In [33]:

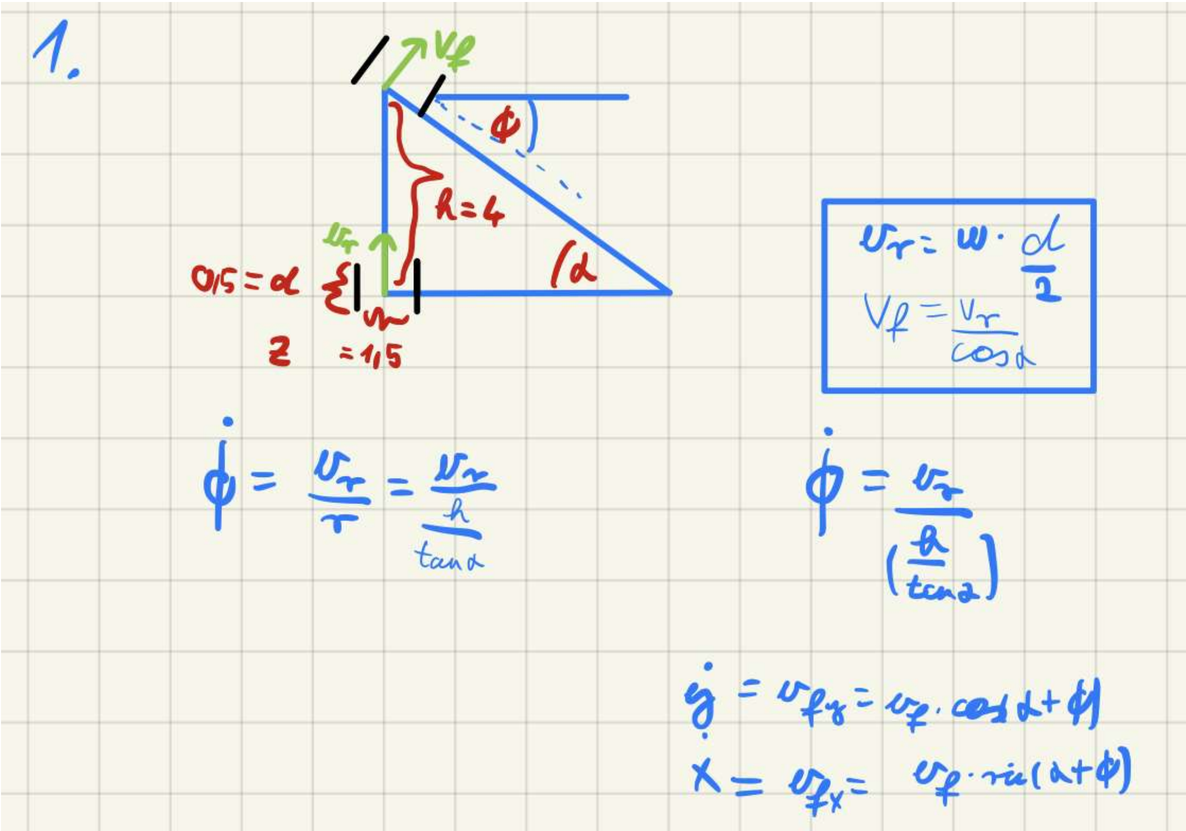
```
from sympy import*
from IPython.display import Image, display, HTML
from scipy import optimize
import matplotlib.pyplot as plt
```

1.

In [34]:

```
Image("figure1.png")
```

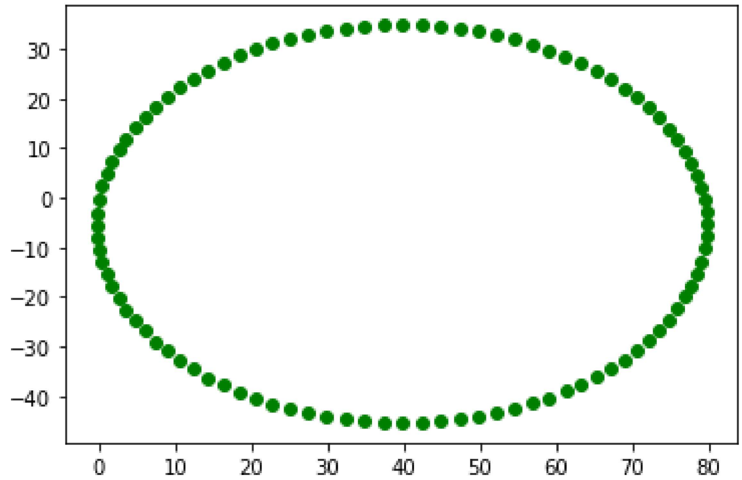
Out[34]:



Plot with alpha according to the sketch

In [56]:

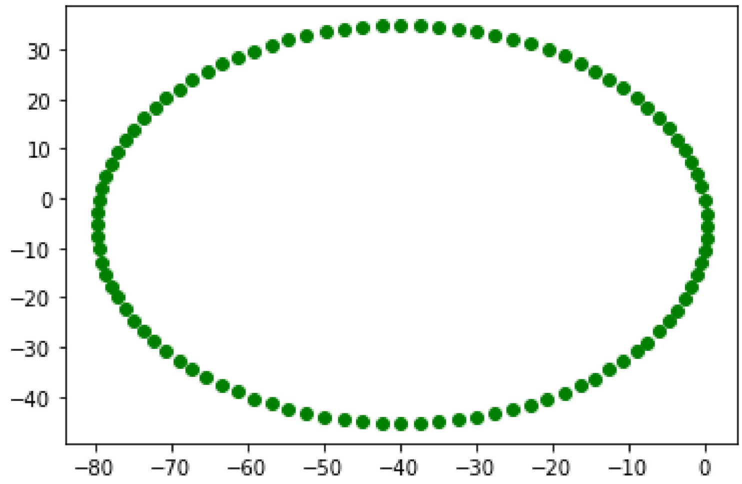
```
#Initialize
h_ = 4 # m
d_ = 0.5 # m
z_ = 0 # m
alfa_ = 0.1 # rad
x_0 = 0 # m
y_0 = 0 # m
teta_0 = 0 # rad
omega_= 10 # rad/s

x, y, teta, xd, yd, tetad, h, d, alfa, omega= symbols('x y teta x^d y^d theta^d h d alpha o

v_r=omega*d/2;
v_f=v_r/cos(alfa)
xd=v_f*sin(alfa+teta)
yd=v_f*cos(alfa+teta)
tetad=v_r/(h/tan(alfa))
x=0
y=0
teta_=0
#Taking integrates using x=x+delta T*xdot, the same with y and theta
for i in range(0,100):
    teta_=teta_+tetad.subs({h:h_,d:d_,alfa:alfa_,omega:omega_,teta:teta_})
    x=x+xd.subs({h:h_,d:d_,alfa:alfa_,omega:omega_,teta:teta_})
    y=y+yd.subs({h:h_,d:d_,alfa:alfa_,omega:omega_,teta:teta_})
    xd=v_f*sin(alfa+teta)
    yd=v_f*cos(alfa+teta)
    tetad=v_r/(h/tan(alfa))
    plt.plot(x, y, color='green', linestyle='solid', linewidth = 3,
         marker='o')
```



Plot according to alpha with right hand rule

In [36]:

```
v_r=omega*d/2;
v_f=v_r/cos(alfa)
xd=-v_f*sin(alfa+teta)
yd=v_f*cos(alfa+teta)
tetad=v_r/(h/tan(alfa))
x=0
y=0
teta_=0
for i in range(0,100):
    teta_=teta_+tetad.subs({h:h_,d:d_,alfa:alfa_,omega:omega_,teta:teta_})
    x=x+xd.subs({h:h_,d:d_,alfa:alfa_,omega:omega_,teta:teta_})
    y=y+yd.subs({h:h_,d:d_,alfa:alfa_,omega:omega_,teta:teta_})
    xd=-v_f*sin(alfa+teta)
    yd=v_f*cos(alfa+teta)
    tetad=v_r/(h/tan(alfa))
    plt.plot(x, y, color='green', linestyle='solid', linewidth = 3,
         marker='o')
```



In [ ]:

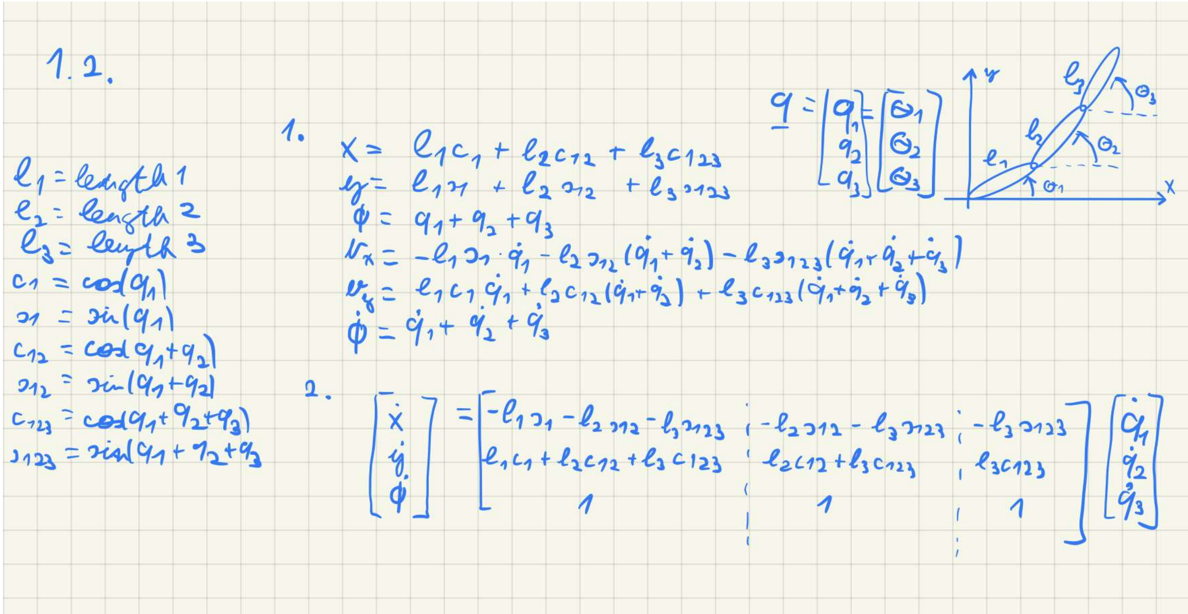
```

```

2.

In [37]:

```
Image("figure2.png")
```

Out[37]:



In [38]:

```
#Initialize the variables
x_2, y_2, teta_2, xd_2, yd_2, tetad_2, l1, l2, l3= symbols('x y teta x^d y^d theta^d l_1 l_


t=symbols('t')
q1=Function('q1')
q2=Function('q2')
q3=Function('q3')
dq1 = Function('\dot{q_1}')
dq2 = Function('\dot{q_2}')
dq3 = Function('\dot{q_3}')

#Calculating the positions and then differentiate by time to get the velocities
x_2=l1*cos(q1(t))+l2*cos(q1(t)+q2(t))+l3*cos(q1(t)+q2(t)+q3(t))
y_2=l1*sin(q1(t))+l2*sin(q1(t)+q2(t))+l3*sin(q1(t)+q2(t)+q3(t))
teta_2=q1(t)+q2(t)+q3(t)
xd_2 = x_2.diff(t).subs({q1(t).diff(t):dq1(t),q2(t).diff(t):dq2(t),q3(t).diff(t):dq3(t)})
yd_2 = y_2.diff(t).subs({q1(t).diff(t):dq1(t),q2(t).diff(t):dq2(t),q3(t).diff(t):dq3(t)})
tetad_2 = teta_2.diff(t).subs({q1(t).diff(t):dq1(t),q2(t).diff(t):dq2(t),q3(t).diff(t):dq3(
```

In [39]:

```
x_2
```

Out[39]:

$$l_1 \cos\left(q_1(t)\right) + l_2 \cos\left(q_1(t) + q_2(t)\right) + l_3 \cos\left(q_1(t) + q_2(t) + q_3(t)\right)$$

In [40]:

```
y_2
```

Out[40]:

$$l_1 \sin\left(q_1(t)\right) + l_2 \sin\left(q_1(t) + q_2(t)\right) + l_3 \sin\left(q_1(t) + q_2(t) + q_3(t)\right)$$

In [41]:

```
teta_2
```

Out[41]:

$$q_1(t) + q_2(t) + q_3(t)$$

In [42]:

```
xd_2
```

Out[42]:

$$-l_1 \dot{q}_1(t) \sin\left(q_1(t)\right) - l_2 \left(\dot{q}_1(t) + \dot{q}_2(t)\right) \sin\left(q_1(t) + q_2(t)\right) - l_3 \left(\dot{q}_1(t) + \dot{q}_2(t) + \dot{q}_3(t)\right) \sin($$

In [43]:

```
yd_2
```

Out[43]:

$$l_1 \dot{q}_1(t) \cos\left(q_1(t)\right) + l_2 \left(\dot{q}_1(t) + \dot{q}_2(t)\right) \cos\left(q_1(t) + q_2(t)\right) + l_3 \left(\dot{q}_1(t) + \dot{q}_2(t) + \dot{q}_3(t)\right) \cos($$

In [44]:

```
tetad_2
```

Out[44]:

$$\dot{q}_1(t) + \dot{q}_2(t) + \dot{q}_3(t)$$

In [45]:

```
#Inverse Matrix
system=Matrix([[-l1*sin(q1(t))-l2*sin(q1(t)+q2(t))-l3*sin(q1(t)+q2(t)+q3(t)),
                -l2*sin(q1(t)+q2(t))-l3*sin(q1(t)+q2(t)+q3(t)),
                -l3*sin(q1(t)+q2(t)+q3(t))],
                   [l1*cos(q1(t))+l2*cos(q1(t)+q2(t))+l3*cos(q1(t)+q2(t)+q3(t)),
                    l2*cos(q1(t)+q2(t))+l3*cos(q1(t)+q2(t)+q3(t)),
                    l3*cos(q1(t)+q2(t)+q3(t))],
                   [1,1,1]])
system.inv()
```

Out[45]:

$$\begin{bmatrix} \frac{\cos(\mathrm{q}_1(t)+\mathrm{q}_2(t))}{l_1 \sin(\mathrm{q}_1(t)+\mathrm{q}_2(t)) \cos(\mathrm{q}_1(t)) - l_1 \sin(\mathrm{q}_1(t)) \cos(\mathrm{q}_1(t)+\mathrm{q}_2(t))} & \frac{\sin(\mathrm{q}_1(t)+\mathrm{q}_2(t))}{l_1 \sin(\mathrm{q}_1(t)+\mathrm{q}_2(t)) \cos(\mathrm{q}_1(t)) - l_1 \sin (\mathrm{q}} \\ \frac{-l_1 \cos(\mathrm{q}_1(t)) - l_2 \cos(\mathrm{q}_1(t)+\mathrm{q}_2(t))}{l_1 l_2 \sin(\mathrm{q}_1(t)+\mathrm{q}_2(t)) \cos(\mathrm{q}_1(t)) - l_1 l_2 \sin(\mathrm{q}_1(t)) \cos(\mathrm{q}_1(t)+\mathrm{q}_2(t))} & \frac{-l_1 \sin(\mathrm{q}_1(t)) - l_2 \sin(\mathrm{q}_1 (}{l_1 l_2 \sin(\mathrm{q}_1(t)+\mathrm{q}_2(t)) \cos(\mathrm{q}_1(t)) - l_1 l_2 \sin} \\ -\frac{\cos(\mathrm{q}_1(t))}{-l_2 \sin(\mathrm{q}_1(t)+\mathrm{q}_2(t)) \cos(\mathrm{q}_1(t)) + l_2 \sin(\mathrm{q}_1(t)) \cos(\mathrm{q}_1(t)+\mathrm{q}_2(t))} & -\frac{\sin(\mathrm{q}_1(t))}{-l_2 \sin(\mathrm{q}_1(t)+\mathrm{q}_2(t)) \cos(\mathrm{q}_1(t)) + l_2 \sin} \end{bmatrix}$$

In [51]:

```
#Simplified form
A_inv=simplify(system.inv())
A_inv
```

Out[51]:

$$\begin{bmatrix} \frac{\cos(\mathrm{q}_1(t)+\mathrm{q}_2(t))}{l_1 \sin(\mathrm{q}_2(t))} & \frac{\sin(\mathrm{q}_1(t)+\mathrm{q}_2(t))}{l_1 \sin(\mathrm{q}_2(t))} & \frac{l_3 \sin(\mathrm{q}_3(t))}{l_1 \sin(\mathrm{q}_2(t))} \\ -\frac{l_1 \cos(\mathrm{q}_1(t)) + l_2 \cos(\mathrm{q}_1(t)+\mathrm{q}_2(t))}{l_1 l_2 \sin(\mathrm{q}_2(t))} & -\frac{l_1 \sin(\mathrm{q}_1(t)) + l_2 \sin(\mathrm{q}_1(t)+\mathrm{q}_2(t))}{l_1 l_2 \sin(\mathrm{q}_2(t))} & -\frac{l_3 (l_1 \sin(\mathrm{q}_2(t)+\mathrm{q}_3(t)) + l_2 \sin}{l_1 l_2 \sin(\mathrm{q}_2(t))} \\ \frac{\cos(\mathrm{q}_1(t))}{l_2 \sin(\mathrm{q}_2(t))} & \frac{\sin(\mathrm{q}_1(t))}{l_2 \sin(\mathrm{q}_2(t))} & \frac{l_2 + \frac{l_3 \sin(\mathrm{q}_3(t))}{\tan(\mathrm{q}_2(t))} + l_3 \cos(\mathrm{q}_3}{l_2} \end{bmatrix}$$

In [52]:

```
Image("figure3.png")
```

Out[52]:

2.

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}}_{q}$$

$$A^{-1} Y = q$$

In [53]:

```
xd_22, yd_22, tetad_22,= symbols('x^d y^d theta^d')
```

In [54]:

```
#Velocities
Velocities=Matrix([[xd_22],[yd_22],[tetad_22]])
Velocities
```

Out[54]:

$$\begin{bmatrix} x^d \\ y^d \\ \theta^d \end{bmatrix}$$

In [55]:

```
#equations in matrix form for joint velocities
A_inv*Velocities
```

Out[55]:

$$\begin{bmatrix} \frac{l_3\theta^d \sin(q_3(t))}{l_1 \sin(q_2(t))} + \frac{x^d \cos(q_1(t)+q_2(t))}{l_1 \sin(q_2(t))} + \frac{y^d \sin(q_1(t)+q_2(t))}{l_1 \sin(q_2(t))} \\ -\frac{l_3\theta^d(l_1 \sin(q_2(t)+q_3(t))+l_2 \sin(q_3(t)))}{l_1 l_2 \sin(q_2(t))} - \frac{x^d(l_1 \cos(q_1(t))+l_2 \cos(q_1(t)+q_2(t)))}{l_1 l_2 \sin(q_2(t))} - \frac{y^d(l_1 \sin(q_1(t))+l_2 \sin}{l_1 l_2 \sin(q_2} \\ \frac{\theta^d\left(l_2+\frac{l_3 \sin(q_3(t))}{\tan(q_2(t))}+l_3\cos(q_3(t))\right)}{l_2} + \frac{x^d \cos(q_1(t))}{l_2 \sin(q_2(t))} + \frac{y^d \sin(q_1(t))}{l_2 \sin(q_2(t))} \end{bmatrix}$$