```
Machine Learning Online Class
 1
 2
        Exercise 5 | Regularized Linear Regression and Bias-Variance
 3
 4
     %
        Instructions
 5
        _____
 6
 7
        This file contains code that helps you get started on the
         exercise. You will need to complete the following functions:
 8
 9
            linearRegCostFunction.m
10
     %
     %
            learningCurve.m
11
           validationCurve.m
12
13
14
     % For this exercise, you will not need to change any code in this file,
15
        or any other files other than those mentioned above.
16
17
18
     % Initialization
19
     clear; close all; clc
20
21
     % ====== Part 1: Loading and Visualizing Data ========
22
     % We start the exercise by first loading and visualizing the dataset.
23
     % The following code will load the dataset into your environment and
     plot
24
     % the data.
25
26
27
      % Load Training Data
28
      fprintf('Loading and Visualizing Data ...\n')
29
30
      % Load from ex5data1:
31
      % You will have X, y, Xval, yval, Xtest, ytest in your environment
      load ('ex5data1.mat');
32
33
     % m = Number of examples
34
35
     m = size(X, 1);
36
37
      % Plot training data
38
      plot(X, y, 'rx', 'MarkerSize', 10, 'LineWidth', 1.5);
39
      xlabel('Change in water level (x)');
      ylabel('Water flowing out of the dam (y)');
40
41
      fprintf('Program paused. Press enter to continue.\n');
42
43
      pause;
44
      % ====== Part 2: Regularized Linear Regression Cost =======
45
     % You should now implement the cost function for regularized linear
46
     % regression.
47
     %
48
49
50
     theta = [1 ; 1];
      1 = linearRedCostFunction([ones(m 1) X] v theta 1):
51
```

```
\sigma = \operatorname{ctheathegeostimiccton}(\operatorname{colestim}) if \kappa_1, \gamma_2, therefore
52
       fprintf(['Cost at theta = [1 ; 1]: %f '...
53
54
                '\n(this value should be about 303.993192)\n'], J);
55
       fprintf('Program paused. Press enter to continue.\n');
56
57
       pause;
58
59
       %% ====== Part 3: Regularized Linear Regression Gradient
 .
       _____
60
       % You should now implement the gradient for regularized linear
61
         regression.
62
       %
63
       theta = [1; 1];
64
       [J, grad] = linearRegCostFunction([ones(m, 1) X], y, theta, 1);
65
66
67
       fprintf(['Gradient at theta = [1; 1]: [%f; %f] '...
                \n(this value should be about [-15.303016; 598.250744]) \n'],
68
 .
                grad(1), grad(2));
69
70
71
       fprintf('Program paused. Press enter to continue.\n');
72
       pause;
73
74
75
       % ====== Part 4: Train Linear Regression =======
76
       % Once you have implemented the cost and gradient correctly, the
77
         trainLinearReg function will use your cost function to train
78
         regularized linear regression.
79
       % Write Up Note: The data is non-linear, so this will not give a great
80
       %
81
                         fit.
82
83
       % Train linear regression with lambda = 0
84
85
       lambda = 0:
       [theta] = trainLinearReg([ones(m, 1) X], y, lambda);
86
87
       % Plot fit over the data
88
       plot(X, y, 'rx', 'MarkerSize', 10, 'LineWidth', 1.5);
89
       xlabel('Change in water level (x)');
90
       ylabel('Water flowing out of the dam (y)');
91
92
       hold on;
       plot(X, [ones(m, 1) X]*theta, '--', 'LineWidth', 2)
93
94
       hold off;
95
96
       fprintf('Program paused. Press enter to continue.\n');
97
       pause;
98
99
100
       % ====== Part 5: Learning Curve for Linear Regression
```

J _

```
101
       % Next, you should implement the learningCurve function.
102
       % Write Up Note: Since the model is underfitting the data, we expect to
103
                         see a graph with "high bias" -- Figure 3 in ex5.pdf
104
105
       %
106
107
       lambda = 0;
108
       [error_train, error_val] = ...
109
           learningCurve([ones(m, 1) X], y, ...
                         [ones(size(Xval, 1), 1) Xval], yval, ...
110
111
                         lambda);
112
       plot(1:m, error_train, 1:m, error_val);
113
       title('Learning curve for linear regression')
114
115
       legend('Train', 'Cross Validation')
116
       xlabel('Number of training examples')
117
       ylabel('Error')
118
       axis([0 13 0 150])
119
120
       fprintf('# Training Examples\tTrain Error\tCross Validation Error\n');
121
           fprintf(' \t%d\t\t%f\n', i, error_train(i), error_val(i));
122
123
       end
124
125
       fprintf('Program paused. Press enter to continue.\n');
126
       pause;
127
128
       %% ======= Part 6: Feature Mapping for Polynomial Regression
       _____
       % One solution to this is to use polynomial regression. You should now
129
130
       % complete polyFeatures to map each example into its powers
131
132
133
       p = 8;
134
       % Map X onto Polynomial Features and Normalize
135
       X poly = polyFeatures(X, p);
136
137
       [X_poly, mu, sigma] = featureNormalize(X_poly); % Normalize
       X_{poly} = [ones(m, 1), X_{poly}];
                                                        % Add Ones
138
139
140
       % Map X_poly_test and normalize (using mu and sigma)
       X poly test = polyFeatures(Xtest, p);
141
       X_poly_test = bsxfun(@minus, X_poly_test, mu);
142
       X_poly_test = bsxfun(@rdivide, X_poly_test, sigma);
143
144
       X_poly_test = [ones(size(X_poly_test, 1), 1), X_poly_test];
      Add Ones
145
       % Map X_poly_val and normalize (using mu and sigma)
146
147
       X_poly_val = polyFeatures(Xval, p);
148
       X_poly_val = bsxfun(@minus, X_poly_val, mu);
149
       X polv val = bsxfun(@rdivide. X polv val. sigma):
```

```
150
      X_poly_val = [ones(size(X_poly_val, 1), 1), X_poly_val];
       Add Ones
151
152
       fprintf('Normalized Training Example 1:\n');
153
       fprintf(' %f \n', X_poly(1, :));
154
155
       fprintf('\nProgram paused. Press enter to continue.\n');
156
       pause;
157
158
159
      %% ======= Part 7: Learning Curve for Polynomial Regression
160
      % Now, you will get to experiment with polynomial regression with
161
•
      multiple
      % values of lambda. The code below runs polynomial regression with
162
163
      % lambda = 0. You should try running the code with different values of
      % lambda to see how the fit and learning curve change.
164
165
166
167
       lambda = 3; % CAMBIAR ESTE LAMBDA PARA VER SU EFECTO %%%% 1 ok, 100
 •
       chungo
                   %% el mejor es 3, como se ve mas adelante
168
169
       [theta] = trainLinearReg(X_poly, y, lambda);
170
      % Plot training data and fit
171
       figure(1);
172
       plot(X, y, 'rx', 'MarkerSize', 10, 'LineWidth', 1.5);
173
       plotFit(min(X), max(X), mu, sigma, theta, p);
174
       xlabel('Change in water level (x)');
175
176
       ylabel('Water flowing out of the dam (y)');
177
      title (sprintf('Polynomial Regression Fit (lambda = %f)', lambda));
178
179
      figure(2);
180
       [error_train, error_val] = ...
181
           learningCurve(X_poly, y, X_poly_val, yval, lambda);
       plot(1:m, error_train, 1:m, error_val);
182
183
184
      title(sprintf('Polynomial Regression Learning Curve (lambda = %f)',
       lambda));
•
       xlabel('Number of training examples')
185
       ylabel('Error')
186
187
       axis([0 13 0 100])
       legend('Train', 'Cross Validation')
188
189
       fprintf('Polynomial Regression (lambda = %f)\n\n', lambda);
190
       fprintf('# Training Examples\tTrain Error\tCross Validation Error\n');
191
192
       for i = 1:m
193
           fprintf(' \t%d\t\t%f\n', i, error_train(i), error_val(i));
194
       end
195
```

1 6 1 1 1 1 1 1 1 1 1 1

```
tprintf('Program paused. Press enter to continue.\n');
196
197
       pause;
198
199
      %% ====== Part 8: Validation for Selecting Lambda ======
200
      % You will now implement validationCurve to test various values of
201
      % lambda on a validation set. You will then use this to select the
      % "best" lambda value.
202
203
      %
204
       [lambda_vec, error_train, error_val] = ...
205
          validationCurve(X_poly, y, X_poly_val, yval);
206
207
208
       close all;
209
       plot(lambda_vec, error_train, lambda_vec, error_val);
       legend('Train', 'Cross Validation');
210
211
       xlabel('lambda');
      ylabel('Error');
212
```

```
function [J, grad] = linearRegCostFunction(X, y, theta, lambda)
 1
     %LINEARREGCOSTFUNCTION Compute cost and gradient for regularized linear
 2
     %regression with multiple variables
 3
         [J, grad] = LINEARREGCOSTFUNCTION(X, y, theta, lambda) computes the
4
         cost of using theta as the parameter for linear regression to fit the
 5
6
         data points in X and y. Returns the cost in J and the gradient in
     grad
7
8
     % Initialize some useful values
9
     m = length(y); % number of training examples
10
     % You need to return the following variables correctly
11
12
     J = 0;
13
     grad = zeros(size(theta));
14
15
     % Instructions: Compute the cost and gradient of regularized linear
16
17
                     regression for a particular choice of theta.
18
19
                     You should set J to the cost and grad to the gradient.
20
     %
21
22
     Reg = lambda / (2 * m) * (theta' * theta - theta(1)^2); % término
•
     regulazacion
23
     J = 1 / (2 * m) * sum((X * theta - y) .^2) + Reg;
24
25
     forma = ones(size(theta));
26
     forma(1) = 0;
     grad = 1 / m * ((X * theta - y)' * X)' + lambda / m * (theta * forma);
27
28
29
•
30
31
     grad = grad(:);
32
```

```
function [error_train, error_val] = ...
1
2
          learningCurve(X, y, Xval, yval, lambda)
3
     %LEARNINGCURVE Generates the train and cross validation set errors needed
     %to plot a learning curve
4
5
         [error_train, error_val] = ...
6
             LEARNINGCURVE(X, y, Xval, yval, lambda) returns the train and
7
             cross validation set errors for a learning curve. In particular,
             it returns two vectors of the same length - error_train and
8
             error_val. Then, error_train(i) contains the training error for
9
     %
             i examples (and similarly for error_val(i)).
10
11
     %
12
     % In this function, you will compute the train and test errors for
13
         dataset sizes from 1 up to m. In practice, when working with larger
14
        datasets, you might want to do this in larger intervals.
15
16
17
     % Number of training examples
18
     m = size(X, 1);
19
20
     % You need to return these values correctly
21
     error_train = zeros(m, 1);
22
     error_val = zeros(m, 1);
23
24
     25
     % Instructions: Fill in this function to return training errors in
26
                     error_train and the cross validation errors in error_val.
27
                     i.e., error_train(i) and
28
                     error_val(i) should give you the errors
29
                     obtained after training on i examples.
30
31
     % Note: You should evaluate the training error on the first i training
32
             examples (i.e., X(1:i, :) and y(1:i)).
33
     %
34
             For the cross-validation error, you should instead evaluate on
             the _entire_ cross validation set (Xval and yval).
35
36
37
     % Note: If you are using your cost function (linearRegCostFunction)
38
             to compute the training and cross validation error, you should
39
             call the function with the lambda argument set to 0.
             Do note that you will still need to use lambda when running
40
41
             the training to obtain the theta parameters.
42
43
     % Hint: You can loop over the examples with the following:
44
45
             for i = 1:m
     %
                 % Compute train/cross validation errors using training examples
46
47
                 % X(1:i, :) and y(1:i), storing the result in
     %
                 % error_train(i) and error_val(i)
48
49
     %
50
             end
51
     %
52
```

```
J _
53
    % ----- Sample Solution -----
54
55
56
    for i = 1:m
57
58
      % Compute train/cross validation errors using training examples
59
      X_{sample} = X(1:i,:);
      y_sample = y(1:i);
60
61
62
      theta = trainLinearReg(X_sample, y_sample, lambda);
63
64
      error_train(i) = linearRegCostFunction(X_sample, y_sample, theta, 0);
      error_val(i) = linearRegCostFunction(Xval, yval, theta, 0);
65
    end
66
67
68
69
70
71
    72
73
    end
```

```
function [X_norm, mu, sigma] = featureNormalize(X)
1
2
     %FEATURENORMALIZE Normalizes the features in X
3
        FEATURENORMALIZE(X) returns a normalized version of X where
4
     % the mean value of each feature is 0 and the standard deviation
5
     % is 1. This is often a good preprocessing step to do when
     % working with learning algorithms.
6
7
8
     mu = mean(X);
9
     X_norm = bsxfun(@minus, X, mu);
10
     sigma = std(X_norm);
11
12
     X_norm = bsxfun(@rdivide, X_norm, sigma);
13
14
15
     16
17
     end
18
```

```
1
     function [X_poly] = polyFeatures(X, p)
2
     %POLYFEATURES Maps X (1D vector) into the p-th power
         [X\_poly] = POLYFEATURES(X, p) takes a data matrix X (size m x 1) and
3
4
     % maps each example into its polynomial features where
5
     % X_{poly}(i, :) = [X(i) \ X(i).^2 \ X(i).^3 ... \ X(i).^p];
6
7
8
9
     % You need to return the following variables correctly.
     X_poly = zeros(numel(X), p);
10
11
12
     % Instructions: Given a vector X, return a matrix X_poly where the p-th
13
                    column of X contains the values of X to the p-th power.
14
15
16
     %
17
18
     for i = 1: p
19
      X_{poly}(:, i) = X' .^i;
20
21
22
23
24
     end
```

```
function [theta] = trainLinearReg(X, y, lambda)
 1
     %TRAINLINEARREG Trains linear regression given a dataset (X, y) and a
 2
 3
     %regularization parameter lambda
         [theta] = TRAINLINEARREG (X, y, lambda) trains linear regression using
4
 5
     % the dataset (X, y) and regularization parameter lambda. Returns the
6
     % trained parameters theta.
7
8
9
     % Initialize Theta
      initial_theta = zeros(size(X, 2), 1);
10
11
12
     % Create "short hand" for the cost function to be minimized
      costFunction = @(t) linearRegCostFunction(X, y, t, lambda);
13
14
15
     % Now, costFunction is a function that takes in only one argument
     options = optimset('MaxIter', 200, 'GradObj', 'on');
16
17
     % Minimize using fmincg
18
19
     theta = fmincg(costFunction, initial_theta, options);
20
21
     end
```

```
function [lambda_vec, error_train, error_val] = ...
 1
          validationCurve(X, y, Xval, yval)
 2
 3
     %VALIDATIONCURVE Generate the train and validation errors needed to
      %plot a validation curve that we can use to select lambda
4
 5
          [lambda_vec, error_train, error_val] = ...
 6
             VALIDATIONCURVE(X, y, Xval, yval) returns the train
7
             and validation errors (in error_train, error_val)
             for different values of lambda. You are given the training set
 8
 •
      (X,
             y) and validation set (Xval, yval).
9
10
11
      % Selected values of lambda (you should not change this)
12
      lambda_vec = [0 0.001 0.003 0.01 0.03 0.1 0.3 1 3 10]';
13
14
     % You need to return these variables correctly.
15
      error_train = zeros(length(lambda_vec), 1);
16
17
      error_val = zeros(length(lambda_vec), 1);
18
     19
20
     % Instructions: Fill in this function to return training errors in
                     error train and the validation errors in error val. The
21
22
     %
                     vector lambda_vec contains the different lambda
•
     parameters
23
                     to use for each calculation of the errors, i.e,
24
                     error_train(i), and error_val(i) should give
     %
25
                     you the errors obtained after training with
                     lambda = lambda\_vec(i)
26
      %
27
28
     % Note: You can loop over lambda_vec with the following:
29
             for i = 1:length(lambda_vec)
30
     %
31
                 lambda = lambda \ vec(i);
     %
                 % Compute train / val errors when training linear
32
     %
33
                 % regression with regularization parameter lambda
     %
                 % You should store the result in error_train(i)
34
35
                 % and error_val(i)
     %
36
     %
                  . . . .
37
     %
38
     %
             end
39
40
     %
41
42
     for i = 1:length(lambda_vec)
43
       lambda = lambda_vec(i);
44
       theta = trainLinearReg(X, y, lambda);
45
       error_train(i) = linearRegCostFunction(X, y, theta, 0);
46
       error_val(i) = linearRegCostFunction(Xval, yval, theta, 0);
47
48
      end
49
```