \sin(q_2)\cos(q_1) + d_5(-\sin(q_2)\cos(\\
- d_7(-((\sin(q_2)\sin(q_4)\cos(q_1) + \cos(q_1)\cos(q_2)\cos(q_4))\cos(q_5) - \sin(q_1)\sin(\\
-154.84m_2 \sin(q_1)\sin(q_2) + 9.8m_4(-a_3 \sin(q_1)\sin(q_2)\sin(q_4) - a_3 \mathrm{si}\\
+ 9.8m_5(-a_3 \sin(q_1)\sin(q_2)\sin(q_4) - a_3 \sin(q_1)\cos(q_2\\
+ d_5(-\sin(q_1)\sin(q_2)\cos(q_4) + \sin(q_1)\sin(q_4)\cos(q_2)) - 19\\
+ 9.8m_6(a_3((\sin(q_1)\sin(q_2)\sin(q_4) + \sin(q_1)\cos(q\\
+ a_3(-\sin(q_1)\sin(q_2)\cos(q_4) + \sin(q_1)\sin(q_4)\cos(q_2))\sin(q_6) - a_3 \sin(\\
- d_3 \sin(q_1)\sin(q_2) + d_5(-\sin(q_1)\sin(q_2\\
+ 9.8m_7(a_3((\sin(q_1)\sin(q_2)\sin(q_4) + \sin(q_1)\cos(q\\
+ a_3(-\sin(q_1)\sin(q_2)\cos(q_4) + \sin(q_1)\sin(q_4)\cos(q_2))\sin(q_6) - a_3 \sin(\\
- d_3 \sin(q_1)\sin(q_2) + d_5(-\sin(q_1)\sin(q_2\\
- d_7(-((\sin(q_1)\sin(q_2)\sin(q_4) + \sin(q_1)\cos(q_2)\cos(q_4))\cos(q_5) + \sin(q_5)\cos(\\
163.17m_1 + 9.8m_2(d_1 + 15.8\cos(q_2)) + 9.8m_4(-a_3 \sin(q_2)\cos\\
+ 9.8m_5(-a_3 \sin(q_2)\cos(q_4) + a_3 \sin(q_2) + a_3 \sin(q_4)\cos(q_2) + d_1 + d_3\\
(q_4) + 19.2\cos(q\\
+ 9.8m_6(a_3(\sin(q_2)\sin(q_4) + \cos(q_2)\cos(q_4))\sin(q_6) + a_3(\sin(q_2)\cos(\\
(q_2) + a_3 \sin(q_4)\cos(q_2) + d_1 + d_3 \cos(q_2) +\\
+ 9.8m_7(a_3(\sin(q_2)\sin(q_4) + \cos(q_2)\cos(q_4))\sin(q_6) + a_3(\sin(q_2)\cos(\\
(q_2) + a_3 \sin(q_4)\cos(q_2) + d_1 + d_3 \cos(q_2) +\\
- d_7((\sin(q_2)\sin(q_4) + \cos(q_2)\cos(q_4))\cos(q_6) - (\sin\\
9.8m_1 + 9.8m_2 + 9.8m_4 + 9.
\end{matrix}\right.
$$

The $g(q)$ vector was calculated below.

In [29]:

```
G_q=Matrix([[P[2].diff(q1)],[P[2].diff(q2)],[P[2].diff(q4)],[P[2].diff(q5)],[P[2].diff(q6)]
G_q
```

Out[29]:

$$
\left[\begin{array}{r}
0 \\
-154.84 m_{2} \sin\left(q_{2}\right) + 9.8 m_{4} \left(-a_{3} \sin\left(q_{2}\right) \sin\left(q_{4}\right) - a_{3} \mathrm{cc} \right.\\
+ 9.8 m_{5} \left(-a_{3} \sin\left(q_{2}\right) \sin\left(q_{4}\right) - a_{3} \cos\left(q_{2}\right) \cos\left(q_{4}\right) + a_{3} \cos\left(q_{2}\right) - d_{3} \sin\left(q_{2}\right) + \right.\\
+ 19.2 \sin\left(q_{4}\right) \cos(\\
+ 9.8 m_{6} \left(a_{3} \left(\sin\left(q_{2}\right) \sin\left(q_{4}\right) + \cos\left(q_{2}\right) \cos\left(q_{4}\right)\right) \cos\left(q_{5}\right) \cos\left(q_{6}\right) + a_{3} \left(- \sin\left(q_{2}\right)\right.\right.\\
\left(q_{2}\right) \cos\left(q_{4}\right) + a_{3} \cos\left(q_{2}\right) - d_{3} \sin\left(q_{2}\right) + d_{5} \left(- \mathrm{si}\right.\\
+ 9.8 m_{7} \left(a_{3} \left(\sin\left(q_{2}\right) \sin\left(q_{4}\right) + \cos\left(q_{2}\right) \cos\left(q_{4}\right)\right) \cos\left(q_{5}\right) \cos\left(q_{6}\right) + a_{3} \left(- \sin\left(q_{2}\right)\right.\right.\\
\left(q_{2}\right) \cos\left(q_{4}\right) + a_{3} \cos\left(q_{2}\right) - d_{3} \sin\left(q_{2}\right) + d_{5} \left(- \mathrm{si}\right.\\
- d_{7} \left(\left(- \sin\left(q_{2}\right) \sin\left(q_{4}\right) - \cos\left(q_{2}\right) \cos\left(q_{4}\right)\right) \sin\left(q_{6}\right) \cos\left(q_{5}\right) + \right.\\
9.8 m_{4} \left(a_{3} \sin\left(q_{2}\right) \sin\left(q_{4}\right) + a_{3} \mathrm{c}\right.\\
+ 9.8 m_{5} \left(a_{3} \sin\left(q_{2}\right) \sin\left(q_{4}\right) + a_{3} \cos\left(q_{2}\right) \cos\left(q_{4}\right) + d_{5} \left(\sin\left(q_{2}\right) \cos\left(q_{4}\right) - \mathrm{sir}\right.\right.\\
+ 9.8 m_{6} \left(a_{3} \left(- \sin\left(q_{2}\right) \sin\left(q_{4}\right) - \cos\left(q_{2}\right) \cos\left(q_{4}\right)\right) \cos\left(q_{5}\right) \cos\left(q_{6}\right) + a_{3} \left(\sin\left(q_{2}\right)\right.\right.\\
\left(q_{2}\right) \cos\left(q_{4}\right) + d_{5} \left(\sin\left(q_{2}\right) \cos\left(q_{4}\right)\right.\\
+ 9.8 m_{7} \left(a_{3} \left(- \sin\left(q_{2}\right) \sin\left(q_{4}\right) - \cos\left(q_{2}\right) \cos\left(q_{4}\right)\right) \cos\left(q_{5}\right) \cos\left(q_{6}\right) + a_{3} \left(\sin\left(q_{2}\right)\right.\right.\\
\left(q_{2}\right) \cos\left(q_{4}\right) + d_{5} \left(\sin\left(q_{2}\right) \cos\left(q_{4}\right) - \sin\left(q_{4}\right) \cos\left(q_{2}\right)\right) - d_{7} \left(\left(\sin\left(q_{2}\right) \sin\left(q_{4}\right) + \cos\left(q_{2}\right) \mathrm{c}\right.\right.\\
-9.8 a_{3} m_{6} \left(\sin\left(q_{2}\right) \cos\left(q_{4}\right) - \sin\left(q_{4}\right) \mathrm{c}\right.\\
+ 9.8 m_{7} \left(-a_{3} \left(\sin\left(q_{2}\right) \cos\left(q_{4}\right) - \sin\left(q_{4}\right) \cos\left(q_{2}\right)\right) \sin\left(q_{5}\right) \cos\left(q_{6}\right) + d_{7}\right.\\
9.8 m_{6} \left(a_{3} \left(\sin\left(q_{2}\right) \sin\left(q_{4}\right) + \cos\left(q_{2}\right) \cos\left(q_{4}\right)\right) \cos\left(q_{6}\right) - a_{3} \left(\sin\right.\right.\\
+ 9.8 m_{7} \left(a_{3} \left(\sin\left(q_{2}\right) \sin\left(q_{4}\right) + \cos\left(q_{2}\right) \cos\left(q_{4}\right)\right) \cos\left(q_{6}\right) - a_{3} \left(\mathrm{si}\right.\right.\\
- d_{7} \left(- \left(\sin\left(q_{2}\right) \sin\left(q_{4}\right) + \cos\left(q_{2}\right) \cos\left(q_{4}\right)\right) \sin\left(q_{6}\right) + \left(- \sin\left(q\right.\right.\right.\\
0
\end{array}\right.
$$

The mass values are substituted.

In [30]:

```
G_q=G_q.subs({m1:m_1,m2:m_2,m4:m_4,m5:m_5,m6:m_6,m7:m_7,d1:33.3,d3:31.6,d5:38.4,a3:8.8,d7:2
F=Matrix([[-5],[0],[0],[0],[0],[0]])
G_q
```

Out[30]:

$$\begin{bmatrix} 0 \\ -336.7476\left(-\sin(q_2)\sin(q_4)-\cos(q_2)\cos(q_4)\right)\sin(q_6)\cos(q_5)+286.83424\left(\sin\right. \\ +286.83424\left(-\sin(q_2)\cos(q_4)+\sin(q_4)\cos(q_2)\right)\sin(q_6)-336.7476\left(-\sin(q_2)\cos(q\right. \\ (q_4)-1943.12832\sin(q_2)\cos(q_4)-3117.85824\sin(q_2)+1943.12832\sin(q_4)\cos(q \\ 286.83424\left(-\sin(q_2)\sin(q_4)-\cos(q_2)\cos(q_4)\right)\cos(q_5)\cos(q_6)-336.7476\left(\sin\right. \\ +286.83424\left(\sin(q_2)\cos(q_4)-\sin(q_4)\cos(q_2)\right)\sin(q_6)-336.7476\left(\sin(q_2)\cos(q_4)-\right. \\ +1943.12832\sin(q_2)\cos(q_4)-1943.12832\sin(q_4)\cos(q_2) \\ 336.7476\left(-\sin(q_2)\cos(q_4)+\sin(q_4)\cos(q_2)\right)\sin(q_5)\sin(q_6)-286.83424\left(\sin\right. \\ 336.7476\left(\sin(q_2)\sin(q_4)+\cos(q_2)\cos(q_4)\right)\sin(q_6)+286.83424\left(\sin(q_2)\sin\right. \\ -336.7476\left(-\sin(q_2)\cos(q_4)+\sin(q_4)\cos(q_2)\right)\cos(q_5)\cos(q_6)-286.83424\left(\sin(q_2\right. \\ 0 \end{bmatrix}$$

Running a loop to calculate to do the inverse kinematics, first calculate the velocity vector, from that using the inverse Jacobian we get $\dot{q}$, by numerical integration we get q and using the transformation matrix we calculate the Y, Z coordiantes in each iteration. The code is pretty much the same as it was in HW4 just the time is adjusted and there is a torque calculation extension. In each cycle the τ values are calculated based on the matrix equation written up on the figure above, the values are plotted below. The loop runs for a while but the results seem reasonable and the circle looks nice.

In [31]:

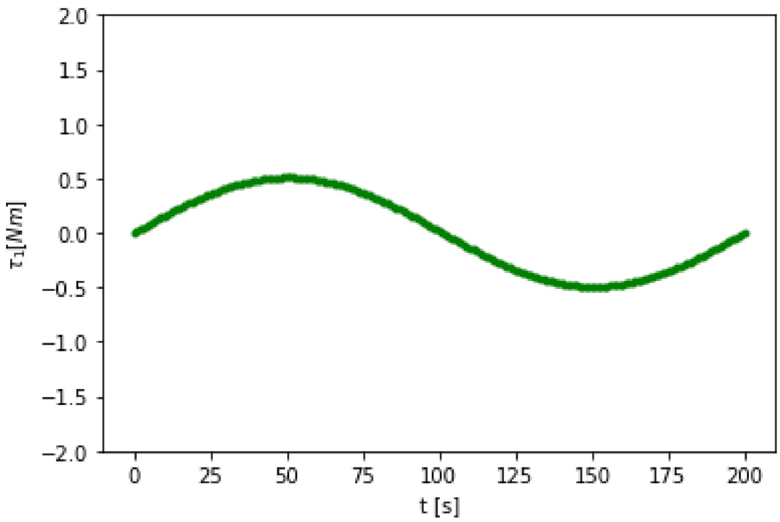
```
theta_dot=2*pi/200
t=0
V=Matrix([[0],[0],[0],[0],[0],[0]])
Q=Matrix([[0.0],[0.0],[pi/2],[0.0],[pi],[0.0]])
i=0
r=10
x=[]
y=[]
z=[]
tau_1=[]
tau_2=[]
tau_4=[]
tau_5=[]
tau_6=[]
tau_7=[]
time=[]
Tau=Matrix([[0],[0],[0],[0],[0],[0]])
Plot=Matrix([[0],[0],[0],[0]])
A=H_07_inv.col(-1)
while(i<=200):

    V[1]=(-r*sin(pi/2+theta_dot*i)*theta_dot).evalf()
    V[2]=(r*cos(pi/2+theta_dot*i)*theta_dot).evalf()
    #print(Plot[2])
    #Plot=(H_07_inv.col(-1)).subs({q1:Q[0],q2:Q[1],q4:Q[2],q5:Q[3],q6:Q[4],q7:Q[5]}).evalf(
    K=M.subs({q1:Q[0],q2:Q[1],q4:Q[2],q5:Q[3],q6:Q[4],q7:Q[5]}).evalf()
    Q_dot=K.inv().evalf()*V
    Q=Q+1*Q_dot

    Tau = G_q.subs({q1:Q[0],q2:Q[1],q4:Q[2],q5:Q[3],q6:Q[4],q7:Q[5]}).evalf()-K.T*F
    tau_1.append(Tau[0]/100)
    tau_2.append(Tau[1]/100)
    tau_4.append(Tau[2]/100)
    tau_5.append(Tau[3]/100)
    tau_6.append(Tau[4]/100)
    tau_7.append(Tau[5]/100)
    time.append(i)

    #Q[0]=Q[0]+0.25*Q_dot[0]
    #Q[1]=Q[1]+0.25*Q_dot[1]
    #Q[2]=Q[2]+0.25*Q_dot[2]
    #Q[3]=Q[3]+0.25*Q_dot[3]
    #Q[4]=Q[4]+0.25*Q_dot[4]
    #Q[5]=Q[5]+0.25*Q_dot[5]
    Plot=(A.subs({q1:Q[0],q2:Q[1],q4:Q[2],q5:Q[3],q6:Q[4],q7:Q[5]})).evalf()
    x.append(Plot[0])
    y.append(Plot[1])
    z.append(Plot[2])
    ## 2D plot essentials
    #print(Plot[2])
    #plt.plot(Plot[0],Plot[1], Plot[2], color='green', linestyle='solid', linewidth = 3,
     #    marker='o')
    #
    i=i+1
```

In [32]:

```
plt.plot(time,tau_1, color='green', linestyle='solid', linewidth = 1,
        marker='.')
plt.ylabel('$\\tau_1 [Nm]$')
plt.xlabel('t [s]')
plt.ylim((-2,2))
```

Out[32]:

```
(-2.0, 2.0)
```

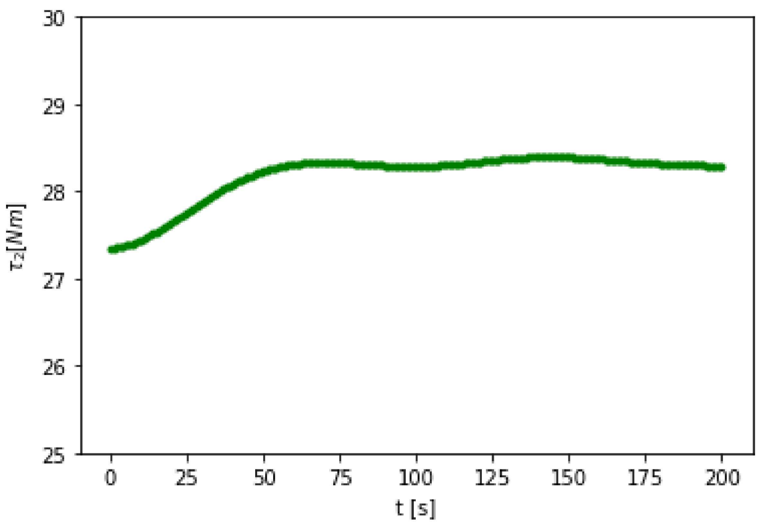


In [33]:

```
plt.plot(time,tau_2, color='green', linestyle='solid', linewidth = 1,
        marker='.')
plt.ylabel('$\\tau_2 [Nm]$')
plt.xlabel('t [s]')
plt.ylim((25,30))
```

Out[33]:

```
(25.0, 30.0)
```

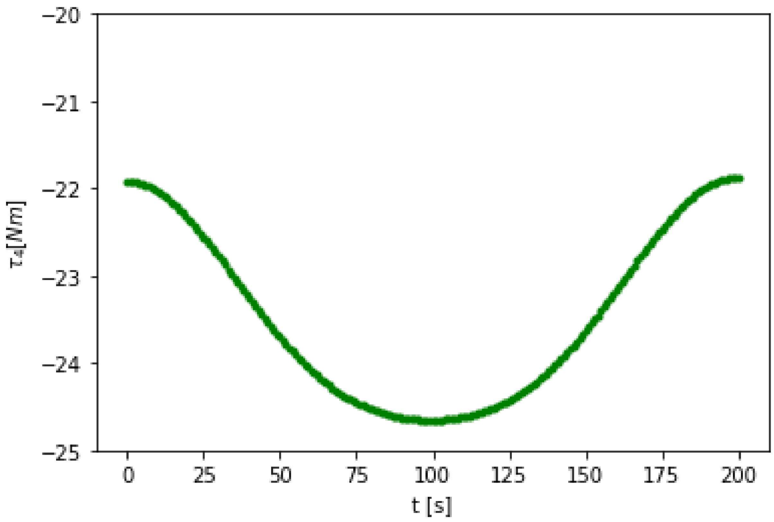


In [34]:

```
plt.plot(time,tau_4, color='green', linestyle='solid', linewidth = 1,
        marker='.')
plt.ylabel('$\\tau_4 [Nm]$')
plt.xlabel('t [s]')
plt.ylim((-25,-20))
```

Out[34]:

```
(-25.0, -20.0)
```

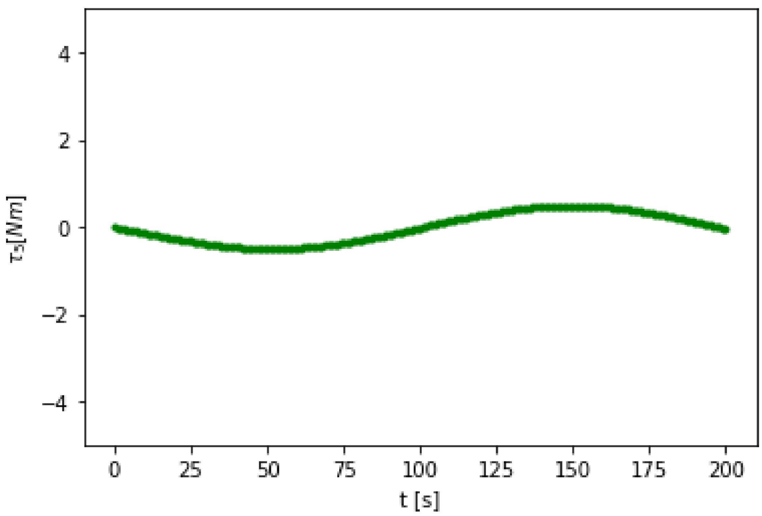


In [35]:

```
plt.plot(time,tau_5, color='green', linestyle='solid', linewidth = 1,
        marker='.')
plt.ylabel('$\\tau_5 [Nm]$')
plt.xlabel('t [s]')
plt.ylim((-5,5))
```

Out[35]:

```
(-5.0, 5.0)
```

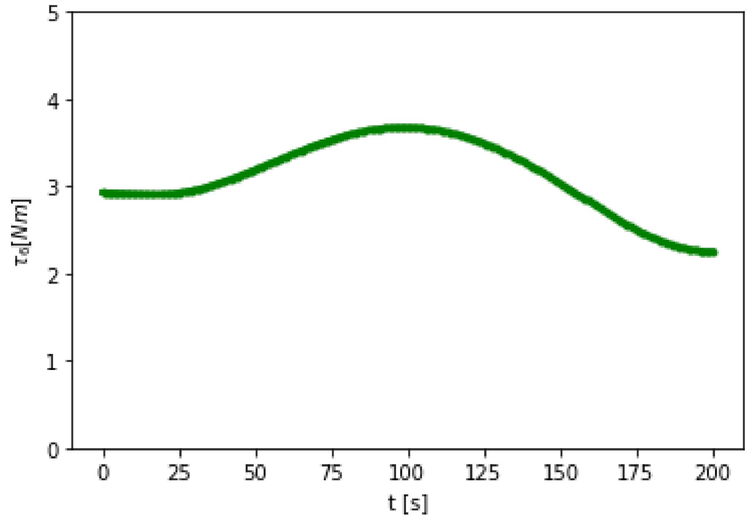


In [36]:

```
plt.plot(time,tau_6, color='green', linestyle='solid', linewidth = 3,
        marker='.')
plt.ylabel('$\\tau_6 [Nm]$')
plt.xlabel('t [s]')
plt.ylim((0,5))
```

Out[36]:

```
(0.0, 5.0)
```

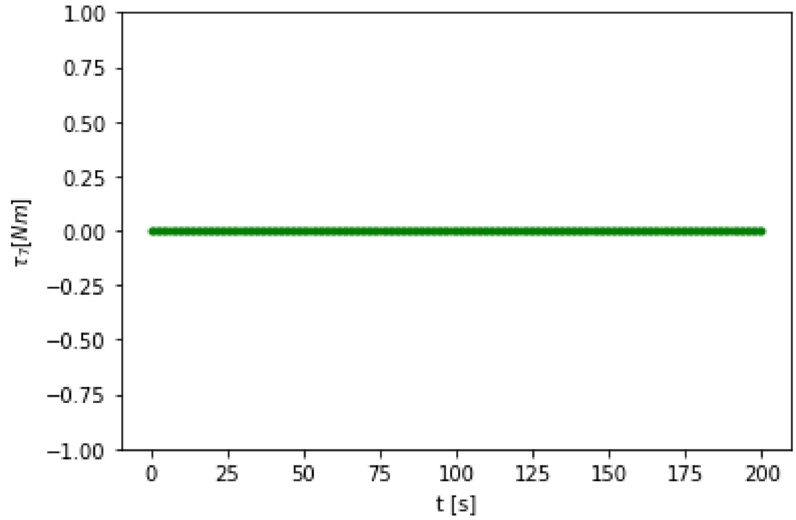


In [37]:

```
plt.plot(time,tau_7, color='green', linestyle='solid', linewidth = 1,
        marker='.')
plt.ylabel('$\\tau_7 [Nm]$')
plt.xlabel('t [s]')
plt.ylim((-1,1))
```

Out[37]:

```
(-1.0, 1.0)
```



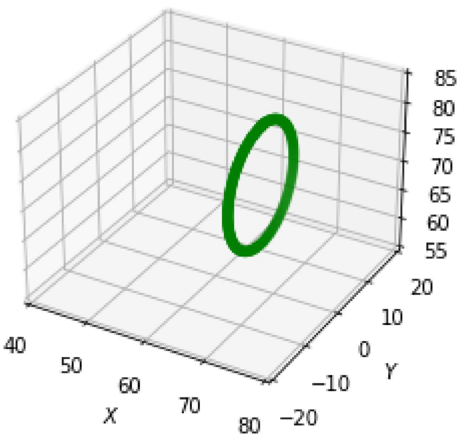
3D plot

In [38]:

```
ax = plt.axes(projection='3d')
ax.set_aspect('equal','box')
ax.axes.set_xlim3d(left=40, right=80)
ax.axes.set_ylim3d(bottom=-20, top=20)
ax.axes.set_zlim3d(bottom=55, top=85)
ax.scatter3D(x, y, z, color='green');
ax.set_xlabel('$X$')
ax.set_ylabel('$Y$')
```

Out[38]:

```
Text(0.5, 0.5, '$Y$')
```



In [ ]: