In [34]:

```
from sympy import*
from IPython.display import Image, display, HTML
from scipy import optimize
import matplotlib.pyplot as plt
import numpy as np
```

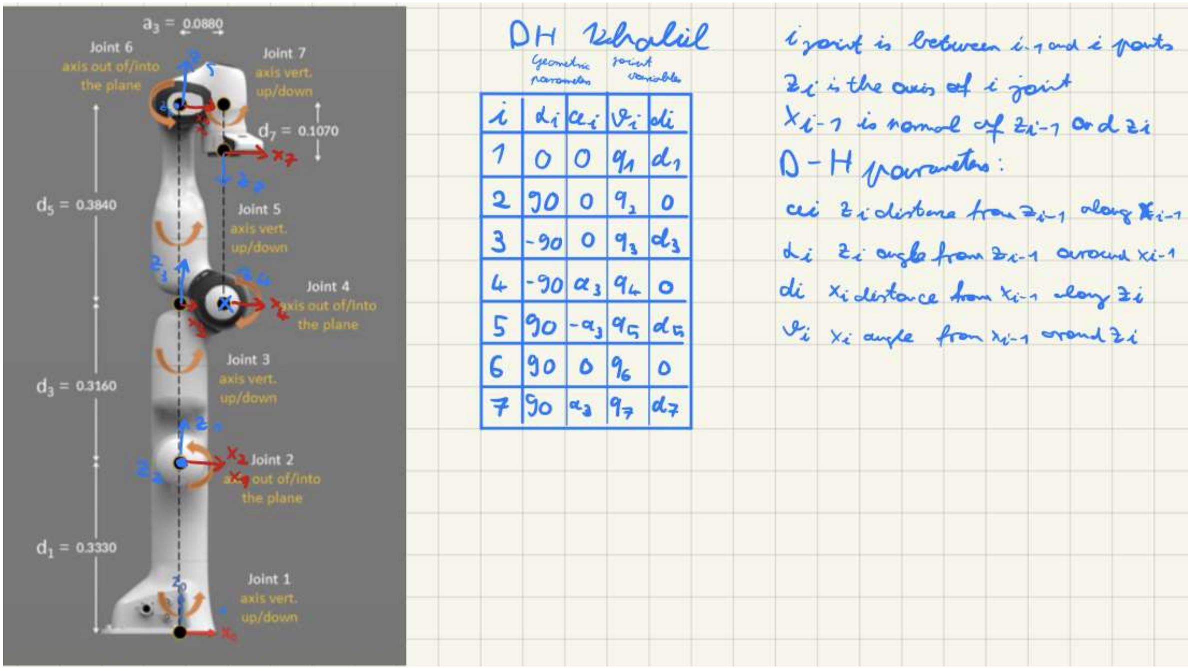
1. Position Kinematics - Panda

First I completed the DH methode with the Khalil convention, I am used to this and it makes more sense for me. I'll be able to check my results as well at the end.

In [35]:

```
Image("Panda_K.png")
```

Out[35]:



In [36]:

```
def DH_Tr_sym(alpha=0, a=0, theta=0, d=0):
    Tr_alpha=Matrix([[1,0,0,0],
                       [0,cos(alpha),-sin(alpha),0],
                       [0,sin(alpha) ,cos(alpha),0],
                       [0,0,0,1]])
    Tr_a=Matrix([[1,0,0,a],
                   [0,1,0,0],
                   [0,0,1,0],
                   [0,0,0,1]])
    Tr_theta=Matrix([[cos(theta),-sin(theta),0,0],
                       [sin(theta),cos(theta),0,0],
                       [0,0,1,0],
                       [0,0,0,1]])
    Tr_d=Matrix([[1,0,0,0],
                   [0,1,0,0],
                   [0,0,1,d],
                   [0,0,0,1]])
    Tr_KHALIL=Tr_alpha@Tr_a@Tr_theta@Tr_d

    return Tr_KHALIL
```

In [37]:

```
q1,q2,q3,q4,q5,q6,q7,d1,d3,d5,d7,a3=symbols('q_1 q_2 q_3 q_4 q_5 q_6 q_7 d_1 d_3 d_5 d_7 a_
```

In [38]:

```
T_01=simplify(DH_Tr_sym(alpha=0, a=0, theta=q1, d=d1))
```

In [39]:

```
T_12=simplify(DH_Tr_sym(alpha=pi/2, a=0, theta=q2, d=0))
```

In [40]:

```
T_23=simplify(DH_Tr_sym(alpha=-pi/2, a=0, theta=q3, d=d3))
```

In [41]:

```
T_34=simplify(DH_Tr_sym(alpha=-pi/2, a=a3, theta=q4, d=0))
```

In [42]:

```
T_45=simplify(DH_Tr_sym(alpha=pi/2, a=-a3, theta=q5, d=d5))
```

In [43]:

```
T_56=simplify(DH_Tr_sym(alpha=pi/2, a=0, theta=q6, d=0))
```

In [44]:

```
T_67=simplify(DH_Tr_sym(alpha=pi/2, a=a3, theta=q7, d=d7))
```

In [45]:

```
T_07=T_01@T_12@T_23@T_34@T_45@T_56@T_67
T_07
```

Out[45]:

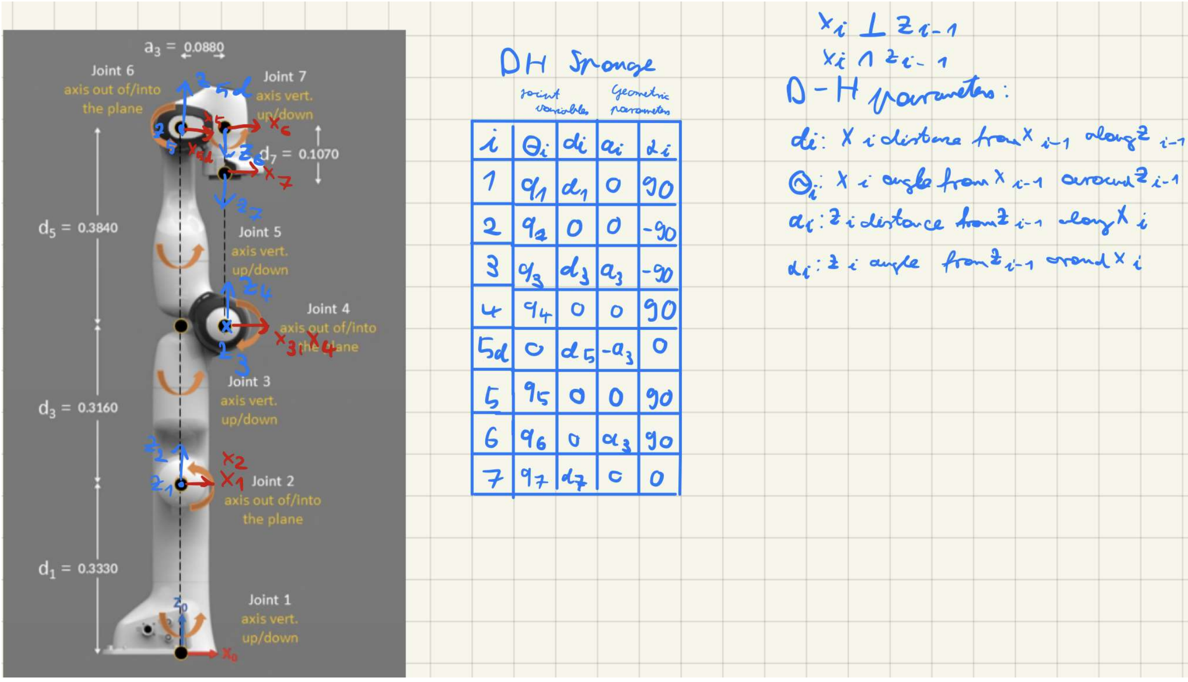
$$\left[\begin{matrix} ((((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1))\,\mathrm{cc} \\ (q_5))\cos(q_6)+((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\sin(q_4)\cdot \\ +(((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1))\,\mathrm{s} \\ (q_5))\sin(q_7) \\ \\ ((((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4))\cos \\ (q_5))\cos(q_6)+((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\sin(q_4)- \\ +(((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4))\,\mathrm{sir} \\ (q_5))\sin(q_7) \\ \\ (((\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\cos(q_5)-\sin(q_2)\sin(q_3)\sin(q_5))\cos(q_6 \\ (q_7)+((\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\sin(q_5)+ \\ \\ 0 \end{matrix} \right.$$

Second I completed the DH methode with the Sponge convention.

In [46]:

```
Image("Panda_S.png")
```

Out[46]:



In [47]:

```
def DH_Tr_sym_UMD(theta=0, d=0 , a=0, alpha=0):
    Tr_theta=Matrix([[cos(theta),-sin(theta),0,0],
                       [sin(theta),cos(theta),0,0],
                       [0,0,1,0],
                       [0,0,0,1]])
    Tr_d=Matrix([[1,0,0,0],
                   [0,1,0,0],
                   [0,0,1,d],
                   [0,0,0,1]])
    Tr_a=Matrix([[1,0,0,a],
                   [0,1,0,0],
                   [0,0,1,0],
                   [0,0,0,1]])
    Tr_alpha=Matrix([[1,0,0,0],
                       [0,cos(alpha),-sin(alpha),0],
                       [0,sin(alpha) ,cos(alpha),0],
                       [0,0,0,1]])


    Tr_KHALIL=Tr_theta@Tr_d@Tr_a@Tr_alpha

    return Tr_KHALIL
```

In [48]:

```
H_01=DH_Tr_sym_UMD(theta=q1, d=d1 , a=0, alpha=pi/2)
H_01
```

Out[48]:

$$\begin{bmatrix} \cos(q_1) & 0 & \sin(q_1) & 0 \\ \sin(q_1) & 0 & -\cos(q_1) & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [49]:

```
H_12=DH_Tr_sym_UMD(theta=q2, d=0 , a=0, alpha=-pi/2)
H_12
```

Out[49]:

$$\begin{bmatrix} \cos(q_2) & 0 & -\sin(q_2) & 0 \\ \sin(q_2) & 0 & \cos(q_2) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [50]:

```
H_23=DH_Tr_sym_UMD(theta=q3, d=d3 , a=a3, alpha=-pi/2)
H_23
```

Out[50]:

$$\begin{bmatrix} \cos(q_3) & 0 & -\sin(q_3) & a_3\cos(q_3) \\ \sin(q_3) & 0 & \cos(q_3) & a_3\sin(q_3) \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [51]:

```
H_34=DH_Tr_sym_UMD(theta=q4, d=0 , a=0, alpha=pi/2)
H_34
```

Out[51]:

$$\begin{bmatrix} \cos(q_4) & 0 & \sin(q_4) & 0 \\ \sin(q_4) & 0 & -\cos(q_4) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [52]:

```
H_45d=DH_Tr_sym_UMD(theta=0, d=d5 , a=-a3, alpha=0)
H_45d
```

Out[52]:

$$\begin{bmatrix} 1 & 0 & 0 & -a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [53]:

```
H_5d5=DH_Tr_sym_UMD(theta=q5, d=0 , a=0, alpha=pi/2)
H_5d5
```

Out[53]:

$$\begin{bmatrix} \cos(q_5) & 0 & \sin(q_5) & 0 \\ \sin(q_5) & 0 & -\cos(q_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [54]:

```
H_56=DH_Tr_sym_UMD(theta=q6, d=0 , a=a3, alpha=pi/2)
H_56
```

Out[54]:

$$\begin{bmatrix} \cos(q_6) & 0 & \sin(q_6) & a_3\cos(q_6) \\ \sin(q_6) & 0 & -\cos(q_6) & a_3\sin(q_6) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [55]:

```
H_67=DH_Tr_sym_UMD(theta=q7, d=d7 , a=0, alpha=0)
H_67
```

Out[55]:

$$\begin{bmatrix} \cos(q_7) & -\sin(q_7) & 0 & 0 \\ \sin(q_7) & \cos(q_7) & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [56]:

```
H_07=H_01@H_12@H_23@H_34@H_45d@H_5d5@H_56@H_67
H_07
```

Out[56]:

$$\left[\begin{array}{c} ((((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1))\,\mathrm{cc} \\ (q_5))\cos(q_6)+((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\sin(q_4)\,\cdot \\ +\,(((-\sin(q_1)\sin(q_3)+\cos(q_1)\cos(q_2)\cos(q_3))\cos(q_4)+\sin(q_2)\sin(q_4)\cos(q_1))\,\mathrm{s} \\ (q_5))\sin(q_7) \\ \\ ((((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4))\cos \\ (q_5))\cos(q_6)+((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\sin(q_4)- \\ +\,(((\sin(q_1)\cos(q_2)\cos(q_3)+\sin(q_3)\cos(q_1))\cos(q_4)+\sin(q_1)\sin(q_2)\sin(q_4))\,\mathrm{sir} \\ (q_5))\sin(q_7) \\ \\ (((\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\cos(q_5)-\sin(q_2)\sin(q_3)\sin(q_5))\cos(q_6 \\ (q_7)+((\sin(q_2)\cos(q_3)\cos(q_4)-\sin(q_4)\cos(q_2))\sin(q_5)+ \\ \\ 0 \end{array}\right.$$

I compared the two results, they are giving the same answers, so it seems fine

In [57]:

```
T_07.subs({q1:pi,q2:pi/2,q3:pi,q4:pi/2,q5:pi,q6:pi/2,q7:pi})
```

Out[57]:

$$\begin{bmatrix} 0 & 0 & 1 & a_3+d_3+d_7 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -2a_3+d_1-d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [58]:

```
H_07.subs({q1:pi,q2:pi/2,q3:pi,q4:pi/2,q5:pi,q6:pi/2,q7:pi})
```

Out[58]:

$$\begin{bmatrix} 0 & 0 & 1 & a_3+d_3+d_7 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -2a_3+d_1-d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In tha last step I completed the five known configuration.

Testing with known configutation. Let $d_1, d_3, d_5 = 1$ and $d_7, a_3 = 0.5$

First test the configuration according to the initial pose, means, all angles are 0. Output should be at $x = 0.5, y = 0, z = 2.5$ with orientation $x = x, y = -y, z = -z$

In [59]:

```
H_07.subs({q1:0,q2:0,q3:0,q4:0,q5:0,q6:0,q7:0,d1:1,d3:1,d5:1,d7:0.5,a3:0.5})
```

Out[59]:

$$\begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Second test the configuration with $q_7 = 90deg$ Output should be at $x = 0.5, y = 0, z = 2.5$ with orientation $x = -y, y = -x, z = -z$

In [60]:

```
H_07.subs({q1:0,q2:0,q3:0,q4:0,q5:0,q6:0,q7:pi/2,d1:1,d3:1,d5:1,d7:0.5,a3:0.5})
```

Out[60]:

$$\begin{bmatrix} 0 & -1 & 0 & 0.5 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Third test the configuration with $q_5 = -90deg$ and $q_7 = -90deg$ Output should be at $x = 0, y = -0.5, z = 2.5$ with orientation $x = x, y = -y, z = -z$

In [61]:

```
H_07.subs({q1:0,q2:0,q3:0,q4:0,q5:-pi/2,q6:0,q7:-pi/2,d1:1,d3:1,d5:1,d7:0.5,a3:0.5})
```

Out[61]:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -0.5 \\ 0 & 0 & -1 & 2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fourth test the configuration with $q_1 = -90deg$ and $q_3 = -90deg$ and $q4 = 90deg$ Output should be at $x = 1, y = 0, z = 2$ with orientation $x = -z, y = -y, z = -x$

In [62]:

```
H_07.subs({q1:pi/2,q2:0,q3:-pi/2,q4:pi/2,q5:0,q6:0,q7:0,d1:1,d3:1,d5:1,d7:0.5,a3:0.5})
```

Out[62]:

$$\begin{bmatrix} 0 & 0 & -1 & 1.0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 2.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fifth test the configuration with $q_2 = -90deg$ and $q_6 = 270deg$ Output should be at $x = 1.5, y = 0, z = 1.5$ with orientation $x = -x, y = -y, z = z$

In [63]:

```
H_07.subs({q1:0,q2:-pi/2,q3:0,q4:0,q5:0,q6:1.5*pi,q7:0,d1:1,d3:1,d5:1,d7:0.5,a3:0.5})
```

Out[63]:

$$\begin{bmatrix} -1 & 0 & 0 & 1.5 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

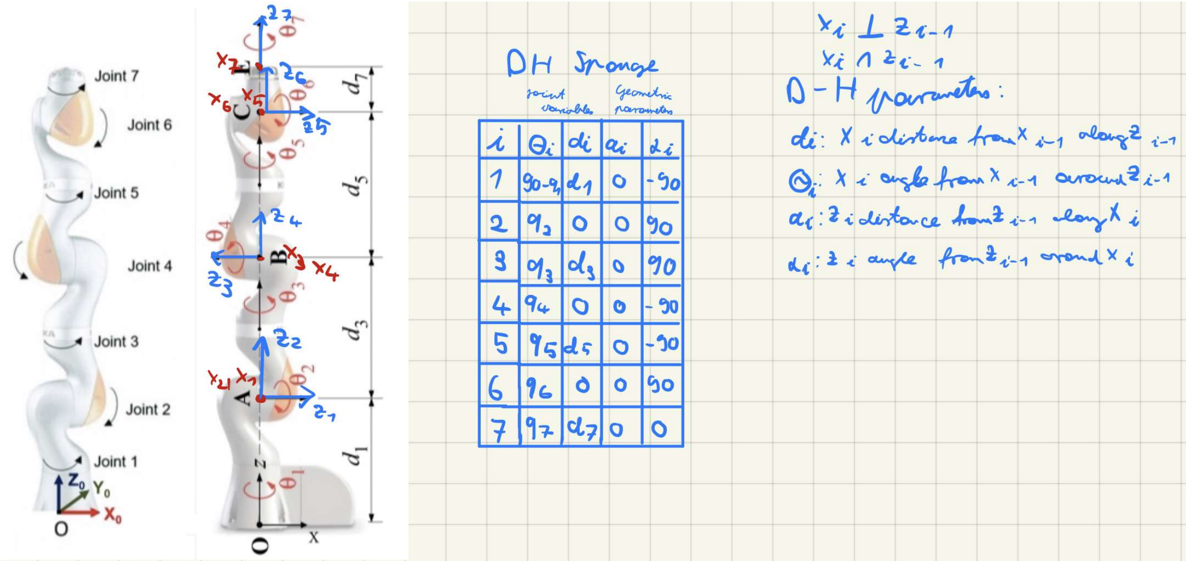
2. Position Kinematics - KUKA

See the table and the drawing below.

In [64]:

```
Image("KUKA.png")
```

Out[64]:



i	Θ_i	d_i	a_i	α_i
1	$90-q_1$	d_1	0	-90
2	q_2	0	0	90
3	q_3	d_3	0	90
4	q_4	0	0	-90
5	q_5	d_5	0	-90
6	q_6	0	0	90
7	q_7	d_7	0	0