



Lecture 15: Sorting Algorithms

CSE 373: Data Structures and Algorithms

Administrivia

Piazza!

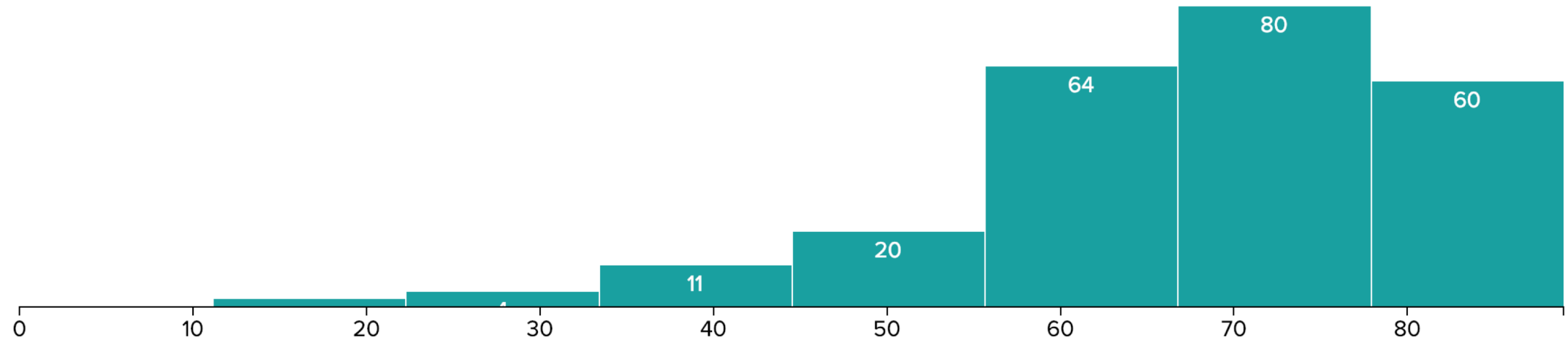
Homework

- HW 5 Part 1 Due Friday 2/22
- HW 5 Part 2 Out Friday, due 3/1
- HW 3 Regrade Option due 3/1

Grades

- HW 1, 2 & 3 Grades into Canvas soon
- Socrative EC status into Canvas soon
- HW 4 Grades published by 3/1
- Midterm Grades Published

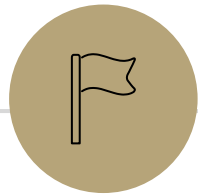
Midterm Stats



MINIMUM	MEDIAN	MAXIMUM	MEAN	STD DEV
17.0	69.0	89.0	67.23	13.17

Midterm 89.0 points

MINIMUM	MEDIAN	MAXIMUM	MEAN	STD DEV
19.1%	77.53%	100.0%	75.54%	14.8%



Sorting

INEFFECTIVE SORTS

```
DEFINE HALFHEARTEDMERGESORT(LIST):  
  IF LENGTH(LIST) < 2:  
    RETURN LIST  
  PIVOT = INT(LENGTH(LIST) / 2)  
  A = HALFHEARTEDMERGESORT(LIST[:PIVOT])  
  B = HALFHEARTEDMERGESORT(LIST[PIVOT:])  
  // UMMMMM  
  RETURN[A, B] // HERE. SORRY.
```

```
DEFINE FASTBOGOSORT(LIST):  
  // AN OPTIMIZED BOGOSORT  
  // RUNS IN O(N LOG N)  
  FOR N FROM 1 TO LOG(LENGTH(LIST)):  
    SHUFFLE(LIST):  
    IF ISSORTED(LIST):  
      RETURN LIST  
  RETURN "KERNEL PAGE FAULT (ERROR CODE: 2)"
```

```
DEFINE JOBINTERVIEWQUICKSORT(LIST):  
  OK SO YOU CHOOSE A PIVOT  
  THEN DIVIDE THE LIST IN HALF  
  FOR EACH HALF:  
    CHECK TO SEE IF IT'S SORTED  
    NO, WAIT, IT DOESN'T MATTER  
    COMPARE EACH ELEMENT TO THE PIVOT  
    THE BIGGER ONES GO IN A NEW LIST  
    THE EQUAL ONES GO INTO, UH  
    THE SECOND LIST FROM BEFORE  
  HANG ON, LET ME NAME THE LISTS  
  THIS IS LIST A  
  THE NEW ONE IS LIST B  
  PUT THE BIG ONES INTO LIST B  
  NOW TAKE THE SECOND LIST  
  CALL IT LIST, UH, A2  
  WHICH ONE WAS THE PIVOT IN?  
  SCRATCH ALL THAT  
  IT JUST RECURSIVELY CALLS ITSELF  
  UNTIL BOTH LISTS ARE EMPTY  
  RIGHT?  
  NOT EMPTY, BUT YOU KNOW WHAT I MEAN  
  AM I ALLOWED TO USE THE STANDARD LIBRARIES?
```

```
DEFINE PANICSORT(LIST):  
  IF ISSORTED(LIST):  
    RETURN LIST  
  FOR N FROM 1 TO 10000:  
    PIVOT = RANDOM(0, LENGTH(LIST))  
    LIST = LIST[PIVOT:] + LIST[:PIVOT]  
    IF ISSORTED(LIST):  
      RETURN LIST  
  IF ISSORTED(LIST):  
    RETURN LIST  
  IF ISSORTED(LIST): // THIS CAN'T BE HAPPENING  
    RETURN LIST  
  IF ISSORTED(LIST): // COME ON COME ON  
    RETURN LIST  
  // OH JEEZ  
  // I'M GONNA BE IN SO MUCH TROUBLE  
  LIST = [ ]  
  SYSTEM("SHUTDOWN -H +5")  
  SYSTEM("RM -RF ./")  
  SYSTEM("RM -RF ~/*")  
  SYSTEM("RM -RF /")  
  SYSTEM("RD /S /Q C:\*") // PORTABILITY  
  RETURN [1, 2, 3, 4, 5]
```

Types of Sorts

Comparison Sorts

Compare two elements at a time

General sort, works for most types of elements

Element must form a “consistent, total ordering”

For every element a , b and c in the list the following must be true:

- If $a \leq b$ and $b \leq a$ then $a = b$
- If $a \leq b$ and $b \leq c$ then $a \leq c$
- Either $a \leq b$ is true or $b \leq a$

What does this mean? `compareTo()` works for your elements

Comparison sorts run at fastest $O(n \log(n))$ time

Niche Sorts aka “linear sorts”

Leverages specific properties about the items in the list to achieve faster runtimes

niche sorts typically run $O(n)$ time

In this class we'll focus on comparison sorts

Sort Approaches

In Place sort

A sorting algorithm is in-place if it requires only $O(1)$ extra space to sort the array

Typically modifies the input collection

Useful to minimize memory usage

Stable sort

A sorting algorithm is stable if any equal items remain in the same relative order before and after the sort

Why do we care?

- Sometimes we want to sort based on some, but not all attributes of an item
- Items that `compareTo()` the same might not be exact duplicates
- Enables us to sort on one attribute first then another etc...

`[(8, "fox"), (9, "dog"), (4, "wolf"), (8, "cow")]`

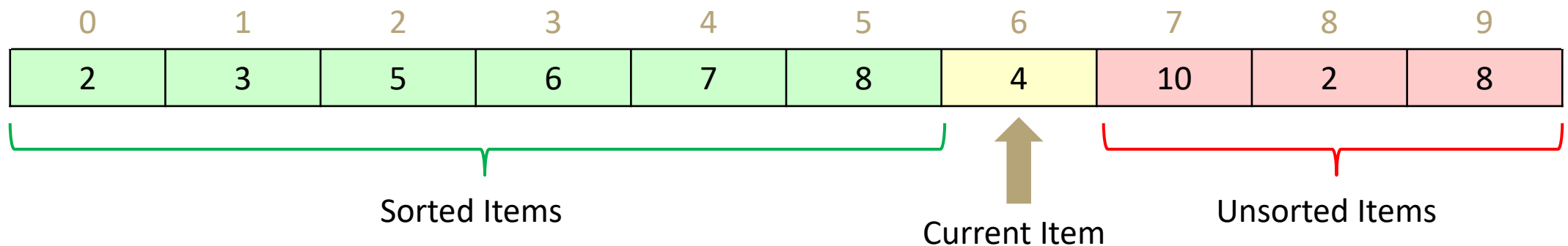
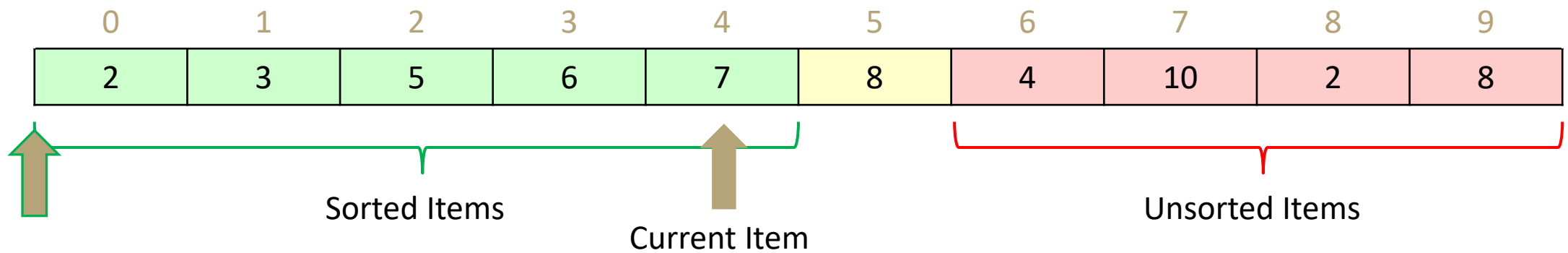
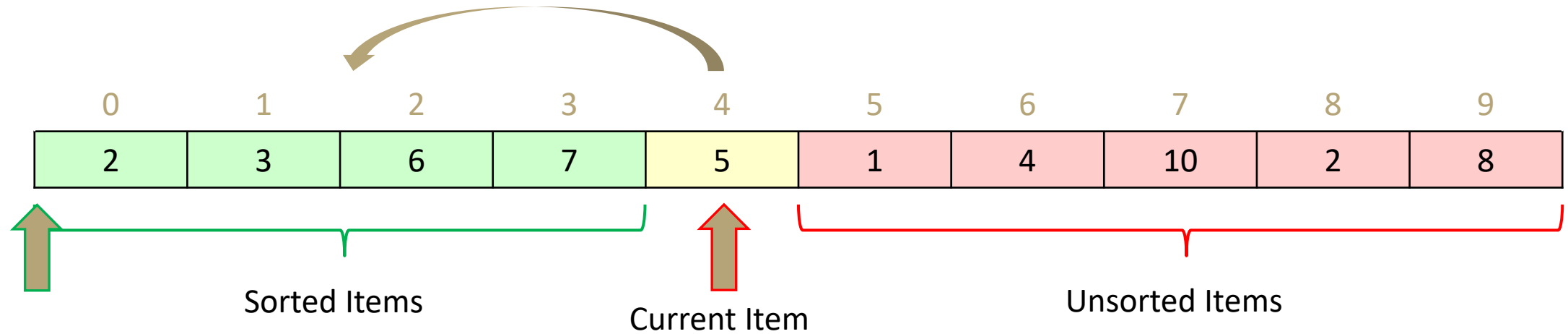
`[(4, "wolf"), (8, "fox"), (8, "cow"), (9, "dog")]` Stable

`[(4, "wolf"), (8, "cow"), (8, "fox"), (9, "dog")]` Unstable

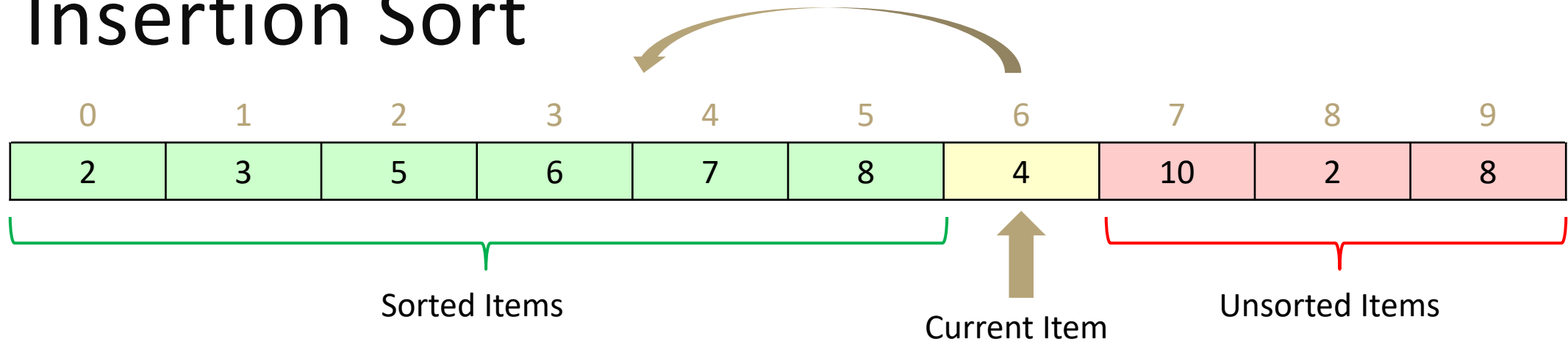
SO MANY SORTS

Quicksort, Merge sort, in-place merge sort, heap sort, insertion sort, intro sort, selection sort, timsort, cubesort, shell sort, bubble sort, binary tree sort, cycle sort, library sort, patience sorting, smoothsort, strand sort, tournament sort, cocktail sort, comb sort, gnome sort, block sort, stackoverflow sort, odd-even sort, pigeonhole sort, bucket sort, counting sort, radix sort, spreadsort, burstsort, flashsort, postman sort, bead sort, simple pancake sort, spaghetti sort, sorting network, bitonic sort, bogosort, stooge sort, insertion sort, slow sort, rainbow sort...

Insertion Sort



Insertion Sort



```
public void insertionSort(collection) {  
    for (entire list)  
        if(currentItem is smaller than largestSorted)  
            int newIndex = findSpot(currentItem);  
            shift(newIndex, currentItem);  
}  
public int findSpot(currentItem) {  
    for (sorted list)  
        if (spot found) return  
}  
public void shift(newIndex, currentItem) {  
    for (i = currentItem > newIndex)  
        item[i+1] = item[i]  
    item[newIndex] = currentItem  
}
```

Worst case runtime? $O(n^2)$

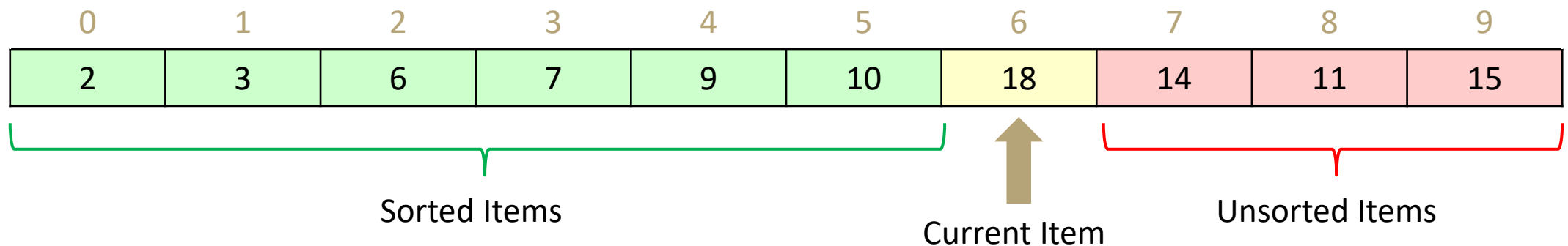
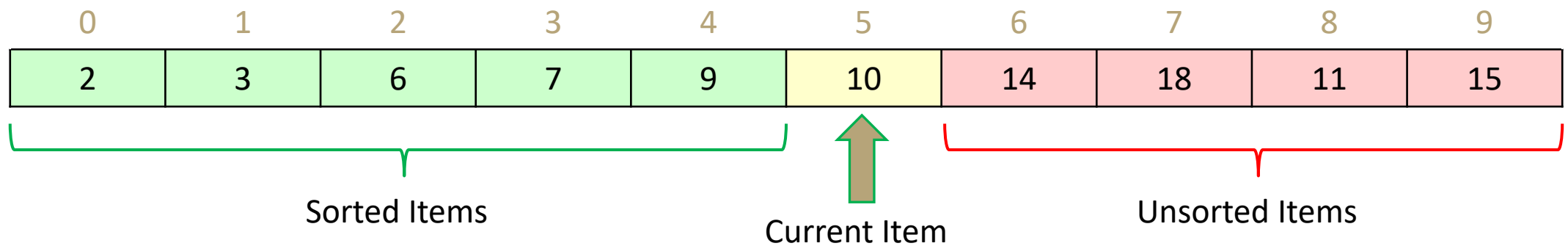
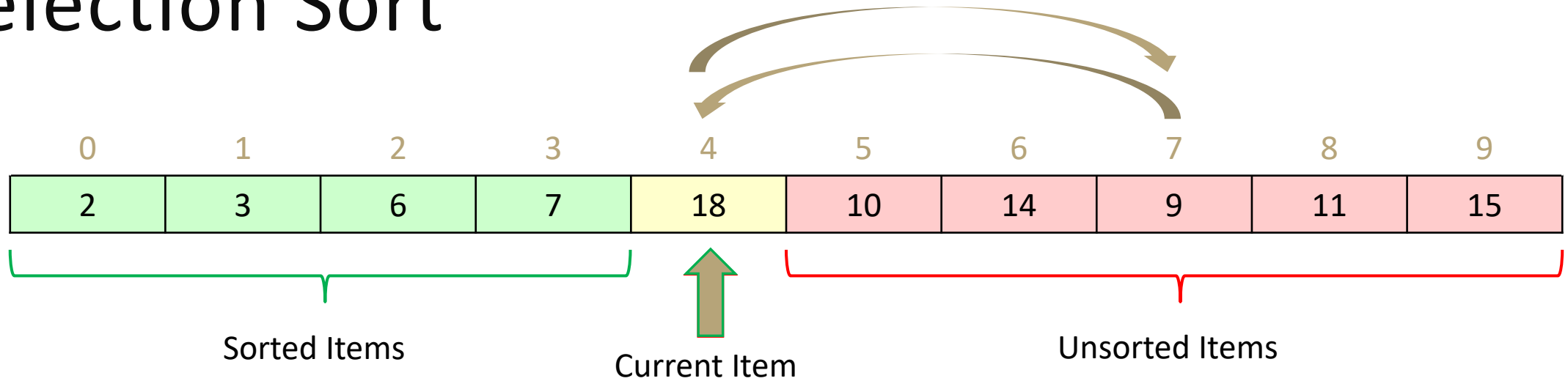
Best case runtime? $O(n)$

Average runtime? $O(n^2)$

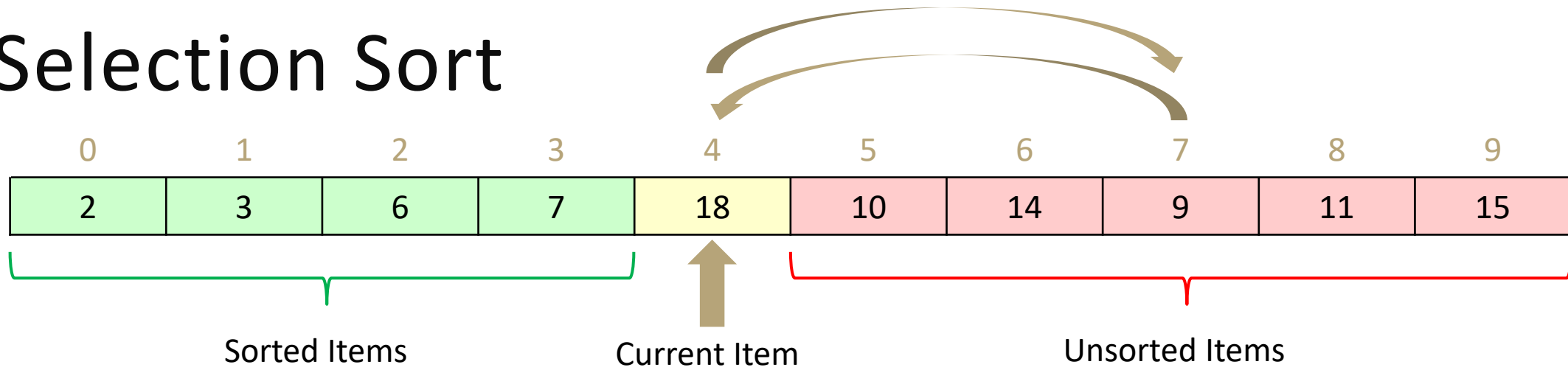
Stable? Yes

In-place? Yes

Selection Sort



Selection Sort



```
public void selectionSort(collection) {  
    for (entire list)  
        int newIndex = findNextMin(currentItem);  
        swap(newIndex, currentItem);  
}  
public int findNextMin(currentItem) {  
    min = currentItem  
    for (unsorted list)  
        if (item < min)  
            min = currentItem  
    return min  
}  
public int swap(newIndex, currentItem) {  
    temp = currentItem  
    currentItem = newIndex  
    newIndex = temp  
}
```

Worst case runtime? $O(n^2)$

Best case runtime? $O(n^2)$

Average runtime? $O(n^2)$

Stable? Yes

In-place? Yes

Heap Sort

1. run Floyd's buildHeap on your data
2. call removeMin n times

```
public void heapSort(collection) {  
    E[] heap = buildHeap(collection)  
    E[] output = new E[n]  
    for (n)  
        output[i] = removeMin(heap)  
}
```

Worst case runtime? $O(n \log n)$

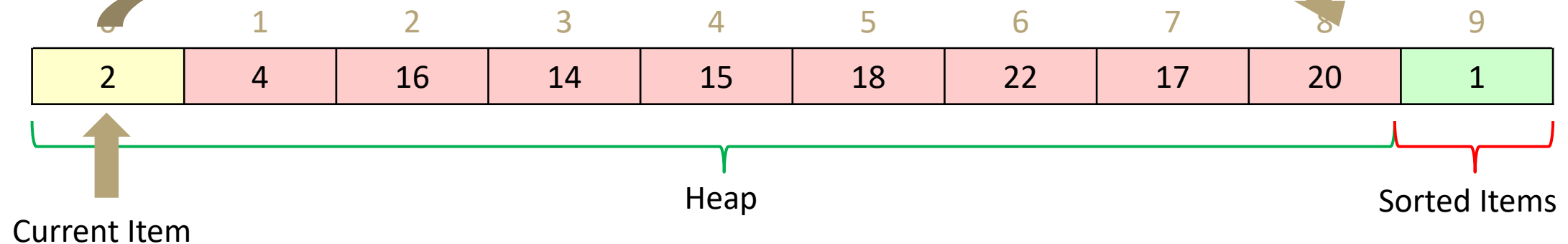
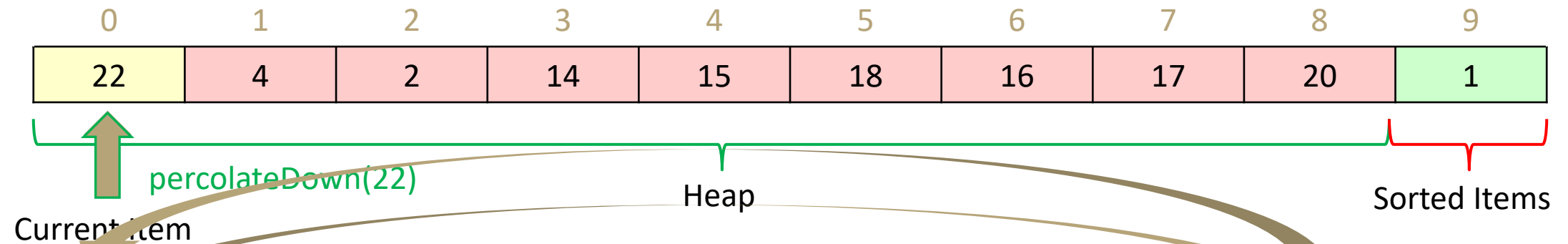
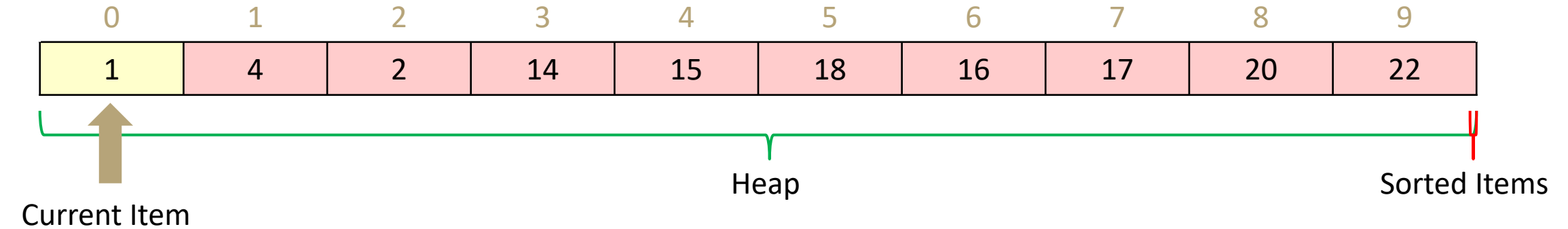
Best case runtime? $O(n \log n)$

Average runtime? $O(n \log n)$

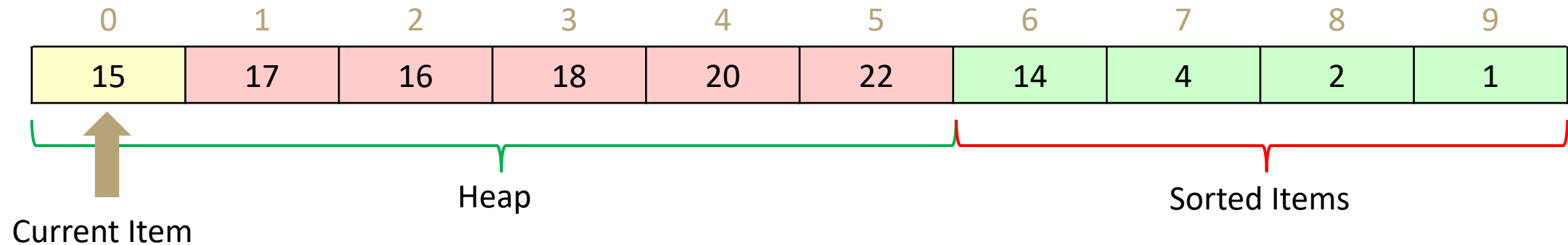
Stable? No

In-place? No

In Place Heap Sort



In Place Heap Sort



```
public void inPlaceHeapSort(collection) {  
    E[] heap = buildHeap(collection)  
    for (n)  
        output[n - i - 1] = removeMin(heap)  
}
```

Complication: final array is reversed!

- Run reverse afterwards ($O(n)$)
- Use a max heap
- Reverse compare function to emulate max heap

Worst case runtime? $O(n \log n)$

Best case runtime? $O(n \log n)$

Average runtime? $O(n \log n)$

Stable? No

In-place? Yes

Divide and Conquer Technique

1. Divide your work into smaller pieces recursively

- Pieces should be smaller versions of the larger problem

2. Conquer the individual pieces

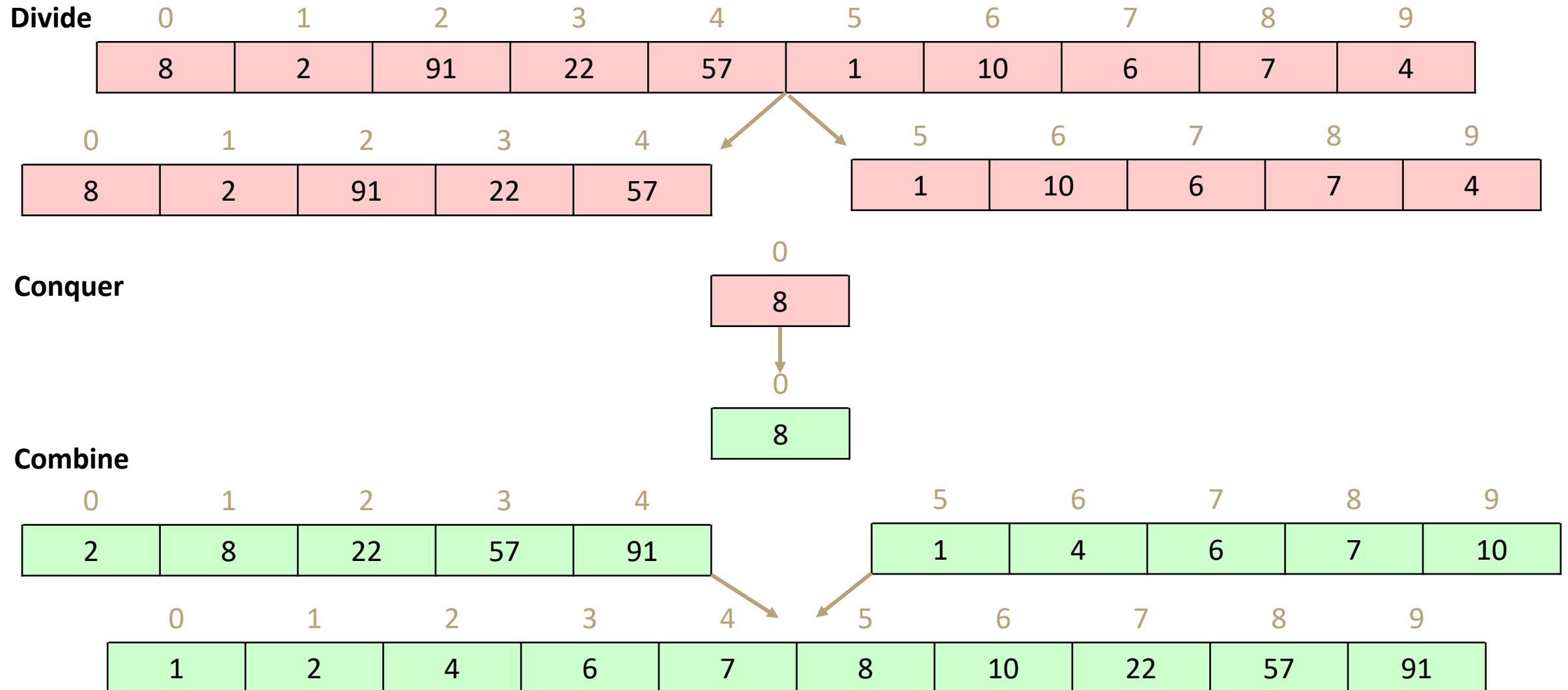
- Base case!

3. Combine the results back up recursively

```
divideAndConquer(input) {  
    if (small enough to solve)  
        conquer, solve, return results  
    else  
        divide input into a smaller pieces  
        recurse on smaller piece  
        combine results and return  
}
```

Merge Sort

https://www.youtube.com/watch?v=XaqR3G_NVoo



Merge Sort

```
mergeSort(input) {
  if (input.length == 1)
    return
  else
    smallerHalf = mergeSort(new [0, ..., mid])
    largerHalf = mergeSort(new [mid + 1, ...])
    return merge(smallerHalf, largerHalf)
}
```

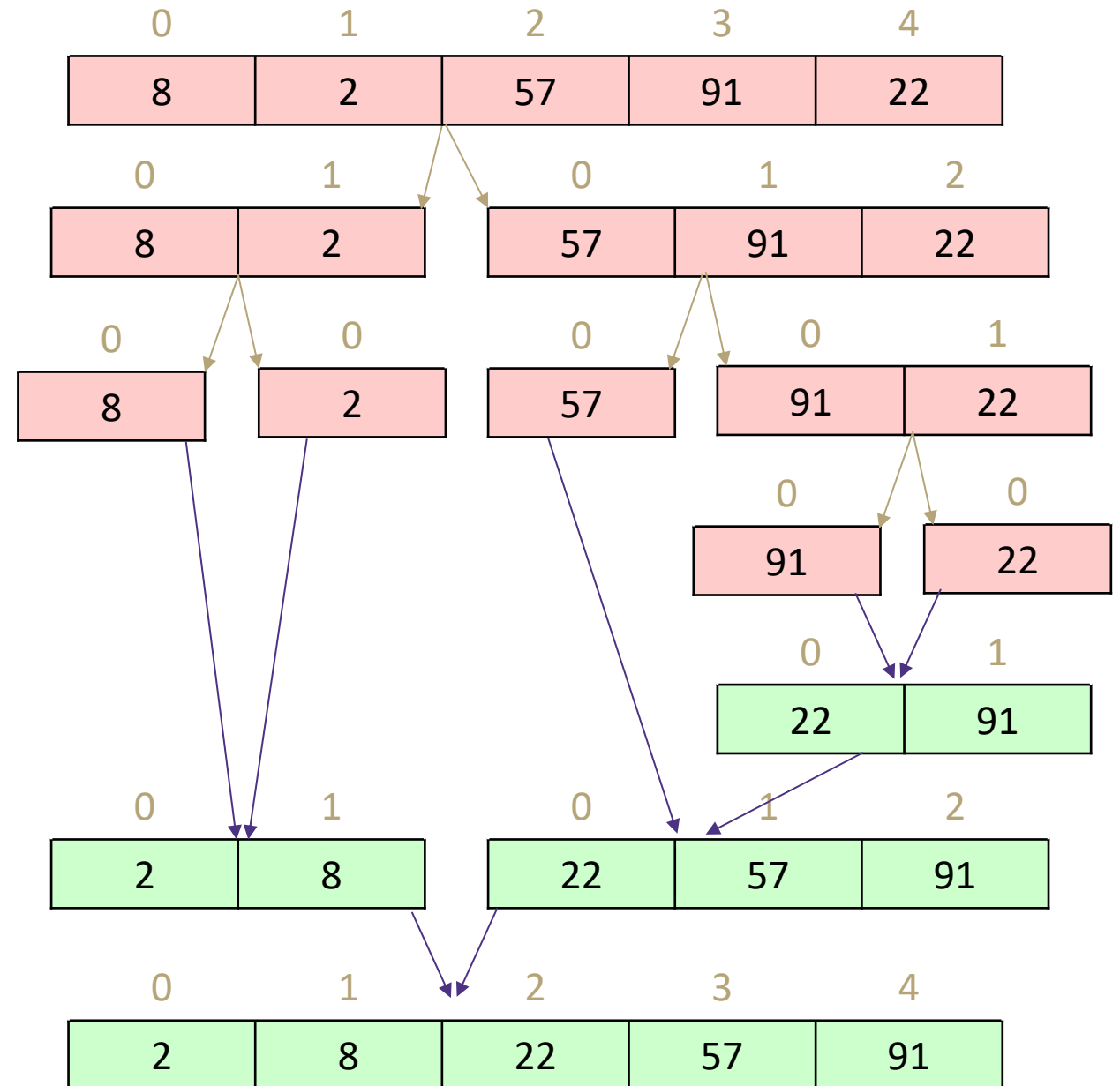
Worst case runtime?

Best case runtime? $T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ 2T(n/2) + n & \text{otherwise} \end{cases}$

Average runtime?

Stable? Yes

In-place? No



Quick Sort

Divide

0	1	2	3	4	5	6	7	8	9
8	2	91	22	57	1	10	6	7	4

0	1	2	3	4	0	0	1	2	3
2	1	6	7	4	8	91	22	57	10

Conquer

0
6

Combine

0
6

0	2	3	4	0	5	6	7	8	9
2	22	57	91	8	1	4	6	7	10

0	1	2	3	4	5	6	7	8	9
1	2	4	6	7	8	10	22	57	91

Quick Sort

```
quickSort(input) {
  if (input.length == 1)
    return
  else
    pivot = getPivot(input)
    smallerHalf = quickSort(getSmaller(pivot, input))
    largerHalf = quickSort(getBigger(pivot, input))
    return smallerHalf + pivot + largerHalf
}
```

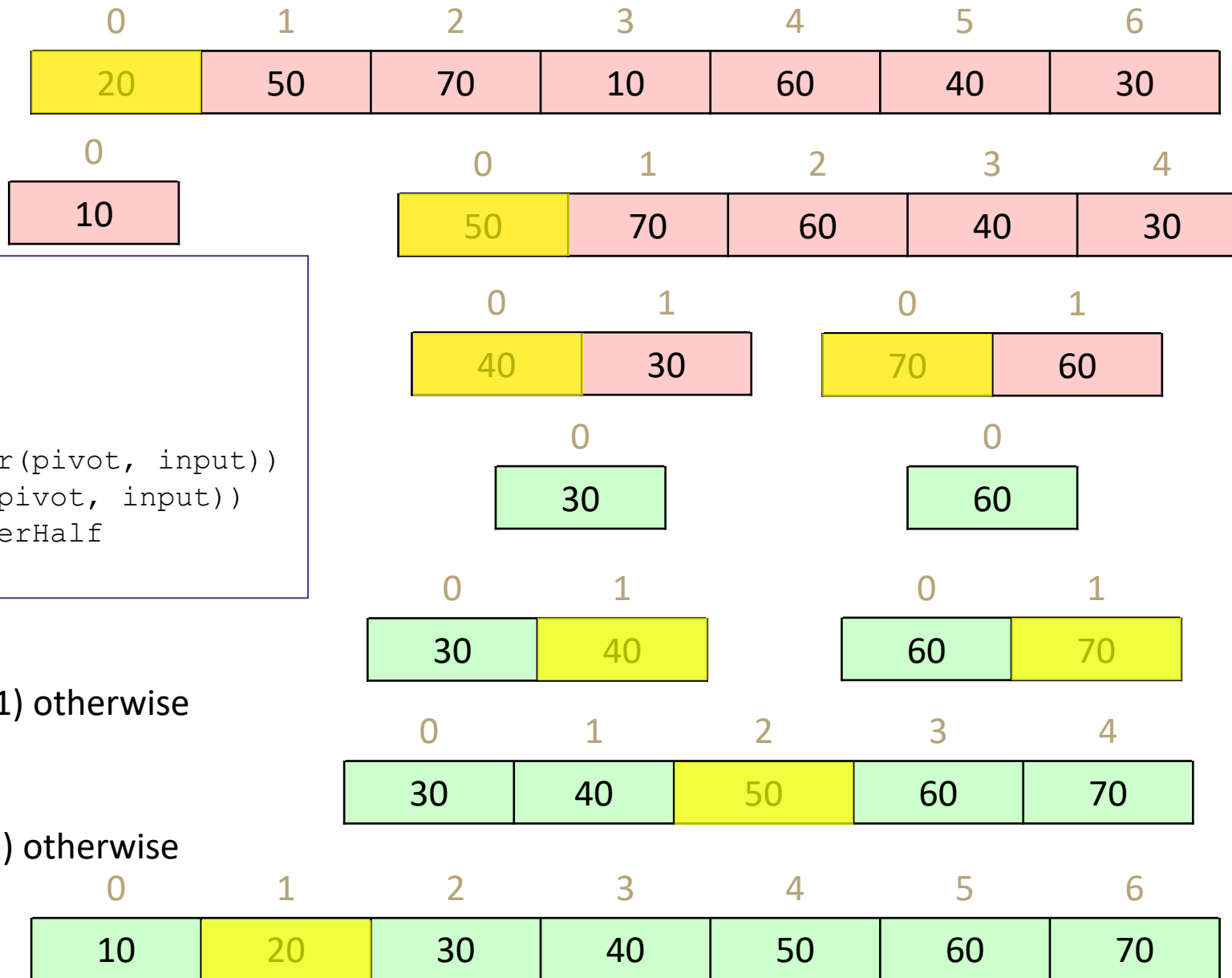
Worst case runtime? $T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ n + T(n - 1) & \text{otherwise} \end{cases}$

Best case runtime? $T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ n + 2T(n/2) & \text{otherwise} \end{cases}$

Average runtime?

Stable? No

In-place? No



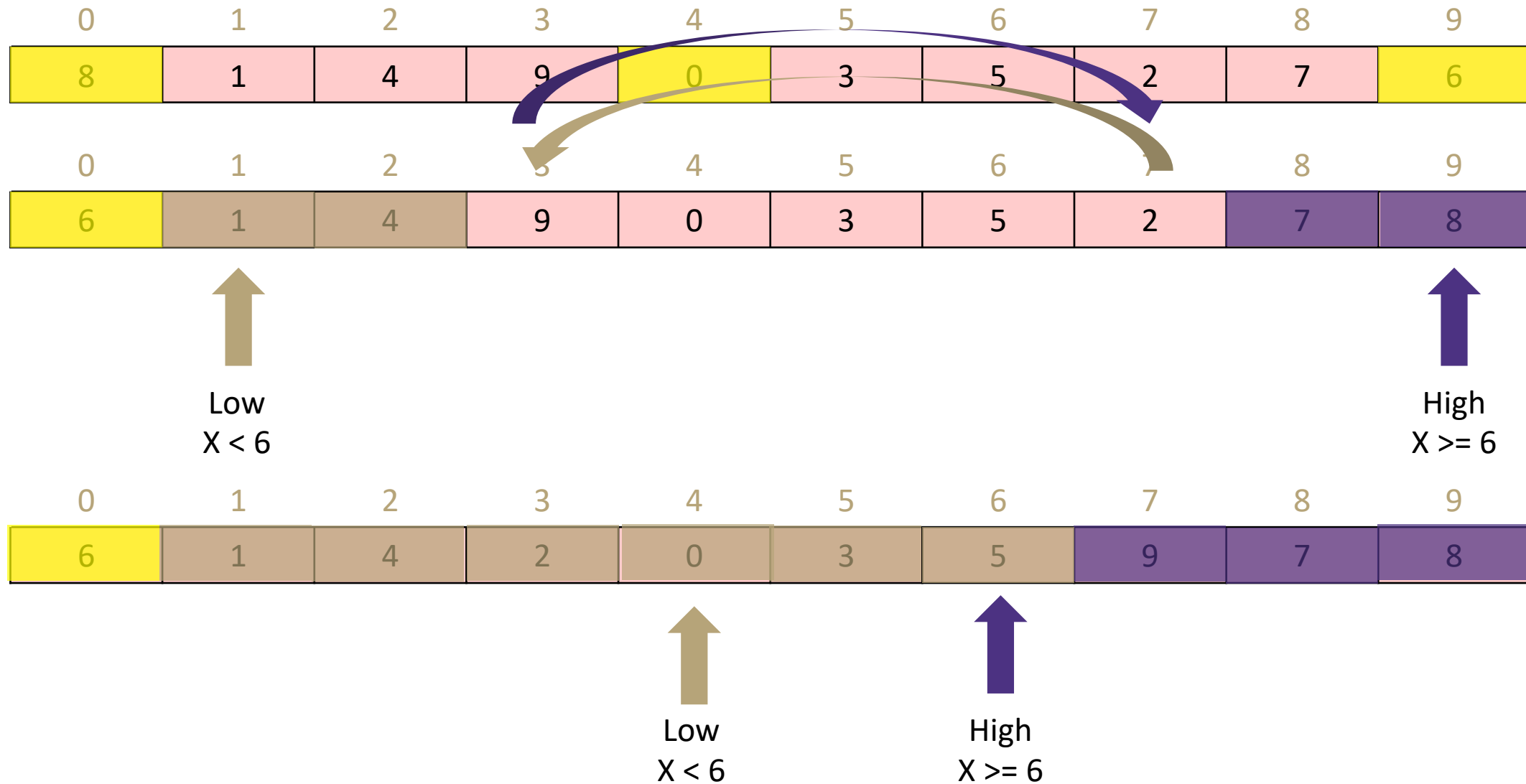
Can we do better?

Pick a better pivot

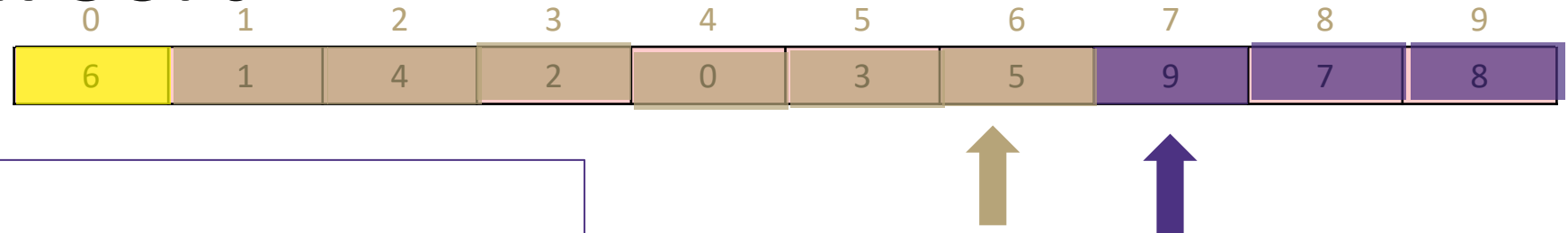
- Pick a random number
- Pick the median of the first, middle and last element

Sort elements by swapping around pivot in place

Better Quick Sort



Better Quick Sort



```
quickSort(input) {  
  if (input.length == 1)  
    return  
  else  
    pivot = getPivot(input)  
    smallerHalf = quickSort(getSmaller(pivot, input))  
    largerHalf = quickSort(getBigger(pivot, input))  
    return smallerHalf + pivot + largerHalf  
}
```

Worst case runtime?

Best case runtime? $T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ n + 2T(n/2) & \text{otherwise} \end{cases}$

Average runtime?

Stable? No

In-place? Yes

